# Time Scale Exploitation to frame Simplified Models in Plasmas

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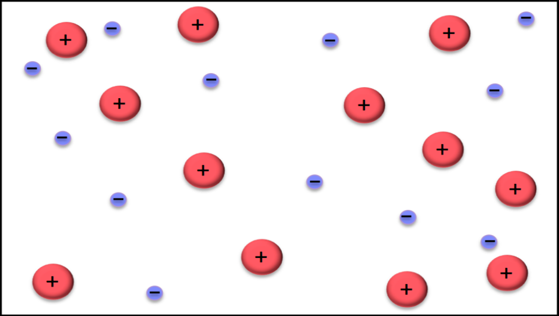
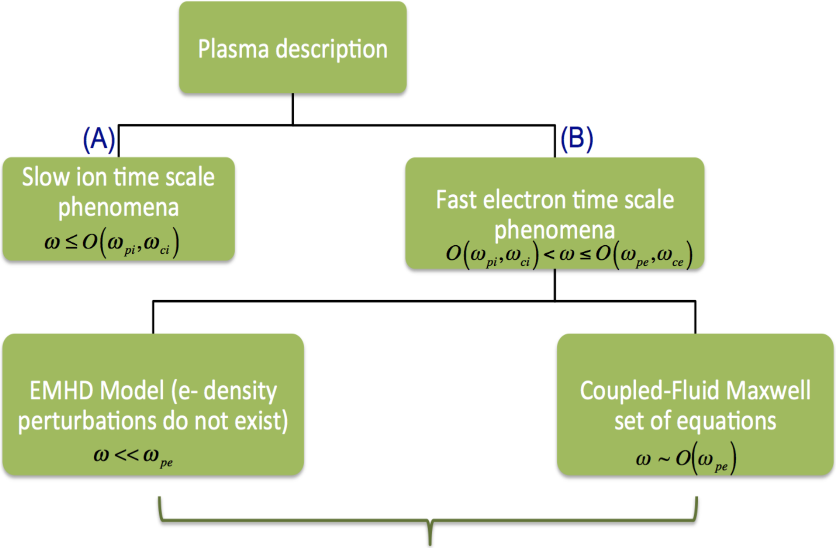
# Plasma



Disparity in masses → different response time

## Exploitation of time scale separation




Issues related to these time scales will be addressed.

## Issues addressed

- Instability studies - KH instability in electron fluid
  Electrons act as a magnetized fluid. Susceptible to fluid instabilities - specific case KH - flow shear driven instability
- Coherent nonlinear solutions - for the coupled laser plasma system using LCPFCT fluid simulation
  Dynamical evolution and stability studies of a particular flat top variety of soliton.
- Physics of dip observed in proton spectra using LSP PIC simulation.

## EMHD Model

- EMHD model describes the dynamics of magnetized electron fluid in the presence of self-consistent and external electric and magnetic field on time scales in between electron and ion gyro frequencies.
- Length and time scale involved in the EMHD phenomena is

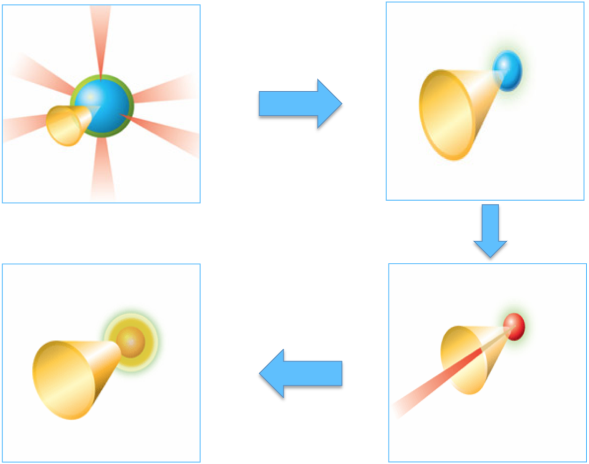
$$\rho_{ce} \ll l \ll \rho_{ci}, d_i$$
$$\omega_{ci}, \omega_{pi} \ll \omega \ll \omega_{pe}, \omega_{pe}^2/\omega_{ce}$$

- Ion dynamics is completely ignored and role of ions is just to provide neutralizing background.

