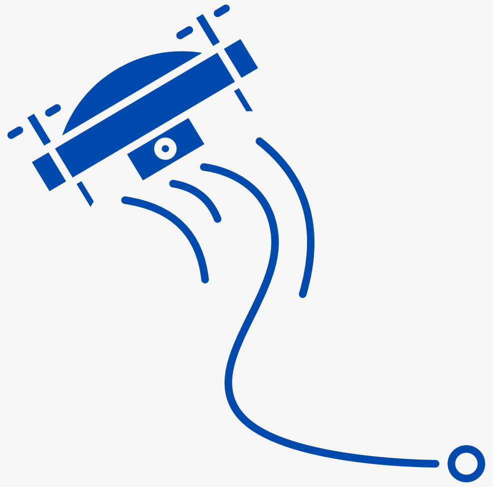
2023 Winter MESTER

2024/02/21 Meeting

Tilt-Rotor VTOL Modeling and Control

20210027 김지유
20193770 우영찬

01.
목차



Task

Task

Task 1 (김지유)

Trim >> Transfer function

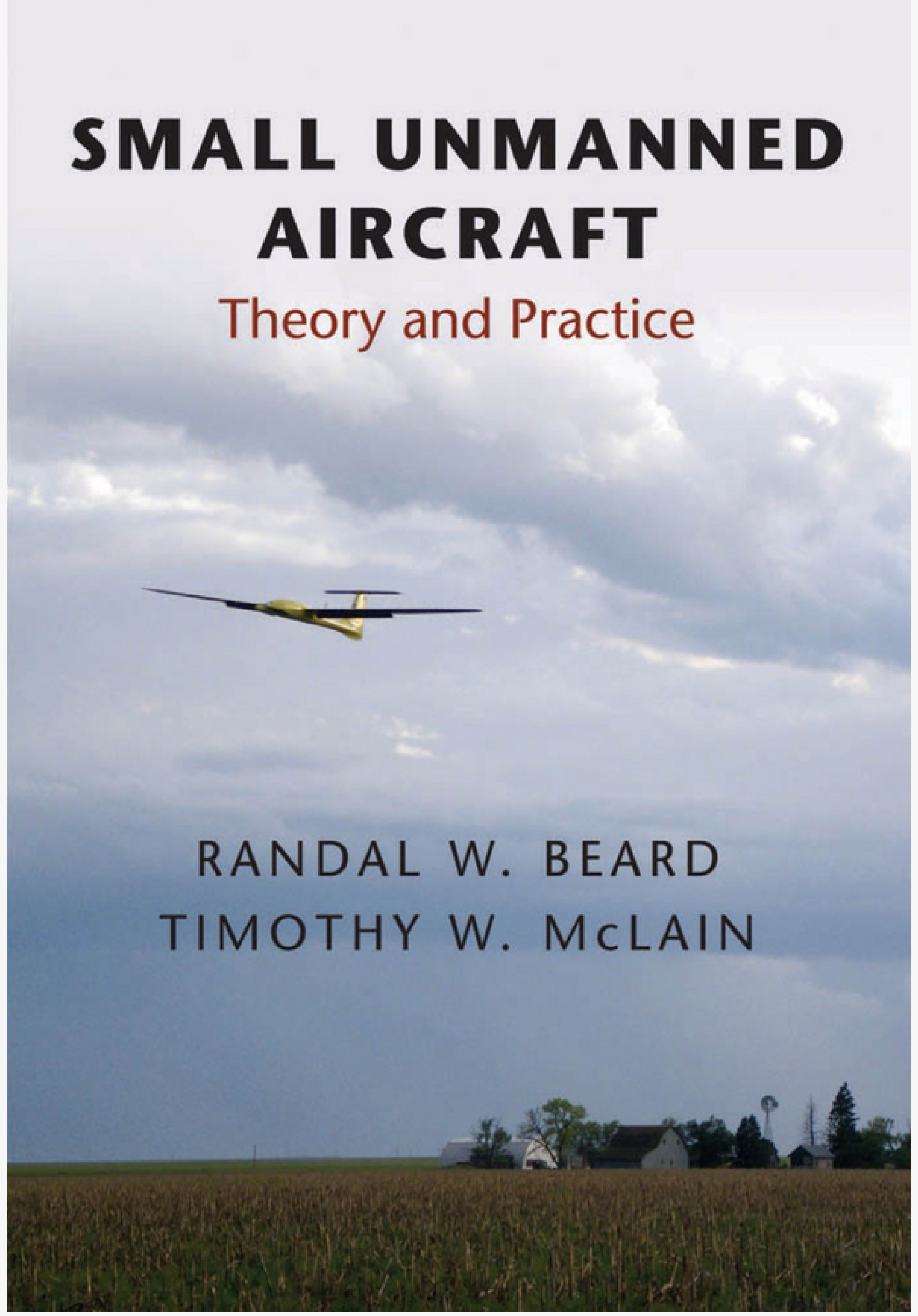
Find A,B Matrix

Task 2 (우영찬)

Matlab & Simulink를 통해
주어진 동역학 모델의 trim 상태 계산
-
Straight, level flight에서 longitudinal linear state-space model의 A, B matrix를 얻을 수 있다.

수행내용

Task 1



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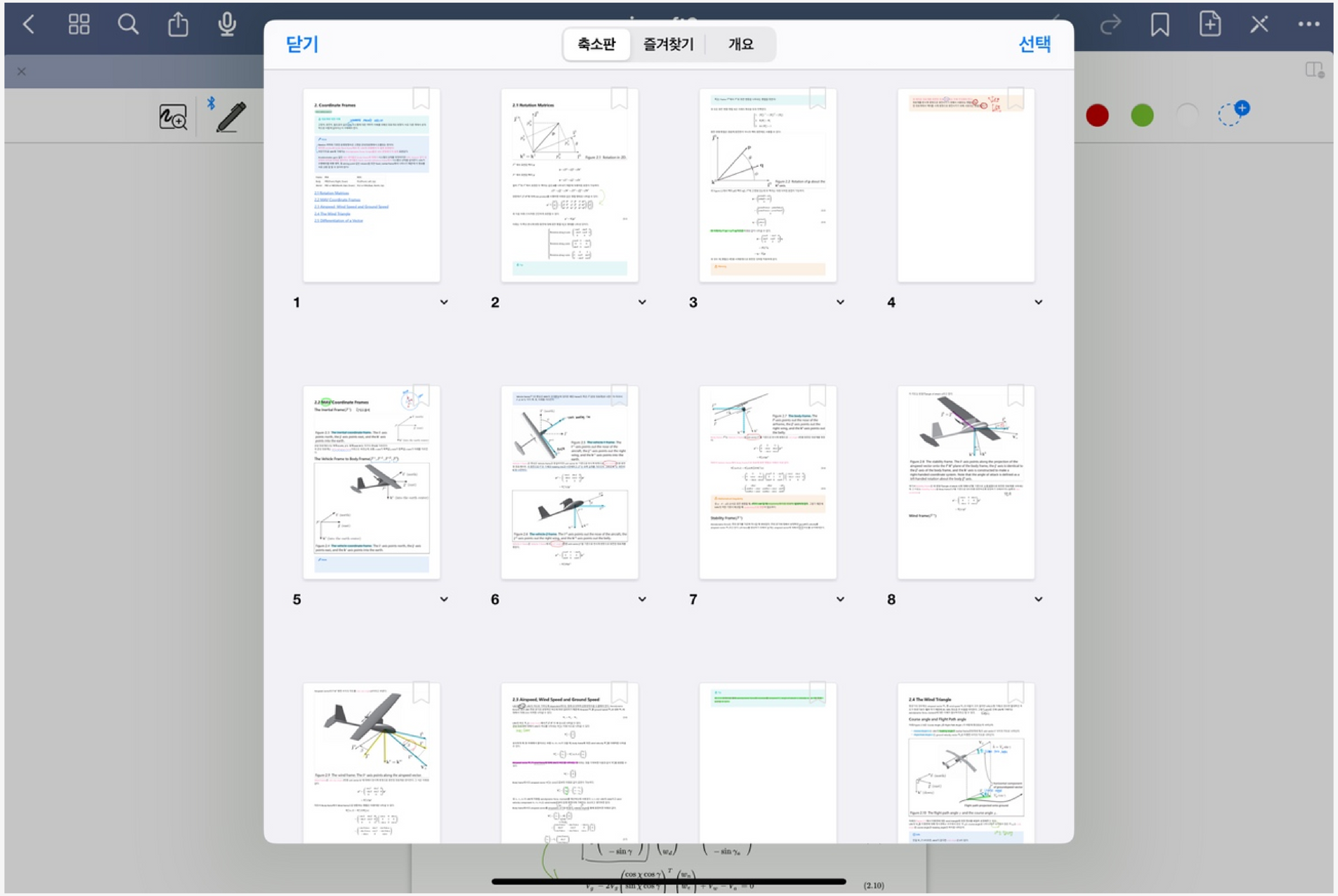
수행내용

