

CARDAMOM DALEC Model ID 813

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1 Mathematical Dynamics

Let $\vec{C}^{(t)} = [C_{\text{lab}}^{(t)}, C_{\text{fol}}^{(t)}, C_{\text{roo}}^{(t)}, C_{\text{woo}}^{(t)}, C_{\text{lit}}^{(t)}, C_{\text{som}}^{(t)}]$ denote the labile (lab), foliar (fol), fine root (roo), wood (woo), litter (lit), and soil organic matter (SOM) carbon pools on day t . The dynamics are given by

$$\begin{aligned}\vec{C}^{(t+1)} &= \text{DALEC}_{813}(\vec{C}^{(t)}) \\ &= \text{DALEC}_{813}^{\text{fire}}(\text{DALEC}_{813}^{\text{pre-fire}}(\vec{C}^{(t)})),\end{aligned}$$

where $\text{DALEC}_{813}^{\text{pre-fire}}(\cdot)$ denotes Carbon dynamics in the absence of fire and $\text{DALEC}_{813}^{\text{fire}}(\cdot)$ denotes Carbon Carbon dynamics due to fires.

1.1 Pre-fire Dynamics

Denote $[C_{\text{lab}}^{(t+1)'}, C_{\text{fol}}^{(t+1)'}, C_{\text{roo}}^{(t+1)'}, C_{\text{woo}}^{(t+1)'}, C_{\text{lit}}^{(t+1)'}, C_{\text{som}}^{(t+1)'}] = \vec{C}^{(t+1)'} = \text{DALEC}_{813}^{\text{pre-fire}}(\vec{C}^{(t)})$. The pre-fire dynamics are given by

$$C_{\text{lab}}^{(t+1)'} = f_{\text{lab}} F_{\text{gpp}}^{(t)} + (1 - \phi_{\text{onset}}^{(t)}) C_{\text{lab}}^{(t)}, \quad (1)$$

$$C_{\text{fol}}^{(t+1)'} = \phi_{\text{onset}}^{(t)} C_{\text{lab}}^{(t)} + (1 - \phi_{\text{fall}}^{(t)}) C_{\text{fol}}^{(t)} + f_{\text{fol}} F_{\text{gpp}}^{(t)}, \quad (2)$$

$$C_{\text{roo}}^{(t+1)'} = (1 - \theta_{\text{roo}}) C_{\text{roo}}^{(t)} + f_{\text{roo}} F_{\text{gpp}}^{(t)}, \quad (3)$$

$$C_{\text{woo}}^{(t+1)'} = (1 - \theta_{\text{woo}}) C_{\text{woo}}^{(t)} + f_{\text{woo}} F_{\text{gpp}}^{(t)}, \quad (4)$$

$$C_{\text{lit}}^{(t+1)'} = (1 - (\theta_{\text{lit}} + \theta_{\text{min}}) \rho^{(t)}) C_{\text{lit}}^{(t)} + \phi_{\text{fall}}^{(t)} C_{\text{fol}}^{(t)} + \theta_{\text{roo}} C_{\text{roo}}^{(t)}, \quad (5)$$

$$C_{\text{som}}^{(t+1)'} = (1 - \theta_{\text{som}} \rho^{(t)}) C_{\text{som}}^{(t)} + \theta_{\text{woo}} C_{\text{woo}}^{(t)} + \theta_{\text{min}} \rho^{(t)} C_{\text{lit}}^{(t)}. \quad (6)$$

Here $F_{\text{gpp}}^{(t)}$ denotes the gross primary production (GPP) of Carbon on day t and is a function of meteorology and atmospheric vapor pressure:

$$F_{\text{gpp}}^{(t)} = F_{\text{gpp(max)}}^{(t)} \times \min\left(1, \frac{W^{(t)}}{\omega}\right), \quad (7)$$

where ω is the water stress threshold, $F_{\text{gpp(max)}}^{(t)}$ denotes the maximum GPP on day t , which is given by the Aggregated Canopy Model (ACM), described in [another document](#), and $W^{(t)}$ is the plant-available water pool on day t . $W^{(t)}$ is given dynamically:

$$W^{(t+1)} = W^{(t)} + P^{(t)} - R^{(t)} - ET^{(t)}, \quad (8)$$

where $P^{(t)}$, $R^{(t)}$, and $ET^{(t)}$ are the precipitation, runoff, and evapotranspiration on day t , respectively. Evapotranspiration and runoff are given by

$$ET^{(t)} = F_{\text{gpp}}^{(t)} \frac{VPD^{(t)}}{v_e}, \quad (9)$$

$$R^{(t)} = \begin{cases} \alpha (W^{(t)})^2 & \text{if } W^{(t)} \leq \frac{1}{2\alpha}, \\ W^{(t)} - \frac{1}{2\alpha} & \text{if } W^{(t)} > \frac{1}{2\alpha}, \end{cases} \quad (10)$$

where v_e is the inherent water-use efficiency, α is a runoff decay constant, and $VPD^{(t)}$ is the vapor pressure deficit on day t .

