Universidade Federal de Viçosa
Centro de Ciências Exatas
Departamento de Matemática

# Gabarito 2ª Lista - MAT 241 - Cálculo III - 2018/II

1) $V = \pi r^2 \left(\frac{4r}{3} + h\right)$

2) (a) $2y^5 + y + 1$ e $\frac{1}{2}$

   (b) $1 - 3h^2$ e $\frac{1}{1-h}$

   (c) $h^5 - h + 1$ e $1$

   (d) $5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4 + y^2(-9x^2 - 9xh - 3)$ e $\frac{-y}{(x+h-y)(x-y)}$

   (e) $-3x^3(2y+h)$ e $\frac{x}{(x-y)(x-y-h)}$

3) (a) $0$ (b) $x^6$ (c) $y^2z^4$ (d) $2xy^2z^2$ (e) $2yx^2z^2$ (f) $2zx^2y^2$

4) (a) $D_f = \{(x,y) \in \mathbb{R}^2; (x-y)(x+y) \geq 0 \text{ e } x \neq -y\}$

   (b) $D_f = \{(x,y) \in \mathbb{R}^2; x \neq y\}$

   (c) $D_f = \{(x,y) \in \mathbb{R}^2; x \neq 0 \text{ e } y \neq 0\}$

   (d) $D_f = \mathbb{R}^2$

   (e) $D_f = \{(x,y) \in \mathbb{R}^2; x \neq 0\}$

   (f) $D_f = \{(x,y) \in \mathbb{R}^2; |x| + |y| \leq 1\}$

   (g) $D_f = \{(x,y) \in \mathbb{R}^2; y \neq x + 2k\pi$ ou $y \neq -x + (2k+1)\pi, k \in \mathbb{Z}\}$

   (h) $D_f = \{(x,y) \in \mathbb{R}^2; x \leq y \leq 1\}$

   (i) $D_f = \mathbb{R}^3$

   (j) $D_f = \mathbb{R}^3$

   (k) $D_f = \{(x,y,z) \in \mathbb{R}^3; x \neq 0 \text{ e } z \neq 0\}$

   (l) $D_f = \{(x,y,z) \in \mathbb{R}^3; y \neq \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\}$

   (m) $D_f = \{(x,y,z) \in \mathbb{R}^3; x^2 + y^2 + z^2 > 1\}$

   (n) $D_f = \{(x,y,z) \in \mathbb{R}^3; z \geq x^2 + y^2\}$

   (o) $D_f = \{(x,y,z) \in \mathbb{R}^3; z^2 \geq x^2 + y^2\}$

   (p) $D_f = \{(x,y,z) \in \mathbb{R}^3; 400 \geq 16x^2 + 25y^2 + z^2\}$

5) (a) $N_0 = \{(x,y) \in \mathbb{R}^2; x^2 + y^2 = 100\}$,
   $N_2 = \{(x,y) \in \mathbb{R}^2; x^2 + y^2 = 96\}$ e
   $N_{10} = \{(0,0)\}$

   (b) $N_0 = \{(0,0)\}$, $N_1 = \{(x,y) \in \mathbb{R}^2; x^2 + y^2 = 1\}$ e
   $N_2 = \{(x,y) \in \mathbb{R}^2; x^2 + y^2 = 4\}$

   (c) $N_0 = \{(0,0)\}$, $N_1 = \{(x,y) \in \mathbb{R}^2; 4x^2 + 9y^2 = 2\}$ e
   $N_4 = \{(x,y) \in \mathbb{R}^2; 4x^2 + 9y^2 = 4\}$

   (d) $N_{-1} = \left\{(x,y) \in \mathbb{R}^2; y = \frac{3}{7}x + \frac{1}{7}\right\}$,
   $N_0 = \left\{(x,y) \in \mathbb{R}^2; y = \frac{3}{7}x\right\}$ e
   $N_1 = \left\{(x,y) \in \mathbb{R}^2; y = \frac{3}{7}x - \frac{1}{7}\right\}$

   (e) $N_0 = \{(0,0)\}$ e
   $N_1 = \{(x,y) \in \mathbb{R}^2; x^2 - y^2 = 1\}$

   (f) $N_0 = \{(x,y) \in \mathbb{R}^2; x = y\}$ e
   $N_1 = \{(x,y) \in \mathbb{R}^2; y = x - 1 \text{ e } y = x + 1\}$

