



Universidade Federal de Viçosa
Centro de Ciências Exatas
Departamento de Matemática

Gabarito 1^a Lista - MAT 135 - Geometria Analítica e Álgebra Linear
2017/II

1. (a) Não, (b) Não, (c) Sim, ordem 4x5, (d) Não.

2. (a) $A_{4 \times 5}$, (b) $a_{21} = -4$, $a_{34} = 1$ e $a_{44} = 0$.

3. $c_{23} = 5$, $d_{41} = 5$.

$$4. A = \begin{bmatrix} -2 & 1 & 0 & -1 \\ 1 & -2 & 3 & 2 \\ 0 & 3 & 0 & 5 \\ -1 & 2 & 5 & 4 \end{bmatrix}.$$

5. (a) $A^2 = I$, (b) $A^3 = A$, (c) $A^{31} = A$, (d) $A^{42} = I$.

6. $x = -1$, $y = 1$.

7. (a) $x = -1$, (b) $x = -4$, $y = -4$, $z = -2$, (c) $\begin{matrix} x = 2, & y = -7, & z = -2 \\ x = -2, & y = -3, & z = 10 \end{matrix}$

$$8. (a) \begin{bmatrix} 22 & -6 & 8 \\ -2 & 4 & 6 \\ 10 & 0 & 4 \end{bmatrix},$$

$$(b) \begin{bmatrix} 7 & 2 & 4 \\ 3 & 5 & 7 \end{bmatrix},$$

$$(c) \begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -6 \end{bmatrix},$$

$$(d) \begin{bmatrix} 10 & -6 \\ -14 & 2 \\ -6 & -8 \end{bmatrix}.$$

$$9. \begin{bmatrix} a & b \\ 0 & a - b \end{bmatrix}.$$

10. (a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$.

(b) 4 matrizes: $\begin{bmatrix} \sqrt{5} & 0 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} -\sqrt{5} & 0 \\ 0 & 3 \end{bmatrix}$, $\begin{bmatrix} \sqrt{5} & 0 \\ 0 & -3 \end{bmatrix}$, $\begin{bmatrix} -\sqrt{5} & 0 \\ 0 & -3 \end{bmatrix}$.

(c) Não, $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$.

11. (b) A matriz identidade.

12. $A^3 = 0$.

13.

14.

15.

16. (a) ± 1 , (b) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ e $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

17. Ortogonais: A , C e D .

Não ortogonais: B .

18. (a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (c) $T_{-\alpha} = (T_{\alpha})^T$.

19.

20.

21.

22.

23. Sim

24.

25. (a) 4^5 , (b) P é invertível, (c) -9 , (d) Q é invertível.

26. -5

27. (a) 576,

(b) $A^{-1} = \begin{bmatrix} 1 & \frac{5}{2} & \frac{17}{8} & -\frac{31}{12} \\ 0 & \frac{1}{2} & \frac{3}{8} & -\frac{5}{12} \\ 0 & 0 & \frac{1}{4} & \frac{1}{6} \\ 0 & 0 & 0 & \frac{1}{3} \end{bmatrix}$,

$$(c) B^{-1} = \begin{bmatrix} -\frac{1}{3} & 0 & 0 & 0 \\ -\frac{1}{4} & -\frac{1}{4} & 0 & 0 \\ -\frac{7}{6} & -\frac{1}{2} & -1 & 0 \\ -\frac{25}{24} & -\frac{3}{8} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix},$$

$$(d) (AB)^{-1} = \begin{bmatrix} -\frac{215}{288} & -\frac{37}{288} & -\frac{67}{144} & -\frac{25}{72} \\ -\frac{23}{32} & -\frac{5}{32} & -\frac{3}{16} & -\frac{1}{8} \\ -\frac{5}{6} & -\frac{1}{6} & -\frac{1}{3} & -\frac{1}{6} \\ \frac{31}{24} & \frac{5}{24} & -\frac{1}{12} & -\frac{1}{6} \end{bmatrix},$$

$$(e) \det C = 0 \text{ ou } \det C = \frac{1}{16}.$$

$$28. \det Q = (-2)^n.$$

$$29. (a) 58, \quad (b) 58, \quad (c) 3364, \quad (d) A^{-1} = \begin{bmatrix} \frac{25}{29} & -\frac{32}{29} & -\frac{13}{29} & \frac{10}{29} \\ \frac{7}{29} & -\frac{2}{29} & \frac{1}{29} & -\frac{3}{29} \\ -\frac{157}{58} & \frac{165}{58} & \frac{77}{58} & -\frac{14}{29} \\ -\frac{73}{58} & \frac{83}{58} & \frac{31}{58} & -\frac{3}{58} \end{bmatrix},$$

