Exclusionary Discounts

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Abstract

We consider a two-period model with two sellers and one buyer in which the efficient outcome calls for the buyer to purchase one unit from each seller in each period. We show that when the buyer's valuations between periods are linked by switching costs and at least one seller is financially constrained, there are plausible conditions under which exclusion arises as the unique equilibrium outcome (the buyer buys both units from the same seller). The exclusionary equilibria are supported by price-quantity offers in which the excluding seller offers its second unit at a price that is below its marginal cost of production. In some cases, the price of this second unit is negative. Our findings contribute to the literatures on exclusive dealing, bundling, and loyalty rebates/payments.

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1   Introduction

Manufacturers often encourage retailers to promote their products by offering discounts if the retailers' purchases meet or exceed certain quantity or share thresholds. Sometimes the discounts apply only to the units that are purchased in excess of the threshold, while at other times, the discounts apply retroactively to all the units purchased, once the threshold is reached.[1] Although both types of discounts have the property that the retailer's incremental price is lower with higher quantities, the latter also has the defining feature that the retailer's outlay schedule jumps down at the threshold, implying that at the threshold the retailer faces a negative incremental price.

Generally the aforementioned discounts are a sign of healthy price competition, and are often required by buyers as a price of doing business; for example, the lower incremental prices may create powerful incentives for a retailer to deploy market strategies that expand output and increase welfare (see Mills, 2004). But, in other circumstances, these discounts may be anticompetitive; the lower incremental prices may instead create incentives for a retailer to promote the sale of products on which it is eligible to earn a discount primarily at the expense of other, substitute products, which would have been more preferred by consumers. As we will show in this paper, when implemented by a dominant firm, who may have easier access to financing than its rival or rivals, these discounts can sometimes exclude equally-efficient rivals, misallocate resources, and lower overall welfare.

First-mover incumbent advantages, 'mistakes' due to uncertainty, linear pricing in contracts, and economies of scale, have all been advanced previously to explain how inefficient exclusion can arise in a vertical setting (see, e.g., Aghion and Bolton, 1987; Mathewson and Winter, 1987; Rasmusen et al, 1991; Innes and Sexton, 1994; and Segal and Whinston, 2000a). In contrast, in this paper, we abstract from these explanations by considering a simple two-period model with two manufacturers and one retailer in which there is complete information and the efficient outcome calls for the retailer to buy one unit from each manufacturer in each period. We show that when the retailer's—or his customers'—valuations between periods are linked by switching costs and at least one seller is financially constrained, there are plausible conditions under which exclusion (the retailer buys from only one manufacturer) arises as the unique equilibrium outcome, even though there are no economies of scale, the sellers move simultaneously, and nonlinear pricing is feasible.[2]

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[1]For example, in the former case, a manufacturer might specify a price of $3 per unit for the first 999 units and a price of $2.50 per unit thereafter, whereas in the latter case, the manufacturer might specify a price of $3 per unit for purchases of less than 1000 units and a price of $2.50 per unit for all purchases of 1000 or more units.

[2]Importantly, exclusive-dealing provisions are not needed to obtain this outcome. This result challenges traditional

We focus in particular in this paper on the role of negative prices in supporting equilibrium outcomes—when are they necessary, when are they superfluous, and when can they not support equilibria. We find, for example, that negative prices can arise (under some conditions) in both efficient and exclusionary equilibria, but whereas they cannot always support efficient equilibria (when these equilibria exist), they can always support exclusionary equilibria (when these equilibria exist). More generally, we find that a common feature of all exclusionary equilibria is that they are supported by price-quantity offers in which the excluding firm offers to sell its incremental unit for a price that is below its marginal cost of production. In some cases, we find that the excluding firm must offer this unit at a negative price if it is to exclude its rival; in other cases, it need not.

That allegedly harmful discounts can arise in both efficient and exclusionary equilibria is an attractive feature of the model. However, it begs the question whether it is possible ex-ante to distinguish procompetitive discounts from anticompetitive discounts. If the dividing line is drawn too aggressively, one risks chilling price competition that would ultimately benefit consumers in the form of lower retail prices, but if the line is drawn the line too passively, the dominant manufacturer may be able to exclude its rival, which leads to fewer product choices and reduces overall welfare.

