1 Numpy

Generate matrices A, with random Gaussian entries, B, a Toeplitz matrix, where $A \in \mathbb{R}^{n \times m}$ and $B \in \mathbb{R}^{m \times m}$, for n = 200, m = 500.

Exercise 1.1: Matrix operations
Calculate $A + A$, $AA^\top$, $A^\top A$ and AB. Write a function that computes $A(B - \lambda I)$ for any λ.

Exercise 1.2: Solving a linear system
Generate a vector b with m entries and solve $Bx = b$.

Exercise 1.3: Norms
Compute the Frobenius norm of A: $\|A\|_F$ and the infinity norm of B: $\|B\|_\infty$. Also find the largest and smallest singular values of B.

Exercise 1.4: Power iteration
Generate a matrix Z, $n \times n$, with Gaussian entries, and use the power iteration to find the largest eigenvalue and corresponding eigenvector of Z. How many iterations are needed till convergence?

Optional: use the `time.clock()` method to compare computation time when varying n.

Exercise 1.5: Singular values
Generate an $n \times n$ matrix, denoted by C, where each entry is 1 with probability p and 0 otherwise. Use the linear algebra library of Scipy to compute the singular values of C. What can you say about the relationship between n, p and the largest singular value?

Exercise 1.6: Nearest neighbor
Write a function that takes a value z and an array A and finds the element in A that is closest to z. The function should return the closest value, not index.

Hint: Use the built-in functionality of Numpy rather than writing code to find this value manually. In particular, use brackets and `argmin`.

2 Scipy

Exercise 2.1: Least squares
Generate matrix $A \in R^{m \times n}$ with $m > n$. Also generate some vector $b \in R^m$.

Now find $x = \arg\min_x \|Ax - b\|_2$.

Print the norm of the residual.

Exercise 2.2: Optimization
Find the maximum of the function

$$f(x) = \sin^2(x - 2)e^{-x^2}$$

Exercise 2.3: Pairwise distances
Let X be a matrix with n rows and m columns. How can you compute the pairwise distances between every two rows?

As an example application, consider n cities, and we are given their coordinates in two columns. Now we want a nice table that tells us for each two cities, how far they are apart.

Again, make sure you make use of Scipy's functionality instead of writing your own routine.

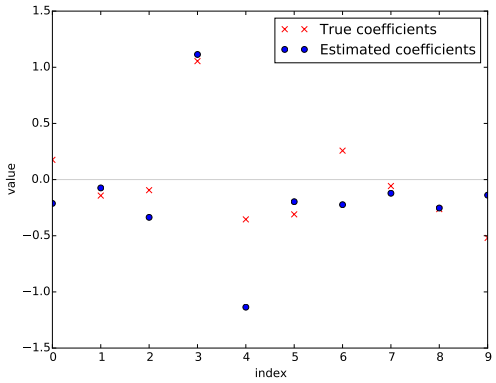

Figure 1: Parameter plot

3 Matplotlib

Exercise 3.1: Plotting a function

Plot the function

$$f(x) = \sin^2(x-2)e^{-x^2}$$

over the interval $[0, 2]$. Add proper axis labels, a title, etc.

Exercise 3.2: Data

Create a data matrix X with 20 observations of 10 variables. Generate a vector b with parameters Then generate the response vector $y = Xb+z$ where z is a vector with standard normally distributed variables.

Now (by only using y and X), find an estimator for b, by solving

$$\hat{b} = \arg\min_b \|Xb - y\|_2$$

Plot the true parameters b and estimated parameters $\hat{b}$. See Figure 1 for an example plot.

Exercise 3.3: Histogram and density estimation

Generate a vector z of 10000 observations from your favorite exotic distribution. Then make a plot that shows a histogram of z (with 25 bins), along with an estimate for the density, using a Gaussian kernel density estimator (see scipy.stats). See Figure 2 for an example plot.

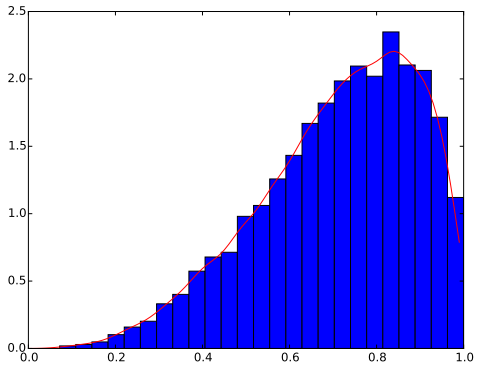

Figure 2: Histogram