Betting in the Shadow of Match-Fixing*

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Abstract

Two bookmakers compete in Bertrand fashion while setting odds on the outcomes of a sporting contest where an influential punter (or betting syndicate) may bribe some player(s) to fix the contest. Zero profit and bribe prevention may not always hold together. When the influential punter is quite powerful, the bookies may coordinate on prices and earn positive profits for fear of letting the 'lemons' (i.e., the influential punter) in. On the other hand, sometimes the bookies make zero profits but also admit match-fixing. When match-fixing occurs, it often involves bribery of only the strong team. The theoretical analysis is intended to address the problem of growing incidence of betting related corruption in world sports including cricket, horse races, tennis, soccer, basketball, wrestling, snooker, etc. ***JEL* Classification:** D42, K42. **Key Words:** Sports betting, bookie, punters, corruption, match-fixing, lemons problem.

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1 Introduction

Match-fixing and gambling related corruption often grab news headlines. Almost any sport – horse races, tennis, soccer, cricket, to name a few – is susceptible to negative external influences.[1] Someone involved in betting on a specific sporting event may have access to player(s) and induce underperformance through bribery. In high visibility sports many unsuspecting punters, bookmakers and the general viewing public may therefore be defrauded in the process.[2]

We adapt the horse race betting models due to Shin (1991; 1992) to analyze match-fixing in team or individual sporting contests. In Shin (1991) a monopolist bookmaker sets odds on each one of two horses winning a race, whereas in Shin (1992) two bookmakers simultaneously set odds, as in Bertrand competition, in an n-horse race game ($n > 2$).[3] In both models, there is an *insider* who knows precisely which horse would win the race, while the remaining are noise punters with their different exogenous beliefs about the horses' winning probabilities that are uncorrelated with the true probabilities. The bookmaker(s) know only the true winning probabilities.[4]

[1]See "Race-fixing probed in Fallon trial" and similar reports at http://www.channel4.-com/news/articles/sports/racefixing+probed+in+fallon+trial+/894147. See also a BBC panorama on this subject (http://news.bbc.co.uk/1/hi/programmes/panorama/2290356.stm).

For tennis, see reports such as "Tennis chiefs battle match-fixers" and "ITF working with ATP, WTA and Grand Slam Committee to halt match-fixing in tennis" (http://news.bbc.co.uk/sport1/hi/tennis/7035003.stm ; http://www.signonsandiego.com/sports/20071009-0552-ten-tennis-gambling.html).

In March 2009, Uefa president Michel Platini publicly issues the following warning: "There is a grave danger in the world of football and that is match-fixing." Uefa general secretary says, "We are setting up this betting fraud detection system across Europe to include 27,000 matches in the first and second division in each national association." See http://news.bbc.co.uk/sport2/hi/football/europe/7964790.stm. Latest, police carried out 50 raids in Germany, the UK, Switzerland and Austria: "Prosecutors believe a 200-strong criminal gang has bribed players, coaches, referees and officials to fix games and then made money by betting on the results." See the report (dated 20 November, 2009), http://news.bbc.co.uk/2/hi/8370748.stm. See Appendix for a copy of the report.

For match-fixing in cricket, see http://news.bbc.co.uk/sport1/low/cricket/719743.stm. For basketball, Wolfers (2006) estimated that nearly 1 percent of all games in NCAA Division one basketball (about 500 games between 1989 and 2005) involved gambling related corruption. A striking account of match-rigging in Sumo wrestling in Japan appears in Duggan and Levitt (2002). And at the time of revising this draft, snooker got tainted with the revelation of betting related bribery (see http://news.bbc.co.uk/2/hi/uk_news/8656637.stm).

[2]Corruption in sports has been only occasionally highlighted by economists without the consideration of its causal relationship to betting – see Duggan and Levitt (2002), and Preston and Szymanski (2003). Wolfers (2006), Winter and Kukuk (2008), and Strumpf (2003) are few exceptions.

[3]To be precise, Shin's (1992) price-setting game is slightly different from one-shot Bertrand game: the bookmakers first submit bids specifying a maximum combined price for bets on all the horses, the low bidder wins and then sets prices for individual bets so that the total for all bets combined does not exceed the winning bid.

[4]In an empirical framework, Shin (1993) provides estimates for the incidence of insider trading in UK betting markets.

Rather than assuming an insider who knows before betting the identity of the winner (as in Shin's models), we consider the prospect of a gambler *influencing* the contestants' winning odds through bribery and match-fixing. Ex ante (before bribing), this gambler, to be called the 'influential punter', is no better informed than the bookmakers and is less privileged than Shin's insider. However, different from Shin's framework, the influential punter may become better informed than the bookmakers through his secret dealings with one of the contestants, if chance presents it and bookmakers' odds make it worthwhile. Thus, we shift the focus from the use of insider *information* (i.e. pure adverse selection) to manipulative *action* that generates inside information for the influential gambler. As actions are choices, our bookmakers can control these by appropriately setting their odds – a possibility absent in models of pure adverse selection such as Shin's.

Moreover, there is an issue of legality. While betting on the basis of inside information may not be illegal, match-fixing through bribery clearly is. However, the dominant focus of the theoretical literature on betting so far (including Shin's works) has been to explain the empirical regularity of the favorite-longshot bias in race-track betting.[5] We raise concern about the unfairness of contests due to match-fixing not only for the sake of unsuspecting bettors but also for the viewing public and the media that rely on the public's interest in sports. How the threat of bribery and manipulation influences betting odds and the eventual occurrence of match-fixing thus form the main subject of our interest.[6]

We focus on the bookmakers' odds setting behavior in the shadow of match-fixing under Bertrand competition. We ask: when is match-fixing a serious threat, and when it is a threat how bookmakers combat or even perversely trigger match-fixing. We are going to argue that competition does not necessarily yield zero profits, nor does it guarantee fair play.

Our model involves two bookmakers (or bookies) and two types of punters (ordinary/naive and influential). The bookies set fixed odds and the punters place bets on the outcome of a sporting contest between two teams (or contestants).[7] The influential punter (or equivalently a large betting syndicate), who shares the same beliefs as the bookies (and the players) about the teams' winning chances, may be able to gain access to some members in one of the teams and bribe them to sabotage the team.[8] When bribing a team, the influential

[5] Ottaviani and Sorensen (2008) is a detailed survey of alternative explanations.

[6] In a parimutuel market setting, Winter and Kukuk (2008) allow a participant jockey to underperform, but they do not consider enforcement: the cheating jokey does not face the prospect of being found out for the deliberate underperformance. Winter-Kukuk model is not thus adequately rich to analyze cheating incentives formally in the style of standard cheating/punishment models.

[7] Fixed-odds betting, as opposed to parimutuel betting, is a more relevant format for analysis of match-fixing where bookies play a significant role without direct involvement in the act of bribery and/or placement of surrogate bets. For a contrast between how odds are set (or determined) in these two betting markets (but without the issues of match-fixing), see Ottaviani and Sorensen (2005).

[8] In contests involving rival firms or lobbies, sabotage is a well-studied theme; see, for instance, Konrad

punter would place a bet on the other team. The anti-corruption authority may investigate the losing team and punish the match-fixing punter and the corrupt player(s) whenever it catches them.[9]

With the threat of match-fixing looming, in selecting odds the bookies take into account both the benefit and the danger of undercutting each other. When the influential punter cannot place too large a bet, the following results occur. If, *ex ante*, teams are relatively more even, competition yields zero expected profits for the bookies without attracting the risk of bribery and match-fixing: prices of both the tickets corresponding to fair odds tend to be too high for the influential punter to bribe and bet.[10] But if (*ex ante*) teams are more uneven, Bertrand competition cannot guarantee elimination of bribery; the bookies make zero profits and the influential punter earns rent. Match-fixing will occur with positive probability, and it can be attributed to *opportunism*. If undercutting triggers match-fixing, its adverse impact (loss) is shared by both bookies, but if match-fixing is not triggered then the gain is exclusive (positive profit). Whenever match-fixing occurs, it involves bribery of only the strong team.

On the other hand, when the influential punter can place a significantly large bet, the adverse impact of match-fixing could be so severe that undercutting becomes very risky. In particular, in contests that are *ex ante* nearly even, the bookies will coordinate on prices strictly above fair odds and sustain, non-cooperatively, positive profits and prevent bribery. Positive profits seem to go against common wisdoms of competition. Here, the *fear* of triggering (the 'lemons' of) match-fixing forces the bookies to coordinate on prices. Ironically, without the corrupting influential punter the bookies would compete away profits. There is also another possibility that the bookies set prices inducing bribery of either team and make zero expected profits. With this latter equilibrium, the chance of match-fixing remains rather high (as the influential punter will bribe whenever he has an access to a team) and ticket prices are set above the respective teams' uncorrupted winning odds to make up for the potential loss to the match-fixing punter. It is difficult to cleanly predict, though, which of the two equilibria – positive profit bribe prevention or zero profit match-fixing – is likely to happen.

Before we proceed to detailed analysis, we would like to note that in practice bookmakers are well aware of the potential risks of the influential punter's involvement and as a precaution

(2000). Our sports contest model is much simpler than the 'effort contest' games (such as the one analyzed by Konrad) in that we assume exogenous winning probabilities of the contestants due to their inherent skills (or characteristics), and sabotage is a deliberate underperformance relative to one's own skills.

[9] The law enforcement is one of *investigation* rather than *monitoring* (Mookherjee and Png, 1992).

[10] In Shin (1991; 1992), prices exaggerate the odds.

they may limit the size of trades at posted prices.[11] Even more, the bookmakers may set new odds seeing the increasing volume of bets being placed on a particular outcome so that the influential punter may face a quantity-price trade-off. Further, odds revisions may generate and disseminate new information even among the ordinary punters leading to an erosion of the value of insider information, similar to the market micro structure literature (Glosten and Milgrom, 1985; Kyle, 1985). While our model does not incorporate these features employed in models of financial economics, we do not see the basic insights of our analysis changing qualitatively even if a more sophisticated and much more complex model were formulated. We also assume exogenous investigation probabilities and fines by the prosecution authorities, to keep the analysis tractable. Nor do we model the role of sports bodies that may regulate the betting market in large to prevent cheating. These considerations are important no doubt, but beyond the scope of the present work.

In section 2 we present the model, followed by an analysis of the betting and bribing decisions in section 3. In section 4 we analyze the Bertrand duopoly competition. Section 5 concludes. The formal proofs appear in the Appendix, and a separate Supplementary material reports an extra derivation.

2 The Model

There are two bookmakers, called the bookies, who set the odds on each of two teams winning a competitive sports match (equivalently, set the prices of two tickets); the match being drawn is not a possibility. Ticket i with price π_i yields a dollar whenever team i wins the contest and yields nothing if team i loses, with $0 \leq \pi_1, \pi_2 \leq 1$. To keep the notations simple, the bookie indices will be omitted from the prices.

There are a continuum of naive punters, to be described as *punters* or sometimes *ordinary punters*, parameterized by individual belief (i.e., the probability) q that team 1 will win ($1-q$ is the probability that team 2 will win); q is distributed 'uniformly' over (0, 1). Ordinary punters stubbornly stick to their beliefs.

There is also a knowledgeable and potentially corrupt/influential punter, to be referred as punter I, who may influence a team's winning chances by bribing its corruptible players to underperform. Punter I gains access to team i with probability $0 \leq \mu_i \leq 1$; with probability $1 - \mu_1 - \mu_2$, he fails to gain any access. At best, punter I can access only one team. The bookies and the prosecution authority know only (μ_1, μ_2).

[11] This may be difficult to implement, however, as any corrupt betting syndicate may have multiple punters on its team.

The distribution of ordinary punters' wealth is 'uniform' over [0, 1], with a collective wealth of y dollars; the wealth of punter I is $z = 1 - y$ dollars.

In the absence of any external influence, the probability that team 1 will win is $0 < p_1 < 1$ and the corresponding probability for team 2 is $p_2 = 1 - p_1$. The bookies, punter I, and the players – all initially observe the draw p_1.[12] The prosecution need not observe p_1, or even when the prosecution observes p_1 it does not employ sophisticated game-theoretic inferences whether match-fixing has occurred or not based on the betting odds and p_1.

The prosecution authority (or ACU, anti-corruption unit) investigates team i only when team i loses the contest. Assume that the probability of investigation of team i, $0 < \alpha_i < 1$, is known to all, and the investigation detects bribery, if any, with probability one. The investigation probability may differ across teams.[13] Given our focus on the bookies' pricing strategies, we take the prosecution to be non-strategic rule-book follower.

On conviction, the corrupt player (or players) will be imposed a total fine $0 < f \leq \bar{f}$ and punter I is imposed a fine $0 < f_I \leq \bar{f}$.[14]

When punter I gets access to team i, by making a bribe promise of b_i *conditional on team i losing* he can lower the probability of team i winning from the *true* probability p_i to $\lambda_i p_i$, where $0 \leq \lambda_i < 1$, provided the corrupt players of team i cooperate with punter I in undermining the team performance. λ_i depends on the susceptibility to corruption and bribery of team i's members, i.e., whether a small or a significant section of the team takes part in undermining the team cause. Also, the particular player (or players) to whom punter I is likely to have an access may be of varied importance to the team's overall performance. We take λ_i to be exogenous and common knowledge.

The bookies, two types of punters and the corruptible team members – all are assumed to be risk-neutral and maximize their respective expected profits/payoffs. Define the 'betting and bribery' game, Γ, as follows:

Stage 1. Nature draws p_1 and reveals it to the bookies, punter I and the players; the ordinary punters draw their respective private signals q. Then the bookies simultaneously announce the prices (π_1, π_2).

Stage 2. Punter I learns about his access to team 1 or team 2 or neither,[15] and subsequently decides whether to bribe the team or not (in the event of gaining access).

[12] Levitt (2004) recognizes that bookmakers are usually more skilled at predicting match outcomes than ordinary punters. In any case, without such confidence in abilities the bookies won't be in the business.

[13] This difference could be due to the teams' different susceptibility to corruption.

[14] The finding of bribery is assumed to reveal the identity of punter I.

[15] The timing of the influential punter's access to teams (after or before the odds are posted) is not going to matter. What is important is that the bookies do not know whether, or to which team, the influential punter will have an access.

Stage 3. The ordinary punters as well as punter I place bets according to their 'eventual' beliefs. When the bookies charge the same price for a given ticket, market is evenly shared, and when they charge unequal prices, lower price captures the whole market. The match is played out according to winning probabilities $(p_1, 1 - p_1)$ or $(\lambda p_1, 1 - \lambda p_1)$ (where team 1 is bribed), or $(1 - \lambda p_2, \lambda p_2)$ (where team 2 is bribed) and the outcome of the match is determined.

Stage 4. Finally, the ACU follows its investigation policy, (α_1, α_2). On successful investigation, fines are imposed on the corrupt player(s) and punter I. $||$

See also Fig. 1.

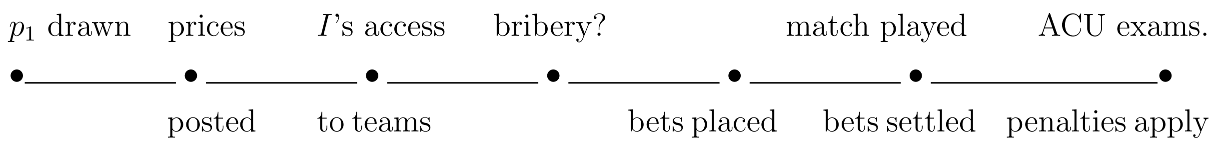

Figure 1: Time line

Thus the bookies move simultaneously in stage 1, then punter I decides on bribery in stage 2 followed by betting in stage 3 and finally the prosecution moves, defining the extensive form. Simultaneous moves (in stage 1) and punter I's betting based on privately held beliefs about the teams' winning odds make the game an imperfect information game between the two bookies and punter I. So we will solve for the **subgame perfect equilibrium** (SPE) strategies in prices, bribery and betting.

3 Betting and Bribing Decisions

3.1 Ordinary Punters' Betting Decision

The ordinary punters adopt the following betting rule:[16]
If $q \geq \pi_1$ but $1 - q < \pi_2$, bet on team 1;
If $1 - q \geq \pi_2$ but $q < \pi_1$, bet on team 2;
If $q \geq \pi_1$ and $1 - q \geq \pi_2$, then bet on team 1 if $\frac{q}{\pi_1} \geq \frac{1-q}{\pi_2}$ and bet on team 2 if $\frac{q}{\pi_1} \leq \frac{1-q}{\pi_2}$;
If $q < \pi_1$ and $1 - q < \pi_2$, do not bet on either team.

3.2 Player Incentives for Bribe-taking and Sabotage

Given the prosecution's investigation strategy, let us consider the incentives of players to accept bribes. In a team context, the incentives concern the corruptible member(s) of a

[16] This is same as the betting rule by the *Outsiders* in Shin (1991).

team. We assume a single corruptible member in each team; the analysis applies equally to a consortium of corruptible members. Suppose the corruptible player of team i (with whom punter I establishes contact) gets the reward w in the event team i wins, and receives nothing if team i loses.[17] Given any belief p_i, a bribe b_i is accepted and honored by the corruptible player by underperforming

$$\begin{aligned} &\text{if and only if} \quad (\lambda_i p_i)w + (1-\lambda_i p_i)(b_i - \alpha_i f) \geq p_i w + (1-p_i)(b_i - \alpha_i f) \\ \text{i.e.,} \quad &\text{if and only if} \quad b_i \geq w + \alpha_i f. \end{aligned} \tag{1}$$

A player can renege on his promise to underperform even after entering into an agreement with punter I. The right-hand side of (1) recognizes this possibility. A player will be penalized for taking bribes, even if he might not have deliberately underperformed.[18]

The minimum bribe required to induce the corruptible player to accept the bait is $\underline{b}_i = w + \alpha_i f$. That is, the *reservation bribe* covers the loss of the prize w and the expected penalty. We assume that punter I holds all the bargaining power so that $b_i = \underline{b}_i$.[19]

3.3 Influential Punter's Betting and Bribing Incentives

First consider the betting incentives. Having learnt the true probabilities p_i and observed the prices π_i $(i = 1, 2)$, if punter I fails to contact either team or decides not to bribe,

$$\begin{aligned} &\text{he will bet } z \text{ on team } i \text{ if} \quad \frac{p_i}{\pi_i} \geq max\{1, \frac{p_j}{\pi_j}\}, \quad i \neq j,^{20} \\ &\text{and will bet on neither if} \quad \frac{p_i}{\pi_i} < 1, \quad i = 1, 2. \end{aligned}$$

The expected profit to punter I from betting exclusively on team i is $E\Pi^I_{0i} = p_i \frac{z}{\pi_i} - z, \quad i = 1, 2$, and zero when he bets on neither.[21] His expected profit from not bribing is $\max\{E\Pi^I_{0i}, E\Pi^I_{0j}, 0\}$.

If, however, punter I contacts a corruptible member of team i, offers him a bribe and places a bet on team j, his expected profit equals: $E\Pi^I(b_i) = (1-\lambda_i p_i)\left[\frac{z}{\pi_j} - \underline{b}_i - \alpha_i f_I\right] - z$.

[17] The prize w includes both direct and indirect rewards, with the latter in the form of lucrative endorsement opportunities for commercials. The player may additionally receive unconditional retainer wage/appearance fee that does not affect the player's bribe-taking incentives. In the case of a two-player contest such as tennis, the index i will refer to the player, with w_i reflecting the particular player's reputation/stake.

[18] To prove that a player has deliberately underperformed is very difficult. On the other hand, bribery can be established based on hard evidence.

[19] Our analysis can be easily extended to bargaining over bribe.

[20] When $\frac{p_i}{\pi_i} = \frac{p_j}{\pi_j} \geq 1$, the punter is indifferent between two teams.

[21] Betting on both teams yields the same profit as exclusive betting, given the betting rule specified above.

Substituting $\underline{b}_i = w + \alpha_i f$,

$$E\Pi^I(b_i) = (1-\lambda_i p_i)z\Big[\frac{1}{\pi_j} - \Omega_i\Big] - z = (1-\lambda_i p_i)z\Big[\frac{1}{\pi_j} - \frac{1}{\phi_i}\Big],$$

$$\text{where} \qquad \Omega_i = \frac{w + \alpha_i(f + f_I)}{z}, \qquad \text{and} \qquad \phi_i = \frac{1-\lambda_i p_i}{1 + (1-\lambda_i p_i)\Omega_i}.$$

Clearly, if $E\Pi^I(b_i) > 0$ and greater than the profit from 'not bribing', he will bribe (upon access). But there are several situations of indifference, for which we impose two tie-breaking rules:

Assumption 1. (**Tie-breaking rule I**) *If 'bribing and betting' and 'betting without bribing' yield identical and positive expected profits for the influential punter, then he will choose bribing and betting.*

(**Tie-breaking rule II**) *If 'bribing and betting' and 'betting without bribing' yield zero expected profits for the influential punter, then he will not bet at all.*

The first rule would bring to bear the full impact of the (negative) influence. Any adverse consequence of bribing for punter I, such as getting caught leading to jail and banning from sports betting etc., is captured by the penalty term f_I. So leaning towards betting and bribing to break the indifference should be reasonable. Tie-breaker-II is to ensure that the bribe prevention prices are well-defined.

To analyze various players' decisions we impose the following assumption. The assumption is based on sound economic principles.

Assumption 2. (***Dutch-book restriction***) *The bookies must always choose prices $0 \le \pi_1, \pi_2 \le 1$, both on- and off-the-equilibrium path, such that $\pi_1 + \pi_2 \ge 1$.*

The Dutch-book restriction (or rather the absence of the Dutch book) can be defended as follows. If instead $\pi_1 + \pi_2 < 1$, it gives rise to the "money pump" scenario implying someone who otherwise might not have bet on the sporting event (for reasons of risk aversion and the likes) can make free money by spending less than a dollar to earn a dollar for sure. It is also conceivable that if one of the bookies violates the Dutch-book restriction, the other bookie can bet large sums of money and drive his competitor out of business. It is reasonable to assume that the bookies will have reserves of funds to engage in this predatory behavior.