## Relevance of KH instability in the context of electron fluid

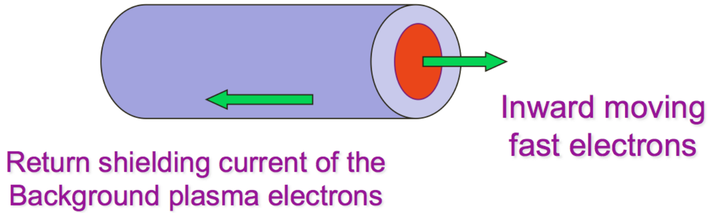
Where do sheared electron flow occur?

FAST IGNITION

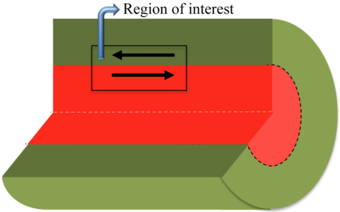


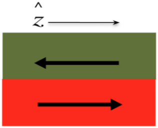
## Current Channel

Schematic of flow configuration after Weibel has occurred

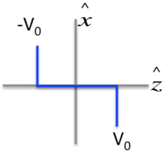


# System Description

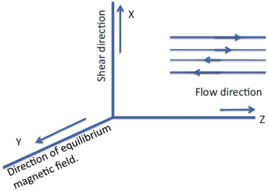


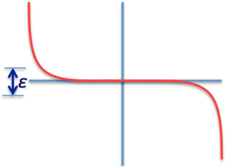




Region of interest




model equilibrium velocity profile

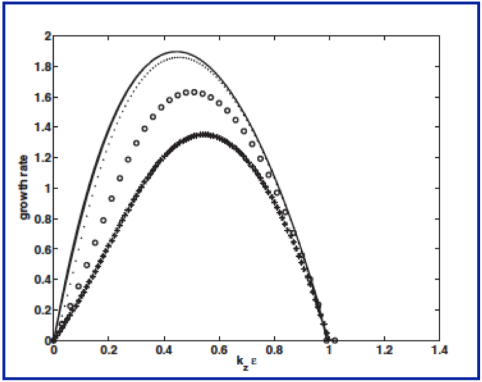



tangent hyperbolic velocity profile

# KH in EMHD (a brief recap)

Realistic case with finite shear width– there exists a threshold wavenumber beyond which the growth rate vanishes. [1]




Growth rate vanishes beyond $k_z \varepsilon \leq 1$

Only one unstable eigen-mode

Excited by $d^2V/dx^2$ in 2-D.

Necessary condition for instability is ($d^2V/dx^2 - V$) should change sign.

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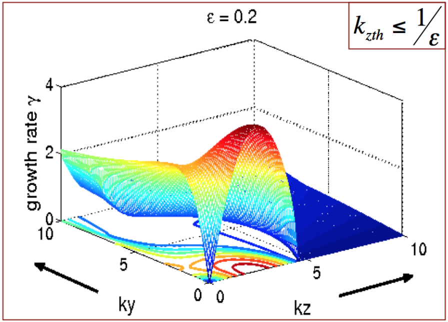
[1] N. Jain *et al. Phys. Plasmas*, 11, 4390 (2004) ; G. Gaur *et al. Phys. Plasmas*, 16, 072310 (2009)

# 3D character of instability



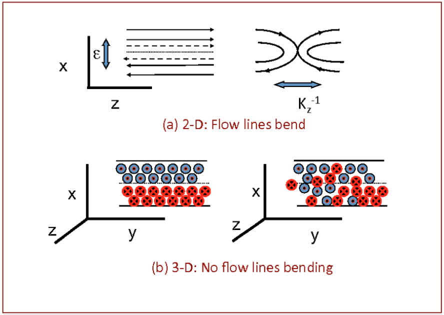

A surface and contour plot of the growth rate for the velocity shear driven mode in the $k_y$-$k_z$ plane. Profile is $v_0=V_0\ tanh(x/\varepsilon)$.

