9: $f(x,y) = 3x^2 + y^2$ $\quad g(x,y) = x^2 + y^2 = 1$

$\nabla f = \begin{bmatrix} 6x \\ 2y \end{bmatrix}$ $\quad \nabla g = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$ $\quad \nabla f = \lambda \nabla g$ $\quad 6x = \lambda 2x$

$2y = \lambda 2y$

$x^2 + y^2 = 1$

1)

$\lambda = 1 \therefore x = 0 \quad y^2 = 1 \quad y = \pm 1$

1.1) $f(0,1) = 3(0)^2 + (1)^2 = 1$

1.2) $f(0,-1) = 3(0)^2 + (-1)^2 = 1$

2)

$\lambda = 1/3 \therefore y = 0 \quad x^2 = 1 \quad x = \pm 1$

2.1) $f(1,0) = 3(1)^2 + (0)^2 = 3$

2.2) $f(-1,0) = 3(-1)^2 + 0^2 = 3$

Maximos	Minimos
$f(1,0) = 3$	$f(0,1) = 1$
$f(-1,0) = 3$	$f(0,-1) = 1$

10: $f(x,y,z) = 3x + 2y + z$ $\quad g(x,y,z) = x+y+z = 1$ $\quad h(x,y,z) = x^2+y^2 = 1$

$\nabla f = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ $\quad \nabla g = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\quad \nabla h = \begin{bmatrix} 2x \\ 2y \\ 0 \end{bmatrix}$ $\quad \nabla f = \lambda \nabla g + \mu \nabla h$

1) $3 = \lambda + \mu 2x$

2) $2 = \lambda + \mu 2y$

3) $1 = \lambda$

4) $x + y + z = 1$

5) $x^2 + y^2 = 1$

6) $(3-1) = 2 = \mu 2x$

7) $(2-1) = 1 = \mu 2y$

8) $x = 1/\mu$

9) $y = 1/2\mu$

10) $\left[\frac{1}{\mu}\right]^2 + \left[\frac{1}{2\mu}\right]^2 = 1$

$\frac{1}{\mu^2} + \frac{1}{4\mu^2} = 1 = \frac{4+1}{4\mu^2}$ $\quad \mu = \pm \frac{\sqrt{5}}{2}$

$x = \pm 2/\sqrt{5}$

$y = \pm 1/\sqrt{5}$

$z = 1 - x - y$