%% Testing "reduction by substitution"

```
b=(eye(2)+z)/2;
b1 = kron(b,eye(4));
b2 = kron(kron(eye(2),b),eye(2));
b3 = kron(eye(4),b);
b1 = kron(b1,eye(2));
b2 = kron(b2,eye(2));
b3 = kron(b3,eye(2));
ba =kron( eye(8),b);
lambda=2;

Left: [E,D]=eig(b1*ba + lamda*(3*ba - 2*ba*(b2 + b3) + b2*b3))
Right: [E,D]=eig(b1*b2*b3)
```

Eigenvectors:
```
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0    1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0    0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0    0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0    0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0    0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0    0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0    0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0    0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0    0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1    0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0    0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0    0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0    0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0    0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0    0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0    0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
```

Eigenvalues:
```
0 0 0 0 0 0 0 1 2 2 2 2 3 3 6 7    0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1
```

```
lambda=10;
[E,D] = eig(b1*ba + lambda*(3*ba -2*ba*(b2 + b3) + b2*b3))
```

Eigenvectors:
```
0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0
1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0
0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1
0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0
0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0
0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0
0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0
```

Eigenvalues:
```
0 0 0 0 0 0 0 1 10 10 10 10 11 11 30 31
```

|b1 b2 b3 b4> = |1111> has eigenvalue = 0, but should not.

It just means lambda was not big enough

Eigenvectors of z1z2z3:

```
    0   0   0   0   1   0   0   0
    1   0   0   0   0   0   0   0
    0   1   0   0   0   0   0   0
    0   0   0   0   0   0   0   1
    0   0   1   0   0   0   0   0
    0   0   0   0   0   1   0   0
    0   0   0   0   0   0   1   0
    0   0   0   1   0   0   0   0
```

Eigenvalues of z1z2z3:

```
   -1   0   0   0   0   0   0   0
    0  -1   0   0   0   0   0   0
    0   0  -1   0   0   0   0   0
    0   0   0  -1   0   0   0   0
    0   0   0   0   1   0   0   0
    0   0   0   0   0   1   0   0
    0   0   0   0   0   0   1   0
    0   0   0   0   0   0   0   1
```

Minima area at: $|001\rangle$, $|010\rangle$, $|100\rangle$, $|111\rangle$ (ie: 1 or 3 z's are -1).

Eigenvectors of: $z1*z2+z2*z3 + z3*z1 + 2*za*(z1+z2+z3)-(z1+z2+z3+2*za)$;

```
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   1   0
    1   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   1   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   1   0   0   0   0   0   0
    0   0   0   0   1   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   1   0   0   0   0   0   0   0   0   0   0
    0   1   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   1   0   0   0
    0   0   0   0   0   0   1   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   1   0   0   0   0   0   0   0   0
    0   0   1   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   1   0   0   0   0
    0   0   0   1   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   1   0   0
    0   0   0   0   0   0   0   0   0   0   1   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0   1
```

Eigenvalues of: $z1*z2+z2*z3 + z3*z1 + 2*za*(z1+z2+z3)-(z1+z2+z3+2*za)$;

```
   -4   0   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0  -4   0   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0  -4   0   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0  -4   0   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0  -2   0   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0  -2   0   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0  -2   0   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0  -2   0   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0  -2   0   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0  -2   0   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0  -2   0   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   4   0   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   4   0   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   4   0   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   4   0
    0   0   0   0   0   0   0   0   0   0   0   0   0   0   0  14
```

Minima area at: |0001>, |0111>, |1011>, |1101> (ie: 1 or 3 z's are +1).

Let's try -(z1*z2+z2*z3 + z3*z1 + 2*za*(z1+z2+z3)-(z1+z2+z3+2*za))

0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

>> D2

D2 =

-14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	-4	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	2	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	4	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	4	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4

Minima occur at: |1111>, |1000>, |1100>, |1110>, |0001>. 5 of them, not all same energy, and 111 gives the same as -1,-1,-1. This has to be wrong.

kron(z1,eye(2))*kron(z2,eye(2))*kron(z3,eye(2))

```
                 *
   1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
   0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
   0  0 -1  0  0  0  0  0  0  0  0  0  0  0  0  0
   0  0  0 -1  0  0  0  0  0  0  0  0  0  0  0  0
   0  0  0  0  1  0  0  0  0  0  0  0  0  0  0  0
   0  0  0  0  0  1  0  0  0  0  0  0  0  0  0  0
   0  0  0  0  0  0 -1  0  0  0  0  0  0  0  0  0
   0  0  0  0  0  0  0 -1  0  0  0  0  0  0  0  0
   0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0
   0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0
   0  0  0  0  0  0  0  0  0  0 -1  0  0  0  0  0
   0  0  0  0  0  0  0  0  0  0  0 -1  0  0  0  0
   0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0
   0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0
   0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1  0
   0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1


   1  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
   0  1  0  0  0  0  0  0  0  0  0  0  0  0  0  0
   0  0 -1  0  0  0  0  0  0  0  0  0  0  0  0  0
   0  0  0 -1  0  0  0  0  0  0  0  0  0  0  0  0
   0  0  0  0 -1  0  0  0  0  0  0  0  0  0  0  0
   0  0  0  0  0 -1  0  0  0  0  0  0  0  0  0  0
   0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0
   0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0
   0  0  0  0  0  0  0  0 -1  0  0  0  0  0  0  0
   0  0  0  0  0  0  0  0  0 -1  0  0  0  0  0  0
   0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0
   0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0
   0  0  0  0  0  0  0  0  0  0  0  0  1  0  0  0
   0  0  0  0  0  0  0  0  0  0  0  0  0  1  0  0
   0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1  0
   0  0  0  0  0  0  0  0  0  0  0  0  0  0  0 -1
```