

# Volume 3: List of Multi-run Quadratizations

Nike Dattani and Andreas Soteriou

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## DECOMPOSITION OF A MONOMIAL

$$b_1 b_2 b_3 \dots b_k = \min(b_1 b_2 \dots b_{k_1}, b_{k_1+1} b_{k_1+2} \dots b_{k_2}, b_{k_2+1} b_{k_2+2} \dots b_{k_3}, \dots, b_{k_n+1} b_{k_n+2} \dots b_k) \quad (1)$$

$$b_1 b_2 b_3 \dots b_k = \min(b_1, b_2, b_3, \dots, b_k) \quad (\text{Example of Eq. 1: Linearization of a degree-}k \text{ monomial}). \quad (2)$$

$$b_1 b_2 b_3 b_4 = \min(b_1 b_2, b_3 b_4) \quad (\text{Example of Eq. 1: Quadratzation of a degree-4 monomial}). \quad (3)$$

$$b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 : \quad (4)$$

$$\longrightarrow 3b_a + b_1 b_2 + b_1 b_3 + b_1 b_4 + b_2 b_3 + b_2 b_4 + b_3 b_4 - 2b_a(b_1 + b_2 + b_3 + b_4) \quad (5)$$

$$\longrightarrow 3b_a + b_5 b_6 + b_5 b_7 + b_5 b_8 + b_6 b_7 + b_6 b_8 + b_7 b_8 - 2b_a(b_5 + b_6 + b_7 + b_8) \quad (6)$$

## DECOMPOSITION OF BINOMIALS OF DEGREE- $k$ TERMS

$$b_1 b_2 b_3 b_4 + b_3 b_4 b_5 b_6 = \min(b_2 b_3 + b_3 b_6, b_1 b_4 + b_4 b_5, b_1 b_2 + b_5 b_6 - b_3 - b_4 + 2) \quad (k, n) = (4, 6). \quad (7)$$

$$b_1 b_2 b_3 b_4 + b_4 b_5 b_6 b_7 : \quad (k, n) = (4, 7). \quad (8)$$

$$\longrightarrow b_2 b_3 + b_5 b_6 + b_a(1 - b_5 - b_6 + b_7) \quad (9)$$

$$\longrightarrow b_1 b_4 + b_4 + b_a \quad (10)$$

$$\longrightarrow b_5 b_6 + b_1 + b_a(1 - b_5 - b_6 + b_7) \quad (11)$$

$$b_1 b_2 b_3 b_4 b_5 + b_3 b_4 b_5 b_6 b_7 : \quad (k, n) = (5, 7). \quad (12)$$

$$\longrightarrow b_2 b_5 + b_5 b_6 + b_5 b_7 + b_6 b_7 + b_a(b_5 + b_6 + b_7 - 1) - b_5 - b_6 - b_7 + 1 \quad (13)$$

$$\longrightarrow b_1 b_3 + b_3 b_7 + b_a(1 + b_5 - b_7) - b_5 + 1 \quad (14)$$

$$\longrightarrow b_1 b_4 + b_4 b_6 - b_5 b_6 + b_5 b_a - b_5 + b_6 + 1 \quad (15)$$

$$b_1 b_2 b_3 b_4 b_5 b_6 + b_2 b_3 b_4 b_5 b_6 b_7 : \quad (k, n) = (6, 7). \quad (16)$$

$$\longrightarrow 2b_3 b_6 \quad (17)$$

$$\longrightarrow 2b_4 b_5 - b_5 b_6 + b_5 \quad (18)$$

$$\longrightarrow b_1 b_4 - b_2 b_5 + b_2 b_6 + b_2 b_7 + b_5 b_7 - b_6 b_7 - b_5 - b_6 + 2 \quad (19)$$

$$\longrightarrow b_1 b_2 - b_1 b_5 + b_1 b_7 + b_2 b_3 + b_3 b_6 - b_3 b_7 - b_4 b_5 - b_5 b_6 - b_3 + b_5 + 2 \quad (20)$$

$$b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7 : \quad (k, n) = (6, 7). \quad (21)$$

