Lista 1 - MAT147 2020/4

1. a) $\lim_{x\to+\infty}\left(1+\frac{1}{2x}\right)^{x^2} = \lim_{x\to+\infty} e^{\ln\left(1+\frac{1}{2x}\right)^{x^2}} = \lim_{x\to+\infty} e^{x^2\ln\left(1+\frac{1}{2x}\right)} = e^{\lim_{x\to+\infty}\frac{\ln\left(1+\frac{1}{2x}\right)}{\frac{1}{x^2}}}$

$\overset{\frac{0}{0}}{\underset{L'H}{=}} e^{\lim_{x\to+\infty}\frac{\frac{-1}{2x^2}\cdot\frac{1}{1+\frac{1}{2x}}}{\frac{-2}{x^3}}} = e^{\lim_{x\to+\infty}\frac{\frac{1}{\cancel{2}x^2\left(\frac{2x-1}{\cancel{x}}\right)}}{\frac{2}{x^3}}} = e^{\lim_{x\to+\infty}\frac{1}{x(2x-1)}\cdot\frac{x^3}{2}}$

$= e^{\lim_{x\to+\infty}\frac{x^2}{4x-4}} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} e^{\lim_{x\to+\infty}\frac{12x}{24}} = e^{\lim_{x\to+\infty}\frac{x}{2}} = e^{+\infty}$

b) $\lim_{x\to0}\frac{\sin x}{2x} = \frac{1}{2}\cdot\lim_{x\to0}\frac{\sin x}{x} \overset{\frac{0}{0}}{\underset{L'H}{=}} \frac{1}{2}\lim_{x\to0}\frac{\cos x}{1} = \frac{1}{2}\cdot 1 = \frac{1}{2}$

c) $\lim_{x\to+\infty}\left(\frac{x+1}{x-1}\right)^{x} = \lim_{x\to+\infty} e^{\ln\left(\frac{x+1}{x-1}\right)^x} = \lim_{x\to+\infty} e^{x\ln\left(\frac{x+1}{x-1}\right)} = e^{\lim_{x\to+\infty} x\ln\left(\frac{x+1}{x-1}\right)}$

$= e^{\lim_{x\to+\infty}\frac{\ln\left(\frac{x+1}{x-1}\right)}{\frac{1}{x}}} \overset{\frac{0}{0}}{\underset{L'H}{=}} e^{\lim_{x\to+\infty}\frac{\frac{(x-1)x-(x+1)x}{(x-1)^2}\cdot\frac{(x-1)}{(x+1)}}{\frac{-1}{x^2}}} = e^{\lim_{x\to+\infty}\frac{\cancel{x}-1-\cancel{x}-1}{(x-1)(x+1)}\cdot x^2}$

$= e^{\lim_{x\to+\infty}\frac{+2\cdot x^2}{+1\cdot(x^2-1)}} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} e^{\lim_{x\to+\infty}\frac{4\cancel{x}}{2\cancel{x}}} = e^{\lim_{x\to+\infty}2} = e^2$

d) $\lim_{x\to+\infty}\frac{\arctan(x)}{\sqrt{x}} = 0$

e) $\lim_{x\to0}\frac{a^x-1}{x} = \lim_{x\to0}\frac{a^x\cdot\ln a}{1} = \ln a\cdot\lim_{x\to0}a^x = \ln a$

f) $\lim_{x\to5}\frac{\sqrt{x-1}-2}{x^2-25} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to5}\frac{\frac{1}{2}\cdot\frac{1}{\sqrt{x-1}}}{2x} = \lim_{x\to5}\frac{1}{2\sqrt{x-1}}\cdot\frac{1}{2x} = \frac{1}{40}$

g) $\lim_{x\to-3}\frac{x^2+2x-3}{2x^2+3x-9} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim_{x\to-3}\frac{2x+2}{4x+3} = \frac{+4}{+9}$

h) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen} x - x}{\tan x - x} \underset{L'H}{\overset{0/0}{=}} \lim\limits_{x\to 0} \dfrac{\cos x - 1}{\sec^2 x - 1} \overset{0/0}{=} \lim\limits_{x\to 0} \dfrac{-\operatorname{sen} x}{2\sec x^2 \cdot \operatorname{tg} x} = \lim\limits_{x\to 0} \dfrac{-\operatorname{sen} x}{\dfrac{2\operatorname{sen} x}{\cos^3 x}}$

$= \lim\limits_{x\to 0} \dfrac{-1}{2} \cdot \cos^3 x = \dfrac{-1}{2}$

i) $\lim\limits_{x\to +\infty} \dfrac{x^2}{\ln x} \underset{L'H}{\overset{\infty/\infty}{=}} \lim\limits_{x\to +\infty} \dfrac{2x}{\dfrac{1}{x}} = \lim\limits_{x\to +\infty} 2x^2 = +\infty$

j) $\lim\limits_{x\to 0} \dfrac{e^{5x}-1}{3x} \underset{L'H}{\overset{0/0}{=}} \lim\limits_{x\to 0} \dfrac{5e^{5x}}{3} = \dfrac{5}{3}$

K) $\lim\limits_{x\to 0} x^{1/\ln x} = \lim\limits_{x\to 0} e^{\ln x^{1/\ln x}} = \lim\limits_{x\to 0} e^{\frac{1}{\ln x}\cdot \ln x} = e$

l) $\lim\limits_{x\to +\infty} \left(1+\dfrac{a}{x}\right)^x = \lim\limits_{x\to +\infty} e^{\ln\left(1+\frac{a}{x}\right)^x} = \lim\limits_{x\to +\infty} e^{x\ln\left(1+\frac{a}{x}\right)} = e^{\lim\limits_{x\to+\infty} x\ln\left(1+\frac{a}{x}\right)}$ (1)

$\lim\limits_{x\to +\infty} \dfrac{\ln\left(1+\frac{a}{x}\right)}{\dfrac{1}{x}} \underset{L'H}{\overset{0/0}{=}} \lim\limits_{x\to +\infty} \dfrac{\dfrac{-a}{x^2}\cdot \dfrac{1}{1+\frac{a}{x}}}{-\dfrac{1}{x^2}} = \lim\limits_{x\to +\infty} \dfrac{+a}{\cancel{x^2}} \cdot \dfrac{1}{1+\frac{a}{x}} \cdot \cancel{x^2} = \lim\limits_{x\to +\infty} a\cdot \dfrac{x+a}{x}$

