Lecture 6:
Finding Features (part 1/2)

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What we will learn today?

- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector
- Scale invariant region selection
  - Automatic scale selection
  - Difference-of-Gaussian (DoG) detector
- SIFT: an image region descriptor

Next lecture (#7)

What we will learn today?

- Local invariant features
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  - Requirements, invariances
- Keypoint localization
  - Harris corner detector

Some background reading:
Rick Szeliski, Chapter 14.1.1; David Lowe, IJCV 2004

Image matching: a challenging problem

Image matching: a challenging problem

by Diva Sian

by swashford

Slide credit: Steve Seitz

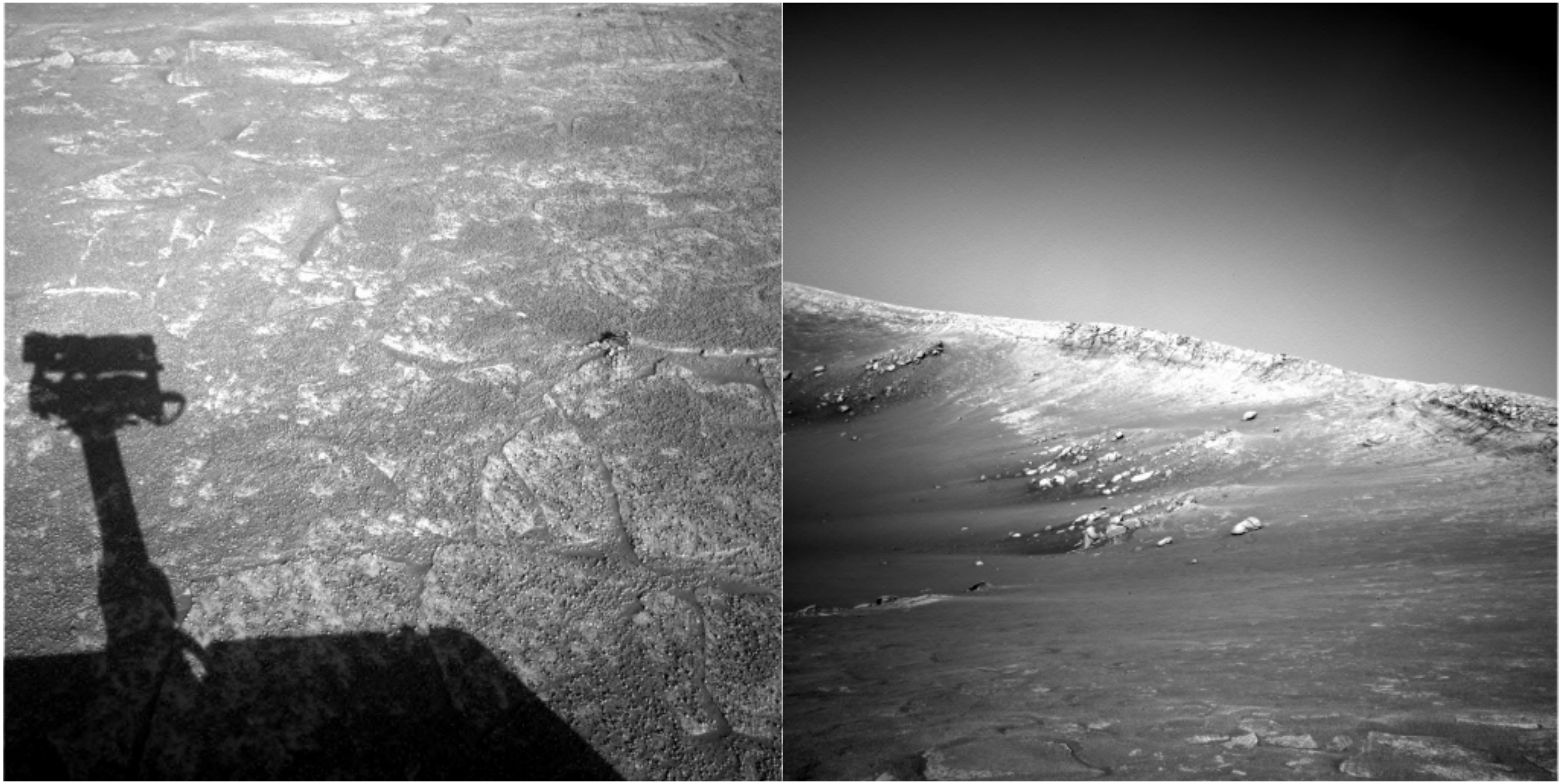
Harder Case

by Diva Sian

by scgbt

Slide credit: Steve Seitz

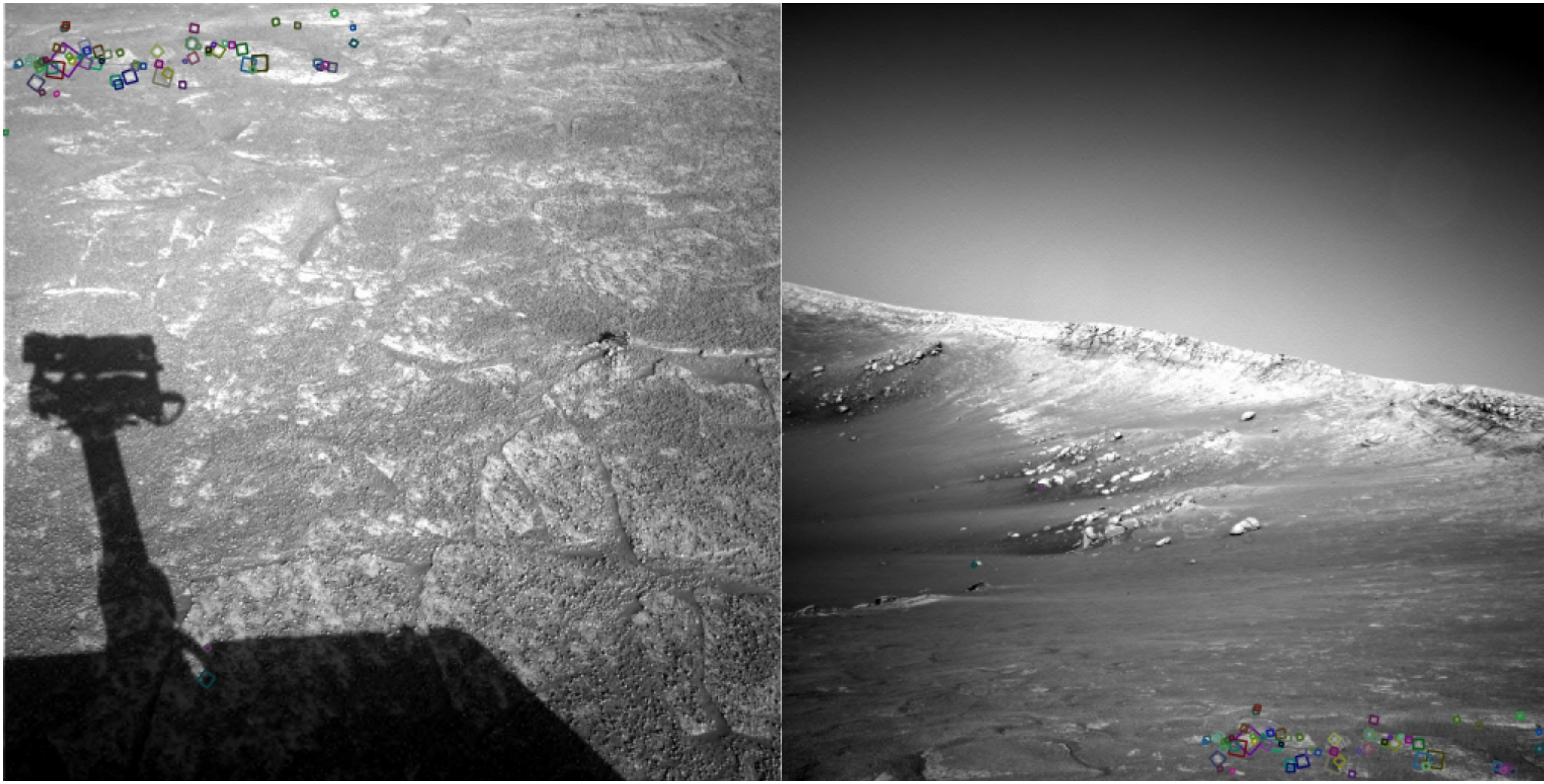
Harder Still?



