School of Mathematics and Statistics
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ENGR 121 **Assignment 4**

- Submission due date: 17/04/2024
- Please ensure that you include all necessary steps and explanations in your submission.
- Refer to the appendix for a list of trigonometric identities and ratios for special angles.

1 For each of the following angles, draw a separate trigonometric unit circle and illustrate the angle on it:

- $\angle A = \dfrac{-3\pi}{4}$
- $\angle B = \dfrac{7\pi}{3}$
- $\angle C = 225^\circ$

2 Find $\sin, \cos, \tan$ of the following angles without using a calculator. Please include all your working.

(a) $\dfrac{7\pi}{3}$

(b) $-\dfrac{7\pi}{4}$

(c) -480°

3 Using the trigonometric identities simplify the following expressions

(a) $\sqrt{\sec^2 t - 1}$

(b) $(\sin x + \cos x)^2 - 1$

(c) $4 \sin A \cos A \cos 2A$

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4 Prove that $\sin^{-1}(x) + \cos^{-1}(x) = \dfrac{\pi}{2}$ for every $x \in [-1, 1]$.

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5 Express $9\cos(\dfrac{t}{2}) - 4\sin(\dfrac{t}{2})$ in the form

$$A\cos(\frac{t}{2} + \varphi).$$

You don't need to calculate the numercal value of φ. Instead, illustrate it on the trigonometric unit circle specifying the quadrant that it lies.

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6 Find the exact value of $\cos\left(\dfrac{\pi}{8}\right)$, i.e. expressing it using only natural numbers and operations $+, -, \times, /, \sqrt{\cdot}$. Explain your answer.

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7 Using the following formulas:

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

and

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

prove that

$$\cos(a) - \cos(b) = -2 \sin \frac{a + b}{2} \sin \frac{a - b}{2}$$

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8 Find the exact value of $\sin 75^\circ \cos 15^\circ$ expressing it using only natural numbers and operations $+, -, \times, /, \sqrt{\cdot}$.

Trigonometric Identities
$\sin^2\theta + \cos^2\theta = 1$
$\frac{1}{\sin\theta} = \csc\theta,\ \frac{1}{\cos\theta} = \sec\theta,\ \frac{1}{\tan\theta} = \cot\theta$
$\tan\theta = \frac{\sin\theta}{\cos\theta},\ \cot\theta = \frac{\cos\theta}{\sin\theta}$
$\sin(-\theta) = -\sin\theta,\ \cos(-\theta) = \cos\theta$
$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta,\ \cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$
$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$ $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin 2\theta = 2\sin\theta\cos\theta$ $\cos 2\theta = \cos^2\theta - \sin^2\theta$ $= 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$
$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$ $\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$

radians	sin	cos	tan	cot
0	0	1	0	-
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{\pi}{2}$	1	0	-	0