Volume 3: List of Multi-run Quadratizations

Nike Dattani and Andreas Soteriou
(Dated: 21st September 2019)

DECOMPOSITION OF A MONOMIAL

$$b_1 b_2 b_3 \dots b_k = \min\left(b_1 b_2 \dots b_{k_1}, b_{k_1+1} b_{k_1+2} \dots b_{k_2}, b_{k_2+1} b_{k_2+2} \dots b_{k_3}, \dots, b_{k_n+1} b_{k_n+2} \dots b_k\right) \tag{1}$$

$$b_1 b_2 b_3 \dots b_k = \min\left(b_1, b_2, b_3, \dots, b_k\right) \qquad \text{(Example of Eq. 1: Linearization of a degree-k monomial).} \tag{2}$$

$$b_1 b_2 b_3 b_4 = \min\left(b_1 b_2, b_3 b_4\right) \qquad \text{(Example of Eq. 1: Quadratization of a degree-4 monomial).} \tag{3}$$

$$b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 : \tag{4}$$
$$\longrightarrow 3b_a + b_1 b_2 + b_1 b_3 + b_1 b_4 + b_2 b_3 + b_2 b_4 + b_3 b_4 - 2b_a(b_1 + b_2 + b_3 + b_4) \tag{5}$$
$$\longrightarrow 3b_a + b_5 b_6 + b_5 b_7 + b_5 b_8 + b_6 b_7 + b_6 b_8 + b_7 b_8 - 2b_a(b_5 + b_6 + b_7 + b_8) \tag{6}$$

DECOMPOSITION OF BINOMIALS OF DEGREE-k TERMS

$$b_1 b_2 b_3 b_4 + b_3 b_4 b_5 b_6 = \min\left(b_2 b_3 + b_3 b_6, b_1 b_4 + b_4 b_5, b_1 b_2 + b_5 b_6 - b_3 - b_4 + 2\right) \qquad (k,n) = (4,6). \tag{7}$$

$$b_1 b_2 b_3 b_4 b_5 b_6 + b_2 b_3 b_4 b_5 b_6 b_7 : \qquad (k,n) = (6,7). \tag{8}$$
$$\longrightarrow 2b_3 b_6 \tag{9}$$
$$\longrightarrow 2b_4 b_5 - b_5 b_6 + b_5 \tag{10}$$
$$\longrightarrow b_1 b_4 - b_2 b_5 + b_2 b_6 + b_2 b_7 + b_5 b_7 - b_6 b_7 - b_5 - b_6 + 2 \tag{11}$$
$$\longrightarrow b_1 b_2 - b_1 b_5 + b_1 b_7 + b_2 b_3 + b_3 b_6 - b_3 b_7 - b_4 b_5 - b_5 b_6 - b_3 + b_5 + 2 \tag{12}$$

$$b_1 b_2 b_3 b_4 + b_5 b_6 b_7 b_8 : \qquad (k,n) = (4,8). \tag{13}$$
$$\longrightarrow b_1 b_2 + b_6 b_8 + b_a\left(1 - b_6 + b_7 - b_8\right) \tag{14}$$
$$\longrightarrow b_3 b_4 + b_6 b_8 + 2b_8 b_a \tag{15}$$
$$\longrightarrow b_2 b_3 + b_5 b_7 + b_a\left(1 - b_6 + b_7\right) \tag{16}$$
$$\longrightarrow b_1 b_4 + b_5 b_7 - b_6 b_8 + b_7 b_a + b_6 \tag{17}$$

$$b_1 b_2 b_3 b_4 b_5 + b_6 b_7 b_8 b_9 b_{10} : \qquad (k,n) = (5,10). \tag{18}$$
$$\longrightarrow b_2 b_3 + b_6 b_9 + b_9 b_a \tag{19}$$
$$\longrightarrow b_1 b_4 + b_8 b_{10} + b_9 b_a \tag{20}$$
$$\longrightarrow b_3 b_5 + b_7 b_{10} + b_1 b_a + b_9 b_a \tag{21}$$
$$\longrightarrow b_4 b_5 + b_6 b_9 + b_9 b_a \tag{22}$$
$$\longrightarrow b_1 b_2 + b_7 b_9 + b_9 b_a \tag{23}$$
$$\longrightarrow b_2 b_5 + b_6 b_8 + b_9 b_a \tag{24}$$
$$\longrightarrow b_2 b_3 + b_8 b_{10} + b_9 b_a \tag{25}$$
$$\longrightarrow b_1 b_4 + b_6 b_7 + b_9 b_a \tag{26}$$

