2017 International Conference on
Unmanned Aircraft Systems (ICUAS)
June 13-16, 2017, Miami, FL, USA

Transition Control of Tilt Rotor Unmanned Aerial Vehicle Based on Multi-Model Adaptive Method*

Zhong Liu[1], Didier Theilliol[2], Liying Yang[3], Yuqing He[3], and Jianda Han[3]

Abstract— **Tilt rotor unmanned aerial vehicle (TRUAV) with ability of hovering and high-speed cruise has attached much attention, but its transition control is still a difficult point because of varying dynamics. This paper proposes a multi-model adaptive control (MMAC) method for a quad-TRUAV, and the stability in the transition procedure could be ensured by considering corresponding dynamics. For safe transition, tilt corridor is considered firstly, and actual flight status should locate within it. Then, the MMAC controller is constructed according to mode probabilities, which are calculated by solving a quadratic programming problem based on a set of input-output plant models. Compared with typical gain scheduling control, this method could ensure transition stability more effectively.**

I. Introduction

Fixed-wing unmanned aerial vehicle (FWUAV) and rotorcraft unmanned aerial vehicle (RUAV) have played important roles in current fields of commerce, agriculture, and military, but their applications are always limited by flexibility, payload, and endurance, which are concerned with their own typical structures and dynamical systems. In this way, plentiful fixed-wing vertical take-off and landing (VTOL) unmanned aerial vehicles (UAVs) are researched for breaking above limitations, and UAVs with tiltable rotors are included [1].

Tilt rotor UAV (TRUAV) is the aircraft that relys on wings and rotors for generating lift together, and accompanied by one or more pairs of tiltable rotors. That is to say, this special vehicle possesses of structure characteristics of FWUAV and RUAV. According to rotor-tilt angle, TRUAV owns two work modes called helicopter mode and airplane mode [2], in which the direction of rotor thrust is vertical and horizontal. In the transition procedure of above two modes, the rotor would tilt from the vertical position to horizontal position or inversely, and the vehicle would be with varying structure and dynamics. So transition control is a obviously key point, and still a difficult point [3].

*This study was co-supported by the National Natural Science Foundation of China (Nos. 61503369 and U1508208)

[1]State Key Laboratory of Robotics, Shenyang Institute of Automation, Chinese Academy of Sciences, 110016 Shenyang, Liaoning Province, P.R.China, University of Chinese Academy of Sciences, 100049, Beijing, P.R.China, liuzhong@sia.cn

[2]CRAN UMR 7039, CNRS, University of Lorraine, Faculte des Sciences et Techniques, B.P. 70239, 54506 VANDOEUVRE-LES-NANCY, France, didier.theilliol@univ-lorraine.fr (corresponding author)

[3]State Key Laboratory of Robotics, Shenyang Institute of Automation, Chinese Academy of Sciences, 110016 Shenyang, Liaoning Province, P.R.China yangliying@sia.cn, heyuqing@sia.cn, jdhang@sia.cn

In the transition procedure, minimum and maximum flight speeds under a certain rotor-tilt angle are limited by the flight height and dynamical system [2]. So a special flight envelope with respect to rotor-tilt angle and flight speed called tilt corridor should be considered firstly for safe transition [4][5]. With the limitation of rotor corridor, conventional linear controller, PID included, designed for a special work mode with fixed rotor tilt-angle could not stabilize the vehicle in the transition procedure anymore because of the varying dynamics, so gain scheduling (GS) control structure is considered in almost all references for TRUAV transition control [3][4][6][7]. In [3], [4], and [6], two sets of controllers are designed for the helicopter mode and airplane mode respectively, and the control variables from these two controllers are mixed by a controller weight concerned with the rotor-tilt angle in the transition procedure. And in [7], divide and conquer GS method is used with the idea of "direct switch" between a set of linear controllers for different TRUAV linear models.

However, above GS control structures do not take the transition dynamics into consideration or cause unnecessary state jumps because of "direct switch" between controllers, so the stability in the transition procedure could not be ensured well, and their focus is the stability of final flight status in the helicopter mode or airplane mode. To fuse more TRUAV dynamics with different rotor-tilt angles into controller design smoothly, multi-model adaptive control (MMAC) is introduced here. MMAC is with the ability of recognizing which environment is currently in existence and servicing it appropriately [8]. It is usually accompanied by multi-model adaptive estimator (MMAE) for mode probability that has been widely used for tracking manoeuvring targets [9] and fault detection and diagnosis [10], and its control effectiveness has been verified by RUAV high-speed control [11] and some other nonlinear controlled plant [12][13].

In this paper, the main contribution is to propose a MMAC method for the transition control of a quad-TRUAV within the limitation of tilt corridor. Based on the a nonlinear model of TRUAV, tilt corridor is obtained by trim calculations [4]. Then, with a set of linear reference models, mode probabilities are estimated by solving a quadratic programming problem. In case of low variance measurement noise, method based on input-output(IO) plant model rather than Kalman filter (KF) is used here with the idea of moving-horizon, which is inspired by [13]. The MMAC controller could be constructed according to mode probabilities, and its control performance in the transition procedure is closely related to the choice of initial linear reference models and

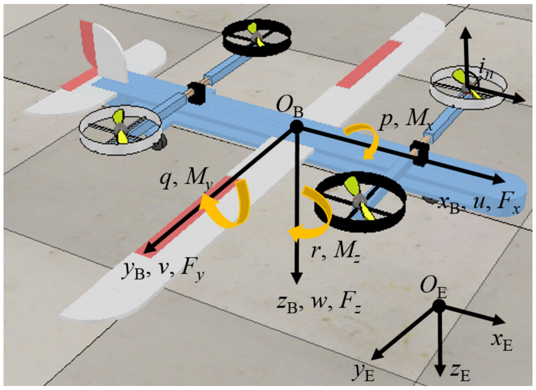

Fig. 1. TRUAV structure and coordinate system

corresponding linear controllers.

