

Research of Transition Corridor and Optimal Trajectory for Front-Tilt-Quadplane eVTOL Aircraft

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Abstract—Urban air mobility is undergoing a booming development. Electrical vertical takeoff and landing (eVTOL) aircraft has drawn great attention from industrial and academic communities recently. In this paper, a Front-Tilt-Quadplane configuration is proposed and dynamics model of full-mode flight is proposed. Control-stability characteristics are analyzed within different modes. Then transition corridor is calculated with limits of attack angle, thrust and stalling speed. Finally, the transition control strategy is analyzed with trajectory optimization technology.

Keywords—Front-Tilt-Quadplane, eVTOL, Transition Corridor, Trajectory Optimization

I. INTRODUCTION

Recently, with the latest advances in the technological fields of structures, automation and control, propulsion, and energy-storage, and the increasing population density, there have been numerous developments in the area of urban air mobility(UAM). Electrical VTOL(eVTOL) vehicle is a promising solution[1].

According to the propulsion configuration, eVTOL vehicles can be divided into three sub-groups:

Currently, diverse configurations are being explored, designed and flight tested. Hybrid configuration and distributed electric propulsion technology are widely adopted. Based on the propulsion configuration, these vehicles can be divided into three sub-groups[2]:

1. tilting propulsion systems: Lilium Eagle, Vahana, Joby Aviation S2.
2. separate vertical and/or forward propulsion system: Kitty Hawk Cora.
3. separate vertical combined with a tilting propulsion system: Uber CRM-001

The tilting propulsion configuration has obvious advantages in range, cruising speed and payload, and has become the most active aircraft in research and development.

Over the last decade, for VTOL vehicles with tilting propulsion systems, the research community has addressed several important problems, such as flight dynamics, flight control, control allocation, and handling qualities. T Lombaerts et al.[2] propose a nonlinear dynamic inversion based attitude

control law for a quad tiltrotor eVTOL vehicle. Incremental nonlinear dynamic inversion method is also discussed. But the research is limited to the hover conditions. M Allenspach et al.[3] introduce a MPC controller and a daisy-chain allocation strategy for a quad tiltrotor UAV, capable of handling the nonlinear dynamics and changing control authorities during the transition. But oscillations of actuator is violent. G Özdoğan et al. [4]propose an unconventional heavy-lift aerial vehicle and present the design and control allocation analysis. Jacob Willis et al.[5] introduce a control scheme consists of two parts: a low-level angular rate controller and variable mixer, and a trajectory tracking state-dependent LQR controller. M Mousaei et al.[6] propose a dynamic control allocation method that allows the system to adapt to actuator failures of tiltrotor VTOL with the ability to rotate each individual propeller. In [7], L Bauersfeld et al. propose a unified nonlinear control scheme with a MPC out-loop controller and control allocation for a tilt-quadrotor VTOL aircraft.

Y Yuan et al. [8] introduce a flight dynamics model of the tiltrotor aircraft incorporating AD (Automatic Differentiation) methods, verify its accuracy using trim comparison with an established model. This model can be further expanded for maneuvering flight. D Milz et al. [9]provide a unified control scheme with an INDI controller for an integrated tilt-wing eVTOL model. The control law is designed to be generic and robust. S Zhang et al. [10]investigate the dynamic characteristics of a distributed propulsion tilt-rotor aircraft by fully considering the variations of the center of gravity and the inertia during the transition. They conclude that the aircraft must accumulate enough speed to provide the necessary positive pitch control moment before the thrust line of the main propellers pass the center of gravity. In [11], a nonlinear hierarchical adaptive control framework is proposed that consists of an outer-loop Total Energy based speed and altitude control cascaded with an inner-loop model reference adaptive attitude control. In [12], N Kang et al. develop the longitudinal flight control scheme for the tilt-rotor VTOL but automatically scheduled on airspeed. M Yayla et al. [13] propose an autonomous flight control strategy for a tilt-prop UAV. The inner loop controller is designed to hold desired attitude and altitude using adaptive feedback linearization control theory. The outer loop controller is designed with Lyapunov-based approach to hold the desired velocity. Besides, desired attitudes and the tilt angle of front rotors are determined with the commanded velocities.