$= \lim\limits_{x\to +\infty} \dfrac{ax+a}{x} \underset{L'H}{\overset{\infty/\infty}{=}} \lim\limits_{x\to +\infty} a = a$ Voltando para (1) $\cdot$ $e^{\lim\limits_{x\to+\infty} x\ln\left(1+\frac{a}{x}\right)} = e^a$

m) $\lim\limits_{x\to 0} x \cdot e^{1/x}$ $u = \dfrac{1}{x}$ $\Rightarrow \lim\limits_{u\to +\infty} \dfrac{e^u}{u} \underset{L'H}{\overset{\infty/\infty}{=}} \lim\limits_{u\to +\infty} e^u = +\infty$

n) $\lim\limits_{x\to 0} \dfrac{\cot\left(\frac{\pi}{2}-x\right)}{\dfrac{1}{\cot(2x)}}$

θ) $\lim\limits_{x \to \frac{\pi}{2}} (\sec x - \tan x) = \lim\limits_{x \to \frac{\pi}{2}} \frac{1}{\cos x} - \frac{\operatorname{sen} x}{\cos x} = \lim\limits_{x \to \frac{\pi}{2}} \frac{1 - \operatorname{sen} x}{\cos x} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim\limits_{x \to \frac{\pi}{2}} \frac{-\cos x}{\operatorname{sen} x} = 0$

2. a) $\int_0^{+\infty} x e^{-x^2} dx \qquad u = x^2 \quad du = 2x\,dx \quad dx = \frac{du}{2x}$

$\lim\limits_{b \to +\infty} \int_0^b \cancel{x} \frac{e^{-u}}{2\cancel{x}} du = \lim\limits_{b \to +\infty} \frac{1}{2} \int_0^b e^{-u} du = \lim\limits_{b \to +\infty} \frac{1}{2} - e^{-u} \Big|_0^b$

$= \frac{1}{2} \lim\limits_{b \to +\infty} \left[ -\overset{0}{e^{-b}} + e^0 \right] = +\frac{1}{2}$

b) $\int_0^{+\infty} e^{-t} \operatorname{sen} t \, dt \qquad u = \operatorname{sen} t \quad du = \cos t$

$\qquad dv = e^{-t} \quad v = -e^{-t}$

$\int_0^{+\infty} e^{-t} \operatorname{sen} t = -e^{-t} \operatorname{sen} t + \int_0^{+\infty} e^{-t} \cos t \, dt$

$= -e^{-t} \operatorname{sen} t - e^{-t} \cdot \cos t - \int_0^{+\infty} e^{-t} \operatorname{sen} t$

$2\int_0^{+\infty} e^{-t} \operatorname{sen} t = -e^{-t} (\operatorname{sen} t + \cos t)$

$\int_0^{+\infty} e^{-t} \operatorname{sen} t = \frac{-e^{-t}}{2} (\operatorname{sen} t + \cos t)$

$\lim\limits_{b \to +\infty} \frac{-\overset{0}{e^{b}}}{2} (\operatorname{sen} b + \cos b) = 0$

$\lim\limits_{b \to 0} \frac{-e^{-b}}{2} (\operatorname{sen} \overset{0}{b} + \cos \overset{1}{b}) = \frac{1}{2}$

c) $\int_{2}^{+\infty} \frac{1}{-1+x^2}\,dx = \int_{+2}^{+\infty} \frac{1}{(x+1)} \cdot \frac{1}{(x-1)}\,dx$ $\quad u = x-1 \quad du = 1$

$dv = (x+1)^{-1} \quad v = \ln(x+1)$

$= \lim_{b \to +\infty} \left[ (x-1)\ln(x+1) \Big|_2^b - \int_2^b \ln(x+1)\,dx \right]$ ①

$u = \ln(x+1) \quad du = \frac{1}{x+1}$

$dv = dx \quad v = x$

① $= \ln(x+1) \cdot x - \int_2^b \frac{x}{x+1}$

d) $\int_{-\infty}^{0} x e^{-x^2}\,dx = \int_{-\infty}^{0} \frac{\cancel{x} e^{-u}}{2\cancel{x}}\,du = -\frac{1}{2}\int_{-\infty}^{0} e^{-u}\,du$ $\quad u = -x^2 \quad du = -2x\,dx$

$dx = -\frac{du}{2x}$

$= -\frac{1}{2}\left[ e^{u} \Big|_{-\infty}^{0} \right] = -\frac{1}{2}\left[ +e^0 + e^{-\infty^{0}} \right] = -\frac{1}{2}$

e) $\int_{-\infty}^{+\infty} \frac{1}{4+x^2}\,dx = \int_{-\infty}^{0} \frac{1}{4+x^2}\,dx + \int_{0}^{+\infty} \frac{1}{4+x^2}\,dx = \lim_{b \to +\infty} \left[ \int_{-b}^{0} \frac{1}{4+x^2}\,dx + \int_{0}^{b} \frac{1}{4+x^2}\,dx \right]$

$\lim_{b \to +\infty} \frac{1}{4} \left[ \int_{-b}^{0} \frac{1}{1+\left(\frac{x}{2}\right)^2}\,dx + \int_{0}^{b} \frac{1}{1+\left(\frac{x}{2}\right)^2}\,dx \right] = \lim_{b \to +\infty} \frac{1}{\cancel{4}_2} \left[ \cancel{2} \arctan\left(\frac{x}{2}\right) \Big|_{-b}^{0} + \cancel{2} \arctan\left(\frac{x}{2}\right) \Big|_{0}^{b} \right]$

$\lim_{b \to +\infty} \frac{1}{2} \left[ \arctan\left(\frac{0}{2}\right) - \arctan\left(\frac{-b}{2}\right) + \left( \arctan\left(\frac{b}{2}\right) - \arctan\left(\frac{0}{2}\right) \right) \right]$

$\lim_{b \to +\infty} \frac{1}{2} \left[ +\arctan\left(\frac{-b}{2}\right) - \arctan\left(\frac{b}{2}\right) \right] = \frac{1}{2} \left[ +\frac{\pi}{2} + \frac{\pi}{2} \right] = \frac{\pi}{2}$

3 - a) $\int_1^{+\infty} \frac{1}{x^5+3x+1}\,dx$ Sabemos que, $\forall x \in [1,+\infty)$, $x^5+3x+1 > x^5$

