Universidade Federal de Viçosa
Centro de Ciências Exatas
Departamento de Matemática

Gabarito $5^{\underline{a}}$ Lista - MAT 135 - Geometria Analítica e Álgebra Linear

1. Somente o conjunto do item (a) é um espaço vetorial com as operações dadas.

2. Os conjuntos dos itens (a) e (d) são subespaços vetoriais e os conjuntos dos itens (b), (c) e (e) não são subespaços vetoriais.

3. (a) $w = 3u - v$.

   (b) Não.

   (c) $k = 12$.

   (d) $c = 16a + 10b$.

4. $v_1 = 1u + \dfrac{11}{3}v + \dfrac{16}{3}w; \quad v_2 = 3u - \dfrac{11}{3}v - \dfrac{10}{3}w; \quad v_3 = 0u + 0v + 0w.$

5. $k = -8$.

6. $-a + 3b + 5c = 0$.

7. (a) L.D.

   (b) L.I.

   (c) L.D.

   (d) L.D.

   (e) L.D.

   (f) L.D.

8. L.I.

9.

10.

11.

12. (a) $W = [(2, 1, -2)]$.

    (b) $W = [(-2, 1, 0), (3, 0, 1)]$.

    (c) $W = [(1, 0, 1, 0), (0, 1, 2, 0)]$.

    (d) $W = [(1, 1, 1, 0), (-1, -1, 0, 1)]$.

13. (a) $[S] = \mathbb{R}^2$.

    (b) $[S] = \{\,(x,y,z) \in \mathbb{R}^3 \,/\, x = y\,\}$.

    (c) $[S] = \{\,(x,y,z) \in \mathbb{R}^3 \,/\, y = 2x\,\}$.

    (d) $[S] = \{\,(x,y,z,t) \in \mathbb{R}^4 \,/\, x + y + z - t = 0\,\}$.

14. Os dois conjuntos geram o seguinte subespaço de $\mathbb{R}^3$:
    $W = \{\,(x,y,z) \in \mathbb{R}^3 \,/\, x - 5y - 3z = 0\,\}$.

15. (a) $\beta = \{\,(1,1)\,\}$ e $\dim(W) = 1$.

    (b) $\beta = \{\,(1,1,1,0), (-1,-1,0,1)\}$ e $\dim(W) = 2$.

    (c) $\beta = \{\,(1,2,3), (0,0,2)\}$ e $\dim(W) = 2$.

    (d) $\beta = \{\,(1,0,0,-1), (0,1,0,1), (0,0,1,-1)\}$ e $\dim(W) = 3$.

16. (a) (i) $\alpha = \{\,(1,0,-1), (0,1,-1)\,\}$ e $\dim(U) = 2$.

    (ii) $\beta = \{\,(1,0,0), (0,1,0)\,\}$ e $\dim(W) = 2$.

    (iii) $\gamma = \{\,(1,0,-1), (0,1,-1), (1,0,0)\,\}$ e $\dim(U + W) = 3$.

    (iv) $\delta = \{\,(1,1,0)\,\}$ e $\dim(U \cap W) = 1$.

    (b) (i) $\alpha = \{\,(1,-1,0,0), (0,0,1,1)\,\}$ e $\dim(U) = 2$.

    (ii) $\beta = \{\,(1,0,0,0), (0,1,0,0)\,\}$ e $\dim(W) = 2$.

    (iii) $\gamma = \{\,(0,0,1,1), (1,0,0,0), (0,1,0,0)\,\}$ e $\dim(U + W) = 3$.

    (iv) $\delta = \{\,(1,-1,0,0)\}$ e $\dim(U \cap W) = 1$.

    (c) (i) $\alpha = \{\,(0,1,0), (0,0,1)\,\}$ e $\dim(U) = 2$.

    (ii) $\beta = \{\,(2,2,0), (1,2,3), (7,12,21)\,\}$ e $\dim(W) = 3$.

    (iii) $\gamma = \{\,(1,0,0), (0,1,0), (0,0,1)\,\}$ e $\dim(U + W) = 3$.

    (iv) $\delta = \{\,(0,1,0), (0,0,1)\,\}$ e $\dim(U \cap W) = 2$.

    (d) (i) $\alpha = \{\,(1,0,0,-1), (0,1,0,1), (0,0,1,-1)\,\}$ e $\dim(U) = 3$.

    (ii) $\beta = \{\,(1,0,0,1), (0,1,0,1), (0,0,1,1)\,\}$ e $\dim(W) = 3$.

    (iii) $\gamma = \{\,(1,0,0,-1), (0,1,0,1), (0,0,1,-1), (1,0,0,1)\,\}$ e $\dim(U + W) = 4$.

    (iv) $\delta = \{\,(1,0,-1,0), (0,1,0,1)\,\}$ e $\dim(U \cap W) = 2$.

17. $\gamma = \{\,(1,1,0,0,0), (1,0,1,0,0), (0,1,0,0,0), (0,0,0,1,0), (0,0,0,0,1)\,\}$.

18. $\gamma = \{\,(1,1,1), (1,2,3), (3,0,2)\,\}$.

19.

20. Sim, a soma é direta.

21. $[(1,0,0)]_\gamma = \begin{bmatrix} 1/3 \\ -1/3 \\ 1/3 \end{bmatrix}$.

22. (a) $[(4,5,3)]_\beta = \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}$, (b) $[(4,5,3)]_\beta = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$, (c) $[(4,5,3)]_\beta = \begin{bmatrix} 41/11 \\ -10/11 \\ 3/11 \end{bmatrix}$.

23. $[u]_\gamma = \begin{bmatrix} b-c \\ b \\ c+a-2b \end{bmatrix}$.

24. $\alpha = \{(-3,5),(1,-1)\}$ e $u = (-1,3)$.

25. $[I]_\gamma^\beta = \begin{bmatrix} 2 & 3 & -2 \\ 1 & 2 & -1 \\ -1/3 & -1 & 2/3 \end{bmatrix}$, $[I]_\beta^\gamma = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 0 \\ -1 & 3 & 3 \end{bmatrix}$, $[u]_\gamma = \begin{bmatrix} 2 \\ 2 \\ -1/3 \end{bmatrix}$.

26. $(a)[v]_\beta = \begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$, $(b)[v]_{\beta'} = \begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix}$.

27. γ não é ortonormal.
$\gamma' = \left\{ \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right) \right\}$ é base ortonormal obtida a partir de γ.

28. $\gamma' = \{\,(1,0),(0,1)\,\}$.

29. $\alpha' = \left\{ \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), \left(-\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, -\frac{1}{\sqrt{6}}\right), \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right) \right\}$ é base ortonormal obtida a partir de α.

30. $\gamma = \left\{ \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right), \left(\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right) \right\}$ é uma base ortonormal para o subespaço W.