   (g) $N_{-1} = \{(x,y) \in \mathbb{R}^2; x^2 + y^2 = 1 + e^{-1}\}$,
   $N_0 = \{(x,y) \in \mathbb{R}^2; x^2 + y^2 = 2\}$ e
   $N_1 = \{(x,y) \in \mathbb{R}^2; x^2 + y^2 = 1 + e\}$

   (h) $N_{-1} = N_1 = \emptyset$ e $N_0 = \{(x,y) \in \mathbb{R}^2; x = 0\}$

   (i) $N_{-1} = N_0 = \emptyset$ e $N_1 = \{(0,0)\}$

   (j) $N_{-1} = \{(x,y,z) \in \mathbb{R}^3; z^2 = 1 - x^2 - y^2\}$,
   $N_0 = \{(0,0,0)\}$ e $N_1 = \emptyset$

   (k) $N_{-1} = \emptyset$, $N_0 = \{(0,0,0)\}$ e
   $N_1 = \{(x,y,z) \in \mathbb{R}^3; 4x^2 + y^2 + 9z^2 = 1\}$

   (l) $N_{-1} = \{(x,y,z) \in \mathbb{R}^3; z = -1 - x^2 - y^2\}$,
   $N_0 = \{(x,y,z) \in \mathbb{R}^3; z = -x^2 - y^2\}$ e
   $N_1 = \{(x,y,z) \in \mathbb{R}^3; z = 1 - x^2 - y^2\}$

   (m) $N_{-1} = \{(x,y,z) \in \mathbb{R}^3; x = -1 + y^2 - z^2\}$,
   $N_0 = \{(x,y,z) \in \mathbb{R}^3; x = y^2 - z^2\}$ e
   $N_1 = \{(x,y,z) \in \mathbb{R}^3; x = 1 + y^2 - z^2\}$

6) (a) Sim, de grau 2

   (b) Sim, de grau $-2$

   (c) Sim, de grau 1

   (d) Sim, de grau $-2$.

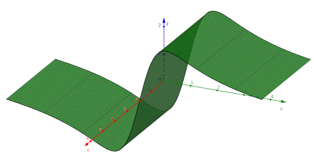
7) (a) 25 graus (b) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

8) Encontre as curvas de nível e os traços nos planos $xz$ e $yz$.

9) (a) $D(f) = \mathbb{R}^2$ e $Im(f) = \{z \in \mathbb{R}; -2 \leq z \leq 2\}$

   (b) Valor 2 em $\{(x,y) \in \mathbb{R}^2; y = 1\}$

   (c)



   (d) $\sqrt{2}$

10) (a) $D(f) = \{(x,y) \in \mathbb{R}^2; x^2 + y^2 \leq 1\}$

(b) $Im(f) = \{z \in \mathbb{R}; 5 \leq z \leq 6\}$

11) A menor distância é $2\sqrt{2}$.

12) (a) i) 0 ii) $\frac{5}{2}$ (b) i) 0 ii) 1 (c) i) 0 ii) $\frac{3}{8}$

13) (a) 0
(b) Não existe
(c) Considere $C_1 : x = \sqrt[3]{t - t^2}$ e $y = t$
(d) Considere $C_2 : x = \sqrt{t^4 + t^2}$ e $y = t$, $t > 0$.

14) (a) $C_1 : x = t$ e $y = 2t$ e $C_2 : x = t$ e $y = t + 1$
(b) $C_1 : x = t$ e $y = t$ e $C_2 : x = t$ e $y = t^2$
(c) $C_1 : x = t$ e $y = t$ e $C_2 : x = 0$ e $y = t$
(d) $C_1 : x = t$ e $y = t$ e $C_2 : x = t$ e $y = t^2$
(e) $C_1 : x = t$ e $y = t$ e $C_2 : x = t$ e $y = t^2$
(f) $C_1 : x = t$ e $y = t$ e $C_2 : x = t$ e $y = 0$

15) (a) Tome $x^2 + y^2 = r^2$
(b) Propriedades de limite e Teorema do Confronto
(c) Use o Teorema do Confronto
(d) Use o Primeiro Limite Fundamental
(e) $x^2 + y^2 = r^2$

16) Demostração.