$\Rightarrow \frac{1}{x^5+3x+1} < \frac{1}{x^5}$

Tomemos $\int_1^{+\infty} \frac{1}{x^5} \to$ Integral tipo p com $p=5>1$. Logo, $\int_1^{+\infty} \frac{1}{x^5}\,dx$ converge

Pelo teorema da comparação, como temos $\frac{1}{x^5+3x+1} < \frac{1}{x^5}$, e $\int_1^{+\infty} \frac{1}{x^5}$ converge, segue que $\int_1^{+\infty} \frac{1}{x^5}\,dx$ também converge.

b) $\int_1^{+\infty} \frac{\cos 3x}{x^3}\,dx$ $-1 \leq \cos 3x \leq 1 \Rightarrow 0 \leq |\cos 3x| \leq 1 \Rightarrow 0 \leq \frac{|\cos 3x|}{x^3} \leq \frac{1}{x^3}$

Tomemos $\int_1^{+\infty} \frac{1}{x^3}\,dx \to$ Integral tipo p com $p=5>1$. Logo, $\int_1^{+\infty} \frac{1}{x^5}\,dx$ converge.

Como temos $\frac{|\cos x|}{x^3} \leq \frac{1}{x^3}$, e $\int_1^{+\infty} \frac{1}{x^3}\,dx$ converge, então segue que, pelo teste da comparação, $\int_1^{+\infty} \frac{\cos 3x}{x^3}\,dx$ também converge.

c) $\int_1^{+\infty} \frac{x^2+1}{x^3+1}\,dx$ $x^2+1 > 1$, $\forall x \in [1,+\infty)$

$\Rightarrow \frac{x^2+1}{x^3+1} > \frac{1}{x^3+1}$

$\int_1^{+\infty} \frac{1}{x^3+1}\,dx = \lim_{b \to +\infty} \int_1^{b} \frac{1}{x^3+1}$

d) $\int_0^{+\infty} \frac{\arctan x}{x^2+1}$

$u = \arctan x$

$du = \frac{1}{1+x^2}\,dx \quad dx = du(x^2+1)$

$$\int_0^{+\infty} \frac{u}{\cancel{1+x^2}} \cdot (\cancel{1+x^2})\,du = \int_0^{+\infty} u\,du = \lim_{b \to +\infty} \frac{u^2}{2}\Big|_0^b = \lim_{b \to +\infty} \frac{(\arctan x)^2}{2}\Big|_0^b$$

$= \frac{\left(\frac{\pi}{2}\right)^2}{2} - 0 = \frac{\pi^2}{8}$. Logo, como $\int_0^{+\infty} \frac{\arctan x}{x^2+1}\,dx = \frac{\pi^2}{8}$, segue que a integral converge.

e) $\int_0^{+\infty} e^{-st}\,dt$

$e^{st}$

$u = -st$

$du = +s\,dt$

$dt = +\frac{du}{s}$

$$= \int_0^{+\infty} \frac{e^{-u}}{s} = \frac{1}{s}\int_0^{+\infty} \frac{1}{e^{u}} = \frac{1}{s} \lim_{b \to +\infty} -e^{-u}\Big|_0^b = \frac{1}{s} \cdot [0+1] = \frac{1}{s}$$

$\int_0^{+\infty} e^{-st}\,dt$ converge para $\frac{1}{s}$ se $s > 0$ e diverge se $s \leq 0$.

Pois, se $s > 0$, a integral continuaria tendo o formato $\int_0^{+\infty} \frac{1}{e^{st}}\,dt$.

Se $s \leq 0$ ocorre uma inversão de sinal e passamos a ter $\int_0^{+\infty} e^{st}\,dt$, que diverge.

4. a) $y = \frac{1}{(x-1)^2} \quad 0 \leq x < 1$

$\int_0^1 \frac{1}{(x-1)^2}\,dx \qquad u = (x-1)$

$du = 1\,dx$

$\int_{-1}^{0} \frac{1}{u^2}\,dx = \lim_{b \to 0} \frac{-1}{u}\Big|_{-1}^{b} = -\infty$ logo, a área não existe pois a integral diverge

b) $\frac{1}{\sqrt{3-x}} \quad 0 \leq x < 3$

$\int_0^3 \frac{1}{\sqrt{3-x}}\,dx \qquad u = 3-x \qquad \int_0^3 \frac{1}{u^{1/2}}\,du = \lim_{b \to 0} 2\sqrt{u}\Big|_b^3$

$du = -dx$

$dx = -1du$

$= \lim_{b \to 0}\left[2\sqrt{3} + 2\sqrt{b}\right] = 2\sqrt{3}$ logo, a área da região é $2\sqrt{3}$

c) $y = \sec^2 x \quad 0 \leq x < \frac{\pi}{2}$

$\int_0^{\pi/2} \sec^2 x\,dx = \lim_{b \to \pi/2} \tan(x)\Big|_0^b = \lim_{b \to \pi/2}\left[\tan(b) - \tan(0)\right] = +\infty$

logo, como a integral diverge, a área não existe.

d) $y = \frac{1}{(x+1)^{2/3}} \quad -2 \leq x \leq 7 \qquad \int_{-2}^{7} \frac{1}{(x+1)^{2/3}}\,dx \qquad u = x+1$

$du = dx$

$\int_{-1}^{8} \frac{1}{u^{2/3}}\,du = \lim_{b \to -1} 3\sqrt[3]{u}\Big|_b^8 = \lim_{b \to -1}\left[3\sqrt[3]{8} - 3\sqrt[3]{b}\right] = 3 \cdot 2 - 3 \cdot -1 = 9$

e)

e) $\dfrac{x-2}{x^2-5x+4}$ . $2 \leq x < 4$ $\displaystyle\int_2^4 \frac{x-2}{x^2-5x+4}\,dx$

$\Delta = 25 - 16 = 9$

$x = \dfrac{5 \pm 3}{2} = \begin{cases} 4 \\ 1 \end{cases}$

$$\frac{x-2}{x^2-5x+4} = \frac{A}{(x-4)} + \frac{B}{(x-1)} \quad ①$$

$$\frac{Ax - A + Bx - 4B}{x^2-5x+4} = \frac{(A+B)x - (A+4B)}{x^2-5x+4}$$

$$\begin{cases} A+B = 1 \\ A+4B = +2 \end{cases}$$

$-3B = -1$ $\quad A = \dfrac{2}{3}$

$B = +\dfrac{1}{3}$

Substituindo A e B em ①, temos

$$\int_2^4 \frac{x-2}{x^2-5x+4}\,dx = \frac{2}{3}\int_2^4 \frac{1}{(x-4)}\,dx + \frac{1}{3}\int_2^4 \frac{1}{(x-1)}\,dx$$

$$\frac{2}{3}\lim_{b \to 4} \overset{-\infty}{\ln(x-4)}\Big|_2^b + \frac{1}{3}\ln(x-1)\Big|_2^4 = -\infty - \frac{2\ln(2)}{3} + \frac{\ln(3)}{3} - 0 = -\infty$$

Como $\displaystyle\lim_{b \to 4} \ln(b-4) = -\infty$, segue que $\displaystyle\int_2^4 \frac{x-2}{x^2-5x+4}\,dx$ diverge, e, portanto, a área não pode ser calculada.