One promising but only little investigated and understood subcategory comprises front-tilt-quadplane eVTOL. In this paper, transition corridor and optimal trajectory for front-tilt-quadplane eVTOL aircraft are analyzed, and the main contributions are listed as follows: (1) Full-mode flight of front-tilt-quadplane aircraft is established and transition corridor is calculated with limits of attack angle, thrust and stalling speed; (2) The transition control strategy is analyzed with trajectory optimization technology. Parameter sensitivity analysis is conducted for transition strategy.

The remainder of this paper is organized as follows. Multimode flight dynamics model is built in Section II. Control-stability characteristics are introduced in Section III. Transition corridor analyses are shown in Section IV. Transition trajectory optimization is described in Section V. Conclusions and future works are summarized in Section VI.

II. MUTIMODE FLIGHT DYNAMICS MODEL

A. The Example UAV

To study flight dynamics and flight control for front-tilt-quadplane eVTOL aircraft, an example front-tilt-quadplane UAV is designed, which is illustrated in Fig. 1.

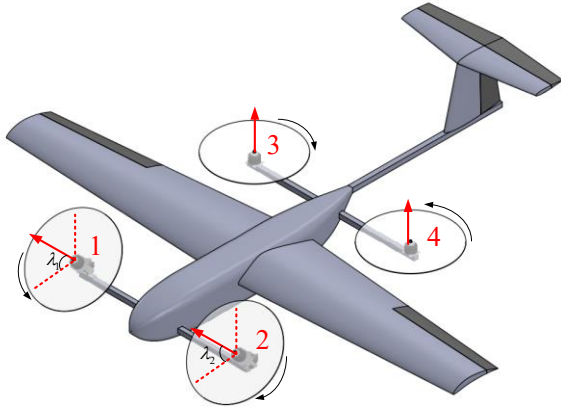


Fig. 1. The example front-tilt-quadplane eVTOL UAV.

B. Gravity Model

The components of gravity in the body-fixed reference frame are given by

$$\mathbf{F}_g = \begin{bmatrix} -mg \sin \theta \\ mg \cos \theta \sin \phi \\ mg \cos \theta \cos \phi \end{bmatrix} \quad (1)$$

where m is the mass of the aircraft, and g is acceleration of gravity.

C. Aerodynamic Forces and Moments

The components of aerodynamic forces in the body-fixed reference frame are given by

$$\mathbf{F}_{\text{plane}} = \begin{bmatrix} -D \cos \alpha \cos \beta - Y \cos \alpha \sin \beta + L \sin \alpha \\ -D \sin \beta + Y \cos \beta \\ -D \sin \alpha \cos \beta - Y \sin \alpha \sin \beta - L \cos \alpha \end{bmatrix} \quad (2)$$

where L , Y and D are lift, side force and drag acted on the aircraft respectively. The aerodynamic force coefficients in wind-axes reference frame are given as follows:

$$\begin{cases} L = \frac{1}{2} \rho V^2 C_L S, & D = \frac{1}{2} \rho V^2 C_D S, & Y = \frac{1}{2} \rho V^2 C_Y S \\ C_L = C_{L0} + C_{L\alpha} \alpha + C_{Lq} \frac{qc}{2V} + C_{L\delta_e} \delta_e + C_{L\delta_a} |\delta_a| \\ C_D = C_{D\min} + K(C_L - C_{L,C_{D\min}})^2 + C_{D\delta_a} |\delta_a| + C_{D\delta_e} |\delta_e| + C_{D\delta_r} |\delta_r| \\ C_Y = C_{Y0} + C_{Y\beta} \beta + C_{Yp} \frac{pb}{2V} + C_{Yr} \frac{rb}{2V} + C_{Y\delta_a} \delta_a + C_{Y\delta_r} \delta_r \end{cases} \quad (3)$$