Remaining parts of this paper are organized as follows. Section II introduces the modeling of the quad-TRUAV and the calculation procedure of tilt corridor; main idea of MMAC for TRUAV transition control is formulated in Section III; simulation results and comparisons with GS method are shown in Section IV; Section V ends the paper with conclusion.

II. TRUAV Model and Tilt Corridor

A. Modeling of TRUAV

The structure of quad-TRUAV and the definitions of coordinate systems are shown in Fig. 1. This TRUAV is with four tiltable rotors mounted on both sides of the fuselage in front of and behind the wings. In this way, the impact caused by the rotor downwash could be alleviated effectively, and would be ignored in this section. The definitions of body-axis coordinate system ($O_\mathrm{B}x_\mathrm{B}y_\mathrm{B}z_\mathrm{B}$) and NED coordinate system ($O_\mathrm{E}x_\mathrm{E}y_\mathrm{E}z_\mathrm{E}$) refer to [14]. Based on these coordinate systems, $\boldsymbol{V} = [u\ v\ w]^\mathrm{T}$, $[\phi\ \theta\ \psi]$, and $\boldsymbol{\omega} = [p\ q\ r]^\mathrm{T}$ are defined as velocities in body-axis coordinate system, Euler angles, and corresponding angular velocities.

In conjunction with all forces and moments, dynamics equation and kinematics equation of a rigid-body are shown as (1)$\sim$(3) [14].

$$\dot{\boldsymbol{V}} = -\boldsymbol{\omega} \times \boldsymbol{V} + \boldsymbol{F}/m + \boldsymbol{F}_\mathrm{g}/m \tag{1}$$

$$\dot{\boldsymbol{\omega}} = -\boldsymbol{I}^{-1}[\boldsymbol{M} - \boldsymbol{\omega} \times (\boldsymbol{I}\boldsymbol{\omega})] \tag{2}$$

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 1 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix} \boldsymbol{\omega} \tag{3}$$

where $\boldsymbol{I} = \mathrm{diag}\{I_{xx}\ I_{yy}\ I_{zz}\}$ is inertia matrix, $\boldsymbol{F}_\mathrm{g} = [-mg\sin\theta\ \mathrm{mg}\sin\phi\cos\theta\ \mathrm{mg}\cos\phi\sin\theta]^\mathrm{T}$, m and g represent mass and acceleration of gravity respectively. For TRUAV nonlinear model, $\boldsymbol{F}$ and $\boldsymbol{M}$ should represent the resultant forces and moments in three directions of body-axis coordinate system generated by five parts of this quad-TRUAV: rotors, wings, fuselage, vertical tail, and horizontal tail.

For modeling of rotor part, generated tension and reaction torque of every rotor that is driven by brushless direct current motor could refer to [15]. The difference is that there would be a matrix to transform these forces and moments into body-axis coordinate system as follows.

$$\boldsymbol{T}_\mathrm{R2B} = \begin{bmatrix} \sin i_n & 0 & -\cos i_n \\ 0 & 1 & 0 \\ \cos i_n & 0 & \sin i_n \end{bmatrix}$$

where i_n is defined as rotor-tilt angle as shown in Fig. 1. What's more, to represent motor dynamics, first-order inertia link is introduced as follows with the form of state-space equation [16].

$$\dot{\Omega} = 1/T_\mathrm{m}(-\Omega + U_\Omega) \tag{4}$$

where T_m is the time constant, U_Ω is the actual control variable set by controller, and Ω is the actual rotor speed. With similar formulation, time constant for rotor-tilt structure is signed by T_{i_n}, and its control variable is signed by U_{i_n}.

As for remaining aerodynamics parts, formulations of all aerodynamic forces and moments could refer to [17] and [18], and they are all with the form of multiplication of dynamic pressure, reference area, and aerodynamic coefficient which is the function of angle of attack, angle of sideslip, and control variables, such as deflections of aileron, elevator, and rudder. To further represent dynamics of these actuators, first-order inertia links with time constant T_s similar to (4) are also introduced here.

In following controller design, the longitudinal nonlinear model of TRUAV will be considered for simplification. This model is with state vector $\boldsymbol{x} = [u\ w\ q\ \theta\ i_n\ \Omega_\mathrm{F}\ \Omega_\mathrm{B}\ D_\mathrm{e}]$ and control variable vector $\boldsymbol{u} = [U_{i_n}\ U_{\Omega_\mathrm{F}}\ U_{\Omega_\mathrm{B}}\ U_{D_\mathrm{e}}]$, where Ω_F and Ω_B are forward and backward rotor speeds, D_e is deflection of elevator, and corresponding actual control variables are shown in vector u.

B. Tilt Corridor

In some references, tilt corridor is measured by lots of flight experiences [2]. However, considering the danger of experience without some evidences, many references adopt an engineering method based on the trim calculations of a nonlinear dynamics model to obtain it [4] [5]. The main idea is that, for one fixed rotor-tilt angle, it is possible to trim the nonlinear dynamics model and obtain researchable maximum and minimum flight speeds based on suitable initial conditions and a range of pitch angle. When more enough rotor-tilt angles are considered with above idea, the tilt corridor can be obtained. In general, tilt corridor represents the essential dynamics characteristics of TRUAV, and serves as the first-degreed limitation in the transition procedure.

After obtaining a tilt corridor, a reasonable transition curve is needed within it to indicate the law of rotor-tilt angle variation with flight speed. It might be a hard work to ensure the TRUAV platform follow this curve, but it provides an important reference especially in initial simulation tests.