5- $\int_1^{+\infty} \frac{1}{x^p}\,dx = \lim_{b \to +\infty} \int_1^b \frac{1}{x^p}\,dx = \lim_{b \to +\infty} \left. \frac{x^{-p+1}}{-p+1} \right|_1^b$

$= \lim_{b \to +\infty} \frac{b^{-p+1}}{-p+1} - \frac{1^{-p+1}}{-p+1}$

Se $p > 1$, a integral será convergente, pois teremos $\lim_{b \to +\infty} \cancelto{0}{\frac{1}{b^{p-1}(1-p)}} - \frac{1}{1-p}$ considerando que $p > 1$.

Se $p \leq 1$, a integral será diverge, pois teremos $\lim_{b \to +\infty} \frac{b^{-p+1}}{-p+1}$

9- a) $(1, 4, 7, 10, \ldots)$

| $a_1$ | $a_2$ | $a_3$ | $a_4$ |
| --- | --- | --- | --- |
| 1 | 4 | 7 | 10 |
| $3 \cdot 1 - 2$ | $3 \cdot 2 - 2$ | | |

$a_n = 3n - 2$

$\lim_{n \to +\infty} 3n - 2 = +\infty$ Logo, a sequência diverge

b) $\left(1 + \frac{1}{2}, 1 + \frac{3}{4}, 1 + \frac{7}{8}, \ldots\right)$

10. a) $a_n = \dfrac{n \operatorname{sen}^2 n}{n^5 + 1}$

$-1 \leq \operatorname{sen}(x) \leq 1$

$0 \leq \operatorname{sen}^2(x) \leq 1 \Rightarrow \dfrac{n \cdot 0}{n^5+1} \leq \dfrac{n \operatorname{sen}^2(n)}{n^5+1} \leq \dfrac{n \cdot 1}{n^5+1}$

$\lim\limits_{n \to +\infty} \dfrac{n \cdot 0}{n^5+1} = 0$

$\lim\limits_{n \to +\infty} \dfrac{n}{n^5+1} = \lim\limits_{n \to +\infty} \dfrac{\cancel{n}(1)}{\cancel{n}(n^4 + 1/n)} \to 0 = 0$

Logo, pelo teorema do confronto, segue que a sequência converge para 0

b) $\sqrt{2m+3} - \sqrt{2n-3}$

$\lim\limits_{m \to +\infty} \sqrt{2m+3} - \sqrt{2m-3} = \lim\limits_{m \to +\infty} \left(\sqrt{2m+3} - \sqrt{2m-3}\right) \cdot \dfrac{\left(\sqrt{2m+3} + \sqrt{2m-3}\right)}{\left(\sqrt{2m+3} + \sqrt{2m-3}\right)}$

$= \lim\limits_{b \to +\infty} \dfrac{\cancel{2m}+3 - \cancel{\sqrt{2m+3} \cdot \sqrt{2m-3}} - \cancel{\sqrt{2m+3} \cdot \sqrt{2m-3}} - \cancel{2m}+3}{\sqrt{2m+3} + \sqrt{2m-3}} = \lim\limits_{b \to +\infty} \dfrac{6}{\sqrt{2m+3} + \sqrt{2m-3}}$

$= 0$ Logo, a sequência converge para 0

c) $\dfrac{1}{n} \operatorname{sen}\left(\dfrac{3\pi}{n^2+1}\right)$

$-1 \leq \operatorname{sen}\left(\dfrac{3\pi}{n^2+1}\right) \leq 1 \Rightarrow \dfrac{-1}{m} \leq \dfrac{\operatorname{sen}\left(\frac{3\pi}{m^2+1}\right)}{m} \leq \dfrac{1}{m}$

$\lim\limits_{m \to +\infty} \dfrac{-1}{m} = 0$

Logo, pelo teorema do confronto, segue que a sequência converge para 0.

$\lim\limits_{m \to +\infty} \dfrac{1}{m} = 0$

d) $a_n = \ln\sqrt{n^3 - n^2}$ $\lim\limits_{n\to+\infty} \ln n\sqrt{n-1} = \lim\limits_{n\to+\infty} \ln n + \ln\sqrt{n-1}$

$= \lim\limits_{n\to+\infty} \ln n + \frac{1}{2}\ln(n-1) = \infty + \infty = +\infty$

Logo, a sequência diverge.

e) $a_n = (-1)^n \frac{n}{n+1}$

f) $a_n = n^2\left(1 - \cos\frac{a}{n}\right)$ $\lim\limits_{n\to+\infty} \frac{\left(1-\cos\frac{a}{n}\right)}{\frac{1}{n^2}}$

$\lim\limits_{x\to+\infty} \frac{\left(1-\cos\frac{a}{x}\right)}{\frac{1}{x^2}} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim\limits_{x\to+\infty} \frac{-ax^{-2}\operatorname{sen}\frac{a}{x}}{\frac{-2}{x^3}} = \lim\limits_{x\to+\infty} \frac{\frac{+a}{\cancel{x^2}}\operatorname{sen}\frac{a}{x}\cdot x^{\cancel{3}}}{+2}$

$= \lim\limits_{x\to+\infty} \frac{a\operatorname{sen}\frac{a}{x}}{\frac{-2}{x}} \overset{\frac{0}{0}}{\underset{L'H}{=}} \lim\limits_{x\to+\infty} \frac{-\frac{a^2}{\cancel{x^2}}\cos\frac{a}{x}}{\frac{-2}{\cancel{x^2}}} = \lim\limits_{x\to+\infty} \frac{a^2}{2}\cdot\cos\overset{1}{\cancel{\frac{a}{x}}} = \frac{a^2}{2}$

Logo, pelo teorema da substituição, a sequência converge para $\frac{a^2}{2}$

g) $\frac{a^n}{n!}$

11. a)

b)

12. $\frac{n!}{n^n}$ $\quad a_1 = 1 \quad a_2 = \frac{2}{4} = \frac{1}{2} \quad a_3 = \frac{6}{27} = \frac{2}{9} \quad a_4 = \frac{24}{256} = \frac{3}{32}$

Logo, como $n!$ e $n^n$ são funções positivas e crescentes no intervalo em questão, temos que $a_1 > a_2 > a_3 > a_4 > \dots > a_m$. Logo, a função é decrescente.