NASA Mars Rover images

Slide credit: Steve Seitz

Answer Below (Look for tiny colored squares)



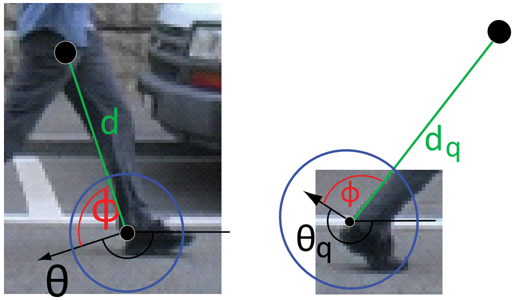
NASA Mars Rover images with SIFT feature matches
(Figure by Noah Snavely)

Slide credit: Steve Seitz

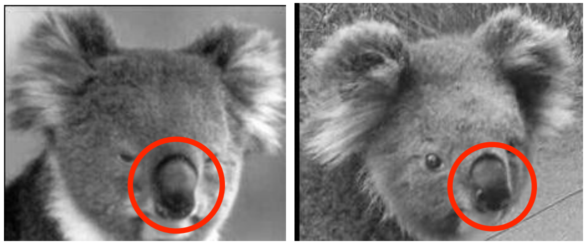
Motivation for using local features

- Global representations have major limitations
- Instead, describe and match only local regions
- Increased robustness to
  - Occlusions

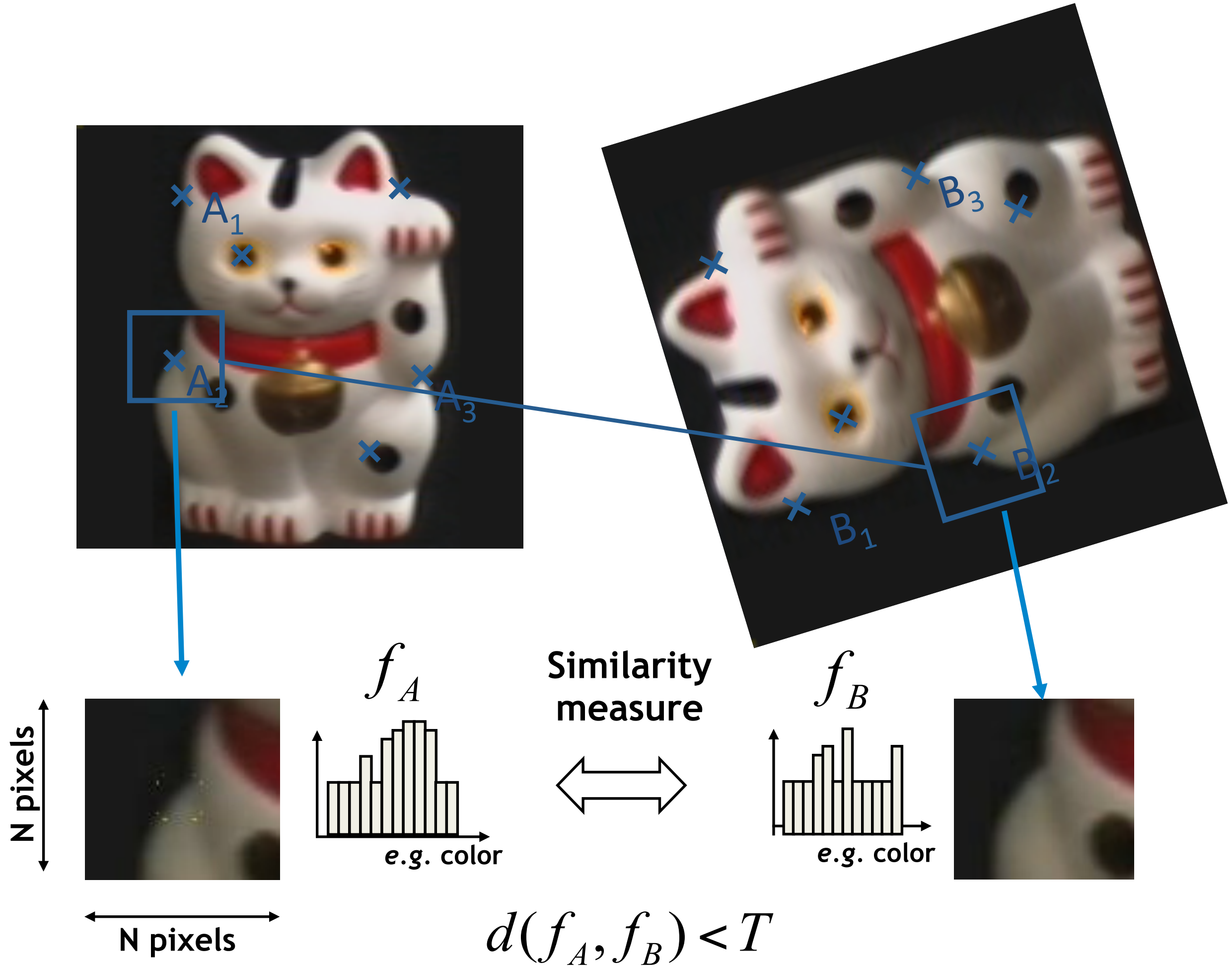
  - Articulation



  - Intra-category variations



General Approach

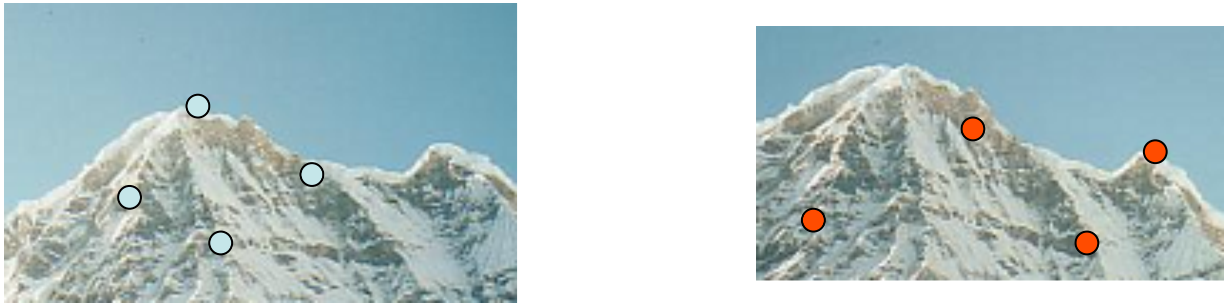


1. Find a set of distinctive key-points
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

Slide credit: Bastian Leibe

Common Requirements

- Problem 1:
  - Detect the same point *independently* in both images



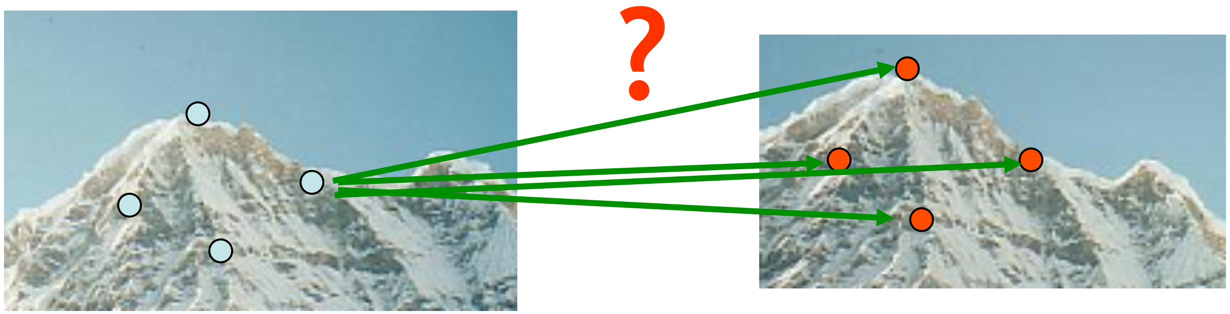
No chance to match!

We need a repeatable detector!

Slide credit: Darya Frolova, Denis Simakov

Common Requirements

- Problem 1:
  - Detect the same point *independently* in both images
- Problem 2:
  - For each point correctly recognize the corresponding one



