Prova T5:

① a) $A=(1,3,2)$ $B=(5,3,2)$ $C=(2,2,2)$

$\vec{AB}=(4,0,0)$

$\vec{AC}=(1,-1,0)$

$$\overrightarrow{AB\times AC}=\left|\begin{array}{ccc|cc} \hat{i} & \vec{j} & \hat{k} & \hat{i} & \vec{j} \\ 4 & 0 & 0 & 4 & 0 \\ 1 & -1 & 0 & 1 & -1 \end{array}\right|$$

$$= 0\hat{i}+0\vec{j}-4\vec{k}-(0\hat{k}+0\hat{i}+0\vec{j}) = 0\hat{i}+0\hat{j}-4\vec{k}$$

$$=(0,0,4)$$

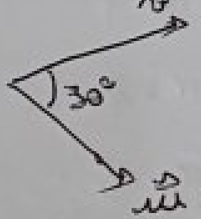
Agora, $||\overrightarrow{AB\times AC}|| = \sqrt{0^2+0^2+4^2} = \sqrt{16} = 4$

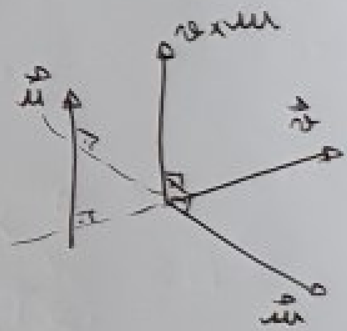
$$A=\frac{||\overrightarrow{AB\times AC}||}{2} = \frac{4}{2} = 2.$$

①

b) $\vec{u}\perp\vec{v}$ $||\vec{u}||=3;\ ||\vec{v}||=5;\ ||\vec{w}||=2$

$\hat{u}\perp\vec{w}$ $\langle u, v\times w\rangle = ?$





$$\cos\theta = \frac{\langle u, v\times w\rangle}{||u||.||v\times w||} \Rightarrow$$

θ é ângulo formado entre u e $v\times w$, ou seja, 0° ou 180°.

Vamos considerar 0°.

$$\cos\theta = \frac{\langle u, v\times w\rangle}{3.||v\times w||} \Rightarrow \langle u, v\times w\rangle = 3.||v\times w||.\cos\theta.$$

Agora,

$$\operatorname{sen} 30° = \frac{||v\times w||}{||v||.||w||} \Rightarrow \frac{1}{2} = \frac{||v\times w||}{5.2} \Rightarrow ||v\times w|| = 5.$$

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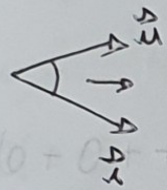
Dessa forma.

$$\langle u, v \times w \rangle = 3 \cdot 5 \cdot 1 = 15 \Rightarrow \langle u, v \times w \rangle = 15$$

1 c) $\vec{u} = (k, 2, 1)$ $\vec{v} = (1, 1, -2)$



agudo (menor do que 90°).

$$\langle \vec{u}, \vec{i} \rangle = \langle (k, 2, 1), (1, 0, 0) \rangle = k > 0.$$

$$\vec{u \times v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ k & 2 & 1 \\ 1 & 1 & -2 \end{vmatrix} \begin{matrix} \vec{i} & \vec{j} \\ k & 2 \\ 1 & 1 \end{matrix} = -2\vec{i} + \vec{j} + k \cdot \vec{k} - (2\vec{k} + \vec{i} - 2k\vec{j})$$

$$= -3\vec{i} + (1 + 2k)\vec{j} + (k-2)\vec{k}$$

$$\Rightarrow \vec{u \times v} = (-3, 1 + 2k, k - 2)$$

$$\| \vec{u \times v} \| = \sqrt{(-3)^2 + (1 + 2k)^2 + (k-2)^2} = \sqrt{9 + 1 + 4k + 4k^2 + k^2 - 4k + 4}$$

$$= \sqrt{5k^2 + 14}.$$

Agora, $\frac{\| \vec{u \times v} \|}{2} = \sqrt{57} \Rightarrow \frac{\sqrt{5k^2 + 14}}{2} = \sqrt{57}$

$$\Rightarrow \frac{5k^2 + 14}{4} = 57 \Rightarrow 5k^2 + 14 = 228 \Rightarrow 5k^2 = 214$$

$\Rightarrow k^2 = \frac{214}{5} \Rightarrow k = \pm \sqrt{\frac{214}{5}}$. Como $k > 0$, então

$$\boxed{k = +\sqrt{\frac{214}{5}}}$$

Dessa forma.