Analisando $m!$, temos, para $m \geq 1$, os seguintes valores: 1, 2, 6, 24, 120, 720, ...

Analisando $m^m$, temos, para $m \geq 1$, os seguintes valores: 1, 4, 27, 256, ...

Logo, como o denominador cresce mais rápido que o numerador,

$\lim_{m \to +\infty} \frac{n!}{n^n} = 0.$

Logo, como a função é monótona e limitada, segue que ela é convergente, e seu limite é 0.

13. a) $a_{m+1} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot \ldots \cdot (2(m+1)-1)}{2 \cdot 4 \cdot 6 \cdot \ldots (2(m+1))} = \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot \ldots (2m+2-1)}{2 \cdot 4 \cdot 6 \cdot \ldots (2m+2)} = \frac{(2m+1)}{(2m+2)} \cdot a_m$

b) $\frac{a_{m+1}}{a_m} = \frac{2m+1}{2m+2}$ como $2m+2 > 2m+1$, segue que $a_m$ é estritamente decrescente.

c) $\lim_{m \to +\infty} \frac{1 \cdot 3 \cdot 5 \cdot 7 \ldots (2m-1)}{2 \cdot 4 \cdot 6 \cdot \ldots 2m}$ como $(2m-1) < 2m$, então quando $m \to +\infty$, o limite tende a 0.

Logo, como a sequência é monótona e limitada, segue que ela converge.

15. $\sum_{m=1}^{+\infty} \frac{1-n}{2^{m+1}}$

a) $S_1 = a_1 \qquad a_1 = \frac{1-1}{2^{1+1}} = 0 \qquad S_1 = 0$

$S_2 = a_1 + a_2 \qquad a_2 = \frac{-1}{2^{2+1}} = \frac{-1}{8} \qquad S_2 = 0 - \frac{1}{8} = \frac{-1}{8}$

$S_3 = a_1 + a_2 + a_3 \qquad a_3 = \frac{-2}{2^4} = \frac{-1}{8} \qquad S_3 = \frac{-1}{8} - \frac{1}{8} = \frac{-1}{4}$

b) $\frac{1-n}{2^{m+1}} = \frac{n+A}{2^{m+1}} + \frac{Bm}{2^m} = \frac{n+A+2Bm}{2^{m+1}}$

$\frac{m(1+2B)+A}{2^{m+1}} \qquad \begin{cases} 1+2B = -1 \\ A = 1 \end{cases} \qquad \begin{matrix} 2B = -2 \\ B = -1 \end{matrix} \qquad$ Logo, $A = 1$ e $B = -1$

c) $a_m = \frac{1+n}{2^{n+1}} - \frac{m}{2^m} = \left(\frac{\cancel{2}}{\cancel{2^2}} - \frac{1}{2}\right) + \left(\frac{\cancel{3}}{\cancel{8}} - \frac{\cancel{2}}{\cancel{4}}\right) + \left(\frac{\cancel{4}}{\cancel{16}} - \frac{\cancel{3}}{\cancel{8}}\right) + \dots +$

$\left(\frac{\cancel{1-(n-1)}}{\cancel{2^{n-1+1}}} + \frac{\cancel{m-1}}{\cancel{2^{m-1}}}\right) + \left(\frac{1+m}{2^{m+1}} - \frac{\cancel{m}}{\cancel{2^m}}\right)$

$S_m = \frac{-1}{2} + \frac{1+m}{2^{m+1}}$

d) $\lim_{m \to +\infty} \frac{-1}{2} + \frac{1+m}{2^{m+1}} = \frac{-1}{2} + \lim_{m \to +\infty} \frac{m+1}{2^{m+1}} \overset{L'H}{\underset{\frac{\infty}{\infty}}{=}} \frac{-1}{2} + \lim_{m \to +\infty} \cancelto{0}{\frac{1}{2^m \cdot \log 2}} = -\frac{1}{2}$

e) Sim, já que $\lim_{m \to +\infty} S_m = S$, sendo $S = \frac{-1}{2}$

$$16.\ a)\ \sum_{m=1}^{+\infty} \frac{1}{m(m+1)} = 1 \qquad \frac{1}{m(m+1)} = \frac{A}{m+1} + \frac{B}{m} = \frac{Am+Bm+B}{m(m+1)}$$

$$\begin{cases} A+B=0 \\ B=1 \end{cases} \quad A=-1 \qquad \sum_{m=1}^{+\infty} \frac{-1}{m+1} + \frac{1}{m} = \left[\frac{-\cancel{1}}{\cancel{2}} + 1\right] + \left[\frac{-\cancel{1}}{\cancel{3}} + \frac{\cancel{1}}{\cancel{2}}\right] + \left[\frac{-\cancel{1}}{\cancel{4}} + \frac{\cancel{1}}{\cancel{3}}\right] + \dots +$$

$$\left[\frac{-\cancel{1}}{\cancel{(m-1)+1}} + \frac{1}{m-1}\right] + \left[\frac{-1}{m+1} + \frac{\cancel{1}}{\cancel{m}}\right] = 1 + \frac{1}{m-1} - \frac{1}{m+1}$$

$$\lim_{m\to+\infty} 1 + \frac{\cancel{1}^{\,0}}{\cancel{m}-1} - \frac{\cancel{1}^{\,0}}{\cancel{m}+1} = 1$$

$$b)\ \sum_{m=1}^{+\infty} \frac{2m+1}{n^2(m+1)^2} = 1 \qquad \frac{2m+1}{n^2(m+1)^2} = \frac{A}{n^2} + \frac{B}{(m+1)^2} = \frac{Am^2+2Am+A+Bm^2}{n^2(m+1)^2}$$

