Short-Time Stability of Proportional Navigation Guidance Loop

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Stability characteristics of proportional navigation (PN) guidance are analyzed by using the short-time stability criterion which is extended here to accommodate time-varying state weights and time-varying bounds of the state norm. As short-time stability is defined over a specified time interval, its application to the stability analysis of a homing guidance loop that operates up to a finite time gives more accurate results than previous studies. Furthermore, within the framework of short-time stability, zero effort miss and acceleration command, which are the most important variables determining guidance performance, can be directly related with guidance loop stability. An application to a PN guidance loop with a 1st-order missile/autopilot time lag shows that the stability condition based on short-time stability is less conservative than the previous results based on hyperstability and Popov stability.

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I. INTRODUCTION

Proportional navigation guidance (PNG) is widely used for terminal homing guidance because of its simplicity, effectiveness, and ease of implementation. However, the nonlinear time-varying characteristic of the governing equation makes it difficult to analyze the stability of PNG in analytic terms.

This difficulty may be reduced by assuming an ideal missile that behaves as a point mass. Previous works based on the ideal missile model are as follows. A qualitative study is performed by Murtaugh and Criel [1] on pure PN (PPN) for which missile acceleration is commanded normal to the instantaneous line of sight (LOS). Guelman derived a closed form solution of PPN by assuming constant closing velocity [2]. For true PN (TPN), for which missile acceleration is normal to the missile velocity vector, Guelman [3–5] provided conditions for reaching the target regardless of initial condition and obtained the bounds of missile acceleration for intercepting a constant-acceleration target. Mahapatra and Shukla [6, 7] derived an approximated closed-form solution of the nonlinear differential equation of TPN for laterally maneuvering target with a constant acceleration. Ha, et al. [8] proved the intercept performance of PNG against a randomly maneuvering target.

Previous studies show that PNG enables an ideal missile to intercept the target when a suitable guidance gain is adopted. Moreover, the intercept performance of PNG is achieved for any initial heading error and target maneuver with the guidance command remaining bounded through the engagement.

Unfortunately, for the PNG loop with a non-ideal missile dynamics, no closed-form solution has been reported. However, it is still possible to investigate the effects of guidance gain and missile dynamics on the stability of guidance loop. One of the benefits of the stability analysis is that guidance loop stability can be predicted without obtaining a closed-form solution.

Even though there does not exist a widely accepted definition of guidance loop stability, instability of guidance loop can be described as follows: The magnitudes of the acceleration command and the state variables associated with the guidance loop grow infinitely as time to go (t_{go}) approaches zero. Guidance loop instability may degrade intercept accuracy because of the saturation of missile acceleration.

The simplest way to study guidance loop stability is to apply Routh–Hurwitz criterion to the linearized guidance loop as in [9]. Although the Routh–Hurwitz stability criterion is applicable to a slow time-varying system under the assumption of frozen time, its application to the analysis of PNG is not appropriate since a PNG loop has fast time-varying element such as range.

A linearized PNG loop consists of a linear time invariant block (missile/autopilot dynamics) and a time-varying block (PNG kinematics). This kind of structure can be analyzed by using absolute stability. Absolute stability is stated in terms of Popov criterion and the circle criterion which guarantee asymptotic stability of an equilibrium point.

Guelman [10] suggested the concept of finite-time absolute stability in which stability is defined over a finite time period. Conditions of finite-time absolute stability obtained by extending the Popov stability theorem is summarized as the following two conditions: 1) $\Lambda > 2$, and 2) $t_{go} > \sup_\omega\{-\Lambda \mathrm{Re}[G(j\omega)/j\omega]\}$ where Λ is the effective guidance gain and $G(s)$ is the missile/autopilot transfer function. In the analysis of finite-time absolute stability, it is assumed that PNG starts from the infinite time to go. This assumption may not be practical in real engagement.

Tanaka and Eguchi [11, 12] defined hyperstable range as a relative range between the target and missile satisfying the hyperstability condition and derived the hyperstable range as a function of frequency response of the time invariant block. Robustness with respect to the external disturbance is also considered. Hyperstable range, denoted by HR, is computed as $\mathrm{HR} = \sup_\omega\{-\mathrm{Re}[G_D(j\omega)]\}$ for a given $G_D(s) \in \{\text{positive real}\}$ which is the transfer function from the LOS angle (input) to the missile position (output). $G_D(s)$ includes dynamics of seeker, autopilot, airframe, guidance logic, and other uncertainties. In [13], it is proved that the hyperstability condition also satisfies the condition of input-output stability.