Task 1



수행내용

Task 1

CHAP2 SUMMARY

2. Coordinate Frames

2.1 Rotation Matrices

좌표계 회전 변환 행렬
벡터 회전 변환 행렬
변환 행렬의 특성

2.2 MAV Coordinate Frames

The Inertial Frame
The Vehicle Frame
The body Frame
Stability Frame
Wind frame

2.3 Airspeed, Wind Speed and Ground Speed

airspeed, wind speed, ground speed의 차이
Body frame에서의 airspeed

2.4 The Wind Triangle

Course angle과 Flight Path angle의 정의
crab angle과 air-mass-referenced flight path angle

수행내용

Task 1

질문

아래의 Figure 2.11에서 지평면에 대한 wind triangle에 대한 정보를 세밀히 표현해주고 있다.
UAV의 $\mathbf{V}_g$를 지평면에 대해 정사영해 $\mathbf{i}^i$ 사이에서 얻은 각 χ는 course angle로 나타나며 $\mathbf{i}^b$ 사이에서 얻은 각 χ_c는 *crab angle*로 course angle과 heading angle의 차이로 나타난다.

ⓘ Info

만일 $\mathbf{V}_w$가 0이라면, wind가 없다면 *crab angle*은 0이 된다.

$$\chi_c \triangleq \chi - \psi$$

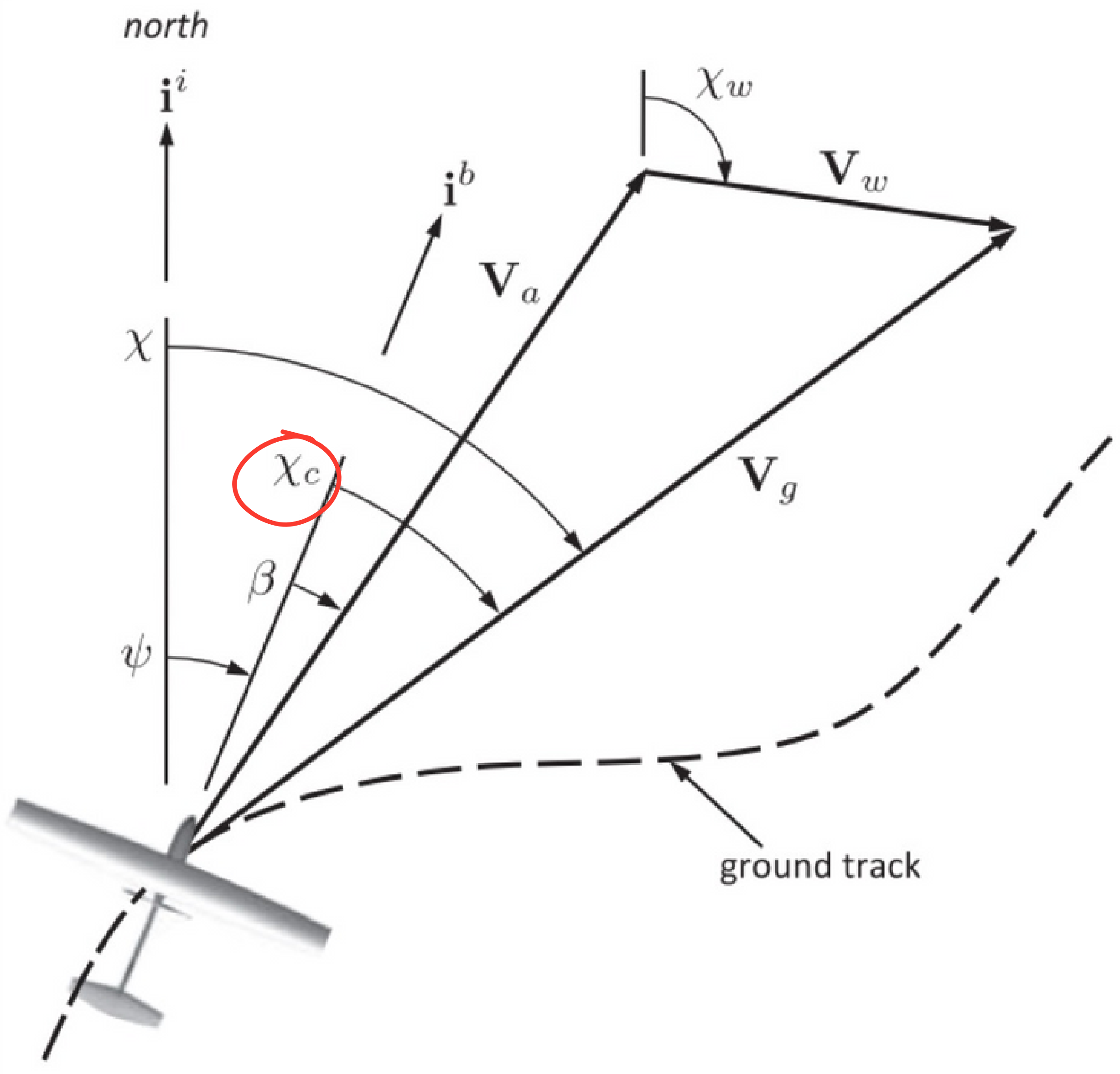


Figure 2.11 Heading is the direction that the MAV is pointed. Course is the direction of travel relative to the earth's surface. The crab angle is the difference between course and heading. In the absence of wind, the crab angle is zero.

수행내용

Task 2

Longitudinal State-space Equations

$$x_{\mathrm{lon}} \triangleq (u,\ w,\ q,\ \theta,\ h)^{\top}$$

$$u_{\mathrm{lon}} \triangleq (\delta_e,\ \delta_t)^{\top}$$

$$\begin{pmatrix} \dot{\bar{u}} \\ \dot{\bar{w}} \\ \dot{\bar{q}} \\ \dot{\bar{\theta}} \\ \dot{\bar{h}} \end{pmatrix} = \begin{pmatrix} X_u & X_w & X_q & -g\cos\theta^* & 0 \\ Z_u & Z_w & Z_q & -g\sin\theta^* & 0 \\ M_u & M_w & M_q & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \sin\theta^* & -\cos\theta^* & 0 & u^*\cos\theta^* + w^*\sin\theta^* & 0 \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{w} \\ \bar{q} \\ \bar{\theta} \\ \bar{h} \end{pmatrix} + \begin{pmatrix} X_{\delta_e} & X_{\delta_t} \\ Z_{\delta_e} & 0 \\ M_{\delta_e} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \bar{\delta}_e \\ \bar{\delta}_t \end{pmatrix}$$