- Nonlocal character for unstable perturbations in the flow-shear plane.
- Extended range of unstable wave vector along the direction of magnetic field

# 3D character of instability contd...

## Explanation :

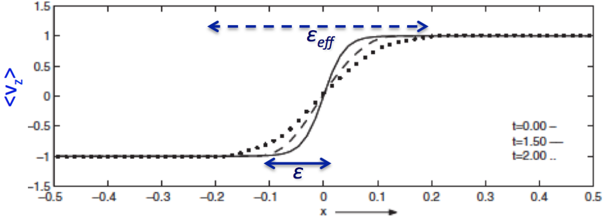
- Source of free energy: shear flow configuration.
- Unstable Eigen functions can't be sharper than the shear width leading to threshold on wave vector.
- Mode with x-y variation- predominantly magnetic energy.




(a) A schematic flow configuration after KH destabilization in 2-D. (b) For perturbation scales along *y* direction the flow lines do not have to bend, they merely get shuffled around to generate short scales along *y*.

## Saturation of the instability

In the nonlinear state the shear width of the original velocity profile broaden to an effective value of the shear width [2]




Analytical order of magnitude estimate of the growth rate $\sim k_z V_0$

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[2] N. Jain *et al. Phys. Plasmas* , 10, 29 (2003)

## Role of relativistic flows on the instability

- Astrophysical: magnetic reconnection
- Fast Ignition - velocities are relativistic - need to understand the relativistic KH
- Velocities close to ‘c’ implies displacement current ought to be retained - density fluctuation may be important - EMHD model can not be used.

# Governing Equations for relativistic case

Relativistic electron fluid equations are

$$m_e n_e \left[ \frac{\partial \gamma_e \vec{\upsilon}_e}{\partial t} + \left( \vec{\upsilon}_e \cdot \nabla \right) \gamma_e \vec{\upsilon}_e \right] = -en_e \vec{E} - \frac{en_e \left( \vec{\upsilon}_e \times \vec{B} \right)}{c} - \nabla p_e \text{....(2)}$$

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \left( n_e \vec{\upsilon}_e \right) = 0 \text{.......................................................(3)}$$

Maxwell's Equations

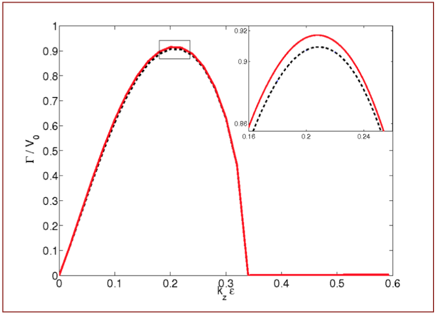
$$\nabla \cdot \vec{E} = -4\pi n_e e$$

$$\nabla \cdot \vec{B} = 0$$

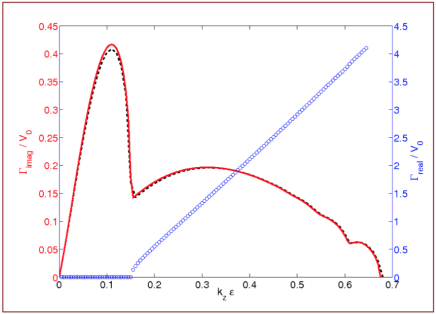
$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = -\frac{4\pi}{c} n_e e \vec{v}_e + \cancel{\frac{1}{c}\frac{\partial \vec{E}}{\partial t}}$$

# Growth rate for Tangent hyperbolic profile



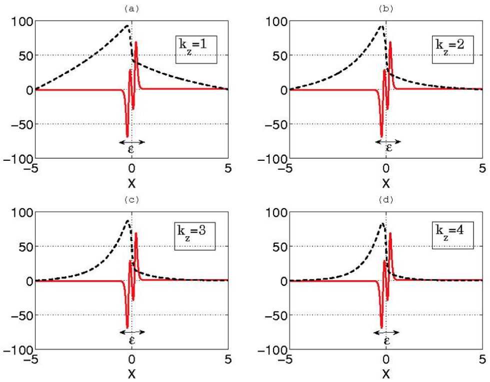

The solid and dashed lines show the plot of relativistic and weakly relativistic growth rates as a function of $k_z \varepsilon$ where shear width $\varepsilon$=0.1, $V_0$ =0.8, $v_0 = V_0 \tanh(x/\varepsilon)$.