We now consider three scenarios relevant for punter I's bribery decisions.

Bribe prevention: Suppose $\pi_i \geq p_i$, $i = 1, 2$ (such that $\max\{E\Pi^I_{0i}, E\Pi^I_{0j}, 0\} = 0$). Then punter I does not bribe team i, if and only if $E\Pi^I(b_i) \leq 0$, i.e.,

$$\pi_j \geq \frac{1 - \lambda_i p_i}{1 + (1 - \lambda_i p_i)\Omega_i} \equiv \phi_i. \tag{2}$$

Tie-breaker-II applies when $E\Pi^I(b_i) = 0$.

Bet reversal: Alternatively, suppose $\pi_i < p_i$ (and $\pi_j > p_j$, by Assumption 2) such that $\max\{E\Pi^I_{0i}, E\Pi^I_{0j}, 0\} = z(\frac{p_i}{\pi_i} - 1) > 0$. Then punter I bribes team i and bets on team j (as opposed to betting on team i), if and only if $E\Pi^I(b_i) \geq z(\frac{p_i}{\pi_i} - 1)$, i.e.,

$$\pi_j \leq \frac{(1 - \lambda_i p_i)\pi_i}{p_i + (1 - \lambda_i p_i)\Omega_i \pi_i} \equiv \psi_i(\pi_i). \tag{3}$$

Tie-breaker-I applies when $E\Pi^I(b_i) = z(\frac{p_i}{\pi_i} - 1)$.

Bet accentuation: Continuing with the assumption that $\pi_i < p_i$ (and $\pi_j > p_j$) such that $\max\{E\Pi^I_{0i}, E\Pi^I_{0j}, 0\} = z(\frac{p_i}{\pi_i} - 1) > 0$, punter I bribes team j and bets on team i if and only if $E\Pi^I(b_j) \geq z(\frac{p_i}{\pi_i} - 1)$, i.e.,

$$\pi_i \leq \frac{(1 - \lambda_j)p_j}{(1 - \lambda_j p_j)\Omega_j} \equiv h_j. \tag{4}$$

Tie-breaker-II applies when $E\Pi^I(b_j) = z(\frac{p_i}{\pi_i} - 1)$.

Interpretations: Condition (2), the **bribe prevention condition**, says that by setting the price of ticket j high enough, team i can be protected from match-fixing, and by doing so for both tickets punter I can be altogether kept out of the market. Indeed, that will be the outcome in any equilibrium featuring bribe prevention. If punter I does not bribe, but bets on team i, he must earn strictly positive profit. This is possible if and only if $\pi_i < p_i$, which is clearly loss-making for the bookies. Thus, if bribery is prevented, punter I will not participate at all.

Condition (3) is the condition for **bribe inducement of team** i, when team i is otherwise attractive to bet on. Essentially by reducing the price of ticket j below a threshold level, the betting incentive of punter I is reversed (hence the term, *bet reversal*). The threshold level will evidently depend on the price of ticket i. In particular, $\psi_i < \phi_i$ for $\pi_i < p_i$; if $\pi_i = p_i$ then $\phi_i = \psi_i$.

How the threshold price for bet reversal compares with the bribe prevention price threshold can be seen in Fig. 2. Here, given $\pi_2 < p_2$ and $\pi_1 > p_1$, team 2 is bribed for $\pi_1 < \Psi_2(\pi_2)$.

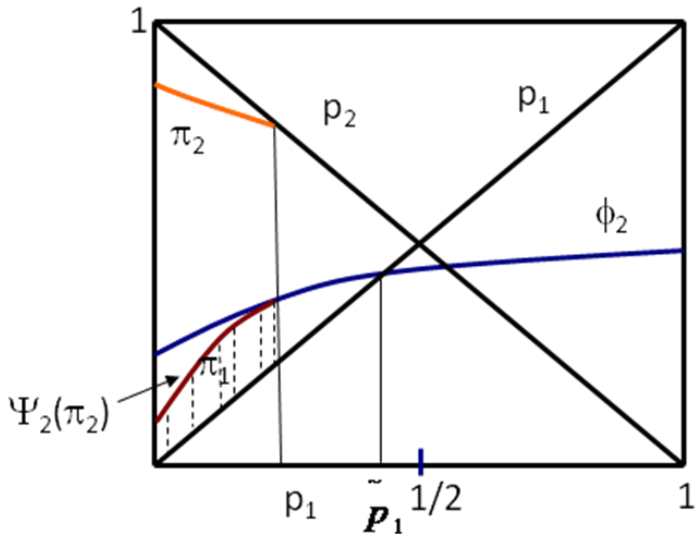

Figure 2: **Bet reversal**

Finally, (4) is the condition for **bribe inducement of team** j, when prima facie team i is attractive to bet on. Here by reducing the price ticket of i below a threshold level (so that bribery can by financed from the potential gains), punter I's incentive to bet on team i is further strengthened by prompting him to bribe team j (*bet accentuation*).

An implication of the tie-breaking rule II, which is also evident in the bribery conditions (3) and (4), is that punter I will bet only if his expected profit is strictly positive. This also means:

Fact 1. *When punter I places a bet, the bookies' expected profits from any potential trade with punter I must be negative.*

4 Bertrand Competition in Bookmaking

In this section, our principal observations will be on two important issues concerning the effects of competition. First, a basic fact of (Bertrand) competition is that firms earn zero profits. But here the profitability of trades by bookmakers – buying and selling of bets – depend on the type of trades. The prices may endogenously lead to match-fixing and informed trading by select punters. So whether price competition will lead to *zero profits* or not cannot be answered independently of the related *match-fixing/corruption* implications: does competition in the betting market imply a corruption-free play of the sports contest? While under certain conditions competition ensures zero profits (to the bookies) *and* prevention of bribery and match-fixing (Proposition 1), either of these two results may fail to obtain in isolation (Propositions 3 and 4) under complementary conditions, that is, bribery/match-fixing may be triggered with positive probability or firms may make positive expected profits. Moreover, for the scenarios that we study positive expected profits and bribery/match-fixing do not occur at the same time. In the remainder of this section, we analyze these possibilities.

Before we say what might happen in equilibrium, we can say the following.

Lemma 1. *There cannot be an equilibrium (featuring bribe inducement or bribe prevention) in which $\pi_1 + \pi_2 < 1$, where (π_1, π_2) are the minimal of two sets of prices charged by the two bookies.*

It can be readily seen that if $\pi_1 + \pi_2 < 1$ were to hold in equilibrium, then it must be the case that either each ticket or at least one ticket is underpriced relative to its corrupted or uncorrupted probability of winning. Any underpriced ticket must be loss-making, and one of the bookies can always profitably deviate by raising the price. Hence, $\pi_1 + \pi_2 < 1$ cannot arise in equilibrium.

4.1 Bribe Prevention with Zero Profit

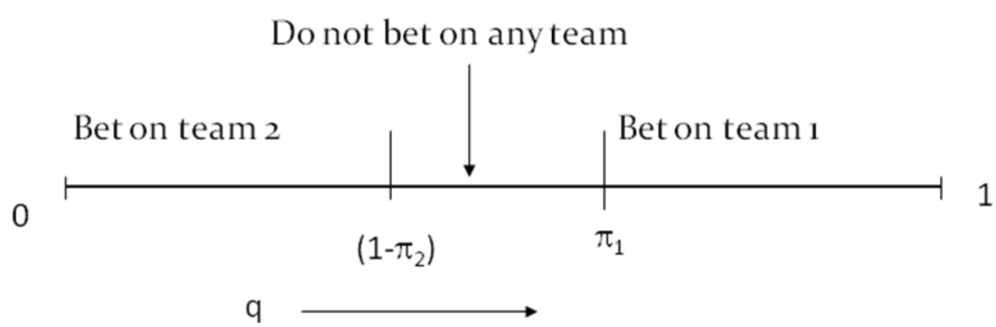

Figure 3: **Ordinary punter's betting rule**

Let us first determine the prices at which expected profit is zero and bribery is prevented. Focusing on identical prices (and therefore suppressing bookie indices for notational simplicity), consider the posting of (π_1, π_2) in the region of $1 \leq \pi_1 + \pi_2 \leq 2$. An ordinary punter's optimal betting rule is indicated in Fig. 3. So the bookie's objective function can be written as:

$$E\Pi^d_{BP} = \frac{y}{2}\Big[\int_{\pi_1}^{1}(1-\frac{p_1}{\pi_1})\,\mathrm{d}q\Big] + \frac{y}{2}\Big[\int_{0}^{1-\pi_2}(1-\frac{p_2}{\pi_2})\,\mathrm{d}q\Big] = y\big[3-\pi_1-\pi_2-\frac{p_1}{\pi_1}-\frac{p_2}{\pi_2}\big].$$

The bribe prevention constraints are: $\pi_1 \geq \max\{p_1, \phi_2(p_1)\}$, $\pi_2 \geq \max\{p_2, \phi_1(p_1)\}$.

From the objective function one might expect that competition in each market should induce $\pi_1 = p_1$ and $\pi_2 = p_2$ (i.e., prices equal the true probabilities of winning) leading to $E\Pi^d_{BP} = 0$. But to ensure such an outcome, the prices must also prevent bribery. To analyze the possibility of such an equilibrium, let us introduce two critical probabilities (refer Fig. 4):

Definition 1. *Let $\tilde{p}_1$ be the unique p_1 such that $\phi_2(p_1) = p_1$, and $\hat{p}_1$ be the unique p_1 such that $\phi_1(p_1) = p_2$.*

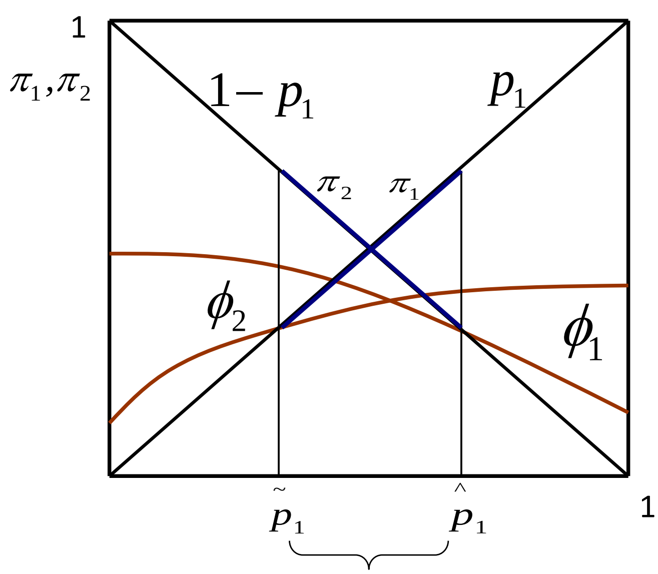

Figure 4: **Region of bribe prevention zero-profit equilibrium**

Further, $\tilde{p}_1$ and $\hat{p}_1$ can be calculated as follows :

$$\begin{aligned}
\tilde{p}_1 &= \frac{1}{2}\Big[\frac{(1-\lambda_2)}{\lambda_2}\frac{1+\Omega_2}{\Omega_2}\Big]\Big[\sqrt{1+\frac{\lambda_2}{1-\lambda_2}\frac{4\Omega_2}{(1+\Omega_2)^2}}-1\Big], \\
\hat{p}_1 &= 1-\frac{1}{2}\Big[\frac{(1-\lambda_1)}{\lambda_1}\frac{1+\Omega_1}{\Omega_1}\Big]\Big[\sqrt{1+\frac{\lambda_1}{1-\lambda_1}\frac{4\Omega_1}{(1+\Omega_1)^2}}-1\Big].
\end{aligned}$$

It can be readily checked that $\phi_2'(p_1) > 0$, $\phi_2''(p_1) < 0$ with $\phi_2(0) > 0$ and $\phi_2(1) < 1$, as shown in Fig. 4. Therefore, a unique $\tilde{p}_1$ must exist and is in $(0, 1)$. It then follows that at all $p_1 < \tilde{p}_1$, $\phi_2(p_1) > p_1$, and at all $p_1 > \tilde{p}_1$, $\phi_2(p_1) < p_1$. Similarly, $\phi_1'(p_1) < 0$, $\phi''(p_1) < 0$ with $\phi_1(0) > 0$, $\phi_1(1) < 1$. Therefore, $\hat{p}_1$ also exists and it is unique. Further, at all $p_1 > \hat{p}_1$, $\phi_1(p_1) > p_2$, and at all $p_1 < \hat{p}_1$, $\phi_1(p_1) < p_2$.

If Ω_i is large enough, which requires z to be small relative to $w + \alpha_i(f + f_I)$, $\tilde{p}_1$ will be smaller than $\hat{p}_1$. In other words, the influential punter should not be 'too powerful' (in terms of wealth); in Fig. 4, as z becomes large, both ϕ_2 and ϕ_1 curves shift upwards, pushing $\tilde{p}_1$ to the right and $\hat{p}_1$ to the left, thus shrinking and even eliminating the $(\tilde{p}_1, \hat{p}_1)$ interval. Until specified otherwise, we will assume:

Assumption 3. (***Not too powerful punter*** *I*) *Let* $\Omega_1 > \frac{2(1-\lambda_1)}{2-\lambda_1}$ *and* $\Omega_2 > \frac{2(1-\lambda_2)}{2-\lambda_2}$ *so that* $\tilde{p}_1 < \frac{1}{2} < \hat{p}_1$.

Fig. 4 is drawn on the basis of Assumption 3. Clearly, over the interval $[\tilde{p}_1, \hat{p}_1]$ the zero-profit prices $\pi_i = p_i$ $(i = 1, 2)$ prevent bribery, and these can be sustained as Bertrand equilibrium by applying the usual logic: unilateral price increase(s) by a bookie do not improve profits, and any price reduction(s) inflict losses (in addition to violating the Dutch-book constraint). The closeness of the contest means if prices are set according to fair odds, both the ticket prices tend to be rather high for the influential punter to bribe one team, bet on the other team and make a profit; hence match-fixing is prevented.

But outside $[\tilde{p}_1, \hat{p}_1]$ we cannot have bribe prevention, along with zero profits, in equilibrium – a direct implication of the constructed interval $[\tilde{p}_1, \hat{p}_1]$. With the contest sufficiently unbalanced, setting prices according to fair odds would mean the weak team's price is rather low so that the influential punter can bribe the strong team, if he has got access, and bet on the weak team. For such prices (reflecting fair odds), significant majority of ordinary punters will also bet on the weak team (ordinary punters' beliefs are uniformly distributed), so when the strong team loses (the probability of which will increase if bribery is successful) the bookies will make losses. This makes simultaneous attainment of zero expected profits for the bookies and bribe prevention untenable.

So now the question is, outside the interval $[\tilde{p}_1, \hat{p}_1]$ can bribery be prevented with certainty while profit remaining positive? The answer is 'no'. Below we provide detailed reasoning; a formal short proof appears in the Appendix.

If bribe prevention with positive profit were to be an equilibrium for sufficiently unbalanced contests, then we must have one of the following three possibilities: (i) both tickets are generating profit; (ii) only one ticket is generating profit, while the other ticket is generating loss, but the overall profit is positive; and (iii) only one ticket yields positive profit, while the other ticket yields zero profit. In all three cases, given Assumption 3 and also as is evident from Fig. 4, only one team, i.e. the weak team, needs to be protected from the influential punter's betting by raising its price well above its uncorrupted probability of win, and thus yielding positive profits for the bookmakers. For instance in the region $[0, \tilde{p}_1)$, ticket 1 needs to be protected. For the other ticket (namely ticket 2 when $p_1 < \tilde{p}_1$), competition will wither away profit. Therefore, possibility (i) is ruled out. Possibility (ii) is also ruled out, because one can raise the price of the loss making ticket and lose the market altogether.

So, we are left with only possibility (iii). For the sake of concreteness consider $p_1 < \tilde{p}_1$. Here as argued above, the profit generating ticket must be ticket 1, due to the bribe prevention constraint $\pi_1 \geq \phi_2$ (recall (2)). But it can be easily seen that competition will force the constraint to bind. Thus, we will have $\pi_1 = \phi_2 > p_1$. For ticket 2 we have $\pi_2 = p_2$. Now, from this proposed equilibrium, we argue, *both* tickets can be undercut without increasing the prospect of bribery and profit will improve. To see that, suppose one of the bookies reduces π_2 slightly below p_2 and takes a small loss on ticket 2. But simultaneously he makes the bribe prevention constraint $\pi_1 = \phi_2$ irrelevant. As punter I now can gainfully bet on ticket 2 without committing bribery, the new bribe prevention constraint should be $\pi_1 > \psi_2$ (recall (3) and tie-breaking rule I). As can be checked from (2) and (3), $\psi_2 < \phi_2$ as long as $\pi_2 < p_2$. Therefore π_1 can be suitably reduced to π_1' (in accordance with the reduction in π_2 below p_2) such that $\psi_2 < \pi_1' < \phi_2$. Thus, bribery is still prevented and the bookie fully captures both markets. As long as price reductions are of small order, loss in ticket 2 will be compensated by gains from ticket 1: starting from zero profit earned on ticket 2 a slight reduction in π_2 from p_2 will involve a small loss (with bribe prevention maintained), even after having to serve almost the double the volume of ordinary punters prior to the price decrease; on the other hand, starting from a positive profit earned on ticket 1, because the required reduction in π_1 from ϕ_2 (following the reduction in π_2 from p_2) will be small (recall, from (3), $\psi_2(\pi_2)$ varies continuously in π_2, and $\psi_2 = \phi_2$ whenever $\pi_2 = p_2$), the new profit from ticket 1 is almost twice as large. Hence, possibility (iii) is also ruled out.

Proposition 1 (Bribe prevention). *Suppose Assumptions 2 and 3 hold.*

(i) For $p_1 \in [\tilde{p}_1, \hat{p}_1]$, the unique and symmetric equilibrium under Bertrand competition is $\pi_1 = p_1$ and $\pi_2 = p_2$, such that bribery is prevented surely and each bookie earns zero expected profit.

(ii) For p_1 outside the interval $[\tilde{p}_1, \hat{p}_1]$, there is no pure strategy equilibrium under Bertrand competition in which bribery is prevented with probability one.

Thus, it is possible that the influential punter will not bribe so that match-fixing is not a threat and Bertrand competition leads to zero profits with prices of bets equalling fair odds. In contrast, in Shin (1992; Proposition 1) the presence of an insider meant distortion in the prices of bets away from fair odds, in particular, prices reflected the favorite-longshot bias. The main reason for the difference in our result is that, different from Shin (1992) and similar other analysis of competitive bookmaking (e.g., Ottaviani and Sorensen, 2005), our bookmakers are as well informed as the influential punter (i.e., the insider) when match-fixing is deterred. The basis for price distortion is thus removed.

4.2 Bribe Inducement with Zero Profit

Outside $[\tilde{p}_1, \hat{p}_1]$, we will look for a (pure strategy pricing) equilibrium in which bribery occurs with positive probability. Also, we will be interested in the qualitative equilibrium properties characterizing bribery. Studying when bribery actually occurs, and not just talk about the implications of its potential threat, should be highly relevant in the context of growing incidence of betting related match-fixing in sports as we have cited in the Introduction.

Let us start by asking: *Which team will be bribed?* The following result will be useful in analyzing the bribery prospect of the longshot.

Lemma 2. *Suppose Assumption 3 holds (i.e., the influential punter is not too powerful). Then at all $p_1 \leq \tilde{p}_1$ it must be that $h_1 < p_1$, where $h_1 \equiv \frac{(1-\lambda_1)p_1}{(1-\lambda_1 p_1)\Omega_1}$ as in (4).*

So if the longshot (in this case team 1) is to be bribed, ticket 2 price must be lowered below a critical level which is less than p_1, and given that $p_2 > p_1$ (when $p_1 \leq \tilde{p}_1 < 1/2$) such bribery will involve transferring a significant amount of rent to punter I (due to $\pi_2 \leq h_1 < p_2$). Indeed, this is going to make bribery of the longshot an unlikely event. In this context, we make a mild assumption on (μ_1, μ_2) and (λ_1, λ_2), that the a priori strong team's (team 2) chance of winning, after allowing for potential corruption, remains above the weak team's uncorrupted chance of winning. In other words, the strong team does not become weaker than where the weak team was in the absence of any corruption possibility. Under symmetry of μ and λ, the assumption is automatically satisfied.

Assumption 4. (***A threshold on strong team's ex-ante winning odds, with corruption***) *For all $p_1 \leq \tilde{p}_1 < 1/2$, the following holds:*

$$p_2[\mu_2\lambda_2 + (1-\mu_1-\mu_2)] + \mu_1(1-\lambda_1 p_1) \quad > \quad p_1,$$

or equivalently

$$p_1\big[(1-\mu_1) + \mu_1\lambda_1\big] \quad < \quad p_2\big[(1-\mu_2) + \mu_2\lambda_2\big].$$

Note, however, that with bribery possible, the ex-ante probability of team 2 winning may well be lower than that of team 1.

Proposition 2 (Bribery of the strong team). *Suppose Assumptions 2, 3 and 4 hold.*

(*i*) *There is no equilibrium of the Bertrand game involving* ***bribery of only the weak team****.*

(*ii*) *There is no equilibrium of the Bertrand game involving* ***bribery of either teams****.*

Thus **if bribery is to happen then it must involve only the strong team**. To understand why only the strong team is bribed, consider first bribery of the longshot (bet accentuation). If team 1 (the longshot) were to be bribed, ideally the bookies would have liked to lure the naive punters to bet on team 1 by setting π_1 appropriately low and/or π_2 high. But to encourage punter I to bribe team 1 and bet on team 2, the bookies must do the opposite – set π_2 low and π_1 high. This conflict makes bribery of the underdog loss-making and unsustainable for the bookies. Next, consider the prospect of either teams being bribed. That would require setting not just π_2 low, but also π_1 low. There are no pair of prices feasible for this to happen, as the Dutch-book restriction will be violated. Thus, if bribery is to occur it must involve bribery of the favorite alone (team 2).