Trim 상태의 u, w, alpha, q, airspeed,
delta_e, delta_t, theta 를 알아야 함

Longitudinal State-space Model Coefficients

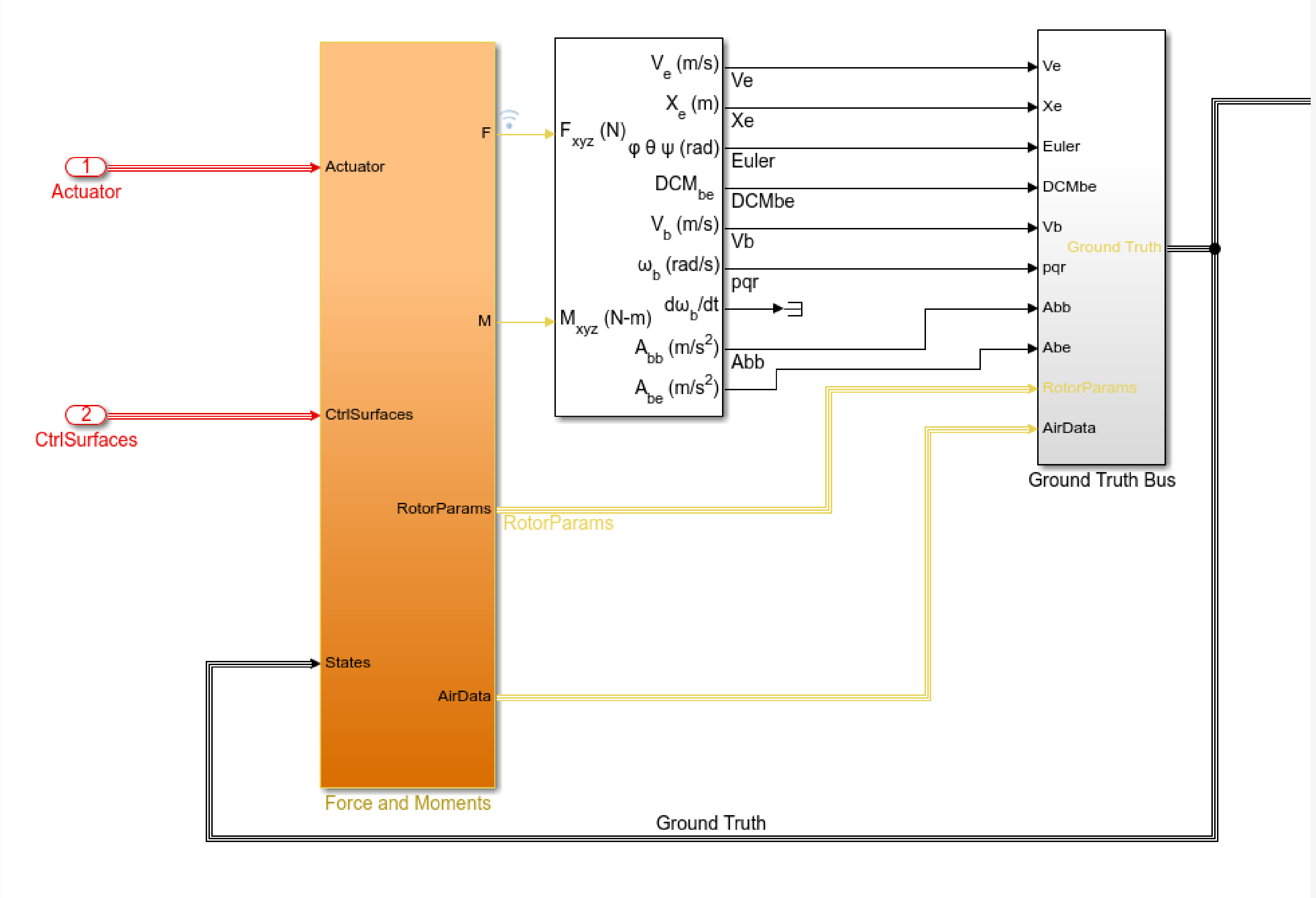
Longitudinal	*Formula*
X_u	$\frac{u^*\rho S}{m}\left[C_{X_0} + C_{X_\alpha}\alpha^* + C_{X_{\delta_e}}\delta_e^*\right] - \frac{\rho S w^* C_{X_\alpha}}{2m} + \frac{\rho S c C_{X_q} u^* q^*}{4mV_a^*} - \frac{\rho S_{\mathrm{prop}} C_{\mathrm{prop}} u^*}{m}$
X_w	$-q^* + \frac{w^*\rho S}{m}\left[C_{X_0} + C_{X_\alpha}\alpha^* + C_{X_{\delta_e}}\delta_e^*\right] + \frac{\rho S c C_{X_q} w^* q^*}{4mV_a^*} + \frac{\rho S C_{X_\alpha} u^*}{2m} - \frac{\rho S_{\mathrm{prop}} C_{\mathrm{prop}} w^*}{m}$
X_q	$-w^* + \frac{\rho V_a^* S C_{X_q} c}{4m}$
X_{δ_e}	$\frac{\rho V_a^{*2} S C_{X_{\delta_e}}}{2m}$
X_{δ_t}	$\frac{\rho S_{\mathrm{prop}} C_{\mathrm{prop}} k^2 \delta_t^*}{m}$
Z_u	$q^* + \frac{u^*\rho S}{m}\left[C_{Z_0} + C_{Z_\alpha}\alpha^* + C_{Z_{\delta_e}}\delta_e^*\right] - \frac{\rho S C_{Z_\alpha} w^*}{2m} + \frac{u^*\rho S C_{Z_q} c q^*}{4mV_a^*}$
Z_w	$\frac{w^*\rho S}{m}\left[C_{Z_0} + C_{Z_\alpha}\alpha^* + C_{Z_{\delta_e}}\delta_e^*\right] + \frac{\rho S C_{Z_\alpha} u^*}{2m} + \frac{\rho w^* S c C_{Z_q} q^*}{4mV_a^*}$
Z_q	$u^* + \frac{\rho V_a^* S C_{Z_q} c}{4m}$
Z_{δ_e}	$\frac{\rho V_a^{*2} S C_{Z_{\delta_e}}}{2m}$
M_u	$\frac{u^*\rho S c}{J_y}\left[C_{m_0} + C_{m_\alpha}\alpha^* + C_{m_{\delta_e}}\delta_e^*\right] - \frac{\rho S c C_{m_\alpha} w^*}{2J_y} + \frac{\rho S c^2 C_{m_q} q^* u^*}{4J_y V_a^*}$
M_w	$\frac{w^*\rho S c}{J_y}\left[C_{m_0} + C_{m_\alpha}\alpha^* + C_{m_{\delta_e}}\delta_e^*\right] + \frac{\rho S c C_{m_\alpha} u^*}{2J_y} + \frac{\rho S c^2 C_{m_q} q^* w^*}{4J_y V_a^*}$
M_q	$\frac{\rho V_a^* S c^2 C_{m_q}}{4J_y}$
M_{δ_e}	$\frac{\rho V_a^{*2} S c C_{m_{\delta_e}}}{2J_y}$