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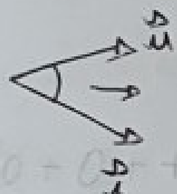

agudo (menor do que 90°).

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d) $r: \begin{cases} P_r = (2, -3, 1) \\ \vec{v}_r = (4, 3, 1) \end{cases}$ $s: \begin{cases} P_s = (-2, -3, 1) \\ \vec{v}_s = (3, -1, 3) \end{cases}$

(I) Não existe $\lambda \in \mathbb{R}$, tal que $\vec{v}_r = \lambda \vec{v}_s$, logo, os vetores não são paralelos e as retas também não são paralelas.

(II) $\overrightarrow{P_r P_s} = (-4, 0, 0)$

$$\langle v_r, v_s \times P_r P_s \rangle = \begin{vmatrix} 4 & 3 & 1 \\ 3 & -1 & 3 \\ -4 & 0 & 0 \end{vmatrix} \begin{matrix} 4 & 3 \\ 3 & -1 \\ -4 & 0 \end{matrix} = 0 - 16 + 0 - (4 + 0 + 0)$$

$$= -16 - 4 = -20.$$

Dessa forma, como $\langle v_r, v_s \times P_r P_s \rangle = -20 \neq 0$, então as retas não são coplanares. Logo, são reversas.

(2) plano $x=2 \rightarrow \vec{\eta}_x = (1,0,0)$

plano $xz \rightarrow \vec{\eta}_{xz} = (0,1,0)$

Seja $\vec{\eta}_\alpha = (a,b,c)$ a normal de α.

• Como α forma $30°$ com $x=2$, então.

$$\cos 30° = \frac{\langle \eta_\alpha, \eta_x \rangle}{\|\eta_\alpha\| \|\eta_x\|} = \frac{\langle (a,b,c),(1,0,0) \rangle}{8 \cdot \sqrt{1^2+0^2+0^2}} = \frac{a}{8}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{a}{8} \Rightarrow \boxed{a = 4\sqrt{3}}.$$

• Como α forma $60°$ com xz então:

$$\cos 60° = \frac{\langle \eta_\alpha, \eta_{xz} \rangle}{\|\eta_\alpha\| \cdot \|\eta_{xz}\|} = \frac{\langle (a,b,c),(0,1,0) \rangle}{8 \cdot \sqrt{0^2+1^2+0^2}} = \frac{b}{8}$$

$\Rightarrow \frac{1}{2} = \frac{b}{8} \Rightarrow \underline{b = 4}$.

Daí, $\vec{\eta}_\alpha = (4\sqrt{3}, 4, c)$. Como $\|\vec{\eta}_\alpha\| = 8$, então,

$$\|\vec{\eta}_\alpha\| = \sqrt{(4\sqrt{3})^2 + 4^2 + c^2} = \sqrt{16 \cdot 3 + 16 + c^2} = 8$$

$$\Rightarrow 64 + c^2 = 64 \Rightarrow c = 0.$$

Logo, $\vec{\eta}_\alpha = (4\sqrt{3}, 4, 0)$. Sendo $A = (3, 2, 1) \in \alpha$.

Equação Geral de α:

$$4\sqrt{3}(x-3) + 4(y-2) + 0(z-1) = 0$$

$$\Rightarrow 4\sqrt{3}x - 12\sqrt{3} + 4y - 8 = 0 \Rightarrow \boxed{4\sqrt{3}x + 4y = 12\sqrt{3} + 8}$$

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3) $S: x^2 + y^2 + z^2 + 2x + 2y - 4z = -2$

$$\Rightarrow x^2 + 2x + 1 + y^2 + 2y + 1 + z^2 - 4z + 4 = -2 + 1 + 1 + 4$$

$$\Rightarrow (x+1)^2 + (y+1)^2 + (z-2)^2 = 4 \; ; \quad O = (-1, -1, 2) \; ; \; r = 2$$

$$r: \left\{ \frac{x-2}{3} = \frac{y+2}{2} = z - 4 \; ; \right. \quad P_r = (2, -2, 4)$$

$$\vec{v}_r = (3, 2, 1)$$

Queremos encontrar o plano π.