The components of aerodynamic moments in the body-fixed reference frame are given as follows:

$$\mathbf{M}_{\text{plane}} = [l \quad M \quad N]^T \quad (4)$$

where l , M and N are rolling moment, pitching moment and yawing moment acted on the aircraft respectively. The aerodynamic moment coefficients are given as follows:

$$\begin{cases} l = \frac{1}{2} \rho V^2 C_l S b, & M = \frac{1}{2} \rho V^2 C_M S c_A, & N = \frac{1}{2} \rho V^2 C_n S b \\ C_l = C_{l0} + C_{l\beta} \beta + C_{lp} \frac{pb}{2V} + C_{lr} \frac{rb}{2V} + C_{l\delta_a} \delta_a + C_{l\delta_r} \delta_r \\ C_m = C_{m0} + C_{m\alpha} \alpha + C_{mq} \frac{qc}{2V} + C_{m\delta_e} \delta_e + C_{m\delta_r} |\delta_r| \\ C_n = C_{n0} + C_{n\beta} \beta + C_{np} \frac{pb}{2V} + C_{nr} \frac{rb}{2V} + C_{n\delta_a} \delta_a + C_{n\delta_r} \delta_r \end{cases} \quad (5)$$

where C_l , C_m and C_n are rolling moment, pitching moment and yawing moment coefficient respectively. p , q and r are the roll, pitch and yaw angular velocity in the body-fixed reference frame respectively. b and c_A are wingspan and average aerodynamic chord length respectively. The coefficients and relevant derivatives of aerodynamic forces and moments are described in more details in [14].

D. Rotor Forces and Moments

According to the blade element theory, the total lift and total torque can be expressed as follows

$$\begin{cases} F_i = \rho n^2 D^4 C_T(J), i = 1, 2, 3, 4 \\ Q_i = \rho n^2 D^5 C_Q(J), i = 1, 2, 3, 4 \end{cases} \quad (6)$$

where C_T and C_Q are the thrust coefficient and moment coefficient of propeller, ρ is air density, D is the propeller diameter, and n is rotation speed of rotor with the unit of r/s. J is the rotor advance ratio defined as $J = V_a/n/D$.

Then the rotor forces in the body-axes frame can be described as

$$\begin{cases} \mathbf{F}_1 = F_1[\cos \lambda_1 & 0 & -\sin \lambda_1]^T \\ \mathbf{F}_2 = F_2[\cos \lambda_2 & 0 & -\sin \lambda_2]^T \\ \mathbf{F}_3 = F_3[0 & 0 & -1]^T \\ \mathbf{F}_4 = F_4[0 & 0 & -1]^T \end{cases} \quad (7)$$

where λ_1 and λ_2 are the tilting angle of rotor 1 and rotor 2.

The total force of the rotor is

$$\mathbf{F}_{\text{rotor}} = \sum_{i=1}^4 \mathbf{F}_i, i = 1, 2, 3, 4 \quad (8)$$

With the rotor forces in the body-axes frame can be described as

$$\begin{cases} \mathbf{Q}_1 = Q_1[-\cos \lambda_1 & 0 & \sin \lambda_1]^T \\ \mathbf{Q}_2 = Q_2[\cos \lambda_2 & 0 & -\sin \lambda_2]^T \\ \mathbf{Q}_3 = Q_3[0 & 0 & -1]^T \\ \mathbf{Q}_4 = Q_4[0 & 0 & 1]^T \end{cases} \quad (9)$$

The total rotor moment can be calculated by

$$\mathbf{M}_{\text{rotor}} = \sum_{i=1}^4 \mathbf{Q}_i + \sum_{i=1}^4 \mathbf{r}_i \times \mathbf{F}_i \quad (10)$$

where \mathbf{r}_i is the vector from the center of mass to the acting point of \mathbf{F}_i . Then the total force and moment can be expressed as

$$\begin{cases} \mathbf{F} = \mathbf{F}_g + \mathbf{F}_{\text{plane}} + \mathbf{F}_{\text{rotor}} \\ \mathbf{M} = \mathbf{M}_g + \mathbf{M}_{\text{plane}} + \mathbf{M}_{\text{rotor}} \end{cases} \quad (11)$$

III. CONTROL AND STABILITY CHARACTERISTICS

In order to obtain the flight dynamics characteristics in different modes, the nonlinear dynamic model is trimmed with different tilting angles. Then the nonlinear model is linearized at these equilibrium points by using “small disturbance” assumption. The trim conditions are shown in Table I.

