Resolved Motion Rate Control of Space Manipulators with Generalized Jacobian Matrix

YOJI UMETANI AND KAZUYA YOSHIDA

Abstract—**In recent years, space has attracted special interest as a new application field of robotics. A robotic teleoperator system installed with space manipulators will play an important role in future space projects, such as constructing space structures or servicing satellites. However, in space environment, the lack of a fixed base raises many problems in controlling space robotic systems. In general, any motion of the manipulator arm will induce reaction forces and moments which disturb position and attitude of the supporting base satellite. To establish a control method for space manipulators taking dynamical interaction between the manipulator arm and the base satellite into account, the authors investigate the kinematics of free-flying multibody systems by introducing the momentum conservation law into the formulation and derive a new Jacobian matrix in generalized form for space robotic arms. By means of the new matrix, they develop a control method for space manipulators based on the resolved motion rate control concept. The proposed method is widely applicable in solving not only free-flying manipulation problems but also attitude control problems. The validity of the method is demonstrated by computer simulations with a realistic model of robot satellite.**

I. Introduction

SPACE ROBOTICS is a new technological field. In future space development, robotization and automation will be key technology and contribute much to the success of space projects, reducing the workload of astronauts, and increasing operational efficiency.

As advanced investigation in this emerging field, various types of space manipulator systems, such as a large-scale manipulator equipped on a space shuttle or manned space station, or rather small ones mounted on an unmanned satellite have been studied and developed in recent years [1]–[10]. Principally, the authors have primary interest in smaller systems installed on a free-flying unmanned satellite which is being planned as a "robot satellite" [8], since it is expected to play an important role in space development as a competent free-flying teleoperator accomplishing precise and dexterous missions on orbit and all fundamental technology in space robotics is involved in it.

One major characteristics of space manipulators which clearly distinguishes them from ground-fixed ones is the lack of a fixed base in space environment. If a space arm is operated for a certain task, position and attitude of the base satellite which is the foundation of the arm are disturbed by reaction forces and moments due to the arm motion, so it cannot accomplish the task smoothly without provision for this disturbance. No space manipulators can avoid the reaction disturbance to some degree, and this undesirable effect is most remarkable in precise and dexterous operation by robot satellites. Therefore, the conventional control method for ground-fixed manipulators cannot be adopted directly to those robot satellites, and a new control method is needed to cope with this problem.

There are many studies on control methods for space manipulators, but most of them assume that the manipulator base is stationary, i.e., the theory is not different at all from that of conventional ground-fixed manipulators in kinematics or dynamics except that there is no gravitational force. Some advanced papers on modeling and control of space vehicles with flexible appendages investigate in detail the kinematics and dynamics of free-flying mechanical link systems and its application for manipulator control [9]. However, their main interest is analysis and suppression of the vibration due to flexible elements. They do not treat inverse kinematics or trajectory tracking control of space manipulators. Only a few studies address the inverse dynamic problem of free-flying space manipulator arms [10], [11]. They assume that the base body is free to translate under the effect of reaction forces but fixed in attitude by utilizing on-board attitude control systems. They have pointed out, on their assumption of partial restriction, that the problem of kinematics and dynamics can be decoupled. However, this approach requires a high-performance attitude control system and much fuel consumption and drawbacks are increased by mechanical complexity and system weight.

On the other hand, as efficient and general treatment of a control method involving inverse kinematics and inverse dynamics, the authors treat the case without any restrictions on the base body, i.e., the manipulator is mounted on a completely free-flying base satellite under non position or attitude control. In order to examine the reaction effect, the authors pay attention to the momentum conservation law which represent the dynamics of a free-flying system. They introduce the relationship of momentum equilibrium into the kinematic formulation, then derive a new Jacobian matrix in generalized form for space manipulators. The authors have

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The authors are with the Department of Mechanical Engineering Science, Tokyo Institute of Technology, 2-12-1, O-okayama, Meguro-ku, Tokyo 152, Japan.

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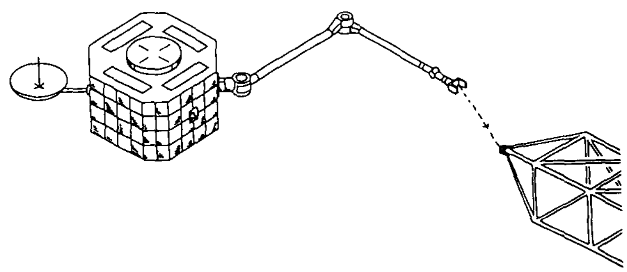


Fig. 1. A schematic illustration of a capture operation in space.

introduced first the idea of this matrix in [12] and [13] then further investigation confirmed that a control method based on the generalized Jacobian is widely applicable for both the completely free-flying (nonrestricted) case and the partially restricted (fixed in attitude only) case.

It should be noted that the most recent paper treats the same problem on free-flying manipulators as indicated in this paper, but with a different approach. Vafa and Dubowsky [14] propose an interesting concept of a "virtual manipulator" method and they discuss work space analysis and inverse kinematic problems of space manipulators. Their method is based on the kinematical investigation of imaginary mechanical links, on the contrary, the present paper treats real mechanical links.

This paper consists of five sections. Section II presents the problem formulation and assumptions which hold throughout this paper. In Section III, the kinematic analysis of space manipulators and the definition of a generalized Jacobian matrix are discussed in closer detail. In addition, the relationship between the proposed new matrix and the conventional Jacobian is also discussed using a simple example in the last part of the section. A kinematic control method based upon the generalized Jacobian matrix is demonstrated by computer simulation with a realistic robot satellite model in Section IV. In this section, 3-axes and 2-axes attitude control problems are also presented as an application of the proposed method.

II. Problem Formulation

A. Description of Problem

As a typical example of on-orbit operations, the authors mainly assume a capture operation. Fig. 1 shows a schematic illustration of a capture operation in which a space robot system is now reaching out its manipulator arm to capture a component part of a certain space structure. The authors regard this task as most important and elementary for space manipulations because, in order to accomplish this operation, the arm should be controlled on the basis that the inverse kinematic problem is clearly resolved. This is the main focus of this paper. The drawback in kinematic problems of free-flying space manipulators is that, as Longman *et al.* [11] and Vafa *et al.* [14] have pointed out in their papers, even the forward kinematics has remarkable difficulty, i.e., the position and orientation of the manipulator end-effector does not have a closed-form solution because it depends on the inertia property which changes according to the configuration, so that the solution cannot be derived without considering the history of the postural change. Moreover, much more difficulty will arise in the inverse problem.

To cope with the problem, the present paper describes the kinematics not by positions or angles but by their motion rates. As discussed in detail in Section III, the relationship between the motion rate of the end-effector and that of joint angles can be linearized excluding from their history and the inverse kinematic problem is solved analytically. From this viewpoint, the authors develop the kinematic formulation paying attention to the inverse problem.

B. Assumptions

In this paper, the authors assume a simple model of a robot satellite which has an articulated manipulator system. In order to clarify the point at issue, they make the following assumptions.

a) The installed manipulator system consists of n links. Each joint has one rotational degree of freedom and is rate-controlled. But the position or attitude of the satellite main body is not controlled at all.

b) At an initial state and during the motion, the position and attitude of the robot system are well known from the inertial coordinate system.

c) There are no mechanical restrictions nor external forces and torques, so that momentum conservations, and equilibrium of forces and moments, strictly hold true during the operation.

d) On the whole, the system is composed of rigid bodies.