The solid and dashed lines show the plot of relativistic and weakly relativistic growth rates as a function of $k_z \varepsilon$ where shear width $\varepsilon$=0.1, $V_0$ =0.9, $v_0 = V_0 \tanh(x/\varepsilon)$.

## Explanation of multiple peaks in growth rate

The necessary condition for instability in the relativistic regime is $(\gamma_0 v_{z0})'' - v_{z0}$ should change sign.




The four subplots show $v_{z0}$-$(\gamma_0 v_{z0})''$ (solid line) and the eigen function (dashed line) as a function of x for tan hyperbolic equilibrium shear flow for different values of $k_z$ for $V_0$ =0.95 and ε=0.1.

# Coherent solutions in the laser plasma system

How do these structures form?

Balance of nonlinearity with Dispersion

Importance:

Coherent entities of fundamental interest

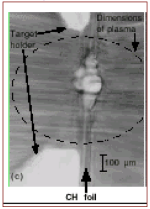
Inward Energy Transport in Laser Based Fusion

*M. Blazquez, Laser Part. Beams, 2002*

Charged Particle Acceleration

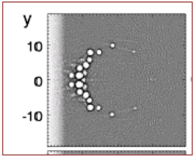
*Mima et al., PRL 1986; Kaw et al., PRL 1992*

*Experiments*



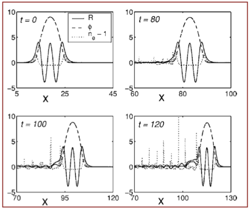

Borghesi et al, 2002

*Soliton necklace*




Bulanov et al, 2002

*Fluid Simulation*




Vikrant et al, 2007 32

# Motivation

- Kaw et al. (1992), Kuehl et al. (1993), Dimant et al.(1998) and many others have studied the equilibrium envelope solitary solutions in the static ion case.
- In the static ion case, there are solutions for which $\beta$ is very small or $\beta = 0$. For these structures, ion dynamics will be important.
- An understanding of the dynamics and stability of soliton solutions in the presence of ion response is incomplete.

We have incorporated ion dynamics and studied the detailed characterization and stability properties of relativistic electromagnetic solitons.

# One dimensional cold plasma model



$$\frac{\partial n_e}{\partial t}+\frac{\partial(n_e u_e)}{\partial x}=0$$

$$\frac{\partial n_i}{\partial t}+\frac{\partial(n_i u_i)}{\partial x}=0$$

$$\left(\frac{\partial}{\partial t}+u_e\frac{\partial}{\partial x}\right)(\gamma_e u_e)=\frac{\partial\phi}{\partial x}-\frac{1}{2\gamma_e}\frac{\partial A_\perp^2}{\partial x}$$

$$\left(\frac{\partial}{\partial t}+u_i\frac{\partial}{\partial x}\right)(\gamma_i u_i)=-\alpha\frac{\partial\phi}{\partial x}-\frac{\alpha^2}{2\gamma_i}\frac{\partial A_\perp^2}{\partial x}$$

$$\frac{\partial^2\phi}{\partial x^2}=n_e-n_i$$

$$\frac{\partial^2 A_\perp}{\partial t^2}-\frac{\partial^2 A_\perp}{\partial x^2}=-\left(\frac{n_i\alpha}{\gamma_i}+\frac{n_e}{\gamma_e}\right)A_\perp$$

$$\alpha=\frac{m_e}{m_i}$$

$$\gamma_e=\sqrt{\frac{1+A_\perp^2}{1-u_e^2}}$$

$$\gamma_i=\sqrt{\frac{1+\alpha^2 A_\perp^2}{1-u_i^2}}$$

## Co-ordinate Transformation

$$\xi=x-\beta t;\ \ \tau=t;\ \ \frac{\partial}{\partial x}=\frac{\partial}{\partial\xi};\ \ \frac{\partial}{\partial t}=\cancelto{0}{\frac{\partial}{\partial\tau}}-\beta\frac{\partial}{\partial\xi}$$

$$\beta=\frac{v_{gr}}{c}$$

# Coherent Nonlinear Solutions

$$\phi'' = \frac{\beta}{\beta - u_e} - \frac{\beta}{\beta - u_i}$$

$$R'' + \frac{R}{1-\beta^2}\left[\left(\lambda^2 - \frac{M^2}{R^4}\right)\frac{1}{1-\beta^2} - \frac{\beta}{\beta - u_e}\frac{1-\beta u_e}{1+\phi} - \alpha\frac{\beta}{\beta - u_i}\frac{1-\beta u_i}{1-\phi\alpha}\right] = 0$$

