and the esthetic value of the hypothesis, e.g., its elegance and simplicity. Many of the classical paradoxes with regard to induction can be resolved if we realize that the crucial factor determining the acceptability of a hypothesis is not confirmation but the extra-evidential extra-logical considerations [10].

This takes the process of induction clearly out of the reach of purely mechanical data processing. We can produce a semblance of inductive inference with a computer by formulating esthetic criteria in machine-legible sentences, but we have to point out first that these rules cannot be produced by a machine, and second that the criteria of esthetic values cannot be properly formulated in terms of abstract symbols. (See the end of Section III.) We may, therefore, conclude that all machine imitation of induction has either to omit some important factor of induction or to smuggle in man-made factors under some disguise.

VII. Pattern Recognition

Pattern recognition can mean various things, but a typical task of pattern recognition may be defined as follows. We are first shown a certain limited number of samples (paradigms) of each of the given number of classes, and then we are shown an object of unknown class and are required to place it in one of the classes. It is obviously an inductive process, hence it is not a logical process or mechanical sorting that the computer can perform without human aid. However, it is very instructive to study more closely the nature of the problem. See [11] for the relationship between pattern recognition and induction.

Pattern recognition is a task of classification, but the classes are defined neither by their intention nor by their extension. They are indicated only by paradigms. If the computer is ever to become capable of doing this job, it has to derive the class-defining properties from the paradigms, so that the task becomes one of intentional sorting, which the computer is capable of doing. But the derivation of class-defining properties from paradigms is not a machine-feasible operation without human aid. This can be seen by the following consideration.

According to the theorem of the Ugly Duckling, any pair of nonidentical objects share an equal number of predicates as any other pair of nonidentical objects, insofar as the number of predicates is finite [10], [12]. That is to say, from a logical point of view there is no such thing as a natural kind. In the case of pattern recognition, the new arrival shares the same number of predicates with any paradigm of any class. This shows that pattern recognition is a logically indeterminate problem. The class-defining properties are generalizations of certain of the properties shared by the paradigms of the class. Which of the properties should be used for generalization is not logically defined. If it were logically determinable, then pattern recognition would have a definite answer in violation of the theorem of the Ugly Duckling.

This conclusion is somewhat disturbing because our empirical knowledge is based on natural kinds of objects. The source of the trouble lies in the fact that we were just counting the number of predicates in the foregoing, treating them as if they were all equally important. The fact is that some predicates are more important than some others. Objects are similar if they share a large number of important predicates.

Important in what scale? We have to conclude that a predicate is important if it leads to a classification that is useful for some purpose. From a logical point of view, a whale can be put together in the same box with a fish or with an elephant. However, for the purpose of building an elegant zoological theory, it is better to put it together with the elephant, and for classifying industries it is better to put it together with the fish. The property characterizing mammals is important for the purpose of theory building in biology, while the property of living in water is more important for the purpose of classification of industries.

The conclusion is that classification is a value-dependent task and pattern recognition is mechanically possible only if we smuggle into the machine the scale of importance of predicates. Alternatively, we can introduce into the machine the scale of distance or similarity between objects. This seems to be an innocuous set of auxiliary data, but in reality we are thereby telling the machine our value judgment, which is of an entirely extra-logical nature. The human mind has an innate scale of importance of predicates closely related to the sensory organs. This scale of importance seems to have been developed during the process of evolution in such a way as to help maintain and expand life [12], [14]. The machine, having no organized life-sustaining inner value structure, cannot form classes of objects or classify objects according to paradigms. In the human mind, on the other hand, paradigmatic symbols, being concrete objects and having their place in the inner value structure, can generate classes as cloud seeds can generate a large cloud around them.

In summary, we may say that artificial intelligence and human intelligence have an unbridgeable gap not only in procedure, but also in functional effect. If one wants, however, at least to narrow the gap, the first thing one could try may be to develop a genuine associative memory—that really deserves the name—to imitate some of the features of paradigmatic symbols.

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A Model for "The Tragedy of the Commons"

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***Abstract*—A formal dynamic model which represents the assumptions in Hardin's paper, "The Tragedy of the Commons," exhibits his conclusions explicitly. The calculus serves as an adequate language for the model, but its conclusions are exhibited graphically with the aid of a simulation language and computation. Technological solutions to the "tragedy" necessarily fail; coercive solutions admit a range of possibilities. Limitation of the use of the "commons" may prove more**

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TABLE I
CAPITAL AND COMMONS

Problem	Commons	Capital	Product
Overgrazing the village green	Grass	Sheep	Wool
Polluting water	Water	Factory	Gadgets
Overuse of wilderness	Wilderness	Men	Recreation

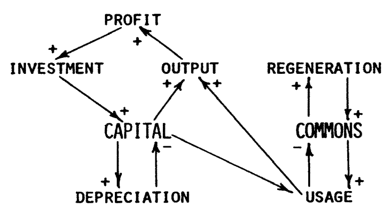

Fig. 1. Cause-and-effect relationships between capital and commons in "The Tragedy of the Commons." Arrow indicates cause-and-effect relationship, with tail at cause and arrowhead at effect. Sign at arrowhead indicates sense of relationship; for example, regeneration increases commons but usage depletes commons.

successful than taxation for the use of the "commons." The model lays both assumptions and conclusions open to critical examination, and serves as a useful teaching tool.