Beard and Mclain - Small Unmanned Aircraft Theory and Practice

수행내용

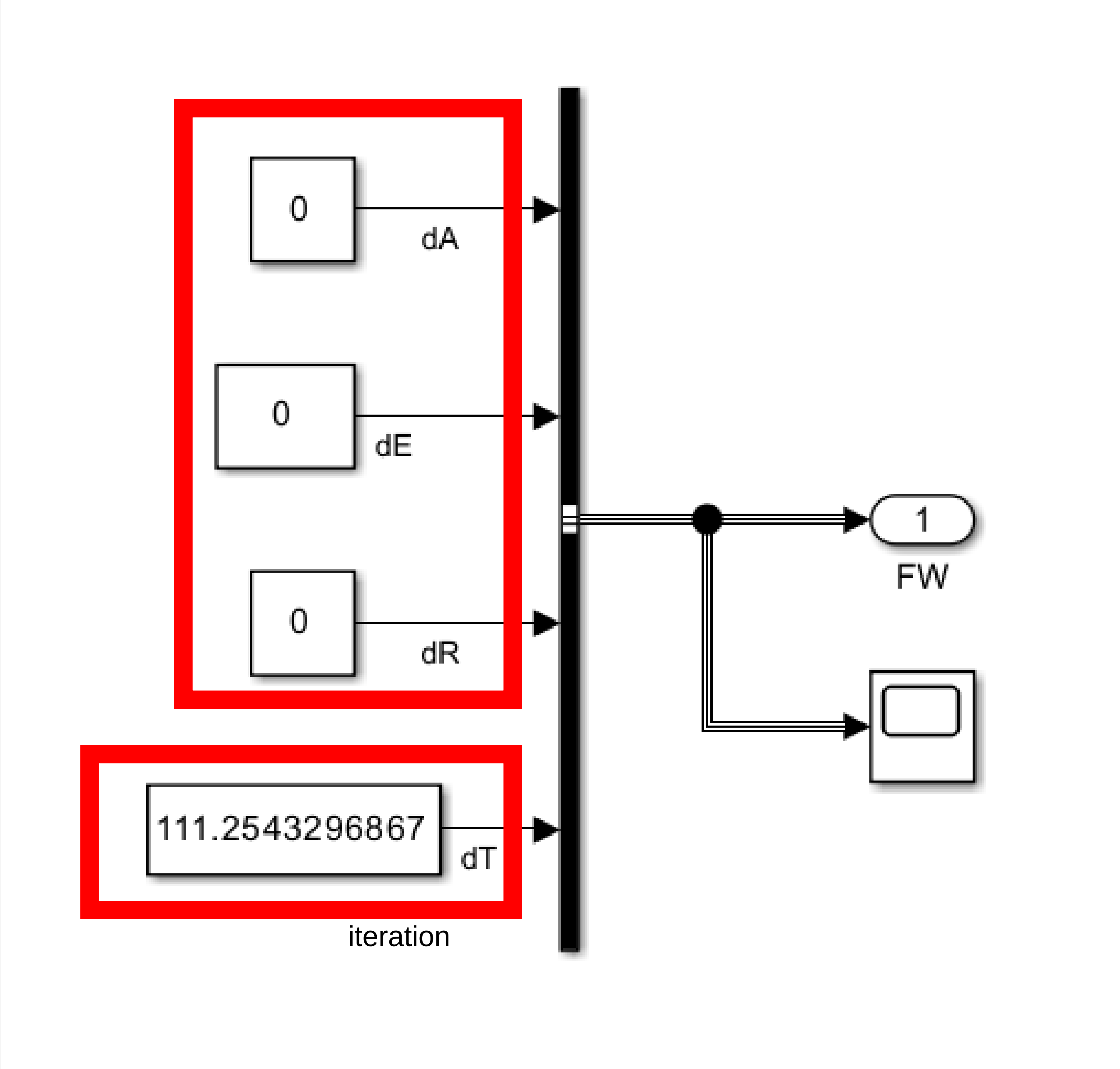
Task 2

Longitudinal State-space Equations



Trim 상태 : 고도 유지(V_Z = 0)
pitch 변화율 = 0
airspeed = 일정

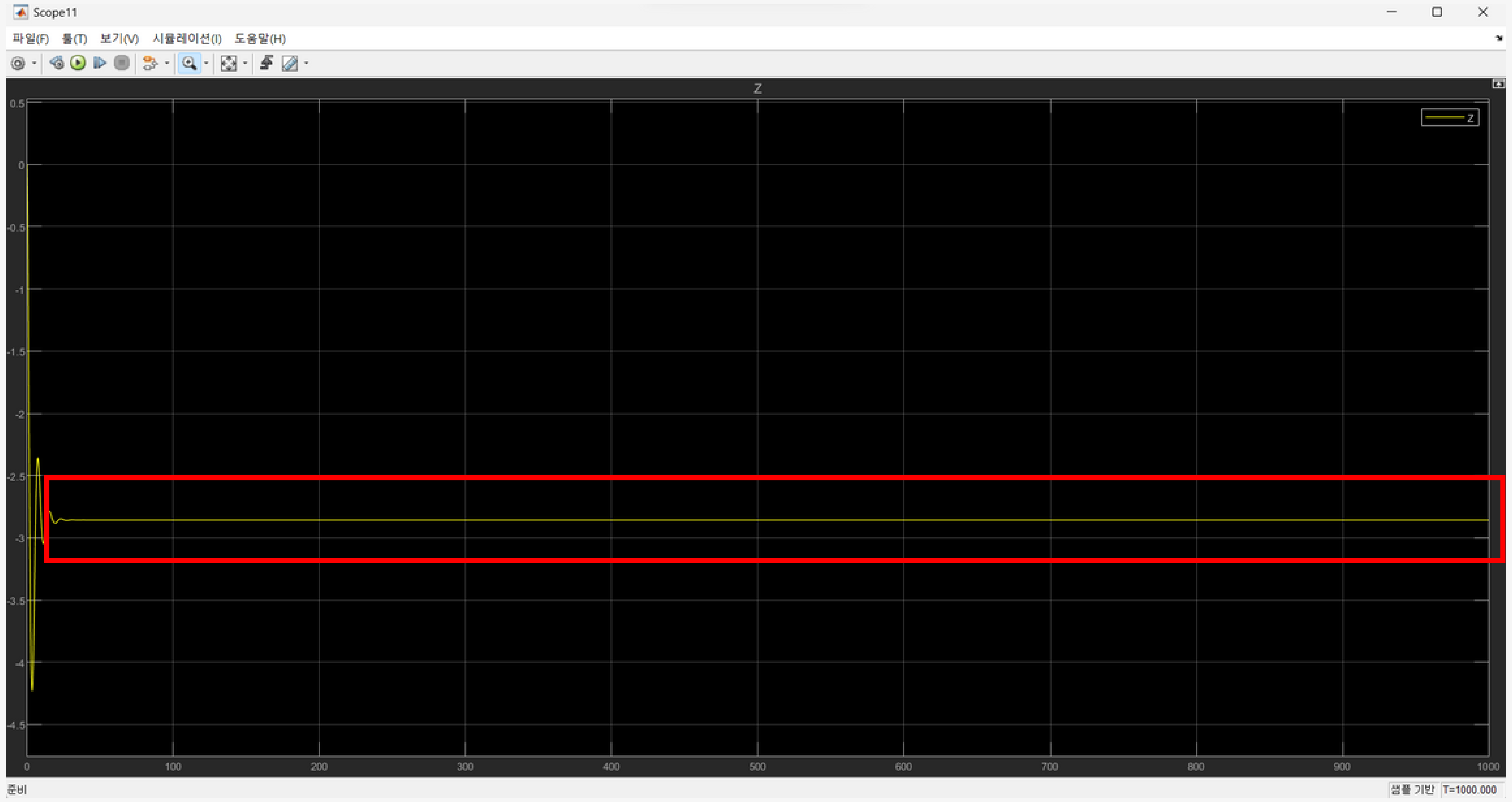
dA=dE=dR=0 일 때 dT의 값에 변화를 주며 Trim 상태를 만듦