As for the above assumptions, the system is regarded a free-flying mechanical chain composed of $n + 1$ rigid bodies.

C. Coordinate System

In order to describe the motion of the entire system from the objective frame, the inertial coordinate system Σ_A is introduced as an absolute coordinate system. In addition the relative coordinate systems fixed to each link Σ_i are defined in Appendix I. The relationship of each coordinate system is represented by a transformation matrix A. Let ${}^jA_i$ be a 3×3 matrix which transforms a vector or a matrix with reference to the ith coordinate system into that with reference to the jth coordinate system. The definition and properties of transformation matrix ${}^jA_i$ are shown in Appendix II.

D. Model and Nomenclature

Fig. 2 shows the model of the robot satellite considered in this paper. The symbols are defined as follows:

$\boldsymbol{r}_i$	position vector of the mass center of link i
$\boldsymbol{r}_G$	position vector of the mass center of the entire system
$\boldsymbol{p}_n$	position vector of the tip of the manipulator
$\boldsymbol{l}_i$	vector pointing from joint i to joint $i + 1$
$\boldsymbol{a}_i$	vector pointing from joint i to the mass center of link i
$\boldsymbol{b}_i$	vector pointing from the mass center of link i to joint $i + 1$
m_i	mass of link i
w	total mass of the system

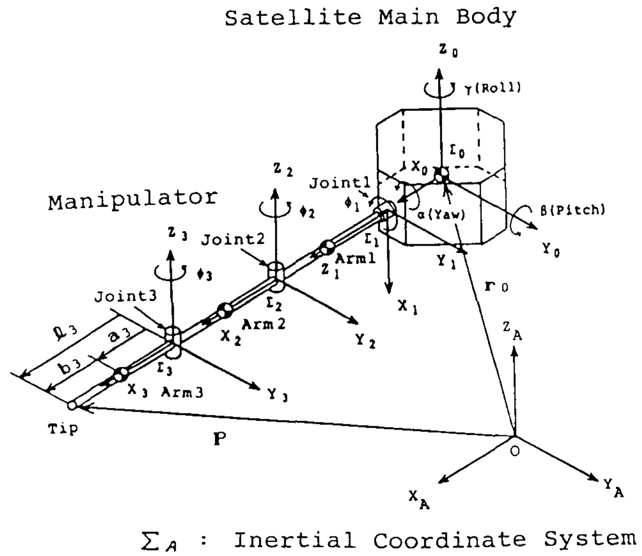


Fig. 2. Model of a robot satellite installed with an articulated manipulator.

- I_i inertia matrix of link i with respect to the mass center
- ω_i angular velocity of link i
- α, β, γ attitude angles of a satellite main body (yaw, pitch, roll)
- ϕ_i rotational angle of joint i

where $i = 0 \cdots n$.

Each vector and matrix is with reference to the absolute coordinate system Σ_A. To clarify the reference coordinate system, vectors and matrices are described with superscript (vector) or super- and subscripts (matrix) as defined in Appendix I. As an exceptional description, vectors or matrices without any indicated scripts can mean those with reference to the absolute coordinate system.

III. Kinematic Formulation

A. Fundamental Equations

Fundamental equations of the system are described generally as follows.

Geometrical definition of the mass center of the system:

$$\sum_{i=0}^{n} m_i \boldsymbol{r}_i = \boldsymbol{r}_G \sum_{i=0}^{n} m_i. \tag{1}$$

Translational momentum conservation:

$$\sum_{i=0}^{n} m_i \dot{\boldsymbol{r}}_i = \text{const.} \tag{2}$$

Rotational momentum conservation:

$$\sum_{i=0}^{n} (\boldsymbol{I}_i \boldsymbol{\omega}_i + m_i \boldsymbol{r}_i \times \dot{\boldsymbol{r}}_i) = \text{const.} \tag{3}$$

Geometrical relation of links:

$$\boldsymbol{r}_i - \boldsymbol{r}_{i-1} = \boldsymbol{a}_i + \boldsymbol{b}_{i-1}. \tag{4}$$

Characteristic equation of manipulator:

$$\boldsymbol{p}_n = \boldsymbol{r}_0 + \boldsymbol{b}_0 + \sum_{i=1}^{n} \boldsymbol{l}_i. \tag{5}$$

Equations (1) and (4) are simultaneous equations for $\boldsymbol{r}_i$ and can be easily solved as

$$^{A}\boldsymbol{r}_i = \sum_{j=1}^{n} K_{ij}(^{A}A_i\, {}^{i}\boldsymbol{a}_i + {}^{A}A_{i-1}\, {}^{i-1}\boldsymbol{b}_{i-1}) + {}^{A}\boldsymbol{r}_G \tag{6}$$

where the coefficient

$$K_{ij} = \begin{cases} \left(\sum_{l=0}^{j-1} m_l\right) \Big/ w & (i \geqq j) \\ -\left(\sum_{l=j}^{n} m_l\right) \Big/ w & (i<j) \end{cases} \tag{7}$$

is a function of the mass ratio of each link. Differentiate (6) with respect to time, then

$$^{A}\dot{\boldsymbol{r}}_i = \sum_{j=1}^{n} K_{ij}(^{A}\dot{A}_i\, {}^{i}\boldsymbol{a}_i + {}^{A}\dot{A}_{i-1}\, {}^{i-1}\boldsymbol{b}_{i-1}) + {}^{A}\dot{\boldsymbol{r}}_G. \tag{8}$$

The differentiation of the transformation matrix A is defined as follows (see Appendix III):

$$^{A}\dot{A}_i = \sum_{k=0}^{i} \dot{\phi}_k \frac{\partial\, {}^{A}A_i}{\partial \phi_k}.$$

By using this definition, $\dot{\boldsymbol{r}}_i$ written as

$$^{A}\dot{\boldsymbol{r}}_i = \sum_{j=0}^{n} \boldsymbol{v}_{ij} \dot{\phi}_j + {}^{A}\boldsymbol{v}_G \tag{9}$$

where

$$\boldsymbol{v}_{ij} = \sum_{k=j}^{n} K_{ik} \left(\frac{\partial\, {}^{A}A_k}{\partial \phi_j}\, {}^{k}\boldsymbol{a}_k + \frac{\partial\, {}^{A}A_{k-1}}{\partial \phi_j}\, {}^{k-1}\boldsymbol{b}_{k-1} \right). \tag{10}$$

$\boldsymbol{v}_G$ is initial velocity of the mass center of the entire system and it is constant as far as (2) is concerned. Equation (9) means that $\dot{\boldsymbol{r}}_i$ is written as a linear combination of vector $\boldsymbol{v}_{ij}$ and angular velocity $\dot{\phi}_j$.

On the other hand, angular velocity of each link with respect to its mass center $\boldsymbol{\omega}_i$ is also written in a similar form.

$$^{A}\omega_i = \sum_{j=0}^{i} {}^{A}A_j\, {}^{j}\omega_j = \sum_{j=0}^{i} (^{A}A_j\, {}^{j}\boldsymbol{u}_j)\dot{\phi}_j + \omega_G \tag{11}$$

where $\boldsymbol{u}_j$ is a unit vector which indicates the rotational axis of joint j. $\boldsymbol{\omega}_G$ is an initial angular velocity of the entire system and it is constant as far as (3) is concerned.

