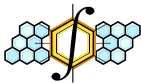# Quarterfinal 2

MWIT-KVIS Integration Bee

November 12, 2023

Rules

- 4 problems (+ sudden death)
- 3 minutes per problem
- CIRCLE your final answer

Problem 1

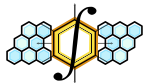$$\int_0^\infty \frac{x\left(1 - \ln(x)\right)}{1 + x^4}\, dx$$

Problem 1 Answer

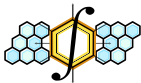$$\int_0^\infty \frac{x\left(1 - \ln(x)\right)}{1 + x^4}\, dx = \boxed{\frac{\pi}{4}}$$

Problem 2

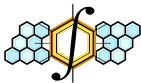$$\int_0^1 x \arcsin\left(\sin\left(\frac{1}{x}\right)\right) dx$$

Problem 2 Answer

$$\int_0^1 x \arcsin\left(\sin\left(\frac{1}{x}\right)\right) \, dx = \boxed{\frac{1}{2}}$$

Problem 3

$$\int \frac{dx}{ax^2 + bx + c}, \text{ when } b^2 - 4ac < 0$$

Problem 3 Answer

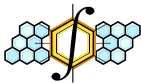$$\int \frac{dx}{ax^2 + bx + c} = \boxed{\frac{2}{\sqrt{4ac - b^2}} \arctan\left(\frac{2ax + b}{\sqrt{4ac - b^2}}\right) + C}$$

Problem 4
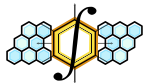
$$\int \ln(1 + x^{1/3}) \, dx$$

Problem 4 Answer

$$\int \ln(1 + x^{1/3})\, dx = \boxed{\frac{1}{6}(3x^{2/3} - 2x - 6x^{1/3}) + (x + 1)\ln(1 + x^{1/3}) + C}$$

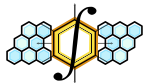# Sudden Death Problem 1

$$\int \frac{3e^{2x} - 3e^x}{e^{3x} + 1} dx$$

Sudden Death Problem 1 Answer

$$\int \frac{3e^{2x} - 3e^x}{e^{3x} + 1}\,dx = \boxed{\ln|e^{2x} - e^x + 1| - 2\ln|e^x + 1| + C}$$

Sudden Death Problem 2

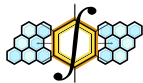$$\int_0^{\pi/2} \left(\frac{x}{\sin x} \right)^2 dx$$

Sudden Death Problem 2 Answer

$$\int_0^{\pi/2} \left(\frac{x}{\sin x} \right)^2 dx = \boxed{\pi \ln 2}$$

Sudden Death Problem 3

$$\int_{-1}^{1} \left(\arcsin(x)\right)^2 \, dx$$
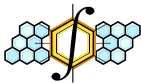
Sudden Death Problem 3 Answer

$$\int_{-1}^{1} (\arcsin(x))^2 \, dx = \boxed{\frac{1}{2}\left(\pi^2 - 8\right)}$$

Sudden Death Problem 4

$$\int \sin x \sin 2x \sin 3x \sin 4x \cos x \cos 2x \cos 3x \cos 4x \, dx$$

Sudden Death Problem 4 Answer

$$\int \sin x \sin 2x \sin 3x \sin 4x \cos x \cos 2x \cos 3x \cos 4x \, dx$$

$$= \frac{1}{16} \left(\frac{x}{8} - \frac{\sin(12x)}{96} - \frac{\sin(16x)}{128} + \frac{\sin(20x)}{160} \right) + C$$