Closed-form matting

$I = \alpha F + (1 - \alpha)B$ Color Line Space Assumption:

> Forground and Background Colors in a small window lie on a straight line in RGB space

Notice

1. Line depends on which window we choose
2. Assumption is based on RGB space, so I doubt that it's not hold on other color space
3. Edge is ok, like black(0,0,0) and white(255,255,255) can be connected on a straight line

$F_i = \beta_i F_1 + (1 - \beta_i)F_2$ $B_i = \gamma_i B_1 + (1 - \gamma_i)B_2$
If Color Line Assumption holds, the the true mate α satifies $\alpha = \alpha^{\mathrm{T}} I_i + b$ For all pixels in the window (I_i is 3-by-1 matrix, α^{T} is 3-by-1 vector, and b is scalar)

Why is this Time?

$$\begin{aligned} I_i &= \alpha_i F_i + (1 - \alpha_i)B_i \\ &= \alpha_i(\beta_i F_1 + (1 - \beta_i)F_2) + (1 - \alpha_i)(\gamma_i B_1 + (1 - \gamma_i)B_2) \\ &= \alpha_i \beta_i F1 + \alpha_i F_2 - \alpha_i \beta_i F_2 + \alpha_i B1 - \alpha_i \gamma_i B_1 + B_2 - \alpha_i B_2 - \gamma_i B_2 + \alpha_i \gamma_i B_2 \end{aligned}$$

So, we can use matrix manuplation to formulate the equation above

$$I_i - B_2 = \begin{bmatrix} F_2 - B_2 \\ F_1 - F_2 \\ B_1 - B_2 \end{bmatrix} \begin{bmatrix} \alpha_i \\ \alpha_i \beta_i \\ \alpha_i \gamma_i \end{bmatrix}$$

which can be like this

$$I_i - B_2 = [\, F_2 - B_2, F_1 - F_2, B_1 - B_2 \,] \begin{bmatrix} \alpha_i \\ \alpha_i \beta_i \\ \gamma_i - \alpha_i \gamma_i \end{bmatrix}$$

and

$$\begin{bmatrix} \alpha_i \\ unknown \\ unknown \end{bmatrix} = \begin{pmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{pmatrix}^{-1} [\, I_i - B_2 \,]$$