TABLE I. TRIM CONDITIONS WITH DIFFERENT TILTING ANGLES

Tilting angle(deg)	Airspeed (m/s)	Mode
0	14	Fixed-wing
30	12	Transition
45	10.8	
60	9.5	
70	8.2	
80	6.3	
90	0.01	quadrotor

The trim results are shown in Fig. 2. We can see from Fig. 2 that the airspeed increases gradually with the decrease of the tilting angle, which is the basic characteristics of tilt-rotor aircraft. All rotor speeds decrease with the increase of airspeed,

the front two rotors speed increase slightly in the fixed-wing mode for cruise flight. Elevator is basically in neutral position.

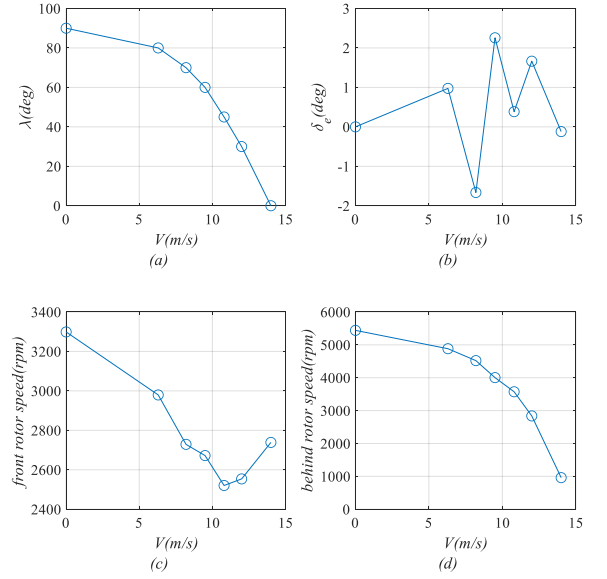


Fig. 2. Trim results with different tilting angles.

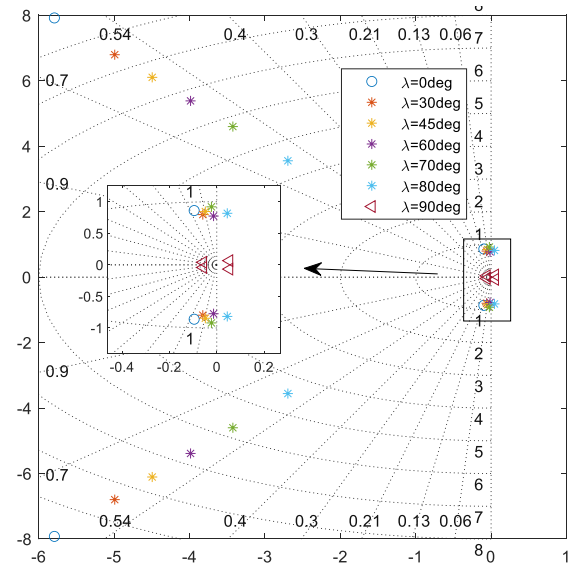


Fig. 3. Eigenvalues distribution with different tilting angles.

The longitudinal eigenvalues distribution with different tilting angles is illustrated in Fig. 3. The transition mode and the fixed-wing mode have the same pattern, with the short-period and phugoid modes. The phugoid mode is slightly unstable with $\lambda = 80$ deg. The quadrotor mode does not have a typical fixed-wing motion mode, which is like a multi-rotor UAV.