It can be shown that the hyperstability condition is identical to the finite-time absolute stability condition when the guidance loop uncertainties are neglected and the closing velocity is assumed to be a constant. Hence, in this work, the stability conditions based on hyperstability and finite-time absolute stability are referred to as Popov stability conditions. Since all the Popov stability conditions are defined over an infinite-horizon time domain, these criteria are not suitable for the stability analysis of a homing guidance loop which is only defined over a finite time interval.

It may be noteworthy that PNG with sliding mode control is claimed to be partially robust to parameter variations of the homing guidance loop as reported in [14]. However, the sliding surface suggested in [14] does not consider the missile attitude dynamics so that the guidance loop stability is not guaranteed near the intercept point. By this reason, the sliding mode approach to PN suffers from the same stability problems as the conventional PN guidance law.

The stability of PNG is analyzed here in the sense of short-time stability and the behavior of state trajectories over a given time interval is studied. The short-time stability of missile/autopilot dynamics, guidance command, and zero effort miss is defined and sufficient conditions for short-time stability are derived. Stability analysis of PNG is then conducted to show that the stability conditions based on short-time stability is less conservative than the Popov stability conditions.

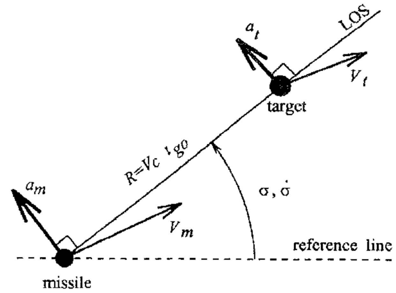

Fig. 1. Missile and target engagement geometry.

Augmented proportional navigation (APN) can be utilized to cope with the target maneuver [15]. As long as the guidance law design is based on the ideal missile model, APN will show the same divergence characteristics as those of PN. Hence, APN is not discussed in this work.

This paper is organized as follows. A mathematical model of PNG loop is described in Section II. The short-time stability theorem is extended in Section III to deal with more general cases. Section IV contains an application of the short-time stability theorem to PNG and Section V presents simulation results to verify the stability analysis of this work. Final comments are made in Section VI.

II. GUIDANCE MODEL

A homing guidance loop includes the dynamics of seeker, guidance computer, and missile dynamics. In this work, the seeker is simply modeled as a pure differentiator which generates the LOS rate of the target without delay. The guidance computer is modeled as an algebraic function that computes the acceleration command by using the information obtained from the seeker. Missile/autopilot dynamics that include vehicle dynamics, autopilot, and actuator is modeled as a simple first-order or second-order system.

Consider a two-dimensional missile/target engagement as shown in Fig. 1, where R is the relative range between target and missile, σ is the LOS angle, and a_m and a_t represent the missile and target acceleration normal to the LOS line, respectively.

The kinematic relation between the target and missile motion is described by a nonlinear equation

$$R\ddot{\sigma} + 2\dot{R}\dot{\sigma} = -a_m + a_t. \qquad (1)$$

If the missile approaches the target with a constant closing velocity of V_c, the range R is simply expressed as $R = V_c t_{go}$ where t_{go} denotes the time to go for the missile to intercept the target. Then, (1) becomes a linear equation of σ as

$$V_c t_{go}\ddot{\sigma} - 2V_c\dot{\sigma} = -a_m + a_t. \qquad (2)$$

Define a new variable δ as $\delta \equiv V_c\dot{\sigma}$, then (2) can be rewritten as

$$\dot{\delta} = \frac{2}{t_{\text{go}}}\delta + \frac{1}{t_{\text{go}}}(-a_m + a_t). \tag{3}$$

In TPN [16], the guidance command a_{mc} is calculated from

$$a_{mc} = \Lambda V_c\dot{\sigma} = \Lambda\delta \tag{4}$$

where Λ is the effective guidance gain for which a value between 3 and 5 is frequently chosen in practice.

Missile/autopilot dynamics may be represented as

$$\dot{x}_m = F_m x_m + G_m a_{mc} \tag{5}$$

$$a_m = H_m x_m. \tag{6}$$

By combining (3)–(6), and assuming a nonmaneuvering target, the PNG loop dynamics can be expressed by the following homogeneous linear time-varying differential equation:

$$\begin{bmatrix} \dot{\delta} \\ \dot{x}_m \end{bmatrix} = \begin{bmatrix} \dfrac{2}{t_{\text{go}}} & -\dfrac{1}{t_{\text{go}}}H_m \\ \Lambda G_m & F_m \end{bmatrix} \begin{bmatrix} \delta \\ x_m \end{bmatrix}. \tag{7}$$

