Volume 3: List of Multi-run Quadratizations

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DECOMPOSITION OF A MONOMIAL

$$b_1b_2b_3\dots b_k = \min\left(b_1b_2\dots b_{k_1}, b_{k_1+1}b_{k_1+2}\dots b_{k_2}, b_{k_2+1}b_{k_2+2}\dots b_{k_3}, ..., b_{k_n+1}b_{k_n+2}...b_k\right) \quad (1)$$

$$b_1b_2b_3\dots b_k = \min\left(b_1, b_2, b_3, \dots, b_k\right) \quad \text{(Example of Eq. 1: Linearization of a degree-k monomial).} \quad (2)$$

$$b_1b_2b_3b_4 = \min\left(b_1b_2, b_3b_4\right) \quad \text{(Example of Eq. 1: Quadratization of a degree-4 monomial).} \quad (3)$$

DECOMPOSITION OF BINOMIALS OF DEGREE-k TERMS

$$b_1b_2b_3b_4 + b_3b_4b_5b_6 = \min\left(b_2b_3 + b_3b_6, b_1b_4 + b_4b_5, b_1b_2 + b_5b_6 - b_3 - b_4 + 2\right) \quad (k,n) = (4,6). \quad (4)$$

$b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7$: $(k,n) = (6,7)$. (5)

$\longrightarrow 2b_3b_6$ (6)

$\longrightarrow 2b_4b_5 - b_5b_6 + b_5$ (7)

$\longrightarrow b_1b_4 - b_2b_5 + b_2b_6 + b_2b_7 + b_5b_7 - b_6b_7 - b_5 - b_6 + 2$ (8)

$\longrightarrow b_1b_2 - b_1b_5 + b_1b_7 + b_2b_3 + b_3b_6 - b_3b_7 - b_4b_5 - b_5b_6 - b_3 + b_5 + 2$ (9)

$b_1b_2b_3b_4 + b_5b_6b_7b_8$: $(k,n) = (4,8)$. (10)

$\longrightarrow b_1b_2 + b_6b_8 + b_a\left(1 - b_6 + b_7 - b_8\right)$ (11)

$\longrightarrow b_3b_4 + b_6b_8 + 2b_8b_a$ (12)

$\longrightarrow b_2b_3 + b_5b_7 + b_a\left(1 - b_6 + b_7\right)$ (13)

$\longrightarrow b_1b_4 + b_5b_7 - b_6b_8 + b_7b_a + b_6$ (14)

$b_1b_2b_3b_4b_5 + b_6b_7b_8b_9b_{10}$: $(k,n) = (5,10)$. (15)

$\longrightarrow b_2b_3 + b_6b_9 + b_9b_a$ (16)

$\longrightarrow b_1b_4 + b_8b_{10} + b_9b_a$ (17)

$\longrightarrow b_3b_5 + b_7b_{10} + b_1b_a + b_9b_a$ (18)

$\longrightarrow b_4b_5 + b_6b_9 + b_9b_a$ (19)

$\longrightarrow b_1b_2 + b_7b_9 + b_9b_a$ (20)

$\longrightarrow b_2b_5 + b_6b_8 + b_9b_a$ (21)

$\longrightarrow b_2b_3 + b_8b_{10} + b_9b_a$ (22)

$\longrightarrow b_1b_4 + b_6b_7 + b_9b_a$ (23)