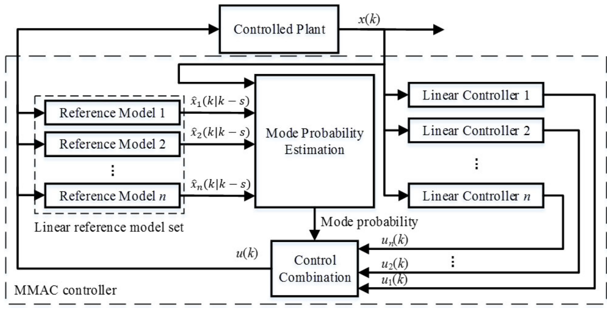

Fig. 2. MMAC structure

III. MMAC Based on IO Plant Model

A. MMAC Formulation

With generality, the nonlinear longitudinal model in Section II could be expressed as $\dot{\boldsymbol{x}} = f(\boldsymbol{x},\ \boldsymbol{u})$. Time t is omitted in the expression of state and control variable vectors, and the following is same. For MMAC controller design, this nonlinear system would be linearized in a set of equilibrium points $(\boldsymbol{x}_{\text{o}i},\ \boldsymbol{u}_{\text{o}i})$ $(i = 1,\ 2,\ ...\ n)$. Resulting linear systems could be regarded as a linear reference model set, and every linear system is a reference model as follows.

$$\dot{\boldsymbol{x}}_i = \boldsymbol{A}_i \boldsymbol{x}_i + \boldsymbol{B}\boldsymbol{u}_i \tag{5}$$

where $\boldsymbol{x}_i = \boldsymbol{x} - \boldsymbol{x}_{\text{o}i}$ and $\boldsymbol{u}_i = \boldsymbol{u} - \boldsymbol{u}_{\text{o}i}$. With equilibrium points near transition curve, reference models in a linear reference model set could represent partial dynamics of helicopter mode, airplane mode, and transition procedure respectively. For the reason of considering the dynamics of actuators, the input matrix of every linear system is constant.

With above linear reference model set, MMAC structure in this paper is shown as Fig. 2. In this structure, reference models would approximate controlled plant with the help of mode probabilities that are estimated by method based on IO plant model. Obviously, the choice of reference models, in other word, the choice of linear reference model set is very important when the controlled plant is a nonlinear system or actual platform. By the way, many MMAEs, such as interacting multi-model (IMM) estimator [10][11], are also based on similar structure by replacing linear controllers by typical estimators, such as KF.

With the linear controller signed by the form of state feedback matrix $\boldsymbol{K}_i$ and mode probabilities signed by vector $\boldsymbol{W} = [w_1\ w_2\ ...\ w_n]$ with unit sum, weighted sum of linear controllers is formulated as the MMAC controller,

$$\boldsymbol{K} = \sum_{i=1}^{n} w_i \boldsymbol{K}_i \tag{6}$$

With these definitions, equation

$$\dot{\boldsymbol{x}} = \sum_{i=1}^{n} w_i [\boldsymbol{A}_i(\boldsymbol{x} - \boldsymbol{x}_{\text{o}i}) + \boldsymbol{B}(\boldsymbol{u} - \boldsymbol{u}_{\text{o}i})] \tag{7}$$

would own similar dynamics with $\dot{\boldsymbol{x}} = f(\boldsymbol{x},\ \boldsymbol{u})$ and approximate it, which will be shown in next subsection accompanying the derivation of mode probability. With control variable vector as follows,

$$\boldsymbol{u} = \textstyle\sum_{i=1}^{n} w_i [\boldsymbol{K}_i(\boldsymbol{x} - \boldsymbol{x}_{\text{o}i}) + \boldsymbol{u}_{\text{o}i}]$$

the closed-loop multi-model system is considered with following form.

$$\dot{\boldsymbol{x}} = \sum_{i=1}^{n} w_i (\boldsymbol{A}_i - \boldsymbol{B}\boldsymbol{K}_i)(\boldsymbol{x} - \boldsymbol{x}_{\text{o}}) \tag{8}$$

where

$$\boldsymbol{x}_{\text{o}} = [\sum_{i=1}^{n} w_i (\boldsymbol{A}_i - \boldsymbol{B}\boldsymbol{K}_i)]^{-1} [\sum_{i=1}^{n} w_i (\boldsymbol{A}_i - \boldsymbol{B}\boldsymbol{K}_i)\boldsymbol{x}_{\text{o}i}] \tag{9}$$

Obviously, with fixed mode probabilities, if there is a common Lyapunov matrix $\boldsymbol{P}$ that can ensure $\boldsymbol{P} > 0$ and

$$\textstyle\sum_{i=1}^{n} w_i (\boldsymbol{A}_i - \boldsymbol{B}\boldsymbol{K}_i)^{\text{T}} \boldsymbol{P} + \boldsymbol{P} \sum_{i=1}^{n} w_i (\boldsymbol{A}_i - \boldsymbol{B}\boldsymbol{K}_i) \leq 0,$$

closed-loop system (8) would be stable. These conditions are actually not serious restrictions [19], and have been applied in some references for linear parameter-varying (LPV) system [20]. In this way, the eigenvalues of closed-loop system matrix $\sum_{i=1}^{n} w_i (\boldsymbol{A}_i - \boldsymbol{B}\boldsymbol{K}_i)$ are all with negative real parts, and its inverse is always existent.