17) (a) 0 (b) Não existe pela regra dos dois caminhos.

18) Neste caso, o limite não existe.

19) Demonstração.

20) A função $f$ é contínua em $\{(x, y) \in \mathbb{R}^2; x^2 + y^2 \neq 1\}$

21) A função é contínua em $\mathbb{R}^2$.

22) Não.

23) $k = 11$

24) (a)
$$\frac{\partial f}{\partial x}(x, y) = \begin{cases} \dfrac{y^2 - x^2 - 8xy^3}{(x^2 + y^2)^2}, & (x, y) \neq (0, 0) \\ \text{não existe}, & (x, y) = (0, 0) \end{cases}$$
$$\frac{\partial f}{\partial y}(x, y) = \begin{cases} \dfrac{12y^2x^2 + 4y^4 - 2xy}{(x^2 + y^2)^2}, & (x, y) \neq (0, 0) \\ 4, & (x, y) = (0, 0) \end{cases}$$
(b)
$$\frac{\partial f}{\partial x}(x, y) = \begin{cases} \dfrac{3x^4 + 9x^2y^2 + 4xy^3}{(x^2 + y^2)^2}, & (x, y) \neq (0, 0) \\ 3, & (x, y) = (0, 0) \end{cases}$$
$$\frac{\partial f}{\partial y}(x, y) = \begin{cases} \dfrac{-6x^2y^2 - 2y^4 - 6x^3y}{(x^2 + y^2)^2}, & (x, y) \neq (0, 0) \\ -2, & (x, y) = (0, 0) \end{cases}$$

25) (a)
$$\frac{\partial f}{\partial x}(x, y) = 10x + 6y + 2e^{2x+y}$$
$$\frac{\partial f}{\partial y}(x, y) = 6x + e^{2x+y}$$

(b)
$$\frac{\partial f}{\partial x}(x, y, z) = yz2^{xyz} \ln 2,$$
$$\frac{\partial f}{\partial y}(x, y, z) = xz2^{xyz} \ln 2$$
$$\frac{\partial f}{\partial z}(x, y, z) = xy2^{xyz} \ln 2$$

(c)
$$\frac{\partial f}{\partial x}(x, y) = 20x^3 + 6y^3 + \frac{2y}{2xy \ln 10}$$
$$\frac{\partial f}{\partial y}(x, y) = 18xy^2 + \frac{2x}{2xy \ln 10}$$

(d)
$$\frac{\partial f}{\partial x}(x, y, z) = \frac{\cos(\ln(xyz^2))}{x},$$
$$\frac{\partial f}{\partial y}(x, y, z) = \frac{\cos(\ln(xyz^2))}{y}$$
$$\frac{\partial f}{\partial z}(x, y, z) = \frac{2\cos(\ln(xyz^2))}{z}.$$

(e)
$$\frac{\partial f}{\partial x}(x, y) = \frac{2x}{1 + (x^2 + y^2)^2}$$
$$\frac{\partial f}{\partial y}(x, y) = \frac{1}{1 + (x^2 + y^2)^2}$$

(f)
$$\frac{\partial f}{\partial x}(x, y, z) = \frac{x}{\sqrt{x^2 - y^2 + z^2}},$$
$$\frac{\partial f}{\partial y}(x, y, z) = \frac{-y}{\sqrt{x^2 - y^2 + z^2}}$$
$$\frac{\partial f}{\partial z}(x, y, z) = \frac{z}{\sqrt{x^2 - y^2 + z^2}}$$

(g)
$$\frac{\partial f}{\partial x}(x, y) = \frac{y^3}{x\sqrt{x^2 - y^6}}$$
$$\frac{\partial f}{\partial y}(x, y) = \frac{-3y^2}{\sqrt{x^2 - y^6}}$$

(h)
$$\frac{\partial f}{\partial x}(x, y, z) = y^2 z \sec(xy^2z) \operatorname{tg}(xy^2z),$$
$$\frac{\partial f}{\partial y}(x, y, z) = 2xyz \sec(xy^2z) \operatorname{tg}(xy^2z)$$
$$\frac{\partial f}{\partial z}(x, y, z) = xy^2 \sec(xy^2z) \operatorname{tg}(xy^2z)$$