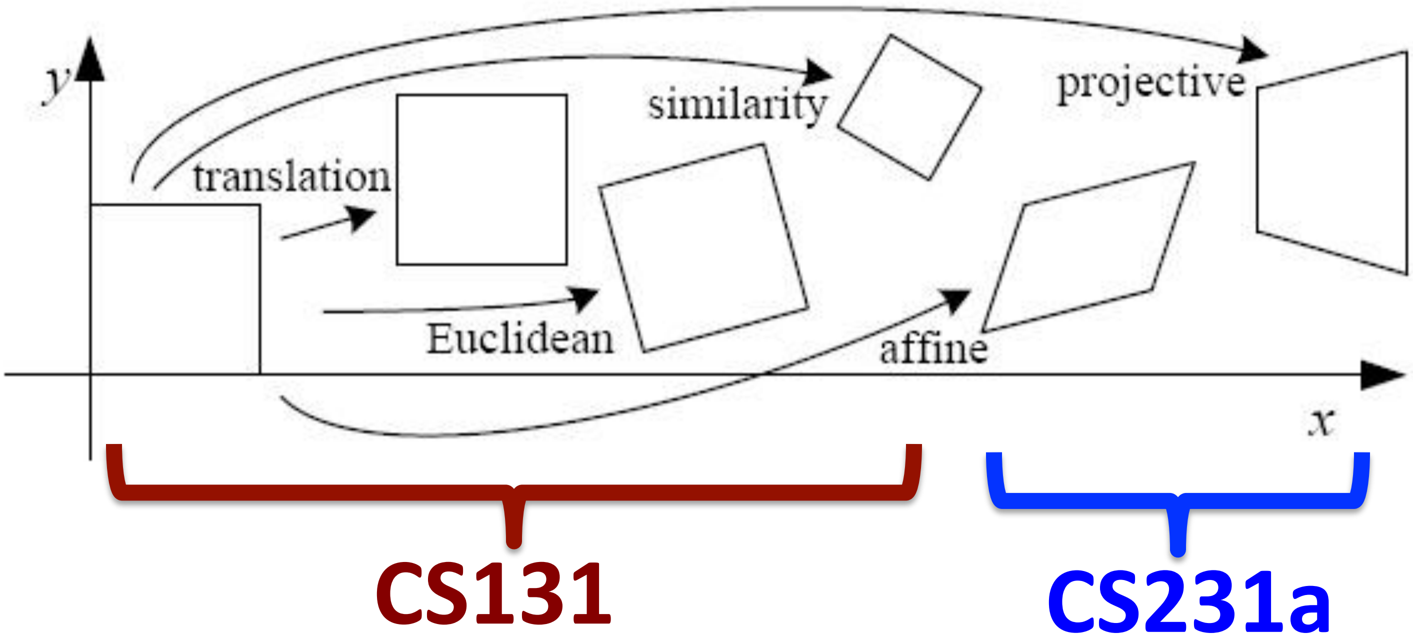
We need a reliable and distinctive descriptor!

Slide credit: Darya Frolova, Denis Simakov

Invariance: Geometric Transformations

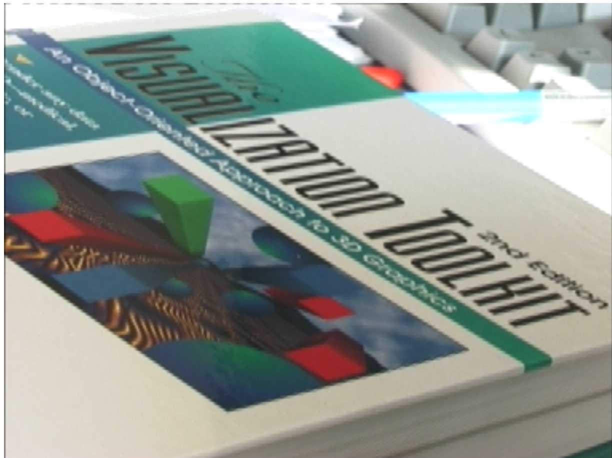
Slide credit: Steve Seitz

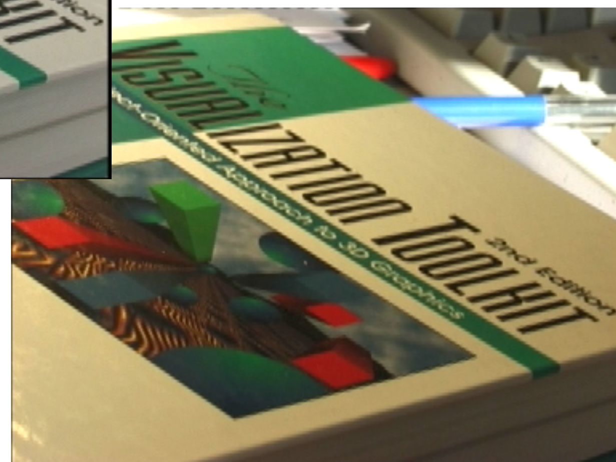
Levels of Geometric Invariance

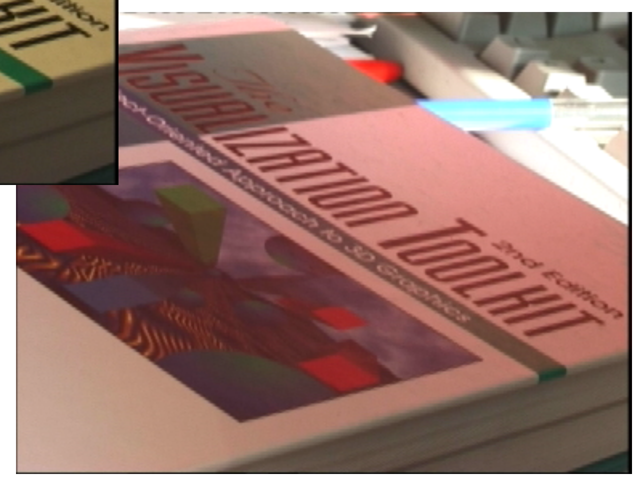


Slide credit: Bastian Leibe

Invariance: Photometric Transformations







- Often modeled as a linear transformation:
  - Scaling + Offset

Slide credit: Tinne Tuytelaars

Requirements

- Region extraction needs to be repeatable and accurate
  - Invariant to translation, rotation, scale changes
  - Robust or covariant to out-of-plane (≈affine) transformations
  - Robust to lighting variations, noise, blur, quantization
- Locality: Features are local, therefore robust to occlusion and clutter.
- Quantity: We need a sufficient number of regions to cover the object.
- Distinctivenes : The regions should contain "interesting" structure.
- Efficiency: Close to real-time performance.

Slide credit: Bastian Leibe

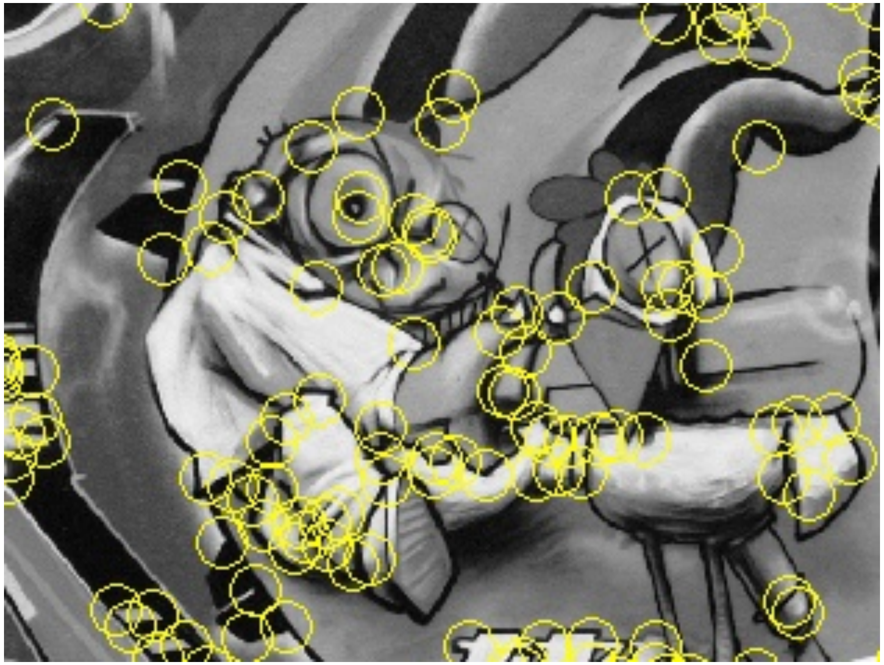
Many Existing Detectors Available

- Hessian & Harris [Beaudet '78], [Harris '88]
- Laplacian, DoG [Lindeberg '98], [Lowe '99]
- Harris-/Hessian-Laplace [Mikolajczyk & Schmid '01]
- Harris-/Hessian-Affine [Mikolajczyk & Schmid '04]
- EBR and IBR [Tuytelaars & Van Gool '04]
- MSER [Matas '02]
- Salient Regions [Kadir & Brady '01]
- Others…

- *Those detectors have become a basic building block for many recent applications in Computer Vision.*

Slide credit: Bastian Leibe

Keypoint Localization

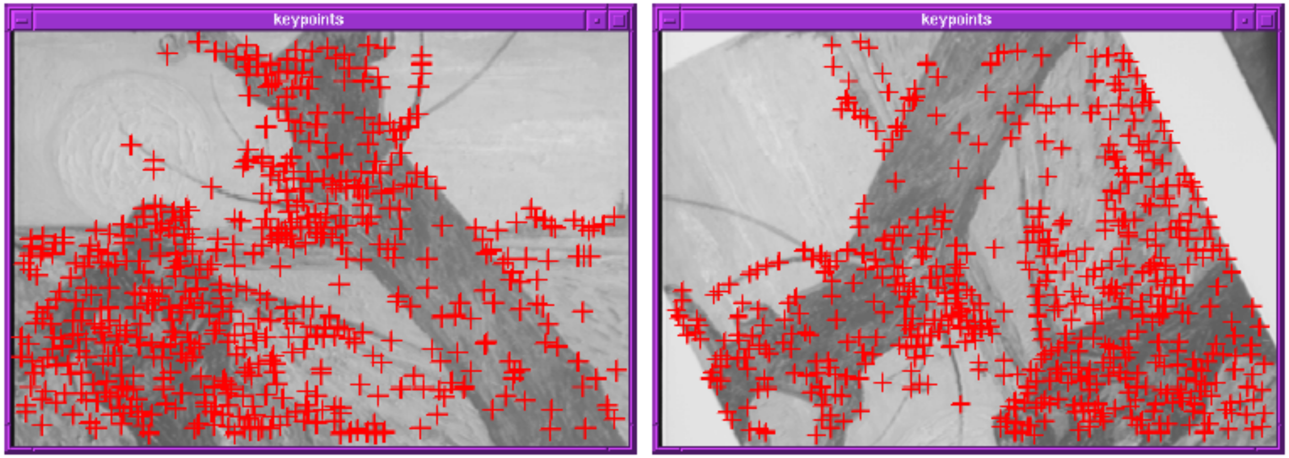


- Goals:
  - Repeatable detection
  - Precise localization
  - Interesting content