B. Mode Probability

Mode probability is actually the degree of approximation between dynamics of current controlled plant and every linear reference model. To quantify this degree of approximation, the IO response of every plant model would be considered. Let $\boldsymbol{y}(k)$ represent the output state of controlled plant in kth instant, $\boldsymbol{x}_{\text{d}i}(k)$ and $\boldsymbol{u}_{\text{d}i}(k)$ represent the discrete state and control variable vectors of ith linear reference model, $\boldsymbol{A}_{\text{d}i}$ and $\boldsymbol{B}_{\text{d}i}$ represent discrete system matrix of system (5).

Without amendment from system output $\boldsymbol{y}(k)$, there would be estimation residual between controller plant and every reference model as follow.

$$\boldsymbol{r}_i(k|k-1) = \boldsymbol{y}(k) - \boldsymbol{C}_{\text{d}} \cdot \hat{\boldsymbol{x}}_{\text{d}i}(k|k-1) \tag{10}$$

where $\boldsymbol{C}_{\text{d}}$ is the output matrix of a linear state space equation, and $\hat{\boldsymbol{x}}_{\text{d}i}(k|k-1) = \boldsymbol{A}_{\text{d}i} \cdot \boldsymbol{x}_{\text{d}i}(k-1) + \boldsymbol{B}_{\text{d}i} \cdot \boldsymbol{u}_{\text{d}i}(k-1)$ that represents 1-step re-estimation of state by discrete linear model of (5). To further emphasize the difference between every reference model and actual plant, s-step re-estimation $\hat{\boldsymbol{x}}_{\text{d}i}(k|k-s)$ is considered as follows.

$$\boldsymbol{r}_i(k|k-s) = \boldsymbol{y}(k) - \boldsymbol{C}_{\text{d}} \cdot \hat{\boldsymbol{x}}_{\text{d}i}(k|k-s)$$

where

$$\hat{\boldsymbol{x}}_{\text{d}i}(k|k-s) = \boldsymbol{A}_{\text{d}i}^{s} \cdot \boldsymbol{x}_{\text{d}i}(k-s) + \begin{bmatrix} \boldsymbol{A}_{\text{d}i}^{s-1}\boldsymbol{B}_{\text{d}i} & \boldsymbol{A}_{\text{d}i}^{s-2}\boldsymbol{B}_{\text{d}i} & ... & \boldsymbol{B}_{\text{d}i} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_{\text{d}i}(k-s) \\ \boldsymbol{u}_{\text{d}i}(k-s+1) \\ \vdots \\ \boldsymbol{u}_{\text{d}i}(k-1) \end{bmatrix}$$

and the superscript of matrix means exponentiation. Then, weighted sum of above residuals could be obtained according to mode probabilities.

$$\boldsymbol{r}(k|k-s) = [\boldsymbol{r}_1(k|k-s)\ \boldsymbol{r}_2(k|k-s)\ ...\ \boldsymbol{r}_n(k|k-s)] \cdot [w_1\ w_2\ ...\ w_n]^{\mathrm{T}}$$

Moreover, define a quadratic cost function as following equation with the idea of moving-horizon.

$$J(w_1, w_2, ..., w_n) = [\textstyle\sum_{j=0}^{m} \boldsymbol{r}(k-j|k-j-s)]^{\mathrm{T}} \cdot \boldsymbol{R} \cdot [\textstyle\sum_{j=0}^{m} \boldsymbol{r}(k-j|k-j-s)] \tag{11}$$

where $\boldsymbol{R}$ is a weight matrix and usually diagonal. In (11), m s-step re-estimations from instant $k-m$ to k are applied for residuals to abandon some historical data of no value. Obviously, minimizing above cost function by mode probabilities with suitable performance parameters s, m, and $\boldsymbol{R}$, (7) could approximate the dynamics of initial nonlinear system $\dot{\boldsymbol{x}} = f(\boldsymbol{x},\ \boldsymbol{u})$.

From above formulation, the estimation of mode probabilities could be summarized by following quadratic programming problem with equality and inequality constraints.

$$\begin{aligned} &\min\ J(w_1, w_2, ..., w_n) \\ &\text{s.t.} \\ &\quad w_1 + w_2 + ... + w_n = 1, \\ &\quad 0 \le w_i \le 1,\ i = 1,\ 2,\ ...\ n \end{aligned} \tag{12}$$

Above problem could be solved by many current algorithms, such as active set method, Lemke method, and so on, and could be implemented in many platforms easily. In following simulations, active set method would be used, in which inequality constraints in every feasible point are transformed into equality constraints. In this way, quadratic programming problem with equality constraints could be formulated as follows

$$\begin{aligned} &\min\ J(\boldsymbol{W}^{(m)}) \\ &\text{s.t.} \\ &\quad \boldsymbol{a}^d \cdot \boldsymbol{W}^{(m)\mathrm{T}} = b^d, \quad d \in D^{(m)} \end{aligned}$$

where superscript (m) represents number of iterations, D is active constraint index set, row vector $\boldsymbol{a}$ and scalar b establish equality constraints. This problem can be solved easily for better feasible point by constructing Lagrange function $L(\boldsymbol{W}^{(m)}) = J(\boldsymbol{W}^{(m)}) + \lambda(\boldsymbol{a}^d \cdot \boldsymbol{W}^{(m)\mathrm{T}} - b^d)$ where λ is Lagrange gain [21].

Note that, with $m = k-1$ and $s = 1$ that mean all historical data is reserved, above method is same as corresponding method in [13]. In this reference, to avoid large workload, recursive least square algorithm has to be applied.

C. Linear Controller Design

Considering the ultimate goal of MMAC controller is the transition control of TRUAV, the linear controller in (6) should be with the ability of tracking reference velocities. As for a typical linear system shown in (5), introduce new states $\boldsymbol{e}_i$ as the integrals of tracking errors [22] as follows.

$$\boldsymbol{e}_i = \int(\boldsymbol{z}_i - \boldsymbol{ref})$$

where $\boldsymbol{z}_i = \boldsymbol{C}\boldsymbol{x}_i$ that represents the trackable states, and $\boldsymbol{ref}$ is reference value vector.

