THE UNIVERSITY OF CHICAGO

Master of Science in Analytics

MSCA 37016 – Advanced Linear Algebra for Machine Learning

Instructions:

- Mark the question number and your final answer clearly (use a textbox.)
- Remember to show and explain your work (*If you can't explain it, you don't understand it.*)
- Please submit your solution through Canvas.

Let

$$\boldsymbol{A} = \begin{bmatrix} 0 & 3 & -1 \\ -1 & 4 & -2 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 0 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

$$v = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$u = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$$

$$a = -2$$

$$b = 1$$

(3 points) Question 1:

1) 2% - Calculate

$$\|v\|_1 v + au$$

2) 1% - Validate your answer using Python.

(4 points) Question 2:

1) 3% - Using the cosine formula, and assuming the angle between vectors v and u is equal to θ, calculate $\cos\theta$.
2) 1% - Validate your answer using Python.

(4 points) Question 3:

1) 3% - Calculate

$$a(\boldsymbol{A} \cdot v)$$

2) 1% - Validate your answer using Python.

(4 points) Question 4:

1) 3% - Let $tr(\boldsymbol{B})$ and $\boldsymbol{L}$ be the trace and lower triangular matrix of matrix $\boldsymbol{B}$, respectively. Calculate

$$\boldsymbol{A} \cdot \boldsymbol{B}^T + tr(\boldsymbol{B}) * \boldsymbol{L}$$

2) 1% - Validate your answer using Python.

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Question 1 :

1) $\|v\|_1 v + au$

$= (|2| + |-1| + |4|) \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + (-2) \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$

$= 7 \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix} + (-2) \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$

$= \begin{bmatrix} 14 \\ -7 \\ 28 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \\ -10 \end{bmatrix}$

$= \boxed{\begin{bmatrix} 18 \\ -9 \\ 18 \end{bmatrix}}$

Question 2:

$v = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$

$u = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$

1)

$$v \cdot u = 2(-2) + (-1)(1) + 4(5)$$

$$= -4 + (-1) + 20$$

$$= 15$$

$$\|v\| = \sqrt{2^2 + (-1)^2 + 4^2} = \sqrt{21}$$

$$\|u\| = \sqrt{(-2)^2 + (1)^2 + 5^2} = \sqrt{30}$$

$$\cos\theta = \frac{v \cdot u}{\|v\|\|u\|}$$

$$= \frac{15}{\sqrt{21}\sqrt{30}} \approx \boxed{.5976}$$

Question 3:

1) $a(A \cdot v) = -2\left(\begin{bmatrix} 0 & 3 & -1 \\ -1 & 4 & -2 \\ 1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}\right)$

$= -2\left(\begin{bmatrix} 0(2) + 3(-1) + (-1)(4) \\ (-1)(2) + (4)(-1) + (-2)(4) \\ 1(2) + 3(-1) + (1)(4) \end{bmatrix}\right)$

$= -2\begin{bmatrix} -7 \\ -14 \\ 3 \end{bmatrix}$

$= \boxed{\begin{bmatrix} 14 \\ 28 \\ -6 \end{bmatrix}}$

Question 4:

- Let $tr(\boldsymbol{B})$ and $\boldsymbol{L}$ be the trace and lower triangular matrix of matrix $\boldsymbol{B}$, respectively. Calculate

$$\boldsymbol{A} \cdot \boldsymbol{B}^T + tr(\boldsymbol{B}) * \boldsymbol{L}$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 3 & -1 \\ -1 & 4 & -2 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 0 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

$$v = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

$$u = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$$

$$a = -2$$

$$b = 1$$

1) $B^T = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix}$

$tr(B) = 2 + 0 + 2 = 4$

$L = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 2 & 2 \end{bmatrix}$

$A \cdot B^T + tr(B) * L$

$= \begin{bmatrix} 0 & 3 & -1 \\ -1 & 4 & -2 \\ 1 & 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & -1 & -1 \\ -1 & 0 & 2 \\ 2 & 1 & 2 \end{bmatrix} + 4 \begin{bmatrix} 2 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 2 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0(2)+3(-1)+(-1)(2) & 0(-1)+(3)(0)+(-1)(1) & 0(-1)+(3)(2)+(-1)(2) \\ (-1)(2)+4(-1)+(-2)(2) & (-1)(-1)+(4)(0)+(-2)(1) & (-1)(-1)+(4)(2)+(-2)(2) \\ (1)(2)+3(-1)+(1)(2) & (1)(-1)+(3)(0)+(1)(1) & (1)(-1)+(3)(2)+(1)(2) \end{bmatrix}$

$+ \begin{bmatrix} 8 & 0 & 0 \\ -4 & 0 & 0 \\ -4 & 8 & 8 \end{bmatrix}$

$= \begin{bmatrix} -5 & -1 & 4 \\ -10 & -1 & 5 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 8 & 0 & 0 \\ -4 & 0 & 0 \\ -4 & 8 & 8 \end{bmatrix}$

$= \begin{bmatrix} 3 & -1 & 4 \\ -14 & -1 & 5 \\ -3 & 8 & 15 \end{bmatrix}$

In [1]:
```
import numpy as np
from scipy import linalg as la
```

In [2]:
```
v = np.array([2,-1,4])
u = np.array([-2,1,5])
a = -2
b =1
A = np.array([[0,3,-1],[-1,4,-2],[1,3,1]])
B = np.array([[2,-1,2],[-1,0,1],[-1,2,2]])
```

In [3]:
```
#Question 1
print('L1 Norm = ',la.norm(v,1))
print('Question 1 = ',la.norm(v,1)*v+a*u)
```

```
L1 Norm =  7.0
Question 1 =  [18. -9. 18.]
```

In [5]:
```
#Question 2
Cosine_Theta = (np.dot(u,v)) / (la.norm(u,2)*la.norm(v,2))
print('L2 of u =',la.norm(u,2))
print('L1 of v =',la.norm(v,2))
print('Dot Product = ',np.dot(u,v))
print('Cosine Theta = ',Cosine_Theta)
```

```
L2 of u = 5.477225575051661
L1 of v = 4.58257569495584
Dot Product =  15
Cosine Theta =  0.5976143046671969
```

In [6]:
```
#Question3
a_A_dot_v = a*np.matmul(A,v)
print('a*(A dot v) = ',a_A_dot_v)
```

```
a*(A dot v) =  [14 28 -6]
```

In [8]:
```
#Question4
Q4 = np.dot(A, B.transpose())+ B.trace()*la.tril(B)
print('Question 4 = \n', Q4)
```

```
Question 4 =
 [[  3  -1   4]
 [-14  -1   5]
 [ -3   8  15]]
```

In [ ]:
```

```