$\Rightarrow$ *Look for two-dimensional signal changes*

Slide credit: Bastian Leibe

Finding Corners



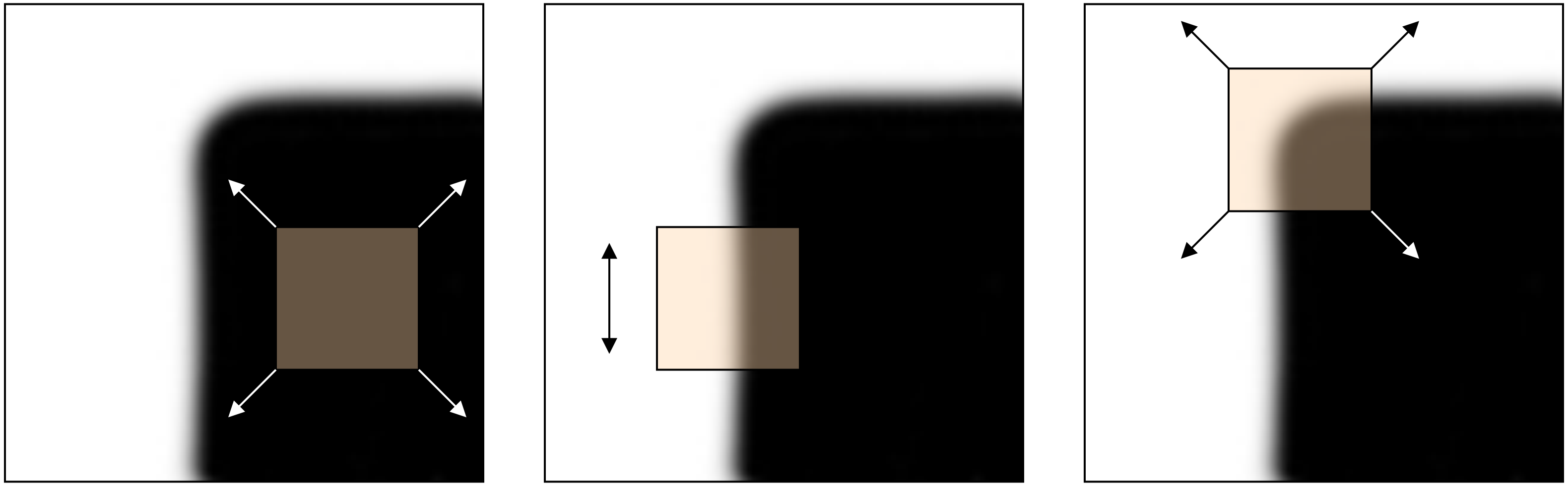
- Key property:
  - In the region around a corner, image gradient has two or more dominant directions
- Corners are *repeatable* and *distinctive*

C.Harris and M.Stephens. "A Combined Corner and Edge Detector.“
Proceedings of the 4th Alvey Vision Conference, 1988.

Slide credit: Svetlana Lazebnik

Corners as Distinctive Interest Points

- Design criteria
  - We should easily recognize the point by looking through a small window (*locality*)
  - Shifting the window in *any direction* should give *a large change* in intensity (*good localization*)



"flat" region:
no change in all directions

"edge":
no change along the edge direction

"corner":
significant change in all directions

Slide credit: Alyosha Efros

Harris Detector Formulation

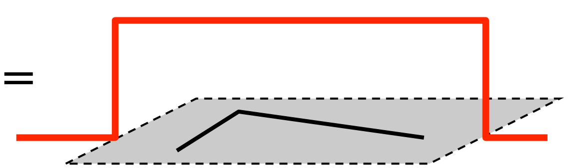
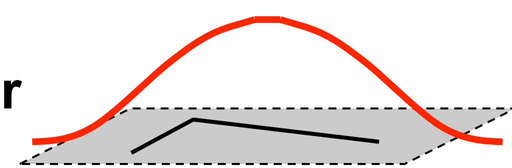
- Change of intensity for the shift [u,v]:

$$E(u,v) = \sum_{x,y} w(x,y)\left[I(x+u, y+v) - I(x,y)\right]^2$$

Window function

Shifted intensity

Intensity

Window function $w(x,y) =$  or 

1 in window, 0 outside

Gaussian

Slide credit: Rick Szeliski

Harris Detector Formulation

- This measure of change can be approximated by:

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a 2×2 matrix computed from image derivatives:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

Gradient with respect to x, times gradient with respect to y

Sum over image region – the area we are checking for corner

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \;\; I_y]$$

Slide credit: Rick Szeliski

Harris Detector Formulation

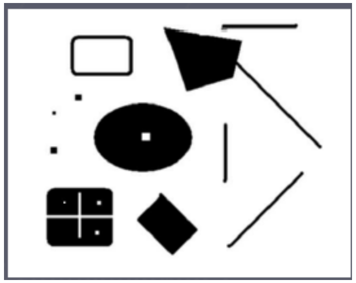
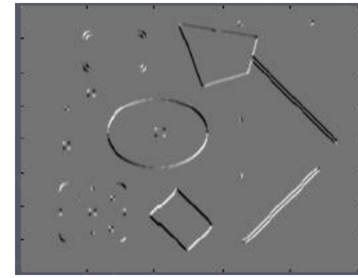


Image I

I_x

I_y



$I_x I_y$

where M is a 2×2 matrix computed from image derivatives:

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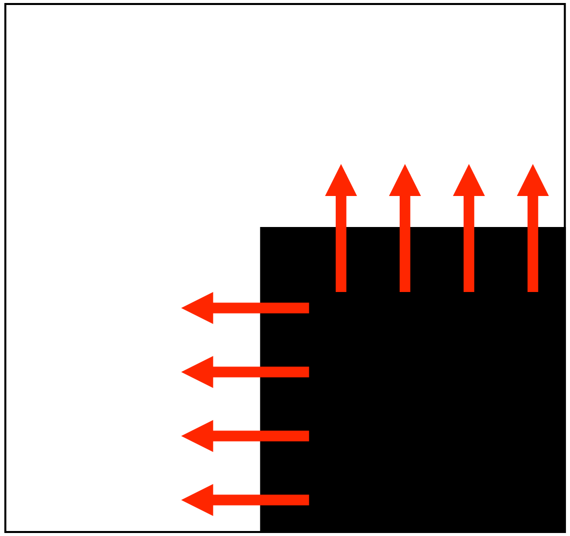
Slide credit: Rick Szeliski

What Does This Matrix Reveal?

- First, let's consider an axis-aligned corner:

$$M = \begin{bmatrix} \sum I_x^2 & \sum I_x I_y \\ \sum I_x I_y & \sum I_y^2 \end{bmatrix} = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$



Slide credit: David Jacobs

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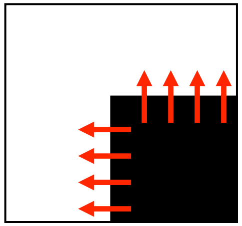


- This means:
  - Dominant gradient directions align with x or y axis
  - If either λ is close to 0, then this is not a corner, so look for locations where both are large.
- What if we have a corner that is not aligned with the image axes?