B. Characteristic Equation of Manipulator

The characteristic equation for ordinary ground-fixed manipulators can be written in general form as

$$\boldsymbol{P} = f(\boldsymbol{\phi}) \tag{12}$$

where

$$P=(p_1, p_2, \cdots p_m)^T \qquad \text{task space}$$

$$\phi=(\phi_1, \phi_2, \cdots, \phi_n)^T \qquad \text{joint space.}$$

In general, the inverse transformation of (12) cannot be solved in a simple way, because of nonlinearlity and configuration dependence. However, it is well known that, by differentiating (12) with respect to time, the transformation between $\dot{\phi}$ and $\dot{P}$ can be linearized and the motion rate of end-effector in the task space can be resolved into that of joint variables in the configuration space [15].

$$\dot{P}=J(\phi)\dot{\phi} \tag{13}$$

where

$$J(\phi)=\partial f/\partial \phi$$

is the Jacobian matrix.

A kinematic control method based upon this formulation with the Jacobian matrix is well known as the Resolved Motion Rate Control (RMRC), and the Jacobian is useful for analyzing kinematic properties of a manipulator.

In this paper, the authors adopt RMRC for two reasons. One is that, as pointed out before, the kinematic problems can be treated linearly with the Jacobian matrix and the inverse kinematics problem which is the main focus of this paper can be easily resolved merely by computing an inverse of the Jacobian matrix. If an initial state and the joint motion rate at each step are determined, the trajectory of the position and orientation of the end-effector in the inertial space can be easily obtained by numerical integration. Secondly, by considering the motion rate, it becomes easy to introduce the relations of momentum conservation into the formulation. Note that in order to describe the reaction effect of the system, it is sufficient to consider only motion rates and momenta and not necessary to consider accelerations and forces or torques.

The definition of a new Jacobian matrix is described in two steps. In the first step, the expanded form of Jacobian for the free-flying link system is derived. Then combined with the momentum conservation law, it will be rearranged into a general form involving kinematics and dynamics.

Differentiate the characteristic equation (5) with respect to time, then

$${}^A\dot{p}_n={}^A\dot{r}_0+{}^A\dot{A}_0\,{}^0b_0+\sum_{i=1}^{n}{}^A\dot{A}_i\,{}^il_i. \tag{14}$$

The conventional Jacobian matrix for ground-fixed manipulators is determined from the third term of this equation. However, in this case the former terms ${}^A\dot{r}_0 + {}^A\dot{A}_0{}^0b_0$ which represent the translation and rotation of the base satellite should be considered in the definition of the Jacobian.

By substituting (8)–(10) into (14), the differentiated characteristic equation is expressed as a linear combination with the attitude and joint angular velocities. As a result, the first-step extended Jacobian matrix which is described in an oblique symbol $\bar{J}$ is defined as follows:

$$\dot{P}=\begin{bmatrix}\dot{p}_n\\ \omega_n\end{bmatrix}=\bar{J}(\phi)\dot{\phi}+\dot{P}_0$$
$$=\begin{bmatrix}\bar{J}_\alpha & \bar{J}_\beta & \bar{J}_\gamma & \bar{J}_{\phi_1} & \cdots & \bar{J}_{\phi_n}\\ {}^AA_\alpha i & {}^AA_\beta j & {}^AA_\gamma k & {}^AA_0 k & \cdots & {}^AA_n k\end{bmatrix}\cdot\begin{bmatrix}\dot{\alpha}\\ \dot{\beta}\\ \dot{\gamma}\\ \dot{\phi}_1\\ \vdots\\ \dot{\phi}_n\end{bmatrix}+\dot{P}_0 \tag{15}$$

where

$$\bar{J}_{\phi_i}=\begin{cases} v_{0i}+\dfrac{\partial\,{}^AA_0}{\partial\phi_i}{}^0b_0+\displaystyle\sum_{j=1}^{n}\dfrac{\partial\,{}^AA_j}{\partial\phi_i}{}^jl_j & (\phi_i=\alpha,\beta,\gamma)\\ v_{0i}\qquad\qquad\qquad\;\; +\displaystyle\sum_{j=i}^{n}\dfrac{\partial\,{}^AA_j}{\partial\phi_i}{}^jl_j & (i=1\cdots n)\end{cases}$$

i, j, k are unit vectors, and $\dot{P}_0 = (v_G^T, \omega_G^T)^T$ is an initial translational movement of the mass center of the entire system.

Note that the auxiliary vector v_{ij} defined in (10) is a function of the mass ratio k_{ij}, so that this Jacobian matrix $\bar{J}$ is also a function of the mass ratio in itself.

This equation can be divided into satellite and manipulator parts, and rearranged as follows:

$$\dot{P}=\bar{J}_S\dot{\phi}_S+\bar{J}_M\dot{\phi}_M+\dot{P}_0 \tag{16}$$

where

$\phi_S=(\alpha, \beta, \gamma)^T$ attitude angles of the satellite main body (yaw, pitch, roll)

$\phi_M=(\phi_1 \cdots \phi_n)^T$ joint angles of the manipulator

$\bar{J}_S$ is an $m \times 3$ matrix, $\bar{J}_M$ is an $m \times n$ matrix, and $\bar{J} = (\bar{J}_S, \bar{J}_M)$.

C. Momentum Conservation

Equation (16) is one of the significant equations for kinematic analysis of space manipulators and, if the angular velocity of each joint $\dot{\phi}_M$ (n variables) and that of attitude angles $\dot{\phi}_S$ (3 variables) are given, the motion rate of the end-effector can be easily solved. However, the inverse problem cannot be solved by only this equation, because the equation has only m linear relations for $n + 3$ unknown variables.

In order to solve the inverse kinematics problem, the unknown attitude angular velocities should be determined from other relations. To solve it, the authors introduce the momentum conservation law as a restriction to be satisfied on free-flying mechanical links.

By substituting (9) and (11) into (3), the momentum conservation is also expressed as a linear combination with the attitude and joint angular velocities.

$$[\bar{I}_\alpha, \bar{I}_\beta, \bar{I}_\gamma, \bar{I}_{\phi 1} \cdots \bar{I}_{\phi n}] \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \\ \dot{\phi}_1 \\ \vdots \\ \dot{\phi}_n \end{bmatrix} = L_0 \tag{17}$$

where

$$\bar{I}_{\phi_i} = \left(\sum_{j=i}^{n} {}^A A_j \, {}^j[I_j]_j \, {}^j A_A \right) {}^A A_i \, {}^i u_i + \sum_{j=0}^{n} m_j \, {}^A r_j \times v_{ij}.$$

Equation (17) can also be divided into two parts.

$$\bar{I}_S \dot{\phi}_S + \bar{I}_M \dot{\phi}_M = L_0 \tag{18}$$

where

$\bar{I}_S$ (3 × 3 matrix), $\bar{I}_M$ (3 × n matrix) is a satellite part and a manipulator part of inertia matrix, respectively, and L_0 is initial momentum of the system.

Note that the inertia matrices in oblique symbol $\bar{I}_S$ and $\bar{I}_M$ are neither symmetrical matrices nor tensors.

D. Generalized Jacobian Matrix

As a result of the above discussion, the authors have derived two significant equations; (16) and (18). These equations consist of $m + 3$ independent linear relations for $n + 3$ variables $\dot{\alpha}$, $\dot{\beta}$, $\dot{\gamma}$, $\dot{\phi}_1$, $\cdots$, $\dot{\phi}_n$. So that by combining the equations and eliminating the unknown attitude variables $\dot{\phi}_S$ from them, the motion rate of end effector $\dot{P}$ and joint variables $\dot{\phi}_M$ are expressed in simple form.

$$\dot{\phi}_S = -\bar{I}_S^{-1} \bar{I}_M \dot{\phi}_M + \bar{I}_S^{-1} L_0 \tag{19}$$

therefore

$$\dot{P} = (\bar{J}_M - \bar{J}_S \bar{I}_S^{-1} \bar{I}_M) \dot{\phi}_M + \dot{P}_0 \tag{20}$$

where, rewrite the constant term $\dot{P}_0 + \bar{J}_S \bar{I}_S^{-1} L_0$ as $\dot{P}_0$.