θ_{roo} and θ_{woo} are the fine root and stem C turnover rates, respectively. $\theta_{\text{lit}}\rho^{(t)}$ and $\theta_{\text{som}}\rho^{(t)}$ are the litter and SOM C turnover rates, respectively, and $\theta_{\text{min}}\rho^{(t)}$ is the rate of litter mineralization to SOM, where $\rho^{(t)}$ is a function of daily and monthly average temperature and precipitation values:

$$\rho^{(t)} = e^{\Theta(T^{(t)} - \bar{T})} \left(\left(\frac{P^{(t)}}{\bar{P}} - 1 \right) s_p + 1 \right) \quad (11)$$

Here Θ and s_p are heterotrophic temperature and precipitation dependence factors, \bar{T} and \bar{P} are long term running averages of temperature and precipitation, and $T^{(t)}$ and $P^{(t)}$ are the average temperature and precipitation on day t .

A constant proportion f_{auto} of GPP is lost due to autotrophic respiration. f_{lab} , f_{fol} , f_{roo} , and f_{woo} are constant proportions of GPP allocated to the labile, foliar, fine root, and wood Carbon pools. The sum of these proportions is equal to 1: $f_{\text{lab}} + f_{\text{fol}} + f_{\text{roo}} + f_{\text{woo}} + f_{\text{auto}} = 1$.

$\phi_{\text{onset}}^{(t)}$ and $\phi_{\text{fall}}^{(t)}$ are phenological functions describing the rates of labile to foliar and foliar to litter pool transfer on day t :

$$\phi_{\text{onset}}^{(t)} = \frac{\sqrt{2}}{\sqrt{\pi}} \times \left(\frac{-\log(1 - c_{\text{lr}})}{c_{\text{ronset}}} \right) \times \exp \left[- \left(\frac{s\sqrt{2}}{c_{\text{ronset}}} \sin \left(\frac{t - d_{\text{onset}} - 0.6245c_{\text{ronset}}}{s} \right) \right)^2 \right], \quad (12)$$

$$\phi_{\text{fall}}^{(t)} = \frac{\sqrt{2}}{\sqrt{\pi}} \times \left(\frac{-\log(1 - c_{\text{ll}})}{c_{\text{rfall}}} \right) \times \exp \left[- \left(\frac{s\sqrt{2}}{c_{\text{rfall}}} \sin \left(\frac{t - d_{\text{fall}} - \psi_f}{s} \right) \right)^2 \right]. \quad (13)$$

Here c_{ronset} and c_{rfall} are the labile release and leaf fall periods, d_{onset} and d_{fall} are the leaf onset and fall days, c_{lr} and c_{ll} are the annual labile C release and leaf loss fractions, respectively, and $s = \frac{365.25}{\pi}$. ψ_f is the solution to a transcendental equation discussed [in another document](#).

1.2 Fire Dynamics

Denote $[C_{\text{lab}}^{(t+1)}, C_{\text{fol}}^{(t+1)}, C_{\text{roo}}^{(t+1)}, C_{\text{woo}}^{(t+1)}, C_{\text{lit}}^{(t+1)}, C_{\text{som}}^{(t+1)}] = \vec{C}^{(t+1)} = \text{DALEC}_{813}^{\text{fire}}(\vec{C}^{(t+1)'})$. The fire dynamics are given by

$$C_i^{(t+1)} = C_i^{(t+1)'} - FE_i^{(t)} - FM_i^{(t)}, \quad (14)$$

for $i = \text{lab, fol, roo, woo, and}$

$$C_{\text{lit}}^{(t+1)} = C_{\text{lit}}^{(t+1)'} - FE_{\text{lit}}^{(t)} + FM_{\text{lab}}^{(t)} + FM_{\text{fol}}^{(t)} + FM_{\text{roo}}^{(t)}, \quad (15)$$

$$C_{\text{som}}^{(t+1)} = C_{\text{som}}^{(t+1)'} - FM_{\text{som}}^{(t)} + FM_{\text{woo}}^{(t)}. \quad (16)$$

Here $FE_i^{(t)}$ and $FM_i^{(t)}$ are fire emission and fire mortality fluxes for pool i on day t :

$$FE_i^{(t)} = C_i^{(t+1)'} BA^{(t)} k_i, \quad (17)$$

$$FM_i^{(t)} = C_i^{(t+1)'} BA^{(t)} (1 - k_i) r, \quad (18)$$

where $BA^{(t)}$ is the burned area on day t , k_i are the combustion factors for pool i , and r is the resilience factor.