By this definition, a closed-loop augmented system with state feedback matrix $\boldsymbol{K}_i = [\boldsymbol{K}_{\mathrm{LQ}i}\ \boldsymbol{K}_{\mathrm{I}i}]$ can be represented as follows.

$$\begin{aligned} \begin{bmatrix} \dot{\boldsymbol{x}}_i \\ \dot{\boldsymbol{e}}_i \end{bmatrix} &= \begin{bmatrix} \boldsymbol{A}_i - \boldsymbol{B}\boldsymbol{K}_{\mathrm{LQ}i} & -\boldsymbol{B}\boldsymbol{K}_{\mathrm{I}i} \\ -\boldsymbol{C} & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_i \\ \boldsymbol{e}_i \end{bmatrix} + \begin{bmatrix} 0 \\ \boldsymbol{I} \end{bmatrix} \boldsymbol{ref} \\ &\triangleq \boldsymbol{A}_{\mathrm{cl}i}\boldsymbol{x}_{\mathrm{a}i} + \boldsymbol{E}\boldsymbol{ref} \end{aligned}$$

where $\boldsymbol{I}$ is a unit matrix. If the eigenvalues of matrix $\boldsymbol{A}_{\mathrm{cl}i}$ are all with negative real parts, $\dot{\boldsymbol{e}} = 0$ and $\boldsymbol{z} = \boldsymbol{ref}$ when $t \to +\infty$. In most cases, a linear quadratic cost function

$$J_i(u_i) = \int_0^{+\infty} (\boldsymbol{x}_{\mathrm{a}i}^{\mathrm{T}} \cdot \boldsymbol{Q}_{\mathrm{LQI}i} \cdot \boldsymbol{x}_{\mathrm{a}i} + \boldsymbol{u}_i^{\mathrm{T}} \cdot \boldsymbol{R}_{\mathrm{LQI}i} \cdot \boldsymbol{u}_i)\mathrm{d}t \tag{13}$$

with $\boldsymbol{u}_i = \boldsymbol{K}_i\boldsymbol{x}_{\mathrm{a}i}$ is always considered by Matlab Control System Toolbox, and a linear quadratic integrator (LQI) controller could be obtained [23].

Note that, with the controller structure described in this subsection, $\boldsymbol{K}_i$ in (9) should be substituted by $\boldsymbol{K}_{\mathrm{LQ}i}$ to consider a stabilization problem for equilibrium point $\boldsymbol{x}_{\mathrm{o}}$. And to track reference values, MMAC controller is still the weighted sum of LQI controllers.

IV. Simulation Results

For nonlinear modeling of quad-TRUAV in Section II, the mass is $m = 2$kg, inertia is $I_{yy} = 0.0675\mathrm{kgm}^2$, rotor radius is $r = 0.12$m, positions of rotor, wing, fuselage, vertical tail, and horizontal tail in $x-$direction and $z-$direction are $(0.28\mathrm{m},\ -0.05\mathrm{m})$, $(0\mathrm{m},\ -0.015\mathrm{m})$, $(0.012\mathrm{m},\ 0.02\mathrm{m})$, $(-0.515\mathrm{m},\ 0.035\mathrm{m})$, and $(-0.495\mathrm{m},\ -0.04\mathrm{m})$ respectively, and time constants are set as $T_{\mathrm{m}} = 0.02$, $T_{i_n} = 0.07$, and $T_{\mathrm{s}} = 0.05$ by experience. For more aerodynamic parameters, refer to [15] and [17].

A. Tilt Corridor Calculation

With the method given in Section II, tilt corridor could be obtained by trim calculations. In this process, pitch angle is set within the range of $[-0.4363,\ 0.4363]$. Calculation results are listed in Table I, and tilt corridor curves are shown in Fig. 3. With the limitation of this tilt corridor, a transition curve could be described as follows, which is chosen near the middle of this corridor and easy to be fitted.

$$i_n = \begin{cases} 1.5708 & u_{\mathrm{F}} < 8 \\ -0.007u_{\mathrm{F}}^2 + 0.1117u_{\mathrm{F}} + 1.124 & 8 \le u_{\mathrm{F}} < 18 \\ -0.2909u_{\mathrm{F}} + 6.109 & 18 \le u_{\mathrm{F}} < 21 \\ 0 & u_{\mathrm{F}} \ge 21 \end{cases} \tag{14}$$

where $u_{\mathrm{F}} = \sqrt{u^2 + v^2}$ represents flight speed. In the transition procedure, U_{i_n} is set by (14) according to current flight speed, and MMAC controller is designed for other control variables.

B. Choice of Linear Reference Model Set

For reasonable mode probabilities and representing dynamics of controlled plant well, linear reference model set plays an important role. However, optimal selection of reference set for MMAC or MMAE is still "largely in the kingdom of black magic" [24]. In this way, MMAC

TABLE I
TRIM RESULTS FOR TILT CORRIDOR

Rotor-tilt angle (rad)	1.5708	1.3963	1.2217	1.0472	0.8727	0.6981	0.5236	0.3491	0.1745	0
Minimum flight speed (m/s)	-5.0147	1.4763	4.0132	12.6998	15.5969	17.4982	18.4970	19.0980	19.8016	19.8008
Maximum flight speed (m/s)	10.0049	14.9965	18.0293	19.0006	19.9714	20.3001	20.9986	21.5139	21.9972	22.0018