$$b_1b_2b_3b_4b_5b_6 + b_5b_6b_7b_8b_9b_{10} : \qquad\qquad (k,n) = (6,10). \quad (27)$$
$$\longrightarrow b_5b_6 + b_5b_7 + b_a(1 - b_{10}) \quad (28)$$
$$\longrightarrow b_1b_2 + b_8b_9 - b_a(b_9 + b_{10}) + b_9 + b_{10} \quad (29)$$
$$\longrightarrow b_3b_4 + b_a(1 + b_7 - b_9 - b_{10}) + b_{10} \quad (30)$$
$$\longrightarrow b_3b_6 - b_5b_{10} + b_6b_7 + b_a(1 - b_9) + b_{10} \quad (31)$$
$$\longrightarrow b_1b_2 + b_7b_{10} + b_a(1 - b_9 - b_{10}) + b_{10} \quad (32)$$
$$\longrightarrow b_3b_4 + b_8b_9 + b_a(1 - b_9 - b_{10} - b_2) - b_2 + b_9 + b_{10} + 1 \quad (33)$$

$$b_1b_2b_3b_4b_5b_6b_7 + b_4b_5b_6b_7b_8b_9b_{10} : \qquad\qquad (k,n) = (7,10). \quad (34)$$
$$\longrightarrow b_4b_7 + b_6b_7 + b_a(1 - b_4 - b_7 + b_{10}) \quad (35)$$
$$\longrightarrow b_2b_5 + b_5b_9 + b_a \quad (36)$$
$$\longrightarrow b_1b_4 + b_4b_8 + b_a \quad (37)$$
$$\longrightarrow b_1b_3 + b_6b_{10} \quad (38)$$
$$\longrightarrow b_3b_6 + b_6b_9 + b_a \quad (39)$$
$$\longrightarrow b_2b_3 + b_8b_{10} \quad (40)$$
$$\longrightarrow b_1b_4 + b_9 \quad (41)$$

$$b_1b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9b_{10} : \qquad\qquad (k,n) = (8,10). \quad (42)$$
$$\longrightarrow b_2b_4 + b_4b_9 - b_a(b_9 + b_{10}) + b_9 + b_{10} \quad (43)$$
$$\longrightarrow b_1b_7 + b_7b_{10} - b_a(b_9 + b_{10}) + b_9 + b_{10} \quad (44)$$
$$\longrightarrow b_5b_8 + b_6b_8 - b_a(b_9 + b_{10}) + b_9 + b_{10} \quad (45)$$
$$\longrightarrow b_3b_6 + b_a(b_3 - b_{10}) + b_{10} \quad (46)$$
$$\longrightarrow b_1b_5 + b_5b_9 - b_a(b_9 + b_{10}) + b_9 + b_{10} \quad (47)$$
$$\longrightarrow b_6b_9 - b_a(b_9 + b_{10}) + b_6 + b_9 + 1 \quad (48)$$
$$\longrightarrow b_1b_2 - b_9b_a + b_{10} + 1 \quad (49)$$

$$b_1b_2b_3b_4b_5b_6b_7b_8b_9 + b_2b_3b_4b_5b_6b_7b_8b_9b_{10} : \qquad\qquad (k,n) = (9,10). \quad (50)$$
$$\longrightarrow b_1b_6 + b_6b_{10} + b_9b_a \quad (51)$$
$$\longrightarrow b_4b_7 + b_7b_8 \quad (52)$$
$$\longrightarrow b_4b_9 + b_a(b_9 - b_4) + b_4 \quad (53)$$
$$\longrightarrow b_2b_3 + b_3b_8 + b_9b_a \quad (54)$$
$$\longrightarrow b_1b_5 + b_2b_5 + b_9b_a \quad (55)$$
$$\longrightarrow b_2b_8 - b_6b_7 + b_8b_{10} + b_9b_a + 1 \quad (56)$$
$$\longrightarrow b_2b_{10} + b_2 \quad (57)$$

DECOMPOSITION OF DEGREE-k, EXACT-k-OF-n TRINOMIALS

$$b_1b_2b_3b_4b_5b_6b_7b_8 + b_2b_3b_4b_5b_6b_7b_8b_9 + b_3b_4b_5b_6b_7b_8b_9b_{10} : \qquad (k,n) = (8,10). \quad (58)$$

$$\longrightarrow \; b_3b_5 + b_5b_9 + b_5b_{10} \tag{59}$$

$$\longrightarrow \; b_1b_4 + b_4b_7 + b_4b_9 \tag{60}$$

$$\longrightarrow \; b_1b_6 + b_2b_6 - b_5b_6 + b_6b_{10} + b_6 \tag{61}$$

$$\longrightarrow \; b_4b_8 + 2b_8 \tag{62}$$

$$\longrightarrow \; b_1b_7 + b_2b_7 - b_a(b_4 + b_6) + b_7b_9 + b_4 + 1 \tag{63}$$

$$\longrightarrow \; b_2b_3 + b_3b_5 + b_3 \tag{64}$$

$$\longrightarrow \; b_1b_2 + b_2b_9 - b_5b_6 - b_5b_a + b_6b_9 + 2 \tag{65}$$

$$\longrightarrow \; b_2b_7 + b_2 + b_{10} \tag{66}$$

$$b_1b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7b_8 + b_5b_6b_7b_8b_9b_{10} : \qquad (k,n) = (6,10). \quad (67)$$