The parallel fluid velocities for electron and ion are

$$u_e = \frac{\beta\left(1+R^2\right) - (1+\phi)\left[(1+\phi)^2 - \left(1-\beta^2\right)\left(1+R^2\right)\right]^{1/2}}{(1+\phi)^2 + \beta^2\left(1+R^2\right)}$$

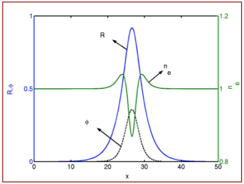
$$u_i = \frac{\beta\left(1+R^2\alpha^2\right) - (1-\phi\alpha)\left[(1-\phi\alpha)^2 - \left(1-\beta^2\right)\left(1+R^2\alpha^2\right)\right]^{1/2}}{(1-\phi\alpha)^2 + \beta^2\left(1+R^2\alpha^2\right)}$$

## Circular Polarization

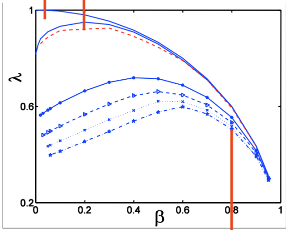
$$\vec{A} = \left(a(\xi)/2\right)\left[\{\hat{y} + i\hat{z}\}\exp(-i\lambda\tau) + c.c.\right]$$