Studying data on German horse race betting, Winter and Kukuk (2008) found some evidence of cheating by the favorites when races are very uneven. Suspecting cheating by one of the favorite jockeys, the bettors tend to bet disproportionately more on the longshots. Winter and Kukuk's empirical observation that the favorite(s) cheat bears some resemblance to our theoretical observation above. However, their analysis is mainly for *parimutuel betting* whereas ours is for *fixed-odds betting*.

To continue with our analysis, when the favorite (team 2) is to be bribed, π_2 must be *above* some threshold level and π_1 must be *below* a similar threshold level. Can the bookies then make *positive expected profits*, and can such profit-generating prices be sustained as a Nash equilibrium? The answer is 'no'. Even though the Dutch-book requirement restricts the scale of undercutting, it turns out that there will always be some incentive for slight undercutting on one or both tickets. If ticket 1 were to generate positive profit alone or along with ticket 2 (remember, team 1's winning prospect is enhanced with the bribery of team 2), then a bookie can easily steal this market without altering punter I's incentive to bribe team 2 (simply lower π_1 slightly). On the other hand, if only ticket 2 generates profit then a coordinated undercutting on both tickets (if necessary) becomes profitable, despite a loss on ticket 1 (a formal argument appears in the proof). In either scenario, positive profit cannot be sustained.

It is also the case that in a bribe inducement equilibrium, favorite must be under-priced ($\pi_2 < p_2$) and the longshot then must be over-priced ($\pi_1 > p_1$) relative to the true probabilities of winning. The reason is, the bookies need to attract (ordinary) punters to bet on a losing cause by lowering the price of the favorite and discourage them from betting on the longshot as its winning chance secretly goes up. These also imply that punter I must be earning strictly positive expected profit in a bribe inducement equilibrium, regardless of whether the bookies make zero or positive expected profits.

Lemma 3. *Suppose Assumption 3 holds. Then there is no equilibrium with match-fixing in which the bookies earn **positive (expected) profits**.*

Lemma 4. *Suppose Assumption 3 holds, and $p_1 \in [0, \tilde{p}_1)$. Then there cannot be a bribe inducement equilibrium such that $\pi_2 \geq p_2$ and $\pi_1 \leq p_1$. In other words, any bribe inducement equilibrium must involve $\pi_2 < p_2$ and $\pi_1 > p_1$, with punter I earning a **positive (expected) profit**.*

In view of Proposition 2 and the above two lemmas, we are left with only the possibility of a zero profit, bribe inducement equilibrium, which we propose next (maintaining Assumption 3):

Equilibrium $\mathcal{E}$. *Suppose $p_1 \in [0, \tilde{p}_1)$. Symmetric[22] equilibrium prices are (π_{10}, π_{20}) with $\pi_{20} < p_2$ and $\pi_{10} > p_1$. On access only team 2 (the strong team) will be bribed; when team 2 is bribed, punter I will bet on team 1 (the weak team), and otherwise he will bet on team 2. Each bookie makes zero expected profit in each market.*

For the construction of $\mathcal{E}$, note that (π_{10}, π_{20}) must ensure zero profits in each market:

$$E\Pi_1^b \;=\; \frac{1}{2}\big\{(1-\pi_{10})\big[1-\frac{p_1^b}{\pi_{10}}\big]y \,+\, \mu_2 z\big[1-\frac{(1-\lambda_2 p_2)}{\pi_{10}}\big]\big\} = 0 \tag{5}$$

$$E\Pi_2^b \;=\; \frac{1}{2}\big\{(1-\pi_{20})\big[1-\frac{p_2^b}{\pi_{20}}\big]y \,+\, (1-\mu_2) z\big[1-\frac{p_2}{\pi_{20}}\big]\big\} = 0 \tag{6}$$

where $p_1^b = [\mu_2(1-\lambda_2 p_2) + (1-\mu_2)p_1]$, and $p_2^b = 1 - p_1^b = [\mu_2\lambda_2 p_2 + (1-\mu_2)p_2]$; p_i^b $(i = 1, 2)$ represents the ex-ante winning probability of team i when team 2 is bribed with probability μ_2.

[22] By symmetry we mean symmetry across bookies. We do not analyze whether there might be an asymmetric equilibrium in which bribery is induced (with positive probability), at least one bookie is the sole server in one market and each bookie makes zero expected profit. Such an equilibrium, if it exists, will be similar in spirit, as far as bribery is concerned, to the equilibrium $\mathcal{E}$. Further, symmetric prices must generate zero profit in each market, otherwise deviation would occur.

In addition, the two prices must satisfy condition (3) for bribery of team 2:

$$\pi_{10} \leq \psi_2(\pi_{20}),$$

and violate condition (4) so that team 1 is *not* bribed:

$$\pi_{20} > h_1.$$

From (5) and (6) we can identify the bounds for π_{10} and π_{20}. In the event of gaining access to team 2, punter I will bribe team 2 and bet on team 1 if the price of ticket 1 is smaller than the corrupted probability of winning of team 1, i.e. $\pi_{10} < (1 - \lambda_2 p_2)$. So the second term in (5), which refers to profit from punter I, must be negative, and therefore, the first term in (5) (i.e. profit from the naive punters) must be positive, which implies $\pi_{10} > p_1^b$. Therefore, π_{10} must lie within the interval[23] $\Bigg(p_1^b,\ (1-\lambda_2 p_2)\Bigg)$. Similarly, in the event of not gaining access to team 2, punter I will bet on team 2 if $\pi_{20} < p_2$. This makes the second term (representing profit from punter I) in (6) negative, and therefore the first term in (6), which represents profit from the naive punters, must be positive implying $\pi_{20} > p_2^b$. That is to say, π_{20} must belong to the interval $\Bigg(p_2^b,\ p_2\Bigg)$.

The above bounds ensure that $\pi_{10} + \pi_{20} > p_1^b + p_2^b = 1$. Further, since $E\Pi_1^b$ and $E\Pi_2^b$ are continuous functions of π_1 and π_2 respectively, there must exist at least one π_{10} and one π_{20} within the above specified intervals solving (5) and (6). In fact, the solution (π_{10}, π_{20}) is unique.[24]

Next we look at various deviation incentives that must be deterred. Unlike the textbook Bertrand model, analyzing deviations is much more complex in our setting due to the interdependence between the two markets: gains from undercutting in one market must be evaluated in light of the possible bribery implication in the other market, and moreover a bookie will have the option of altering the two prices in various combinations (lower/increase, lower/stay-put, lower/lower, etc.). In addition, if a deviation alters the status-quo bribery or no-bribery situation, the deviation profit of the bookie can rise or fall discontinuously.

[23] It is evident that $p_1 < p_1^b < 1 - \lambda_2 p_2$.

[24] Eqs. (5) and (6) are quadratic in π_{10} and π_{20} respectively, solving which we obtain:

$$\begin{aligned}
\pi_{10} &= \frac{1}{2y}\{(y + k_1) \pm \sqrt{(y - k_1)^2 + 4y\mu_2\lambda_2 p_2 z}\} \\
\pi_{20} &= \frac{1}{2y}\{(y + k_2) \pm \sqrt{(y - k_2)^2 + 4y(1 - \mu_2)p_1 z}\},
\end{aligned}$$

where $k_1 = p_1^b y + \mu_2 z$ and $k_2 = p_2^b y + (1 - \mu_2)z$. For each price, one of the two roots exceeds 1.

Starting at zero-profit prices (π_{10}, π_{20}), neither bookie would gain by deviating unless it alters the bribery incentive of punter I. We must therefore protect our posited equilibrium against the following three deviations altering the bribery incentive:

Starting from $h_1 < \pi_{20} < p_2$ so that **team 1 is not bribed**, and $p_1 < \pi_{10} \leq \psi_2(\pi_{20})$ so that **team 2 is bribed**,

(i) Deviation leading to bribery of team 1 instead of team 2: π_2 is reduced to π_2' and π_1 is chosen to be some appropriate π_1' such that $\pi_2' \leq h_1$ and $\min\{\pi_1', \pi_{10}\} > \psi_2(\pi_2')$;[25]

(ii) Deviation leading to bribery of either teams: π_2 is reduced to π_2' and π_1 is appropriately chosen to be some π_1' such that $\pi_2' \leq h_1$ and $p_1 < \min\{\pi_1', \pi_{10}\} \leq \psi_2(\pi_2')$.

(iii) Deviation leading to bribery of neither teams: π_2 is reduced to π_2' and π_1 is reduced to π_1' (or unchanged so that $\pi_1' = \pi_{10}$) such that $h_1 < \pi_2'$ and $p_1 < \psi_2(\pi_2') < \pi_1'$.

It turns out that deviations **(i)** and **(ii)** are either infeasible (i.e. violate the Dutch-book restriction) or clearly unprofitable (Assumption 4 and Lemma 2 will be used to establish this in the proof of Proposition 3 in the Appendix). It is deviation **(iii)** that requires additional attention. It is quite possible that if π_{10} is sufficiently high, then one of the bookies can undercut in such a manner that punter I will no longer find it optimal to bribe, and yet the deviating bookie will make positive expected profit. We therefore identify an upper bound for π_{10}, below which deviation **(iii)** will not be profitable. The following two definitions will be used to determine the relevant upper bound.

Definition 2 (**A bound for** π_{10})**.** *Fix any* $0 < p_1 < \tilde{p}_1$*. Let* $(\tilde{\pi}_1, \tilde{\pi}_2)$ *be such that* $\tilde{\pi}_1 + \tilde{\pi}_2 = 1$ *and* $\tilde{\pi}_1 = \psi_2(\tilde{\pi}_2)$*.*

By construction the prices $(\tilde{\pi}_1, \tilde{\pi}_2)$ are unique, and $\tilde{\pi}_1 > p_1,\ \tilde{\pi}_2 < p_2$.

Definition 3 (**An alternative bound for** π_{10})**.** *Fix any* $0 < p_1 < \tilde{p}_1$*, and assume that* $\tilde{\pi}_1 \geq \frac{y}{1+y}$*. There exists a unique pair of prices,* $(\pi_1, \pi_2) = (\pi_1^{M0}, \pi_2^{M0})$*, at which the dual objectives of* $E\Pi^M = 0$ *and* $\pi_1 = \psi_2(\pi_2)$ *will be met, where* $E\Pi^M$ *represents no-bribery, monopoly profit with punter I betting on team 2 and given by*

$$E\Pi^M = y\big[3 - \pi_1 - \pi_2 - \frac{p_1}{\pi_1} - \frac{p_2}{\pi_2}\big] + z\big[1 - \frac{p_2}{\pi_2}\big], \tag{7}$$

where $p_1 < \pi_1^{M0} < \phi_2$ *and* $0 < \pi_2^{M0} < p_2$*.*

[25] Note that as π_2 is lowered to π_2', $\psi_2(\pi_2') < \psi_2(\pi_{20})$; it is conceivable that $\psi_2(\pi_2') < \pi_{10} < \psi_2(\pi_{20})$ in which case the deviating bookie may even set $\pi_1' \geq \pi_{10}$. That is, π_1' does not have to be lower than π_{10}.

In Fig. 5a displayed, the relevant upper bound on π_{10} will be $\tilde{\pi}_1$ (see Definition 2). Uniqueness of $(\tilde{\pi}_1, \tilde{\pi}_2)$ is evident from the construction. Next in Fig. 5b, the relevant upper bound on π_{10} is π_1^{M0} (see Definition 3). Recall, by engaging in deviation **(iii)** a bookie expects to receive the monopoly profit, $E\Pi^M$, with bribery eliminated. It can be checked that the iso-profit curve, $E\Pi^M = 0$, intersects the $\pi_1 + \pi_2 = 1$ line at two points $(\pi_1 = p_1, \pi_2 = p_2)$ and $(\pi_1 = \frac{y}{1+y}, \pi_2 = \frac{1}{1+y})$. Depending on the parameter values we will have $p_1 < \tilde{\pi}_1 < \frac{y}{1+y}$ (as drawn in Fig. 5a), or $p_1 < \frac{y}{1+y} < \tilde{\pi}_1$ (as drawn in Fig. 5b). Since $E\Pi^M$ is defined over the region $\pi_1 + \pi_2 \geq 1$, we discard the segment of the iso-profit curve that falls below the $\pi_1 + \pi_2 = 1$ line. It can also be verified (and we establish this formally in the Appendix) that the iso-profit curve will be concave at $\pi_1 \geq \frac{y}{1+y}$ (assuming $p_1 < \frac{y}{1+y}$). Further, any price pair that lies outside (or to the left of) the curve representing $E\Pi^M = 0$ will yield negative profit (under monopoly and no-bribery), and any price pair lying inside (or to the right) will give positive profit under monopoly and no-bribery.[26]

When $\tilde{\pi}_1 < \frac{y}{1+y}$, π_1^{M0} does not exist, as in Fig. 5a. Point w represents a (zero-profit) bribe inducement equilibrium, where $\pi_{10} < \tilde{\pi}_1$. To the south-west of point w there is no price pair available at which the incentive for bribing the favorite team (team 2) is violated and at the same time the Dutch-book restriction is met. Put another way, by moving south-west there is no way one can cross over to the other side of the ψ_2 curve and still be on the right-hand side of the $\pi_1 + \pi_2 = 1$ line. But if the equilibrium was at point w' instead of w, in which case $\pi_{10} > \tilde{\pi}_1$, then a deviation to a point like d would have been possible. Point d satisfies the Dutch-book restriction and it also yields a positive profit, no-bribery monopoly outcome. Thus, when $\tilde{\pi}_1 < \frac{y}{1+y}$, the restriction $\pi_{10} \leq \tilde{\pi}_1$ is both necessary and sufficient to rule out deviation **(iii)**. The set of sustainable bribe inducement equilibrium prices is given by the shaded area (not including $\pi_1 = p_1^b, \pi_2 = p_2^b$).

The possibility of $\tilde{\pi}_1 \geq \frac{y}{1+y}$ is drawn in Fig. 5b. The equilibrium price is again denoted by point w, which shows that $\pi_{10} < \pi_1^{M0}$. Starting from point w if one moves south-west (i.e. undercutting on both tickets), one cannot violate the incentive for bribing the favorite team (by crossing over the ψ_2 curve) without crossing over the $E\Pi^M = 0$ curve. That is to say, deviation to no-bribery prices will only fetch negative profit. Similarly, if π_{10} was greater than π_1^{M0}, as is the case with point w', then deviation to point d, where bribery does not occur, is perfectly possible and it will be profitable as well. As before, $\pi_{10} \leq \pi_1^{M0}$ is thus the necessary and sufficient condition for ruling out deviation **(iii)**. The set of sustainable equilibrium prices is given by the shaded area (not including $\pi_1 = p_1^b, \pi_2 = p_2^b$). As a

[26] For ease of exposition we ignored the bribery indifference condition while drawing the iso-profit curve. But the underlying probability of a team winning that determines the no-bribery monopoly profit will depend on whether the prices remain above the bribery indifference curve ψ_2. We consider this aspect in our formal argument.

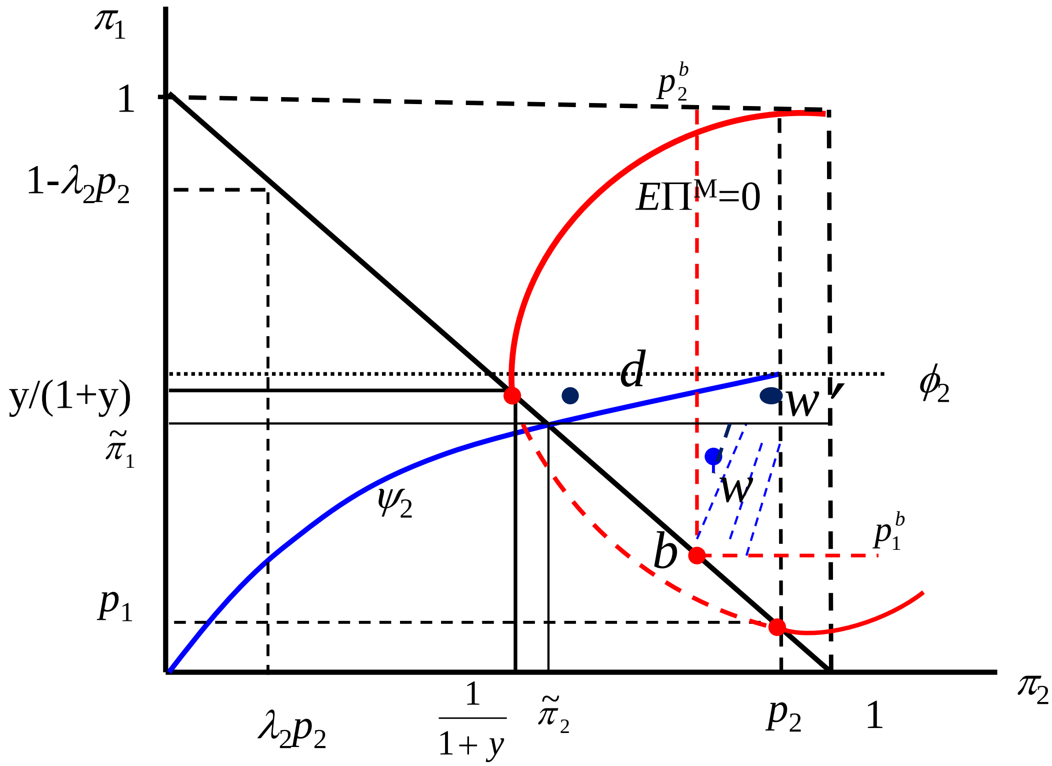

Figure 5a: Zero profit bribe inducement equilibrium

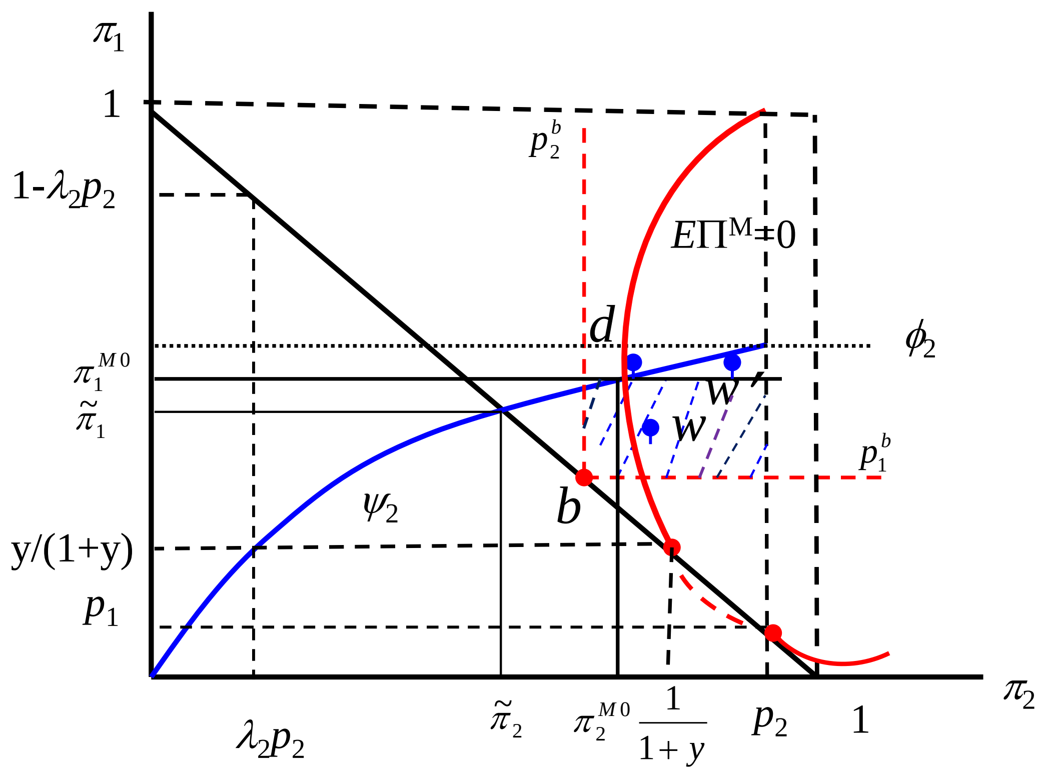

Figure 5b: Zero profit bribe inducement equilibrium

numerical illustration of the equilibrium for this case as well as the first case, we provide an example in the Appendix (see Example 1).

Proposition 3 (**Match-fixing Equilibrium**). *Suppose Assumptions 2, 3 and 4 hold and consider any $p_1 < \tilde{p}_1$. Then $(\pi_{10} > p_1, \pi_{20} < p_2)$ satisfying (5) and (6), and thus meeting the Dutch-book restriction, constitute a unique competitive equilibrium denoted by $\mathcal{E}$ if and only if*

$$\pi_{10} \leq \psi_2(\pi_{20}), \quad \pi_{20} > h_1, \quad \textit{and} \quad \begin{cases} \pi_{10} \leq \tilde{\pi}_1 & \textit{if} \quad \tilde{\pi}_1 < \frac{y}{1+y}; \\ \pi_{10} \leq \pi_1^{M0} & \textit{if} \quad \tilde{\pi}_1 \geq \frac{y}{1+y}. \end{cases}$$

In equilibrium,

(*i*) *punter I will bribe the strong team, team 2, whenever he gets an access to team 2;*

(*ii*) *each bookie will earn zero expected profit; and*

(*iii*) *punter I will earn strictly positive expected profit.*

Thus, under plausible economic situations price competition among bookmakers in the sports betting market may endogenously lead to match-fixing initiated by a corrupt punter. The prices are set by the bookmakers in response to the match-fixing possibility, and a continuum of punters respond by betting optimally while a privileged punter bribes the players and fixes the match. And in conformity with common perceptions, with contests uneven the strong team is bribed. Finally, with the possibility of profitable match-fixing open, the influential punter can also profitably bet even when he actually fails to get an access to the strong team and thus fails to bribe. That is, ticket prices are such that the influential punter profits either way. This is in contrast with the bribe prevention equilibrium of Proposition 1 in which the influential punter is fully kept out.