We examine these tradeoffs by considering first the consequences in our model of a ban on offers in which a seller charges a price that is below its marginal cost of production. We then consider the welfare effects of a ban on negative prices. We find that the former always yields a first-best outcome in the model (the buyer will always purchase one unit from each seller). In contrast, we find that the latter does not always eliminate the possibility of exclusion. It does, however, reduce the set of circumstances under which such equilibria arise, and in that sense, it too is welfare improving. The latter is also easier to implement than the former when marginal costs are unknown or not easily verified, a consideration that is important in minimizing Type 2 errors.

Our findings contribute to several literatures. The majority of models of exclusionary conduct come from the literature on exclusive dealing, where a dominant manufacturer is explicit in its requirement of exclusivity.[3] In these models (as in our model), the dominant firm finds itself in an environment in which profits are linked across time or across markets and in which there

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antitrust analysis governing exclusive-dealing arrangements, which tends to focus on whether a manufacturer has an explicit contractual arrangement that requires its distributors to deal exclusively with it. As Tom et. al. (2000) note, "In recent years, however, some manufacturers have begun to use subtler arrangements in which incentives replace requirements." They argue that these incentives should be treated in antitrust like any other exclusionary conduct.

[3]Although our focus is on exclusionary conduct, it is well known that exclusive-dealing arrangements may also have efficiency rationales. See, for example, Marvel (1982), Ornstein (1989), and Segal and Whinston (2000b).

are contracting externalities that keep the dominant firm from bearing the full cost of inducing exclusion. For example, Bernheim and Whinston (1998) show that exclusion can arise when there are 'noncoincident markets,' where the exclusive-dealing arrangements serve to extract rents from markets other than the ones in which they are employed; Rasmusen et al. (1991) and Segal and Whinston (2000a) show that exclusion can arise when there are economies of scale in production and no one retailer is large enough to ensure the survival of the entrant; and Aghion and Bolton (1987) show that exclusion can arise when a dominant manufacturer leverages its first-mover advantage to extract rents from a more efficient entrant, when the entrant's marginal costs are unknown.[4]

More recently, Nalebuff (2004) and Greenlee et al (2004) show how bundling by a multi-product firm can lead to the exclusion of an equally efficient rival, albeit one that produces a single-product.[5] In these models, as in many of the above models of exclusive dealing, the dominant manufacturer is assumed to have a first-mover advantage. We differ in this paper in that we assume there are no economies of scale and the sellers' offers are made simultaneously. In addition, in our model, there are no coordination issues among the buyers as there is only one buyer, and the ability of the entrant to offer its own bundle does not ensure its survival as it does in the bundling models.

The literatures on loyalty rebates, all-units discounts, and market-share discounts have thus far mostly focused on how these discounts can facilitate rent-shifting between a manufacturer and its retailers, or between manufacturers, and do not consider exclusionary motives.[6] Papers by D. Spector, B. Kobayashi, and A. Heimler in "A Symposium on Loyalty Rebates" (2005) discuss anticompetitive uses of loyalty rebates but do not rigorously model the market settings in which the rebates arise in equilibrium of a well-specified game. Like us, these authors suggest various screens in rule-of-reason cases that would delineate anti-competitive from permissable loyalty rebates.

The rest of the paper proceeds as follows. Section 2 describes the model. Section 3 characterizes necessary and sufficient conditions for equilibria to exist and discusses the role of lock-in and

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[4]Erutku (2006) extends Aghion and Bolton to the case of multiple competing retailers and shows how the dominant manufacturer can induce exclusion by offering rebates to retailers conditional on their signing exclusivity agreements.

[5]Whinston (1990) was the first to formalize the notion that a dominant firm could extend its market power from one market into another by conditioning the sale of one product to the sales of another. More recently, Carlton and Waldman (2002) examine how tying can be used to foreclose competitors in industries with network externalities.

[6]See, for example, Greenlee and Reitman (2004), who view loyalty rebates as a means of extracting surplus from heterogeneous buyers when other types of nonlinear pricing are infeasible; Kolay et al (2004), who show how all-units discounts can be used by a manufacturer to extract more surplus from its retailers when the retailers differ in size or there is uncertainty in market demand; and Marx and Shaffer (2004), who show how discounts conditioned on a manufacturer's share of a retailer's purchases can enhance rent-extraction from a rival manufacturer who moves second. Marvel and Yang (2006) find that, by putting all consumers "in play" as opposed to only those at the margin, market-share discounts can increase competition relative to a regime in which firms are restricted to linear prices.

financing constraints in supporting exclusion. Section 4 focuses on the sellers' equilibrium contract terms and considers the effects of various restrictions on the space of allowable contracts. Section 5 considers extensions to the model, and Section 6 discusses the policy implications of our findings.