IV. TRANSITION CORRIDOR

The tilt transition corridor is essentially a 2D feasible region defined by airspeed and tilting angle. In order to facilitate the solution of the transition corridor, this work transforms this problem into a classic nonlinear optimization problem. Taking the airspeed as the optimization index, the maximum and minimum airspeeds satisfying the constraints and equation

conditions are calculated under different tilt angles respectively. The optimization problem is described as

$$\begin{aligned} \min / \max : |V| &= \sqrt{2q / \rho} \\ \text{s.t.} : \begin{cases} \sum F_x \geq 0 \\ \sum F_z = 0 \\ \sum M_y = 0 \end{cases} \text{ and } \begin{cases} 0^\circ \leq \theta \leq 12^\circ, -15^\circ \leq \delta_e \leq 15^\circ \\ 0 \leq V \leq 45 \\ 0 \leq T_i \leq 30 \end{cases} \end{aligned} \quad (12)$$

The problem is solved with “*fminsearch()*” of MATLAB. The tilt transition corridor calculated is shown in Fig. 4.

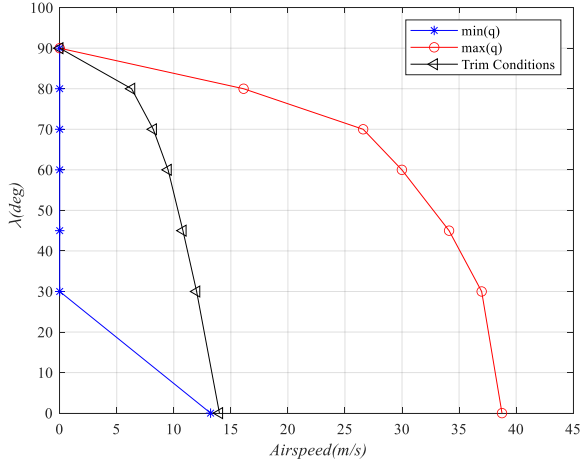


Fig. 4. The transition corridor of the example UAV.

We can see that the trim conditions of Section III are all in the transition corridor. The upper and lower limits of the states and controls at the corridor is shown in Fig. 5. It can be seen that with the increase of the tilting angle, the dynamic pressure and the pitch angle decrease. The front two rotors speed are reduced, and the behind two rotor speed increase to balance the gravity.

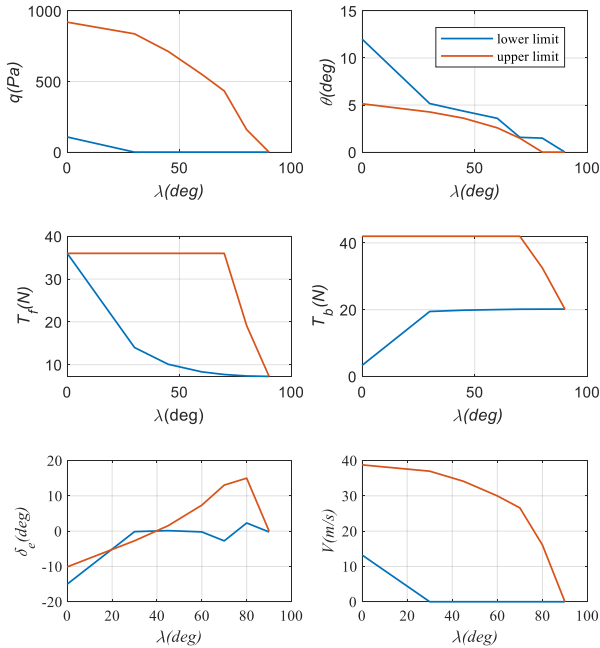


Fig. 5. Upper and lower limits of the states and controls at the corridor.

V. TRANSITION TRAJECTORY OPTIMIZATION

In order to obtain a transition strategy that can satisfy both rapidity and smoothness, trajectory optimization is conducted to calculation an optimal transition process in this section.

In this work, the aircraft dynamics model is transformed into the state equation, and the tilting angle velocity is used as the control quantity, and the initial and final values and process constraints are set according to the flight characteristics and actual physical constraints. Gaussian pseudospectrum is adopted to solve the nonlinear programming (NLP) problem.

Limited by the length of the article, the expression of the optimization problem is not expanded here. The evaluation criteria in this section are the least amount of manipulation and the shortest transition time, i.e. the optimization index is given as

$$J_c = c \cdot t + \int_{t_0}^{t_f} x_1 \cdot \sum T_i + x_2 \cdot \dot{\lambda}^2 + x_3 \cdot \delta_e^2 \quad (13)$$

where t_0 is the start time, t_f is the end time, c, x_1, x_2, x_3 are weights of total time and manipulation.