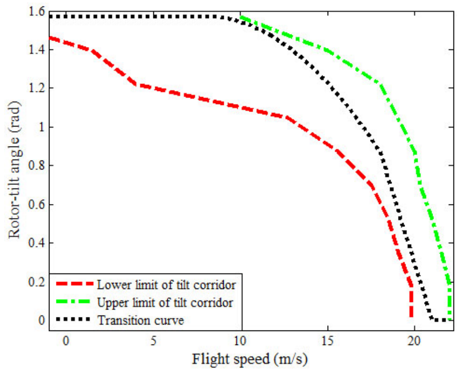

Fig. 3. Tilt corridor and transition curve

controllers (6) with some linear reference model sets listed in Table II will be considered and compared, and these sets are all formed by some reference models and numbered as Set 1∼4. Nonlinear system gotten in Section II would be linearized with corresponding rotor-tilt angles and flight speeds, which are all chosen near transition curve (14) in previous subsection. To illustrate difference between these four sets, 1-step re-estimations are considered, and weighted sums of estimation residuals (10) of four states, which could be formulated as $\boldsymbol{r}(k|k-1) = [\boldsymbol{r}_1(k|k-1)\ \boldsymbol{r}_2(k|k-1)\ \dots\ \boldsymbol{r}_n(k|k-1)] \cdot [w_1\ w_2\ \dots\ w_n]^{\mathrm{T}}$, are shown in Fig. 4. Their mean values are shown in Fig. 5 by hists.

Based on these comparisons, Set 4 is chosen for following research which is with smaller estimation residuals than other sets. Note that, the residual curves of Set 3 are with severe oscillations in the second half of the simulation time. That is due to that Reference Model 5 and Reference Model 6 in this set are similar with each other, so corresponding mode probabilities would also with some oscillations in this period which may eventually affect the MMAC controller. That is to say, the number of linear reference models and their "uniformity" for representing dynamics should all be taken into account when linear reference model set is determined.

C. MMAC controller

With nonlinear model as the controlled plant and linear reference model Set 4 shown in Table II, MMAC controller could be designed by solving quadratic programming problem (12) for mode probabilities and minimizing cost function (13) for linear controllers.

When reference value of forward velocity is set as 21 m/s, according to transition curve (14), TRUAV would transit into airplane mode in stable status from helicopter mode with

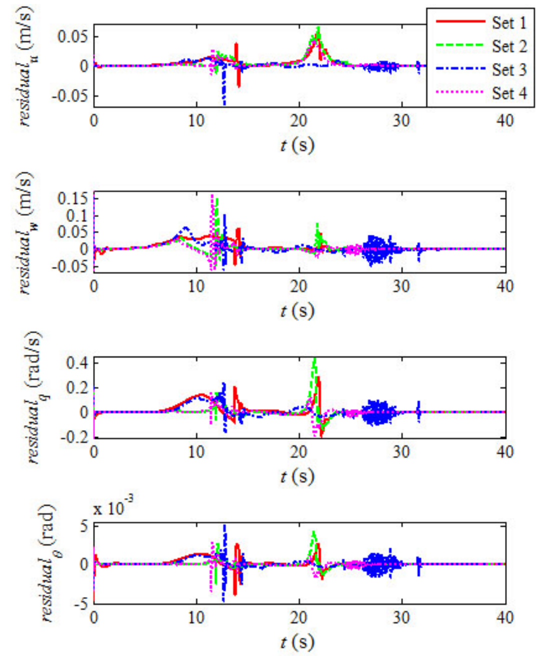

Fig. 4. Estimation residuals of states

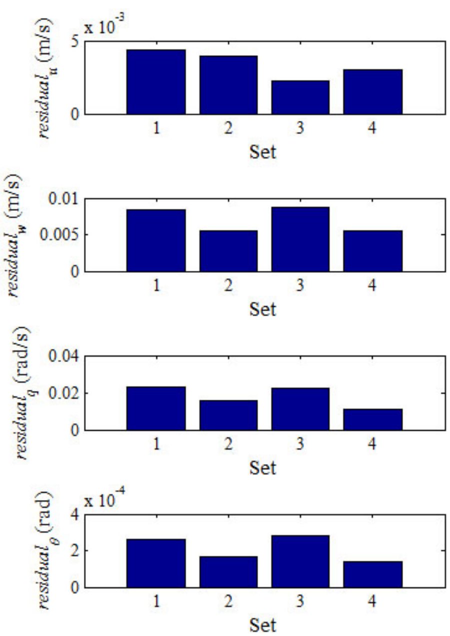

Fig. 5. Mean values of estimation residuals

TABLE II
SOME REFERENCE SETS FOR MMAC

	Reference Model 1		Reference Model 2		Reference Model 3		Reference Model 4		Reference Model 5		Reference Model 6	
	Rotor-tilt angle (rad)	Flight speed (m/s)	Rotor-tilt angle (rad)	Flight speed (m/s)	Rotor-tilt angle (rad)	Flight speed (m/s)	Rotor-tilt angle (rad)	Flight speed (m/s)	Rotor-tilt angle (rad)	Flight speed (m/s)	Rotor-tilt angle (rad)	Flight speed (m/s)
Set 1	1.5708	0	1.2217	15.1	0.6981	18.6	0	21	-	-	-	-
Set 2	1.5708	0	1.5708	8	1.2217	15.1	0.6981	18.6	0	21	-	-
Set 3	1.5708	0	1.2217	15.1	0.8727	18	0.5236	19.2	0.1745	20.4	0	21
Set 4	1.5708	0	1.5708	8	1.2217	15.1	0.8727	18	0.3491	19.8	0	21