One may be tempted to conclude that the above result is an evidence against the conventional wisdom that more competition means less corruption (Rose-Ackerman, 1978). But there is little to relate our model to this usual corruption/competition story where incentives for bribe-giving may be created because of an exclusive valuable asset (say a monopoly service provision right), access to which can be restricted by regulation and where some key government official decides arbitrarily, without transparency, who should get access.

4.3 Positive Profit, Bribe Prevention Equilibrium

In this section we show that if Assumption 3 is violated and if in particular $\hat{p}_1 < \frac{1}{2} < \tilde{p}_1$, there are some interesting implications, especially for contests that are "close". Violation of

Assumption 3 occurs if Ω_i is relatively small, i.e., z is large relative to $(w + \alpha_i(f + f_I))$ so that *the influential punter is quite powerful in terms of wealth.* Fig. 6 presents this case.[27] We will restrict attention to the case of "close" contests: $p_1 \in (\hat{p}_1, \tilde{p}_1)$.

Here we identify the possibility of two types of equilibrium. In one, either team will be bribed by punter I upon access but the bookies earn zero expected profit. In the other, bribery is prevented with certainty and the bookies earn positive expected profit each. The first possibility is similar to our bribe inducement equilibrium in highly uneven contests (Proposition 3) except that now even the weak team is bribed and then punter I will bet on the strong team; in the event of not getting access to any team he will bet on neither. He will also earn strictly positive expected profit. This equilibrium can be sustained if the bookies, unilaterally, cannot profitably undercut (substantially) in one or both prices to eliminate punter I's bribery incentive for *at least* one team. Such deviation will also require reducing one of the ticket prices below the true winning probability, say π_1 below p_1, and thereby allowing punter I to bet on team 1 even without bribery. Clearly, ticket 1 will be loss-making, which must be more than compensated by profits from the sale of ticket 2. This seems to be a very costly deviation strategy and therefore the proposed equilibrium might be sustained. Since a proof of this claim will be fairly involved and similar to the proof of Proposition 3, we do not include the proof or discuss it further (see the Supplementary material for a formal treatment of this claim). Instead, we focus on the other equilibrium in which bribery is prevented and yet the bookies earn strictly positive profits, which contrasts the finding of Proposition 1 when Assumption 3 is maintained.[28]

Our proposed bribe prevention equilibrium is $(\pi_1 = \phi_2, \pi_2 = \phi_1)$; this generates strictly positive profit since $\phi_2 > p_1, \phi_1 > p_2$. More precisely each bookie's equilibrium profit is:

$$E\Pi^d_{BP} = \frac{y}{2}\big[3 - \phi_2 - \phi_1 - \frac{p_1}{\phi_2} - \frac{p_2}{\phi_1}\big] \equiv k.$$

k can be large around $p_1 = 1/2$.[29] We need to show that this equilibrium is immune to all possible undercutting, which are undercutting on both tickets and undercutting on each ticket separately. In what follows we provide an informal argument by suggesting conditions that will ensure immunity against all undercutting. The formal proof and precise conditions are provided in the Appendix.

[27] Bribe prevention then requires ticket prices to be set high, with ϕ_1, ϕ_2 shifting upwards; contrast Fig. 6 with Fig. 4, especially the reversal of positions of $\tilde{p}_1$ and $\hat{p}_1$.

[28] It is difficult to ascertain whether these two equilibria will hold in isolation or simultaneously. Numerical simulation might shed some light on this.

[29] Note that even if the volume of bets on the two tickets may be fairly even if the ticket prices, for $p_1 = 1/2$, are symmetric (or fairly close), large prices of bets means the bookies' profits may be substantial.

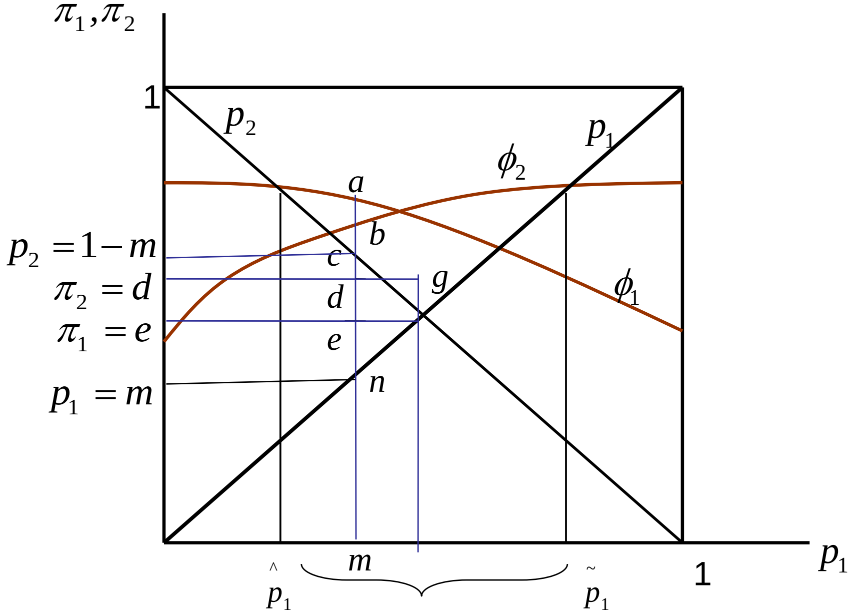

Figure 6: **Positive profit, bribe prevention prices**

Undercutting on both tickets: First consider undercutting on both tickets. In Fig. 6, let us select p_1 to be m, at which $\phi_2 = b$ and $\phi_1 = a$. Suppose one bookie deviates from the equilibrium by undercutting π_2 slightly below a, and π_1 slightly below b. As long as the reduced π_i is strictly greater than p_i, by the violation of bribe prevention constraint (2) punter I will be strictly better off by bribing. Therefore, team 1 will be bribed with probability μ_1 and team 2 with probability μ_2. With both markets being monopolized by the undercutting bookie, he will face significant losses with probability $\mu_1 + \mu_2$, against significant gains with probability $(1 - \mu_1 - \mu_2)$. Intuitively, it then seems that his expected overall profit is likely to be smaller than the non-deviation duopoly profit if $\mu_1 + \mu_2$ is sufficiently high. Let $\bar{\mu}$ be such that the deviation profit $E\Pi_{BI}$ is just equal to the duopoly profit k, and for $\mu_1 + \mu_2 > \bar{\mu}$, $E\Pi_{BI} < k$. Thus, a lower bound on the total probability of access seems in order. In the Appendix we formally specify $\bar{\mu}$.

Undercutting on a single ticket: Suppose the deviating bookie lowers π_1 slightly from b while maintaining $\pi_2 = \phi_1 = a$. As long as $\pi_1 > p_1$ (which is indeed possible at $p_1 = m$ in Fig. 6), any slight reduction in π_1 from ϕ_2 will trigger bribery of team 2 with probability μ_2 (but team 1 will not be bribed). In the event of bribery capturing market 1 becomes a curse, and therefore if μ_2 is sufficiently high the bookie will be deterred from such undercutting. Let the critical value of μ_2 be denoted as μ_2^*, such that at all $\mu_2 \geq \mu_2^*$, $E\Pi_{BI} \leq k$. Symmetrically, let μ_1^* be the critical value of μ_1 such that at all $\mu_1 \geq \mu_1^*$ slight undercutting on ticket 2 is deterred.

Thus, we need to have lower bounds on individual μ_is as well as their sum $(\mu_1 + \mu_2)$ to support the proposed equilibrium. In addition to slight undercutting, we need to consider large-scale undercutting as well. For example, in Fig. 6 π_1 can be reduced from b to any point between b and n, while π_2 can be reduced at the same time to any point between a and c. As long as these prices do not violate the Dutch-book restriction, the deviating bookie will earn monopoly profit (from both markets) with the prospect of match-fixing for either team.[30] Identifying conditions under which this deviation (monopoly) profit falls short of the duopoly profit k proves to be difficult under the general case. But we can say that if the deviation-profit is found to be increasing at ϕ_2 and ϕ_1 (and not decreasing at $\pi_1 < \phi_2$ and $\pi_2 < \phi_1$), then restricting attention to small-scale undercutting is sufficient. In the following proposition we provide a sufficient condition to ensure that indeed that is the case. Then with this monotonicity condition and lower bound restrictions on μ_1 and μ_2 we can support the bribe prevention, positive profit equilibrium. In Example 2 in the Appendix, we demonstrate this type of equilibrium numerically.

Proposition 4 (Bribe prevention and positive profit). *Suppose $\hat{p}_1 < \frac{1}{2} < \tilde{p}_1$, and $p_1 \in (\hat{p}_1, \tilde{p}_1)$. That is, the influential punter has significant wealth and the contests are "close". If*

(i) μ_1 and μ_2 exceed some threshold levels (to be precisely determined in the Appendix), and

(ii)

$$\sqrt{\rho + \frac{z\mu_2(1-\lambda_2 p_2)}{y}} \geq \phi_2, \quad \textit{and} \quad \sqrt{(1-\rho) + \frac{z\mu_1(1-\lambda_1 p_1)}{y}} \geq \phi_1, \tag{8}$$

where $\rho = \mu_1\lambda_1 p_1 + \mu_2(1-\lambda_2 p_2) + (1-\mu_1-\mu_2)p_1$ is the ex-ante probability of team 1 winning,

then $\pi_1 = \phi_2$ and $\pi_2 = \phi_1$ is a bribe prevention equilibrium in which each bookie makes a positive expected profit.

The above is a possibility result which, to our knowledge, is new. One would normally expect competition to drive down symmetrically informed bookmakers' profits to zero (as was the result in Shin's (1992) exogenous insider information model, for instance). Our intuition is that *high* chances of corruption make undercutting a risky proposition as it may create a

[30] One can see that undercutting of π_1 and π_2 need not end at p_1 and p_2. For example, π_1 can be reduced to as low as e and π_2 can be reduced to d; that is, $\pi_2 = d < p_2$ but $\pi_1 = e > p_1$ and yet the Dutch-book is satisfied, i.e. $\pi_1 + \pi_2 = e + d > 1$. To see this we can map $\pi_1 = e$ onto the horizontal axis and arrive at point g which is above the 45° line and has coordinates (e, d). Since point g is outside the unit simplex, $e + d$ must be greater than 1. Thus the price combination satisfies Dutch-book and can induce bribery.

'lemon' (Akerlof, 1970) and give rise to an ***adverse selection*** problem similar to the credit rationing story of Stiglitz and Weiss (1981). Potential entry by the influential punter who has *significant* wealth and who can fix the match works as a disciplining influence deterring deviation by the bookies from the implicit 'collusive' equilibrium. Bookmakers' tendencies to resist excessive price reduction is similar to banks resisting lowering of interest rates that may invite high-risk borrowers.

It may also be noted that positive profits for the bookies are not generated due to any asymmetry of information between the bookies about the teams' winning odds, although such a situation can be easily visualized when sometimes bookies may have differing private information due to their expertise (or the lack of it) on underworld/illegal betting syndicates. Our result thus differs from that of Dell'Ariccia, Friedman and Marquez (1999) who studied Bertrand competition between two incumbent banks (who are somewhat better but asymmetrically informed about the potential customers' riskiness due their differing existing market shares) and an entrant (with no information) and showed that the dominant incumbent bank would earn positive profits due to its superior information.

4.4 Uneven Contest and the Possibility of Bribing the Favorite

Now we consider uneven contests continuing with the assumption that $\hat{p}_1 < \frac{1}{2} < \tilde{p}_1$. In particular, we restrict our attention to $p_1 < \hat{p}_1$ (i.e., highly uneven contests). What can we say about match-fixing or bribe prevention? We discuss the following possibilities informally.

Bribe prevention: As argued in section 4.1, here too bribe prevention with zero profit is not possible, because at all $p_1 < \hat{p}_1$, $p_1 < \phi_2$ and $\phi_1 < p_2$. Prevention of bribery of team 1 requires setting $\pi_1 \geq \phi_2$ if $\pi_2 = p_2$. But then ticket 1 will yield positive profit.

Similarly, bribe prevention with positive profit is also not possible. If the prices are set such $\pi_2 = p_2$ and $\pi_1 = \phi_2 > p_1$ (yielding positive profit on ticket 1), one of the bookies can slightly undercut on ticket 2 (from π_2 to π_2') and simultaneously undercut on ticket 1 in such a manner that $\pi_1' > \psi_2(\pi_2')$. This undercutting is clearly profitable and yet bribery is avoided, as shown earlier in section 4.1. Thus, here too bribe prevention with positive profit is ruled out.

Bribe inducement: We can confirm that Lemma 4 will continue to hold ruling out any bribe inducement equilibrium with $\pi_2 \geq p_2$ and $\pi_1 \leq p_1$. Therefore, if the underdog (i.e., team 1) is to be bribed, we must have $\pi_2 < h_1$ and also $\pi_2 < p_2$. But ticket 2 will then be clearly loss-making, similar to our argument in the proof of Proposition 2 part (i). It can be checked that this result does not rely on Assumption 3 and therefore it goes through here as well.

Can there be any bribe inducement equilibrium involving bribery of either team or only the favorite? To answer this let us note that when punter I is too powerful (meaning z is large relative to $w + \alpha(f + f_I)$), Ω_i falls below 1 and our Lemma 2 may no longer be true; h_1 can be larger than p_1 or even larger than p_2. In section 4.3, a low value of h_1 was particularly helpful in eliminating the incentive to bribe the underdog (i.e., bribery of the bet accentuation variety), which in turn helped us rule out the possibility of bribing either team (Proposition 2 part (ii)) and protect the equilibrium involving bribery of the favorite alone (Proposition 3). Here too, we can protect this result, as long as $h_1 \leq p_1$. From (4) we can determine that if $\Omega_1 \in [\frac{1-\lambda_1}{1-\lambda_1 p_1}, 1]$, then $h_1 \leq p_1$. Then our bribe inducement negative result of Proposition 2 part (ii) and bribe inducement positive result of Proposition 3 both go through. Basically, if Ω_1 falls below 1 but does not fall too far, our qualitative results of section 4.3 remain unaffected.

What happens if Ω_1 falls further (which may result from a fall in $w+\alpha(f+f_I)$)? Clearly, h_1 will rise above p_1. As long as $h_1 < p_2$, the match-fixing equilibrium still can be protected by imposing a suitable upper bound on h_1. However, if one considers even smaller values of Ω_1, h_1 will rise further and one of the sufficient conditions of our match-fixing equilibrium, $\pi_{20} > h_1$, may now fail. In effect, ticket 2 price does not have to be reduced too far to trigger bribery of team 1. This is a deviation that could be profitable and thus the match-fixing equilibrium may be destroyed, and along with it the existence of a pure-strategy equilibrium may be ruled out.

5 Conclusion

Table 1: Summary of match-fixing results

	Close contests	**Highly uneven contests**
Punter I is not too powerful	Bribe prevention with zero profit	Bribe inducement of the favorite and zero profit
Punter I is powerful	Bribe prevention with positive profit, and/or bribe inducement of either team and zero profit	Bribe inducement of the favorite and zero profit

Match-fixing in a number of sports and its implications for betting have attracted a great deal of media attention in recent times. Building on Shin's (1991; 1992) horse race betting model with fixed odds, we analyze the match-fixing and bribing incentives of a potentially corrupt gambler and show how competition in bookmaking affects match-fixing, taking the anti-corruption authority's investigation strategy as exogenous. At the set prices, the bookies are obliged to honor the bets using deep pockets. The bookies' pricing decisions determine whether the corrupt influence comes into play or kept out. We show that the competitive equilibrium may not always ensure zero profit, nor does it always prevent bribery. And when match-fixing is induced, often the strong team will be bribed, though in some close contests either team may be bribed. In Table 1 we present a broad summary of our results.

We did not comment on the favorite-longshot bias. In the bribe prevention equilibrium of Proposition 1, the bias clearly disappears. But in other cases (Propositions 3 and 4), the favorite-longshot bias reappears. We also think that studying the monopoly case should be of interest. The monopolist can control the influential punter's incentive without having to worry about losing the market to a rival. It is conceivable that sometimes the monopolist may even want to engineer match-fixing. Characterization of the monopolist's optimal strategy, i.e. whether to prevent or induce match-fixing, depending on the type of contest (close or uneven) is the subject matter of our related work, Bag and Saha (2009).

Appendix

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Match-fixing inquiry probes 200 European football gam

About 200 European football games are under investigation in a match-fixing inquiry, German prosecutors have said.

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Friedhelm Althans, Bochum Police: 'Group betting with large sums of cash'

At least three of the games were in the Champions League and another 12 were in the Uefa Europa League, officials said.

Uefa representative Peter Limacher called it the biggest match-fixing scandal ever to hit Europe.

On Thursday police carried out about 50 raids in Germany, the UK, Switzerland and Austria, making 17 arrests and seizing cash and property.

Fifteen of those arrested were in Germany and the other two in Switzerland.

Matches under investigation were played in Germany, Belgium, Switzerland, Croatia, Slovenia, Turkey, Hungary, Bosnia-Hercegovina and Austria.

Officials said the games included the 40 which Uefa revealed in September were under investigation.

'Stunned'

Prosecutors believe a 200-strong criminal gang has bribed players, coaches, referees and officials to fix games and then made money by betting on the results.

The investigation is being carried out by German authorities and supported by Uefa, the European football body.

All the matches under suspicion are believed to have taken place this year, although prosecutors did not specify if they were qualifying games or group-round matches.

> **"Uefa will be demanding the harshest of sanctions before the competent courts for any individuals, clubs or officials who are implicated in this**

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Proof of Proposition 1. First note that $\tilde{p}_1$ and $\hat{p}_1$ can be verified to be as follows:

$$\begin{aligned}\tilde{p}_1 &= \frac{1}{2}\Big\{\frac{(1-\lambda_2)}{\lambda_2}\frac{1+\Omega_2}{\Omega_2}\Big\}\Big[\sqrt{1+\frac{\lambda_2}{1-\lambda_2}\frac{4\Omega_2}{(1+\Omega_2)^2}}-1\Big],\\ \hat{p}_1 &= 1-\frac{1}{2}\Big\{\frac{(1-\lambda_1)}{\lambda_1}\frac{1+\Omega_1}{\Omega_1}\Big\}\Big[\sqrt{1+\frac{\lambda_1}{1-\lambda_1}\frac{4\Omega_1}{(1+\Omega_1)^2}}-1\Big].\end{aligned}$$

Part (i): By construction, for any $p_1 \in [\tilde{p}_1, \hat{p}_1]$, the ticket prices ($\pi_1 \geq p_1, \pi_2 \geq p_2$) imply that $\pi_1 \geq \phi_2$ and $\pi_2 \geq \phi_1$ (see Fig. 4). So by condition (2), the two teams will be immune to match-fixing. Now the standard Bertrand competition would lead to the zero-profit equilibrium prices ($\pi_1 = p_1, \pi_2 = p_2$) for both the bookies that also satisfy the Dutch-book restriction; deviation in the form of unilateral price reduction(s) from ($\pi_1 = p_1, \pi_2 = p_2$) is not feasible as it violates the Dutch-book restriction. The equilibrium is unique just like in textbook Bertrand competition game.

Part (ii): Consider $p_1 < \tilde{p}_1$, where the focus is restricted to bribery of team 2 only; symmetric argument will apply to $p_1 > \hat{p}_1$. If there is a pure strategy bribe prevention equilibrium, it must be from one of the following two price configurations:

[1] $\pi_1 = \phi_2(p_1)$ and $\pi_2 = p_2$. (We can rule out $\pi_1 > \phi_2$ and/or $\pi_2 > p_2$ due to Bertrand competition in each of ticket 1 and 2 respectively.)

[2] $\pi_1 < \phi_2$ and $\pi_2 < p_2$; but π_1 is such that punter I finds bribing team 2 (when accessed) less profitable than betting on team 2. That is, $E\Pi^I(b_2) = (1-\lambda_2 p_2)z\big[\frac{1}{\pi_1} - \frac{1}{\phi_2}\big] < E\Pi^I_{02} = z\big[\frac{p_2}{\pi_2} - 1\big]$, or $\pi_1 > \psi_2$ where ψ_2 is defined in (3).

From (2) and (3) check that $\psi_2 < \phi_2$ if $\pi_2 < p_2$, and $\psi_2 = \phi_2$ if $\pi_2 = p_2$. Thus, the second set of price configuration is: $\pi_1 \in (\psi_2(p_1), \phi_2(p_1))$, $\pi_2 < p_2$.

Of these configurations, [2] cannot be equilibrium: starting from $\pi_2 < p_2$, a bookie can raise only π_2 and avoid the loss on ticket 2.

For configuration [1], starting from ($\pi_1 = \phi_2(p_1), \pi_2 = p_2$) we show that a deviation in the form of slight undercutting on both tickets by one of the bookies will be profitable. In the posited equilibrium each bookie earns the profit $E\Pi_C = \frac{y}{2}(1-\phi_2)\left[1-\frac{p_1}{\phi_2}\right] > 0$. Now consider the following deviation: $\pi_2 = p_2 - \epsilon$ and $\pi_1 = \phi_2 - \delta$, such that $\psi_2(p_2-\epsilon) < \pi_1 = \phi_2 - \delta < \phi_2$. As long as $\epsilon > 0$, $\psi_2(p_2-\epsilon) < \phi_2$ since ψ_2 is increasing in π_2 (easy to check), and then some $\delta > 0$ can be chosen to satisfy the above inequality. Further, as ϵ becomes small, the permissible δ will also become small.