Zero effort miss is defined as the distance by which the missile will miss the target if the target flies along its present course without maneuver and the missile makes no further corrective maneuvers. Consider small deviations from the collision course, then the zero effort miss, denoted by M, is computed from

$$M(t) = z + vt_{\text{go}} + R_m^T x_m \tag{8}$$

where z and v are the relative position and relative velocity, respectively. The last term of (8) is the influence of the current non-zero missile states on the zero effort miss. Let $\{F_m, G_m, H_m\}$ be the state-space realization of the missile/autopilot dynamics and a_m be the first state variable of this realization. Then, $R_m(t)$ is calculated from

$$R_m^T(t) = -H_m(e^{F_m t_{\text{go}}} - F_m t_{\text{go}} - I)F_m^{-2}. \tag{9}$$

For a small time to go and fast missile/autopilot dynamics, $R_m(t)$ is small enough to be neglected and the zero effort miss is approximated as

$$M(t) \approx z + vt_{\text{go}}. \tag{10}$$

The polar coordinate equivalence of (10) is derived as

$$M(t) \approx R(t)\Delta\sigma \tag{11}$$

where $\Delta\sigma$, the angle of LOS rotation during t_{go}, is computed from $\Delta\sigma \approx \dot{\sigma}(t)t_{\text{go}}$.

Equation (11) enables to derive the relationship between the state variable δ and zero effort miss. For a constant closing velocity, $R(t)$ is given as $R(t) = V_c t_{\text{go}}$ and the zero effort miss becomes

$$M(t) \approx V_c\dot{\sigma}(t)t_{\text{go}}^2. \tag{12}$$

By substituting $\delta = V_c\dot{\sigma}$ into (12), we obtain

$$\delta(t) \approx \frac{\text{ZEM}}{t_{\text{go}}^2}. \tag{13}$$

Equation (13) shows that $\delta(t)$ remains finite if the zero effort miss decreases faster than t_{go}^2 as the time to go approaches zero.

III. SHORT-TIME STABILITY CRITERION

Short-time stability is originally concerned with the question whether the state trajectory remains within a predetermined fixed region in the state space during a given fixed time interval [17]. Sufficient conditions for short-time stability that guarantee the Euclidean norm of the states to remain within a constant bound is given in [18]. We extend the previous results on short-time stability to handle more general cases including time-varying state weights and time-varying bounds.

DEFINITION 1 (Short-Time P-Stability) A linear system characterized by the homogeneous equation

$$\dot{x}(t) = F(t)x(t), \qquad x \in \mathbb{R}^n, \quad F(t) \in \mathbb{R}^{n\times n} \tag{14}$$

is said to be short-time P-stable with respect to a positive definite matrix $P(t) \in \mathbb{R}^{n\times n}$, $C(t) \in \mathbb{R}_+$, $C_0 \in \mathbb{R}_+$, and $T \in \mathbb{R}_+$, if

$$x^T(t_0)P(t_0)x(t_0) \le C_0 \tag{15}$$

implies

$$x^T(t)P(t)x(t) \le C(t) \tag{16}$$

on the interval $[t_0, t_0 + T]$.

Definition 1 is an extension to the definition of short-time stability given in [18]. A positive definite matrix $P(t)$ is a time-varying weight on the state and a scalar $C(t)$ specifies a time-varying bound of the state norm at time t.

A sufficient condition for short-time P-stability is summarized in the following theorem.

THEOREM 1 (Sufficient Condition of Short-Time P-Stability) *Suppose that a symmetric positive definite matrix $P(t) \in \mathbb{R}^{n\times n}$ is decomposed as $P(t) = N^T(t)N(t)$, where $N(t) \in \mathbb{R}^{n\times n}$ is a nonsingular matrix, and a matrix $U(t)$ is defined as*

$$U(t) \equiv (\dot{N}N^{-1})^T + \dot{N}N^{-1} + (NFN^{-1})^T + NFN^{-1}. \tag{17}$$

Then, the system described by (14) *is short-time P-stable with respect to $P(t)$, $C(t)$, C_0, T, if a matrix $H(t)$ defined as*

$$H(t) \equiv \left[-U + \frac{\dot{C}(t)}{C(t)}I\right] \tag{18}$$

is not negative definite for all $t \in [t_0, t_0 + T]$.

Alternative condition for short-time P-stability can be derived in an integration form as in the following theorem.

THEOREM 2 (Sufficient Condition of Short-Time P-Stability) *For the matrix $P(t)$ and $U(t)$ defined in Theorem 1, the system described by (14) is short-time P-stable with respect to $P(t)$, $C(t)$, C_0, T, if an inequality condition*

$$\int_{t_0}^{T} \lambda_M(\rho)\,d\rho \le \ln \frac{C(t)}{C_0} \tag{19}$$

where $\lambda_M(t)$ is the maximum eigenvalue of $U(t)$, is satisfied for all $t \in [t_0, t_0 + T]$.

Proofs of Theorem 1 and 2 are given in the Appendices. It is shown in the next section that Theorem 1 offers a more convenient way to evaluate short-time stability but Theorem 2 gives less conservative results.