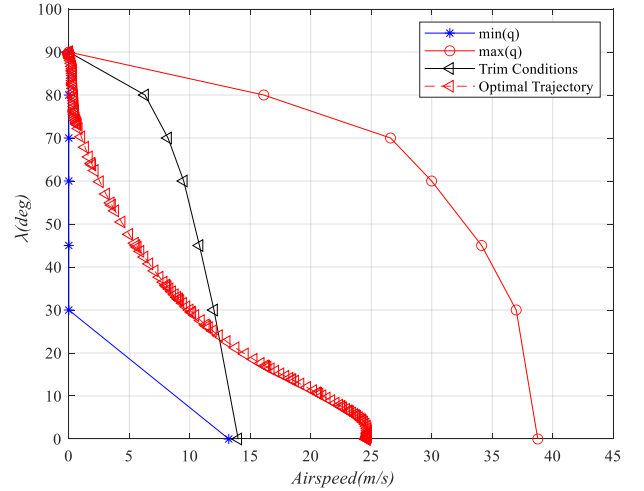


Fig. 6. Optimal trajectory VS the trim results in transition corridor.

The optimal transition trajectory is shown in Fig. 6. Corresponding optimal states and controls are shown in Fig. 7. The tilt angle has distinct start-stop process, which eliminates the constant tilt speed assumption. The increase of airspeed is not linear. The theta and pitch angle rate are reasonable with fluctuations only in the start and end times. The thrust of the rotors are reasonable except the end of the transition process, the front rotors should rise and the behind rotors should stop, which should be optimized in follow-up research.

VI. CONCLUSION

In this paper, a front-tilt-quadplane configuration eVTOL was proposed and full mode flight dynamics model was built. Then the model was trimmed with different tilting angle and control-stability characteristics were analyzed. Based on trim method, transition corridor was calculated with limits, which laid foundation for mode transition strategy and flight control design. Then the transition control strategy is analyzed with trajectory optimization technology.

In the future, we will continue to optimize the transition corridors and transition strategies, and will focus on the multimode flight control law and control allocation of the tilting angles, the control surfaces and the rotors.

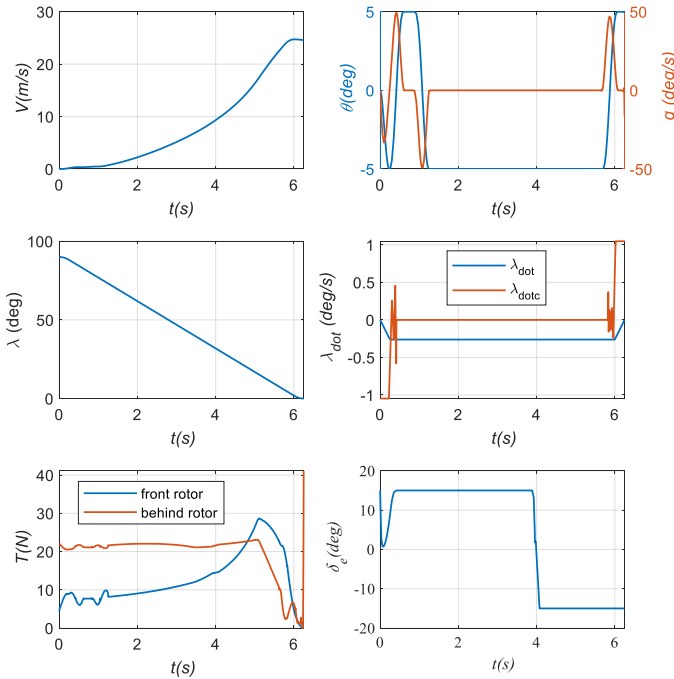


Fig. 7. States and controls of the optimal trajectory.