With this deviation bribery of team 2 is not induced; but it brings monopoly in both markets. For ϵ and δ arbitrarily small, the deviation profit must be greater than $E\Pi_C$ (the reasoning for which is given in the text). This is a contradiction. Thus, at no prices bribery is prevented with certainty. **Q.E.D.**

Proof of Lemma 2. Consider $p_1 \le \tilde{p}_1 < 1/2$. That $h_1 \equiv \frac{(1-\lambda_1)p_1}{(1-\lambda_1 p_1)\Omega_1} < p_1$, i.e., $\frac{1-\lambda_1}{1-\lambda_1 p_1} < \Omega_1$ follows from the fact that at $p_1 = 1/2$ we have $\frac{1-\lambda_1}{1-\lambda_1 p_1} = \frac{2(1-\lambda_1)}{(2-\lambda_1)} < \Omega_1$ (by Assumption 3) and $\frac{1-\lambda_1}{1-\lambda_1 p_1}$ is increasing in p_1. **Q.E.D.**

Proof of Proposition 2. (i) Consider $p_1 \in [0,\, \tilde{p}_1)$ where $\tilde{p}_1 < 1/2$. If bribery of only the underdog (team 1, in this instance) were an equilibrium, we must have, by (3) and (4),

$$\begin{aligned} & \pi_2 \;\le\; h_1, \pi_2 < p_2, \\ \text{and} \qquad & \pi_1 \;>\; \psi_2(\pi_2) \ge p_1, \end{aligned}$$

where (π_1, π_2) corresponds to the symmetric price, if the markets are shared, and corresponds to the minimum price of each ticket, if prices are asymmetric leading to monopoly of at least one market.

Punter I will always bet on team 2, either by bribing team 1 or without bribing, and will never bet on team 1. Thus, if market 2 is shared, the expected profit of each bookie from ticket 2 (ticket index used as subscript in $E\Pi_2$) is

$$E\Pi_2 = \big[\frac{y}{2}(1-\pi_2) + \frac{z}{2}\big]\big[1 - \frac{\mu_1(1-\lambda_1 p_1) + (1-\mu_1)p_2}{\pi_2}\big],$$

and if market 2 is not shared, one of the bookies gets $2E\Pi_2$ from market 2.

In either case, since $\pi_2 < p_2 < (1-\lambda_1 p_1)$, it must be that $E\Pi_2 < 0$ at all π_2 that permits bribery of team 1 alone. Therefore, this cannot be part of an equilibrium: a bookie can always deviate and set a higher price for ticket 2 dumping all of his ticket 2 sale onto the other bookie and avoid this loss.

A symmetric argument will apply for $p_1 \in (\hat{p}_1, 1]$ which is same thing as $p_2 \in [0, \tilde{p}_2]$, ruling out bribery of only team 2 which is the weak team. Finally, as Proposition 1 has shown, for $p_1 \in [\tilde{p}_1, \hat{p}_1]$ there cannot be any bribery in equilibrium, thus completing the proof.

(ii) Consider $p_1 \le \tilde{p}_1 < 1/2$. If bribery of either teams is to be induced in equilibrium, we must have, by (3) and (4),

$$\begin{aligned} & \pi_2 \;\le\; h_1, \;\; \pi_2 < p_2, \\ \text{and} \qquad & p_1 < \pi_1 \;\le\; \psi_2(\pi_2), \end{aligned}$$

where (π_1, π_2) refers to the symmetric price, if the markets are shared, and refers to the minimum price of each ticket, if prices are asymmetric leading to monopoly of at least one market.

Lemma 2 together with $\tilde{p}_1 < \frac{1}{2}$ imply $h_1 < p_1 < p_2$, and by construction $\psi_2(\pi_2) < \phi_2 < p_2$ (see Fig. 2). Therefore, the equilibrium prices $\pi_1 + \pi_2 \leq \psi_2(\pi_2) + h_1 < p_2 + p_1 = 1$ (the strict inequality follows by Lemma 2), contradicting Lemma 1. **Q.E.D.**

Proof of Lemma 3. Suppose not, and assume there is a positive-profit equilibrium. Suppose the prices are symmetric and they are (π_1, π_2). Given Proposition 2, we only need to consider the case where only the favorite (i.e., team 2) is bribed. There are three possibilities for a positive-profit equilibrium: positive profit (i) from both tickets, (ii) from ticket 1 only (zero profit from ticket 2), and (iii) from ticket 2 only (zero profit from ticket 1).

In all scenarios we must have, by (3) and (4),

$$\begin{aligned} p_1 &< \pi_1 \leq \psi_2(\pi_2), \\ h_1 &< \pi_2 < p_2, \\ \pi_1 &\geq \pi_{10}, \;\; \pi_2 \geq \pi_{20}, \quad \text{with at least one strict inequality} \\ \text{and} \quad \pi_1 + \pi_2 &> 1, \end{aligned}$$

where (π_{10}, π_{20}) yield zero expected profit on each ticket (for both duopoly and monopoly). Formally, (π_{10}, π_{20}) is obtained from (5) and (6).

If scenario (i) or (ii) occurs in equilibrium, then a bookie can slightly undercut ticket 1 price, leave the bribery incentives unchanged and capture market 1; this will be a profitable deviation. So neither (i) nor (ii) can be part of an equilibrium.

So consider scenario (iii). Suppose (π_{10}, π_2) is an equilibrium price vector such that profit from ticket 1 is zero and profit from ticket 2 is strictly positive. We will then have, by (3) and (4), $h_1 < \pi_2 < p_2$, and either (iii.a) $p_1 < \pi_{10} < \psi_2(\pi_2)$, or (iii.b) $p_1 < \pi_{10} = \psi_2(\pi_2)$.

In the case of (iii.a), a bookie can profitably deviate by slightly reducing π_2 to $\pi_2 - \epsilon$ while leaving π_1 unchanged and maintaining $\pi_1 < \psi_2(\pi_2 - \epsilon)$ (for ϵ small enough). Ticket 2 (which yields positive profit) will be monopolized, while ticket 1 still yields zero profit; clearly this will be a profitable deviation. Thus, (iii.a) is ruled out.

Finally, consider (iii.b), and the following deviation: π_2 is reduced to $\pi_2 - \epsilon$ and π_1 is reduced to $\pi_{10} - \delta$ such that $\pi_{10} - \delta = \psi_2(\pi_2 - \epsilon)$, where ϵ is small. Both tickets 1 and 2 will be monopolized by the deviating bookie, and bribery incentives will be unaffected: $\delta(\epsilon) = \pi_{10} - \psi_2(\pi_2 - \epsilon)$ is a continuous and increasing function of ϵ (as $\psi_2(.)$ is continuous and increasing), and in particular as $\epsilon \to 0$, $\delta(\epsilon) \to 0$.

Now compare the gain in market 2 with the loss in market 1. The gain in market 2 is

$$
\begin{aligned}
EG(\epsilon) &= (1-\pi_2+\epsilon)\big[1-\frac{p_2^b}{\pi_2-\epsilon}\big]y+(1-\mu_2)z\big[1-\frac{p_2}{\pi_2-\epsilon}\big] \\
&\qquad -(1-\pi_2)\big[1-\frac{p_2^b}{\pi_2}\big]\frac{y}{2}-(1-\mu_2)\frac{z}{2}\big[1-\frac{p_2}{\pi_2}\big] \\
&= (1-\pi_2)\big[1-\frac{p_2^b}{\pi_2-\epsilon}\big]\frac{y}{2}+(1-\mu_2)\frac{z}{2}\big[1-\frac{p_2}{\pi_2-\epsilon}\big]+A, \qquad (9) \\
&\text{where} \quad p_2^b=[\mu_2\lambda_2+(1-\mu_2)]p_2, \\
&\text{and} \quad A=\epsilon\big[y\big\{1-\frac{p_2^b}{2(\pi_2-\epsilon)}\big(1+\frac{1}{\pi_2}\big)\big\}-(1-\mu_2)\frac{z}{2}\frac{p_2}{\pi_2(\pi_2-\epsilon)}\big].
\end{aligned}
$$

In the market for ticket 1, the equilibrium price (π_{10}) yields zero profit. By deviating from this price one earns an expected profit from ticket 1 equal to

$$
\begin{aligned}
E\Pi_1^M(\epsilon) &= (1-\pi_{10}+\delta)\big[1-\frac{p_1^b}{\pi_{10}-\delta}\big]y+\mu_2 z\big[1-\frac{(1-\lambda_2p_2)}{\pi_{10}-\delta}\big] \\
&= (1-\pi_{10})\big[1-\frac{p_1^b}{\pi_{10}-\delta}\big]y+\mu_2 z\big[1-\frac{(1-\lambda_2p_2)}{\pi_{10}-\delta}\big]+\delta\big[1-\frac{p_1^b}{\pi_{10}-\delta}\big]y, \qquad (10)
\end{aligned}
$$

where $p_1^b=\mu_2(1-\lambda_2p_2)+(1-\mu_2)p_1$. As $\epsilon\to 0$, $\delta(\epsilon)\to 0$, and from (10) it is clear that $E\Pi_1^M$ approaches twice the level of expected profit per bookie from ticket 1 market before the deviation, which is equal to zero. On the other hand, from (9) we see that $\lim_{\epsilon\downarrow 0}EG(\epsilon)=(1-\pi_2)\big[1-\frac{p_2^b}{\pi_2}\big]\frac{y}{2}+(1-\mu_2)\frac{z}{2}\big[1-\frac{p_2}{\pi_2}\big]>0$ (as π_2 yields positive profit for ticket 2, by the premise of the equilibrium). In the limit, the overall gain from deviation to a bookie in the two markets combined is $\lim_{\epsilon\downarrow 0}EG(\epsilon)>0$, hence (iii.b) cannot arise in equilibrium.

Now suppose firms charge different prices on each ticket, and let (π_1,π_2) be the minimum of each prices. Then by similar reasoning positive profits from ticket 1, and positive from both tickets are both ruled out. For the case of positive profit only from ticket 2 we can apply the same reasoning as above. In particular, if $\pi_1=\psi_2(\pi_2)$, one bookie can undercut in both markets and again it can be shown that loss in market 1 is approximately zero, but gain in market 2 is distinctly positive, as argued above for the symmetric case. **Q.E.D.**

Proof of Lemma 4. Suppose not. Suppose there is an equilibrium (π_1,π_2) with $\pi_1\le p_1$ and $\pi_2\ge p_2$, and it induces bribery of team 2. Suppose prices are asymmetric giving rise to monopoly in market 1. Then the expected profit from ticket 1 is

$$E\Pi_1^b=(1-\pi_1)y\big[1-\frac{\mu_2(1-\lambda_2p_2)+(1-\mu_2)p_1}{\pi_1}\big]+\mu_2 z\big[1-\frac{(1-\lambda_2p_2)}{\pi_1}\big].$$

Alternatively, if prices are symmetric, expected (duopoly) profit from ticket 1 is $E\Pi^{bd} = E\Pi_1^b/2$. In either case, since $\pi_1 \leq p_1 < (1-\lambda_2 p_2)$, it follows that $E\Pi_1^b < 0$. Then a bookie can raise π_1 and avoid making losses, a contradiction. **Q.E.D.**

Based on Definition 3, next we establish a technical result on (π_1^{M0}, π_2^{M0}) to be used below in the proof of Proposition 3.

Lemma 5 (Existence and Uniqueness of (π_1^{M0}, π_2^{M0})). *The bound π_1^{M0} exists and it is unique, if and only if $\tilde{\pi}_1 \geq \frac{y}{1+y}$.*

Proof. Recall from Definition 3, (π_1^{M0}, π_2^{M0}) simultaneously solve $\pi_1' = \psi_2(\pi_2')$ and $E\Pi^M = 0$, where $E\Pi^M$ is given by (7). Substituting $\pi_2 = 1 - \pi_1$ in (7) write

$$E\Pi^M = \frac{1}{\pi_1(1-\pi_1)}\big[-(1+y)\pi_1^2 + (y + p_1(1+y))\pi_1 - p_1 y\big].$$

Below we first observe certain properties of the $E\Pi^M(.,.)$ function with direct reference to the (π_2, π_1)-plane so that the proof of this lemma can be understood with the help of Figs. 5a and 5b; these Figures are not exhaustive though.

Properties:

[1] There are only two solutions to $E\Pi^M(\pi_1, \pi_2) = 0$ and $\pi_1 + \pi_2 = 1$, viz.,

$$(\pi_1 = p_1, \pi_2 = p_2), \quad (\pi_1 = \frac{y}{1+y}, \pi_2 = \frac{1}{1+y}).$$

Given these and since $E\Pi^M\big|_{\pi_1+\pi_2=1} < 0$ at $\pi_1 = 0$ and $\pi_1 = 1$, we conclude that:

If $p_1 < \frac{y}{1+y}$ then

$$E\Pi^M > 0 \qquad \text{at all } \ \pi_1 \in (p_1, \frac{y}{1+y}) \ \text{ and } \ \pi_2 = 1 - \pi_1;$$
$$E\Pi^M < 0 \qquad \text{at all } \ \pi_1 < p_1 \ \text{ as well as } \ \pi_1 > \frac{y}{1+y} \ \underline{\text{and}} \ \pi_2 = 1 - \pi_1.$$

Alternatively, if $p_1 > \frac{y}{1+y}$ then

$$E\Pi^M > 0 \qquad \text{at all } \ \pi_1 \in (\frac{y}{1+y}, p_1) \ \text{ and } \ \pi_2 = 1 - \pi_1;$$
$$E\Pi^M < 0 \qquad \text{at all } \ \pi_1 < \frac{y}{1+y} \ \text{ as well as } \ \pi_1 > p_1 \ \underline{\text{and}} \ \pi_2 = 1 - \pi_1.$$

[2] Graphically, on the (π_2, π_1)-plane the iso-profit curve $E\Pi^M = 0$ intersects the $\pi_1 + \pi_2 = 1$ at two points identified in property [1]. The segment of the iso-profit curve lying strictly below the $\pi_1 + \pi_2 = 1$ line violates the Dutch-book restriction and therefore is discarded in our search for (π_1^{M0}, π_2^{M0}), as indicated by the dotted part (of the curve) in Figs. 5a,b.

[3] $E\Pi^M$ is continuous at all $\{(\pi_1, \pi_2) | \pi_1 + \pi_2 \geq 1, \pi_1 \leq 1, \pi_2 \leq 1\}$. Further, from $\frac{\partial E\Pi^M}{\partial \pi_2} = -y + \frac{p_2}{{\pi_2}^2}$ we can conclude that

$E\Pi^M$ *is increasing* (*decreasing*) *in* π_2 *at all* $\pi_2 < (>) \sqrt{p_2/y}$.

Similarly, from $\frac{\partial E\Pi^M}{\partial \pi_1} = y\big[-1 + \frac{p_1}{{\pi_1}^2}\big]$ we can conclude that

$E\Pi^M$ *is increasing* (*decreasing*) *in* π_1 *at all* $\pi_1 < (>) \sqrt{p_1}$.

[4] Consider a subset of the feasible prices: $\pi_2 \leq p_2$ and $\pi_1 \geq p_1$ in the region $\pi_1 + \pi_2 \geq 1$. Then properties [1] and [3] together imply that all points to the right of the iso-profit curve $E\Pi^M = 0$ yield $E\Pi^M > 0$ and all points to the left of the iso-profit curve $E\Pi^M = 0$ yield $E\Pi^M < 0$ (since $E\Pi^M$ is increasing in π_2).

[5] We also specifically note the following ((*b*) follows from property [4]):

(a) At $\pi_2 = p_2$ and $\pi_1 = 1$, $\quad E\Pi^M = 0$.

(b) Given $\pi_2 = p_2$, at all $\pi_1 \in (p_1, 1)$ we have $\quad E\Pi^M = y\frac{(\pi_1 - p_1)}{\pi_1}(1 - \pi_1) > 0$; in particular, at $\pi_2 = p_2$ and $\pi_1 = \phi_2$, $\quad E\Pi^M > 0 \quad$ (since $p_1 < \phi_2 < 1$ at $p_1 < \tilde{p}_1$). ||

Next, we search for (π_1, π_2) such that $E\Pi^M = 0$ where

$$\pi_2 \leq p_2, \;\; \pi_1 \geq p_1, \;\; \text{and} \;\; \pi_1 + \pi_2 \geq 1.$$

We claim the following:

For any $\pi_1 \geq \max\{p_1, \frac{y}{1+y}\}$, *if* $E\Pi^M(\pi_1, \bar{\pi}_2 = 1 - \pi_1) \leq 0$ *then there exists a unique* $\pi_2 \leq p_2$ *such that* $E\Pi^M(\pi_1, \pi_2) = 0$.

By property [1], at any $\pi_1 \geq \max\{p_1, \frac{y}{1+y}\}$ we have $E\Pi^M(\pi_1, \bar{\pi}_2 = 1 - \pi_1) \leq 0$, and by property [5.*b*] we have $E\Pi^M(\pi_1, p_2) > 0$. Now, since $E\Pi^M(., \pi_2)$ is continuous in π_2 over $[1 - \pi_1, \, p_2]$ for any $\pi_1 \geq \max\{p_1, \frac{y}{1+y}\}$, by applying the intermediate value theorem we can conclude the following:

[A1] *For every* $\pi_1 \in [\max\{p_1, \frac{y}{1+y}\}, 1]$, *there must exist some* $\hat{\pi}_2(\pi_1) \in [1 - \pi_1, \, p_2]$ *such that*

$$E\Pi^M(\hat{\pi}_2(\pi_1), \pi_1) = 0.$$

Implicitly $\hat{\pi}_2(\pi_1)$ solves $E\Pi^M(\pi_1, \pi_2) = 0$. Note in particular that $\hat{\pi}_2(\pi_1 = p_1) = p_2$, $\hat{\pi}_2(\pi_1 = 1) = p_2$, $\hat{\pi}_2(\frac{y}{1+y}) = \frac{1}{1+y}$ and also $\hat{\pi}_2(\phi_2) < p_2$ (the last inequality being implied by properties [5.b] and [4]). The solution $\hat{\pi}_2(\pi_1)$ is also unique because, by property [3], $E\Pi^M$ is increasing in $\pi_2 < \sqrt{p_2/y}$. This establishes our above claim.

Further, by applying the implicit function theorem we can conclude:

[A2] $\hat{\pi}_2(.)$ *is a continuous function.*

(Implicit function theorem is applicable since $\frac{\partial E\Pi^M}{\partial \pi_2} \neq 0$.)

Now we are going to show that (refer Definitions 2 and 3):

> *If* $\tilde{\pi}_1 \geq \frac{y}{1+y}$ *(or equivalently* $\tilde{\pi}_2 \leq \frac{1}{1+y}$*), there must exist a unique value of* $\pi_1 \geq \tilde{\pi}_1$*, namely* π_1^{M0}*, such that* $\pi_1 = \psi_2(\hat{\pi}_2(\pi_1))$.

First consider the case $p_1 > \frac{y}{1+y}$. Since $\tilde{\pi}_1 > p_1$, it follows that $\tilde{\pi}_1 > \frac{y}{1+y}$. It is also clear that $\psi_2(\hat{\pi}_2(p_1)) = \psi_2(p_2) = \phi_2 > p_1$, and $\psi_2(\hat{\pi}_2(\phi_2)) < \phi_2$ because $\hat{\pi}_2(\phi_2) < p_2$ (as observed above) and $\psi_2(.)$ is increasing in π_2. Now define the composite function

$$\eta(\pi_1) = \psi_2(\hat{\pi}_2(\pi_1)) - \pi_1,$$

which will be continuous in π_1 (using **[A2]**), and using the fact that $\eta(p_1) > 0$ and $\eta(\phi_2) < 0$, we can appeal to the intermediate value theorem to conclude that there must exist at least one $\pi_1^{M0} \in (p_1, \phi_2)$ such that $\eta(\pi_1^{M0}) = 0$. Denote $\pi_2^{M0} = \hat{\pi}_2(\pi_1^{M0})$; it can be verified that $\pi_2^{M0} \in (\tilde{\pi}_2, p_2)$. (Note that this case is not covered by Figs. 5a,b.)

Next, consider the case $p_1 \leq \frac{y}{1+y}$, and continue to assume that $\tilde{\pi}_1 \geq \frac{y}{1+y}$ (or equivalently $\tilde{\pi}_2 \leq \frac{1}{1+y}$). We note that $\psi_2(\hat{\pi}_2(\frac{y}{1+y})) = \psi_2(\frac{1}{1+y}) \geq \psi_2(\tilde{\pi}_2) = \tilde{\pi}_1 \geq \frac{y}{1+y}$. That is, $\eta(\frac{y}{1+y}) \geq 0$, where $\eta(\pi_1)$ is the same composite function as defined in the earlier case. On the other hand, $\psi_2(\hat{\pi}_2(\phi_2)) < \phi_2$ (already noted) implying $\eta(\phi_2) < 0$. Given that $\tilde{\pi}_1 < \phi_2$ (by definition) and $\tilde{\pi}_1 \geq \frac{y}{1+y}$, we have $\frac{y}{1+y} < \phi_2$. By the intermediate value theorem, once again there exists at least one $\pi_1^{M0} \in [\frac{y}{1+y}, \phi_2)$ such that $\eta(\pi_1^{M0}) = 0$. Again, denote $\pi_2^{M0} = \hat{\pi}_2(\pi_1^{M0})$. (This case is shown in Fig. 5b.)

At the minimal (as well as maximal) π_1^{M0} such that $\eta(\pi_1^{M0}) = 0$, it is easy to see that we must have $\eta'(\pi_1) < 0$. This implies $\psi_2'(.)\frac{\mathrm{d}\hat{\pi}_2}{\mathrm{d}\pi_1} < 1$. Since $\frac{1}{\psi_2'(\pi_2)} = {\psi_2}^{-1\prime}(\pi_1)$ (because of continuity and monotonicity of $\psi_2(.)$), at π_1^{M0} the following must hold when $\frac{\mathrm{d}\hat{\pi}_2}{\mathrm{d}\pi_1} > 0$:

$$\frac{\mathrm{d}\hat{\pi}_2}{\mathrm{d}\pi_1} < {\psi_2}^{-1\prime}(.), \quad \text{ or equivalently } \quad \left.\frac{\mathrm{d}\pi_1}{\mathrm{d}\pi_2}\right|_{E\Pi^M=0} > \psi_2'(.). \tag{11}$$

Note that this condition is met at π_1^{M0} and the inequality is maintained thereafter. That is to say, π_1^{M0} is unique. This is confirmed by checking the curvatures of the zero iso-profit curve and the $\psi_2(.)$ curve and the fact that $\hat{\pi}_2(\phi_2) < p_2$ and ${\psi_2}^{-1}(\phi_2) = p_2$ (see Fig. 5b). On the (π_2, π_1)-plane the iso-profit curve is concave when rising and convex when declining. This can be verified (which we leave out) by differentiating the following slope expression:

$$\left.\frac{\mathrm{d}\pi_1}{\mathrm{d}\pi_2}\right|_{E\Pi^M=0} = \left(\frac{{\pi_1}^2}{{\pi_2}^2}\right)\left[\frac{\frac{p_2}{y} - {\pi_2}^2}{{\pi_1}^2 - p_1}\right].$$

As shown in Fig. 5b these two curves can intersect only once and the iso-profit curve will cut the bribery indifference curve from below, which confirms the slope condition (11). If they were to intersect twice, the iso-profit curve must fall below the bribery indifference curve at $\pi_2 = p_2$; but we know that is impossible, because at $\pi_2 = p_2$ and $\pi_1 \in (p_1, \phi_2)$ the no-bribery monopoly profit is strictly positive by property [5.b].