IV. SHORT-TIME STABILITY ANALYSIS OF PNG

The concept of short-time stability discussed in Section III is of practical use for qualitative studies of PNG since the guidance loop operates during a finite interval of time. In this section, the PNG loop dynamics are reformulated to apply the short-time stability theorems. Sufficient conditions for short-time stability of PNG with a 1st-order missile/autopilot dynamics are then derived.

LEMMA 1 *Consider a linear time-varying system characterized by*

$$\dot{x}(t) = F(t)x(t) + G(t)u(t). \tag{20}$$

If system parameters $F(t)$, $G(t)$ and input $u(t)$ satisfy

$$\|F(t)\| > \epsilon_F, \qquad \|G(t)\| < \delta_G, \qquad \|u(t)\| < \delta_u \tag{21}$$

where $\epsilon_F > 0$, $\delta_G < \infty$, $\delta_u < \infty$, then the boundedness of $\|\dot{x}(t)\|$ implies the boundedness of $\|x(t)\|$.

Proof of Lemma 1 is straightforward and omitted here. The above lemma justifies the use of $\|\dot{x}_m(t)\|$ instead of $\|x_m(t)\|$ in testing the boundedness of $\|x_m(t)\|$.

In the spirit of Lemma 1, define a new state vector ξ as

$$\xi \equiv \dot{x}_m \tag{22}$$

then x_m is rewritten in terms of ξ and δ as

$$x_m = F_m^{-1}\xi - \Lambda F_m^{-1} G_m \delta. \tag{23}$$

The guidance loop dynamics given by (7) is also expressed in terms of ξ and δ:

$$\dot{\delta} = \left(\frac{2}{t_{go}} + \frac{\Lambda}{t_{go}} H_m F_m^{-1} G_m\right)\delta - \frac{1}{t_{go}} H_m F_m^{-1} \xi \tag{24}$$

$$\dot{\xi} = \left(F_m - \frac{\Lambda}{t_{go}} G_m H_m F_m^{-1}\right)\xi + \left(\frac{2\Lambda}{t_{go}} G_m + \frac{\Lambda^2}{t_{go}} G_m H_m F_m^{-1} G_m\right)\delta. \tag{25}$$

Without loss of generality, the dc gain of the missile/autopilot transfer function from the guidance command to the missile acceleration is assumed to be unity, or $H_m F_m^{-1} G_m = -1$. Hence, (24) and (25) are rewritten as

$$\begin{bmatrix} \dot{\delta} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \dfrac{2-\Lambda}{t_{go}} & \dfrac{1}{t_{go}} H_m F_m^{-1} \\ \dfrac{\Lambda(2-\Lambda)}{t_{go}} G_m & F_m - \dfrac{\Lambda}{t_{go}} G_m H_m F_m^{-1} \end{bmatrix} \begin{bmatrix} \delta \\ \xi \end{bmatrix}. \tag{26}$$

Suppose that the missile/autopilot dynamics is modeled as a 1st-order system

$$\frac{a_m(s)}{a_{mc}(s)} = \frac{1}{\tau s + 1} \tag{27}$$

which is equivalent to a state-space realization with $F_m = -1/\tau$, $G_m = 1/\tau$, $H_m = 1$. By using a transformation $\xi = \epsilon\zeta$, (26) is transformed to an equivalent state equation

$$\begin{bmatrix} \dot{\delta} \\ \dot{\zeta} \end{bmatrix} = \begin{bmatrix} \dfrac{2-\Lambda}{t_{go}} & \dfrac{\tau}{t_{go}}\epsilon \\ \dfrac{\Lambda(2-\Lambda)}{\tau t_{go}} \dfrac{1}{\epsilon} & -\dfrac{1}{\tau} + \dfrac{\Lambda}{t_{go}} \end{bmatrix} \begin{bmatrix} \delta \\ \zeta \end{bmatrix} \tag{28}$$

where ϵ is a free parameter to be used for reducing the conservatism of the stability conditions. It can be easily proved that ξ is bounded if and only if ζ is bounded for a finite ϵ.

Note that the physical meaning of short-time stability depends upon the parameters such as $P(t)$, $C(t)$, and C_0. In this work, $P(t)$, $C(t)$, and C_0 are determined based on the following definition of guidance loop stability.

DEFINITION 2 (Guidance Loop Stability) A guidance loop is said to be stable if the following properties are satisfied.

PROPERTY 1 *Missile/autopilot states are bounded.*

PROPERTY 2 *Acceleration command is bounded.*

PROPERTY 3 *Zero effort miss decreases with the order of t_{go}^2 as time to go approaches to zero.*

For PNG, Property 2 of Definition 2 implies Property 3 because the zero effort miss should decrease at least with the order of t_{go}^2 for the acceleration command to remain finite. Definition 2 can be formulated as a short-time P-stability problem by selecting a suitable weight $P(t)$ and a suitable bound for $x^T Px$.