REFERENCES

- [1] K. I. Swartz, "Charging forward: New evtol concepts advance," *Vertiflite*, vol. 4, pp. 24–29, 2017.
- [2] T. Lombaerts, J. Kaneshige, S. Schuet, G. Hardy, B. L. Aponso, and K. H. Shish, "Nonlinear dynamic inversion based attitude control for a hovering quad tiltrotor eVTOL vehicle," in *AIAA Scitech 2019 forum*, 2019, p. 0134.
- [3] M. Allenspach and G. J. J. Ducard, "Model Predictive Control of a Convertible Tiltrotor Unmanned Aerial Vehicle," in *2020 28th Mediterranean Conference on Control and Automation (MED)*, Saint-Raphaël, France, Sep. 2020, pp. 715–720. doi: 10.1109/MED48518.2020.9183353.
- [4] G. Ozdogan and K. Leblebicioglu, "Design, Modeling, and Control Allocation of a Heavy-Lift Aerial Vehicle Consisting of Large Fixed Rotors and Small Tiltrotors," *IEEE/ASME Trans. Mechatron.*, vol. 27, no. 5, pp. 4011–4021, Oct. 2022, doi: 10.1109/TMECH.2022.3150713.
- [5] J. Willis, J. Johnson, and R. W. Beard, "State-Dependent LQR Control for a Tilt-Rotor UAV," in *2020 American Control Conference (ACC)*, Denver, CO, USA, Jul. 2020, pp. 4175–4181. doi: 10.23919/ACC45564.2020.9147931.
- [6] M. Mousaei, J. Geng, A. Keipour, D. Bai, and S. Scherer, "Design, Modeling and Control for a Tilt-rotor VTOL UAV in the Presence of Actuator Failure," in *2022 IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Kyoto, Japan, Oct. 2022, pp. 4310–4317. doi: 10.1109/IROS47612.2022.9981806.
- [7] L. Bauersfeld, L. Spannagl, G. Ducard, and C. Onder, "MPC Flight Control for a Tilt-Rotor VTOL Aircraft," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 57, no. 4, pp. 2395–2409, Aug. 2021, doi: 10.1109/TAES.2021.3061819.
- [8] Y. Yuan, D. Thomson, and D. Anderson, "Application of Automatic Differentiation for Tilt-Rotor Aircraft Flight Dynamics Analysis," *Journal of Aircraft*, vol. 57, no. 5, pp. 985–990, Sep. 2020, doi: 10.2514/1.C035811.
- [9] D. Milz and G. Looye, "Tilt-Wing Control Design for a Unified Control Concept," in *AIAA SCITECH 2022 Forum*, San Diego, CA & Virtual, Jan. 2022. doi: 10.2514/6.2022-1084.
- [10] S. Zhang, Y. Yang, X. Wang, J. Zhu, and X. Zhang, "Modeling and Dynamic Analysis of a Distributed Propulsion Tilt-Rotor Aircraft," in *AIAA AVIATION 2022 Forum*, Chicago, IL & Virtual, Jun. 2022. doi: 10.2514/6.2022-3871.
- [11] M. Hsu and H. H. Liu, "Design of a Nonlinear Hierarchical Adaptive Controller for a Novel Tilt-Rotor VTOL AquaUAV," in *AIAA Scitech 2021 Forum, VIRTUAL EVENT*, Jan. 2021. doi: 10.2514/6.2021-1283.
- [12] N. Kang, J. Whidborne, L. Lu, and J. Enconniere, "Scheduled Flight Control System of Tilt-Rotor VTOL PAV," in *AIAA SCITECH 2023 Forum*, National Harbor, MD & Online, Jan. 2023. doi: 10.2514/6.2023-1530.
- [13] M. Yayla, A. T. Kutay, M. Senipek, and O. Gungor, "An Adaptive Flight Controller Design for a Tilt-Prop Fixed Wing UAV for All Flight Modes," in *AIAA Scitech 2020 Forum*, Orlando, FL, Jan. 2020. doi: 10.2514/6.2020-0593.
- [14] R. W. Beard and T. W. McLain, *Small unmanned aircraft: Theory and practice*. Princeton university press, 2012.