Equation (20) has the form that the manipulator part of the extended Jacobian $\bar{J}_M$ is compensated for a disturbance of reactive movement of the base body. The magnitude of the reactive disturbance is in proportion to the inertia ratio of manipulator and satellite parts $\bar{I}_S^{-1} \bar{I}_M$, so that the larger the satellite inertia, the smaller the disturbance. In this case, the inside terms of the parentheses approaches $\bar{J}_M$ and, in the further limit, it converges to the conventional Jacobian for ground-fixed manipulators, as will be proved in the next subsection using a simple example. As a result, (20) includes the conventional Jacobian matrix and it can be regarded as a general expression for Jacobian matrix of manipulators considering the reaction effect of a free-flying base body. Therefore, the authors summarize the term as $\bar{J}^*$ and name it a "generalized Jacobian matrix" for space manipulators mounted on a free-flying satellite.

$$\dot{P} = \bar{J}^* \dot{\phi}_M + \dot{P}_0 \tag{21}$$

where $\bar{J}^*$ is an $n \times n$ square matrix in case of no redundancy between the task space coordinates and the degree of manipulator freedom.

Equation (21) represents the relationship between the motion rate of the end-effector and that of joint angles and is expressed in a set of closed-form linear equations; the inverse transformation can be solved if $\bar{J}^*$ is nonsingular.

$$\dot{\phi}_M = [\bar{J}^*]^{-1} (\dot{P} - \dot{P}_0). \tag{22}$$

Consequently, utilizing the new generalized Jacobian matrix defined here, the inverse kinematics problem which is significant for space manipulations is solved analytically and a resolved motion rate control method of space manipulators can be developed.

E. Simple Example

In this subsection, we will show the simplest example to demonstrate the relationship between the conventional Jacobian and the proposed new matrix.

We assume a system which has a 2-DOF manipulator (n = 2) and moves on a single plane, and consider only the translation on the x–y plane and the rotation around the z (roll) axis. In this case, the mass ratio coefficients in (6) are

$$\begin{aligned} K_{01} &= -(m_1 + m_2)/w & K_{02} &= -m_2/w \\ K_{11} &= m_0/w & K_{12} &= -m_2/w \\ K_{21} &= m_0/w & K_{22} &= (m_0 + m_1)/w. \end{aligned} \tag{23}$$

Let the initial movements $\dot{P}_0$, L_0 be zero, then $\bar{J}$ and $\bar{I}$ in (15) and (17) are derived and rearranged as follows:

$$\begin{aligned} \bar{J} &= [\bar{J}_S \,\vdots\, \bar{J}_M] \\ &= \begin{bmatrix} -l_0 S_0 & -l_1 S_1 & -l_2 S_2 \\ l_0 C_0 & l_1 C_1 & l_2 C_2 \end{bmatrix} \left[\begin{array}{c:cc} K_a & 0 & 0 \\ K_b & K_b & 0 \\ K_c & K_c & K_c \end{array} \right] \end{aligned} \tag{24}$$

$$\begin{aligned} \bar{I} &= [\bar{I}_S \,\vdots\, \bar{I}_M] \\ &= [h_0, h_1, h_2, hc_1, hc_2, hc_3] \left[\begin{array}{c:cc} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 2 & 1 & 0 \\ 2 & 2 & 1 \\ 2 & 1 & 1 \end{array} \right] \end{aligned} \tag{25}$$

where

$$\begin{aligned} K_a &= (m_0 b_0 / l_0)/w & hc_1 &= (M_0 b_0 a_1 + M_1 b_0 b_1) \cos \phi_1 \\ K_b &= (m_0 + m_1 b_1 / l_1)/w & hc_2 &= (M_1 a_1 a_2 + M_2 b_1 a_2) \cos \phi_2 \\ K_c &= (m_0 + m_1 + m_2 b_2 / l_2)/w & hc_3 &= M_1 b_0 a_2 \cos (\phi_1 + \phi_2) \end{aligned}$$

$$h_0 = I_0 + M_0 b_0^2 \qquad M_0 = m_0(m_1+m_2)/w$$
$$h_1 = I_1 + M_0 a_1^2 + M_2 b_1^2 + 2M_1 a_1 b_1 \qquad M_1 = m_0 m_2/w$$
$$h_2 = I_2 + M_2 a_2^2 \qquad M_2 = (m_0+m_1)m_2/w$$
$$C_i = \cos\left(\sum_{j=0}^{i} \phi_j\right) \qquad S_i = \sin\left(\sum_{j=0}^{i} \phi_j\right).$$

Therefore, the proposed generalized Jacobian matrix (2 × 2 in this case) is described as follows:

$$\dot{P} = \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \bar{J}^* \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix}$$

$$\begin{aligned} \bar{J}^* &= \bar{J}_M - \bar{J}_S \bar{I}_S^{-1} \bar{I}_M \\ &= \begin{bmatrix} -l_1 S_1 & -l_2 S_2 \\ l_1 C_1 & l_2 C_2 \end{bmatrix} \begin{bmatrix} K_b & 0 \\ K_c & K_c \end{bmatrix} \\ &\quad - \begin{bmatrix} -K_a l_0 S_0 - K_b l_1 S_1 - K_c l_2 S_2 \\ K_a l_0 C_0 + K_b l_1 C_1 + K_c l_2 C_2 \end{bmatrix} \\ &\quad \cdot [\bar{I}_{M1}/\bar{I}_S,\ \bar{I}_{M2}/\bar{I}_S] \end{aligned} \tag{26}$$

where

$$\bar{I}_{M1} = h_1 + h_2 + hc_1 + 2hc_2 + hc_3$$
$$\bar{I}_{M2} = h_2 + hc_2 + hc_3.$$

Note that $\bar{I}_S$ includes h_0 (inertia of body 0) but $\bar{I}_{M1}$ and $\bar{I}_{M2}$ do not include it.

Let us consider an extreme case that the base satellite is so massive that it can be regarded as $m_0 \gg m_1, m_2$ and $h_0 \gg h_1, h_2$. In this case, it is obvious that $\bar{I}_{M1}/\bar{I}_S$, $\bar{I}_{M2}/\bar{I}_S$ approaches 0 and K_b, K_c approaches 1. Therefore, $\bar{J}^*$ approaches the following simple matrix:

$$\bar{J}^* \rightarrow \begin{bmatrix} -l_1 S_1 - l_2 S_2 & -l_2 S_2 \\ l_1 C_1 + l_2 C_2 & l_2 C_2 \end{bmatrix}. \tag{27}$$

This matrix is a familiar expression as the conventional Jacobian matrix for a 2-link ground-fixed manipulator.

Consequently, this example proves that the proposed generalized Jacobian matrix includes the conventional Jacobian as an extreme case when the base body is remarkably massive.