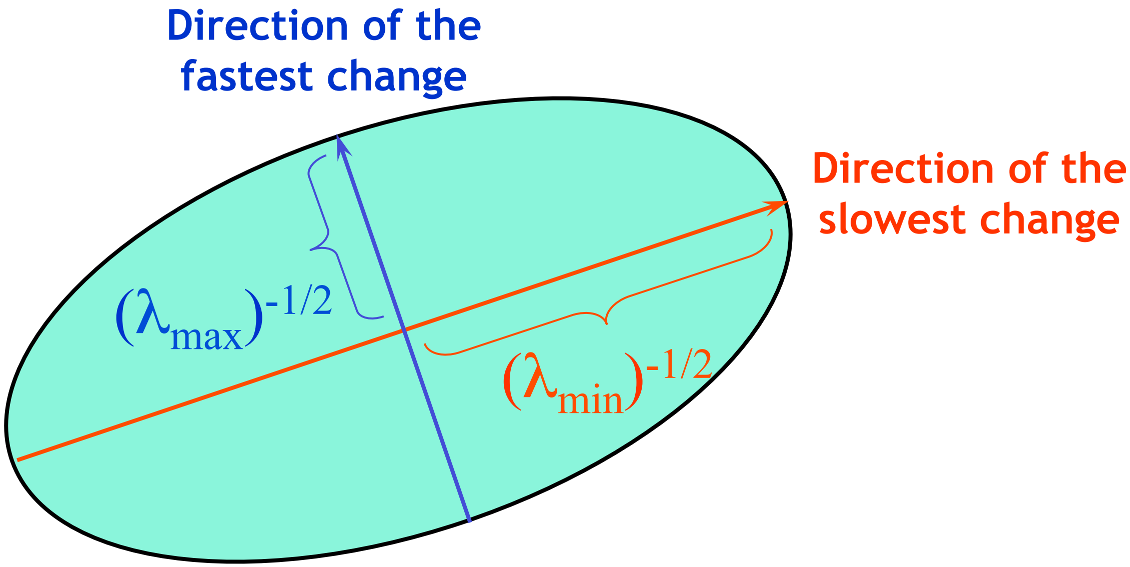
Slide credit: David Jacobs

General Case

- Since M is symmetric, we have $$M = R^{-1}\begin{bmatrix}\lambda_1 & 0 \\ 0 & \lambda_2\end{bmatrix}R$$

  (Eigenvalue decomposition)

- We can visualize M as an ellipse with axis lengths determined by the eigenvalues and orientation determined by R

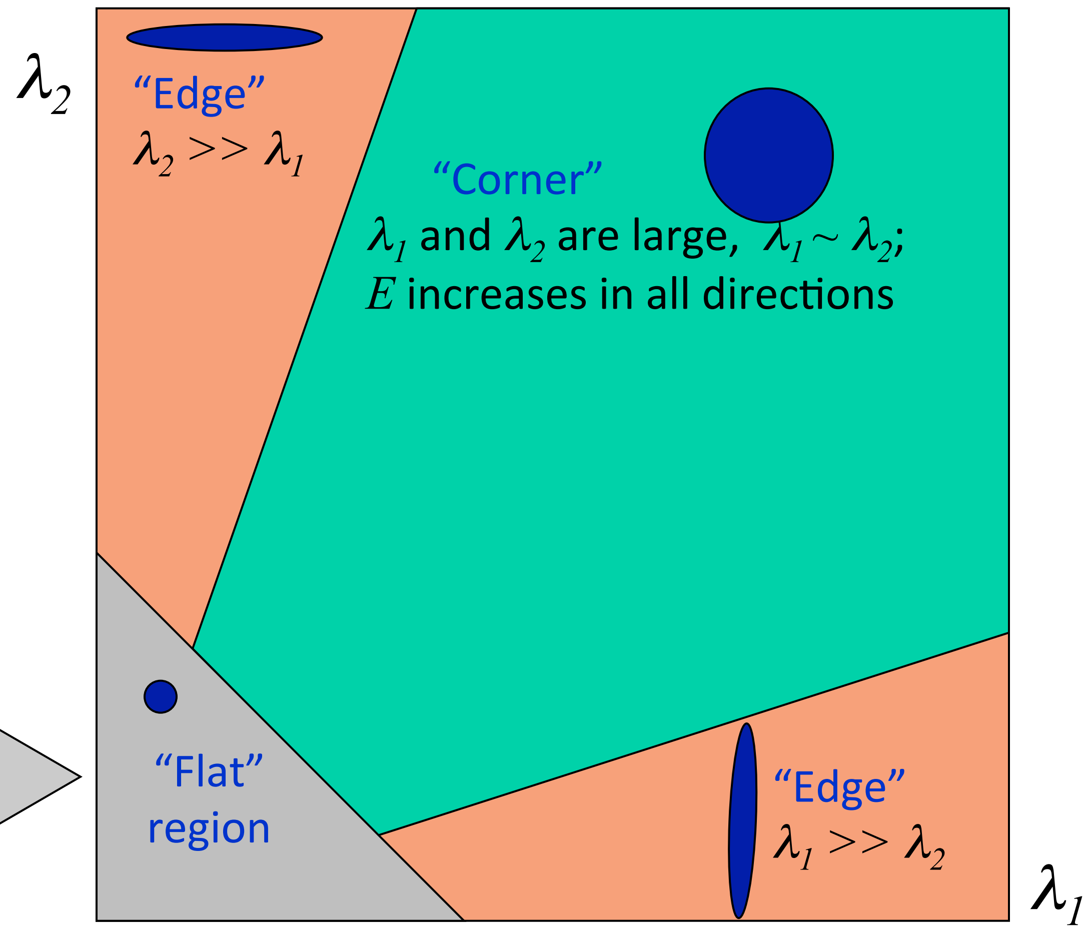


adapted from Darya Frolova, Denis Simakov

Interpreting the Eigenvalues

- Classification of image points using eigenvalues of M:

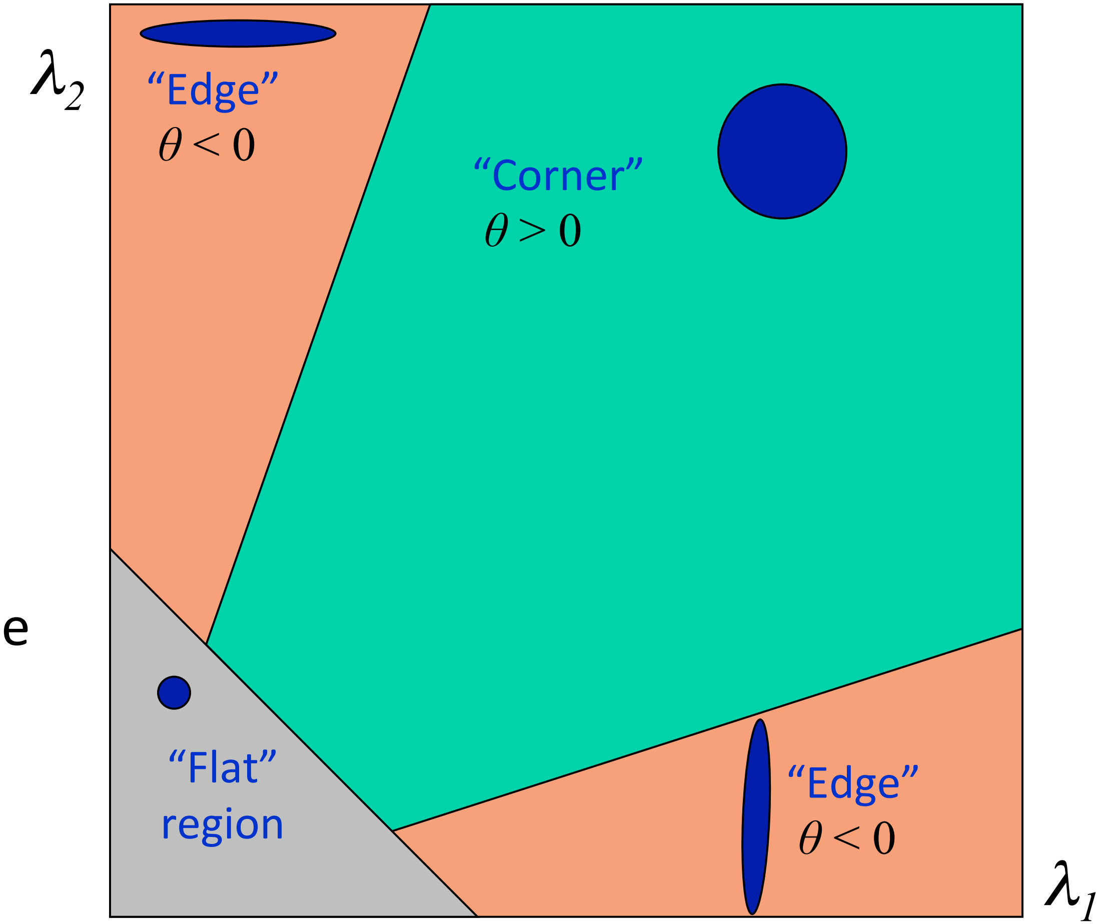
λ_1 and λ_2 are small;
E is almost constant in all directions



Slide credit: Kristen Grauman

Corner Response Function

$$\theta = \det(M) - \alpha\,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$



- Fast approximation
  - Avoid computing the eigenvalues
  - α: constant (0.04 to 0.06)

Slide credit: Kristen Grauman

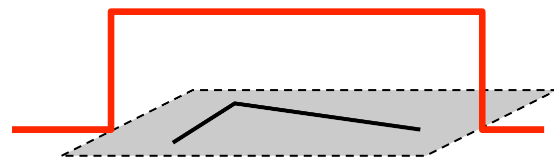
Window Function $w(x,y)$

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Option 1: uniform window
  - Sum over square window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