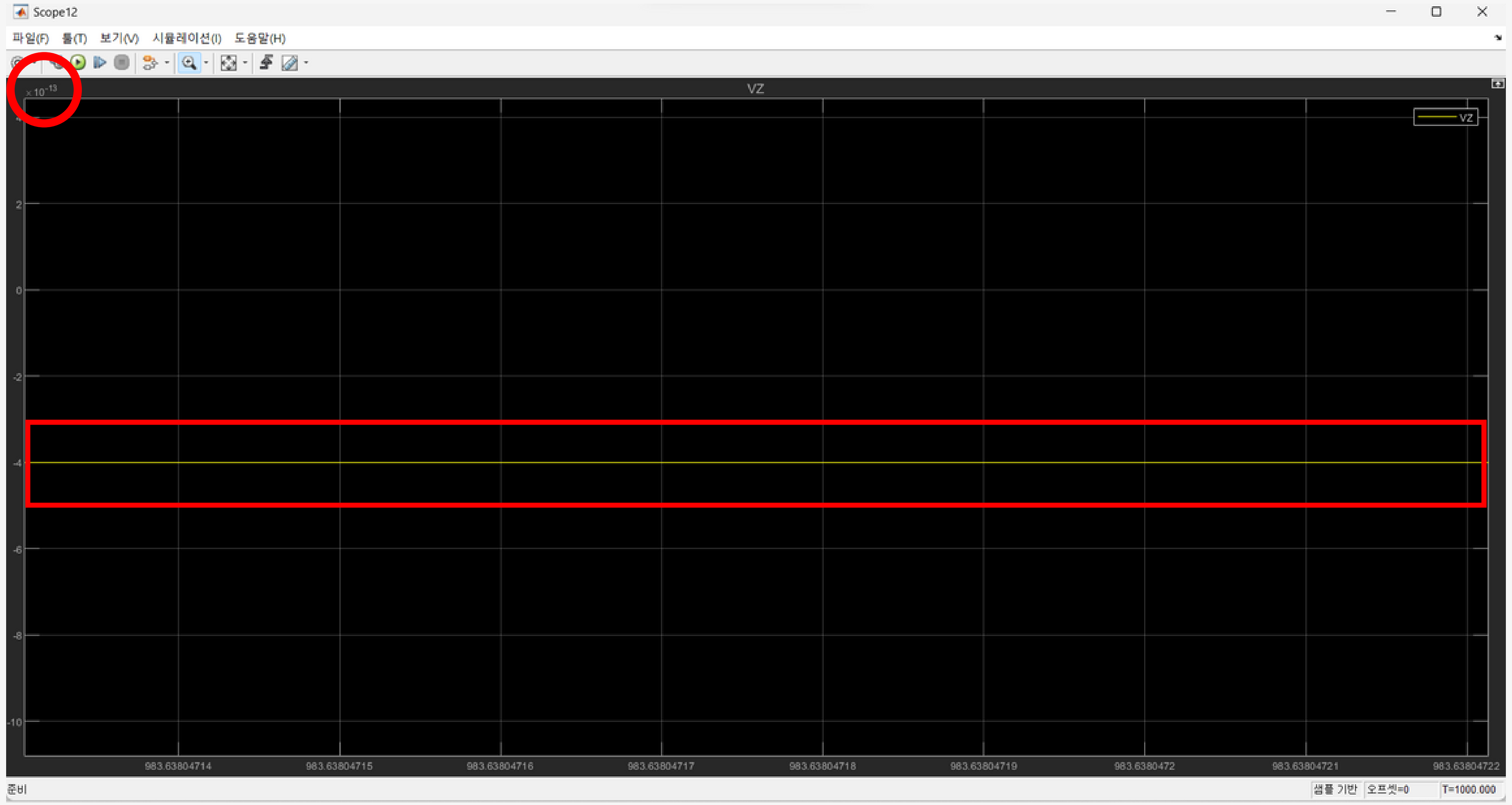


(To Voltage) dT = 0.6675

수행내용

Task 2

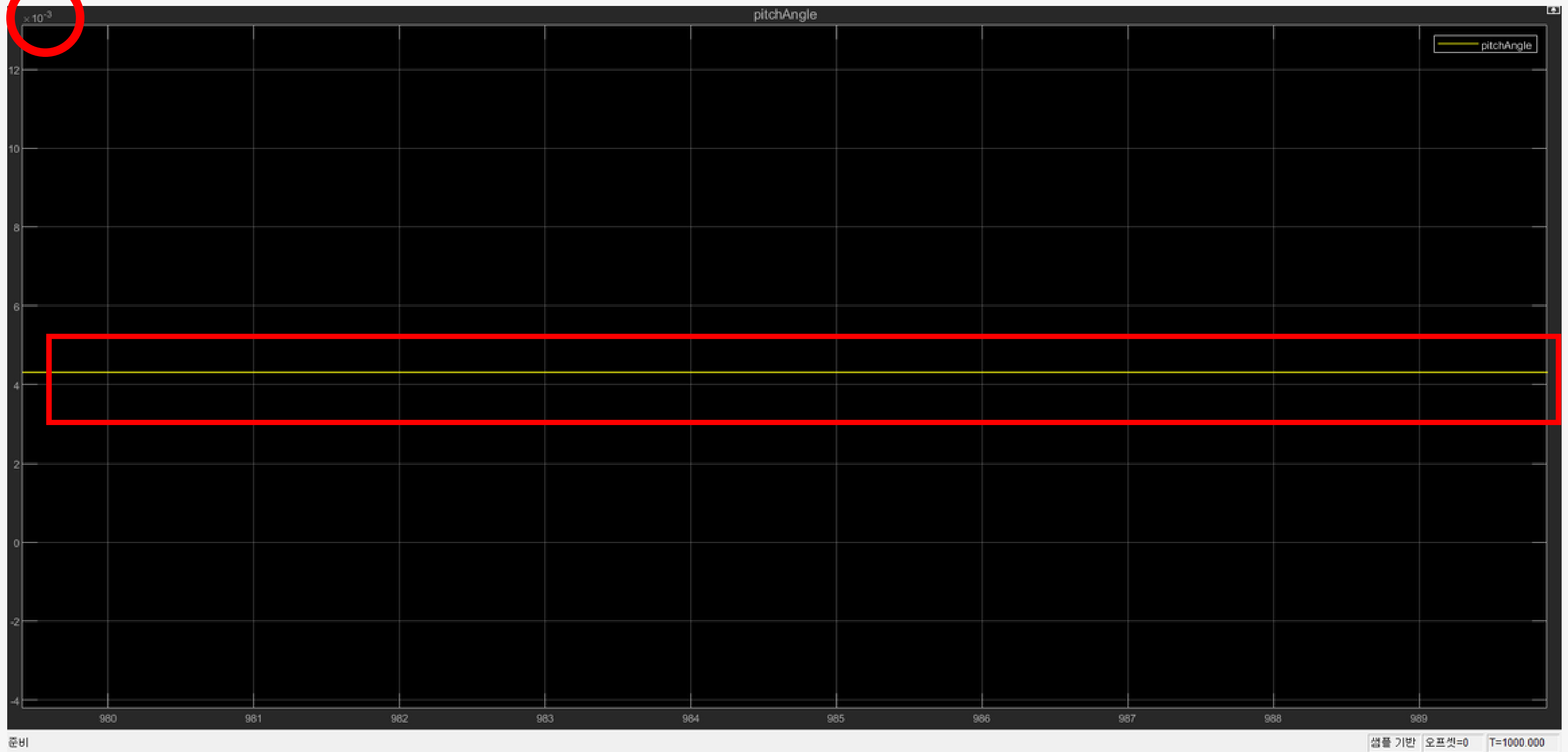


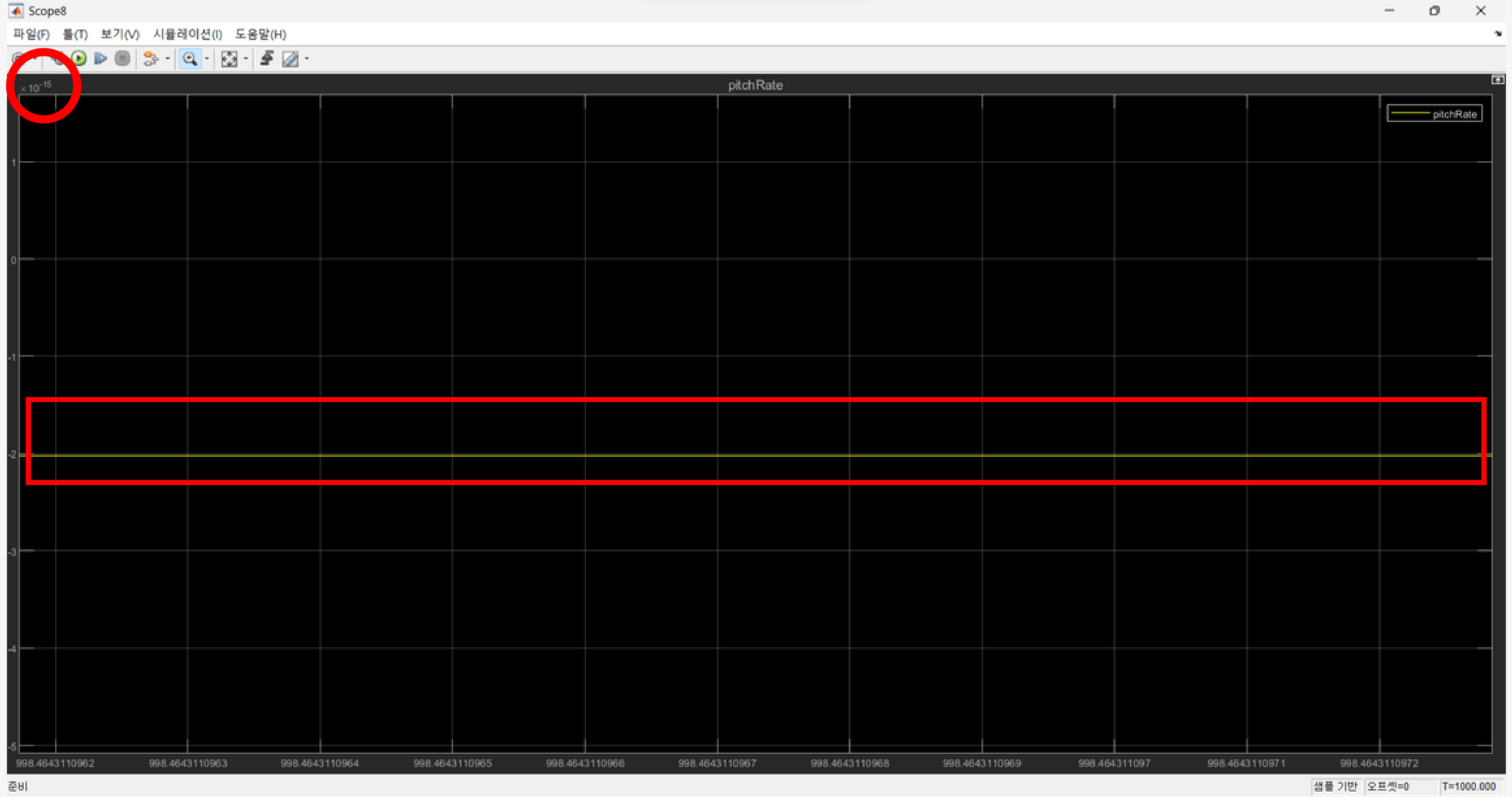


고도 유지 상태
Z = 2.8577 (일정)
V_Z = -4.25e-13

수행내용

Task 2

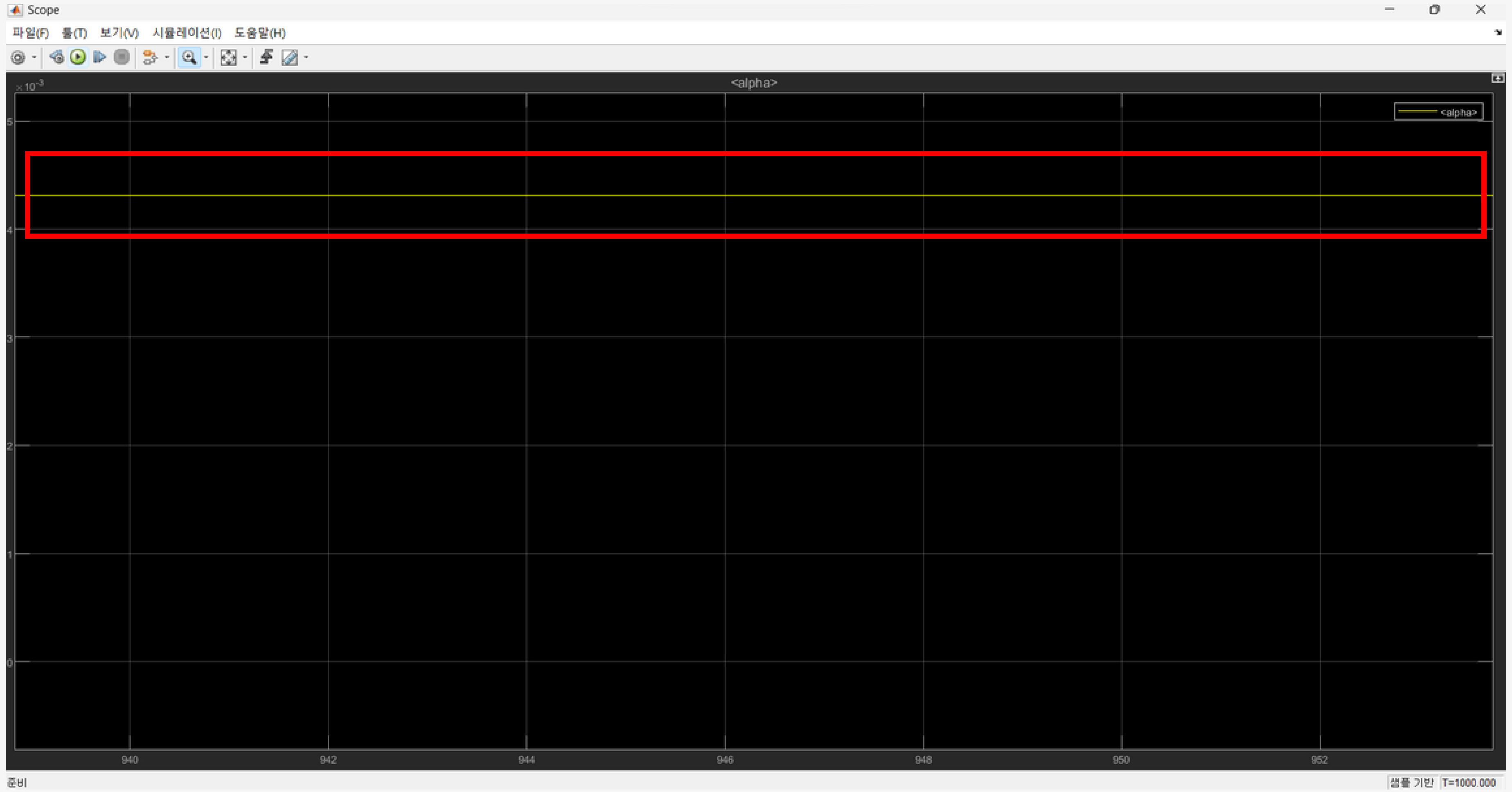


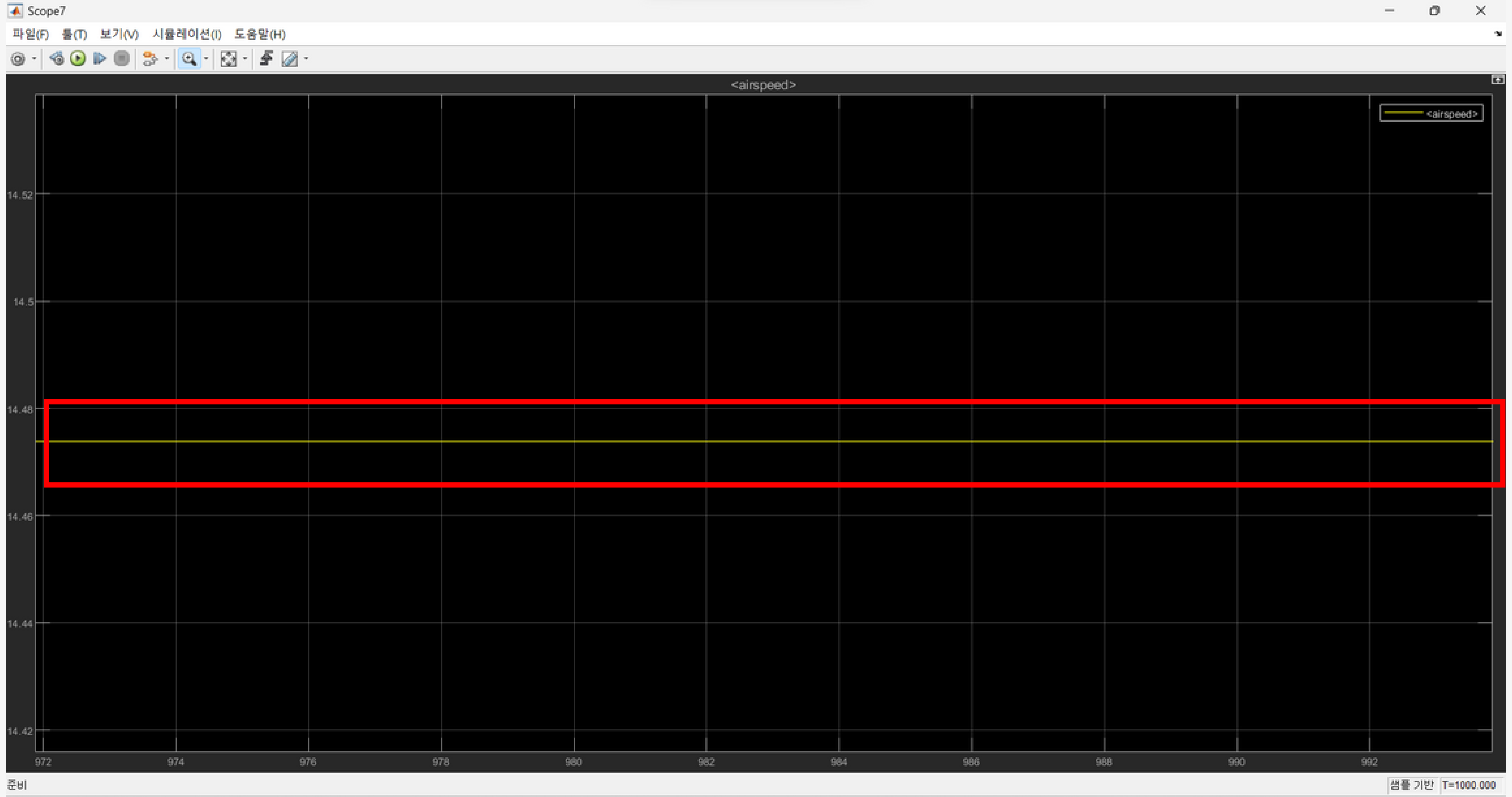


pitchAngle = 0.0043 (일정)
pitchRate = 2.2231e-15

수행내용

Task 2





alpha = 0.0043 (일정)
airspeed = 14.4739 (일정)

수행내용

Task 2 직접 계산

$$
\begin{pmatrix} \dot{\bar{u}} \\ \dot{\bar{w}} \\ \dot{\bar{q}} \\ \dot{\bar{\theta}} \\ \dot{\bar{h}} \end{pmatrix} = \begin{pmatrix} X_u & X_w & X_q & -g\cos\theta^* & 0 \\ Z_u & Z_w & Z_q & -g\sin\theta^* & 0 \\ M_u & M_w & M_q & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ \sin\theta^* & -\cos\theta^* & 0 & u^*\cos\theta^* + w^*\sin\theta^* & 0 \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{w} \\ \bar{q} \\ \bar{\theta} \\ \bar{h} \end{pmatrix} + \begin{pmatrix} X_{\delta_e} & X_{\delta_t} \\ Z_{\delta_e} & 0 \\ M_{\delta_e} & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \bar{\delta}_e \\ \bar{\delta}_t \end{pmatrix}
$$

```matlab
%%
g=9.81;
u_trim=14.4739;   q_trim=0;   w_trim=0;   Va_trim=14.4739;  deltat_trim=0.6675;    deltae_trim=0;  alpha_trim=0.0043; theta_trim=0.0043;
rho=1.2250;     S=0.55;   m=6.0230;   c=0.2750;   k=650;   S_prop=pi/4*(0.3052)^2;  C_prop=0.03338*(14.4739/30.52)^3-0.1265*(14.4739/30.52)^2-0.1210*(14.4739/30.52)+0.1142;