On the other hand, if $\tilde{\pi}_1 < \frac{y}{1+y}$, which is shown in Fig. 5a, at all $\pi_1 \in [\tilde{\pi}_1, \frac{y}{1+y})$, $E\Pi^M(\pi_1, 1-\pi_1) \geq 0$ (by property [1]) and therefore there cannot be any $\hat{\pi}_2(\pi_1)$ such that $\psi_2(\hat{\pi}_2(\pi_1)) = \pi_1$, because $\hat{\pi}_2(\pi_1)$ does not exist (in our feasible region). That is to say, π_1^{M0} does not exist. Now consider any $\pi_1 \in [\frac{y}{1+y}, \phi_2]$, if this range is non-empty. Here too, π_1^{M0} does not exist because $\eta(\pi_1) < 0$. **Q.E.D.**

Proof of Proposition 3 (Match-fixing Equilibrium). The uniqueness of (π_{10}, π_{20}) is guaranteed by the uniqueness of solution to eqs. (5) and (6) (see footnote 24). The three conditions together are both necessary and sufficient.

The necessity of the first two conditions is obvious. If $\pi_{10} > \psi_2(\pi_{20})$ and/or $\pi_{20} \leq h_1$, team 2 will not be bribed and/or team 1 will be bribed due to violations of conditions (3) and/or (4). For the third condition, the necessity part will be established along with sufficiency. For the first two deviations, only sufficiency needs to be established. In what follows we address each deviation.

1. **Ruling out of deviation (i)**. Suppose bookie 1 engages in deviation **(i)**. As a result, he captures all of ticket 2 sale. As for ticket 1 market, there are three subcases:

   (a) $\pi_1' > \pi_{10}$

   (b) $\pi_1' = \pi_{10}$

   (c) $\pi_1' < \pi_{10}$.

In subcase (a), bookie 1 off-loads all of ticket 1 sale to bookie 2. Ticket 2 will be bought by punter I (whether he manages to bribe team 1 or not) and a section of the ordinary punters. The deviation profit (i.e. the monopoly expected profit) of bookie 1 from the sale of ticket 2 is

$$\left[y(1-\pi_2')+z\right]\left[1-\frac{\mu_1(1-\lambda_1 p_1)+(1-\mu_1)p_2}{\pi_2'}\right].$$

This expression is negative because $\pi_2' < \pi_{20} < p_2 < \mu_1(1-\lambda_1 p_1)+(1-\mu_1)p_2$. Thus deviation subcase (a) is ruled out.

In the subcases (b) and (c), bookie 1 at least shares the market for ticket 1. That is, he will be selling both tickets. We know, under this deviation $\pi_2' \leq h_1$ and by Lemma 2 $h_1 < p_1$, so together $\pi_2' < p_1$. As for ticket 1 price, $\pi_{10} \leq \psi_2(\pi_{20}) < \phi_2$ for $\pi_{20} < p_2$ (from (2) and (3), it follows that $\psi_2(\pi_{20}) < \phi_2$). Also, $\phi_2 < p_2$ for $\tilde{p}_1 < 1/2$ (see Fig. 4), and so $\pi_{10} < p_2$. Thus $\pi_1' + \pi_2' \leq \pi_{10} + \pi_2' < p_2 + p_1 = 1$, which is a violation of the Dutch-book restriction, hence the deviations (b) and (c) cannot be feasible.

2. **Ruling out of deviation (ii)**. Next, consider deviation **(ii)** (again by bookie 1), so that $p_1 < \min\{\pi_1', \pi_{10}\} \leq \psi_2(\pi_2')$ and $\pi_2' \leq h_1 < p_1$ (the last inequality follows from Lemma 2). Punter I will bribe whichever team he gets access to, and bet on the other team; and if I gets access to neither team, he will bet on team 2. As before bookie 1 will capture all of market 2, and will face one of three following scenarios in market 1 depending on π_1' :

   (d) $\pi_1' > \pi_{10}$

   (e) $\pi_1' = \pi_{10}$

   (f) $\pi_1' < \pi_{10}$.

   As indicated in footnote 25, π_1' can be higher or lower than (or equal to) π_{10}. If $\pi_1' < \pi_{10}$ then the deviating bookie also captures all of market 1; if $\pi_1' = \pi_{10}$ then he shares market 1 equally with bookie 2; if $\pi_1' > \pi_{10}$ then he off-loads all of market 1 to bookie 2 (who is sticking to his prices). We must deter all these three possibilities within type-**(ii)** deviation.

   First consider the subcases (e) and (f), where $\pi_1' \leq \pi_{10}$. Here too bookie 1 sells both tickets, and once again we will have the violation of the Dutch-book restriction: $\pi_1' + \pi_2' \leq \pi_{10} + \pi_2' < p_2 + p_1 = 1$ (note that $\pi_{10} < p_2$ follows precisely the same way as derived above when ruling out subcases (b) and (c) for deviation **(i)**). So the deviations (e) and (f) will not be feasible.

Finally, consider subcase (d) where bookie 1 does not sell ticket 1. Bookie 1's payoff from deviation is calculated as follows:

$$(1-\pi_2')\big[1-\frac{\mu_2\lambda_2 p_2+\mu_1(1-\lambda_1 p_1)+(1-\mu_1-\mu_2)p_2}{\pi_2'}\big]y$$
$$+(1-\mu_1-\mu_2)z\big[1-\frac{p_2}{\pi_2'}\big]+\mu_1 z\big[1-\frac{1-\lambda_1 p_1}{\pi_2'}\big]. \tag{12}$$

Profit from punter I (i.e., the last two bracketed terms of (12)) is negative, given that $\pi_2' < p_2 < (1-\lambda_1 p_1)$. The profit from the naive punters (which is given by the first term) will also be negative if

$$\pi_2' < \mu_2\lambda_2 p_2+\mu_1(1-\lambda_1 p_1)+(1-\mu_1-\mu_2)p_2. \tag{13}$$

We claim that the RHS of (13) is strictly greater than p_1, i.e.,

$$\begin{array}{rrcl} & p_2\big[(1-\mu_2)+\mu_2\lambda_2\big]+p_1\big[\mu_1-\mu_1\lambda_1\big] & > & p_1, \\ \text{or,} & p_2\big[(1-\mu_2)+\mu_2\lambda_2\big] & > & p_1\big[(1-\mu_1)+\mu_1\lambda_1\big], \end{array}$$

which is true by Assumption 4. On the other hand, using Lemma 2, $\pi_2' \leq h_1 < p_1$. So (13) is established and hence profit from the naive punters is negative, making the proposed deviation unprofitable.

3. **Ruling out of deviation (iii)**. Finally, consider deviation **(iii)** (again by bookie 1), following which there will be no attempt at bribery. There are two cases to consider.

   Case 1 ($\tilde{\pi}_1 < \frac{y}{1+y}$). Deviation to no-bribery prices is ruled out if and only if $\pi_{10} \leq \tilde{\pi}_1$. First consider sufficiency. We begin by noting that since $\pi_{10}+\pi_{20} > 1$, $\pi_{10} \leq \tilde{\pi}_1$ implies $\pi_{20} > \tilde{\pi}_2 (= 1-\tilde{\pi}_1)$. Now let the deviation prices (π_1', π_2') be such that $\pi_1' \leq \pi_{10} \leq \tilde{\pi}_1$, $\pi_2' < \pi_{20}$, and contrary to the claim suppose $\pi_1' > \psi_2(\pi_2')$, which can be written as $\pi_2' < \psi_2^{-1}(\pi_1')$. But $\psi_2^{-1}(\pi_1') \leq \psi_2^{-1}(\tilde{\pi}_1) = \tilde{\pi}_2$, and hence $\pi_2' < \tilde{\pi}_2$. Therefore, $\pi_1'+\pi_2' < 1$ which violates the Dutch-book restriction, and thus this deviation is not possible.

   Now consider the necessity part. Suppose $\pi_{10} > \tilde{\pi}_1$ (as in w' in Fig. 5a); then there is a pair of prices (π_1', π_2') such that $\pi_1' \leq \pi_{10}$, $\pi_2' < \pi_{20}$, satisfying $\pi_1' > \psi_2(\pi_2')$, the Dutch-book restriction, and yielding, as shown in the proof of Lemma 5, $E\Pi^M > 0$ (see point d in Fig. 5a). Thus, the proposed deviation is profitable. ||

Case 2 ($\tilde{\pi}_1 \geq \frac{y}{1+y}$). Deviation to no-bribery prices is ruled out if and only if $\pi_{10} \leq \pi_1^{M0}$. We begin with sufficiency. Suppose there is a pair of prices (π_1', π_2') such that $\pi_1' \leq \pi_{10}$, $\pi_2' < \pi_{20}$, satisfying $\pi_1' > \psi_2(\pi_2')$, $\pi_1' + \pi_2' \geq 1$ and yielding, contrary to our claim, $E\Pi^M > 0$. As $E\Pi^M(\pi_1', \pi_2') > 0$, we must have $\hat{\pi}_2(\pi_1') < \pi_2'$ (because by reducing π_2', profit can be reduced to zero; see Fig. 5b). Then we should also have $\psi_2(\hat{\pi}_2(\pi_1')) < \pi_1'$ leading to $\eta(\pi_1') = \psi_2(\hat{\pi}_2(\pi_1')) - \pi_1' < 0$. But this is a contradiction to the fact that $\eta(\pi_1') \geq 0$ at $\pi_1' \leq \pi_1^{M0}$ (as π_1^{M0} was the minimal π_1 at which $\eta = 0$, as argued in the proof of Lemma 5).

Now consider the necessity part. Suppose $\pi_{10} > \pi_1^{M0}$ (as in w' in Fig. 5b); then there is a price pair (π_1', π_2') such that $\pi_1' \leq \pi_{10}$, $\pi_2' < \pi_{20}$, satisfying $\pi_1' > \psi_2(\pi_2')$, the Dutch-book restriction, and yielding $E\Pi^M > 0$ (see point d in Fig. 5b). Thus, the proposed deviation is profitable. ||

This completes the proof of Proposition 3. **Q.E.D.**

Proof of Proposition 4. Below we derive conditions that would guarantee the particular type of positive-profit, price coordination equilibrium in which bribery is prevented. Example 2 towards the end of this Appendix shows that the conditions are not vacuous.

Slight undercutting on both tickets: By undercutting on both tickets, $\pi_1' \in (p_1, \phi_2), \pi_2' \in (p_2, \phi_1)$, bookie 1 earns the following profit:

$$
\begin{aligned}
E\Pi_{BI} \;&=\; \mu_1 \underbrace{y\Big[\int_{\pi_1'}^{1} \Big(1 - \frac{\lambda_1 p_1}{\pi_1'}\Big)\,\mathrm{d}q + \int_0^{1-\pi_2'} \Big(1 - \frac{(1-\lambda_1 p_1)}{\pi_2'}\Big)\,\mathrm{d}q\Big]}_{\equiv k_1} \\
&\quad + \mu_2 \underbrace{y\Big[\int_{\pi_1'}^{1} \Big(1 - \frac{(1-\lambda_2 p_2)}{\pi_1'}\Big)\,\mathrm{d}q + \int_0^{1-\pi_2'} \Big(1 - \frac{\lambda_2 p_2}{\pi_2'}\Big)\,\mathrm{d}q\Big]}_{\equiv k_2} \\
&\quad + (1-\mu_1-\mu_2)y\Big[3 - \pi_1' - \pi_2' - \frac{p_1}{\pi_1'} - \frac{p_2}{\pi_2'}\Big] \\
&\quad + z\Big[\mu_1\Big\{1 - \frac{(1-\lambda_1 p_1)}{\pi_2'}\Big\} + \mu_2\Big\{1 - \frac{(1-\lambda_2 p_2)}{\pi_1'}\Big\}\Big]. \qquad (14)
\end{aligned}
$$

Let $\pi_1' = \phi_2 - \epsilon_1$ and $\pi_2' = \phi_1 - \epsilon_2$, ϵ_1 and ϵ_2 both arbitrarily small. Since $y[3 - \pi_1' - \pi_2' - \frac{p_1}{\pi_1'} - \frac{p_2}{\pi_2'}] \approx 2k$, we rewrite (14) as:

$$
E\Pi_{BI} \approx \mu_1 k_1 + \mu_2 k_2 + (1-\mu_1-\mu_2)2k - z\Big[\mu_1 \underbrace{\frac{(1-\lambda_1 p_1)}{\phi_1}}_{>1;\ \phi_1 < 1-\lambda_1 p_1} + \mu_2 \underbrace{\frac{(1-\lambda_2 p_2)}{\phi_2}}_{>1;\ \phi_2 < 1-\lambda_2 p_2} - (\mu_1+\mu_2)\Big]. \quad (15)
$$

The first (and second) term(s) in (15) indicate expected profit from naive punters when team 1 (team 2) is bribed. The third term captures the no-bribery profit; this is twice the bribe prevention duopoly profit due to monopolization of both markets. The fourth term is the expected net payout to punter I which is positive-valued. The overall value of $E\Pi_{BI}$ <u>*varies inversely*</u> with $\mu_1 + \mu_2$, if one changes μ_1 and μ_2 in the same proportion. If $\mu_1 + \mu_2$ is sufficiently large (say, $\mu_1 + \mu_2 \to 1$) the magnitude of $E\Pi_{BI}$ will crucially depend on the magnitude of $\mu_1 k_1 + \mu_2 k_2$. If $\max\{k_1, k_2\}$ is not too large relative to k (or is smaller than k), then clearly $E\Pi_{BI} < k = E\Pi^d_{BP}$. On the other hand, by letting $\mu_1 + \mu_2 \to 0$ we will get $E\Pi_{BI} = 2k > k$. Therefore, by the intermediate value theorem:

$$\textit{There exists } \mu_1 + \mu_2 = \bar{\mu} \textit{ such that } E\Pi_{BI}(\bar{\mu}) = k.$$

Thus, slight undercutting on both tickets is unprofitable, if $\mu_1 + \mu_2 \geq \bar{\mu}$. This uses the observation that $E\Pi_{BI}$ is inversely related to $\mu_1 + \mu_2$, holding μ_1/μ_2 fixed. For instance, see Fig. 7 towards the end of this Appendix: starting from any (μ_1, μ_2) such that $\mu_1 + \mu_2 = 0.25$, if one moves North-East along the ray connecting the origin, this specific deviation will be ruled out.

Slight undercutting on ticket 1 alone: Now consider the possibility that the price of ticket 1 is reduced below ϕ_2, while the price of ticket 2 is held at ϕ_1. The market for ticket 1 is captured, but then team 2 will be bribed with probability μ_2 in which case punter I will bet on ticket 1. Let us set in equation (15), $\pi'_2 = \phi_1$, $\mu_1 = 0$, and adjust for sharing of market 2. Then we write the first bookie's deviation payoff as

$$\begin{aligned} E\Pi_{BI} \;\; &\approx \;\; \mu_2 k_2 + (1-\mu_2)k - \mu_2 z[\frac{(1-\lambda_2 p_2)}{\phi_2} - 1] \\ &\quad -\mu_2 \frac{y}{2}\{(1-\phi_1)(1-\frac{\lambda_2 p_2}{\phi_1})\} + (1-\mu_2)\frac{y}{2}\{(1-\phi_2)(1-\frac{p_1}{\phi_2})\}. \end{aligned}$$

The first, third and fourth terms together capture the bribery profit; here since market 2 is not captured, the profit is less than k_2. The second and fifth terms together give the profit in the event of no-bribery. Here there is a gain over the duopoly bribe prevention profit, k, due to undercutting in market 1. The third term indicates the net loss to punter I. Therefore, so long as the sum of the last three terms is negative, we will have $E\Pi_{BI} < k$ provided μ_2 satisfies the following condition:

$$\mu_2 \geq \frac{\frac{y}{2}\big\{(1-\phi_2)(1-\frac{p_1}{\phi_2})\big\}}{\frac{y}{2}\big\{(1-\phi_2)(1-\frac{p_1}{\phi_2})\big\} + \frac{y}{2}\big\{(1-\phi_1)(1-\frac{\lambda_2 p_2}{\phi_1})\big\} + z\big\{\frac{(1-\lambda_2 p_2)}{\phi_2} - 1\big\} + (k-k_2)} \equiv \mu_2^*.$$

Upon simplification we get

$$\mu_2^* = \frac{y\{(1-\phi_2)(1-\frac{p_1}{\phi_2})\}}{yp_2(1-\lambda_2)\left[\frac{2}{\phi_2}-\frac{1+\phi_1}{\phi_1}\right]+2z\Omega_2(1-\lambda_2 p_2)}. \tag{16}$$

Slight undercutting on ticket 2 alone: The analysis is similar to the previous case. Now ticket 2 price is lowered slightly below ϕ_1, while $\pi_1' = \phi_2$. Bookie 1's deviation profit can be calculated (by setting $\mu_2 = 0$ in (15)) as

$$\begin{aligned} E\Pi_{BI} \;\approx\;& \mu_1 k_1 + (1-\mu_1)k - \mu_1 z\left[\frac{(1-\lambda_1 p_1)}{\phi_1} - 1\right] \\ & -\mu_1 \frac{y}{2}\{(1-\phi_2)(1-\frac{\lambda_1 p_1}{\phi_2})\} + (1-\mu_1)\frac{y}{2}\{(1-\phi_1)(1-\frac{p_2}{\phi_1})\}. \end{aligned}$$

The deviation can be ruled out if

$$\mu_1 \geq \frac{\frac{y}{2}\{(1-\phi_1)(1-\frac{p_2}{\phi_1})\}}{\frac{y}{2}\{(1-\phi_1)(1-\frac{p_2}{\phi_1})\}+\frac{y}{2}\{(1-\phi_2)(1-\frac{\lambda_1 p_1}{\phi_2})\}+z\{\frac{(1-\lambda_1 p_1)}{\phi_2}-1\}+(k-k_1)} \equiv \mu_1^*.$$

Upon simplification we get

$$\mu_1^* = \frac{y\{(1-\phi_1)(1-\frac{p_2}{\phi_1})\}}{yp_1(1-\lambda_1)\left[\frac{2}{\phi_1}-\frac{1+\phi_2}{\phi_2}\right]+2z\Omega_1(1-\lambda_1 p_1)}. \tag{17}$$

Large-scale undercutting on both tickets: However, the above conditions do not apply to large-scale deviations. What if the prices are significantly reduced and profit rises? Let ρ denote the probability of team 1 winning (from the bookie's point of view) when either team may be bribed, where $\rho = \mu_1\lambda_1 p_1 + \mu_2(1-\lambda_2 p_2) + (1-\mu_1-\mu_2)p_1$.

The deviating bookie's bribe inducement problem is to maximize

$$E\Pi_{BI} = y\left[3-\pi_1-\pi_2-\frac{\rho}{\pi_1}-\frac{(1-\rho)}{\pi_2}\right] - z\left[\mu_1\frac{(1-\lambda_1 p_1)}{\pi_2}+\mu_2\frac{(1-\lambda_2 p_2)}{\pi_1}-(\mu_1+\mu_2)\right], \tag{18}$$

subject to $p_1 \leq \pi_1 < \phi_2$ and $p_2 \leq \pi_2 < \phi_1$.

The unconstrained solutions (ignoring the two constraints) are:

$$\pi_1^* = \sqrt{\rho + \frac{z\mu_2(1-\lambda_2 p_2)}{y}}, \qquad \pi_2^* = \sqrt{(1-\rho) + \frac{z\mu_1(1-\lambda_1 p_1)}{y}}.$$

If $\pi_1^* \geq \phi_2$ and $\pi_2^* \geq \phi_1$ as in (8), then $E\Pi_{BI}$ must be non-decreasing at $\pi_1 \leq \phi_2$ and $\pi_2 \leq \phi_1$.

Therefore, the deviating bookie would like to capture both markets only by undercutting slightly.

There are two other possible deviations – large-scale undercutting on ticket 1 only, and large-scale undercutting on ticket 2 only. For the specific equilibrium in this proposition, it can be shown that such deviations cannot yield higher profits for the bookies given the two conditions in (8) (see Supplementary material). **Q.E.D.**

Example 1. To illustrate the existence of the equilibrium in Proposition 3, we consider some numerical parameter configurations in Table 2 that will satisfy the (sufficient) equilibrium conditions.

Suppose conditions are symmetric for both teams. First assume $\Omega_1 = \Omega_2 = 2$, $\mu_1 = \mu_2 = 0.15$, $\lambda_1 = \lambda_2 = 0.5$, and $z = 0.3$ (and $y = 0.7$); see the first column. In the second column λ_1, λ_2 are reduced to 0.3. In the third column λ_1 and λ_2 are both further reduced to zero, but at the same time z is raised to 0.5 (and y lowered to 0.5) leading to a fall in Ω_1 and Ω_2 to 1.2.