  - Problem: not rotation invariant

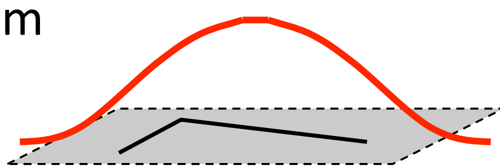


1 in window, 0 outside

- Option 2: Smooth with Gaussian
  - Gaussian already performs weighted sum

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

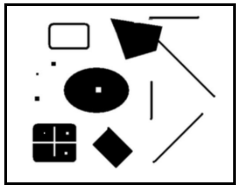
  - Result is rotation invariant



Gaussian

Slide credit: Bastian Leibe

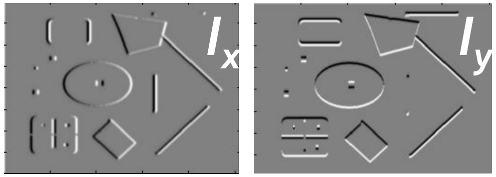
Summary: Harris Detector [Harris88]



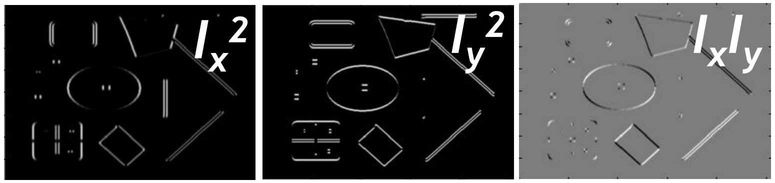
- Compute second moment matrix (autocorrelation matrix)

$$M(\sigma_I, \sigma_D) = g(\sigma_I) * \begin{bmatrix} I_x^2(\sigma_D) & I_x I_y(\sigma_D) \\ I_x I_y(\sigma_D) & I_y^2(\sigma_D) \end{bmatrix}$$

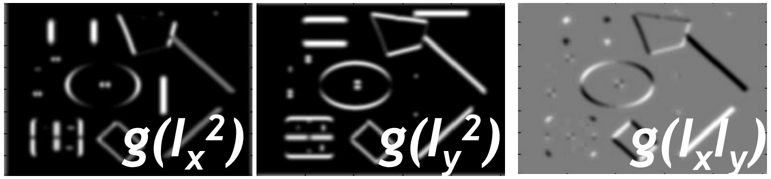
1. Image derivatives



2. Square of derivatives



3. Gaussian filter $g(\sigma_I)$



4. Cornerness function - two strong eigenvalues

$$\theta = \det[M(\sigma_I, \sigma_D)] - \alpha[\mathrm{trace}(M(\sigma_I, \sigma_D))]^2$$
$$= g(I_x^2)g(I_y^2) - [g(I_x I_y)]^2 - \alpha[g(I_x^2) + g(I_y^2)]^2$$

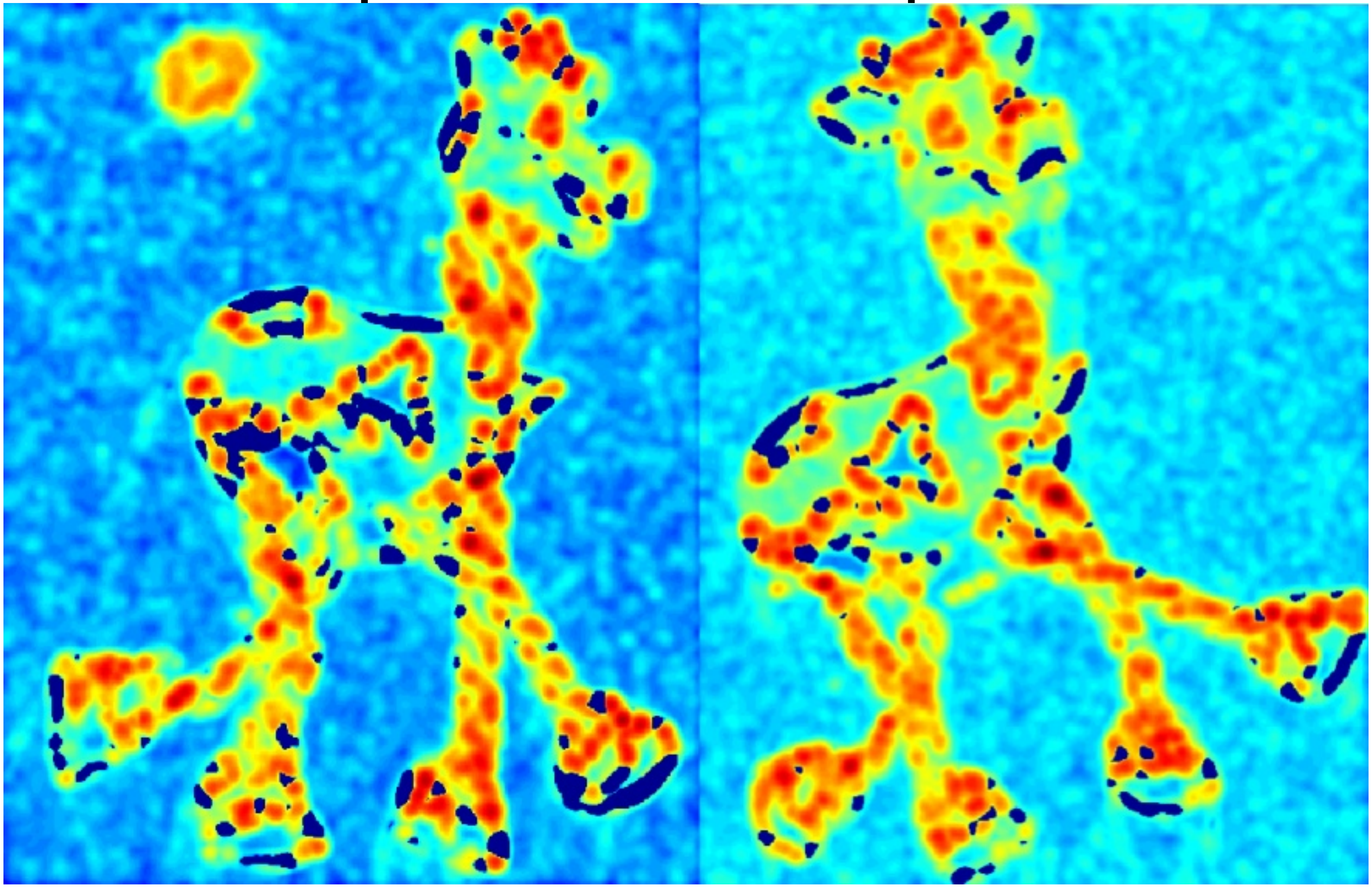
5. Perform non-maximum suppression

Slide credit: Krystian Mikolajczyk

Harris Detector: Workflow

Slide adapted from Darya Frolova, Denis Simakov

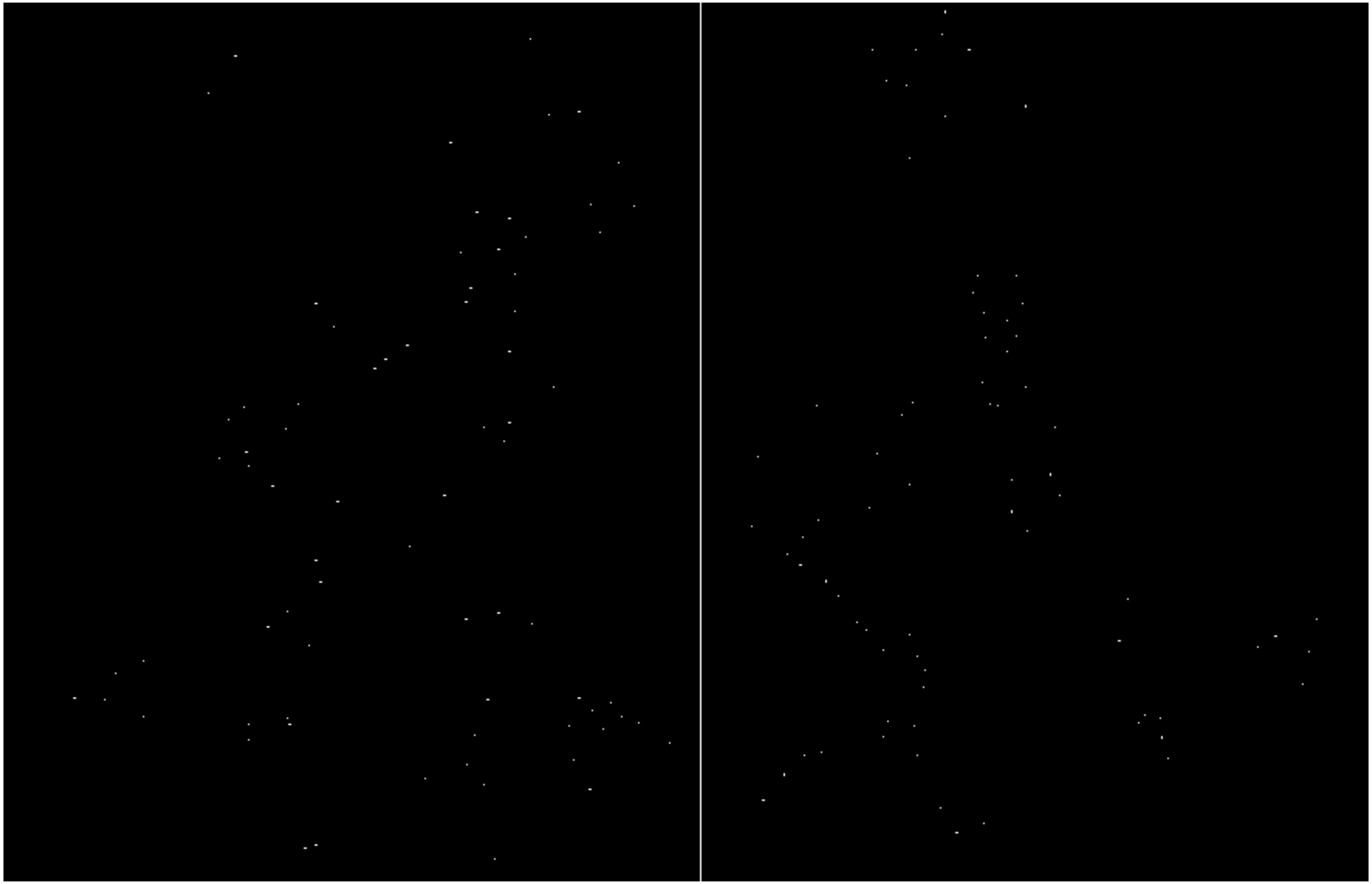
Harris Detector: Workflow
- computer corner responses θ



