Universidade Federal de Viçosa

Centro de Ciências Exatas

Departamento de Matemática

Gabarito $6^{\underline{a}}$ Lista - MAT 135 - Geometria Analítica e Álgebra Linear

1. $(10, -2, 17)$.

2. $\left(\dfrac{\sqrt{2}}{2}, 3\dfrac{\sqrt{6}}{2}, 2\sqrt{3}\right)$.

3. (b) $A' = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$ e $A = \begin{pmatrix} \dfrac{1}{2} & \dfrac{1}{2} & -\dfrac{1}{2} \\ -\dfrac{1}{2} & \dfrac{1}{2} & \dfrac{1}{2} \\ \dfrac{1}{2} & -\dfrac{1}{2} & \dfrac{1}{2} \end{pmatrix}$.

   (c) São as colunas de A, respectivamente: $\left(\dfrac{1}{2}, -\dfrac{1}{2}, \dfrac{1}{2}\right)$, $\left(\dfrac{1}{2}, \dfrac{1}{2}, -\dfrac{1}{2}\right)$ e $\left(-\dfrac{1}{2}, \dfrac{1}{2}, \dfrac{1}{2}\right)$.

   (d) $(-3, 1, 4)$.

4. (a) Como eles são ortogonais dois a dois e $\dim \mathbb{R}^3 = 3$, eles são L.I.

   (b) $A' = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ -2 & 0 & 1 \end{pmatrix}$ e $A = \begin{pmatrix} \dfrac{1}{6} & \dfrac{1}{6} & -\dfrac{1}{3} \\ \dfrac{1}{2} & -\dfrac{1}{2} & 0 \\ \dfrac{1}{3} & \dfrac{1}{3} & \dfrac{1}{3} \end{pmatrix}$.

   (c) São as colunas de A, respectivamente: $\left(\dfrac{1}{6}, \dfrac{1}{2}, \dfrac{1}{3}\right)$, $\left(\dfrac{1}{2}, -\dfrac{1}{2}, \dfrac{1}{3}\right)$ e $\left(-\dfrac{1}{3}, 0, \dfrac{1}{3}\right)$.

   (d) $\left(-\dfrac{11}{6}, \dfrac{3}{2}, \dfrac{4}{3}\right)$.

5.

6. (a) $p(x) = -2x + x^2$, $v_1 = (-1, 1)$ autovetor associado ao autovalor $\lambda_1 = 0$ e $v_2 = (1, 1)$ autovetor associado ao autovalor $\lambda_2 = 2$.

   (b) $p(x) = 8 - 6x + x^2$, $v_1 = (1, 1)$ autovetor associado ao autovalor $\lambda_1 = 4$ e $v_2 = (-1, 1)$ autovetor associado ao autovalor $\lambda_2 = 2$.

(c) $p(x) = -3-2x+x^2$, $v_1 = \left(-\frac{1}{2}, 1\right)$ autovetor associado ao autovalor $\lambda_1 = 3$ e $v_2 = \left(\frac{1}{2}, 1\right)$ autovetor associado ao autovalor $\lambda_2 = -1$.

(d) $p(x) = -x^3$, $v_1 = (1, 0, 0)$ autovetor associado ao autovalor $\lambda_1 = 0$.

(e) $p(x) = -(-1+x)(-3+x)(2+x)$, $v_1 = \left(\frac{3}{4}, \frac{3}{8}, 1\right)$ autovetor associado ao autovalor $\lambda_1 = 1$, $v_2 = \left(0, \frac{5}{2}, 1\right)$ autovetor associado ao autovalor $\lambda_2 = 3$ e $v_3 = (0, 0, 1)$ autovetor associado ao autovalor $\lambda_3 = -2$.

(f) $p(x) = -2x^2 - x^3$, $v_1 = (1, 0, 1)$ e $v_2 = (0, 1, 0)$ autovetores associados ao autovalor $\lambda_1 = 0$ e $v_3 = (-1, 3, 1)$ autovetor associado ao autovalor $\lambda_2 = -2$.

7. (a) $\beta_1 = \{\, (-1, 1) \,\}$ e $\beta_2 = \{\, (1, 1) \,\}$.

   (b) $\beta_1 = \{\, (1, 1) \,\}$ e $\beta_2 = \{\, (-1, 1) \,\}$.

   (c) $\beta_1 = \left\{ \left(-\frac{1}{2}, 1\right) \right\}$ e $\beta_2 = \left\{ \left(\frac{1}{2}, 1\right) \right\}$.

   (d) $\beta_1 = \{\, (1, 0, 0) \,\}$.

   (e) $\beta_1 = \{\, (0, 0, 1) \,\}$, $\beta_2 = \left\{ \left(\frac{3}{4}, \frac{3}{8}, 1\right) \right\}$ e $\beta_3 = \left\{ \left(0, \frac{5}{2}, 1\right) \right\}$.

   (f) $\beta_1 = \{\, (1, 0, 1), (0, 1, 0) \,\}$ e $\beta_2 = \{\, (-1, 3, 1) \,\}$.

8. (a) Diagonalizável. $P = \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$.

   (b) Diagonalizável. $P = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$.

   (c) Diagonalizável. $P = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ 1 & 1 \end{pmatrix}$.

   (d) Não diagonalizável.

   (e) Diagonalizável. $P = \begin{pmatrix} 0 & \frac{3}{4} & 0 \\ 0 & \frac{3}{8} & \frac{5}{2} \\ 1 & 1 & 1 \end{pmatrix}$.

   (f) Diagonalizável. $P = \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \\ 1 & 0 & 1 \end{pmatrix}$.

9. (a) $\lambda_1 = 4$, $\beta_1 = \{\, (1, 1) \,\}$ e $\lambda_2 = 3$, $\beta_2 = \{\, (1, 2) \,\}$.

   (b) $\lambda_1 = 6$, $\beta_1 = \{\, (3, 2, 3) \,\}$ e $\lambda_2 = -2$, $\beta_2 = \{\, (0, 1, -1), (1, 0, -1) \,\}$.

10. (a) $P = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ e $D = \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix}$.

(b) $P = \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ e $D = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$.

(c) $P = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \\ 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ e $D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

11. $\lambda = -4$.