14: $f(x,y) = x^2 + 2y^2$ $\quad g(x,y) = x^2 + y^2 = 1$

$\nabla f = \begin{bmatrix} 2x \\ 4y \end{bmatrix}$ $\quad \nabla g = \begin{bmatrix} 2x \\ 2y \end{bmatrix}$ $\quad \nabla f = \lambda \nabla g$

$2x = \lambda 2x$

$4y = \lambda 2y$

$x^2 + y^2 = 1$

1)

$\lambda = 1 \therefore y = 0 \quad x^2 \leq 1 \quad -1 \leq x \leq 1$

2)

$x = 0 \therefore \lambda = 1/2 \quad y^2 \leq 1 \quad -1 \leq y \leq 1$

Punto critico

$f_x = 2x = 0 \quad x = 0 \quad P_c = (0,0)$

$f_y = 4y = 0 \quad y = 0 \quad f(0,0) = (0)^2 + 2(0)^2 = 0$

$0 < 1 < 2 \therefore$

Minimo	Maximos
$f(0,0) = 0$	$f(0,1) = 2$
	$f(0,-1) = 2$

15- $f(x,y,z) = x + 2y + 3z \quad g(x,y,z) = x - y + z = 1 \quad h(x,y,z) = x^2 + y^2 = 1$

$\nabla f = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\quad \nabla g = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ $\quad \nabla h = \begin{bmatrix} 2x \\ 2y \\ 0 \end{bmatrix}$ $\quad \nabla f = \lambda \nabla g + \mu \nabla h$

1) $1 = \lambda + \mu 2x$

2) $2 = -\lambda + \mu 2y$

3) $3 = \lambda$

4) $x - y + z = 1$

5) $x^2 + y^2 = 1$

6) $-2 = \mu 2x \quad // 1)$

7) $5 = \mu 2y \quad // 2)$

8) $x = -1/\mu \quad // 6)$

9) $y = 5/2\mu \quad // 7)$

10) $(-1/\mu)^2 + (5/2\mu)^2 = 1 \quad // 5), 8), 9)$

$\left[\frac{-1}{\mu}\right]^2 + \left[\frac{5}{2\mu}\right]^2 = 1 = \frac{1}{\mu^2} + \frac{25}{4\mu^2} = \frac{4}{4\mu^2} + \frac{25}{4\mu^2} = \frac{29}{4\mu^2} \therefore = 1$

11) $\mu = \pm\sqrt{\frac{29}{4}} = \pm\frac{\sqrt{29}}{2} \quad // 10)$

12) $x = -1/(\pm\sqrt{29}/2) = \pm\frac{2}{\sqrt{29}} \quad // 8)$

13) $y = 5/2(\pm\sqrt{29}/2) = \pm 5/\sqrt{29} \quad // 9)$

14) $z = 1 + y - x \quad // 5)$

a) $x = 2/\sqrt{29} \quad y = 5/\sqrt{29}$

b) $x = 2/\sqrt{29} \quad y = -5/\sqrt{29}$

c) $x = -2/\sqrt{29} \quad y = 5/\sqrt{29}$

d) $x = -2/\sqrt{29} \quad y = -5/\sqrt{29}$

$z = 1 + 5/\sqrt{29} = 2/\sqrt{29} = \sqrt{29}/\sqrt{29} + 3/\sqrt{29} = (\sqrt{29} + 3)/\sqrt{29}$

$z = 1 - 5/\sqrt{29} - 2/\sqrt{29} = \sqrt{29}/\sqrt{29} - 7/\sqrt{29} = (\sqrt{29} - 7)/\sqrt{29}$

$z = 1 + 5/\sqrt{29} + 2/\sqrt{29} = \sqrt{29}/\sqrt{29} + 7/\sqrt{29} = (\sqrt{29} + 7)/\sqrt{29}$

$z = 1 - 5/\sqrt{29} + 2/\sqrt{29} = \sqrt{29}/\sqrt{29} - 3/\sqrt{29} = (\sqrt{29} - 3)/\sqrt{29}$