Slide adapted from Darya Frolova, Denis Simakov

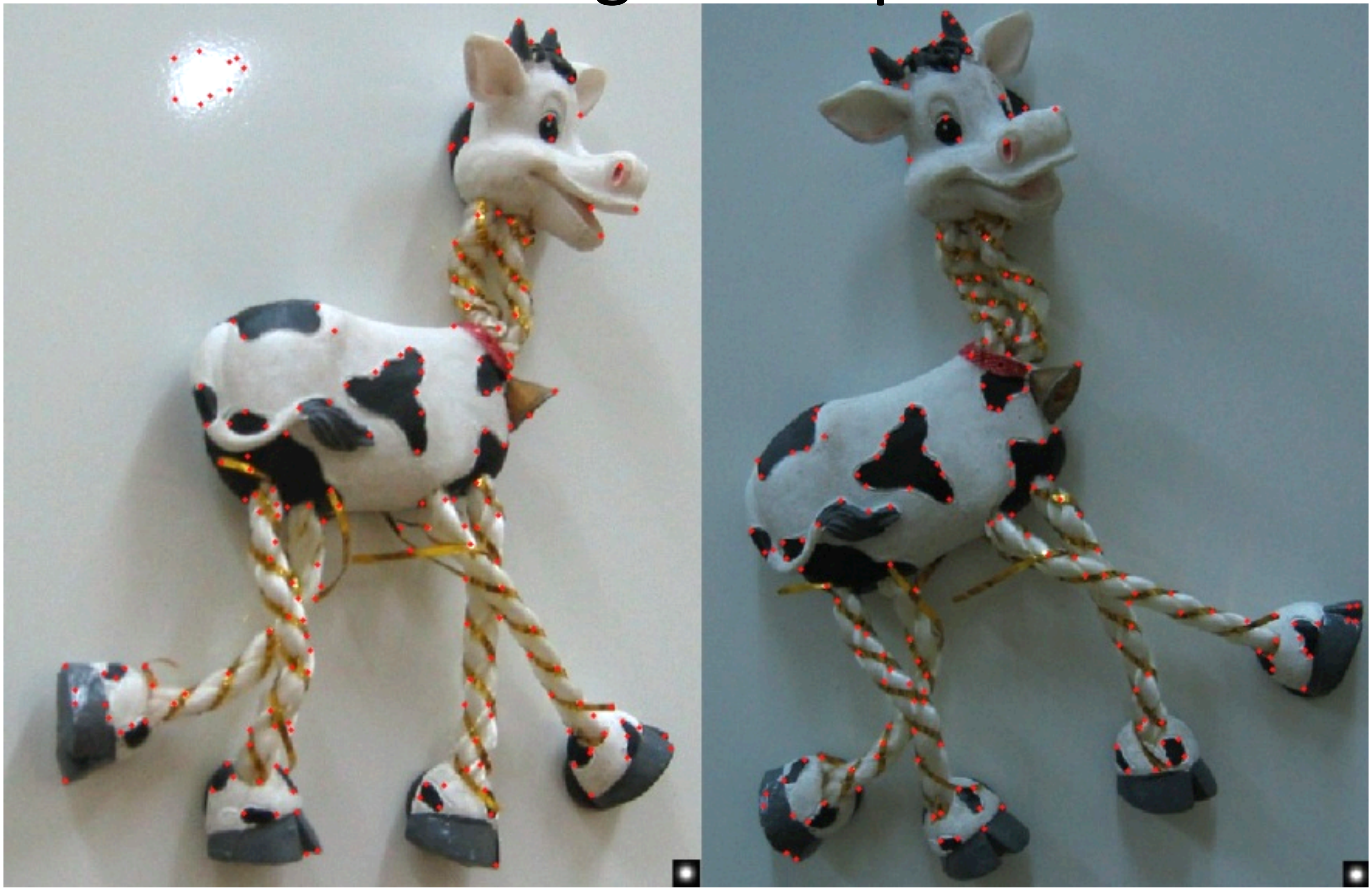
Harris Detector: Workflow

- Take only the local maxima of θ, where θ>threshold



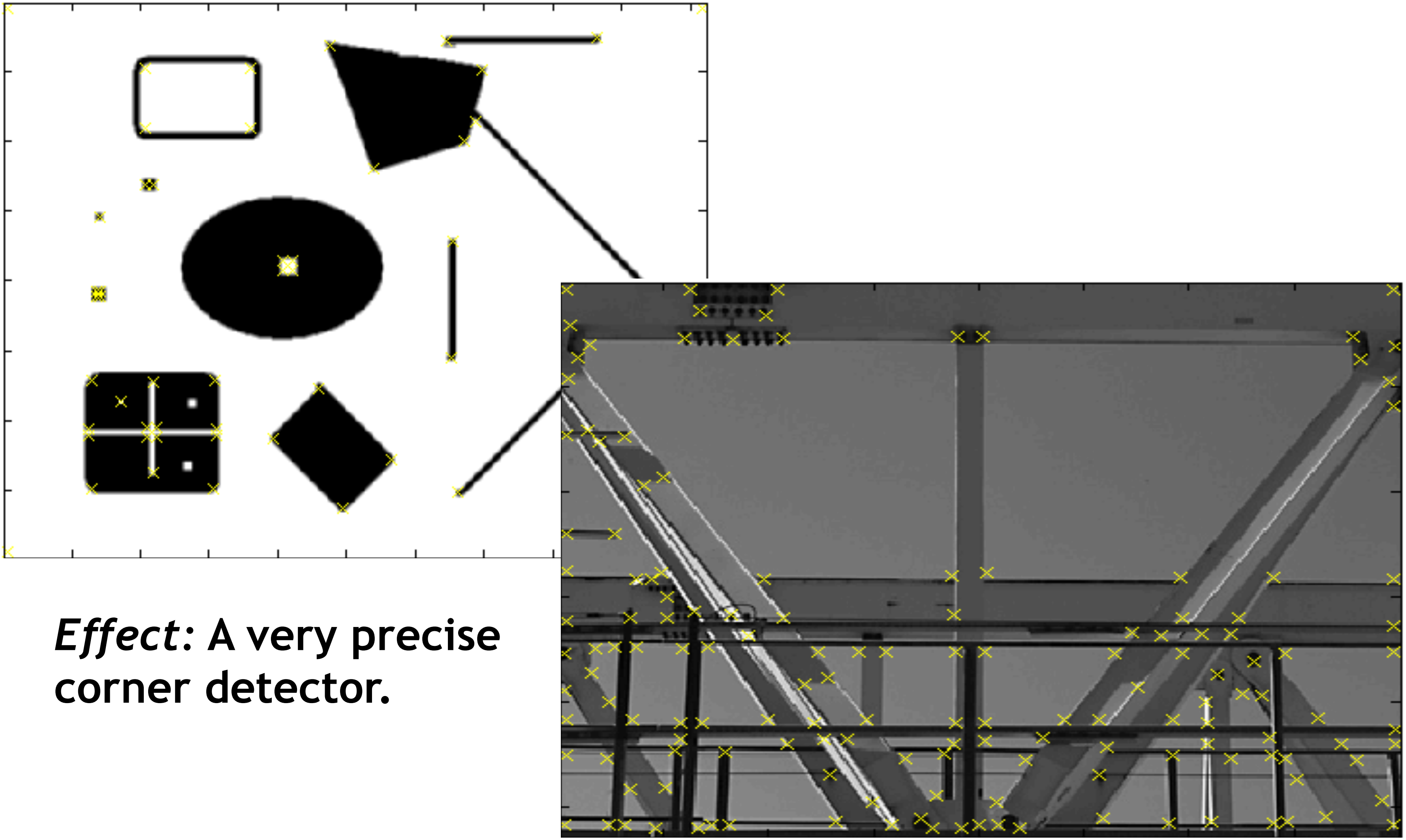
Slide adapted from Darya Frolova, Denis Simakov