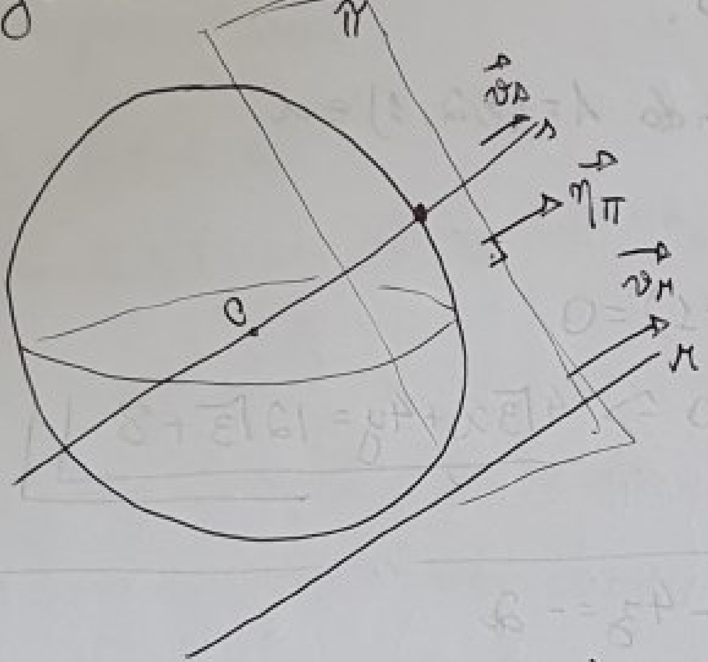
Como $\pi \perp r \Rightarrow \vec{\eta}_\pi \parallel \vec{v}_r$, vamos considerar $\vec{\eta}_\pi = (3, 2, 1)$

sendo $\lambda = 1$. $(\eta_\pi = \lambda \cdot v_r)$

Agora, seja a reta s, de modo que:

$\vec{v}_s = \vec{v}_r = (3, 2, 1)$ e $P_s = O = (-1, -1, 2)$

$$s: \begin{cases} x = -1 + 3t \\ y = -1 + 2t, \quad t \in \mathbb{R}. \\ z = 2 + t \end{cases}$$



A reta s toca a esfera S em dois pontos, vamos encontrá-los:
Existe $t_0 \in \mathbb{R}$, tal que, quando $t = t_0$, o ponto $(-1+3t_0, -1+2t_0, 2+t_0) \in s \cap S$.

$P_0 = (x_0, y_0, z_0) = (-1+3t_0, -1+2t_0, 2+t_0)$

Assim, $(x_0+1)^2 + (y_0+1)^2 + (z_0-2)^2 = 4$

$\Rightarrow (-1+3t_0+1)^2 + (-1+2t_0+1)^2 + (2+t_0-2)^2 = 4$

$\Rightarrow (3t_0)^2 + (2t_0)^2 + t_0^2 = 4 \Rightarrow 9t_0^2 + 4t_0^2 + t_0^2 = 4$

$\Rightarrow 14t_0^2 = 4 \Rightarrow t_0^2 = \frac{4}{14} \Rightarrow t_0 = \pm \frac{2}{\sqrt{14}} \Rightarrow t_0 = \pm \frac{2\sqrt{14}}{14}$

$\Rightarrow \boxed{t_0 = \pm \frac{\sqrt{14}}{7}}$

Utilizando $t_0 = \frac{\sqrt{14}}{7}$, temos:

$$P_0 = \left(-1 + \frac{3\sqrt{14}}{7}, -1 + \frac{2\sqrt{14}}{7}, 2 + \frac{\sqrt{14}}{7}\right)$$

O plano π será tangente na esfera S no ponto P_0.

Equação do plano π: $\vec{\eta}_\pi = (3, 2, 1)$

$$\pi: 3\left(x + 1 - \frac{3\sqrt{14}}{7}\right) + 2\left(y + 1 - \frac{2\sqrt{14}}{7}\right) + 1\left(z - 2 - \frac{\sqrt{14}}{7}\right) = 0$$

$$\Rightarrow 3x + 2y + z + 3 - \frac{9\sqrt{14}}{7} + 2 - \frac{4\sqrt{14}}{7} - 2 - \frac{\sqrt{14}}{7} = 0$$

$$\Rightarrow 3x + 2y + z = -3 + \frac{14\sqrt{14}}{7}$$

$$\pi: \boxed{3x + 2y + z = -3 + 2\sqrt{14}}$$

OU: Utilizando $t_0 = -\frac{\sqrt{14}}{7}$.