INTRODUCTION

Garrett Hardin's seminal paper, "The Tragedy of the Commons," [1] describes in general terms the problems surrounding the depletion of common-property resources. He then applies his analysis to the population problem in particular. The model proposed here offers no results not already anticipated by Hardin; it does, however, exhibit Hardin's assumptions and conclusions in a formal way, laying each open to examination and criticism.

Let us consider the "tragedy" in general terms. Men use common-property resources such as space, water, air, etc. In a system where these resources are free, the gain to the resource user is not reduced by a resource cost. Normally, therefore, the user will be induced to use yet more of the "commons."

Let us consider this economic–ecological system (truly an *eco*system) as being made up of two interwoven parts: capital and commons. "Commons" will represent any renewable common-property resource, and "capital" will represent any means of production of goods. Table I shows some examples. Fig. 1 shows the cause-and-effect relationships between the elements of the system. Capital is regulated by investment and depreciation, and commons by natural regeneration and usage. One can deduce the behavior of the system by examining the feedback loops (closed circuits of causal relationships) which characterize the system. The stability of the commons-capital system hangs on the balance between the positive driving forces of investment and regeneration and the negative restraining forces of depreciation and usage.

In this correspondence we shall develop a model for "The Tragedy of the Commons" and formally examine Hardin's assumptions and conclusions.

DYNAMIC MODELS: THE CALCULUS

A model is nothing more than statements about cause and effect. Models can be carried in one's mind, but mental models often suffer from incompleteness and inaccuracy. Formal models exhibit cause-and-effect relationships explicitly, and dynamic models illustrate the time dependence of these relationships as well.

In fact, one simple explicit language for the expression of dynamic models is the calculus. The model then appears as a system of differential equations. For example, we could write

$$\frac{dC}{dt} = \frac{C}{\tau_r} - \min(\alpha K, C) \tag{1}$$

where C represents commons, τ_r is the regeneration time constant for the commons, and K represents capital. Here we assume that the regeneration of commons follows first-order kinetics (a rate proportional to the remaining commons) and that the commons is depleted at a rate proportional to working capital, but no more than the existing commons.

In the logistic growth curve often used by ecologists; however, the regeneration rate of a species tends toward zero as the population of the species approaches the carrying capacity of the environment for that species. Accordingly, we represent the regeneration time constant for the commons not as a constant, but as a monotonically increasing function of commons[1]

$$\tau = \frac{\tau_0 C_0}{C_0 - C} \tag{2}$$

so that

$$\frac{dC}{dt} = 0.4C\left(\frac{C_0 - C}{C_0}\right) - \min(\alpha K, C).$$

The rate of change of capital depends on investment and depreciation

$$\frac{dK}{dt} = (a - b)K$$

where a and b represent investment and depreciation, respectively. As long as $a > b$, K grows; if K grows enough, the commons usage rate may become total, that is, $\min(\alpha K, C) = C$. Then

$$\frac{dC}{dt} = -0.6C - \frac{0.4C^2}{C_0}$$

which is necessarily negative, leading ultimately to no commons.

What is the effect of regulation? Suppose that a limit of annual commons use is imposed and that that limit is some fraction of the original commons, say, βC_0. Then $dC/dt = 0$ establishes a condition for minimum stable commons

$$C = \frac{-0.4 \pm \sqrt{0.16 - 1.6\beta}}{-0.8}.$$

Clearly $\beta \leq 0.1$. That is, the commons will always decrease to zero—there will be a "tragedy"—unless a firm limit of no more than 10 percent of the commons used annually is imposed. For this, the most generous limit, C(at equilibrium) $= C_0/2$, or half the commons will remain. This conclusion, representing the ultimate or equilibrium state of the system, is independent of the speed by which that equilibrium is reached (the constant $0.4 = \tau_0$ in (2)).

Consider an alternative solution: the taxation of industry at a rate proportional to used commons. This solution admits, by analysis analogous to that just undertaken, of a range of possibilities for equilibria of both commons and capital. One can choose from a range extending from little capital and much commons to much capital and little commons.

[1] In the program actually used, this equation appears as $\tau = 2.5[C_0/(C_0 - C + 1)]$ to avoid a singularity.