2   Model

We consider a two-period model in which there is one buyer and two sellers. For ease of exposition, we call one seller 'the entrant' and the other seller 'the incumbent.' In each period, the buyer wants at most two units of the products sold by the sellers. It can purchase both units from the incumbent, one unit from each seller, one unit from one seller, or no units. For now, we assume the buyer cannot purchase both units from the entrant (we will relax this assumption in section 5). Thus, one can think of the buyer's demand as consisting of a captive unit and a contestable unit, where the latter can be supplied by both but the former can only be supplied by the incumbent.[7]

Let the buyer's valuations for the set of permissable combinations in period one be denoted by

$$V_{II}^1, \ V_{IE}^1, \ V_{IO}^1, \ \text{and} \ V_{EO}^1,$$

where the subscripts II, IE, IO, and EO denote whether the buyer purchases both units from the incumbent, one unit from each seller, one unit from only the incumbent, or one unit from only the entrant, respectively.[8] We assume its valuation is zero if it does not purchase from either seller.

We assume the following relationships hold among the buyer's valuations in the first period:

$$V_{IO}^1 > c, \quad V_{EO}^1 > c, \quad V_{II}^1 > V_{IO}^1 + c, \quad V_{IE}^1 > \max\{V_{IO}^1 + c, V_{EO}^1 + c\}, \tag{1}$$

$$V_{IE}^1 > V_{II}^1, \quad \text{and} \quad V_{IE}^1 \le V_{IO}^1 + V_{EO}^1,$$

where c denotes the marginal cost of production, which is assumed to be the same for both sellers. Thus, for example, (1) implies that the buyer's willingness-to-pay for two units of the incumbent's product exceeds by more than c its willingness-to-pay for only one unit of the incumbent's product, and its willingness-to-pay for one unit of each seller's product exceeds by more than c its willingness-to-pay for only one unit from one of the sellers. Moreover, (1) implies that the willingness-to-pay of the buyer for one unit of each seller's product exceeds its willingness-to-pay for two units from the incumbent but is weakly less than the sum of its willingness-to-pay for each stand-alone unit.

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[7]Alternatively, one can think of the entrant as being capacity constrained at one unit per period.

[8]One can think of the buyer either as being a final consumer of the sellers' products or as being a downstream firm whose valuations are derived from the revenue it expects to earn from reselling the products in its retail markets.

In period two, we assume the buyer—or its customers—becomes locked-in to the seller or sellers from whom it purchased in period one, making switching between the sellers prohibitively costly.[9] Thus, for example, we assume the entrant's product has positive value to the buyer in period two if and only if the entrant supplies the contestable unit in period one (in the next section we show that our qualitative results hold even if switching is possible, provided it is sufficiently costly).

The buyer's valuations in period two thus depend on its purchases in period one. Let these valuations be denoted $V_{II}^2(k)$, $V_{IE}^2(k)$, $V_{IO}^2(k)$, and $V_{EO}^2(k)$, where $k \in \{IE, I\}$ denotes whether the buyer purchases from both sellers or just the incumbent in period one.[10] Our assumptions imply

$$V_{II}^2(I) > V_{IO}^2(I) + c, \quad V_{IO}^2(I) = V_{IE}^2(I) > c, \quad V_{EO}^2(I) = 0,$$

and

$$V_{IE}^2(IE) = V_I^2 + V_E^2, \quad V_{II}^2(IE) = V_{IO}^2(IE) = V_I^2, \quad \text{and} \quad V_{EO}^2(IE) = V_E^2, \tag{2}$$

where V_I^2 and V_E^2 denote the buyer's valuations for the incumbent and the entrant's product in period two, respectively, when the buyer has purchased from both sellers in period one.