C_L0=0.81857;    C_Lalpha=4.09127;    C_Ldeltae=0.50787;      C_Lq=7.31097;

C_Dmin=0.06047;    K=0.1328;    C_LCDmin=0.4806; C_Ddeltae=0.016043;
C_D0=C_Dmin+K*(C_L0-C_LCDmin); C_Dalpha=C_Dmin+K*(C_Lalpha-C_LCDmin); C_Dq=C_Dmin+K*(C_Lq-C_LCDmin);

C_X0=C_L0*sin(alpha_trim)-C_D0*cos(alpha_trim);    C_Xalpha=C_Lalpha*sin(alpha_trim)-C_Dalpha*cos(alpha_trim);
C_Xdeltae=C_Ldeltae*sin(alpha_trim)-C_Ddeltae*cos(alpha_trim);       C_Xq=C_Lq*sin(alpha_trim)-C_Dq*cos(alpha_trim);
C_Z0=-C_L0*cos(alpha_trim)-C_D0*sin(alpha_trim);    C_Zalpha=-C_Lalpha*cos(alpha_trim)-C_Dalpha*sin(alpha_trim);
C_Zdeltae=-C_Ldeltae*cos(alpha_trim)-C_Ddeltae*sin(alpha_trim);   C_Zq=-C_Lq*cos(alpha_trim)-C_Dq*sin(alpha_trim);
C_m0=0.00763;    C_malpha=-1.76966;    C_mdeltae=-1.83747;   C_mq=-19.22663;
J_y=0.2670;
%%
X_u = u_trim *rho * S / m * (C_X0 + C_Xalpha * alpha_trim + C_Xdeltae * deltae_trim) - rho * S * w_trim * C_Xalpha / (2 * m ) + rho *S*c*C_Xq*u_trim*q_trim/(4*m*Va_trim)-rho*S_prop*C_prop*u_trim/m;
X_w = -q_trim+w_trim*rho*S/m*(C_X0+C_Xalpha*alpha_trim+C_Xdeltae*deltae_trim)+rho*S*c*C_Xq*w_trim*q_trim/(4*m*Va_trim)+rho*S*C_Xalpha*u_trim/(2*m )-rho*S_prop*C_prop*w_trim/m;
X_q = -w_trim+rho*Va_trim*S*C_Xq*c/(4*m );

X_deltae = rho*Va_trim^2*S*C_Xdeltae/(2*m );
X_deltat = rho*S_prop*C_prop*k^2*deltat_trim/m;