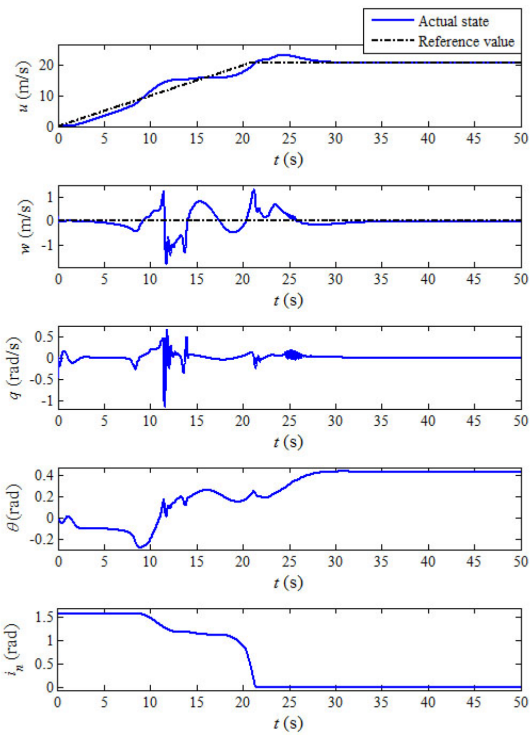

Fig. 6. State and rotor-tilt angle form helicopter mode to airplane mode

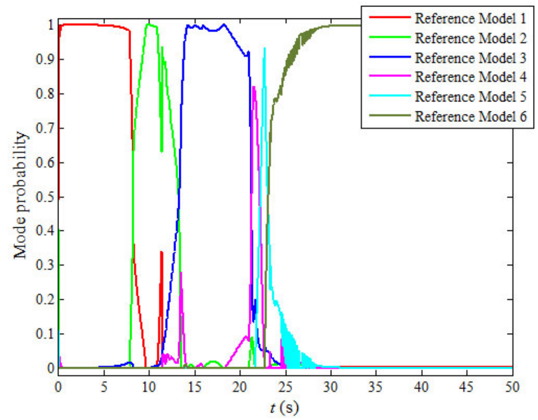

Fig. 7. Mode probabilities form helicopter mode to airplane mode

zero initial condition. State and rotor-tilt angle curves are shown in Fig. 6, and the ramp reference value is to ensure the forward acceleration process is slow enough, because the control variable of rotor-tilt angle is set according to flight speed. These state curves show the effectiveness of MMAC controller. Further considering mode probability shown in Fig. 7, in the transition procedure, different linear controllers could ensure the stability of controlled plant by considering corresponding dynamics.

Typical GS TRUV controller is also designed according to [3] for comparison. In this control structure, LQI controllers for helicopter mode and airplane mode are all formed, and helicopter controller weight is equal to $2 \cdot i_n/\pi$ for GS. With 16 m/s reference value of forward speed, the stable status would be located in transition procedure according to (14), and $i_n = 1.124$ in the end. Under this condition, GS controller could not ensure the stability of current TRUAV as shown in Fig. 8. However, by taking corresponding dynamics into account, MMAC controller could still work well. From

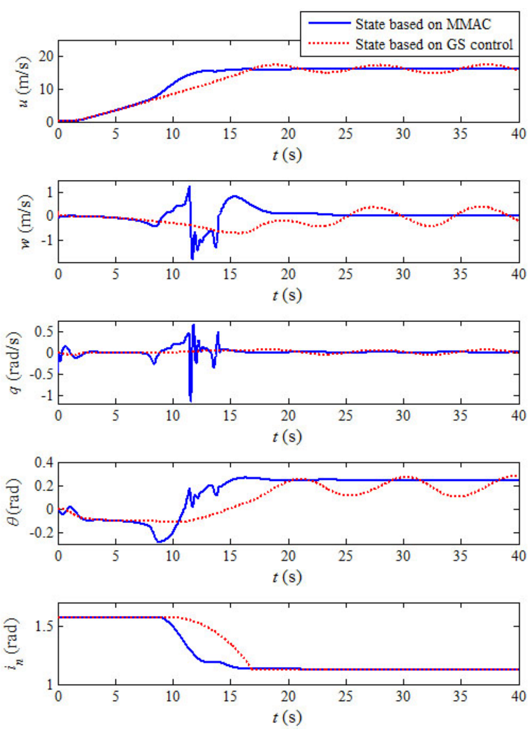

Fig. 8. State and rotor-tilt angle in transition procedure

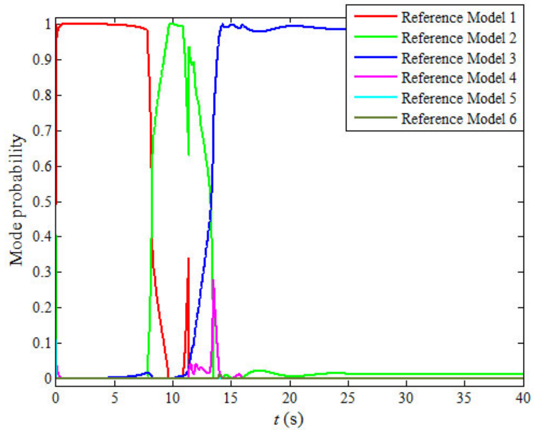

Fig. 9. Mode probabilities in transition procedure

these curves, the control effectiveness of MMAC controller is shown further. And mode probabilities shown in Fig. 9 make clear that the linear controller of Reference Model 3 is the "main force" of final MMAC controller.

Although with better stability in the transition procedure, the controlled plant with MMAC controller is with much poorer transient responses compared with GS method actually. Linear controllers applied in above simulations are designed respectively, and added with weights as MMAC controller. With varying mode probabilities, $\boldsymbol{x}_0$ shown in (9) is also time varying. Further considering different control performances between $\boldsymbol{K}_1 \sim \boldsymbol{K}_6$, the poor transient response is inevitable. To optimize it, global stability analysis with MMAC method is critical. Polytopic LPV controller design may be considered here, but its control performance facing nonlinear plant still needs to be verified.