IV. Control Problems

A computer simulation is executed in order to verify the control method by means of the proposed Jacobian matrix. We illustrate five types of typical manipulator operation corresponding to (A1, A2) direct kinematics problems (see Figs. 3 and 4), (B) the inverse kinematics problem (see Fig. 5), (C1) 3-axes attitude control problem (see Fig. 6), (C2) a 2-axes attitude control problem (see Fig. 7). For simplicity, they assume that the simulation model has a 3-DOF articulated manipulator system and the dimensions and inertia properties of the system are assumed in Table I as a realistic specification.

A. Direct Kinematics

In this subsection, two types of essential examples for the direct kinematics problem are illustrated. The first one is a deployment operation of the manipulator. In an initial state, the manipulator was assumed to be folded around the satellite main body ($\phi_1 = 0°$, $\phi_2 = -135°$, $\phi_3 = -90°$: see Fig. 3(a)), then operated to stretched posture (ϕ_1, ϕ_2, $\phi_3 = 0°$). Fig. 3(b) represents a given operation rate of manipulator joints. The reactive rotational and translational movement of the base satellite calculated by (19) and (9) with numerical integration is shown in Fig. 3(c) and (d), respectively. The simulated course of postural change during this operation is shown in Fig. 3(e). In this case, the manipulator is operated on the x–y plane, so that the reactive movement occurs only around the roll axis and on the x–y plane, however, it should be emphasized that the reaction disturbance, especially on satellite attitude, is too serious to be neglected.

The second is quite an interesting operation. If the arm is operated in sequence, as shown in Fig. 4(a), the satellite attitude can be changed, even though the final joint state is exactly the same as the initial state (see Fig. 4(b)). This is evidence of the statement that the end-effector position and orientation depend on the motion history in Subsection II-A. Vafa and Dubowsky [14] have proposed that this kind of cyclic motion can be used for satellite attitude control.

B. Inverse Kinematics

The capture operation is the main focus of this paper. The authors arbitrarily assume an initial state of the system and a capture trajectory (which, for simplicity, is a straight line in the following simulation) as shown in Fig. 5(a). The motion rate of end-effector along the prescribed trajectory is given as in Fig. 5(b).

The desired motion rate of manipulator joints for this operation is calculated by (22) and shown in Fig. 5(c). The reactive rotational and translational movement of the base satellite and the postural change of the system during the operation are shown in Fig. 5(d)–(f).

The results show that, although the position and attitude of the base satellite is greatly influenced by the reaction of the arm operation, the end-effector can follow precisely the prescribed trajectory by using the proposed control method.

C. Attitude Control Problem

On the basis of the general formulation for free-flying systems presented here, the attitude control problem of the

TABLE I
SPECIFICATION OF THE SYSTEM

	Satellite Link 0	Link 1	Manipulator Link 2	Link 3
Mass (kg)	2000.0	20.0	50.0	50.0
l (m)	3.5	0.25	2.5	2.5
I_x (kg·m^2)	1400.0	0.10	0.25	0.25
I_y (kg·m^2)	1400.0	0.10	26.0	26.0
I_z (kg·m^2)	2040.0	0.10	26.0	26.0

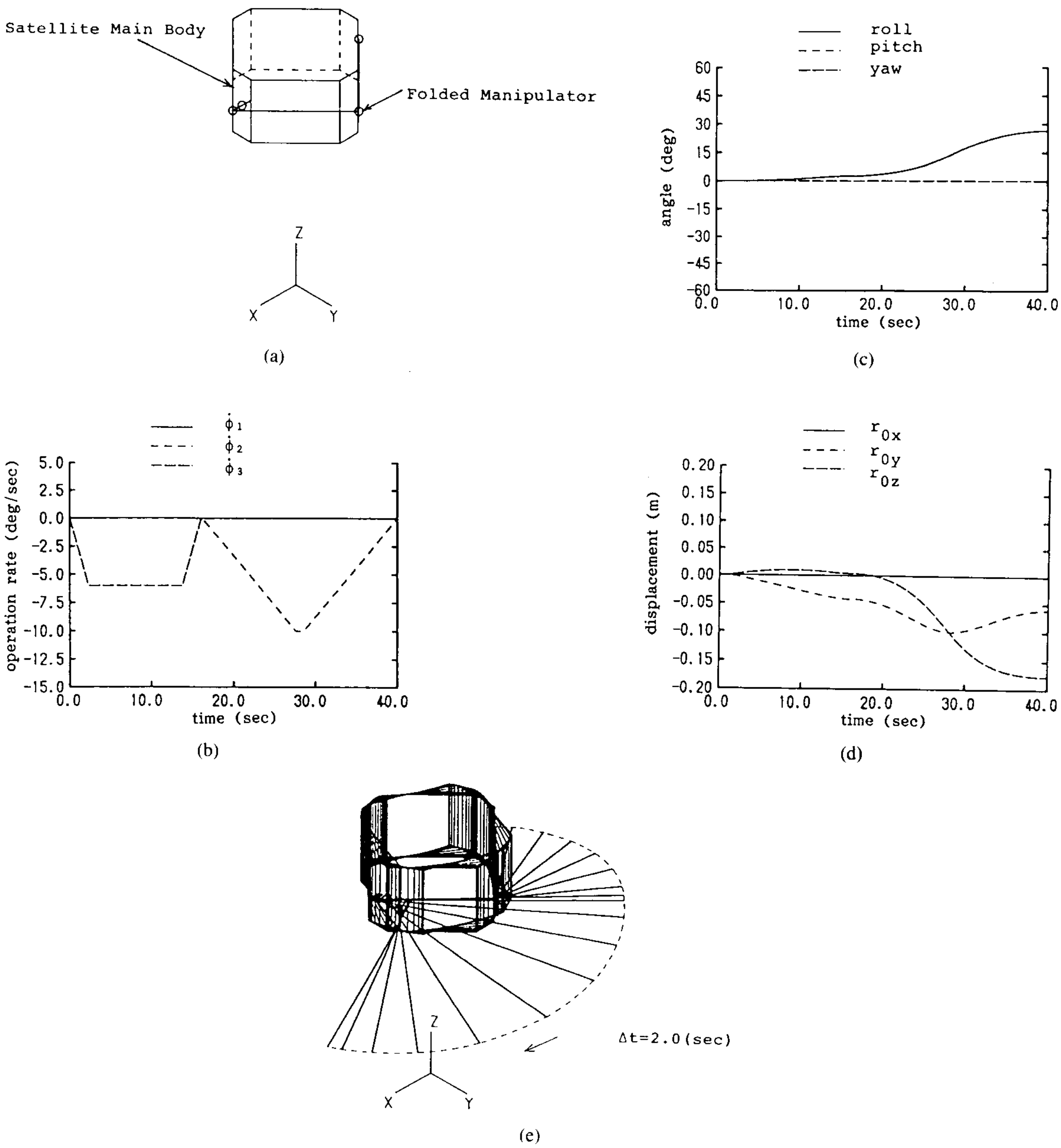


Fig. 3. Simulation (A1): direct kinematics problem—deployment operation. (a) Initial posture. (b) Given operation rate of manipulator joints. (c) Rotational movement of satellite main body. (d) Translational movement of satellite main body. (e) Course of postural change.

