

problems are available in Connect.

Q9.1*

(Online Retailer) Customers send e-mails to a help desk of an online retailer every 2 minutes, on average, and the standard deviation of the interarrival time is also 2 minutes. The online retailer has three employees answering e-mails. It takes on average 4 minutes to write a response e-mail. The standard deviation of the processing times is 2 minutes.

- Estimate the average customer wait before being served. [9.5]
- How many e-mails would there be, on average, that have been submitted to the online retailer but not yet answered? [9.5]

Interarrival time (a): 2m

number of servers (m): 3

activity time (p): 4m

$$\rho = \frac{P}{a \times m} = \frac{4}{2 \times 3} = \frac{2}{3} = 0.666 \dots$$

$$CV_a = \text{Interarrival time} \frac{\text{St-Dev}}{\text{Average}} = \frac{2}{2} \quad CV_p = \text{processing time} \frac{\text{St-Dev}}{\text{Average}} = \frac{2}{4} = \frac{1}{2}$$

$$\begin{aligned} a) T_q &= \frac{P}{m} \times \frac{\rho^{\sqrt{2(m+1)}-1}}{1-\rho} \times \frac{CV_a^2 + CV_p^2}{2} \\ &= \frac{4}{3} \times \frac{(0.666)^{\sqrt{2(3+1)}-1}}{(0.333)} \times \frac{1 + 1/4}{2} = \underline{1.191 \text{ mins}} // \end{aligned}$$

$$b) I_q = T_q / a = 1.191 / 2 = \underline{0.5955 \text{ emails}} //$$

0.3

b. How much does the home security company have to pay the 1000 P
(Video Store) A small video store has nine copies of the DVD *Captain Underpants, The Movie* in its store. There are 15 customers every day who request this movie for their children. If the movie is not on the shelf, they leave and go to a competing store. Customers arrive evenly distributed over 24 hours.

The average rental duration is 36 hours.

- What is the likelihood that a customer going to the video store will find the movie available?
- Assume each rental is \$5. How much revenue does the store make per day from the movie?
- Assume each child that is not able to obtain the movie will receive a \$1 bill. How much money would the store have to give out to children requesting *Captain Underpants* every day?
- Assume the demand for the movie will stay the same for another six months. What would be the payback time (not considering interest rates) for purchasing an additional copy of the movie at \$50? Consider the extra revenues related to question b and the potential cost savings (question c).

$$p = 36 \text{ h} \quad a = 24/15 = 1.6 \text{ h} \quad m = 9 \text{ (copies)}$$

$$a) \text{ Arrival rate } (r) = p/a = 36/1.6 = 22.5$$

$$P_m = 0.6244$$

$$1 - 0.6244 = 0.3756$$

$$B) \text{ Flow rate} = \frac{(1 - P_m)}{a} = \frac{0.3756}{1.6} = 0.235 \text{ movies/h}$$