For the parameter specification in column 1, $\tilde{p}_1 = 0.28$ and $\hat{p}_1 = 0.72$. This column represents the case of $\tilde{\pi}_1 < \frac{y}{1+y} = 0.41$. Now consider $p_1 = 0.05 < \tilde{p}_1$. At this probability the zero-profit prices of ticket 1 and ticket 2 are 0.15 and 0.935 respectively, obtained by solving equations (5) and (6). π_{20} is strictly less than $p_2 = 0.95$ and it gives rise to the highest bribe inducement price of π_1, $\psi_2 = 0.255$; ψ_2 is defined in (3). That ψ_2 is strictly greater than $\pi_{10} = 0.15$ implies that if team 2 is accessed it will be bribed. Further, that team 1 will not be bribed is evident from the fact that $\pi_{20} > h_1 = 0.013$. Further, it is ensured that undercutting on both tickets and inducing bribery of either team, are not possible. Minimum prices to do so ($\pi_2 = h_1 = 0.013$ and $\pi_1 = \psi_2(h_1) = 0.007$) do not satisfy the Dutch-book restriction. It is also not possible to deviate to 'no-bribery' scenario, because $\pi_{10} < \tilde{\pi}_1 = 0.23$.

Also note that the zero-profit prices lie within the intervals specified earlier: $\pi_{20} > \mu_2\lambda_2 p_2 + (1-\mu_2)p_2 = (1-p_1^b) = 0.87$ and $\mu_2(1-\lambda_2 p_2) + (1-\mu_2)p_1 = p_1^b = 0.13 < \pi_{10} < (1-\lambda_2 p_2) = 0.525$.

In the two successive rows (in column 1) we consider two higher values of p_1 ($p_1 = 0.10$ and $p_1 = 0.16$ respectively). With higher p_1 the gap between the two (zero profit) prices gets narrower; π_{10} increases and π_{20} decreases. But in all cases $\pi_{20} < p_2$, and π_{10} remains strictly less than $\psi_2(\pi_{20})$, confirming the inducement of bribery of team 2. Bribery of team 1 is also ruled out for π_2 being greater than h_1. Deviation to no-bribery is also ruled out because $\pi_{10} < \tilde{\pi}_1$. However, at higher p_1 beyond 0.16 one of the constraints will be violated and our proposed equilibrium prices don't exist.

Table 2: Zero profit, bribe inducement (of team 2) prices

Parameters		Parameters		Parameters	
$\Omega_1=\Omega_2=2$, $z=0.3$, $\mu_1=\mu_2=0.15$, $\lambda_1=\lambda_2=0.5 \rightarrow$ $\tilde{p}_1=0.28$, $\hat{p}_1=0.72$ $y/(1+y)=0.41$		$\Omega_1=\Omega_2=2$, $z=0.3$, $\mu_1=\mu_2=0.15$, $\lambda_1=\lambda_2=0.3 \rightarrow$ $\tilde{p}_1=0.3$, $\hat{p}_1=0.7$ $y/(1+y)=0.41$		$\Omega_1=\Omega_2=1.2$, $z=0.5$, $\mu_1=\mu_2=0.15$, $\lambda_1=\lambda_2=0 \rightarrow$ $\tilde{p}_1=0.45$, $\hat{p}_1=0.55$ $y/(1+y)=0.33$	
Bribe inducement range of p_1: $p_1 < 0.28$		Bribe inducement range of p_1: $p_1 < 0.3$		Bribe inducement range of p_1: $p_1 < 0.45$	
Prob.	**Zero profit prices**	**Prob.**	**Zero profit prices**	**Prob.**	**Zero profit prices**
$p_1=0.05$ $p_2=0.95$ $p_1^b=0.13$ $p_2^b=0.87$	$\pi_{10}=0.15$, $\pi_{20}=0.935$ Constraints: $\tilde{\pi}_1=0.23$ $\Psi_2(\pi_{20})=0.255$ $h_1=0.013$ $\Psi_2(h_1)=0.007$	$p_1=0.05$, $p_2=0.95$ $p_1^b=0.15$ $p_2^b=0.85$	$\pi_{10}=0.192$, $\pi_{20}=0.935$ Constraints: $\tilde{\pi}_1=0.263$ $\Psi_2(\pi_{20})=0.292$ $h_1=0.018$ $\Psi_2(h_1)=0.013$	$p_1=0.05$ $p_2=0.95$ $p_1^b=0.19$ $p_2^b=0.81$	$\pi_{10}=0.35$, $\pi_{20}=0.94$ Constraints: $\tilde{\pi}_1=0.37$ $\pi_1^{M0}=0.385$ $\Psi_2(\pi_{20})=0.452$ $h_1=0.042$ $\Psi_2(h_1)=0.042$
$p_1=0.10$ $p_2=0.90$ $p_1^b=0.17$ $p_2^b=0.83$	$\pi_{10}=0.196$, $\pi_{20}=0.885$ Constraints $\tilde{\pi}_1=0.24$ $\Psi_2(\pi_{20})=0.264$ $h_1=0.026$ $\Psi_2(h_1)=0.015$	$p_1=0.10$, $p_2=0.90$ $p_1^b=0.20$ $p_2^b=0.80$	$\pi_{10}=0.236$, $\pi_{20}=0.875$ Constraints: $\tilde{\pi}_1=0.27$ $\Psi_2(\pi_{20})=0.293$ $h_1=0.036$ $\Psi_2(h_1)=0.028$	$p_1=0.10$ $p_2=0.90$ $p_1^b=0.24$ $p_2^b=0.76$	$\pi_{10}=0.385$, $\pi_{20}=0.91$ Constraints: $\tilde{\pi}_1=0.378$ $\pi_1^{M0}=0.396$ $\Psi_2(\pi_{20})=0.457$ $h_1=0.083$ $\Psi_2(h_1)=0.083$
$p_1=0.16$ $p_2=0.84$ $p_1^b=0.22$ $p_2^b=0.78$	$\pi_{10}=0.251$, $\pi_{20}=0.82$ Constraints: $\tilde{\pi}_1=0.254$ $\Psi_2(\pi_{20})=0.264$ $h_1=0.043$ $\Psi_2(h_1)=0.028$	$p_1=0.13$, $p_2=0.87$ $p_1^b=0.22$ $p_2^b=0.78$	$\pi_{10}=0.26$, $\pi_{20}=0.875$ Constraints: $\tilde{\pi}_1=0.275$ $\Psi_2(\pi_{20})=0.299$ $h_1=0.047$ $\Psi_2(h_1)=0.037$	$p_1=0.11$ $p_2=0.89$ $p_1^b=0.24$ $p_2^b=0.76$	$\pi_{10}=0.393$, $\pi_{20}=0.9$ Constraints: $\tilde{\pi}_1=0.379$ $\pi_1^{M0}=0.397$ $\Psi_2(\pi_{20})=0.459$ $h_1=0.09$ $\Psi_2(h_1)=0.09$
Comments: (i) At or above $p_1=0.17$ up to $p_1=0.28$ no (pure strategy) equilibrium exists. (ii) The bribe inducement equilibrium exists at all $p_1<0.17$.		Comments: (i) At or above $p_1=0.15$ up to $p_1=0.30$ no (pure strategy) equilibrium exists. (ii) The bribe inducement equilibrium exists at all $p_1<0.15$		Comments: (i) At or above $p_1=0.12$ up to $p_1=0.45$ no (pure strategy) equilibrium exists. (ii) The bribe inducement equilibrium exists at all $p_1<0.12$.	

Column 2 shows the effect of a decline in λ_i, while still representing the case $\tilde{\pi}_1 < \frac{y}{1+y}$. Here λ_1 and λ_2 fall to 0.3 and corruption becomes more costly. If punter I bribes team 2 and bets on ticket 1, the bookies' expected loss to punter I will rise; to offset that they must get a higher expected profit from the naive punters. Therefore, π_{10} must rise. At $p_1 = 0.05$, π_{10} rises to 0.192. Here too we see that all the relevant constraints are satisfied to ensure that no possible deviation will occur to undermine the zero-profit bribe inducement equilibrium.

In column 3, we consider a special case, where $\lambda_1 = \lambda_2 = 0$; that bribing will lead to certain defeat of the team. We also increase the size of the influential punter's wealth z from 0.3 to 0.5; this implicitly reduces Ω_1 and Ω_2 to 1.2 and $\frac{y}{1+y}$ to 0.33. Here we get $\tilde{\pi}_1 > \frac{y}{1+y}$ so that the relevant upper bound (for preventing no-bribery deviation) is π_1^{M0} instead of $\tilde{\pi}_1$. Greater z and smaller (λ_1, λ_2) enhance the potential damage from bribery; for the same reasoning applied to column 2, price of ticket 1 increases further. In fact, it rises sharply to 0.35. Here, $\pi_1^{M0} > \tilde{\pi}_1$ in all three values of p_1 (0.05, 0.10, and 0.11). π_{10} is strictly less than π_1^{M0} in all three situations. All other relevant constraints are also satisfied. If one considers $p_1 \geq 0.12$, it is easy to see that our proposed (pure strategy) equilibrium does not exist.

In the last row, we note the ranges of p_1 (restricting our attention only to $p_1 \leq \tilde{p}_1$) at which the bribe inducement equilibrium does not exist. We also know that there cannot be any other (pure strategy equilibrium). Comparing column 1 with column 2 we see that as λ falls from 0.5 to 0.3 the impossibility of an equilibrium increases. The range for the non-existence of equilibrium increases from $[0.17, 0.28]$ to $[0.15, 0.30]$. Further, in column 3 where λ drops to zero (and also z increases) the range for 'no equilibrium' significantly expands to $[0.12, 0.45]$. It is also worth emphasizing that if the bribe inducement equilibrium exists at some p_1, then at all smaller values of p_1 (assuming $p_1 < \tilde{p}_1$) the bribe inducement equilibrium should exist. This is an insight we gain from our numerical exercise. ||

Example 2. Suppose $\lambda_1 = \lambda_2 = 0$, $\Omega_1 = \Omega_2 = \Omega$. Then $\phi_2 = \phi_1 = \frac{1}{1+\Omega}$, and $\tilde{p}_1 = \frac{1}{1+\Omega}$ and $\hat{p}_1 = \frac{\Omega}{1+\Omega}$. If $\Omega < 1$, then $\hat{p}_1 < \tilde{p}_1$. As in Proposition 4, we consider $p_1 \in (\hat{p}_1, \tilde{p}_1)$.

Next, it can be shown that at the proposed bribe prevention equilibrium $\pi_1 = \pi_2 = \frac{1}{1+\Omega}$, each bookie's expected profit $E\Pi_{BP} = \frac{y}{2}\left[\frac{\Omega(1-\Omega)}{1+\Omega}\right]$. Now consider a unilateral deviation from the proposed equilibrium by slight undercutting on both tickets. This gives an expected profit approximately, $E\Pi_{BI} = y\left[\frac{\Omega(1-\Omega)}{1+\Omega}\right] - z(\mu_1 + \mu_2)\Omega$. Such deviation is unprofitable, if $E\Pi_{BP} \geq E\Pi_{BI}$ or equivalently,

$$\frac{y}{2z}\left[\frac{1-\Omega}{1+\Omega}\right] \leq (\mu_1 + \mu_2).$$

Further, slight undercutting on ticket 1 and ticket 2 each is unprofitable if $\mu_2 > \mu_2^*$ and

Table 3: Feasible (μ_1,μ_2) for positive profit, bribe prevention

Parameter specification – Case 1: $\lambda_1=\lambda_2=0$, $z=0.25$, $\Omega=0.8 \rightarrow$ $\hat{p}_1 = \frac{\Omega}{1+\Omega} = 0.44$, $\tilde{p}_1 = \frac{1}{1+\Omega} = 0.56$ $\pi_1 = \pi_2 = 0.56$		**Parameter specification – Case 2:** $\lambda_1=\lambda_2=0$, $z=0.4$, $\Omega=0.5 \rightarrow$ $\hat{p}_1 = \frac{\Omega}{1+\Omega} = 0.33$, $\tilde{p}_1 = \frac{1}{1+\Omega} = 0.67$ $\pi_1 = \pi_2 = 0.67$	
	$\mu_1 + \mu_2 \geq 0.167$, and		$\mu_1 + \mu_2 \geq 0.25$, and
p_1=0.45	$\mu_2 \geq 0.087$, $\mu_1 \geq 0.005$ $\mu_2 \geq -0.16 + 0.51\mu_1$, $\mu_2 \leq 0.44 + 1.42\mu_1$	p_1=0.36	$\mu_2 \geq 0.155$, $\mu_1 \geq 0.016$ $\mu_2 \geq 0.065 + 0.28\mu_1$ $\mu_2 \leq 0.31 + 1.60\mu_1$
p_1=0.47	$\mu_2 \geq 0.071$, $\mu_1 \geq 0.022$ $\mu_2 \geq -0.19 + 0.54\mu_1$ $\mu_2 \leq 0.42 + 1.51\mu_1$	p_1=0.43	$\mu_2 \geq 0.124$, $\mu_1 \geq 0.055$ $\mu_2 \geq 0.012 + 0.35\mu_1$ $\mu_2 \leq 0.22 + 1.924\mu_1$
p_1=0.5	$\mu_2 \geq 0.048$, $\mu_1 \geq 0.048$ $\mu_2 \geq -0.23 + 0.6\mu_1$ $\mu_2 \leq 0.38 + 1.67\mu_1$	p_1=0.5	$\mu_2 \geq 0.09$, $\mu_1 \geq 0.09$ $\mu_2 \geq -0.05 + 0.43\mu_1$ $\mu_2 \leq 0.11 + 2.33\mu_1$
p_1=0.52	$\mu_2 \geq 0.031$, $\mu_1 \geq 0.064$ $\mu_2 \geq -0.26 + 0.64\mu_1$ $\mu_2 \leq 0.36 + 1.78\mu_1$	p_1=0.56	$\mu_2 \geq 0.06$, $\mu_1 \geq 0.12$ $\mu_2 \geq -0.10 + 0.51\mu_1$ $\mu_2 \leq -0.01 + 2.79\mu_1$
p_1=0.54	$\mu_2 \geq 0.014$, $\mu_1 \geq 0.079$ $\mu_2 \geq -0.29 + 0.68\mu_1$ $\mu_2 \leq 0.33 + 1.9\mu_1$	p_1=0.63	$\mu_2 \geq 0.022$, $\mu_1 \geq 0.151$ $\mu_2 \geq -0.18 + 0.61\mu_1$ $\mu_2 \leq -0.20 + 3.50\mu_1$

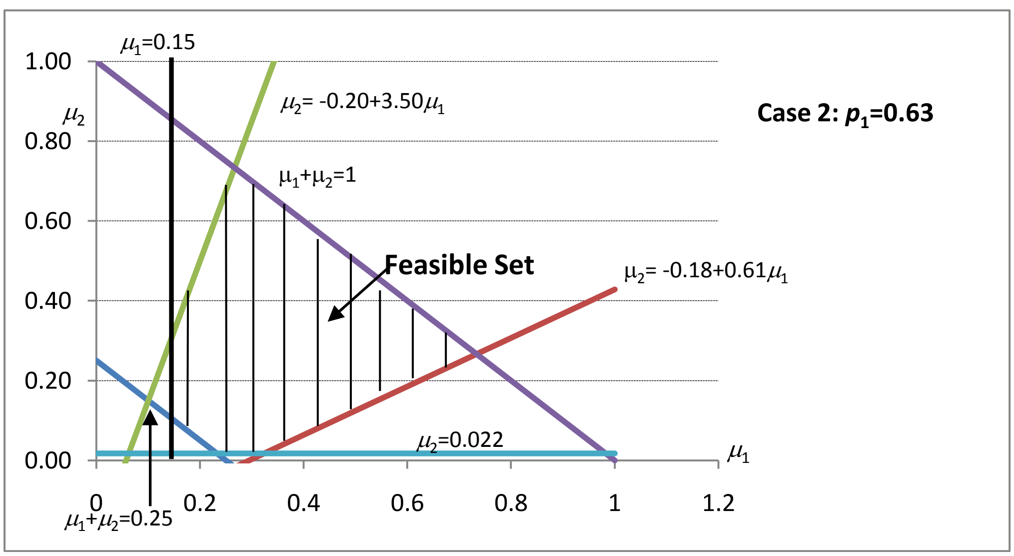

Figure 7: Feasible access probabilities

$\mu_1 > \mu_1^*$ respectively, where μ_2^* and μ_1^* are given in (16) and (17) respectively. For this example, these critical values of μ_2 and μ_1 reduce to

$$\mu_2^* = \frac{y[1 - p_1(1+\Omega)]}{(1+\Omega)[yp_2 + 2z]}, \qquad \mu_1^* = \frac{y[1 - p_2(1+\Omega)]}{(1+\Omega)[yp_1 + 2z]}.$$

In addition, to prevent large-scale undercutting condition (8) must be satisfied, which in this case give rise to the following two restrictions:

$$\begin{aligned} \mu_2 &\geq \frac{y}{yp_2 + z}\left[\frac{1}{(1+\Omega)^2} - p_1\right] + \frac{yp_1}{[yp_2 + z]}\mu_1, \\ \mu_2 &\leq \left[1 - \frac{1}{p_2(1+\Omega)^2}\right] + \frac{[yp_1 + z]}{yp_2}\mu_1. \end{aligned}$$

Now we construct two sets of numerical examples in Table 3 and verify that the set of (μ_1, μ_2), which satisfies each of the above-mentioned conditions, is indeed non-empty. Two columns present two cases or examples each with a series of probability values considered. In column 1, we set $w + \alpha(f + f_I) = 0.2$ and $z = 0.25$. This specification gives rise to $\Omega = 0.8$, and $\hat{p}_1 = \frac{\Omega}{1+\Omega} = 0.44$, $\tilde{p}_1 = \frac{1}{1+\Omega} = 0.56$. Moreover, $\phi_2 = \phi_1 = \frac{1}{1+\Omega} = 0.56$, which is equal to our proposed equilibrium prices. We then consider several values of p_1 from the interval $(0.44, 0.56)$ and show that at each of these p_1 there is a non-empty set of μ_1 and μ_2 such that no deviation from $\pi_1 = \pi_2 = 0.56$ is profitable. Since these prices exceed p_1 and p_2, expected profit for each bookie under bribe prevention is strictly positive. Under this parameter specification one constraint on (μ_1, μ_2) that would commonly occur at all $p_1 \in (0.44, 0.56)$ is $\mu_1 + \mu_2 \geq 0.167$; this ensures that slight undercutting on *both* tickets is not profitable. Then there are four additional constraints to examine, which will vary depending on p_1.

Consider $p_1 = 0.45$. There are individual restrictions on μ_2 and μ_1 to rule out undercutting on a single ticket, as given in (16) and (17). The other two constraints are given by (8), which rules out large-scale undercutting. The same constraints are then reproduced at higher values of p_1 in the interval $(\hat{p}_1, \tilde{p}_1)$. As can be seen, in each cases, the feasible set of (μ_1, μ_2) is non-empty.

Next, in column 2 we set $z = 0.4$ leaving everything else unchanged. As punter I's wealth increases, Ω falls (to 0.5) leading to an expansion of the interval $(\hat{p}_1, \tilde{p}_1)$ to $(0.33, 0.67)$. Here, slight undercutting on both tickets is prevented at all (μ_1, μ_2) such that $\mu_1 + \mu_2 \geq 0.25$. Other constraints on μ_1 and μ_2 are derived considering five different values of p_1 from 0.36 and 0.63. At each of these values of p_1 the feasible set of (μ_1, μ_2) that would support the proposed bribe prevention equilibrium is shown to be non-empty. Fig. 7 illustrates the case of $p_1 = 0.63$ on the (μ_1, μ_2) plane for the scenario considered in column 2, i.e. $(z = 0.4, \Omega = 0.5)$. ||

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Supplementary material

Large-scale deviation proofness (Supporting materials for Proposition 4)

We consider two other deviations – large-scale undercutting on ticket 1 only, and large-scale undercutting on ticket 2 only, which we show can be ruled out if large-scale undercutting on both tickets is ruled out.

Large-scale undercutting on ticket 2 alone: As before, we should take into account of undercutting only on one ticket. Suppose only ticket 2 is undercut, and ticket 1's price is held at ϕ_2. In this case, punter I will bribe team 1 (on access) and bet on team 2. Market 1 is shared, but market 2 is fully captured. Let the probability of team 1 winning in this case be denoted as ρ_1. We can easily calculate

$$\rho_1 = \mu_1\lambda_1 p_1 + (1-\mu_1)p_1.$$

Now maximize

$$E\Pi_{BI} = \frac{y}{2}\left[(1-\phi_2)(1-\frac{\rho_1}{\phi_2}\right] + y\left[2-\rho_1-\pi_2-\frac{(1-\rho_1)}{\pi_2}\right] - \mu_1 z\left[\frac{(1-\lambda_1 p_1)}{\pi_2}-1\right]$$

with respect to π_2 subject to the constraints: $\pi_2 < \phi_1$, $\pi_1 = \phi_2$ and $\pi_2 + \phi_2 \geq 1$.

The unconstrained solution for π_2 is

$$\pi_2 = \sqrt{(1-\rho_1) + \frac{z\mu_1(1-\lambda_1 p_1)}{y}}.$$

Large-scale undercutting on ticket 1 alone: Similarly consider the case where only ticket 1 is undercut, and ticket 2's price is held at ϕ_1. Here, team 2 will be bribed (on access) and bets will be placed on team 1 (by punter I). Market 2 is shared, but market 1 is captured. Let the new probability of team 1 winning be denoted as ρ_2, where

$$\rho_2 = \mu_2(1-\lambda_2 p_2) + (1-\mu_2)p_1.$$

The bookie should then choose π_1 to maximize

$$E\Pi_{BI} = y\left[1+\rho_2-\pi_1-\frac{(\rho_2)}{\pi_1}\right] + \frac{y}{2}\left[(1-\phi_1)(1-\frac{(1-\rho_2)}{\phi_1}\right] - \mu_2 z\left[\frac{(1-\lambda_2 p_2)}{\pi_1}-1\right]$$

subject to the constraints: $\pi_1 < \phi_2$, $\pi_2 = \phi_1$ and $\pi_1 + \phi_1 \geq 1$.