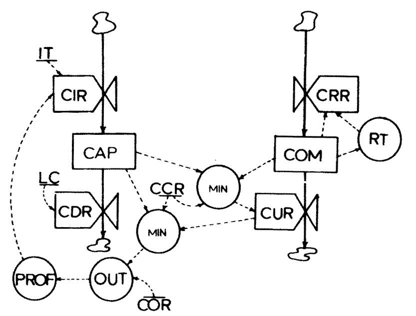

Fig. 2. DYNAMO flow diagram for model.

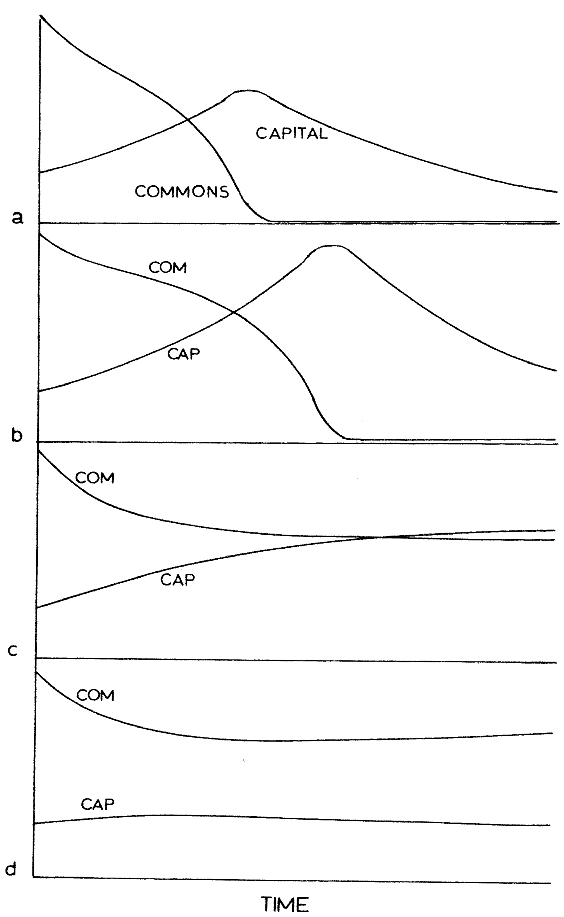

Fig. 3. Simulations of model for "The Tragedy of the Commons." (a) Standard simulation. (b) Simulation with reduced commons–capital ratio, indicating that there are no technical solutions to "tragedy." (c) Simulation with limit placed on commons usage, indicating one possible coercive solution to "tragedy." (d) Simulation with internalization of common-property resource costs, indicating another possible coercive solution to "tragedy."

It is, therefore, clear that coercive solutions are required to save the commons. Merely technological solutions, changing the relationship between capital and commons or the regeneration rate of the commons (the constants α in (1) or τ_0 in (2)) are of no avail.

Dynamic Models: System Dynamics

The cause-and-effect structure shown in Fig. 1 can easily be translated into a computer program using the techniques of system dynamics [2] and the DYNAMO computer language [3]. The result is the flow diagram of Fig. 2.[2]

The standard simulation of this model is shown in Fig. 3(a). The behavior of the system, as Hardin suggests, is characterized by the depletion of the commons accompanied by a rise and fall (overshoot and collapse) of capital.

Hardin suggests also that this is a problem for which there is "no technical solution." To test this, we simulate the same model, but with a lower commons–capital ratio; that is, less commons is used per unit of capital to ensure the same output. One could also simulate the effect of raising the regeneration rate of the commons through technological means; the result is qualitatively similar to that shown in Fig. 3(b). Our result indicates that capital rises higher and commons falls less rapidly before the "remorseless working of things"[3] precipitates the same tragedy.

What solutions to the tragedy are available? Hardin points only to coercive solutions: "mutual coercion mutually agreed upon." Within the context of the present model, we examine two possible coercive solutions: a) limiting the use of the commons; b) charging for (taxing, internalizing external diseconomies) the use of the commons. Figs. 3(c) and 3(d) show simulations of the model under the conditions of implementing these coercive solutions. In each case, a balance between commons and capital is reached. In the language of control and feedback, a negative feedback loop including capital and commons is established which brings the system into a steady-state condition.

The limit imposed in Fig. 3(c) is the most generous limit, 10 percent of the commons per year. The choice of taxes levied, shown in Fig. 3(d), is arbitrary.

Conclusions

The formal dynamic model proposed here, whether expressed in the calculus or in a simulation language, offers no results not already anticipated by Hardin; it does, however, exhibit Hardin's assumptions formally, laying them open to examination, criticism, and interpretation.

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[2] A technical appendix to this paper, containing DYNAMO flow diagram, equations, and parameter changes used in the simulations shown in Fig. 3, can be obtained by writing directly to the author.
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