We also make some additional assumptions which are analogous to those made in period one:

$$V_{IE}^2(IE) > V_{II}^2(I) \quad \text{and} \quad V_{IE}^2(IE) \leq V_{IO}^2(I) + V_{EO}^2(IE). \tag{3}$$

Given our assumptions in (1)–(3), it is easy to see that the efficient outcome calls for the buyer to purchase one unit from each seller in period one and, given that it has done so, to purchase one unit from each seller in period two (i.e., overall surplus over the two periods is maximized at $V_{IE}^1 - 2c + V_{IE}^2(IE) - 2c$). However, it is also easy to see that the buyer's willingness-to-pay for two units of the incumbent's product in period two is increasing in the incumbent's first-period sales, which may provide the impetus for the incumbent to seek the entrant's exclusion in period one.

Turning to the specifics of the game, we assume the sellers have complete information (i.e., they know each other's costs and the buyer's valuations) and can make take-it-or-leave-it offers to the buyer at the beginning of each period. These offers, which we assume are made simultaneously, consist of menus of price-quantity pairs. Let $T_I^i = (T_I^i(1), T_I^i(2))$ and $T_E^i = T_E^i(1)$ denote the incumbent and the entrant's offer in period i, respectively, where $T_I^i(j)$ denotes the incumbent's price in period i if the buyer purchases j units from it, and analogously for the entrant's offer.

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[9]There are many reasons why the buyer might become locked-in to a seller's product. We refer the interested reader to the literature on switching costs. See, for example, Klemperer (1987a) and Farrell and Shapiro (1988).

[10]For the sake of completeness, we assume that the buyer's valuation for the entrant's product is zero in the uninteresting (and out-of-equilibrium) case in which neither seller supplies the contestable unit in period one.

We assume the buyer's payment is zero if it purchases zero units, but we place no other restrictions on the allowable form of contracts (later, we will allow for contract restrictions). We also assume the buyer cannot purchase more units than it consumes. For example, the buyer cannot purchase two units from the incumbent *and* one unit from the entrant. This assumption is meant to capture in a simplified way a real-world seller's ability to prevent a buyer from taking advantage of its generous quantity discounts and then reselling the unused units in a secondary market.[11]

We use subgame perfection as our solution concept, and we begin by solving for the equilibrium in period two as a function of the buyer's first-period choices. Given our assumptions, this is relatively straightforward to do. In any equilibrium in period two, the sellers' contracts will be chosen to extract fully the buyer's surplus in all possible subgames.[12] For example, if the incumbent supplied the contestable unit in period one, then the sellers' contracts in period two will be such that in equilibrium the incumbent earns $V_{II}^2(I) - 2c$, and the buyer and the entrant earn zero. And if the entrant supplied the contestable unit in period two then the sellers' contracts will be such that in equilibrium the incumbent earns $V_I^2 - c$, the entrant earns $V_E^2 - c$, and the buyer earns zero. Thus, the incumbent's second-period gains from lock-in are $V_{II}^2(I) - 2c - (V_I^2 - c) = V_{II}^2(I) - V_I^2 - c > 0$.

The more interesting interactions take place in period one, where we assume the entrant is financially constrained in its ability to bid for the buyer's patronage (later, we will relax this assumption when considering the role of seller financing in inducing exclusion). Financing constraints are a feature common in many long-term lending contracts. As Tirole (2006), Clementi and Hopenhayn (2006), and others have pointed out, in settings such as ours, the availability of 'free' cash (e.g, from installed-base revenues) is key to the ability to finance projects, such as paying a retailer for access to second-period customers.[13] Since a large firm is likely to have access to more cash than a small firm, we assume, for ease of exposition, that the entrant faces a financing constraint (an upper bound on how much it can bid for the buyer's patronage) whereas the incumbent does not.

We model the entrant's constraint by assuming it must earn a minimum payoff of θ in period one. That is, we require that $T_E^1(1) \geq \theta + c$, where $\theta \in (-(V_E^2 - c), 0]$. The upper bound, $\theta = 0$, implies that the entrant is unable to borrow in period one; the lower bound, $\theta = -(V_E^2 - c)$, implies that the entrant can borrow in period one up to its maximum payoff in period two. The upper

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[11]For example, in some industries the buyer is prohibited from doing so by contract. Equivalently, our assumption that the buyer cannot throw units away is tantamount to assuming that its disposal costs are prohibitively high.

[12]This follows because both units are captive in period two and the sellers make take-it-or-leave-it offers.

