

# Quarterfinal 4

MWIT-KVIS Integration Bee

November 12, 2023



# Rules

- 4 problems (+ sudden death)
- 3 minutes per problem
- · CIRCLE your final answer



$$\int (11x^{11} + 10x^5 + 9x^3)\sqrt{x^9 + 2x^3 + 3x} \, dx$$



#### Problem 1 Answer

$$\int (11x^{11} + 10x^5 + 9x^3)\sqrt{x^9 + 2x^3 + 3x} \, dx = \left[ \frac{2}{3} (x^{11} + 2x^5 + 3x^3)^{3/2} + C \right]$$



$$\int \frac{\ln\left(x+1\right)}{\sqrt{x}}\,dx$$



#### Problem 2 Answer

$$\int \frac{\ln(x+1)}{\sqrt{x}} dx = \boxed{2\ln(x+1)\sqrt{x} - 4\sqrt{x} - 4\arctan\sqrt{x} + C}$$



$$\int_0^\infty (x-1)(x-3)(x-5)x^4 e^{-x} \, dx$$



# Problem 3 Answer

$$\int_0^\infty (x-1)(x-3)(x-5)x^4e^{-x} dx = 960.$$



$$\int \frac{x^{2023}}{x^2+1} \, dx$$



#### Problem 4 Answer

$$\int \frac{x^{2023}}{x^2 + 1} \, dx = \left| \sum_{k=1}^{1011} \frac{(-1)^{k+1} x^{2k}}{2k} - \frac{1}{2} \ln(x^2 + 1) + C \right|$$



$$\int \frac{e^{x} \cos \left(\ln \left(\arctan \left(e^{x}\right)\right)\right)}{\left(1+e^{2x}\right)\arctan \left(e^{x}\right)} \, dx$$



#### Sudden Death Problem 1 Answer

$$\int \frac{e^{x} \cos \left(\ln \left(\arctan \left(e^{x}\right)\right)\right)}{\left(1+e^{2x}\right)\arctan \left(e^{x}\right)} \, dx = \boxed{\sin \left(\ln \left(\arctan \left(e^{x}\right)\right)\right) + C}$$



$$\int (x^2+2)\frac{\sin x}{x^3}\,dx$$



#### Sudden Death Problem 2 Answer

$$\int (x^2 + 2) \frac{\sin x}{x^3} \, dx = \left| -\frac{\cos x}{x} - \frac{\sin x}{x^2} + C \right|$$



$$\int \frac{e^{x} dx}{\sqrt{3e^{x} - 2 - e^{2x}}}$$



# Sudden Death Problem 3 Answer

$$\int \frac{e^{x} dx}{3e^{x} - 2 - e^{2x}} = \boxed{\arcsin(2e^{x} - 3) + C.}$$



$$\int \arcsin\left(\sqrt{x}\right) dx$$



#### Sudden Death Problem 4 Answer

$$\int \arcsin\left(\sqrt{x}\right) dx = \left| \frac{1}{2} \left( (2x - 1) \arcsin(\sqrt{x}) + \sqrt{x(1 - x)} \right) + C \right|$$