(i)
$$\frac{\partial f}{\partial x}(x, y) = \frac{y}{2\sqrt{xy}} \cosh(\sqrt{xy})$$
$$\frac{\partial f}{\partial y}(x, y) = \frac{x}{2\sqrt{xy}} \cosh(\sqrt{xy})$$

(j) $\dfrac{\partial f}{\partial x}(x,y,z) = \dfrac{yz}{6\sqrt[6]{(xyz)^5}}$,

$\dfrac{\partial f}{\partial y}(x,y,z) = \dfrac{xz}{6\sqrt[6]{(xyz)^5}}$

$\dfrac{\partial f}{\partial z}(x,y,z) = \dfrac{xy}{\sqrt[6]{(xyz)^5}}$

26) Demonstração.

27) Demonstração.

28) Demonstração.

29) Demonstração.

30) $f(x,y) = 3x^2y - 9xy^2 + 7x + y^3 + C$, com $C \in \mathbb{R}$

31) (a) $\nabla z = \left(\dfrac{-2x}{(x^2+y^2)^2}, \dfrac{-2y}{(x^2+y^2)^2}\right)$

(b) $\nabla w = (-y\,\mathrm{sen}(xy), -x\,\mathrm{sen}(xy) + z\cos(yz), y\cos(yz))$

(c) $\nabla w = \left(\dfrac{1}{x}, \dfrac{1}{y}, \dfrac{1}{z}\right)$

(d) $\nabla w = \left(\dfrac{y}{z}, \dfrac{x}{z}, \dfrac{-xy}{z^2}\right)$

(e) $\nabla w = (w_1, w_2, 0)$, onde

$w_1 = \cos(3y)(-2\,\mathrm{sen}(2x)\,\mathrm{senh}(4x) + 4\cos(2x)\cosh(4x))$

$w_2 = -3\,\mathrm{senh}(3y)\cos(2x)\,\mathrm{senh}(4x)$.

(f) $\nabla w = (ye^z + yze^x, xe^y + ze^x, xye^z + ye^x)$

32) (a) $\dfrac{\partial f}{\partial x}(1,1) = 2$ e $\dfrac{\partial f}{\partial y}(1,1) = 1$

(b) Não.

33) $\mathbb{R}^2 \setminus \{(0,0)\}$

34) As derivadas parciais são contínuas.

35) Use o limite do erro.

36) $\dfrac{\partial F}{\partial t}(0,0,1) = 18$, $\dfrac{\partial F}{\partial u}(0,0,1) = -18$ e $\dfrac{\partial F}{\partial v}(0,0,1) = 54$.

37) $F'(t) = f_x(e^{t^2}, \mathrm{sen}\, t)2te^{t^2} + f_y(e^{t^2}, \mathrm{sen}\, t)\cos t$
$F'(0) = 5$

38) $g'(x) = f_x(x, x^3+2) + 3x^2 f_y(x, x^3+2)$

39) $\dfrac{\partial z}{\partial u} = 2u\dfrac{\partial f}{\partial x} + v\dfrac{\partial f}{\partial y}$ e $\dfrac{\partial z}{\partial v} = 2v\dfrac{\partial f}{\partial x} + u\dfrac{\partial f}{\partial y}$

40) Demonstração.

41) $\dfrac{\partial^2 F}{\partial\rho\partial\varphi}\left(1, \dfrac{\pi}{3}, \dfrac{\pi}{4}\right) = -\sqrt{6}e$.

42) Demonstração.

43) Aumentando.

44) $\dfrac{\partial F}{\partial u}(0,2) = 6$ e $\dfrac{\partial F}{\partial v}(0,2) = -11$.

45) Demonstração.

46) $\dfrac{5}{6}\sqrt{13}$

47) $\dfrac{\partial f}{\partial v}(3,1) = -\dfrac{11}{6}$
$r : x = 3 + 11t,\ y = 1 - 12t,\ z = \dfrac{\pi}{4} + 6t$.