$$\longrightarrow b_5b_6 + b_5b_7 - b_5b_8 + b_6b_8 - b_7b_8 + b_8 \quad (22)$$

$$\longrightarrow b_1b_4 + b_a(b_4 - b_7) + b_7 \quad (23)$$

$$\longrightarrow b_2b_3 + b_2b_7 - b_5b_6 - b_7b_a + b_5 + b_7 \quad (24)$$

$$\longrightarrow b_2b_3 + b_7b_a + b_3 \quad (25)$$

$$b_1b_2b_3b_4b_5 + b_4b_5b_6b_7b_8 : \quad (k, n) = (5, 8). \quad (26)$$

$$\longrightarrow b_1b_2 + b_6b_8 + b_a(1 - b_6 + b_7 - b_8) \quad (27)$$

$$\longrightarrow b_3b_5 + b_8b_a + b_5 \quad (28)$$

$$\longrightarrow b_4b_7 + b_4 \quad (29)$$

$$\longrightarrow b_7b_8 + b_a(1 + b_6 - b_7 - b_8) + b_3 \quad (30)$$

$$b_1b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7b_8 : \quad (k, n) = (6, 8). \quad (31)$$

$$\longrightarrow b_1b_6 + b_6b_8 + b_a(1 - b_6 + b_7 - b_8) \quad (32)$$

$$\longrightarrow b_2b_5 + b_4b_5 + b_4b_a \quad (33)$$

$$\longrightarrow b_3b_4 + b_3b_7 - b_a + 1 \quad (34)$$

$$\longrightarrow b_2b_4 + b_7b_8 \quad (35)$$

$$\longrightarrow b_3b_4 + b_4 \quad (36)$$

$$b_1b_2b_3b_4b_5b_6b_7 + b_2b_3b_4b_5b_6b_7b_8 : \quad (k, n) = (7, 8). \quad (37)$$

