

5.

$$V = xyz \text{ (cm}^3\text{)} \quad x_0 = 75, \quad y_0 = 60, \quad z_0 = 40 \quad dx = dy = dz = 0.2$$

$$f_x = yz \quad f_y = xz \quad f_z = xy$$

$$dV = \frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz = yz(0.2) + xz(0.2) + xy(0.2)$$

$$= (60)(40)(0.2) + (75)(40)(0.2) + (75)(60)(0.2) = 480 + 600 + 900 = 1980 \text{ cm}^3$$

$$\boxed{dV_{(75,60,40)} = 1980 \text{ cm}^3}$$

$$6: \quad z = e^x \sin(y) \quad x = st^2 \quad y = s^2t \quad f(x) = e^x \quad g(y) = \sin(y)$$

$$\frac{\partial z}{\partial s} = f(x) \frac{\partial g(y)}{\partial s} + g(y) \frac{\partial f(x)}{\partial s} = f(x) \frac{\partial g(y)}{\partial y} \frac{\partial y}{\partial s} + g(y) \frac{\partial f(x)}{\partial x} \frac{\partial x}{\partial s}$$

$$= 2e^x \cos(y) st + \sin(y) e^x t^2$$

$$\boxed{\frac{\partial z}{\partial s} = 2e^{st^2} \cos(s^2t) st + \sin(s^2t) e^{st^2} t^2}$$

$$\frac{\partial z}{\partial t} = f(x) \frac{\partial g(y)}{\partial t} + g(y) \frac{\partial f(x)}{\partial t} = f(x) \frac{\partial g(y)}{\partial y} \frac{\partial y}{\partial t} + g(y) \frac{\partial f(x)}{\partial x} \frac{\partial x}{\partial t}$$

$$= e^x \cos(y) s^2 + 2\sin(y) e^x st$$

$$\boxed{\frac{\partial z}{\partial t} = e^{st^2} \cos(s^2t) s^2 + 2\sin(s^2t) e^{st^2} st}$$

$$7: \quad x^3 - 6xy + y^3 = 0$$

$$\frac{dy}{dx} = -\frac{f_x}{f_y} = -\frac{3x^2 - 6y}{3y^2 - 6x} = -\frac{x^2 - 2y}{y^2 - 2x}$$

$$\boxed{\frac{dy}{dx} = -\frac{x^2 - 2y}{y^2 - 2x}}$$