For example, we can select $P(t)$ as an identity matrix and $C(t)$ as a constant, then we obtain $U(t)$ as

$$U(t) = \begin{bmatrix} \dfrac{2(2-\Lambda)}{t_{\rm go}} & \dfrac{\Lambda(2-\Lambda)+\tau^2\epsilon^2}{\epsilon\tau t_{\rm go}} \\ \dfrac{\Lambda(2-\Lambda)+\tau^2\epsilon^2}{\epsilon\tau t_{\rm go}} & \dfrac{2(\tau\Lambda - t_{\rm go})}{\tau t_{\rm go}} \end{bmatrix}. \tag{29}$$

Then, the PNG loop stability under a 1st-order missile/autopilot dynamics can be analyzed by applying Theorem 1 and 2 of the previous section.

A. Stability Condition I

For $U(t)$ given by (29), sufficient conditions for $-U(t)$ not being a negative definite, which is stated in Theorem 1, are reduced to

$$\Lambda \geq 2 \tag{30}$$

$$t_{\rm go} \geq \tau\Lambda + \frac{1}{4\tau(\Lambda-2)}\left[\frac{\Lambda(2-\Lambda)}{\epsilon} + \tau^2\epsilon\right]^2. \tag{31}$$

If the scale factor ϵ is selected as $\epsilon = \sqrt{\Lambda(\Lambda-2)}/\tau$ to minimize the lower bound of $t_{\rm go}$ in (31), then (31) is simplified as

$$t_{\rm go} \geq \tau\Lambda. \tag{32}$$

Note that the conditions given by (30) and (32) are identical to the Popov stability conditions.

B. Stability Condition II

The sufficient condition given by Theorem 2 for a constant bound becomes

$$\int_{t_0}^{t} \lambda_M(\rho)d\rho \leq 0 \tag{33}$$

where $\lambda_M(t)$ is the maximum eigenvalue of $U(t)$. By substituting $\epsilon = \sqrt{\Lambda(\Lambda-2)}/2$ to (29), $U(t)$ is simplified as

$$U(t) = \begin{bmatrix} \dfrac{2(2-\Lambda)}{t_{\rm go}} & 0 \\ 0 & \dfrac{2(\tau\Lambda - t_{\rm go})}{\tau t_{\rm go}} \end{bmatrix}. \tag{34}$$

Define $\eta \equiv (t_f - t)/\tau = t_{\rm go}/\tau$, $\eta_0 \equiv (t_f - t_0)/\tau$, and $\eta_1 \equiv 2(\Lambda - 1)$. Then $U(t)$ is rewritten as

$$U(\eta) = \begin{bmatrix} \dfrac{2(2-\Lambda)}{\tau\eta} & 0 \\ 0 & \dfrac{2(\Lambda-\eta)}{\tau\eta} \end{bmatrix}. \tag{35}$$

The maximum eigenvalue of $U(\eta)$, $\lambda_M(\eta)$, which is a function of time to go, is solved as

$$\lambda_M(\eta) = \frac{2(\Lambda-\eta)}{\tau\eta}, \qquad \text{for} \quad \eta \leq \eta_1 \tag{36}$$

$$\lambda_M(\eta) = \frac{2(2-\Lambda)}{\tau\eta}, \qquad \text{for} \quad \eta > \eta_1. \tag{37}$$

Hence, (33) becomes

$$\int_{\eta}^{\eta_0} \left(\frac{\Lambda-\rho}{\rho}\right) d\rho \leq 0 \qquad \text{for} \quad \eta \leq \eta_0 \leq \eta_1 \tag{38}$$

$$\int_{\eta}^{\eta_1} \left(\frac{\Lambda-\rho}{\rho}\right) d\rho + \int_{\eta_1}^{\eta_0} \left(\frac{2-\Lambda}{\rho}\right) d\rho \leq 0 \qquad \text{for} \quad \eta < \eta_1 < \eta_0 \tag{39}$$

$$\int_{\eta}^{\eta_0} \left(\frac{2-\Lambda}{\rho}\right) d\rho \leq 0 \qquad \text{for} \quad \eta_1 \leq \eta \leq \eta_0. \tag{40}$$

The inequality of (38) is satisfied if $\eta \geq \Lambda$, which is the same as the Popov stability conditions. However, the integration of (38) produces

$$\Lambda \ln\frac{\eta_0}{\eta} - (\eta_0 - \eta) \leq 0 \tag{41}$$

which is less conservative than the Popov stability conditions. For $\eta < \eta_1 < \eta_0$, (39) gives

$$-(\eta_1 - \eta) + \Lambda \ln\frac{\eta_1}{\eta} + (2-\Lambda)\ln\frac{\eta_0}{\eta_1} \leq 0. \tag{42}$$

Also, for $\eta_1 \leq \eta$, (40) gives

$$(2-\Lambda)\ln\frac{\eta_0}{\eta} \leq 0 \tag{43}$$

which is satisfied if $\Lambda \geq 2$.