$$\longrightarrow b_6b_7 + b_6b_8 + b_a(1 - b_6 + b_7 - b_8) \quad (38)$$

$$\longrightarrow b_2b_3 + b_3b_4 \quad (39)$$

$$\longrightarrow b_1b_4 + b_4b_8 - b_6b_a + b_6 \quad (40)$$

$$\longrightarrow b_2b_3 + b_2 \quad (41)$$

$$\longrightarrow b_1b_5 + b_3b_5 + b_6b_a \quad (42)$$

$$b_1b_2b_3b_4 + b_5b_6b_7b_8 : \quad (k, n) = (4, 8). \quad (43)$$

$$\longrightarrow b_1b_2 + b_6b_8 + b_a(1 - b_6 + b_7 - b_8) \quad (44)$$

$$\longrightarrow b_3b_4 + b_6b_8 + 2b_8b_a \quad (45)$$

$$\longrightarrow b_2b_3 + b_5b_7 + b_a(1 - b_6 + b_7) \quad (46)$$

$$\longrightarrow b_1b_4 + b_5b_7 - b_6b_8 + b_7b_a + b_6 \quad (47)$$

$$b_1b_2b_3b_4b_5 + b_6b_7b_8b_9b_{10} : \quad (k, n) = (5, 10). \quad (48)$$

$$\longrightarrow b_2b_3 + b_6b_9 + b_9b_a \quad (49)$$

$$\longrightarrow b_1b_4 + b_8b_{10} + b_9b_a \quad (50)$$

$$\longrightarrow b_3b_5 + b_7b_{10} + b_1b_a + b_9b_a \quad (51)$$

$$\longrightarrow b_4b_5 + b_6b_9 + b_9b_a \quad (52)$$

$$\longrightarrow b_1b_2 + b_7b_9 + b_9b_a \quad (53)$$

$$\longrightarrow b_2b_5 + b_6b_8 + b_9b_a \quad (54)$$

$$\longrightarrow b_2b_3 + b_8b_{10} + b_9b_a \quad (55)$$

$$\longrightarrow b_1b_4 + b_6b_7 + b_9b_a \quad (56)$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 + b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (6, 10). \quad (57) \\
& \longrightarrow b_5 b_6 + b_5 b_7 + b_a(1 - b_{10}) & (58) \\
& \longrightarrow b_1 b_2 + b_8 b_9 - b_a(b_9 + b_{10}) + b_9 + b_{10} & (59) \\
& \longrightarrow b_3 b_4 + b_a(1 + b_7 - b_9 - b_{10}) + b_{10} & (60) \\
& \longrightarrow b_3 b_6 - b_5 b_{10} + b_6 b_7 + b_a(1 - b_9) + b_{10} & (61) \\
& \longrightarrow b_1 b_2 + b_7 b_{10} + b_a(1 - b_9 - b_{10}) + b_{10} & (62) \\
& \longrightarrow b_3 b_4 + b_8 b_9 + b_a(1 - b_9 - b_{10} - b_2) - b_2 + b_9 + b_{10} + 1 & (63)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 + b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (7, 10). \quad (64) \\
& \longrightarrow b_4 b_7 + b_6 b_7 + b_a(1 - b_4 - b_7 + b_{10}) & (65) \\
& \longrightarrow b_2 b_5 + b_5 b_9 + b_a & (66) \\
& \longrightarrow b_1 b_4 + b_4 b_8 + b_a & (67) \\
& \longrightarrow b_1 b_3 + b_6 b_{10} & (68) \\
& \longrightarrow b_3 b_6 + b_6 b_9 + b_a & (69) \\
& \longrightarrow b_2 b_3 + b_8 b_{10} & (70) \\
& \longrightarrow b_1 b_4 + b_9 & (71)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 + b_3 b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (8, 10). \quad (72) \\
& \longrightarrow b_2 b_4 + b_4 b_9 - b_a(b_9 + b_{10}) + b_9 + b_{10} & (73) \\
& \longrightarrow b_1 b_7 + b_7 b_{10} - b_a(b_9 + b_{10}) + b_9 + b_{10} & (74) \\
& \longrightarrow b_5 b_8 + b_6 b_8 - b_a(b_9 + b_{10}) + b_9 + b_{10} & (75) \\
& \longrightarrow b_3 b_6 + b_a(b_3 - b_{10}) + b_{10} & (76) \\
& \longrightarrow b_1 b_5 + b_5 b_9 - b_a(b_9 + b_{10}) + b_9 + b_{10} & (77) \\
& \longrightarrow b_6 b_9 - b_a(b_9 + b_{10}) + b_6 + b_9 + 1 & (78) \\
& \longrightarrow b_1 b_2 - b_9 b_a + b_{10} + 1 & (79)
\end{aligned}$$

$$\begin{aligned}
& b_1 b_2 b_3 b_4 b_5 b_6 b_7 b_8 b_9 + b_2 b_3 b_4 b_5 b_6 b_7 b_8 b_9 b_{10} : & (k, n) = (9, 10). \quad (80) \\
& \longrightarrow b_1 b_6 + b_6 b_{10} + b_9 b_a & (81) \\
& \longrightarrow b_4 b_7 + b_7 b_8 & (82) \\
& \longrightarrow b_4 b_9 + b_a(b_9 - b_4) + b_4 & (83) \\
& \longrightarrow b_2 b_3 + b_3 b_8 + b_9 b_a & (84) \\
& \longrightarrow b_1 b_5 + b_2 b_5 + b_9 b_a & (85) \\
& \longrightarrow b_2 b_8 - b_6 b_7 + b_8 b_{10} + b_9 b_a + 1 & (86) \\
& \longrightarrow b_2 b_{10} + b_2 & (87)
\end{aligned}$$

DECOMPOSITION OF DEGREE- $k$ , EXACT- $k$ -OF- $n$  TRINOMIALS

$$b_1b_2b_3b_4b_5 + b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7 : \quad (k, n) = (5, 7). \quad (88)$$

$$\longrightarrow b_1b_5 + 2b_5b_6 \quad (89)$$

$$\longrightarrow b_2b_4 + b_2b_7 + b_3b_4 + b_6b_7 + b_a(b_6 + b_7 - 1) - b_6 - b_7 + 1 \quad (90)$$

$$\longrightarrow b_1b_3 - b_2b_3 - b_2b_4 - b_2b_6 + b_a(-b_2 + b_4 - b_5 + b_6 - 1) \quad (91)$$

$$+ b_3b_5 + b_3b_7 + b_4b_5 + b_5b_7 + b_6b_7 + b_3 - b_4 - b_5 - b_6 - 2b_7 + 5 \quad (92)$$

$$\longrightarrow b_2b_3 + b_2b_6 - b_4b_5 + b_5b_6 + b_6b_7 + b_a(2b_6 + b_7) - b_6 + 1 \quad (93)$$

$$\longrightarrow b_1b_4 + b_4b_5 - b_5b_7 + b_a(b_5 - 2b_6 - 1) - b_2 + b_4 + 2b_6 + 2 \quad (94)$$