$$(e) 58, \quad (f) 3. \quad \begin{bmatrix} 36 & 23 & 35 & 32 \\ 23 & 25 & -2 & 17 \\ 35 & -2 & 95 & 47 \\ 32 & 17 & 47 & 50 \end{bmatrix}.$$

$$30. p(x) = x^3 - 2x^2 - x + 3 \text{ e } A^{-1} = -\frac{1}{3}(A^2 - 2A - I).$$

$$31. (a) -123, \quad (b) 1 + a + b + c, \quad (c) -c^4 + c^3 - 16c^2 + 8c - 2, \quad (d) -5 \quad (e) -120, \quad (f) -120.$$

$$32. (a) x = 0, -1, 1/2, \quad (b) x = 40/11, \quad (c) x = \frac{3}{4} \pm \frac{1}{4}\sqrt{33}.$$

$$33. \det(A) = a_{41} a_{32} a_{23} a_{14}.$$

34.

$$35. \text{ (a) } A^{-1} = \begin{bmatrix} \frac{29}{152} & \frac{11}{152} & -\frac{1}{8} \\ -\frac{21}{152} & \frac{13}{152} & \frac{1}{8} \\ \frac{27}{152} & \frac{5}{132} & \frac{1}{8} \end{bmatrix},$$

$$\text{(b) } A^{-1} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\text{(c) } A^{-1} = \begin{bmatrix} -1 & 1 & 0 & 0 \\ -\frac{1}{2} & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & -\frac{5}{2} & 1 & -\frac{1}{2} \\ -1 & 2 & -1 & 0 \end{bmatrix},$$

$$\text{(d) } A^{-1} = \begin{bmatrix} 0 & 0 & \frac{1}{4} & 0 \\ 0 & -1 & \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{5}{6} & -\frac{13}{24} & 0 \\ \frac{2}{9} & \frac{16}{9} & -\frac{53}{36} & \frac{1}{3} \end{bmatrix},$$

36.

$$37. \text{ (a) } A^{-1} = \begin{bmatrix} -\frac{1}{8} & \frac{3}{8} & -\frac{1}{8} \\ -\frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{2} & -\frac{1}{4} & 0 \end{bmatrix},$$

$$(b) A^{-1} = \begin{bmatrix} \frac{2}{7} & \frac{1}{14} \\ -\frac{1}{7} & \frac{3}{14} \end{bmatrix},$$

$$(c) A^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \end{bmatrix},$$

$$(d) A^{-1} = \frac{1}{51} \begin{bmatrix} -5 & -28 & -6 \\ -2 & 16 & 1 \\ -2 & -1 & 1 \end{bmatrix}.$$

38. (a) $X = A^{-1}B^{-1}$, (b) $X = A^{-1}B^T$, (c) $X = A^{-1}$,
 (d) $X = B^T - A$, (e) $X = A^{-1}BAB^{-1}$, (f) $X = (B^T)^{-1}A$.

39. **Correção:** A matriz A a ser considerada neste exercício é

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ 2 & 3 & 5 \end{bmatrix}.$$

Assim, teremos:

$$E_1 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}.$$

40. Por exemplo, consideramos as matrizes

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix},$$

$$E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{2}{3} \end{bmatrix}, \quad E_6 = \begin{bmatrix} 1 & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{4}{3} \\ 0 & 0 & 1 \end{bmatrix}.$$

$$41. (a) A = \begin{bmatrix} 20 & 15 & 30 \end{bmatrix} \cdot \begin{bmatrix} 50 & 15 & 6 & 70 & 25 \\ 500 & 1 & 5 & 20 & 30 \\ 200 & 8 & 7 & 50 & 40 \end{bmatrix}.$$

(b) Os elementos de AB representam o valor total de compra e o preço total de transporte de todos os materiais utilizados na construção de todos os estabelecimentos.

42. Faça os produtos AB e AC , onde $A = \begin{bmatrix} 6 & 7 & 5 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 25 & 15 & 70 \\ 30 & 25 & 40 \\ 60 & 10 & 55 \\ 15 & 30 & 60 \end{bmatrix}$ e $C = \begin{bmatrix} 7,5 & 5 & 4,5 & 6,5 \end{bmatrix}$.

43. (a) F, (b) V, (c) V, (d) V, (e) F, (f) V, (g) V, (h) F, (i) V, (j) F, (k) F, (l) V, (m) F, (n) F, (o) F, (p) V, (q) V, (r) V, (s) F, (t) V, (u) F, (v) F, (w) V, (x) F, (y) V, (z) F.