$$\text{Revenue} = 0.235 \times 24 \times \$5 = \$28.17/\text{day}$$

* Erlang Loss Table 이용

Erlang Loss Table

$r = p/a$	1	2	3	4	5	6	7	8	9	10
1.0	0.5000	0.2000	0.0625	0.0154	0.0031	0.0005	0.0001	0.0000	0.0000	0.0000
1.5	0.6000	0.3103	0.1343	0.0480	0.0142	0.0035	0.0008	0.0001	0.0000	0.0000
2.0	0.6667	0.4000	0.2105	0.0952	0.0367	0.0121	0.0034	0.0009	0.0002	0.0000
2.5	0.7143	0.4717	0.2822	0.1499	0.0697	0.0282	0.0100	0.0031	0.0009	0.0002
3.0	0.7500	0.5294	0.3462	0.2061	0.1101	0.0522	0.0219	0.0081	0.0027	0.0008
3.5	0.7778	0.5765	0.4021	0.2603	0.1541	0.0825	0.0396	0.0170	0.0066	0.0023
4.0	0.8000	0.6154	0.4507	0.3107	0.1991	0.1172	0.0627	0.0304	0.0133	0.0053
4.5	0.8182	0.6480	0.4929	0.3567	0.2430	0.1542	0.0902	0.0483	0.0236	0.0105
5.0	0.8333	0.6757	0.5297	0.3983	0.2849	0.1918	0.1205	0.0700	0.0375	0.0184
5.5	0.8462	0.6994	0.5618	0.4358	0.3241	0.2290	0.1525	0.0949	0.0548	0.0293
6.0	0.8571	0.7200	0.5902	0.4696	0.3604	0.2649	0.1851	0.1219	0.0751	0.0431
6.5	0.8667	0.7380	0.6152	0.4999	0.3939	0.2991	0.2174	0.1501	0.0978	0.0598
7.0	0.8750	0.7538	0.6375	0.5273	0.4247	0.3313	0.2489	0.1788	0.1221	0.0787
7.5	0.8824	0.7679	0.6575	0.5521	0.4530	0.3615	0.2792	0.2075	0.1474	0.0995
8.0	0.8889	0.7805	0.6755	0.5746	0.4790	0.3898	0.3082	0.2356	0.1731	0.1217
8.5	0.8947	0.7918	0.6917	0.5951	0.5029	0.4160	0.3356	0.2629	0.1989	0.1446
9.0	0.9000	0.8020	0.7064	0.6138	0.5249	0.4405	0.3616	0.2892	0.2243	0.1680
9.5	0.9048	0.8112	0.7198	0.6309	0.5452	0.4633	0.3860	0.3143	0.2491	0.1914
10.0	0.9091	0.8197	0.7321	0.6467	0.5640	0.4845	0.4090	0.3383	0.2732	0.2146
10.5	0.9130	0.8274	0.7433	0.6612	0.5813	0.5043	0.4307	0.3611	0.2964	0.2374
11.0	0.9167	0.8345	0.7537	0.6745	0.5974	0.5227	0.4510	0.3828	0.3187	0.2596
11.5	0.9200	0.8410	0.7633	0.6869	0.6124	0.5400	0.4701	0.4033	0.3400	0.2811
12.0	0.9231	0.8471	0.7721	0.6985	0.6264	0.5561	0.4880	0.4227	0.3604	0.3019
12.5	0.9259	0.8527	0.7804	0.7092	0.6394	0.5712	0.5049	0.4410	0.3799	0.3220
13.0	0.9286	0.8579	0.7880	0.7192	0.6516	0.5854	0.5209	0.4584	0.3984	0.3412
13.5	0.9310	0.8627	0.7952	0.7285	0.6630	0.5987	0.5359	0.4749	0.4160	0.3596
14.0	0.9333	0.8673	0.8019	0.7373	0.6737	0.6112	0.5500	0.4905	0.4328	0.3773
14.5	0.9355	0.8715	0.8081	0.7455	0.6837	0.6230	0.5634	0.5052	0.4487	0.3942
15.0	0.9375	0.8755	0.8140	0.7532	0.6932	0.6341	0.5761	0.5193	0.4639	0.4103
15.5	0.9394	0.8792	0.8196	0.7605	0.7022	0.6446	0.5880	0.5326	0.4784	0.4258
16.0	0.9412	0.8828	0.8248	0.7674	0.7106	0.6546	0.5994	0.5452	0.4922	0.4406
16.5	0.9429	0.8861	0.8297	0.7739	0.7186	0.6640	0.6102	0.5572	0.5053	0.4547
17.0	0.9444	0.8892	0.8344	0.7800	0.7262	0.6729	0.6204	0.5687	0.5179	0.4682