$$b_1b_2b_3b_4b_5b_6 + b_2b_3b_4b_5b_6b_7 + b_3b_4b_5b_6b_7b_8 : \quad (k, n) = (6, 8). \quad (95)$$

$$\longrightarrow b_1b_3 + b_3b_5 + b_3b_8 + b_a(1 + b_6 - b_7) \quad (96)$$

$$\longrightarrow b_2b_6 + b_6b_7 + b_a(-b_6 + b_7) + b_6 \quad (97)$$

$$\longrightarrow b_1b_5 - b_3b_4 + b_4b_5 + b_5b_6 + b_4 \quad (98)$$

$$\longrightarrow -b_1b_3 + b_1b_6 - b_1b_7 + b_2b_4 - b_3b_7 - b_3b_8 + b_4b_5 + b_4b_6 - b_4b_7 + b_4b_8 \quad (99)$$

$$+ b_5b_8 - b_6b_8 + b_7b_8 + b_a(b_2 - b_4 + b_7 + b_8) + b_1 - b_5 - b_6 + 3 \quad (100)$$

$$\longrightarrow b_1b_4 + b_6b_7 + b_7b_8 \quad (101)$$

$$\longrightarrow b_2b_7 + b_7b_8 + b_8b_a + b_2 \quad (102)$$

$$b_1b_2b_3b_4b_5b_6b_7b_8 + b_2b_3b_4b_5b_6b_7b_8b_9 + b_3b_4b_5b_6b_7b_8b_9b_{10} : \quad (k, n) = (8, 10). \quad (103)$$

$$\longrightarrow b_3b_5 + b_5b_9 + b_5b_{10} \quad (104)$$

$$\longrightarrow b_1b_4 + b_4b_7 + b_4b_9 \quad (105)$$

$$\longrightarrow b_1b_6 + b_2b_6 - b_5b_6 + b_6b_{10} + b_6 \quad (106)$$

$$\longrightarrow b_4b_8 + 2b_8 \quad (107)$$

$$\longrightarrow b_1b_7 + b_2b_7 - b_a(b_4 + b_6) + b_7b_9 + b_4 + 1 \quad (108)$$

$$\longrightarrow b_2b_3 + b_3b_5 + b_3 \quad (109)$$

$$\longrightarrow b_1b_2 + b_2b_9 - b_5b_6 - b_5b_a + b_6b_9 + 2 \quad (110)$$

$$\longrightarrow b_2b_7 + b_2 + b_{10} \quad (111)$$

$$b_1b_2b_3b_4b_5b_6 + b_3b_4b_5b_6b_7b_8 + b_5b_6b_7b_8b_9b_{10} : \quad (k, n) = (6, 10). \quad (112)$$

$$\longrightarrow b_2b_5 + b_5b_6 + b_5b_9 - b_9b_{11} + b_9 \quad (113)$$

$$\longrightarrow b_1b_4 + b_4b_6 + b_6b_{10} - b_9b_{11} + b_9 - 2b_{11} + 2 \quad (114)$$

$$\longrightarrow b_3b_4 + b_3b_7 + b_9b_{10} - b_9b_{11} + b_9 \quad (115)$$

$$\longrightarrow b_2b_3 + b_8b_{11} - b_9b_{11} + b_8 + b_9 \quad (116)$$

$$\longrightarrow b_1b_6 + b_6b_7 - b_9b_{11} + b_{10}b_{11} + b_9 \quad (117)$$

$$\longrightarrow b_1b_3 + b_7b_{10} - b_9b_{11} + b_7 + b_9 \quad (118)$$

$$\longrightarrow b_2b_4 + b_4b_7 - b_9b_{11} + 2b_{10}b_{11} + b_9 \quad (119)$$

$$\longrightarrow b_2b_6 + b_6b_7 + b_6b_8 - b_9b_{11} + b_9 \quad (120)$$

$$\longrightarrow b_1b_4 - b_2b_5 - b_3b_{10} + b_5b_8 - b_7b_{11} + b_8b_{10} - b_9b_{11} + b_{10}b_{11} + b_9 + 2 \quad (121)$$

$$b_1b_2b_3b_4 + b_4b_5b_6b_7 + b_7b_8b_9b_{10} : \quad (k, n) = (4, 10). \quad (122)$$

