

Quiz 4:

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1) by excluding the zeros in between, the series becomes

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \sim \text{which is } \sum_{n=1}^{\infty} \frac{1}{n} + \sum_{n=0}^{\infty} 0$$

$\sum_{n=1}^{\infty} \frac{1}{n}$ could be represented by $f(x) = \frac{1}{x}$ and series is

$$\text{bounded by } \lim_{n \rightarrow \infty} \int_2^n \frac{1}{x} dx \Rightarrow \lim_{n \rightarrow \infty} \left[\ln(x) \right]_2^n = \lim_{n \rightarrow \infty} \ln(n) - \ln(2)$$

With $\lim_{n \rightarrow \infty} \ln(n)$ approaching ∞ , the series converges.

We choose $\int_2^{\infty} \frac{1}{x} dx$ because it's monotonic and bounds

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

2) $\sum_{n=1}^{\infty} \frac{\ln(n)}{\sqrt{n}}$ is bounded by $\int_2^{\infty} \frac{\ln(x)}{\sqrt{x}} dx$ by integrating by

$$\text{Parts, } \begin{array}{l} u = \ln(x) \quad dv = x^{-1/2} \\ du = \frac{1}{x} \quad v = 2\sqrt{x} \end{array} \Rightarrow \left[2\sqrt{x} \ln(x) \right]_2^{\infty} - \int_2^{\infty} \frac{2\sqrt{x}}{x}$$

$$\lim_{n \rightarrow \infty} \left[2\sqrt{x} \ln(x) \right]_2^n = \lim_{n \rightarrow \infty} 2\sqrt{n} \ln(n) - 2\sqrt{2} \ln(2)$$

$2\sqrt{n} \ln(n)$ tends to ∞ , thus diverges and makes the series diverge as well

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{\ln(n)}{\sqrt{n}} = \frac{\infty}{\infty} = \frac{-2 \frac{1}{n}}{\sqrt{n}^3} = \frac{1}{n\sqrt{n}^3} = 0$$

so, the series diverges although a_n goes to zero

$$3) \sum_{n=1}^{\infty} \frac{1}{1+n^{-2}} = \sum_{n=1}^{\infty} \frac{1}{1+\frac{1}{n^2}} \quad a_n = \frac{1}{1+\frac{1}{n^2}}$$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{1}{1+\frac{1}{n^2}} = \frac{1}{1+0} = \boxed{1}$$

the series doesn't pass the divergence test and is therefore divergent.

End time = 3:05 PM