Harris Detector: Workflow
- Resulting Harris points



Slide adapted from Darya Frolova, Denis Simakov

Harris Detector – Responses [Harris88]



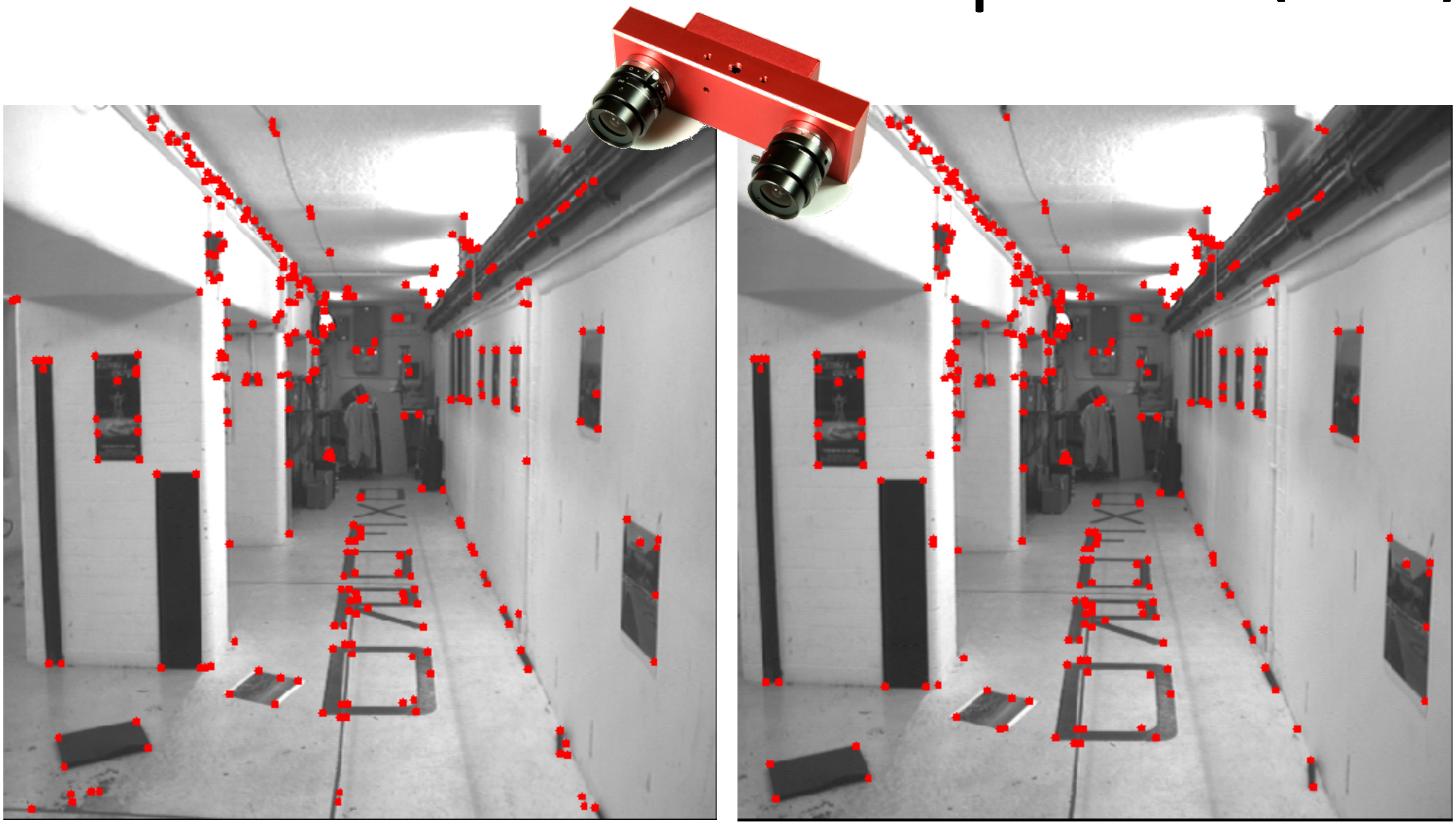
Effect: **A very precise corner detector.**

Slide credit: Krystian Mikolajczyk

Harris Detector – Responses [Harris88]

Slide credit: Krystian Mikolajczyk

Harris Detector – Responses [Harris88]



- Results are well suited for finding stereo correspondences

Slide credit: Kristen Grauman

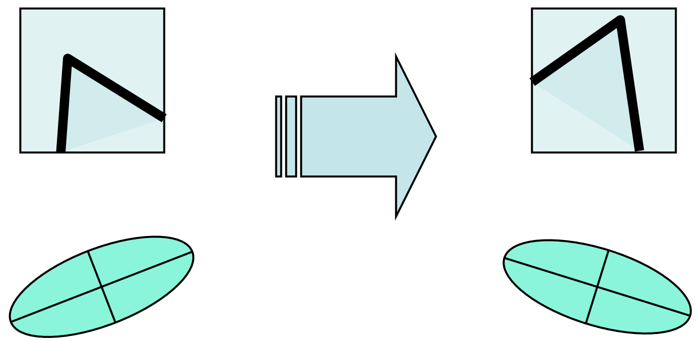
Harris Detector: Properties

- Translation invariance?

Slide credit: Kristen Grauman

Harris Detector: Properties

- Translation invariance
- Rotation invariance?



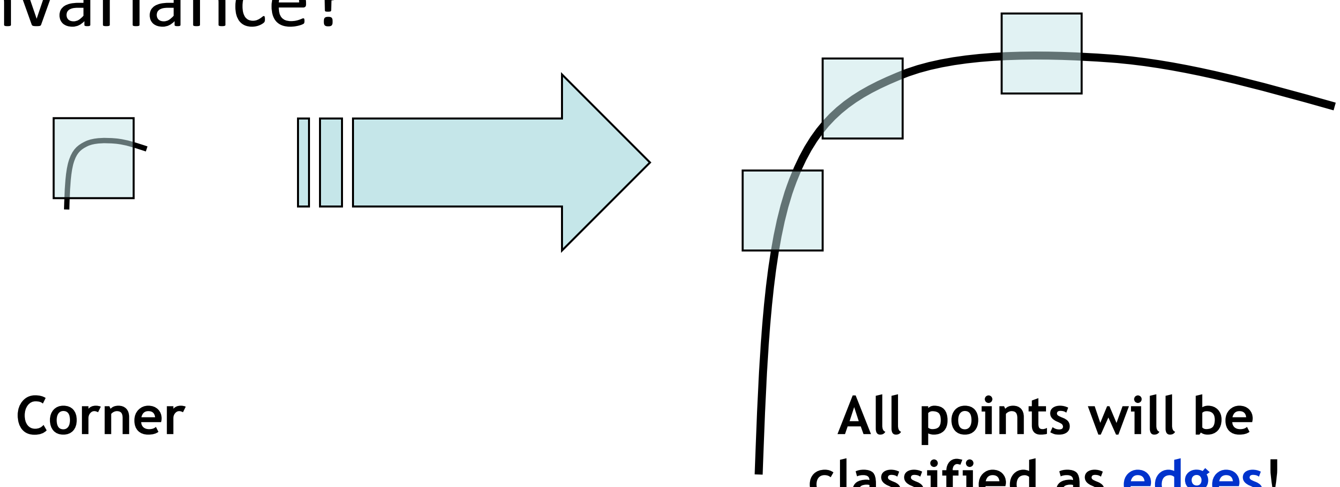
Ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner response θ **is invariant to image rotation**

Slide credit: Kristen Grauman

Harris Detector: Properties

- Translation invariance
- Rotation invariance
- Scale invariance?



Not invariant to image scale!

Slide credit: Kristen Grauman

What we have learned today?

- Local invariant features
  - Motivation
  - Requirements, invariances
- Keypoint localization
  - Harris corner detector
- Scale invariant region selection
  - Automatic scale selection
  - Difference-of-Gaussian (DoG) detector
- SIFT: an image region descriptor

Next lecture (#7)

Some background reading:
Rick Szeliski, Chapter 14.1.1; David Lowe, IJCV 2004