THE UNIVERSITY OF CHICAGO

Master of Science in Analytics

MSCA 37016 – Advanced Linear Algebra for Machine Learning

Instructions:

- Mark the question number and your final answer clearly (use a textbox.)
- Remember to show and explain your work (*If you can't explain it, you don't understand it.*)
- Please submit your solution through Canvas.

(4 points) Question 1:

1) 2% - Choose a q value which gives no solution.
2) 2% - Choose a q value which gives infinitely many solutions.

$$3x + 6y = 1$$
$$6x + 12y = q$$

(7 points) Question 2:

1) 5% - Solve the following system of equations using Gaussian Elimination.

$$2x + 3y + z = 12$$
$$-2x + 3y - 2z = 1$$
$$x - y + 4z = 16$$

2) 2% Validate your answer using Python.

(4 points) Question 3:

4% - Find the rank of each of the following matrices. Verify your rank calculation using Python.

a) $A = \begin{bmatrix} 1 & 3 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 3 & -1 \end{bmatrix}$

b) $A = \begin{bmatrix} -1 & 1 & 0 & -1 \\ -2 & 2 & 1 & -4 \\ -1 & 1 & -2 & 3 \end{bmatrix}$

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Question 1:

$$\begin{bmatrix} 3 & 6 & | & 1 \\ 6 & 12 & | & q \end{bmatrix} \quad L_1 \times 2 \Rightarrow L_1 \quad \begin{bmatrix} 6 & 12 & | & 2 \\ 6 & 12 & | & q \end{bmatrix}$$

columns of A are dependent

1) No solution when q is not 2.

eg) when $\boxed{q = 3}$

$$\begin{bmatrix} 3 & 6 & | & 1 \\ 6 & 12 & | & 3 \end{bmatrix} \quad L_1 \times 2 - L_2 \to L_2 \quad \begin{bmatrix} 3 & 6 & | & 1 \\ 0 & 0 & | & -1 \end{bmatrix}$$

$0y = -1$

$\therefore$ NO solution

2) Infinite solution when $\boxed{q = 2}$

because the solutions would be all points on the line

$$\begin{bmatrix} 3 & 6 & | & 1 \\ 6 & 12 & | & 2 \end{bmatrix} \quad L_1 \times 2 - L_2 \to L_2 \quad \begin{bmatrix} 3 & 6 & | & 1 \\ 0 & 0 & | & 0 \end{bmatrix}$$

$0y = 0$ $\therefore$ infinite solutions

let $x = t$

$$y = \frac{1 - 3t}{6}$$

Question 2: $\begin{aligned} 2x+3y+z &= 12 \\ -2x+3y-2z &= 1 \\ x-y+4z &= 16 \end{aligned}$

$$\left[\begin{array}{ccc|c} 2 & 3 & 1 & 12 \\ -2 & 3 & -2 & 1 \\ 1 & -1 & 4 & 16 \end{array}\right]$$

① $L_1 \Leftrightarrow L_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 16 \\ -2 & 3 & -2 & 1 \\ 2 & 3 & 1 & 12 \end{array}\right]$$

② $(-2)L_1 - L_2 \Rightarrow L_2$

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 16 \\ 0 & -1 & -6 & -33 \\ 2 & 3 & 1 & 12 \end{array}\right]$$

③ $2L_1 - L_3 \Rightarrow L_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 16 \\ 0 & -1 & -6 & -33 \\ 0 & -5 & 7 & 20 \end{array}\right]$$

④ $5L_2 - L_3 \rightarrow L_3$

$$\left[\begin{array}{ccc|c} 1 & -1 & 4 & 16 \\ 0 & -1 & -6 & -33 \\ 0 & 0 & -37 & -185 \end{array}\right]$$

$x - y + 4z = 16$ $\quad x \Rightarrow -1$

$-y - 6z = -33$ $\quad y \Rightarrow 3$

$-37z = -185$ $\quad z \Rightarrow 5$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 5 \end{bmatrix}$$

Question 3:

a) $A = \begin{bmatrix} 1 & 3 & 1 & 2 & 0 \\ 0 & 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 3 & -1 \end{bmatrix}$ $L_3 - L_4 \to L_4$ $\begin{bmatrix} \textcircled{1} & 3 & 1 & 2 & 0 \\ 0 & 0 & \textcircled{2} & 1 & 3 \\ 0 & 0 & 0 & \textcircled{3} & 2 \\ 0 & 0 & 0 & 0 & \textcircled{3} \end{bmatrix}$

0 - pivot

4 pivot columns $r = 4$

b) $A = \begin{bmatrix} -1 & 1 & 0 & -1 \\ -2 & 2 & 1 & -4 \\ -1 & 1 & -2 & 3 \end{bmatrix}$

① $2L_1 - L_2 \to L_2$ $\begin{bmatrix} -1 & 1 & 0 & -1 \\ 0 & 0 & -1 & 2 \\ -1 & 1 & -2 & 3 \end{bmatrix}$

② $L_1 - L_3 \to L_3$ $\begin{bmatrix} -1 & 1 & 0 & -1 \\ 0 & 0 & -1 & 2 \\ 0 & 0 & 2 & -4 \end{bmatrix}$

③ $-2L_2 - L_3 \to L_3$ $\begin{bmatrix} \textcircled{-1} & 1 & 0 & -1 \\ 0 & 0 & \textcircled{-1} & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$r = 2$

In [2]:
```
import numpy as np
from scipy import linalg as la
from numpy import linalg as LA
```

In [6]:
```
#Question2
A = np.array([[2,3,1],[-2,3,-2],[1,-1,4]])
b = np.array([12,1,16])
lu,piv = la.lu_factor(A)
x_b = la.lu_solve((lu,piv),b)
print("x for b (Using lu_solve()): \n {} \n".format(x_b))

print('A dot x_b =',np.matmul(A,x_b))
```

```
x for b (Using lu_solve()):
 [-1.  3.  5.]

A dot x_b = [12.  1. 16.]
```

In [13]:
```
#Question3
A1 = np.array([[1,3,1,2,0],[0,0,2,1,3],[0,0,0,3,2],[0,0,0,3,-1]])
A2 = np.array([[-1,1,0,-1],[-2,2,1,-4],[-1,1,-2,3]])
print("Rank of A1:",LA.matrix_rank(A1))
print("Rank of A2:",LA.matrix_rank(A2))
```

```
Rank of A1: 4
Rank of A2: 2
```

In [ ]:
```

```