Z_u = q_trim + u_trim*rho*S/m*(C_Z0+C_Zalpha*alpha_trim+C_Zdeltae*deltae_trim)-rho*S*C_Zalpha*w_trim/(2*m ) +u_trim*rho*S*C_Zq*c*q_trim/(4*m*Va_trim);
Z_w = w_trim*rho*S/m*(C_Z0+C_Zalpha*alpha_trim+C_Zdeltae*deltae_trim)+rho*S*C_Zalpha*w_trim/(2*m ) +rho*w_trim*S*C_Zq*c*q_trim/(4*m*Va_trim);
Z_q = u_trim+rho*Va_trim*S*C_Zq*c /(4*m );

Z_deltae = rho*Va_trim^2*S*C_Zdeltae/(2*m );

M_u = u_trim*rho*S*c/J_y*(C_m0+C_malpha*alpha_trim+C_mdeltae*deltae_trim)-rho*S*c*C_malpha*w_trim/(2*J_y)+rho*S*c^2*C_mq*q_trim*u_trim/(4*J_y*Va_trim);
M_w = w_trim*rho*S*c/J_y*(C_m0+C_malpha*alpha_trim+C_mdeltae*deltae_trim)+rho*S*c*C_malpha*u_trim/(2*J_y)+rho*S*c^2*C_mq*q_trim*w_trim/(4*J_y*Va_trim);
M_q = rho*Va_trim*S*c^2*C_mq/(4*J_y );

M_deltae = rho*Va_trim^2*S*c*C_mdeltae/(2*J_y);
%%
% u w q theta h
A=[X_u X_w X_q -g*cos(theta_trim) 0; Z_u Z_w Z_q -g*sin(theta_trim) 0; M_u M_w M_q 0 0; 0 0 1 0 0; sin(theta_trim) -cos(theta_trim) 0 u_trim*cos(theta_trim)+w_trim*sin(theta_trim) 0]
B=[X_deltae X_deltat; Z_deltae 0; M_deltae 0; 0 0; 0 0;]
% u alpha q theta h
A1=[X_u X_w*Va_trim*cos(alpha_trim) X_q -g*cos(theta_trim) 0; Z_u/(Va_trim*cos(alpha_trim)) Z_w Z_q/(Va_trim*cos(alpha_trim)) -g*sin(theta_trim)/(Va_trim*cos(alpha_trim)) 0; M_u M_w*Va_trim*cos(alpha_trim) M_q 0 0; 0 0 1 0 0; sin(theta_t
B1=[X_deltae X_deltat; Z_deltae/(Va_trim*cos(alpha_trim)) 0; M_deltae 0; 0 0; 0 0;]
```