$$\longrightarrow b_3b_4 + b_4b_5 + b_8b_{10} + b_a(1 - b_9) \quad (123)$$

$$\longrightarrow b_1b_2 + b_a(1 + b_3 - b_9) + b_5b_6 + b_7b_9 \quad (124)$$

$$\longrightarrow b_1b_3 + b_6b_7 + b_7b_{10} + b_a \quad (125)$$

$$\longrightarrow b_4b_6 + b_7b_9 + b_a(1 - b_9) + b_4 \quad (126)$$

$$\longrightarrow b_2b_3 + b_5b_7 + b_8b_9 + b_a(1 - b_9) \quad (127)$$

$$\longrightarrow b_2b_3 + b_8b_{10} + b_a(1 - b_9) + b_6 - b_9 + 1 \quad (128)$$

$$\longrightarrow b_1b_2 + b_5b_7 + b_8b_{10} + b_a(1 - b_9) - b_3 + 1 \quad (129)$$

$$\longrightarrow b_5b_6 + b_a(1 - b_9) + b_3 + b_9 \quad (130)$$

$$\longrightarrow b_7b_8 + b_2 + b_7 \quad (131)$$

$$b_1b_2b_3b_4b_5 + b_3b_4b_5b_6 + b_4b_5b_6b_7b_8 : \quad (k, n) = (5, 8). \quad (132)$$

$$\longrightarrow b_2b_4 + b_4b_6 + b_4b_7 + b_a(b_7 + b_8) \quad (133)$$

$$\longrightarrow b_1b_3 + b_3b_6 + b_6b_7 \quad (134)$$

$$\longrightarrow b_3b_5 + b_4b_5 - b_6b_8 + b_5 + b_8 + b_9(1 + b_7) \quad (135)$$

$$\longrightarrow b_2b_5 + b_6b_8 + b_6 \quad (136)$$

$$\longrightarrow b_1b_3 + b_3 + b_8 \quad (137)$$

### DECOMPOSITION OF DEGREE- $k$ , EXACT- $k$ -OF- $n$ QUADRINOMIALS

$$b_1b_2b_3b_4b_5b_6b_7 + b_2b_3b_4b_5b_6b_7b_8 + b_3b_4b_5b_6b_7b_8b_9 + b_4b_5b_6b_7b_8b_9b_{10} : \quad (k, n) = (7, 10). \quad (138)$$

$$\longrightarrow b_1b_2 + b_2b_5 + b_5b_7 + b_5b_{10} - b_a(b_9 + b_{10}) + b_9 + b_{10} \quad (139)$$

$$\longrightarrow b_5b_6 + b_6b_8 + b_6b_9 + b_6b_{10} - b_a(b_9 + b_{10}) + b_9 + b_{10} \quad (140)$$

$$\longrightarrow b_1b_7 + b_5b_7 + b_6b_7 + b_7b_9 - b_a(b_9 + b_{10}) + b_9 + b_{10} \quad (141)$$

$$\longrightarrow b_4b_5 + b_4b_8 + b_4b_{10} + b_a(b_4 - b_9 - b_{10}) + b_9 + b_{10} \quad (142)$$

$$\longrightarrow b_1b_3 + b_3b_4 + b_3b_9 - b_6b_7 + b_6b_{10} + b_a(1 - b_9 - b_{10}) - b_6 + b_{10} + 2 \quad (143)$$

$$\longrightarrow b_2b_3 + b_2b_8 + b_3b_9 - b_5b_7 + b_6b_9 - b_a(1 + b_{10}) - b_7 + b_9 + b_{10} + 3 \quad (144)$$

$$\longrightarrow b_2b_3 - b_6b_7 + b_6b_8 + b_7b_8 - b_a(b_9 + b_{10}) + b_8 + b_9 + b_{10} + 1 \quad (145)$$

$$\longrightarrow -b_1b_2 + b_1b_4 + b_2b_5 - b_4b_9 + b_5b_8 + b_5b_9 - b_a(b_9 + b_{10}) - b_4 + b_5 - b_7 + 2b_9 + b_{10} + 2 \quad (146)$$

$$\longrightarrow b_2b_8 + b_8b_{10} + b_a(b_8 - b_9 - b_{10}) + b_1 + b_9 + b_{10} \quad (147)$$