$$P_0 = \left(-1 - \frac{3\sqrt{14}}{7}, -1 - \frac{2\sqrt{14}}{7}, 2 - \frac{\sqrt{14}}{7}\right)$$

$$\pi: 3\left(x + 1 + \frac{3\sqrt{14}}{7}\right) + 2\left(y + 1 + \frac{2\sqrt{14}}{7}\right) + 1\left(z - 2 + \frac{\sqrt{14}}{7}\right) = 0$$

$$\Rightarrow 3x + 2y + z + 3 + \frac{9\sqrt{14}}{7} + 2 + \frac{4\sqrt{14}}{7} - 2 + \frac{\sqrt{14}}{7} = 0$$

$$\Rightarrow 3x + 2y + z = -3 - \frac{14\sqrt{14}}{7}$$

$$\pi: \boxed{3x + 2y + z = -3 - 2\sqrt{14}}$$

4

a) $\alpha: x+y+z=2$ $\quad \beta: x-2y-z=1$

Como $r = \alpha \cap \beta$, ela satisfaz a equacão de ambos planos.

$$\begin{cases} x+y+z=2 \\ x-2y-z=1 \end{cases} \Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & -2 & -1 & 1 \end{array}\right) \underset{L_2 \to L_2 - L_1}{\sim}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & -3 & -2 & -1 \end{array}\right) \Rightarrow \begin{cases} x+y+z=2 \quad \text{I} \\ -3y-2z=-1 \to -3y = -1+2z \end{cases}$$

$$\boxed{y = \frac{1}{3} - \frac{2z}{3}}$$

Em I, tem-se:

$$x + \frac{1}{3} - \frac{2z}{3} + z = 2 \Rightarrow x + \frac{z}{3} = 2 - \frac{1}{3} \Rightarrow \boxed{x = \frac{5}{3} - \frac{z}{3}}$$

$$r: \begin{cases} x = \frac{5}{3} - \frac{z}{3} \\ y = \frac{1}{3} - \frac{2z}{3} \end{cases} \quad z = t \quad r: \begin{cases} x = \frac{5}{3} - \frac{t}{3} \\ y = \frac{1}{3} - \frac{2t}{3} \\ z = t \end{cases}, t \in \mathbb{R}$$

$P_r = \left(\frac{5}{3}, \frac{1}{3}, 0\right)$ $\quad \vec{v}_r = \left(\frac{-1}{3}, \frac{-2}{3}, 1\right)$

b) Quando $t = 1$: $\quad O = \left(\frac{4}{3}, -\frac{1}{3}, 1\right)$

Como a esfera S é tangente à reta s, ao calcularmos a distância do centro de S até a reta s teremos o raio da esfera. (Fórmula dada na prova).
Usando a dica show.

$\vec{P}_s = (1, -1, 2)$

$\vec{v}_s = (1, 1, 1)$

Agora, $d(O, s) = \dfrac{\|\vec{OP_s} \times \vec{v_s}\|}{\|\vec{v_s}\|}$

$\vec{OP_s} = \left(-\frac{1}{3}, -\frac{1}{3}, 1\right)$

$$\vec{OP_s} \times \vec{v_s} = \left|\begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ -\frac{1}{3} & -\frac{1}{3} & 1 \\ 1 & 1 & 1 \end{matrix}\right| \begin{matrix} \vec{i} & \vec{j} \\ -\frac{1}{3} & -\frac{1}{3} \\ 1 & 1 \end{matrix} = -\frac{\vec{i}}{3} + \vec{j} - \frac{\vec{k}}{3} - \left(-\frac{\vec{k}}{3} + \vec{i} - \frac{\vec{j}}{3}\right)$$

$$= -\frac{4}{3}\vec{i} + \frac{4}{3}\vec{j} + 0\vec{k}$$

$$= \left(-\frac{4}{3}, -\frac{4}{3}, 0\right)$$

$$\|\vec{OP_s \times v_s}\| = \sqrt{\left(-\frac{4}{3}\right)^2 + \left(-\frac{4}{3}\right)^2 + 0^2} = \sqrt{\frac{16}{9} + \frac{16}{9}} = \sqrt{\frac{2 \cdot 16}{9}} = \frac{4\sqrt{2}}{3}$$

$$\|\vec{v_s}\| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

Assim, $d(O, s) = \dfrac{\frac{4\sqrt{2}}{3}}{\sqrt{3}} = \dfrac{4\sqrt{2}}{3\sqrt{3}}$.

Ou seja, $r = \dfrac{4\sqrt{2}}{3\sqrt{3}}$. Montando a equação da esfera, Temos:

$$S: \left(x - \frac{4}{3}\right)^2 + \left(y + \frac{1}{3}\right)^2 + (z - 1)^2 = \left(\frac{4\sqrt{2}}{3\sqrt{3}}\right)^2 =$$