Exam

Exercise 2

$\alpha = p \vee q \vee r$

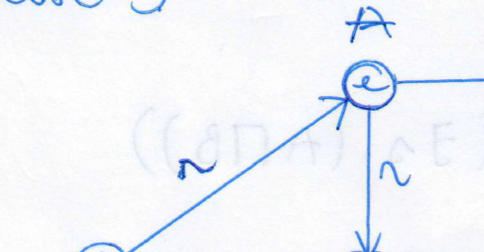
$\beta = (p \wedge q) \vee (\neg p \wedge r) = \beta_1 \vee \beta_2$

p	$\neg p$	q	r	$\beta_1 = p \wedge q$	$\beta_2 = \neg p \wedge r$	$\beta = \beta_1 \vee \beta_2$	$\alpha = p \vee q \vee r$
T	F	T	T	T	F	T	T
T	F	T	F	T	F	T	T
T	F	F	T	F	F	F	T
T	F	F	F	F	F	F	T
F	T	T	T	F	T	T	T
F	T	T	F	F	F	F	T
F	T	F	T	F	T	T	T
F	T	F	F	F	F	F	F

Every model of β is also model of α

$\Rightarrow \alpha$ is a logical consequence of β

Exercise 3



1) $I = (\Delta^I, \cdot^I)$

$\Delta^I = \{d, e, f, g)$

$\cdot^I = \{ A^I = \{d, e, f\} ; B^I = \{f\} ;$

$r^I = \{(d,e) ; (e,g)\}$

$s^I = \{(g,d) ; (e,f) ; (g,g)\} \}$

2) $(A \sqcup B)^I = \cancel{\{} A^I \cup B^I = \{d, e, f\}$

$(\neg A)^I = \Delta^I \setminus A^I = \{g\}$

$\neg \exists r.(A \sqcap B)$

$(A \sqcap B)^I = \{f\}$

$(\exists r.(A \sqcap B))^I = \{x \in \Delta^I, y \cdot (x, y) \in r^I \wedge y \in (A \sqcap B)^I\}$

$= \{\}$

$\neg \exists r.(A \sqcap B)^I = \Delta^I \setminus (\exists r.(A \sqcap B))^I = \Delta^I$

$= \{d, e, f, g\}$

3) Find 3 concepts of C that $C^I = S$

a) $S = \{d; e\}$

$C_1 = \cancel{\{} A \sqcap \neg B$

$C_2 = \exists r.\top$

$C_3 = \forall s.B \sqcap \neg B$

b) $S = \{e; f; g\}$

$C_1 = \neg(\exists r.A) = \forall r.\neg A$

$C_2 = B \sqcup \exists s.\top$

$C_3 = (A \sqcap B) \sqcup (\neg A) \sqcup (\exists s.(A \sqcap B))$

Exercise 4:

1) TBox is ∅ and C is $(A \sqcup B \sqcup \exists R.B) \sqcap (\neg A \sqcap \neg B \sqcap \forall R.\neg B)$

$$(A \sqcup B \sqcup \exists R.B) \sqcap (\neg A \sqcap \neg B \sqcap \forall R.\neg B)(a_0)$$

$$(\neg A \sqcap \neg B \sqcap \forall R.\neg B)(a_0)$$

$$(A \sqcup B \sqcup \exists R.B)(a_0)$$

$A(a_0)$

$\neg A(a_0)$

$\neg B(a_0)$ ($\sqcap$-rule)

$\forall R.\neg B(a_0)$

X

($\sqcup$-rule) $B \sqcup \exists R.B\ (a_0)$

$B(a_0)$

$\neg A(a_0)$

($\sqcap$-rule) $\neg B(a_0)$

$\forall R.\neg B(a_0)$

X

$\exists R.B(a_0)$ ($\sqcup$-rule)

$\neg A(a_0)$

$\neg B(a_0)$

$\forall R.\neg B(a_0)$

$\downarrow$

$R(a_0, a_1)$ ($\exists$-rule)

$B(a_1)$

$\neg A(a_0)$

$\neg B(a_0)$

$\neg B(a_1)$ ($\forall$-rule)

X

C is not satisfiable

2)

TBox is $\emptyset$ and C is $(\exists R.\ \exists S.\ \exists R.\ A) \sqcap (\forall R. \forall S. \forall R \neg A)$

$(\exists R.\ \exists S.\ \exists R.\ A) \sqcap (\forall R. \forall S.\ \forall R. \neg A)(a_0)$

$(\forall R. \forall S.\ \forall R. \neg A)(a_0)$

$(\exists R.\ \exists S.\ \exists R.\ A)(a_0)$

∀r.X rule
The link means that we have already had $r(x_1, x_2)$ before ~~cond~~ add $X(x_2)$

$R(a_0, a_1)$ (∃-rule)

$\exists S.\ \exists R.\ A\ (a_1)$

$S(a_1, a_2)$ (∃-rule)

$\exists R.\ A\ (a_2)$

$R(a_2, a_3)$ (∃-rule)

$\underline{A(a_3)}$

$(\forall R. \forall S.\ \forall R\ \neg A)(a_0)$ (⊓-rule)

$\forall S\ \forall R\ \neg A\ (a_1)$ (∀-rule).

