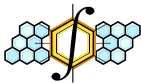


Quarterfinal 1

MWIT-KVIS Integration Bee

November 12, 2023



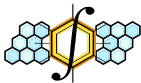
Rules

- 4 problems (+ sudden death)
- 3 minutes per problem
- CIRCLE your final answer



Problem 1

$$\int \sin x \sin 2x \sin 3x \sin 4x \cos x \cos 2x \cos 3x \cos 4x \, dx$$



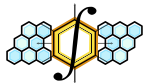
Problem 1 Answer

$$\int \sin x \sin 2x \sin 3x \sin 4x \cos x \cos 2x \cos 3x \cos 4x \, dx$$
$$= \frac{1}{16} \left(\frac{x}{8} - \frac{\sin(12x)}{96} - \frac{\sin(16x)}{128} + \frac{\sin(20x)}{160} \right) + C$$



Problem 2

$$\int_0^{\pi/2} \left(\frac{x}{\sin x} \right)^2 dx$$



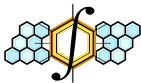
Problem 2 Answer

$$\int_0^{\pi/2} \left(\frac{x}{\sin x} \right)^2 dx = \boxed{\pi \ln 2}$$



Problem 3

$$\int_{-1}^1 (\arcsin(x))^2 dx$$



Problem 3 Answer

$$\int_{-1}^1 (\arcsin(x))^2 dx = \boxed{\frac{1}{2} (\pi^2 - 8)}$$



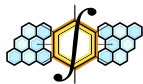
Problem 4

$$\int \frac{3e^{2x} - 3e^x}{e^{3x} + 1} dx$$



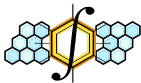
Problem 4 Answer

$$\int \frac{3e^{2x} - 3e^x}{e^{3x} + 1} dx = \boxed{\ln|e^{2x} - e^x + 1| - 2\ln|e^x + 1| + C}$$



Sudden Death Problem 1

$$\int_0^1 x \arcsin \left(\sin \left(\frac{1}{x} \right) \right) dx$$



Sudden Death Problem 1 Answer

$$\int_0^1 x \arcsin \left(\sin \left(\frac{1}{x} \right) \right) dx = \boxed{\frac{1}{2}}$$



Sudden Death Problem 2

$$\int \frac{dx}{ax^2 + bx + c}, \text{ when } b^2 - 4ac < 0$$



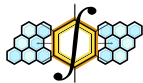
Sudden Death Problem 2 Answer

$$\int \frac{dx}{ax^2 + bx + c} = \boxed{\frac{2}{\sqrt{4ac - b^2}} \arctan \left(\frac{2ax + b}{\sqrt{4ac - b^2}} \right) + C}$$



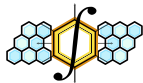
Sudden Death Problem 3

$$\int \ln(1 + x^{1/3}) dx$$



Sudden Death Problem 3 Answer

$$\int \ln(1 + x^{1/3}) dx = \boxed{\frac{1}{6}(3x^{2/3} - 2x - 6x^{1/3}) + (x + 1) \ln(1 + x^{1/3}) + C}$$



Sudden Death Problem 4

$$\int_0^{\infty} \frac{x(1 - \ln(x))}{1 + x^4} dx$$



Sudden Death Problem 4 Answer

$$\int_0^{\infty} \frac{x(1 - \ln(x))}{1 + x^4} dx = \boxed{\frac{\pi}{4}}$$