17.5	0.9459	0.8922	0.8388	0.7859	0.7334	0.6814	0.6301	0.5795	0.5298	0.4811
18.0	0.9474	0.8950	0.8430	0.7914	0.7402	0.6895	0.6394	0.5899	0.5413	0.4935
18.5	0.9487	0.8977	0.8470	0.7966	0.7467	0.6972	0.6482	0.5998	0.5522	0.5053
19.0	0.9500	0.9002	0.8508	0.8016	0.7529	0.7045	0.6566	0.6093	0.5626	0.5167
19.5	0.9512	0.9027	0.8544	0.8064	0.7587	0.7115	0.6647	0.6183	0.5726	0.5275
20.0	0.9524	0.9050	0.8578	0.8109	0.7644	0.7181	0.6723	0.6270	0.5822	0.5380
20.5	0.9535	0.9072	0.8611	0.8153	0.7697	0.7245	0.6797	0.6353	0.5913	0.5480
21.0	0.9545	0.9093	0.8642	0.8194	0.7749	0.7306	0.6867	0.6432	0.6001	0.5576
21.5	0.9556	0.9113	0.8672	0.8234	0.7798	0.7364	0.6934	0.6508	0.6086	0.5668
22.0	0.9565	0.9132	0.8701	0.8272	0.7845	0.7420	0.6999	0.6581	0.6167	0.5757
22.5	0.9574	0.9150	0.8728	0.8308	0.7890	0.7474	0.7061	0.6651	0.6244	0.5842
23.0	0.9583	0.9168	0.8754	0.8343	0.7933	0.7525	0.7120	0.6718	0.6319	0.5924
23.5	0.9592	0.9185	0.8780	0.8376	0.7974	0.7575	0.7177	0.6783	0.6391	0.6003
24.0	0.9600	0.9201	0.8804	0.8408	0.8014	0.7622	0.7232	0.6845	0.6461	0.6079
24.5	0.9608	0.9217	0.8827	0.8439	0.8053	0.7668	0.7285	0.6905	0.6527	0.6153
25.0	0.9615	0.9232	0.8850	0.8469	0.8090	0.7712	0.7336	0.6963	0.6592	0.6224
25.5	0.9623	0.9246	0.8871	0.8497	0.8125	0.7754	0.7385	0.7019	0.6654	0.6292
26.0	0.9630	0.9260	0.8892	0.8525	0.8159	0.7795	0.7433	0.7072	0.6714	0.6358
26.5	0.9636	0.9274	0.8912	0.8552	0.8192	0.7835	0.7479	0.7124	0.6772	0.6422
27.0	0.9643	0.9287	0.8931	0.8577	0.8224	0.7873	0.7523	0.7174	0.6828	0.6483
27.5	0.9649	0.9299	0.8950	0.8602	0.8255	0.7910	0.7565	0.7223	0.6882	0.6543
28.0	0.9655	0.9311	0.8968	0.8626	0.8285	0.7945	0.7607	0.7269	0.6934	0.6600
28.5	0.9661	0.9323	0.8985	0.8649	0.8314	0.7979	0.7646	0.7315	0.6985	0.6656
29.0	0.9667	0.9334	0.9002	0.8671	0.8341	0.8013	0.7685	0.7359	0.7034	0.6710
29.5	0.9672	0.9345	0.9019	0.8693	0.8368	0.8045	0.7722	0.7401	0.7081	0.6763
30.0	0.9677	0.9356	0.9034	0.8714	0.8394	0.8076	0.7758	0.7442	0.7127	0.6813
30.5	0.9683	0.9366	0.9050	0.8734	0.8420	0.8106	0.7793	0.7482	0.7172	0.6863
31.0	0.9688	0.9376	0.9064	0.8754	0.8444	0.8135	0.7827	0.7521	0.7215	0.6910
31.5	0.9692	0.9385	0.9079	0.8773	0.8468	0.8164	0.7860	0.7558	0.7257	0.6957
32.0	0.9697	0.9394	0.9093	0.8791	0.8491	0.8191	0.7892	0.7594	0.7297	0.7002

$$C) \text{ Avg Cost} = \frac{P_m}{a} = 0.39 \text{ /h}$$

$$0.39 \times 24 \times \$1 = \$9.36$$

$$D) \text{ Total time for 7 days } m = 9+1 = 10$$

$$P_m = 0.5842$$

$$\text{Rev} = \$31.19$$

$$\text{Penalty} = \$8.16$$

7일(m=9)	10일(m=10)	
28.17	31.19	
9.36	8.16	
18.80	22.43	22.43
		- 18.80
		3.63

$$\therefore \text{투자회수기간} = \$50 / 3.63 = 13.77 \text{ 일}$$