$$\begin{cases} A+B=0 \\ 2A=2 \\ A=1 \end{cases} \quad B=-1 \qquad \sum_{m=1}^{+\infty} \frac{1}{m^2} - \frac{1}{(m+1)^2} = \left[1 - \frac{\cancel{1}}{\cancel{4}}\right] + \left[\frac{\cancel{1}}{\cancel{4}} - \frac{\cancel{1}}{\cancel{9}}\right] + \left[\frac{\cancel{1}}{\cancel{9}} - \frac{1}{16}\right] + \dots$$

$$+ \left[\frac{\cancel{1}}{\cancel{(m-1)^2}} - \frac{\cancel{1}}{\cancel{(m-1+1)^2}}\right] + \left[\frac{\cancel{1}}{\cancel{m^2}} - \frac{1}{(m+1)^2}\right] = 1 - \frac{1}{(m+1)^2}$$

$$\lim_{m\to+\infty} S_m = \lim_{m\to+\infty} 1 - \frac{\cancel{1}^{\,0}}{(\cancel{m}+1)^2} = 1$$

$$c)\ \sum_{n=2}^{+\infty} \frac{1}{n^2-1} = \frac{3}{4} \qquad \frac{1}{n^2-1} = \frac{A}{(m+1)} + \frac{B}{(m-1)} = \frac{Am-A+Bm+B}{n^2-1}$$

$$\begin{cases} A+B=0 \Rightarrow A=-B \\ -A+B=1 \Rightarrow B=\frac{1}{2} \end{cases} \Rightarrow A=-\frac{1}{2} \qquad \frac{1}{2}\sum_{m=1}^{+\infty} \frac{1}{(m-1)} - \frac{1}{(m+1)} = \frac{1}{2}\left[\left(1 - \frac{\cancel{1}}{\cancel{3}}\right) + \left(\frac{1}{2} - \frac{\cancel{1}}{\cancel{4}}\right) + \left(\frac{\cancel{1}}{\cancel{3}} - \frac{\cancel{1}}{\cancel{5}}\right) + \right.$$

$$\left. \dots + \left(\frac{\cancel{1}}{\cancel{m-2}} - \frac{1}{m}\right) + \left(\frac{1}{m-1} - \frac{1}{m+1}\right)\right] = \frac{1}{2}\left[\frac{-1}{m} + \frac{1}{m-1} - \frac{1}{m+1} + 1 + \frac{1}{2}\right]$$

$$\lim_{m\to+\infty} S_m = \frac{1}{2} \lim_{m\to+\infty} \frac{3}{2} - \frac{\cancel{1}^{\,0}}{\cancel{m}} - \frac{\cancel{1}^{\,0}}{\cancel{m}+1} + \frac{\cancel{1}^{\,0}}{\cancel{m}-1} = \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$$

17. a) $0,412412412 = \sum_{n=1}^{+\infty} \frac{412}{10^{3n}}$

$$S_n = \frac{\frac{412}{10^3}}{1-\frac{1}{1000}} = \frac{412}{\cancel{10^3}}\cdot\frac{\cancel{1000}}{999} = \frac{412}{999}$$

b) $0,0213434 = \frac{21}{10^3} + \sum_{n=1}^{+\infty} \frac{34}{10^{2n+3}} = \frac{21}{10^3} + \frac{34}{10^3}\sum_{n=1}^{+\infty} \frac{1}{10^{2n}}$

$$S_n = \frac{21}{10^3} + \frac{34}{10^3}\cdot\frac{\frac{1}{10^2}}{1-\frac{1}{10^2}} = \frac{21}{10^3} + \frac{34}{10^3}\cdot\frac{1}{\cancel{10^2}}\cdot\frac{\cancel{100}}{99} = \frac{21}{10^3} + \frac{34}{99} = \frac{1079+34}{99\cdot 10^3} = \frac{2113}{99000}$$

18. a) $\sum_{n=1}^{+\infty} \frac{2+\operatorname{sen} 3n}{n}$

$$-1 \le \operatorname{sen} 3n \le 1 \Rightarrow 1 \le 2+\operatorname{sen} 3n \le 3$$

$$\Rightarrow \frac{1}{n} \le \frac{2+\operatorname{sen} 3n}{n} \le \frac{3}{n}$$

$\sum_{n=1}^{+\infty} \frac{1}{n}$ é a série harmônica, que diverge. Logo, como $\sum_{n=1}^{+\infty} \frac{1}{n} \le \sum_{n=1}^{+\infty} \frac{2+\operatorname{sen} 3n}{n}$, segue, pelo teste da comparação, que a série $\sum_{n=1}^{+\infty} \frac{2+\operatorname{sen} 3n}{n}$ diverge

b) $\sum_{n=1}^{+\infty} \sqrt[n]{n!}$

$$n \le n! \Rightarrow \sqrt[n]{n} \le \sqrt[n]{n!}$$

$\sum_{n=1}^{+\infty} \sqrt[n]{n}$ $\quad \lim_{x\to+\infty} \sqrt[x]{x} = \lim_{x\to+\infty} x^{1/x} = e^{\lim_{x\to+\infty} \frac{\ln x}{x}\,\frac{+\infty}{+\infty}} \overset{L'H}{=} e^{\lim_{x\to+\infty} \frac{\frac{1}{x}}{1}} = e^0 = 1$

Logo, pelo teste do termo geral, $\sum_{n=1}^{+\infty} \sqrt[n]{n}$ diverge, pois $L = 1 \neq 0$.

Como $\sum_{n=1}^{+\infty} \sqrt[n]{n} \le \sum_{n=1}^{+\infty} \sqrt[n]{n!}$ e a primeira diverge, então segue que

$\sum_{n=1}^{+\infty} \sqrt[n]{n!}$ também diverge

c) $\sum_{n=1}^{+\infty} \frac{3}{n^2+n}$

$n^2+n > n^2$

$\frac{1}{n^2+n} < \frac{1}{n^2}$

$\frac{3}{n^2+n} < \frac{3}{n^2}$

$\sum_{n=1}^{+\infty} \frac{3}{n^2} = 3\sum_{n=1}^{+\infty} \frac{1}{n^2}$ → Série p com $p>1$, portanto, converge.