48) (a) $\dfrac{\partial f}{\partial v}(P_0) = 1 + 2\sqrt{3}$

(b) $\dfrac{\partial f}{\partial v}(P_0) = 1$

(c) $\dfrac{\partial f}{\partial v}(P_0) = \sqrt{2}$

(d) $\dfrac{\partial f}{\partial v}(P_0) = 11 + \sqrt{2}$

(e) $\dfrac{\partial f}{\partial v}(P_0) = \dfrac{\sqrt{6}}{4}$

49) (a) $-2$

(b) $-\dfrac{9\sqrt{3}}{2}$

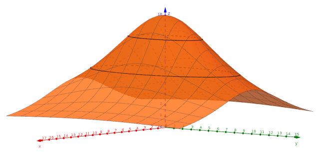
50) (a) Direção $\vec{v} = \nabla f(1,3) = \left(32\ln(2\pi), \dfrac{32}{3}\right)$ e valor $\|\vec{v}\|$.

(b) Direção $\vec{v} = \nabla w(1,0,1) = \vec{0}$ e valor $\|\vec{v}\| = 0$

51) Demonstração.

52) (a) $N_5 = \{(x,y) \in \mathbb{R}^2; x^2+y^2 = 64\}$,
$N_8 = \{(x,y) \in \mathbb{R}^2; x^2+y^2 = 16\}$
$N_{10} = \{(0,0)\}$

(b)



(c) $\vec{v} = \left(0, \dfrac{4}{5}\right)$

(d) $-\dfrac{4}{5}$

53) (a) $\dfrac{\partial f}{\partial v} = \dfrac{164}{2197}$

(b) $\vec{v} = \left(0, \dfrac{5}{2197}, \dfrac{12}{2197}\right)$

(c) $\vec{v} = \left(0, \dfrac{5}{2197}, \dfrac{12}{2197}\right)$

54) (a) Direção $-\vec{v} = \nabla f(4,9) = (16, 54)$

(b) $\vec{v} = (27, 8)$

55) $\vec{v} = (1,1)$

56) $-\dfrac{34}{\sqrt{6}}$

57) $\left(-\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$ ou $\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$

58) $8x + 2y - z = 9$

59) (a) $\pi_1 : x + y = 2$ e $\pi_2 : x + y = -2$

(b) $P = \left(\dfrac{3}{4}, 0, 0\right)$

60) (a) $\alpha : x + y - z = 1$
$r(t) = (2+t, -1+t, -t),\ t \in \mathbb{R}$

(b) $\alpha : x + z = 1$
$r(t) = (t, \pi, -t),\ t \in \mathbb{R}$

(c) $\alpha : 4x + 2y + z = 8$
$r(t) = (1 + 4t, 1 + 2t, 2 + t),\ t \in \mathbb{R}$

(d) $\alpha : 13x + 15y + z = -15$
$r(t) = (2 + 13t, -3 + 15t, 4 + t),\ t \in \mathbb{R}$

(e) $\alpha : \dfrac{x_0 x}{a^2} + \dfrac{y_0 y}{b^2} + \dfrac{z_0 z}{c^2} = 1$
$r(t) = \left(x_0 + \dfrac{x_0 t}{a^2}, y_0 + \dfrac{y_0 t}{b^2}, z_0 + \dfrac{z_0 t}{c^2}\right),\ t \in \mathbb{R}$

(f) $\alpha : \dfrac{x_0 x}{a^2} + \dfrac{y_0 y}{b^2} = 1$
$r(t) = \left(x_0 + \dfrac{x_0 t}{a^2}, y_0 + \dfrac{y_0 t}{b^2}, 0\right),\ t \in \mathbb{R}$

(g) $\alpha : 2x - y + 2z = 0$
$r(t) = (2t, 2 - t, 1 + 2t),\ t \in \mathbb{R}$

61) $x + 6y - 2z = 3.$

62) $\alpha_1 : 10x - 5y + 10z = 10\sqrt{10}$
$\alpha_2 : 10x - 5y + 10z = -10\sqrt{10}$

63) $x + y = -5$

64) Demonstração.