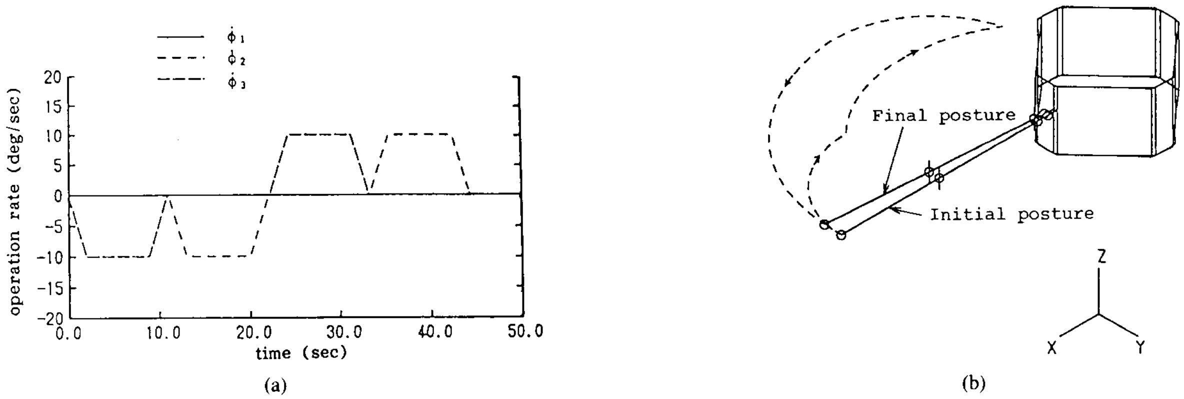


Fig. 4. Simulation (A2): direct kinematics problem—attitude change by a cyclic arm operation. (a) Given operation rate of manipulator joints. (b) Course of postural change.

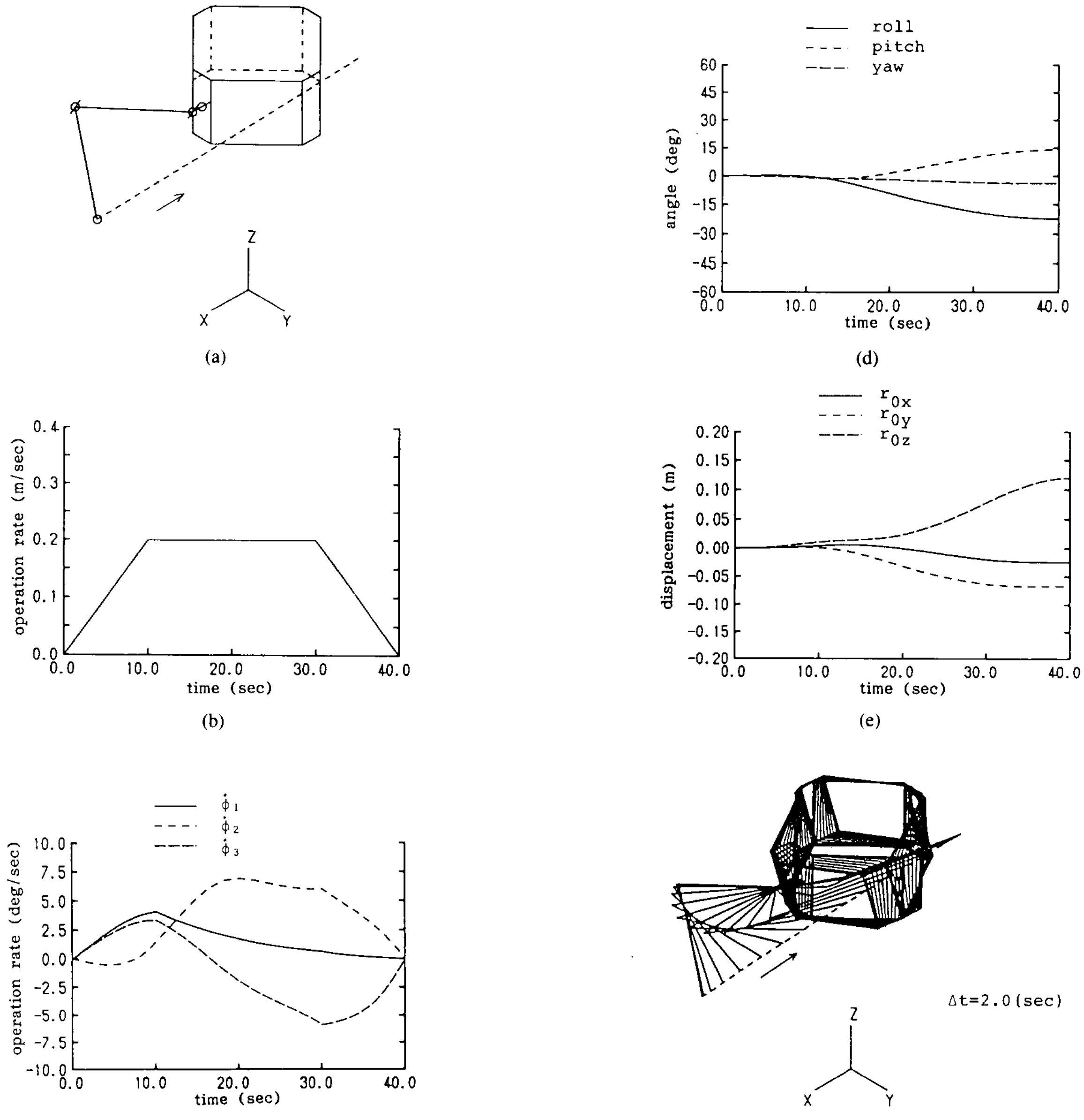
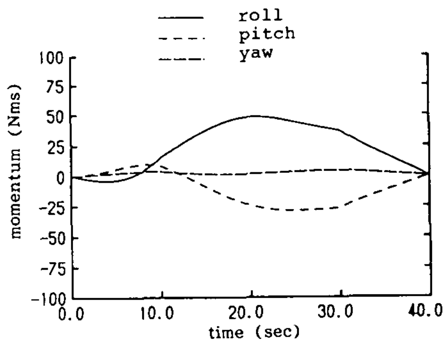
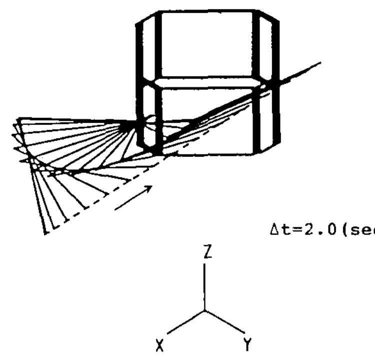


Fig. 5. Simulation (b): inverse kinematics problem—capture operation without attitude control. (a) Initial posture and prescribed trajectory. (b) Motion rate of end-effector along the prescribed trajectory. (c) Calculated operation rate of manipulator joints. (d) Rotational movement of satellite main body. (e) Translational movement of satellite main body. (f) Course of postural change.



(a)



(b)

Fig. 6. Simulation (C1): inverse kinematics problem—capture operation under 3-axes attitude control. (a) Required counter momentum. (b) Course of postural change.

base satellite discussed in previous papers [9], [10] can also be treated.

