Some of the following from: <http://web.pdx.edu/~sytsmam/limno/Limno09.8.Stratification.pdf>

Note: there is an R Package called RLakeAnalyzer

# Stability of a Parcel of Water

The stability of a parcel of water in a water column may be described by the Richardson number:

g(dρ/dz)Ri =ρ(du/dz)2

g = acceleration of gravity

ρ = density

u = horizontal velocity

z = depth

When the Richardson number is less than 0.25, internal waves spontaneously appear, and break. Mixing ensues until gradients are reduced and the system again stabilizes.

But Note that this requires data on horizontal velocity, and especially the vertical gradient in horizontal velocity, which in general I don’t have.

An alternative is the Brunt Vaisala frequency

The two are clearly closely related, except where there is a lot of vertical velocity structure, which is unlikely in lakes. This is depth-specific.

# Derived from basic principals:

Assume a 1cm cubed block of water embedded in a lake. How much work is involved in moving up up 1 cm (ignoring friction and other dissipative forces)?

Work = f\*D

Force here is determined by difference in density, scaled by acceleration due to gravity.

The mass we are moving is 1 \* p. Net force on that. However, is the difference in densities top to bottom

which is just g \* dp/dz p

# Stability of a lake

The stability of a lake is the amount of work that would be required to mix the heat in the lake uniformly over depth.

The calculation of the stability of a lake is based on a comparison between the vertical location of the center of mass of the stratified lake compared to the vertical location of the center of mass of the same lake after complete mixing (and with the same total heat content).

[See discussion of Schmidt stability index, p164ff.]The concept of stability was introduced by Schmidt in 1915. Few authors have calculated S since, however, the term has become engrained in limnological theory and vocabulary. It is defined as the amount of work needed to mix the entire body of water to uniform temperature without addition or subtraction of heat, or the inertial resistance to complete mixing caused by vertical density differences.

A0 = surface area of the lake

Z = the height above the bottom

Zg = the center of volume in [cm] above the bottom

Az = the area of the lake at depth z

ρz = density of water at depth zρ

m = density at complete mixing

In practice, of course, this is calculated as a sum over convenient depths, thus based on a rectangular approximation to the integral.

If density is uniform from top to bottom, stability is zero; no work must be performed to promote heterogeneity. As surface waters warm the so-called center of gravity of the lake moves deeper into the water column as a result of vertical differences in density.

An Alternative formula in the same reference is:

(But this looks like it may be the negative of the last index????)(

St = Schmidt stability

Am = Surface area of the lake (cm2)

z = height above the bottom (cm)

Zm = maximum depth of the lake (cm)

Zg = center of volume in (cm) above bottom

AZ = area of the lake at height Z (cm2)

ρZ = density of water at depth z (g cm-3)

Issue here is, again, for most of these lakes, we don’t have sufficient data on lake morphometry. I could use a “deep hole” approximation, calculating stability for an idealized water column at the point of observation. Alternatively, I could generate some sort of assumption about morphometry based only on max depth and mean depth.

# Heat Content / Heat Budget

Heat storage is a simple calculation based upon mass, temperature, and specific heat. In general, we assume that a gram of water = 1 cc or 1 ml and that its heat content is milliliters x degrees C. Thus, 1 ml of water at 8 C contains 8 calories of heat. This makes calculating heat content of a lake relatively straightforward.

# Other Summaries of Stratification

Depth of thermocline

Intensity of thermocline

Date of onset of stratification (2C)

Date of disappearance of stratification (2C)

Duration of Stratification

# rLake Analyzer

Jordan S.Read, David P.Hamilton, Ian D.Jones, Kohji Muraoka, Luke A.Winslow, RyanKroiss, Chin H.Wua, Evelyn Gaisere. 2011. Derivation of lake mixing and stratification indices from high-resolution lake buoy data. Environmental Modelling & Software. Volume 26, Issue 11, November 2011, Pages 1325-1336 <https://doi.org/10.1016/j.envsoft.2011.05.006>

<https://cran.r-project.org/web/packages/rLakeAnalyzer/rLakeAnalyzer.pdf>

# Issues / Questions

1. Is climate change affecting lake thermal properties?
   1. Stability
   2. Depth pf Thermocline
   3. Period of stratification
      1. Onset, duration, timing of fall overturn
2. Are other lake water quality indicators correlated with thermal properties?