수행내용

Task 2 직접 계산

$$\begin{pmatrix}\dot{\bar{u}}\\ \dot{\bar{w}}\\ \dot{\bar{q}}\\ \dot{\bar{\theta}}\\ \dot{\bar{h}}\end{pmatrix}=\begin{pmatrix}X_u & X_w & X_q & -g\cos\theta^* & 0\\ Z_u & Z_w & Z_q & -g\sin\theta^* & 0\\ M_u & M_w & M_q & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ \sin\theta^* & -\cos\theta^* & 0 & u^*\cos\theta^*+w^*\sin\theta^* & 0\end{pmatrix}\begin{pmatrix}\bar{u}\\ \bar{w}\\ \bar{q}\\ \bar{\theta}\\ \bar{h}\end{pmatrix}+\begin{pmatrix}X_{\delta_e} & X_{\delta_t}\\ Z_{\delta_e} & 0\\ M_{\delta_e} & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\bar{\delta}_e\\ \bar{\delta}_t\end{pmatrix}$$

```
A =

   -0.1754   -0.4229   -0.1042   -9.8099         0
   -1.3546         0   13.6596   -0.0422         0
    0.0002   -8.8872  -13.2764         0         0
         0         0    1.0000         0         0
    0.0043   -1.0000         0   14.4738         0


B =

   -0.1624  133.9701
   -5.9516         0
 -133.5616         0
         0         0
         0         0
```

$$\begin{pmatrix}\dot{\bar{u}}\\ \dot{\bar{\alpha}}\\ \dot{\bar{q}}\\ \dot{\bar{\theta}}\\ \dot{\bar{h}}\end{pmatrix}=\begin{pmatrix}X_u & X_w V_a^*\cos\alpha^* & X_q & -g\cos\theta^* & 0\\ \frac{Z_u}{V_a^*\cos\alpha^*} & Z_w & \frac{Z_q}{V_a^*\cos\alpha^*} & \frac{-g\sin\theta^*}{V_a^*\cos\alpha^*} & 0\\ M_u & M_w V_a^*\cos\alpha^* & M_q & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ \sin\theta^* & -V_a^*\cos\theta^*\cos\alpha^* & 0 & u^*\cos\theta^*+w^*\sin\theta^* & 0\end{pmatrix}\times\begin{pmatrix}\bar{u}\\ \bar{\alpha}\\ \bar{q}\\ \bar{\theta}\\ \bar{h}\end{pmatrix}+\begin{pmatrix}X_{\delta_e} & X_{\delta_t}\\ \frac{Z_{\delta_e}}{V_a^*\cos\alpha^*} & 0\\ M_{\delta_e} & 0\\ 0 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\bar{\delta}_e\\ \bar{\delta}_t\end{pmatrix}$$

```
A1 =

   -0.1754   -6.1207   -0.1042   -9.8099         0
   -0.0936         0    0.9438   -0.0029         0
    0.0002 -128.6315  -13.2764         0         0
         0         0    1.0000         0         0
    0.0043  -14.4736         0   14.4738         0


B1 =

   -0.1624  133.9701
   -0.4112         0
 -133.5616         0
         0         0
         0         0
```

수행내용

Task 2 직접 계산 vs 모델선형기 앱

동작점: 선형 분석 작업 공간의 "op_dE0dT111"

크기: 2개 입력, 1개 출력, 4개 상태

선형화 결과:

A =

	x1	x2	x3	x4
x1	0	1	0	0
x2	-5.684e-14	-14.68	0.08472	-9.824
x3	-9.793	-0.08065	-0.5585	0.4033
x4	-1.421e-14	15.1	-1.463	-3.796

B =

	u1	u2
x1	0	0
x2	-163.2	3.415e-17
x3	-0.6519	0.06584
x4	-7.262	-4.031e-18

C =

	x1	x2	x3	x4
y1	1	0	0	0

D =

	u1	u2
y1	0	0

상태 이름:
x1 - phi theta psi(2)
x2 - p,q,r (2)
x3 - ub,vb,wb(1)
x4 - ub,vb,wb(3)

입력 채널 이름:
u1 - dE
u2 - dT

출력 채널 이름:
y1 - pitchAngle

A =

-0.1754	-0.4229	-0.1042	-9.8099	0
-1.3546	0	13.6596	-0.0422	0
0.0002	-8.8872	-13.2764	0	0
0	0	1.0000	0	0
0.0043	-1.0000	0	14.4738	0

B =

-0.1624	133.9701
-5.9516	0
-133.5616	0
0	0
0	0

<직접 계산한 A, B 행렬>

A =

-0.5585	0.4033	-0.0806	-9.7930
-1.4630	-3.7960	15.1000	-0.0000
0.0847	-9.8240	-14.6800	-0.0000
0	0	1.0000	0

$\bar{u}$
$\bar{w}$
$\bar{q}$
$\bar{\theta}$

B =

-0.6519	0.0658
-7.2620	-0.0000
-163.2000	0.0000
0	0

$\bar{\delta}_e$
$\bar{\delta}_t$

<시뮬링크 모델선형기 앱>

감사합니다

수행내용

Task 2

질문

연구 목표

연구 목표

논문 공부
&기초 개념 공부

시뮬링크 예제
following

PID 튜닝
&
Tilt Corridor

제어기 수정&설계

MATLAB 그래프와
3D Animation을
통해 성능 검증

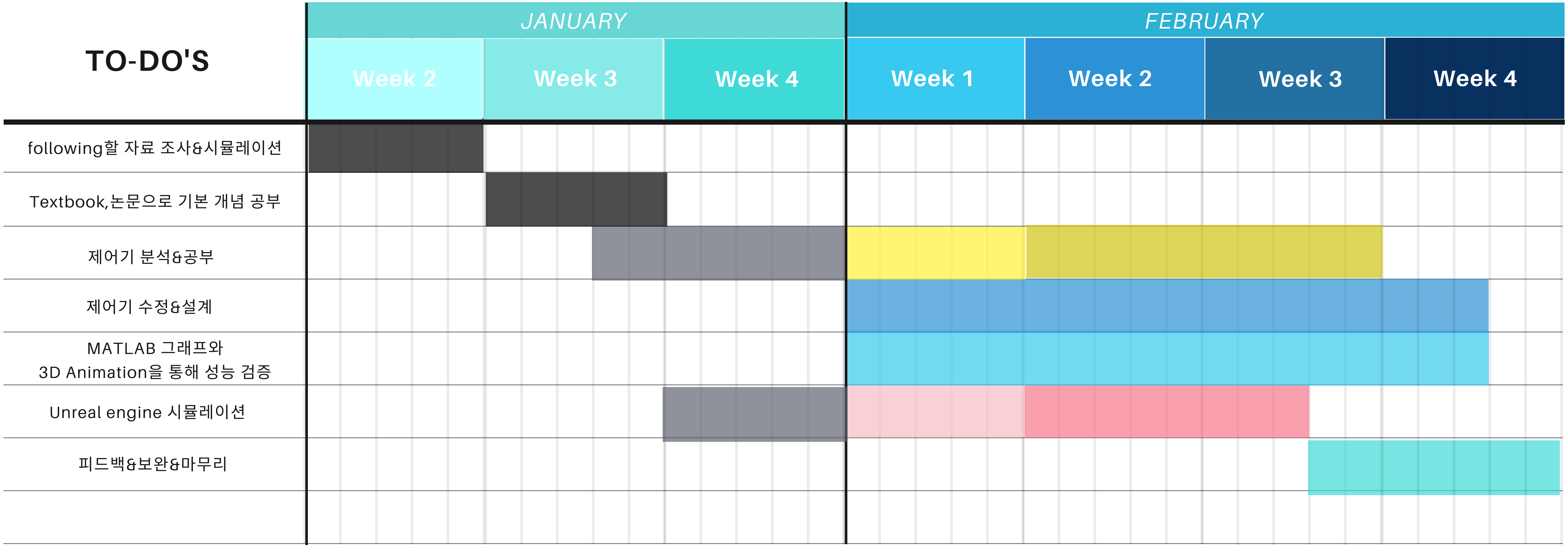
coordinate

Unreal Engine
시뮬레이션

스케줄

gantt chart

Schedule



현재 연구 논의 내용

현재 연구 논의 내용

문의 사항

문의사항
Q&A