V. Conclusion

This paper proposes a new method for TRUAV transition control based on multi-model adaptive method facing nonlinear controlled plant within the limitation of tilt corridor. By solving a quadratic programming problem for mode probabilities, MMAC controller is formulated by weighted sum of linear controllers which could ensure reference value tracking. Simulation results and comparisons with GS method show its control effectiveness especially in transition procedure. In the future work, the global stability of controlled plant should be considered further, and some new optimization tools for this purpose will be introduced.

References

[1] A. S. Saeed, A. B. Younes, S. Islam, J. Dias, L. Seneviratne, and G. Cai, "A review on the platform design, dynamic modeling and control of hybrid UAVs," in *IEEE International Conference on Unmanned Aircraft Systems (ICUAS)*. IEEE, 2015, pp. 806-815.

[2] Z. Liu, Y. He, L. Yang, and J. Han, "Control techniques of tilt rotor unmanned aerial vehicle systems: A review," *Chinese Journal of Aeronautics*, 2016, DOI: 10.1016/j.cja.2016.11.001, in press.

[3] K. Muraoka, N. Okada, and D. Kubo, "Quad iilt wing vTOL UAV: aerodynamic characteristics and prototype flight test," in *AIAA Infotech@Aerospace Conference*. AIAA, 2009, pp. 1-10.

[4] K. Muraoka, N. Okada, D. Kubo, and M. Sato, "Transition flight of quad tilt wing VTOL UAV," in *28th congress of the international council of the aeronautical sciences*. ICAS, 2012, pp. 1-10.

[5] Y. Song, and H. Wang, "Design of flight control system for a small unmanned tilt rotor aircraft," *Chinese Journal of Aeronautics*, vol. 22, no. 3, pp. 250-256, 2009.

[6] S. Verlling, and J. Zilly, "Modeling and control of a VTOL glider," bachelor dissertation, Swiss Federal Institute of Technology Zurich, 2013.

[7] W. Zhao, and C. Underwood, "Robust transition control of a Martian coaxial tiltrotor aerobot," *Acta Astronautica*, vol. 99, pp. 111-129, 2014.

[8] K. S. Narendra, and J. Balakrishnan, "Adaptive control using multiple models," *IEEE Transactions on Automatic Control*, vol. 42, no. 2, pp. 171-187, 1997.

[9] A. Munir, and D. P. Atherton, "Adaptive interacting multiple model algorithm for tracking a manoeuvring target," *Radar, Sonar and Navigation, IEE Proceedings*, vol. 142, no. 1, pp. 11-17, 1995.

[10] J. P. Georges, D. Theilliol, J. C. Ponsart, and C. Aubrun, "Networked control system with intermittent observations: FDI/FTC design based on interacting multiple model approah," *Mediterranean Conference on Control & Automation*, IEEE, 2009, pp. 528-533.

[11] H.H. Afshari, and D. Al-Ani, "A new adaptive control scheme based on the interacting multiple model (IMM) estimation," *Journal of Mechanical Science and Technology*, vol. 30, no. 6, pp. 2759-2767, 2016.

[12] M. Kuure-Kinsey, and B. W. Bequette, "Multiple model predictive control: a state estimation based approach," in *2007 American Control Conference*, IEEE, 2007, pp. 3739-3744.

[13] T. Slavov, "Algorithm for multiple model adaptive control based on input-output plant model," *Cybernetics & Information Technologies*, vol. 12, no. 1, pp. 13-33, 2013.

[14] G. Cai, B. Chen, and T. H. Lee, *Unmanned rotorcraft system*. Springer, 2011, pp. 23-34, pp. 97-135.

[15] P. Pounds, R. Mahony, and P. Corke, "Modelling and control of a large quadrotor robot," *Control Engineering Practice*, vol. 18, no. 7, pp. 691-699, 2010.

[16] C. Papachristos, K. Alexis, and A. Tzes, "Model predictive hovering-translation control of an unmanned Tri-TiltRotor," in *IEEE International Conference on Robotics & Automation*, IEEE, 2013, pp. 5425-5432.

[17] K. Kleinhesselink, "Stability and control modeling of tiltrotor aircraft," master dissertation, University of Maryland, 2007.

[18] A. Vuruskan, B. Yuksek, U. Ozdemir, and A. Yukselen, "Dynamic modeling of a fixed-wing VTOL UAV," in *International Conference on Unmanned Aircraft Systems*, IEEE, 2014, pp. 483-491.

[19] Y. Lu, and Y. Arkun, "Quasi-Min-Max MPC algorithms for LPV systems," *Automatica*, vol. 36, pp. 527-540, 2000.

[20] P. Apkarian, P. Gahinet, and G. Becker, "Self-scheduled H∞ control of linear parameter-varying systems," *Automatica*, vol. 31, pp. 1251-1261, 1995.

[21] B. Chen, *Optimization Theory and Algorithm*. Tsinghua University Press, 2005, pp. 415-431.

[22] P. C. Young, and J. C. Willems, "An approach to the linear multivariable servomechanism problem," *International Journal of Control*, vol. 15, no. 5, pp. 961-979, 1972.

[23] MathWork, *Control System Toolbox User's Guide*. The MathWorks, Inc., 2007, pp. 161-172.

[24] X. Li, "Optimal selection of estimate for multiple-model estimation with uncertain parameters," *IEEE Transactions on Aerospace & Electronic Systems*, vol. 34, no. 2, pp. 653-657.