1.3 Leaf Area Index

Leaf Area Index (LAI) on day t is proportional to the foliar pool on day t :

$$\text{LAI}^{(t)} = \frac{C_{\text{fol}}^{(t)}}{c_{\text{lma}}}, \quad (19)$$

where c_{lma} is the leaf Carbon mass per unit area (sq. m).

2 Comparison to DALEC_813.c

2.1 Dynamic State Variables

$\text{POOLS}[\text{p}+0] = C_{\text{lab}}^{(t)}$	Eqn. (1) = Line 206
$\text{POOLS}[\text{p}+1] = C_{\text{fol}}^{(t)}$	Eqn. (2) = Line 207
$\text{POOLS}[\text{p}+2] = C_{\text{roo}}^{(t)}$	Eqn. (3) = Line 208
$\text{POOLS}[\text{p}+3] = C_{\text{woo}}^{(t)}$	Eqn. (4) = Line 209
$\text{POOLS}[\text{p}+4] = C_{\text{lit}}^{(t)}$	Eqn. (5) = Line 200
$\text{POOLS}[\text{p}+5] = C_{\text{som}}^{(t)}$	Eqn. (6) = Line 208
$\text{POOLS}[\text{p}+6] = W^{(t)}$	Eqn. (8) = Line 219
$\text{POOLS}[\text{nxp}+0] = C_{\text{lab}}^{(t+1)'}$, then $C_{\text{lab}}^{(t+1)}$	Eqn. (14) = Line 236
$\text{POOLS}[\text{nxp}+1] = C_{\text{fol}}^{(t+1)'}$, then $C_{\text{fol}}^{(t+1)}$	
$\text{POOLS}[\text{nxp}+2] = C_{\text{roo}}^{(t+1)'}$, then $C_{\text{roo}}^{(t+1)}$	
$\text{POOLS}[\text{nxp}+3] = C_{\text{woo}}^{(t+1)'}$, then $C_{\text{woo}}^{(t+1)}$	
$\text{POOLS}[\text{nxp}+4] = C_{\text{lit}}^{(t+1)'}$, then $C_{\text{lit}}^{(t+1)}$	Eqn. (15) = Line 239
$\text{POOLS}[\text{nxp}+5] = C_{\text{som}}^{(t+1)'}$, then $C_{\text{som}}^{(t+1)}$	Eqn. (16) = Line 241

2.2 Fluxes and Supporting Equations

$\text{FLUXES}[\text{f}+0] = F_{\text{gpp}}^{(t)}$	Eqn. (7) = Line 169
$\text{FLUXES}[\text{f}+3] = f_{\text{fol}} F_{\text{gpp}}^{(t)}$	
$\text{FLUXES}[\text{f}+4] = f_{\text{lab}} F_{\text{gpp}}^{(t)}$	
$\text{FLUXES}[\text{f}+5] = f_{\text{roo}} F_{\text{gpp}}^{(t)}$	
$\text{FLUXES}[\text{f}+6] = f_{\text{woo}} F_{\text{gpp}}^{(t)}$	
$\text{FLUXES}[\text{f}+7] = \phi_{\text{onset}}^{(t)} C_{\text{lab}}^{(t)}$	
$\text{FLUXES}[\text{f}+9] = \phi_{\text{fall}}^{(t)} C_{\text{fol}}^{(t)}$	
$\text{FLUXES}[\text{f}+10] = \phi_{\text{fall}}^{(t)} C_{\text{fol}}^{(t)}$	
$\text{FLUXES}[\text{f}+11] = \theta_{\text{roo}} C_{\text{roo}}^{(t)}$	
$\text{FLUXES}[\text{f}+12] = \theta_{\text{lit}} \rho^{(t)} C_{\text{lit}}^{(t)}$	
$\text{FLUXES}[\text{f}+13] = \theta_{\text{som}} \rho^{(t)} C_{\text{lit}}^{(t)}$	
$\text{FLUXES}[\text{f}+14] = \theta_{\text{min}} \rho^{(t)} C_{\text{lit}}^{(t)}$	
$\text{FLUXES}[\text{f}+1] = \rho^{(t)}$	Eqn (11) = Line 174
$\text{FLUXES}[\text{f}+8] = \phi_{\text{fall}}^{(t)}$	Eqn (13) = Line 186
$\text{FLUXES}[\text{f}+15] = \phi_{\text{onset}}^{(t)}$	Eqn (12) = Line 188
$\text{FLUXES}[\text{f}+28] = ET^{(t)}$	Eqn. (9) = Line 171
$\text{FLUXES}[\text{f}+29] = R^{(t)}$	Eqn. (10) = Lines 215, 217
$\text{FLUXES}[\text{f}+17+\text{nn}] = FE_i^{(t)}$	Eqn. (17) = Line 231
$\text{FLUXES}[\text{f}+13+\text{nn}] = FM_i^{(t)}$	Eqn. (18) = Line 232
$\text{LAI}[\text{n}] = \text{LAI}^{(t)}$	Eqn. (19) = Line 157