Let L_c be a counter momentum exerted on the satellite main body by satellite-mounted reaction wheels or thrusters. Rearrange the momentum conservation law (18) with L_c, then

$$\bar{I}_S\dot{\phi}_S+\bar{I}_M\dot{\phi}_M+L_C=\mathbf{0}. \tag{28}$$

Now, the change of satellite attitude is controlled to zero; $\dot{\phi}_S = \mathbf{0}$, then (16) and (28) are rewritten as

$$\dot{P}=\bar{J}_M\dot{\phi}_M+\dot{P}_0 \tag{29}$$

$$\bar{I}_M\dot{\phi}_M+L_C=\mathbf{0}. \tag{30}$$

These equations represent the essence of previous studies on the assumption of a base attitude restriction. As Longman *et al.* [11] have pointed out, the manipulator kinematics (29) and the dynamic effect on the base body (30) are decoupled, i.e., the inverse kinematics can be solved only by (29). Equation (30) corresponds to Longman's concept of compensating moments. From these equations, the required counter momentum is solved as

$$^{A}L_C=-\bar{I}_M\bar{J}_M^{-1}(\dot{P}-\dot{P}_0). \tag{31}$$

Let us show the simulation data of the attitude control problem. The initial state, the capture trajectory, and the motion rate of the end-effector are assumed to be the same as the above capture operation. The desired motion rate of manipulator joints and the required momentum for attitude control are solved by (29)–(31) and shown in Fig. 6(a) and (b). As is obvious from the figure, the proposed method is also useful for the attitude control problem.

In addition, the method is applicable to more complicated problems. The above case is a 3-axes control problem. However, in practical space operations, it is not always best to control the satellite attitude around all three axes. In many cases, contrary restrictions exist; some part of the satellite should indicate the same direction throughout the mission, for example, to maintain communication. On the other hand, energy consumption for attitude control should be avoided as much as possible to preclude taxing the power supply system. In these cases, it is effective to control the satellite around two axes and allowing rotation around the third. For example, in a case where rotation around the roll axis is permitted, set the variables $\dot{\alpha}$ (yaw) $= 0$, $\dot{\beta}$ (pitch) $= 0$, and L_c (roll) $= 0$ in (29) and (30), then the change of attitude around the roll axis $\dot{\gamma}$ and the required counter momentum around the other two axes L_c (pitch) and L_c (yaw) are solved by these equations.

The simulation results for the 2-axes control problem are illustrated with the same conditions as above. The required momentum around the yaw and pitch axes and the change of attitude around the roll axis are shown in Fig. 7 (a) and (b), respectively. Fig. 7(c) shows that the roll axis of the satellite indicates the same direction during the operation.

If comparing two cases of required momentum, Fig. 6(a) and Fig. 7(a), it is obvious that in the case of 2-axes attitude

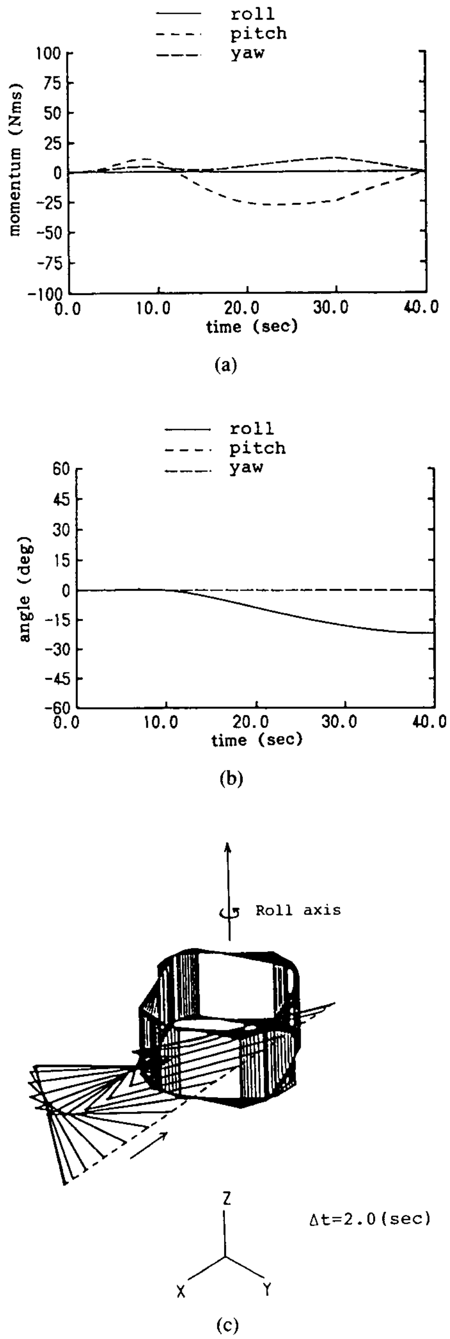


Fig. 7. Simulation (C2): inverse kinematics problem—capture operation under 2-axes attitude control. (a) Required counter momentum. (b) Rotational movement of satellite main body. (c) Course of postural change.

control, the total required momentum—which corresponds to the energy consumption for attitude control—is lower than in the 3-axes control case. The discussion on the amount of required momentum will be useful for designing the capacity of on-board attitude control assemblies in a practical application.

VI. Concluding Remarks

This paper discussed the kinematics of space free-flying multibody systems and proposed the generalized Jacobian matrix for space manipulators, taking dynamical interactions between the arm and the satellite into account, which proved to be an extended and generalized expression of the conventional Jacobian matrix for ground-fixed manipulators. The authors developed a resolved motion rate control method for space manipulators with the proposed Jacobian and applied it to capture operations or trajectory tracking problems. In addition, the presented method was shown to be widely applicable to the problems of three-axes and two-axes satellite attitude control simultaneous with manipulator operation.

The significance of the method was demonstrated by computer simulations, assuming a realistic specification of a robot satellite. Simulation results on the magnitude of satellite attitude change due to the manipulator reaction (free-flying case) and the amount of required counter momentum (attitude control case) are useful for designing control software and mechanical hardware of practical space robot systems.

The authors have, in this paper, limited discussions within the off-line simulation of rate control problems. However, the proposed generalized Jacobian matrix will provide the basis for further advanced and intelligent control methods of space manipulators.

Appendix I

Description of Links, Joints, Coordinates, Vectors, and Matrices

Let link 0 be a satellite main body, link i ($i = 1 \cdots n$) the ith arm of the manipulator in order, and joint i a joint which connects link $i-1$ and link i. The ith coordinate system Σ_i: (x_i, y_i, z_i) for $i = 1 \cdots n$ is assigned to be an orthogonal coordinate system fixed on link i which originates in joint i and the axis z_i corresponds to its rotational axis. Exceptionally, the 0th coordinate system Σ_0 is fixed on a satellite main body which originates in its gravity center, and the absolute (inertial) coordinate system Σ_A is fixed in the space.

To clarify the reference coordinate system of vectors and matrices, vectors are affixed with a superscript on the left side of the symbol such that ${}^i\boldsymbol{r}_j$ indicates a vector $\boldsymbol{r}_j$ with reference to the ith coordinate system, and matrices are affixed with super- and subscripts such that ${}^i[\boldsymbol{I}_j]_i$ indicates a matrix $\boldsymbol{I}_j$ with reference to the ith coordinate system.