Define η^* to be the greatest value of $\eta \equiv t_{\rm go}/\tau$ for which a particular stability condition does not hold, then the stability region, denoted by SR, is stated by using η^* as

$$\text{SR} \equiv [\eta^*, \eta_0]. \tag{44}$$

For the Popov stability conditions and stability condition I, η^* is calculated to be equal to Λ; thus, the stability region becomes

$$\text{SR} = [\Lambda, \eta_0]. \tag{45}$$

The stability region with respect to stability condition II is found by computing η^* from (41) and (42) as

$$\eta^* = \sup\{\eta \mid \eta - \Lambda\ln\eta > \eta_0 - \Lambda\ln\eta_0,\ \eta \in [0, \eta_0]\} \qquad \text{for} \quad \eta_0 \leq \eta_1 \tag{46}$$

$$\eta^* = \sup\{\eta \mid \eta - \Lambda\ln\eta > (\Lambda-2)\ln\eta_0 + \eta_1(1 - \ln\eta_1),\ \eta \in [0, 2(\Lambda-1)]\} \qquad \text{for} \quad \eta_0 > \eta_1. \tag{47}$$

The values of η^* obtained from (46) and (47) for several values of Λ and η_0 are listed in Table I.

The values of η^* based on stability conditions I and II of this work and the Popov stability conditions are

TABLE I
η^* Based on Short-Time Stability Condition II

guidance gain	nondimensional total flight time (η_0)						
	3	5	10	20	40	100	200
$\Lambda = 3$	3.00	1.76	1.13	0.81	0.60	0.41	0.32
$\Lambda = 5$		5.00	2.19	1.18	0.71	0.38	0.24

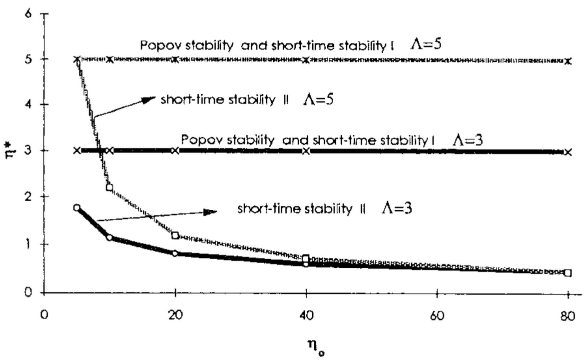

Fig. 2. Values of η^* based on short-time stability and Popov stability.

compared in Fig. 2 for guidance gains of 3 and 5. As discussed earlier, the values of η^* determined from stability condition I and the Popov stability condition are identical and not dependent upon η_0 which is the total flight time or initial time to go. On the other hand, η^* determined from stability condition II for a fixed guidance gain decreases as η_0 increases. This observation implies that the bound of the stable region in the sense of short-time stability can be extended by increasing η_0.

Fig. 2 also shows that for all η_0, the values of η^* based on stability condition II are smaller than those based on stability condition I and the Popov stability conditions. This demonstrates that stability condition II gives less conservative results than stability condition I and the Popov stability conditions.

By comparing η^*s for guidance gains of 3 and 5, we observe that the selection of a smaller guidance gain reduces η^* (or, extends the stability region) under all stability criteria considered in this work. For stability condition II, however, the variation of η^* due to guidance gain diminishes as η_0 increases.

V. SIMULATION

Computer simulations are conducted to check the results of the short-time stability analysis. Linearized kinematic relation is assumed for the relative motion of target and missile. For all simulations, an initial zero effort miss of 10 *m* and zero initial rate of zero effort miss are also assumed.

Figs. 3 and 4 show the time history of acceleration command and zero effort miss, respectively, for guidance gain of 3. The simulation is conducted for four different values of the total flight time: 3 s, 5 s,

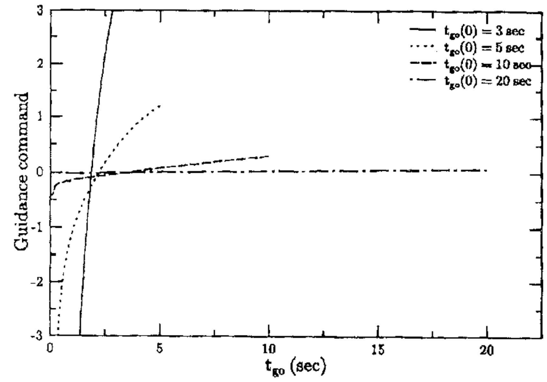

Fig. 3. Time history of guidance command ($\Lambda = 3$).