$$\longrightarrow \; b_2b_5 + b_5b_6 + b_5b_9 - b_9b_{11} + b_9 \tag{68}$$

$$\longrightarrow \; b_1b_4 + b_4b_6 + b_6b_{10} - b_9b_{11} + b_9 - 2b_{11} + 2 \tag{69}$$

$$\longrightarrow \; b_3b_4 + b_3b_7 + b_9b_{10} - b_9b_{11} + b_9 \tag{70}$$

$$\longrightarrow \; b_2b_3 + b_8b_{11} - b_9b_{11} + b_8 + b_9 \tag{71}$$

$$\longrightarrow \; b_1b_6 + b_6b_7 - b_9b_{11} + b_{10}b_{11} + b_9 \tag{72}$$

$$\longrightarrow \; b_1b_3 + b_7b_{10} - b_9b_{11} + b_7 + b_9 \tag{73}$$

$$\longrightarrow \; b_2b_4 + b_4b_7 - b_9b_{11} + 2b_{10}b_{11} + b_9 \tag{74}$$

$$\longrightarrow \; b_2b_6 + b_6b_7 + b_6b_8 - b_9b_{11} + b_9 \tag{75}$$

$$\longrightarrow \; b_1b_4 - b_2b_5 - b_3b_{10} + b_5b_8 - b_7b_{11} + b_8b_{10} - b_9b_{11} + b_{10}b_{11} + b_9 + 2 \tag{76}$$

$$b_1b_2b_3b_4 + b_4b_5b_6b_7 + b_7b_8b_9b_{10} : \qquad (k,n) = (4,10). \quad (77)$$

$$\longrightarrow \; b_3b_4 + b_4b_5 + b_8b_{10} + b_a(1 - b_9) \tag{78}$$

$$\longrightarrow \; b_1b_2 + b_a(1 + b_3 - b_9) + b_5b_6 + b_7b_9 \tag{79}$$

$$\longrightarrow \; b_1b_3 + b_6b_7 + b_7b_{10} + b_a \tag{80}$$

$$\longrightarrow \; b_4b_6 + b_7b_9 + b_a(1 - b_9) + b_4 \tag{81}$$

$$\longrightarrow \; b_2b_3 + b_5b_7 + b_8b_9 + b_a(1 - b_9) \tag{82}$$

$$\longrightarrow \; b_2b_3 + b_8b_{10} + b_a(1 - b_9) + b_6 - b_9 + 1 \tag{83}$$

$$\longrightarrow \; b_1b_2 + b_5b_7 + b_8b_{10} + b_a(1 - b_9) - b_3 + 1 \tag{84}$$

$$\longrightarrow \; b_5b_6 + b_a(1 - b_9) + b_3 + b_9 \tag{85}$$

$$\longrightarrow \; b_7b_8 + b_2 + b_7 \tag{86}$$

DECOMPOSITION OF DEGREE-k, EXACT-k-OF-n QUADRINOMIALS

$$b_1b_2b_3b_4b_5b_6b_7 + b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9 + b_4b_5b_6b_7b_8b_9b_{10} : \qquad (k,n) = (7,10). \quad (87)$$

$$\longrightarrow \ b_1b_2 + b_2b_5 + b_5b_7 + b_5b_{10} - b_a(b_9 + b_{10}) + b_9 + b_{10} \qquad (88)$$

$$\longrightarrow \ b_5b_6 + b_6b_8 + b_6b_9 + b_6b_{10} - b_a(b_9 + b_{10}) + b_9 + b_{10} \qquad (89)$$

$$\longrightarrow \ b_1b_7 + b_5b_7 + b_6b_7 + b_7b_9 - b_a(b_9 + b_{10}) + b_9 + b_{10} \qquad (90)$$

$$\longrightarrow \ b_4b_5 + b_4b_8 + b_4b_{10} + b_a(b_4 - b_9 - b_{10}) + b_9 + b_{10} \qquad (91)$$

$$\longrightarrow \ b_1b_3 + b_3b_4 + b_3b_9 - b_6b_7 + b_6b_{10} + b_a(1 - b_9 - b_{10}) - b_6 + b_{10} + 2 \qquad (92)$$

$$\longrightarrow \ b_2b_3 + b_2b_8 + b_3b_9 - b_5b_7 + b_6b_9 - b_a(1 + b_{10}) - b_7 + b_9 + b_{10} + 3 \qquad (93)$$

$$\longrightarrow \ b_2b_3 - b_6b_7 + b_6b_8 + b_7b_8 - b_a(b_9 + b_{10}) + b_8 + b_9 + b_{10} + 1 \qquad (94)$$

$$\longrightarrow \ -b_1b_2 + b_1b_4 + b_2b_5 - b_4b_9 + b_5b_8 + b_5b_9 - b_a(b_9 + b_{10}) - b_4 + b_5 - b_7 + 2b_9 + b_{10} + 2 \qquad (95)$$

$$\longrightarrow \ b_2b_8 + b_8b_{10} + b_a(b_8 - b_9 - b_{10}) + b_1 + b_9 + b_{10} \qquad (96)$$