$\forall R\ \neg A\ (a_2)$ (∀-rule).

$\underline{\neg A\ (a_3)}$ (∀-rule)

C is not satisfiable

3)

TBox $\mathcal{T} = \{ D = A \sqcup \neg G$;
$E = \neg A \sqcup \forall S. \neg B$
$G = \exists S. B \}$

$= \{ D = A \sqcup \forall S. \neg B$
$E = \neg A \sqcup \forall S. \neg B$
$G = \exists S. B \}$

$C = \exists R.\neg B \sqcap \forall R.\neg A \sqcap \forall R.D$

$= \exists R.\neg(\neg A \sqcup \forall S.\neg B) \sqcap \forall R.\neg A \sqcap \forall R.(A \sqcup \forall S.\neg B)$

$= \exists R.(A \sqcap \exists S.B) \sqcap \forall R.\neg A \sqcap \forall R.(A \sqcup \forall S.\neg B)$

Assume $C^I = \{a_0\}$

We need to check $T' = T \cup \{C(a_c)\}$

$(A \sqcup \forall S.\neg B)(a_c)$

$(\neg A \sqcup \forall S.\neg B)(a_0)$

$\exists S.B(a_0)$

$\exists R(A \sqcap \exists S.B) \sqcap \forall R.\neg A \sqcap \forall R.(A \sqcup \forall S.\neg B)(a_0)$

This exercise is very strange, both TBox and C are not consistent.

For TBox

$\exists S.B(a_0)$

$S(a_0, a_1)$

$B(a_1)$ ($\exists$-rule)

$\neg A \sqcup \forall S.\neg B(a_c)$ ($\sqcap$-rule)

$S(a_c, a_1)$

$B(a_1)$

$\neg A(a_c)$ ($\sqcup$-rule)

$(A \sqcup \forall S.\neg B)(a_c)$ ($\sqcap$-rule)

$S(a_0, a_1)$

$B(a_1)$

$\neg A(a_c)$

$A(a_c)$ ($\sqcup$-rule)

X

$S(a_0, a_1)$

$B(a_1)$

$\neg A(a_c)$

$\forall S.\neg B$

$\downarrow$

$S(a_0, a_1)$

$B(a_1)$

$\neg A(a_c)$

$\neg B(a_1)$

X

$S(a_c, a_1)$

$B(a_1)$

$\forall S \neg B(a_c)$

$\downarrow$

$S(a_c, a_1)$

$B(a_1)$

$\neg B(a_1)$ ($\forall$-rule)

X

For C:

$\exists R (A \sqcap \exists S.B) \sqcap \forall R.\neg A \sqcap \forall R.(A \sqcup \forall S.\neg B)\ (a_0)$

$\forall R (A \sqcup \forall S.\neg B)\ (a_0)$

$\forall R.\neg A \quad (a_0)$

$\exists R.(A \sqcap \exists S.B)\ a_0$

$R(a_0, a_1)$

$(A \sqcap \exists S.B)_{a_1}$ ($\exists$-rule)

$\downarrow$

$R(a_0, a_1)$

$A(a_1)$

$\exists S.B(a_1)$ ($\sqcap$-rule)

$\forall R.\neg A\ (a_0)$ ($\sqcap$-rule)

$\downarrow$

$R(a_0, a_1)$

$\underline{A(a_1)}$

$\exists S.B(a_1)$

$\underline{\neg A(a_1)}$ ($\forall$-rule)

X

Exercise 5

1) $O = (TBox, ABox)$

$TBox = \{ A \sqsubseteq B \sqcap \forall r.C \}$

$ABox = \{ A(a) \}$

O is consistent

$\Delta^I = \{ a ; b \}$

$A^I = \{ a \}$

$B^I = \{ a ; b \}$

$r^I = \{ \quad \}$

We have

$(\forall r.C)^I = \{ a ; b \}$

$-(B \sqcap \forall r.C)^I = B^I \cap (\forall r.C)^I = \{ a ; b \}$

$A^I \subset (B \sqcap \forall r.C)^I$

2) $O' = O \cup \{ r(a,b), \neg C(b) \}$

$ABox = \{ A(a), r(a,b), \neg C(b) \}$

O~~s~~ is not consistent:

We have $r(a,b)$ an $\neg C(b)$

$\Rightarrow a \notin (\forall r.C)^I$

$\Rightarrow a \notin (B \sqcap \forall r.C)^I$

However $a \in A^I$

$\Rightarrow A^I \not\subset (B \sqcap \forall r.C)^I$

$\Rightarrow A \not\sqsubseteq B \sqcap \forall r.C$