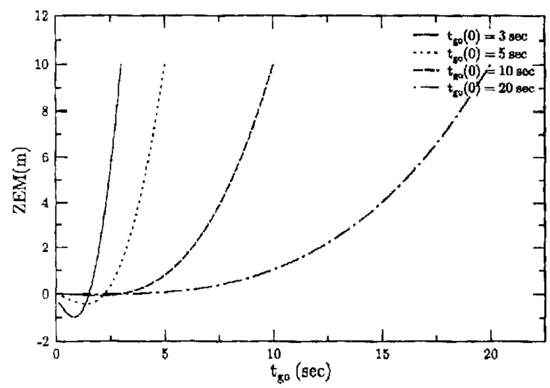

Fig. 4. Time history of zero effort miss ($\Lambda = 3$).

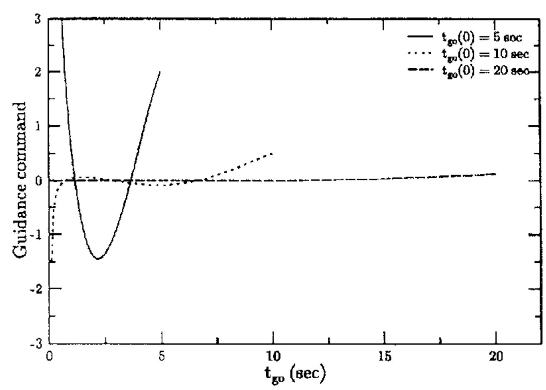

Fig. 5. Time history of guidance command ($\Lambda = 5$).

10 s, and 20 s. Fig. 3 shows that in the case of $\Lambda = 3$ the guidance command diverges right after guidance begins. Note that all stability criteria considered in this work give $\eta^* = 3$ for this case.

Guidance command divergence is delayed as the total flight time is increased to 5 s or 10 s. Moreover, guidance command divergence is not observed if the total flight time is increased up to 20 s. These observations show that short-time stability condition II explains the behavior of guidance loop more adequately than the Popov stability conditions.

Figs. 5 and 6 show acceleration command and zero effort miss for $\Lambda = 5$. These results show the

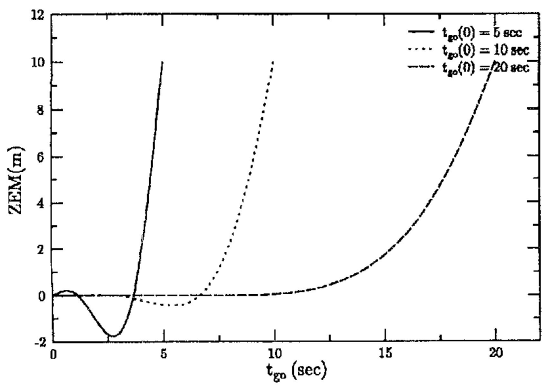

Fig. 6. Time history of zero effort miss ($\Lambda = 5$).

same characteristics of PNG as observed in the case of $\Lambda = 3$.

VI. CONCLUSIONS

In this work, the stability characteristics of PNG under missile/autopilot dynamics are analyzed by applying the short-time stability conditions, which are derived from the extended version of the short-time stability theorem. Stability region in terms of time to go, for which the conditions for short-time stability are satisfied, is obtained for the case of 1st-order missile/autopilot dynamics. Moreover, the relationship between the stability region and the guidance loop parameters such as guidance gain, autopilot time constant, and total flight time is analyzed.

Stability analysis results show that the influence of guidance gain on the stability region depends on the total flight time. In case of short initial ranges, a smaller guidance gain is preferred to extend the stability region. However, the effect of guidance gain on the stability region is reduced as the total flight time increases. The lower bound of the stability region η^* is also observed to be proportional to the time constant of the missile/autopilot dynamics; hence, the guidance loop with a smaller time constant maintains stability longer than the one with a larger time constant. Moreover, as the total flight time is decreased, η^* based on short-time stability is observed to approach to the bounds dictated by Popov stability.

It is emphasized that the stability analysis based on short-time stability greatly reduces the conservatism of the stability conditions proposed in the previous studies. Another advantage of the approach proposed in this work is that the time responses of the guidance loop variables such as missile/autopilot state, guidance command, and zero effort miss can be investigated without solving highly nonlinear differential equations.