Appendix II

Transformation Matrix

Let ${}^{i-1}A_i$ be a 3×3 matrix which transforms vectors or matrices with reference to the coordinate system Σ_i into Σ_{i-1}. For $i = 1 \cdots n$, the transformation matrix is defined as

$$ {}^{i-1}A_i = E^{z\phi_i}E^{x\psi_i} = \begin{bmatrix} C\phi_i & -S\phi_i C\psi_i & S\phi_i S\psi_i \\ S\phi_i & C\phi_i C\psi_i & -C\phi_i S\psi_i \\ 0 & S\psi_i & C\psi_i \end{bmatrix} $$

$$ {}^{i}A_{i-1} = E^{x(-\psi_i)}E^{z(-\phi_i)} = \begin{bmatrix} C\phi_i & S\phi_i & 0 \\ -C\psi_i S\phi_i & C\psi_i C\phi_i & S\psi_i \\ S\psi_i S\phi_i & -S\psi_i C\phi_i & C\psi_i \end{bmatrix} $$

where $E^{u\phi}$ is a rotational transformation tensor around the $\boldsymbol{u}$ axis and ψ_i is a twist angle of the joint. The transformation of the absolute coordinate system (Σ_A) and the satellite-fixed coordinate system (Σ_0) is a roll-pitch-yaw transformation, therefore

$$
\begin{aligned}
{}^A A_0 &= E^{z\gamma} E^{y\beta} E^{x\alpha} \\
&= \begin{bmatrix} C\gamma & -S\gamma & 0 \\ S\gamma & C\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\beta & 0 & S\beta \\ 0 & 1 & 0 \\ -S\beta & 0 & C\beta \end{bmatrix} \\
&\quad \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\alpha & -S\alpha \\ 0 & S\alpha & C\alpha \end{bmatrix}
\end{aligned}
$$

or

$$
{}^0 A_A = E^{x(-\alpha)} E^{y(-\beta)} E^{z(-\gamma)}.
$$

By using the matrix A, the transformations of vectors and matrices are described as follows:

$$
{}^k \boldsymbol{r}_j = {}^k A_i \, {}^i \boldsymbol{r}_j
$$

(transform a vector $\boldsymbol{r}_j$ with reference to Σ_i into Σ_k)

$$
{}^k [\boldsymbol{I}_j]_k = {}^k A_i \, {}^i [\boldsymbol{I}_j]_i \, {}^i A_k
$$

(transform a matrix $\boldsymbol{I}_j$ with reference to Σ_i into Σ_k).

Matrix A satisfies the following relation:

$$
{}^j A_i = {}^j A_{j-1} \, {}^{j-1} A_{j-2} \cdot \cdots \cdot {}^{i+2} A_{i+1} \, {}^{i+1} A_i \qquad (j > i)
$$

$$
[{}^j A_i]^{-1} = [{}^j A_i]^T = {}^i A_j .
$$

Appendix III

Differentiation of the Transformation Matrix

The differentiation of ${}^{i-1}A_i$ with respect to time is defined as

$$
\begin{aligned}
{}^{i-1}\dot{A}_i &= \dot{\phi}_i \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} C\phi_i & -S\phi_i & 0 \\ S\phi_i & C\phi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&\quad \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\psi_i & -S\psi_i \\ 0 & S\psi_i & C\psi_i \end{bmatrix} \\
&= \dot{\phi}_i D^z E^{z\phi_i} E^{x\psi_i} \equiv \dot{\phi}_i \frac{\partial \, {}^{i-1}A_i}{\partial \phi_i}
\end{aligned}
$$

or

$$
{}^i \dot{A}_{i-1} = \dot{\phi}_i E^{x(-\psi_i)} D^{(-z)} E^{z(-\phi_i)} \equiv \dot{\phi}_i \frac{\partial \, {}^i A_{i-1}}{\partial \phi_i}
$$

where D^u is a differential operator.

The differentiation of the satellite attitude transformation is defined as

$$
\begin{aligned}
{}^A \dot{A}_0 &= \frac{d}{dt} E^{z\gamma} E^{y\beta} E^{x\alpha} \\
&= \dot{\alpha} E^{z\gamma} E^{y\beta} D^x E^{x\alpha} \\
&\quad + \dot{\beta} E^{z\gamma} D^y E^{y\beta} E^{x\alpha} \\
&\quad + \dot{\gamma} D^z E^{z\gamma} E^{y\beta} E^{x\alpha} \\
&\equiv \dot{\alpha} \frac{\partial \, {}^A A_0}{\partial \alpha} + \dot{\beta} \frac{\partial \, {}^A A_0}{\partial \beta} + \dot{\gamma} \frac{\partial \, {}^A A_0}{\partial \gamma} .
\end{aligned}
$$

For simplicity of the description, the authors summarize it as

$$
\begin{aligned}
{}^A \dot{A}_0 &= (\dot{\alpha}, \dot{\beta}, \dot{\gamma}) \cdot \left(\frac{\partial \, {}^A A_0}{\partial \alpha}, \frac{\partial \, {}^A A_0}{\partial \beta}, \frac{\partial \, {}^A A_0}{\partial \gamma} \right) \\
&= \dot{\phi}_0 \frac{\partial \, {}^A A_0}{\partial \phi_0} .
\end{aligned}
$$

With these definitions, the differentiation of ${}^A A_i$ is described as follows:

$$
\begin{aligned}
{}^A \dot{A}_i &= \frac{d}{dt} ({}^A A_0 \, {}^0 A_1 \, {}^1 A_2 \cdots {}^{i-1} A_i) \\
&= {}^A \dot{A}_0 \, {}^0 A_1 \, {}^1 A_2 \cdots {}^{i-1} A_i \\
&\quad + {}^A A_0 \, {}^0 \dot{A}_1 \, {}^1 A_2 \cdots {}^{i-1} A_i \\
&\quad + \cdots \\
&\quad + {}^A A_0 \, {}^0 A_1 \, {}^1 A_2 \cdots {}^{i-1} \dot{A}_i \\
&= \dot{\phi}_0 \frac{\partial \, {}^A A_0}{\partial \phi_0} {}^0 A_i + \dot{\phi}_1 \, {}^A A_0 \frac{\partial \, {}^0 A_1}{\partial \phi_1} {}^1 A_i \\
&\quad + \cdots + \dot{\phi}_i \, {}^A A_{i-1} \frac{\partial \, {}^{i-1} A_i}{\partial \phi_i} \\
&= \sum_{j=0}^{i} \dot{\phi}_j \frac{\partial \, {}^A A_i}{\partial \phi_j}
\end{aligned}
$$

where

$$
{}^A A_k \frac{\partial \, {}^k A_j}{\partial \phi_j} {}^j A_i \equiv \frac{\partial \, {}^A A_i}{\partial \phi_j} .
$$

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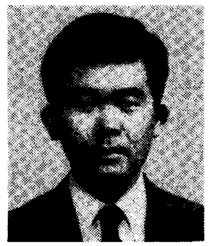
Yoji Umetani was born in Osaka, Japan, on December 12, 1932. He received the B.Eng. degree in 1956 from Kyoto University and the Dr.Eng. degree in 1969 from Tokyo Institute of Technology.

From 1959 to 1970 he was with the Research Institute on Industrial Science, the University of Tokyo as Lecturer. In 1970 he became Associate Professor, in 1975 Professor, both at the Department of Mechanical Engineering Science, Tokyo Institute of Technology. He is also Concurrent Professor at the Institute of Space and Astronautical Science. Since 1987 he has been Chairman of the Research Forum on Space Robotics and Automation.

Dr. Umetani is a member of Robotic Industries of America, Japan Society for Aeronautical and Space Science, the Robotic Society of Japan, and Japan Society for Artificial Intelligence.



Kazuya Yoshida was born in Tokyo, Japan, on December 29, 1960. He received the B.Eng. and M.Eng. degrees in 1984 and 1986, respectively, from Tokyo Institute of Technology.

In 1985 he started research on space robotics and his current interests include modeling, kinematics, dynamics, and control of free-flying multi-link systems. Since 1986 he has been Research Associate at the Department of Mechanical Engineering Science, Tokyo Institute of Technology.

Mr. Yoshida is a member of the Robotics Society of Japan, the Society of Instrument and Control Engineers, and the Research Forum on Space Robotics and Automation.