Model Description

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1 Introduction

The lot-sizing models presented in the following chapters are aimed to solve the medium and long-term planning problem of a footwear injection machine.

Given the existence of set-ups every time a mold is exchanged, it is necessary to understand which orders and items should be produced each week.

The goal is to provide a more comprehensive view of the planning horizon to the machine planner and create solutions that meet the requirements of the production system while minimizing costs.

2 Multi-Item Lot-Sizing Problem

This first Multi-Item Lot-Sizing problem was based on a model proposed by Pochet [1]. In the typical multi-item lot-sizing problem, there are m items, n periods, demands for item i in period t, individual production limits, production costs, storage costs, fixed costs, machine production rates, set-up times, and machine capacity. It was also included backlogging and cost variables that allowed the items to be delivered later than the defined date.

The created model is the following:

Indices

I	types and sizes of shoes
O	production orders
W	planning horizon weeks
WT	planning horizon weeks including week 0
M	injection moulds

Parameters

d_{iot}	$i \in I, o \in O, t \in W$	Demand of Item i and Order o in Week t
p_i	$i \in I$	Fixed production cost of Item i
h_i	$i \in I$	Fixed storage cost of Item i
w_i	$i \in I$	Fixed backlogging cost of Item i
a_i	$i \in I$	Time to produce Item i
b_m	$m \in M$	Time to set-up Mould m
$U_{m,i}$	$m \in M, i \in I$	Utilization of Mould m to produce Item i
B_t	$t \in W$	Total machine time in Week t
$MouldTime$		Total time per mould
M		A big number

Decision Variables

x_{iot}	$i \in I, o \in O, t \in W$	Quantity of Item i in Order o produced in Week t
st_{iot}	$i \in I, o \in O, t \in W$	Quantity of Item i in Order o in stock in Week t
r_{iot}	$i \in I, o \in O, t \in W$	Quantity of Item i in Order o backlogged in Week t
y_{mt}	$m \in M, t \in W$	Activation/Set-Up of Mould m in Week t

Objective Function

$$ \tag{2.1}$$

$$\text{Minimize} \sum_{i \in I, o \in O, t \in W} (x_{iot} \times p_i + st_{iot} \times h_i + r_{iot} \times w_i) \tag{2.2}$$

$$ \tag{2.3}$$

Constraints

$$ \tag{2.4}$$

$$x_{iot} + st_{iot-1} + r_{iot} = d_{iot} + r_{iot-1} + st_{iot} \quad \forall i \in I, o \in O, t \in W \tag{2.5}$$

$$\sum_{o \in O} x_{iot} \times a_i \leq \sum_{(m,i') \in MI : i' = i} (MouldTime \times y_{mt} \times U_{mi'}) \quad \forall i \in I, t \in W$$

$$ \tag{2.6}$$

$$\sum_{i \in I, o \in O} (x_{iot} \times a_i) + \sum_{m \in M} (y_{mt} \times b_i) \leq B_t \quad \forall t \in T$$

$$ \tag{2.7}$$

$$x_{iot} \geq 0 \quad \forall i \in I, o \in O, t \in W \tag{2.8}$$

$$st_{iot} \geq 0 \quad \forall i \in I, o \in O, t \in W \tag{2.9}$$

$$r_{iot} \geq 0 \quad \forall i \in I, o \in O, t \in W \tag{2.10}$$

$$ \tag{2.11}$$

The objective function focuses on minimizing production costs, inventories and delays. Constraint 1 are the product conservation equations, constraint 2 ensures that the mould set-up that the item uses has been done. It also limits the production capacity of the mould. Constraint 3 is the machine capacity constraint, linking the production of different items. Constraints 4,5 and 6 are the non negativity constraints.

After presenting the first solution to the planning managers, it became apparent that the model did not fit the company's reality. The industry in which AMF operates has high volatility in customer orders and in the delivery of materials by suppliers.

One of the factors to take into account is the availability of raw materials. Any delay or anticipation in the arrival of raw materials can influence planning and must be considered.

The other factor to take into account is the productivity of the machine. The company has the ability to hold stocks at low costs and to anticipate the order deliveries and therefore focuses on getting as many orders as possible. One of the planning goals is to reduce the number of weeks it takes to produce all the selected production orders. This can lead to loading the machine close to 100% for every week, offering little flexibility.

After discussions with management and the AMF planning managers, it was agreed that the optimal utilization of the machine would be around 100% for the first two weeks and 70% for the remaining weeks. This planning method allows the machine to be used at maximum capacity in the weeks immediately ahead, where usually no changes occur, and in the following weeks, there is the ability to change or add orders that arise at short notice.

3 Standard Lot-Sizing Problem for AMF Safety Shoes

After the realizations made in the previous chapter, some modifications to the existing model were implemented to meet the company's demands.

The model adaptation required five new parameters and three decision variables. In addition, existing constraints were revised and a new one was added.

