

8:

$$f(x, y) = x^3 - 3xy + 4y^2 \quad \vec{a} = (1, \pi/6)$$

$$D_{\vec{a}} f(x, y) = f_x(x, y) \cos(\pi/6) + f_y(x, y) \sin(\pi/6)$$

$$f_x = 3x^2 - 3y \quad f_y = 8y - 3x \quad \cos(\pi/6) = 0.866$$

$$\sin(\pi/6) = 1/2$$

$$D_{\vec{a}} f(x, y) = (3x^2 - 3y) 0.866 + 1/2 (8y - 3x)$$

$$= 2.598x^2 - 2.598y + 4y - 1.5x$$

$$D_{\vec{a}} f(x, y) = 2.598x^2 - 1.5x + 1.402y$$

$$9: f(x, y, z) = x \sin(yz)$$

$$f_x = \sin(yz) \quad f_y = xz \cos(yz) \quad f_z = xy \cos(yz)$$

$$\nabla f = \begin{bmatrix} \sin(yz) \\ xz \cos(yz) \\ xy \cos(yz) \end{bmatrix}$$

$$|\nabla f| = \sqrt{1^2 + 2^2 + (-1)^2} = \sqrt{6}$$

$$\vec{a} = \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix} \quad P(1, 3, 0)$$

$$\nabla f(1, 3, 0) = \begin{bmatrix} \sin(0) \\ 1 \cdot 3 \cos(0) \\ 1 \cdot 0 \cos(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$$

$$D_{\vec{a}} f(1, 3, 0) = [0, 0, 3] \cdot \begin{bmatrix} 1/\sqrt{6} \\ 2/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix} = -\frac{3}{\sqrt{6}}$$

$$D_{\vec{a}} f(1, 3, 0) = -\frac{3}{\sqrt{6}}$$