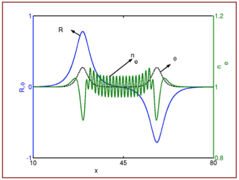
$$a(\xi) = R.\exp(i\theta)$$

# Circularly Polarized Soliton Solutions (e-only)




Single peak

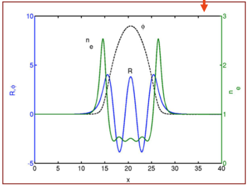



Paired structure

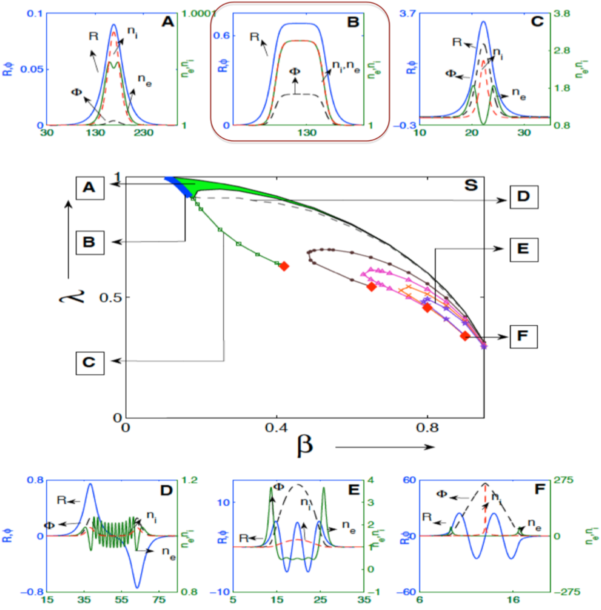
$$\lambda = \omega(1-\beta^2)$$

Poornakala et al.,
Phys. Plasmas 2002

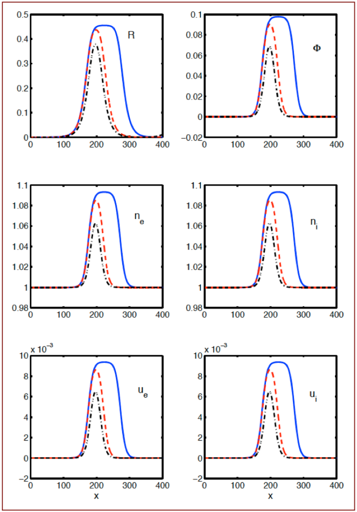



Multiple peak

# Eigen Value Spectrum : Ion Dynamics



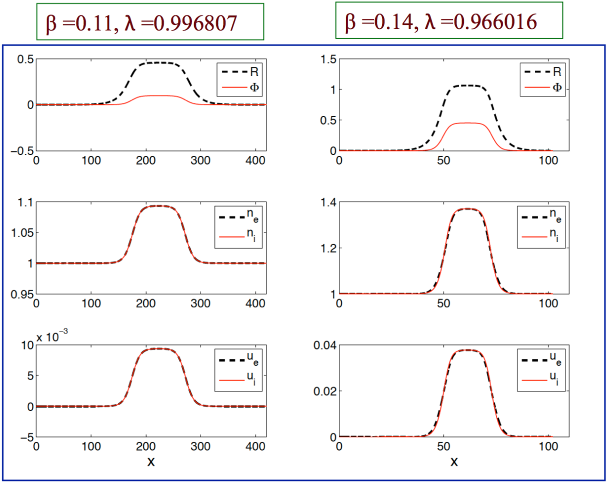
# Continuous transition : flat top to single peak




0.996807
0.99682
0.997
λ

β =0.11

Solution with a lower value of $\beta$ has smaller amplitude and larger width



## Small amplitude solutions

Retaining terms up to $R^3$, wave eq. is of the form

$$R'' + AR + BR^3 = 0$$

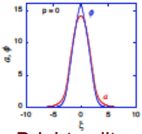
$B > 0$ Bright Soliton

$B < 0$ Dark Soliton

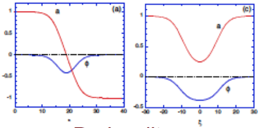
As B changes sign we get bright and dark solitonic structures.

For $\beta = \sqrt{\alpha}$, B---> 0.

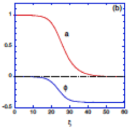
However, if we retain higher order terms, flat top solutions form at this boundary.



Bright soliton



Dark solitons



Shock wave

# Mathematical representation of flat-top solitons

Center for Energy Research

UCSD

Small amplitude solitons: $n_e \approx n_i = n; \quad u_e \approx u_i = u;\ \gamma_e \approx \sqrt{1+R^2};\ \ \gamma_i \approx 1$

Using these in integrated momentum equations and eliminating $\phi$ we get

$$n = 1+\varepsilon;\ \varepsilon = \alpha\frac{R^2}{2\beta^2} + \left[\frac{\alpha^2 R^4\left(1-\beta^2\right)}{4\beta^4}\right] + \ldots..$$

$$\frac{n}{\gamma_e} = 1 - \frac{R^2}{2} + \frac{\alpha R^2}{2\beta^2} + \frac{\alpha^2 R^4}{4\beta^4}\left(1-\beta^2\right) - \frac{\alpha^2 R^4}{4\beta^4} + \ldots$$

$$\frac{n}{\gamma_i} = 1 - \frac{\alpha^2 R^2}{2} + \frac{\alpha R^2}{2\beta^2} + \frac{\alpha^2 R^4}{4\beta^4}\left(1-\beta^2\right) - \frac{\alpha^3 R^4}{4\beta^4} + \ldots$$

Substitution of $n/\gamma_e$, $n/\gamma_i$ and $\varepsilon$ gives

$$R'' + \frac{R}{1-\beta^2}\left[\frac{\lambda^2}{1-\beta^2} - (1+\alpha)\right] + \frac{R^3}{2\left(1-\beta^2\right)}\left[\frac{\beta^2\left(1+\alpha^3\right) - \alpha(1+\alpha)}{\beta^2}\right]$$

$$+\frac{R^5}{4\left(1-\beta^2\right)}\left[\frac{\beta^2\alpha\left(1+\alpha^3\right) - \alpha^2(1+\alpha)\left(1-\beta^2\right)}{\beta^4}\right] = 0$$

# Mathematical representation contd ....

Integrating once and making the substitution $f = R^2$ we get elliptic type eq.