Como $\sum_{n=1}^{+\infty} \frac{3}{n^2+n} < \sum_{n=1}^{+\infty} \frac{3}{n^2}$, e $\sum_{n=1}^{+\infty} \frac{3}{n^2}$ converge, segue que a série $\sum_{n=1}^{+\infty} \frac{3}{n^2+n}$ também converge

19_ I) Falsa. Tomemos $a_n = \frac{1}{n^2}$. $\sum_{n=1}^{+\infty} \frac{1}{n^2}$ é uma série p, com $p>1$ e portanto, converge. $\sum_{n=1}^{+\infty} \sqrt{\frac{1}{n^2}} = \sum_{n=1}^{+\infty} \frac{1}{n}$, que é a série harmônica, que diverg.

II) Falsa.

III) Pela definição, se a soma da série é um número finito, então ela converge. Se uma série diverge, $\lim_{n \to +\infty} a_n \neq 0$. Logo, como $S_n$ converge, então $\lim_{n \to +\infty} a_n = 0$.

IV) Verdadeira. Se $\lim_{n \to +\infty} a_n \neq 0$, então a série $\sum_{n=1}^{+\infty} a_n$ diverge

V) Falsa. Tomando $p(n) = n$ e $q(n) = n^2$, temos $\sum_{n=1}^{+\infty} \frac{n}{n^2} = \sum_{n=1}^{+\infty} \frac{1}{n}$, que diverge

21. a) $\sum_{n=1}^{+\infty} \frac{1}{\sqrt[n]{n}}$ $\quad \lim_{n\to+\infty} \frac{1}{\sqrt[n]{n}} = 1 \neq 0$ logo, a série diverge.

b) $\sum_{n=1}^{+\infty} \frac{1}{\sqrt[3]{n^2+3}}$

$$n^2+3 > n^2$$
$$\sqrt[3]{n^2+3} > \sqrt[3]{n^2}$$
$$\frac{1}{\sqrt[3]{n^2+3}} < \frac{1}{\sqrt[3]{n^2}}$$

$$\lim_{n\to+\infty} \frac{\frac{1}{\sqrt[3]{n^2+3}}}{\frac{1}{\sqrt[3]{n^2}}} = \lim_{n\to+\infty} \frac{\sqrt[3]{n^2}}{\sqrt[3]{n^2+3}} = \lim_{n\to+\infty} \left[\frac{n^2}{n^2}\cdot\left(\frac{1}{1+\frac{3}{n^2}}\right)\right]^{1/3} = \sqrt[3]{1} = 1 > 0$$

logo, pelo teste da comparação por limite, a série diverge, pois $\sum_{n=1}^{+\infty} \frac{1}{\sqrt[3]{n^2}}$ é uma série p com $p \leq 1$, que diverge

c) $\sum_{n=1}^{+\infty} \frac{\cos n + 3}{6^n}$ $\quad -1 \leq \cos n \leq 1 \Rightarrow 2 \leq \cos n + 3 \leq 4 \Rightarrow \frac{2}{6^n} \leq \frac{\cos n+3}{6^n} \leq \frac{4}{6^n}$

$\sum_{n=1}^{+\infty} \frac{4}{6^n} = \sum_{n=1}^{+\infty} 4\cdot\left(\frac{1}{6}\right)^n \to$ Série geométrica com $r < 1$, logo, converge.

Portanto, pelo teste da comparação, a série converge.

d) $\sum_{n=2}^{+\infty} \frac{n}{\ln n}$ $\quad \lim_{n\to+\infty} \frac{n}{\ln n} \overset{L'H}{\underset{\frac{\infty}{\infty}}{=}} \lim_{n\to+\infty} \frac{1}{\frac{1}{n}} = \lim_{n\to+\infty} n = +\infty$. Logo, como $L \neq 0$, a série diverge.

e) $\sum_{n=1}^{+\infty} 2^{-n}+5^{-n} = \sum_{n=1}^{+\infty}\left(\frac{1}{2}+\frac{1}{5}\right)^n = \sum_{n=1}^{+\infty}\left(\frac{5+2}{10}\right)^n = \sum_{n=1}^{+\infty}\left(\frac{7}{10}\right)^n \to$ Série geométrica com $r \leq 1$, portanto, converge

f) $\sum_{n=1}^{+\infty}\left(\frac{1}{5^n}+n\right) = \sum_{n=1}^{+\infty}\left(\frac{1}{5}\right)^n + \sum_{n=1}^{+\infty} n \to$ Diverge

↳ converge. geométrica com $r<1$

Logo, a série diverge

g) $\sum_{n=1}^{+\infty}\frac{n+1}{n^2}$ $\quad \lim_{n\to+\infty}\frac{\frac{n+1}{n^2}}{\frac{1}{n}} = \lim_{n\to+\infty}\frac{n+1}{n} = \lim_{n\to+\infty}\frac{\cancel{n}}{\cancel{n}}\left(\frac{1+\cancel{\frac{1}{n}}^{0}}{1}\right) = 1 > 0$ Logo,

como $\sum_{n=1}^{+\infty}\frac{1}{n}$ diverge, pelo teste da comparação por limite, $\sum_{n=1}^{+\infty}\frac{n+1}{n^2}$ diverge

h) $\sum_{n=1}^{+\infty}\frac{3+(-1)^n}{3^n} = \sum_{n=1}^{+\infty}3\cdot\left(\frac{1}{3}\right)^n + \sum_{n=1}^{+\infty}\left(\frac{-1}{3}\right)^n$ $\quad$ Logo, a série converge.

↳ Geométrica com $|r|<1$ $\quad$ ↳ Geométrica com $|r|<1$

e) $\sum_{n=1}^{+\infty}\frac{1}{n2^n}$ $\quad n\cdot 2^n \geq 2^n$ $\quad \frac{1}{n\cdot 2^n} < \frac{1}{2^n}$ $\quad \sum_{n=1}^{+\infty}\left(\frac{1}{2}\right)^n$ é uma série geométrica com $|r|<1$ e portanto, converge.