65) $y = 0$, $2y - \sqrt{2}x = 5\sqrt{2}$ e $2y + \sqrt{2}x = -5\sqrt{2}$

66) $x + y + z = \dfrac{11\sqrt{7}}{6\sqrt{11}}$ ou $x + y + z = -\dfrac{11\sqrt{7}}{6\sqrt{11}}$

67) $x - 2y + 2z = 7$ e $x + 2y + 2z = 7$

68) (a) $(3, 1, 2)$ (b) $x + y + z = 6$

69) (a) Máximo: $(-1, -3)$;
Mínimo: $(1, 3)$;
Sela: $(-1, 3)$ e $(1, -3)$

(b) Máximo: $(1, 1)$

(c) Máximo: $(-1, -1)$;
Mínimo: $(1, 1)$;
Sela: $(-1, 1)$ e $(1, -1)$

(d) Mínimo em pontos da forma $(x, x)$, $x \in \mathbb{R}$;

(e) Mínimo: $(0, 0)$

(f) Mínimo: $(-1, -1)$; Máximo: $\left(\frac{1}{2}, \frac{1}{2}\right)$

(g) Não tem

70) Temperatura mínima 0 ocorre em $(0, 0)$.
Temperatura máxima 67 ocorre em $(4, 3)$.

71) Mais quentes: $\left(-\dfrac{\sqrt{2+\sqrt{2}}}{2}, -\dfrac{\sqrt{2-\sqrt{2}}}{2}\right)$
$\left(\dfrac{\sqrt{2+\sqrt{2}}}{2}, \dfrac{\sqrt{2-\sqrt{2}}}{2}\right)$
Mais frios: $\left(-\dfrac{\sqrt{2+\sqrt{2}}}{2}, \dfrac{\sqrt{2-\sqrt{2}}}{2}\right)$
$\left(\dfrac{\sqrt{2+\sqrt{2}}}{2}, -\dfrac{\sqrt{2-\sqrt{2}}}{2}\right)$

72) Largura da base: $\dfrac{a}{3}$. Inclinação das faces: $\dfrac{\pi}{3}$.

73) Máximo: $\left(\dfrac{\sqrt{5}}{5}, \dfrac{3\sqrt{5}}{5}\right)$. Mínimo: $\left(-\dfrac{\sqrt{5}}{5}, -\dfrac{3\sqrt{3}}{5}\right)$

74) Máximo: $(0, 1)$. Mínimo: $\left(0, -\dfrac{1}{2}\right)$.

75) Máximos: $\left(-2\dfrac{\sqrt{5}}{5}, -\dfrac{\sqrt{5}}{5}\right)$ e $\left(2\dfrac{\sqrt{5}}{5}, \dfrac{\sqrt{5}}{5}\right)$
Mínimos: $\left(-\dfrac{\sqrt{5}}{5}, 2\dfrac{\sqrt{5}}{5}\right)$ e $\left(\dfrac{\sqrt{5}}{5}, -2\dfrac{\sqrt{5}}{5}\right)$

76) $\left(\dfrac{c}{3}, \dfrac{c}{2}\right)$

77) $x^2 + \dfrac{y^2}{4} + \dfrac{z^2}{9} = 1$

78) $r = \sqrt[3]{\dfrac{1}{\pi}}$ e $h = 2\sqrt[3]{\dfrac{1}{\pi}}$

79) $\dfrac{a}{3}$

80) Mais próximos: $\left(-2^{-6}, -2^{-6}\right)$, $\left(-2^{-6}, 2^{-6}\right)$, $\left(2^{-6}, -2^{-6}\right)$ e $\left(2^{-6}, 2^{-6}\right)$.
Mais afastados: $(-1, 0)$, $(1, 0)$, $(0, -1)$ e $(0, 1)$.

81) 6

82) $\dfrac{6}{11}$

83) $\dfrac{\sqrt{127}}{8}$

84) Comprimento e largura iguais à $\sqrt[3]{2V}$ e altura igual à $\dfrac{\sqrt[3]{2V}}{2}$.

85) O cubo de lado $\dfrac{a}{3}$.

86) $x = \dfrac{a}{3}$, $y = \dfrac{b}{3}$ e $z = \dfrac{c}{3}$.

87) $\dfrac{4\sqrt{3}}{27}$

88) $\left(\dfrac{7}{3}, \dfrac{1}{3}, -\dfrac{5}{3}\right)$

89) $\left(\dfrac{3}{7}, \dfrac{6}{7}, \dfrac{9}{7}\right)$

90) $3\sqrt[3]{k}$

91) $\left(\dfrac{\sqrt[3]{18V}}{3}, \dfrac{\sqrt[3]{18V}}{3}, \dfrac{\sqrt[3]{18V}}{2}\right)$