$$\frac{f'^2}{f^2} + \frac{8C}{6} f^2 + 2Bf + 4A = 0$$ where

$$A = \frac{\left[\frac{\lambda^2}{1-\beta^2} - (1+\alpha)\right]}{1-\beta^2};\; B = \left[\frac{\beta^2(1+\alpha^3) - \alpha(1+\alpha)}{\beta^2(1-\beta^2)}\right];\; C = \frac{\beta^2\alpha(1+\alpha^3) - \alpha^2(1+\alpha)(1-\beta^2)}{\beta^4(1-\beta^2)}$$

The solution of the elliptic type equation is

$$f(\xi) = \frac{2 f_1 f_2}{(f_1 + f_2) - (f_1 - f_2)\cosh\left(2\eta\sqrt{f_1|f_2|}\,\xi\right)}$$ where $\eta = \sqrt{C/3}$

and $f_1$ and $f_2$ are the solutions of the quadratic equation:

$$\frac{8C}{6} f^2 + 2Bf + 4A = 0$$

When $f_1 \to f_2$, we get flat-top solutions.

$$\lambda^2 = (1+\alpha)(1-\beta^2) - \frac{3\left[\beta^2(1+\alpha^3) - \alpha(1+\alpha)\right]^2}{16\alpha\beta^2\left[(1+\alpha^3) - \alpha(1+\alpha)(1-\beta^2)\right]}$$

This is the eigen value condition for flat top structures.

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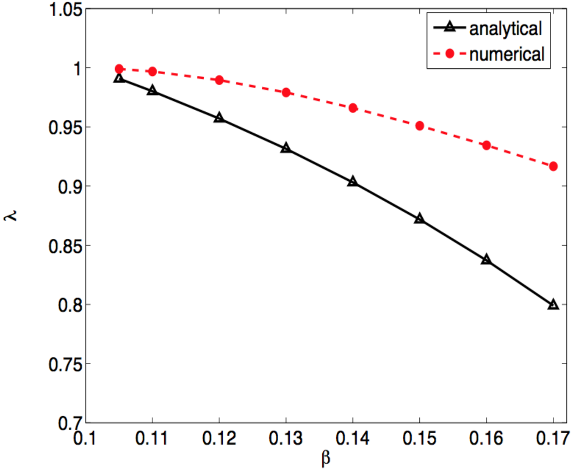
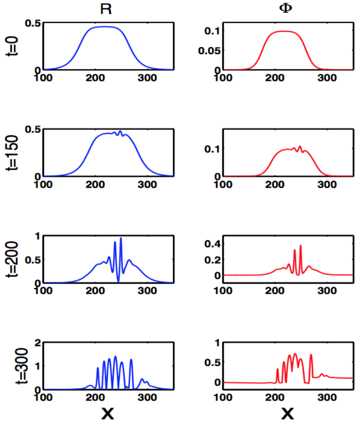
# Eigen value condition for flat top soliton solution



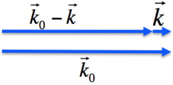
# Dynamical evolution of flat-top




$\beta = 0.11$,
$\lambda = 0.996807$

# Backward instability

Identical scale for perturbed scalar potential and vector potential

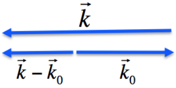



Forward Scattering

$$|\vec{k}_0 - \vec{k}| \approx |\vec{k}_0|$$

$$\vec{k} \approx 0$$

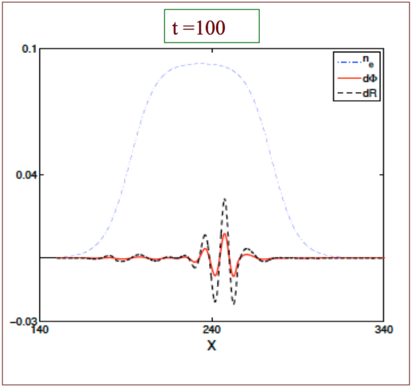
*Frequencies identical implies Long scale in electrostatic potential*