New Parameters

gas_{iot}	$i \in I, o \in O, t \in W$	Raw material of Item i and Order o arriving in Week t
EB_t	$t \in W$	Extra machine time in Week t
$MouldTimeExtra$		Extra time per mould
α		Price of activating the extra time in a week
β		Price of using a week to production

New Decision Variables

e_t	$t \in W$	Activation of extra time on Week t
ms_t	$t \in W$	Activation of production in Week t
$disp_{iot}$	$i \in I, o \in O, t \in W$	Availability of raw material for Item i in Order o in Week t

Objective Function

$$\tag{3.1}$$

$$\text{Minimize} \sum_{i \in I, o \in O, t \in W} r_{iot} \times w_i + \sum_{t \in W} (e_t \times \alpha + ms_t \times \beta \times t) \tag{3.2}$$

$$\tag{3.3}$$

New Constraints

$$\tag{3.4}$$

$$ms_t \times x_{iot} + st_{iot-1} + r_{iot} = d_{iot} + r_{iot-1} + st_{iot} \quad \forall i \in I, o \in O, t \in W \tag{3.5}$$

$$\sum_{o \in O} x_{iot} \times a_i \leq \sum_{(m,i') \in MI : i'=i} ((MouldTime + e_t \times MouldTimeExtra) \times y_{mt} \times U_{mi'}) \quad \forall i \in I, t \in W$$

$$\tag{3.6}$$

$$\sum_{i \in I, o \in O} (x_{iot} \times a_i) + \sum_{m \in M} (y_{mt} \times b_i) \leq B_t + EB_t \times e_t \quad \forall t \in W$$

$$\tag{3.7}$$

$$gas_{iot-1} + disp_{iot-1} - x_{iot} = disp_{iot} \quad \forall i \in I, o \in O, t \in W \tag{3.8}$$

$$disp_{iot} \geq 0 \quad \forall i \in I, o \in O, t \in W \tag{3.9}$$

In the Standard Lot-Sizing problem AMF two main changes have been made. The first is the addition of extra time each week to determine what percentage of machine utilization is desired by AMF. The second is the raw material requirement in order to manufacture a product.

The objective function continues to minimize costs, except in this case, it considers the costs of delays, the costs of using extra time, and the costs of using the machine. The parameters α and β where defined during the test sessions.

In constraint 1 the binary variable of machine activation in the week was added. In constraints 2 and 3 the possibilities of activation of overtime were added, and consequent increase of the total production capacity and per mold.

Restriction 4 is used for the conservation of raw materials.

After several test sessions with members of the company, we understood that the model was adapted to AMF's reality and allowed for the flexibility needed in a real production system. Despite the success in adapting the model, there was an issue that the model did not solve. As mentioned in chapter 1, the company can negotiate with its customers to deliver orders earlier than planned. However, the delivery windows vary from customer to customer and from order to order. Therefore, the model must take this information into account to present the best long-term planning solution.

4 Time-Windows Lot-Sizing Problem for AMF Safety Shoes

To solve the problem presented in the last chapter, the previous model was reconstructed not to have a single delivery week but a time window in which no stocks or delays would be charged. This reconstruction was based in a model proposed by Someone [1]

New Parameters

d_{io}	$i \in I, o \in O$	Raw material of Item i and Order o (previously with defined week t)
$EWeek_{io}$	$i \in I, o \in O$	Earliest possibly week to deliver item i in order o
$LWeek_{io}$	$i \in I, o \in O$	Latest possibly week to deliver item i in order o

New Decision Variables

e_t	$t \in W$	Activation of extra time on Week t
ms_t	$t \in W$	Activation of production in Week t
TD_{iot}	$i \in I, o \in O, t \in W$	Demand for Item i in Order o in Week t

Objective Function

$$\tag{4.1}$$

$$\text{Minimize} \sum_{i \in I, o \in O, t \in W} (r_{iot} \times w_i + st_{iot} \times h_i) + \sum_{t \in W} e_t \times \alpha \tag{4.2}$$

$$\tag{4.3}$$

New Constraints

$$\tag{4.4}$$

$$ms_t \times x_{iot} + (st_{iot-1} - r_{iot-1}) - TD_{iot} = -r_{iot} + st_{iot} \qquad \forall i \in I, o \in O, t \in W \tag{4.5}$$

$$TD_{iot} = 0 \qquad \forall i \in I, o \in O, t \in 1...EWeek[i,o] - 1 \tag{4.6}$$

$$TD_{iot} = 0 \quad \forall i \in I, o \in O, t \in LWeek[i,o] + 1...last(Weeks) \tag{4.7}$$

$$TD_{iot} 0 \qquad \forall i \in I, o \in O, t \in EWeek[i,o]...LWeek[i,o] \tag{4.8}$$

$$\sum_{t \in EWeek[i,o]...LWeek[i,o]} TD_{iot} = dio \qquad \forall i \in I, o \in O$$

$$\tag{4.9}$$

$$\tag{4.10}$$

Adding the parameters EWeek and LWeek allows us to define the time windows within which we can deliver the product. This leads to a change in the objective function, where instead of minimizing the use of weeks, we now minimize the costs with stocks and backlogs without forgetting the use of extra time.

Constraints 1 was changed in order to work with the new model and we added constraints 7, 8, 9 and 10 that have the job of defining which dates the product can be demanded.ALTERAR

The results from this model were interesting, but they require prior in-depth knowledge of the customers' needs and capabilities. The large amount of time required to obtain this information slows down the planning processes. Due to this limitation, the model used by the company was the one presented in Chapter 3.

References

[1] D. Adams. *The Hitchhiker's Guide to the Galaxy*. San Val, 1995.