The unconstrained solution for π_1 is

$$\pi_1 = \sqrt{\rho_2 + \frac{z\mu_2(1-\lambda_2 p_2)}{y}}.$$

It can be readily seen that since $\lambda_1 p_1 < p_1 < 1 - \lambda_2 p_2$, we have $\rho_1 < \rho < \rho_2$ and $(1-\rho_2) < (1-\rho) < (1-\rho_1)$. This implies

$$\begin{aligned}
\sqrt{\rho + \frac{z\mu_2(1-\lambda_2 p_2)}{y}} &< \sqrt{\rho_2 + \frac{z\mu_2(1-\lambda_2 p_2)}{y}}, \\
\sqrt{(1-\rho) + \frac{z\mu_2(1-\lambda_2 p_2)}{y}} &< \sqrt{(1-\rho_2) + \frac{z\mu_2(1-\lambda_2 p_2)}{y}}.
\end{aligned}$$

Therefore, if large-scale undercutting on both tickets is ruled out by ensuring $\sqrt{\rho + \frac{z\mu_2(1-\lambda_2 p_2)}{y}} \geq \phi_2$ and $\sqrt{(1-\rho) + \frac{z\mu_1(1-\lambda_1 p_1)}{y}} \geq \phi_1$, then large-scale single price undercutting is also ruled out. $||$

4.4 Bribe Inducement of Either Team

We assume $\hat{p_1} < \frac{1}{2} < \tilde{p}_1$ and restrict our attention to $p_1 \in (\hat{p_1}, \tilde{p}_1)$. There is a possible equilibrium in which bookies earn zero profit, but punter I will bribe either team upon access and bet on the other team that is not bribed; in the event of not getting access to any team he will bet on neither. We propose the equilibrium as follows.

Equilibrium $\breve{\mathcal{E}}$. *Suppose $p_1 \in (\hat{p}_1, \tilde{p}_1)$. Symmetric equilibrium prices are $(\breve{\pi}_{10}, \breve{\pi}_{20})$ with $p_2 < \breve{\pi}_{20} < \phi_1$ and $p_1 < \breve{\pi}_{10} < \phi_2$. On access punter I will bribe either team and bet on the rival of the bribed team; otherwise he will bet on neither. Each bookie makes zero expected profit in each market.*

By definition, $(\breve{\pi}_{10}, \breve{\pi}_{20})$ satisfy the following equations:

$$E\Pi_1^b = \frac{1}{2}\left\{(1-\breve{\pi}_{10})\left[1 - \frac{\rho}{\breve{\pi}_{10}}\right]y + \mu_2 z\left[1 - \frac{(1-\lambda_2 p_2)}{\breve{\pi}_{10}}\right]\right\} = 0 \tag{19}$$

$$E\Pi_2^b = \frac{1}{2}\left\{(1-\breve{\pi}_{20})\left[1 - \frac{(1-\rho)}{\breve{\pi}_{20}}\right]y + \mu_1 z\left[1 - \frac{(1-\lambda_1 p_1)}{\breve{\pi}_{20}}\right]\right\} = 0 \tag{20}$$

where $\rho = [\mu_1\lambda_1 p_1 + \mu_2(1-\lambda_2 p_2) + (1-\mu_1-\mu_2)p_1]$ is the ex-ante winning probability of team 1, when either team can be bribed. Clearly, when $(\breve{\pi}_{10}, \breve{\pi}_{20})$ exists, it must be that $\rho < \breve{\pi}_{10} < (1-\lambda_2 p_2)$ and $(1-\rho) < \breve{\pi}_{20} < (1-\lambda_1 p_1)$. Combining these restrictions with

$p_1 < \breve{\pi}_{10} < \phi_2$ and $p_2 < \breve{\pi}_{20} < \phi_1$ we write the following:

$$\max\{p_1, \rho\} < \breve{\pi}_{10} < \min\{\phi_2, (1 - \lambda_2 p_2)\}$$
$$\max\{p_2, (1 - \rho)\} < \breve{\pi}_{20} < \min\{\phi_1, (1 - \lambda_1 p_1)\}.$$

To sustain $(\breve{\pi}_{10}, \breve{\pi}_{20})$ as a Bertrand equilibrium we need to prevent all possible deviations. Undercutting on any prices that leaves bribery incentives of punter I unchanged will clearly be unprofitable. That is to say, if the deviation prices π_1' and π_2' are such that $\pi_1' \geq p_1$ and $\pi_2' \geq p_2$ the bribery incentives of punter I will not change (by the violation of bribe prevention condition (2)), and the deviating bookie will make a loss. Therefore, we need to consider only those deviations where the bribery scenario will change. These scenarios are the following.

(a) Deviation leading to bribery of team 1 alone: (i) π_2 is reduced to π_2' and π_1 is reduced to π_1' (or held unchanged at $\breve{\pi}_{10}$) such that $\pi_2' < p_2 \leq h_1$ or $\pi_2' \leq h_1 < p_2$ and $\pi_1' > \psi_2(\pi_2')$.

(ii) π_2 is reduced to π_2' (or held unchanged at $\breve{\pi}_{20}$) and π_1 is reduced to π_1' such that $h_2 < \pi_1' < p_1$ and $\pi_2' \leq \psi_1(\pi_1')$.

(b) Deviation leading to bribery of team 2 alone: (i) π_1 is reduced to π_1' and π_2 is reduced to π_2' (or held unchanged at $\breve{\pi}_{20}$) such that $\pi_1' < p_1 \leq h_2$ or $\pi_1' \leq h_2 < p_1$ and $\pi_2' > \psi_1(\pi_1')$.

(ii) π_1 is reduced to π_1' (or held unchanged at $\breve{\pi}_{10}$) and π_2 is reduced to π_2' such that $h_1 < \pi_2' < p_2$ and $\pi_1' \leq \psi_2(\pi_2')$.

(c) Deviation leading to bribery of neither teams, but punter I bets on team 2: π_2 is reduced to π_2' and π_1 is reduced to π_1' (or unchanged at $\breve{\pi}_{10}$) such that $h_1 < \pi_2' < p_2$ and $p_1 < \psi_2(\pi_2') < \pi_1'$.

(d) Deviation leading to bribery of neither teams, but punter I bets on team 1: π_1 is reduced to π_1' and π_2 is reduced to π_2' (or unchanged at $\breve{\pi}_{20}$) such that $h_2 < \pi_1' < p_1$ and $p_2 < \psi_1(\pi_1') < \pi_2'$.

Consider deviation **(a)**. In the proposed equilibrium $\breve{\pi}_{10} > p_1$ and $\breve{\pi}_{20} > p_2$; here team 2 is bribed because $\breve{\pi}_{10} < \phi_2$ and team 1 is bribed because $\breve{\pi}_{20} < \phi_1$. If the deviation prices are $\pi_2' > p_2$ and $\pi_1' > p_1$, these bribery incentives do not change. Therefore, to make team 2 immune from bribery, π_2 must be reduced below p_2 and it should be reduced to the extent that the bribery threshold level $\psi_2(\pi_2')$ (as dictated by the bet reversal condition) falls below π_1' $(\leq \breve{\pi}_{10})$. In other words, ticket 1 becomes too costly to bet on to finance bribery of team 2, given that team 2 can be bet on without committing bribery and the expected profit of punter I from doing so is strictly positive. Now at the same time if the bribery incentive for team 1 is to be retained, π_2 must be less than or equal to h_1 as dictated by the bet accentuation condition. Since in this case (i.e. $\hat{p}_1 < \frac{1}{2} < \tilde{p}_1$) h_1 may or may not be smaller than p_2. In part **(i)** we specified the required reduction in π_2 to make this deviation possible.

In part **(ii)** we specified an alternative configuration of prices to induce bribery of team 1. Here, π_1 is reduced below p_1 so that punter I can always bet on team 1. To induce bet reversal (i.e. to bribe team 1 and only then bet on team 2), π_2 needs to be appropriately reduced (or maintained) below $\psi_1(\pi_1')$; at the same time to make team 2 immune to bribery π_1' must be held above h_2 (provided of course $h_2 < p_2$). If, of course, $h_2 > p_2$ this deviation is not feasible.

Deviation **(b)** can be analogously explained. Here punter I never bribes team 1. In part **(b.i)** he always bets on team 1 (either by bribing team 2, or by not bribing). In part **(b.ii)** he bets on team 1 only when he bribes team 2; in the event of not bribing he bets on team 2. Deviations **(c)** and **(d)** describe two remaining scenarios where neither team is bribed, but punter I must be allowed to bet either on team 2 or on team 1, respectively.

It is quite likely that at a given parameter configuration not all deviations will be feasible. One can then investigate the feasible set of deviations, and ascertain necessary and sufficient conditions for the proposed equilibrium. Instead of doing that we specify a simple sufficient condition that would make *any* of these deviations unprofitable.

Proposition 5 (Bribe inducement of either teams). *Consider $p_1 \in (\hat{p}_1, \tilde{p}_1)$ and suppose $(\breve{\pi}_{10}, \breve{\pi}_{20})$ exists and $p_1 < \breve{\pi}_{10} < \phi_2$, $p_2 < \breve{\pi}_{20} < \phi_1$. $(\breve{\pi}_{10}, \breve{\pi}_{20})$ is an equilibrium if*

$$\breve{\pi}_{10} + \psi_2^{-1}(\breve{\pi}_{10}) < 1, \quad \textit{and} \quad \breve{\pi}_{20} + \psi_1^{-1}(\breve{\pi}_{20}) < 1. \tag{21}$$

Proof of Proposition 5. Define $\psi_2^{-1}(\breve{\pi}_{10}) = \pi_2^0$ and $\psi_1^{-1}(\breve{\pi}_{20}) = \pi_1^0$. By the continuity and monotonicity of $\psi_i(\pi_i)$ function π_2^0 and π_2^0 exist. Suppose bookie 1 deviates from the proposed equilibrium prices by reducing π_2 alone. If he wishes to take deviation type **(a.i)** he must reduce π_2 to *at least* π_2^0 (provided $\pi_2^0 < p_2$), because π_2^0 corresponds to $\breve{\pi}_{10} = \psi_2(\pi_2^0)$ at which punter I is just indifferent between betting on team 1 (by bribing team 2) and betting on team 2 (without engaging in bribery). By tie-breaking rule-I, punter I bribes. Since deviation **(a.i)** requires bet accentuation (i.e. bribing team 1, when team 2 can be bet on even without bribery) π_2 must be reduced below π_2^0 so that $\pi_2 < p_2 \leq h_1$ or $\pi_2 \leq h_1 < p_2$. But if $\breve{\pi}_{10} + \psi_2^{-1}(\breve{\pi}_{10}) < 1$ such deviation prices will violate the Dutch-book restriction and generate negative profit for bookie 1. Since this is the minimum price reduction considered, we can conclude that deviation **(a.i)** is not profitable; by the same logic deviations **(b.ii)** and **(c)** are also not profitable.

Next, consider $\psi_1^{-1}(\breve{\pi}_{20}) = \pi_1^0$. Suppose bookie 1 wishes to take deviation **(a.ii)**. He must reduce π_1 to *at least* π_1^0, because at π_1^0 (assuming $\pi_1^0 < p_1$), $\psi_1(\pi_1^0) = \breve{\pi}_{20}$ and then punter I is indifferent between bribing team 1 and betting on team 1. By tie-breaking rule-I he bribes team 1. But if $\breve{\pi}_{20} + \psi_1^{-1}(\breve{\pi}_{20}) < 1$, then such deviations will violate the Dutch-book restriction and also yield negative profit for bookie 1. Thus, deviations **(a.ii)** is ruled out, and similarly deviations **(b.i)**, **(d)** are also ruled out. **Q.E.D.**

Intuitively, for any deviation to take place at least one price needs to be reduced from the equilibrium level, and it must be reduced to a critical level. The critical level for π_2 is $\psi_2^{-1}(\breve{\pi}_{10})$ if π_1 is held unchanged, and similarly for π_1 the critical level is $\psi_1^{-1}(\breve{\pi}_{20})$ when π_2 is held unchanged. Consider a reduction in π_2. If it is reduced to say $\pi_2' = \psi_2^{-1}(\breve{\pi}_{10})$ (holding π_1 unchanged), then we have $\breve{\pi}_{10} = \psi_2(\pi_2')$. That is, punter I is indifferent between betting on team 2 or bribing team 2, by the bet reversal condition (3). By tie-breaking rule-I, indeed punter I will choose to bribe. Now if team 1 is *not* to be bribed, π_2' must be above h_1; see deviation **(b.ii)**. If $\breve{\pi}_{10} + \psi_2^{-1}(\breve{\pi}_{10}) = \breve{\pi}_{10} + \pi_2' < 1$, such deviation will be unprofitable due to the violation of the Dutch-book restriction. Alternatively, π_2 can be further reduced below $\psi_2^{-1}(\breve{\pi}_{10})$ (holding π_1 unchanged at $\breve{\pi}_{10}$), so that $\breve{\pi}_{10} > \psi_2^{-1}(\breve{\pi}_{10})$, then punter I will prefer to bet on team 2 and not bribe it. Further, to induce him to bribe team 1, π_2' must be less than h_1. This follows the specification of deviation **(a.i)**. Again, if $\breve{\pi}_{10} + \psi_2^{-1}(\breve{\pi}_{10}) < 1$ we will have a violation of the Dutch-book restriction.

Similarly, if π_2 is held unchanged at $\breve{\pi}_{20}$ and π_1 is reduced, then π_1 must be reduced to at least $\pi_1' = \psi_1^{-1}(\breve{\pi}_{20})$, so that punter I remains indifferent between bribing team 1 and betting on team 1; by tie-breaking rule-I he bribes. To eliminate this bribing incentive, π_1 needs to be reduced below $\psi_1^{-1}(\breve{\pi}_{20})$ and in addition to induce him to bribe team 2, π_1 must be less than h_2, as specified in deviation **(b.i)**. If $\breve{\pi}_{20} + \psi_1^{-1}(\breve{\pi}_{20}) < 1$, such a deviation will be unprofitable. Since any other deviations must involve similar (or greater) price reductions in either ticket 1 or ticket 2 or both, no deviations can be feasible or profitable, as long as condition (21) holds.

Example 3: Now to verify that an equilibrium specified in Proposition 5 exists, let us provide an example. Suppose $\lambda_1 = \lambda_2 = 0$, and $\Omega_1 = \Omega_2 = \Omega$. This implies $\phi_1 = \phi_2 = \frac{1}{1+\Omega}$.

Further, $\tilde{p}_1 = \frac{1}{1+\Omega}$, $\hat{p}_1 = \frac{\Omega}{1+\Omega}$; for $\hat{p}_1 < \frac{1}{2} < \tilde{p}_1$ we need $\Omega < 1$. The ex-ante probability of team 1 winning when both teams can be bribed is $\rho = \mu_2 + (1 - \mu_1 - \mu_2)p_1$. Now let us restrict our attention to $p_1 \in (\frac{\Omega}{1+\Omega}, \frac{1}{1+\Omega})$ and derive $\breve{\pi}_{10}$ and $\breve{\pi}_{20}$ by solving the following two equations:

$$
\begin{aligned}
E\Pi_1^b &= \frac{y}{2}(1-\pi_1)\left[1 - \frac{\rho}{\pi_1}\right] + \mu_2 \frac{z}{2}\left[1 - \frac{1}{\pi_1}\right] = 0 \\
E\Pi_2^b &= \frac{y}{2}(1-\pi_2)\left[1 - \frac{(1-\rho)}{\pi_2}\right] + \mu_1 \frac{z}{2}\left[1 - \frac{1}{\pi_2}\right] = 0.
\end{aligned}
$$

It is straightforward to check that

$$
\breve{\pi}_{10} = \rho + \mu_2 \frac{z}{y}, \qquad \breve{\pi}_{20} = 1 - \rho + \mu_1 \frac{z}{y}. \tag{22}
$$

We also need to impose the following restrictions $(\breve{\pi}_{10}, \breve{\pi}_{20})$:

$$
\begin{aligned}
p_1 &< \breve{\pi}_{10} < \phi_2 : \quad p_1 < \rho + \mu_2 \frac{z}{y} < \frac{1}{1+\Omega}, \\
p_2 &< \breve{\pi}_{20} < \phi_1 : \quad p_2 < 1 - \rho + \mu_1 \frac{z}{y} < \frac{1}{1+\Omega}.
\end{aligned}
$$

These two conditions together imply that p_1 must be in the following interval (after substituting $\rho = \mu_2 + (1 - \mu_1 - \mu_2)p_1$):

$$
1 - \frac{\mu_1}{\mu_1 + \mu_2}\left[1 + \frac{z}{y}\right] < p_1 < \frac{\mu_2}{\mu_1 + \mu_2}\left[1 + \frac{z}{y}\right], \tag{23}
$$

and, in addition, to ensure $\breve{\pi}_{io} < \phi_j$, Ω must satisfy the restriction

$$
\Omega < \frac{1}{\max\{\breve{\pi}_{10}, \breve{\pi}_{20}\}} - 1. \tag{24}
$$

Now let us derive the sufficient condition to prevent any deviations from the equilibrium prices $(\breve{\pi}_{10}, \breve{\pi}_{20})$. Recall from the bet reversal condition $\psi_2(\pi_2) = \frac{\pi_2}{p_2 + \Omega \pi_2}$ (when $\lambda_2 = 0$) and $\psi_1(\pi_1) = \frac{\pi_1}{p_1 + \Omega \pi_1}$ (when $\lambda_1 = 0$). Thus

$$
\begin{aligned}
\psi_2^{-1}(\breve{\pi}_{10}) &= \frac{p_2 \breve{\pi}_{10}}{1 - \Omega \breve{\pi}_{10}}, \\
\psi_1^{-1}(\breve{\pi}_{20}) &= \frac{p_1 \breve{\pi}_{20}}{1 - \Omega \breve{\pi}_{20}}.
\end{aligned}
$$

The condition (21) (i.e. $\breve{\pi}_{i0} + \psi_j^{-1}(\breve{\pi}_{i0}) < 1$) translates into the following two inequalities:

$$1 - (1 + \Omega + p_2)\breve{\pi}_{10} + \Omega\breve{\pi}_{10}^2 > 0, \tag{25}$$

$$1 - (1 + \Omega + p_1)\breve{\pi}_{10} + \Omega\breve{\pi}_{20}^2 > 0. \tag{26}$$

From (25) one obtains the following upper bound on Ω and $\breve{\pi}_{10}$:

$$\Omega \leq \frac{1 - \breve{\pi}_{10}(1 + p_2)}{\breve{\pi}_{10}(1 - \breve{\pi}_{10})}.$$

By symmetry from (26), we obtain

$$\Omega \leq \frac{1 - \breve{\pi}_{20}(1 + p_1)}{\breve{\pi}_{20}(1 - \breve{\pi}_{20})}.$$

By combining the above two conditions, we must have

$$\Omega \leq \min\left\{\frac{1 - \breve{\pi}_{10}(1 + p_2)}{\breve{\pi}_{10}(1 - \breve{\pi}_{10})}, \frac{1 - \breve{\pi}_{20}(1 + p_1)}{\breve{\pi}_{20}(1 - \breve{\pi}_{20})}\right\}. \tag{27}$$

Note that $\breve{\pi}_{10}$ and $\breve{\pi}_{20}$ do not directly depend on Ω, though it depends on z. As $\Omega = \frac{w+\alpha(f+f_I)}{z}$, for any given z, z being strictly greater than $w+\alpha(f+f_I)$, we can determine from condition (27) the highest value of $w + \alpha(f + f_I)$ to sustain our proposed bribe inducement equilibrium.

Numerical illustration: For illustration let us consider $\mu_1 = \mu_2 = 1/4$, $z = 1/6$, $y = 5/6$ and $\lambda_1 = \lambda_2 = 0$. With this specification the range of p_1 we can consider (by condition (23))

$$1 - \frac{1}{2}\left[1 + \frac{1}{5}\right] < p_1 < \frac{1}{2}\left[1 + \frac{1}{5}\right] \quad \text{or} \quad \frac{2}{5} < p_1 < \frac{3}{5}.$$

Let us first consider $p_1 = 1/2$, at which we get $\rho = 1/2$ and then we derive

$$\breve{\pi}_{10} = \breve{\pi}_{20} = \frac{11}{20} > p_1 = p_2.$$

Now calculate the range of Ω that supports this equilibrium. Condition (24) gives $\Omega < \frac{81}{99}$, and condition (27) gives $\Omega < 70/99$. So if $\Omega < 70/99 (= 0.707)$, or equivalently $w+\alpha(f+f_I) = \Omega z < 35/297 = 0.117$, our equilibrium is sustained.

Similarly, consider $p_1 = 0.45$ holding other parameters unchanged. At this p_1 we have

$\rho = 0.475$ and consequently the proposed bribe inducement prices are

$$\breve{\pi}_{10} = 0.525 > p_1(= 0.45) \quad \text{and} \quad \breve{\pi}_{20} = 0.575 > p_2(= 0.55).$$

Next, condition (24) gives $\Omega < \frac{1}{.575} - 1 = 0.74$, and condition (27) also gives $\Omega < \min\{0.68, 0.746\} = 0.68$. So if $\Omega < 0.68$, (i.e. $w + \alpha(f + f_I) = \Omega z < 34/300(= 0.113)$), our equilibrium is sustained.

Symmetrically, at $p_1 = 0.55$ we get

$$\breve{\pi}_{10} = 0.575 > p_1(= 0.55) \quad \text{and} \quad \breve{\pi}_{10} = 0.525 > p_2(= 0.45),$$

and the upper bound on Ω will be exactly same as in the case of $p_1 = 0.45$, i.e. $\Omega < 0.68$.

Finally consider the limiting case of $p_1 = 0.4$, at which $\rho = 9/20$. Equilibrium prices become

$$\breve{\pi}_{10} = 0.5 > p_1(= 0.40) \quad \text{and} \quad \breve{\pi}_{20} = 0.6 = p_2.$$

Condition (24) gives $\Omega < \frac{1}{.6} - 1 = 0.66$, while condition (27) gives $\Omega < \min\{\frac{4}{5}, \frac{4}{6}\} = 4/6 = 0.66$. Symmetrically at $p_1 = 0.6$ we get the same upper bound on Ω. Thus, we can conclude that under the parameter configuration we assumed, if $\Omega < 0.66$ at all $p_1 \in (0.4, 0.6)$ we can sustain the equilibrium where either team can be bribed and bookies make zero expected profit.