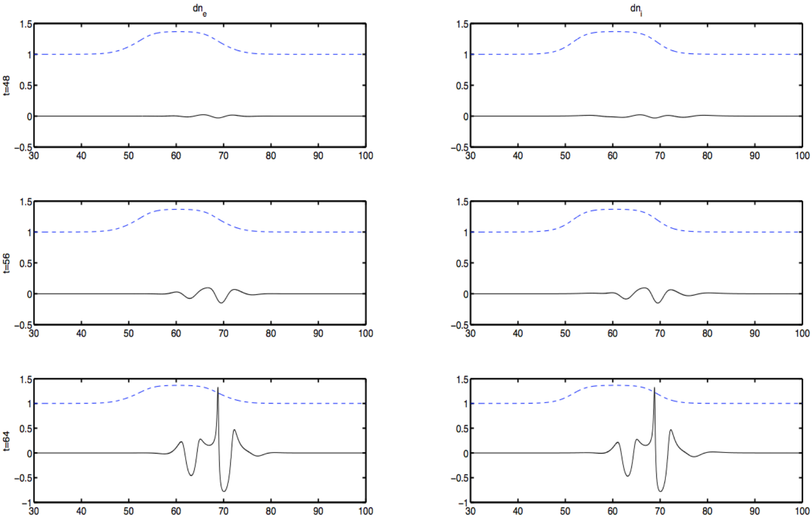

Backward Scattering

$$\vec{k} \approx 2\vec{k}_0$$

*Scales of vector and scalar potential are identical*



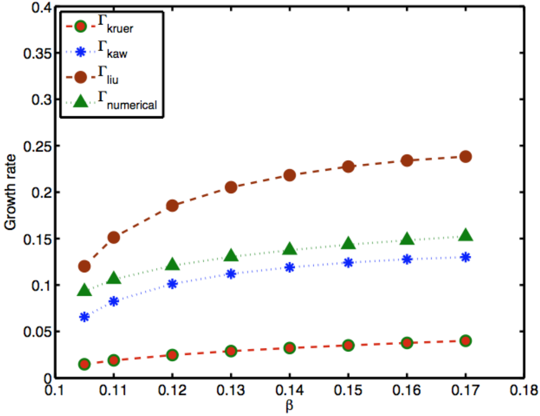
Ion and electron perturbed densities are in phase



Growth rate of quasi-mode of Brillouin Scattering

| $\beta$ | $\lambda$ | $\omega_0$ | $k_0$ | $\upsilon_{os}$ | $\Gamma_{kruer}$ | $\Gamma_{Kaw}$ | $\Gamma_{Liu}$ | $\Gamma_{numerical}$ |
|---|---|---|---|---|---|---|---|---|
| 0.105 | 0.9989473 | 1.0101 | 0.1061 | 0.2933 | 0.0656 | 0.1202 | 0.0931 | 0.04844 |
| 0.11 | 0.9968069 | 1.0090 | 0.1110 | 0.4137 | 0.0825 | 0.1512 | 0.1061 | 0.06552 |
| 0.12 | 0.9895694 | 1.0040 | 0.1205 | 0.5638 | 0.1012 | 0.1855 | 0.1208 | 0.10643 |
| 0.13 | 0.9790741 | 0.9959 | 0.1295 | 0.6594 | 0.1120 | 0.2053 | 0.1304 | 0.14585 |
| 0.14 | 0.9660160 | 0.9853 | 0.1379 | 0.7274 | 0.1190 | 0.2182 | 0.1375 | 0.18114 |
| 0.15 | 0.9509680 | 0.9729 | 0.1459 | 0.7814 | 0.1241 | 0.2275 | 0.1434 | 0.22129 |
| 0.16 | 0.9344057 | 0.9590 | 0.1534 | 0.8186 | 0.1277 | 0.2340 | 0.1482 | 0.19200 |
| 0.17 | 0.9167206 | 0.9440 | 0.1605 | 0.8465 | 0.1300 | 0.2383 | 0.1523 | 0.30170 |

# Brillouin instability [3]




[3] S. Sundar *et al.* *Phys. Plasmas* , 18, 112112 (2011)

# Conclusions

- Incorporation of ion response rules out the existence of static solutions.
- Destabilization process of Flat-top solitons has been identified as backward Brillouin scattering process.
- Electron quiver velocity plays the role of the effective temperature.

# Future work

- Comprehensive study of depletion feature of proton layer.
- Role of proton contamination thickness on the energy spectra, conversion efficiency and mean energy.
- Comparison of 1D3V results with 2D3V and in-depth study of spectra in 2D3V simulation.


Thank you