2.3 Parameters

$\text{pars}[0] = \theta_{\min}$
 $\text{pars}[1] = \% \text{GPP lost to autotrophic respiration } (f_{\text{auto}})$
 $\text{pars}[2] = \% \text{NPP allocated to foliar pool}$
 $f_{\text{fol}} = (1 - \text{pars}[1])\text{pars}[2]$
 $\text{pars}[12] = \% (\text{NPP not allocated to foliar pool}) \text{ allocated to labile pool}$
 $f_{\text{lab}} = (1 - \text{pars}[1])(1 - \text{pars}[2])\text{pars}[12]$
 $\text{pars}[3] = \% (\text{NPP not allocated to foliar or labile pools}) \text{ allocated to fine root pool}$
 $f_{\text{roo}} = (1 - \text{pars}[1])(1 - \text{pars}[2])(1 - \text{pars}[12])\text{pars}[3]$
 $f_{\text{woo}} = (1 - \text{pars}[1])(1 - \text{pars}[2])(1 - \text{pars}[12])(1 - \text{pars}[3])$
 $\text{pars}[4] = \frac{1}{c_{\text{ll}}}$
 $\text{pars}[5] = \theta_{\text{woo}}$
 $\text{pars}[6] = \theta_{\text{roo}}$
 $\text{pars}[7] = \theta_{\text{lit}}$
 $\text{pars}[8] = \theta_{\text{som}}$
 $\text{pars}[9] = \Theta$
 $\text{pars}[10] = \text{one of the nitrogen constants in the ACM}$
 $\text{pars}[11] = d_{\text{onset}}$
 $\text{pars}[13] = c_{\text{ronset}}$
 $\text{pars}[14] = d_{\text{fall}}$
 $\text{pars}[15] = c_{\text{rfall}}$
 $\text{pars}[16] = c_{\text{lma}}$
 $\text{pars}[17] \text{ through } \text{pars}[22] = [C_{\text{lab}}^{(0)}, C_{\text{fol}}^{(0)}, C_{\text{roo}}^{(0)}, C_{\text{woo}}^{(0)}, C_{\text{lit}}^{(0)}, C_{\text{som}}^{(0)}]$ (initial condition)
 $\text{pars}[23] = v_e$
 $\text{pars}[24] = \frac{1}{\alpha}$
 $\text{pars}[25] = \omega$
 $\text{pars}[26] = W^{(0)}$ (initial condition)
 $\text{pars}[27] = k_{\text{fol}}$
 $\text{pars}[28] = k_{\text{lab}} = k_{\text{roo}} = f_{\text{woo}}$
 $\text{pars}[27]/2 + \text{pars}[28]/2 = k_{\text{lit}}$
 $\text{pars}[29] = k_{\text{som}}$
 $\text{pars}[30] = 1 - r$
 $\text{pars}[31] = \frac{1}{c_{\text{lr}}}$
 $\text{pars}[32] = s_p$

2.4 Meteorological Data and Other Inputs

$\text{DATA.MET}[\text{m}+0] = t$	eqn. (12) and (13)
$\text{DATA.MET}[\text{m}+1] = \text{min. temp on day } t$	ACM
$\text{DATA.MET}[\text{m}+2] = \text{max. temp on day } t$	ACM
$\text{DATA.MET}[\text{m}+2]*0.5+\text{DATA.MET}[\text{m}+1]*0.5 = T^{(t)}$	eqn. (11)
$\text{DATA.MET}[\text{m}+3] = \text{radiation}$	ACM
$\text{DATA.MET}[\text{m}+4] = \text{CO}_2$	ACM
$\text{DATA.MET}[\text{m}+5] = \text{yearday}$	ACM
$\text{DATA.MET}[\text{m}+6] = BA^{(t)}$	eqn. (17) and (18)
$\text{DATA.MET}[\text{m}+7] = VPD^{(t)}$	eqn. (9)
$\text{DATA.MET}[\text{m}+8] = P^{(t)}$	eqn. (8) and (11)
$\text{DATA.meantemp} = \bar{T}$	eqn. (11)
$\text{DATA.meanprec} = \bar{P}$	eqn. (11)