Logo, $\sum_{n=1}^{+\infty}\frac{1}{n2^n}$ também converge

23- a) $\sum_{k=1}^{+\infty}\frac{3^k+5^k}{15^k} = \frac{3}{4}$ $\quad \sum_{k=1}^{+\infty}\frac{3^k}{3^k\cdot 5^k} + \sum_{k=1}^{+\infty}\frac{5^k}{3^k 5^k} = \sum_{k=1}^{+\infty}\left(\frac{1}{5}\right)^k + \sum_{k=1}^{+\infty}\left(\frac{1}{3}\right)^k$

$= \frac{1/5}{1-\frac{1}{5}} + \frac{1/3}{1-\frac{1}{3}} = \frac{1/5}{\frac{4}{5}} + \frac{1/3}{\frac{2}{3}} = \frac{1}{4}+\frac{1}{2} = \frac{3}{4}$

b) $\frac{1}{3}+\frac{2}{3^2}+\frac{1}{3^3}+\frac{2}{3^4}+\frac{1}{3^5}+\frac{2}{3^6}+\dots = \frac{5}{8}$ $\quad \sum_{n=1}^{+\infty}\frac{1}{3^{2n-1}} + \sum_{n=1}^{+\infty}\frac{2}{3^{2n}} = \sum_{n=1}^{+\infty}3\cdot\left(\frac{1}{9}\right)^n + \sum_{n=1}^{+\infty}2\left(\frac{1}{9}\right)^n$

$= \frac{1/3}{1-\frac{1}{9}} + \frac{2/9}{1-1/9} = \frac{1/3}{\frac{8}{9}} + \frac{2/9}{8/9} = \frac{3}{8}+\frac{2}{8} = \frac{5}{8}$

c) $\displaystyle\sum_{k=1}^{+\infty} \frac{\sqrt{k+1}-\sqrt{k}}{\sqrt{k^2+k}} = 1$ $\quad \displaystyle\sum_{k=1}^{+\infty} \frac{\sqrt{k+1}}{\sqrt{k}\cdot\sqrt{k+1}} - \sum_{k=1}^{+\infty} \frac{\sqrt{k}}{\sqrt{k}\cdot\sqrt{k+1}} = \sum_{k=1}^{+\infty} \frac{1}{\sqrt{k}} - \sum_{k=1}^{+\infty} \frac{1}{\sqrt{k+1}}$

$$= \left(\frac{1}{1} - \cancel{\frac{1}{\sqrt{2}}}\right) + \left(\cancel{\frac{1}{\sqrt{2}}} - \cancel{\frac{1}{\sqrt{3}}}\right) + \left(\cancel{\frac{1}{\sqrt{3}}} - \cancel{\frac{1}{\sqrt{4}}}\right) + \dots + \left(\cancel{\frac{1}{\sqrt{k-1}}} - \cancel{\frac{1}{\sqrt{k}}}\right) + \left(\cancel{\frac{1}{\sqrt{k}}} - \frac{1}{\sqrt{k+1}}\right)$$

$= 1 - \frac{1}{\sqrt{k+1}} = S_m \quad \lim_{k\to+\infty} 1 - \frac{1}{\sqrt{k+1}}^{\,0} = 1$

d) $\displaystyle\sum_{k=1}^{+\infty} \frac{2k-1}{3^k} = 1$ $\quad \displaystyle\sum_{k=1}^{+\infty} \frac{k}{3^{k-1}} - \sum_{k=1}^{+\infty} \frac{k+1}{3^k} = \left(\frac{1}{1} - \cancel{\frac{2}{3}}\right) + \left(\cancel{\frac{2}{3}} - \cancel{\frac{3}{9}}\right) + \left(\cancel{\frac{3}{9}} - \cancel{\frac{4}{27}}\right)$

$$+ \dots + \left(\cancel{\frac{k-1}{3^{k-2}}} - \cancel{\frac{k}{3^{k-1}}}\right) + \left(\cancel{\frac{k}{3^{k-1}}} + \frac{k+1}{3^k}\right)$$

$= 1 + \frac{k+1}{3^k} = S_m \quad \lim_{k\to+\infty} S_m = \lim_{k\to+\infty} 1 + \frac{k+1}{3^k} = 1 + \lim_{k\to+\infty} \frac{k+1}{3^k}\ \frac{\infty}{\infty} \overset{L'H}{=} 1 + \lim_{k\to+\infty} \frac{1}{\log 3 \cdot 3^k}^{\,0}$

$= 1$

e) $\displaystyle\sum_{m=2}^{+\infty} \frac{1}{2^m} = \sum_{m=2}^{+\infty} \left(\frac{1}{2}\right)^m = \frac{a_1}{1-r} = \frac{1/4}{1-1/2} = \frac{1/4}{1/2} = \frac{1}{2}$

f) $\displaystyle\sum_{m=1}^{+\infty} \frac{1}{m(m+1)(m+2)} =$

g) $\sum_{n=1}^{+\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}$ $\frac{A}{(2n-1)} + \frac{B}{(2n+1)} = \frac{2An + A + 2Bn - B}{(2n-1)(2n+1)}$

$\begin{cases} 2A+2B=0 & A=-B \\ A-B=1 & \Rightarrow B=-\frac{1}{2} \end{cases}$ $A=\frac{1}{2}$ $\sum_{n=1}^{+\infty} \frac{1/2}{2n-1} - \sum_{n=1}^{+\infty} \frac{1/2}{2n+1} = \left(\frac{1}{2} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{10}}\right) + \left(\cancel{\frac{1}{10}} - \frac{1}{14}\right) +$

$+ \cdots + \left(\frac{1}{2} \cdot \cancel{\frac{1}{2n-2-1}} + \frac{1}{2} \cdot \cancel{\frac{1}{2n-2+1}}\right) + \left(\cancel{\frac{1}{2} \cdot \frac{1}{2n-1}} - \frac{1}{2} \cdot \frac{1}{2n+1}\right)$

$= \frac{1}{2} - \frac{1}{4n+2} = S_n$ $\lim_{n \to +\infty} S_n = \lim_{n \to +\infty} \frac{1}{2} - \cancel{\frac{1}{4n+2}}^{0} = \frac{1}{2}$

24. a) F. $\{-1^n\}$ é limitada, mas não é convergente.

b) F. $\left\{\frac{(-1)^n}{n}\right\}$ é limitada, mas não é monótona. $-1, \frac{1}{2}, \frac{-1}{3}, \frac{1}{4}, \ldots$

c) F. $\{n\}$ é monótona, mas não é limitada 1, 2, 3, 4, 5, 6

d) F. $\{n\}$ é divergente e monótona.

e) F

f) F

g) F. $a_n = \frac{1}{n}$ $b_n = n^2$ $\lim_{n \to +\infty} \frac{n^2}{n} = +\infty$

h) V.

i) V.

j) F

k) F. $\sum \frac{1}{n^2}$ converge $\sum \sqrt{\frac{1}{n^2}} = \sum \frac{1}{n}$, que diverge