APPENDIX A. PROOF OF THEOREM I

A positive definite matrix $P(t) \in \mathbb{R}^{n\times n}$ can be written in a factored form as $P(t) = N^T(t)N(t)$ for some nonsingular matrix $N(t) \in \mathbb{R}^{n\times n}$ and the time derivative of $x^T(t)P(t)x(t)$ becomes

$$\frac{d}{dt}x^T(t)P(t)x(t) = x^T(t)N^T(t)U(t)N(t)x(t) \tag{48}$$

where $U(t)$ is defined by

$$\begin{aligned} U(t) \equiv {} & [\dot{N}(t)N^{-1}(t)]^T + \dot{N}(t)N^{-1}(t) \\ & + [N(t)F(t)N^{-1}(t)]^T + N(t)F(t)N^{-1}(t). \end{aligned} \tag{49}$$

Define $y \equiv Nx$, then (48) becomes

$$\frac{d}{dt}y^T(t)y(t) = y^T(t)U(t)y(t). \tag{50}$$

If there exits a positive number $\beta(t)$ that satisfies

$$y^T(t)U(t)y(t) \le \beta(t)y^T(t)y(t) \tag{51}$$

(50) becomes

$$\frac{d}{dt}y^T(t)y(t) \le \beta(t)y^T(t)y(t). \tag{52}$$

By integrating (52), we obtain

$$y^T(t)y(t) \ge y^T(t_0)y(t_0)e^{\int_{t_0}^{t}\beta(\rho)d\rho} \tag{53}$$

or

$$y^T(t)y(t) \ge C_0 e^{\int_{t_0}^{t}\beta(\rho)d\rho}. \tag{54}$$

If $\beta(t)$ is selected to satisfy

$$e^{\int_{t_0}^{t}\beta(\rho)d\rho} = \frac{C(t)}{C_0} \tag{55}$$

then (54) becomes

$$y^T(t)y(t) \ge C(t) \tag{56}$$

and the condition of short-time P-stability is satisfied.

From (55), we obtain

$$\beta(t) = \frac{\dot{C}(t)}{C(t)}. \tag{57}$$

By substituting (57) into (51), we obtain

$$y^T(t)U(t)y(t) \le \frac{\dot{C}(t)}{C(t)}y^T(t)y(t). \tag{58}$$

Note that (58) is satisfied if

$$H(t) = \left[-U(t) + \frac{\dot{C}(t)}{C(t)}I\right] \tag{59}$$

is not negative definite for all $t \in [t_0, t_0 + T]$.

APPENDIX B. PROOF OF THEOREM II

It is noted that

$$y^T(t)U(t)y(t) \le \lambda_M(t)y^T(t)y(t) \tag{60}$$

where $\lambda_M(t)$ is the maximum eigenvalue of $U(t)$. Hence, (50) can be rewritten as

$$\frac{d}{dt}y^T(t)y(t) \le \lambda_M(t)y^T(t)y(t). \tag{61}$$

By integrating (61), we obtain

$$y^T(t)y(t) \le C_0 e^{\int_{t_0}^{t} \lambda_M(\rho)\,d\rho}. \tag{62}$$

If $\lambda_M(t)$ is selected as

$$e^{\int_{t_0}^{t} \lambda_M(\rho)\,d\rho} \le \frac{C(t)}{C_0} \tag{63}$$

then, the condition of short-time P-stability given in Definition 1 is satisfied. Note that (63) can be rewritten as

$$\int_{t_0}^{t} \lambda_M(\rho)\,d\rho \le \ln\frac{C(t)}{C_0}. \tag{64}$$

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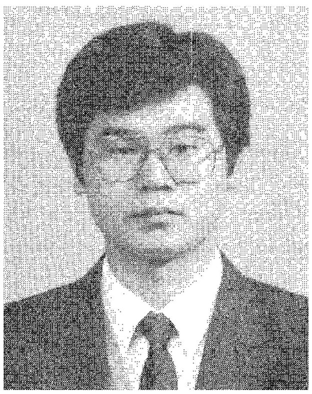

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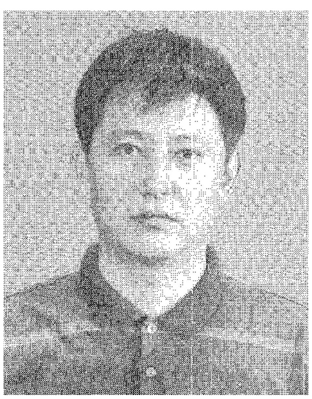

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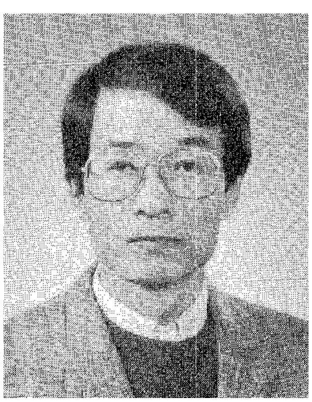

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