[13]Clementi and Hopenhayn (2006) show that financial constraints arise endogenously as a feature of long-term lending contracts, and that "such constraints relax as the value of the borrower's claim to future cash flows increases."

bound implies a tight constraint on the entrant. The lower bound implies essentially no constraint. For intermediate values, we say that the entrant's financing constraint is 'tighter' the higher is θ.

One can think of θ as the maximum payment the entrant can finance today to compensate the buyer for the lock-in of its captive unit tomorrow. Alternatively, the entrant could promise not to exploit fully the buyer's captive unit tomorrow by committing (if it could) to tomorrow's price today. This suggests that there is a duality between the entrant's ability to borrow money today to compensate the buyer and its ability to compensate the buyer by promising a low price tomorrow.[14] For example, an offer of $T_E^1(1) = c + \theta$ in period one, where $\theta = -(V_E^2 - c)$, is equivalent to an offer of $T_E^1(1) = c$ today and a commitment not to charge a price higher than c tomorrow.[15] Hence, one can think of the entrant's financing constraint (if it binds) more generally as representing all impediments to the entrant's ability to compensate the buyer fully for its expected future lock-in.

3 Solving the Model

We can now state our main result, which proves existence and characterizes the equilibria:

Proposition 1 *Equilibria in which the buyer purchases one unit from each seller exist if and only if $\theta \leq V_{IE}^1 - V_{II}^1 - (V_{II}^2(I) - V_I^2 - c)$. In all such equilibria, contracts T_I^* and T_E^* are such that*

$$T_E^*(1) \;=\; V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), \quad T_I^*(1) \;=\; V_{IE}^1 - V_{EO}^1,$$

and

$$c - \left(V_{II}^2(I) - V_I^2 - c\right) \;\geq\; T_I^*(2) - T_I^*(1) \;\geq\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right).$$

In contrast, equilibria in which the buyer purchases both units from the incumbent exist if and only if $\theta \geq V_{IE}^1 - V_{II}^1 - (V_{II}^2(I) - V_I^2 - c)$. In all such equilibria, contracts T_I^{**} and T_E^{**} are such that

$$T_E^{**}(1) \;=\; \theta + c, \quad T_I^{**}(2) \;=\; V_{II}^1 - V_{EO}^1 + \theta + c,$$

and

$$T_I^{**}(2) - T_I^{**}(1) \;\leq\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right).$$

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[14]We abstract from this duality in the model with our assumption that the sellers cannot commit to their second-period contracts in period one. In reality, of course, some commitment may be possible, but even in these cases, contracts between buyers and sellers are often incomplete (due to unanticipated and unverifiable shocks), giving one side or the other the ability to engage in opportunism. This naturally may make both sides wary of committing too early on price in fear of what might happen if the environment subsequently changes. Contracting today over prices in future periods which may be far off may also be fraught with difficulties, especially in fast-paced industries that are characterized by many innovations. In such cases, it may simply not be possible to contract over future prices.

[15]We assume there is no discounting across the two periods strictly for the ease of exposition.

There do not exist equilibria in which the buyer purchases one unit from one seller or no units.

Proof: See Appendix A.

Proposition 1 implies that whether exclusionary equilibria exist depends on the relation between the entrant's financing constraint and the expression $V_{IE}^1 - V_{II}^1 - (V_{II}^2(I) - V_I^2 - c)$. If the former weakly exceeds the latter, then exclusionary equilibria exist. Otherwise, efficient equilibria exist.

The inequality $\theta \geq V_{IE}^1 - V_{II}^1 - (V_{II}^2(I) - V_I^2 - c)$, which hereafter we call "the key condition that enables exclusion," has an intuitive interpretation, which can be seen by rewriting it as

$$V_{II}^2(I) - V_I^2 - c \;\; \geq \;\; V_{IE}^1 - V_{II}^1 - \theta. \tag{4}$$

Condition (4) implies that exclusionary equilibria can be supported if and only if the incumbent's second-period gain from excluding the entrant in the first period exceeds the cost of compensating the buyer for the loss of the entrant's product, where $V_{IE}^1 - V_{II}^1$ is the foregone surplus that would have been added by the entrant's product in period one and $-\theta$ is the maximum amount the entrant can 'pay' the buyer in period one not to give in to the exclusion. In other words, when the incumbent's second-period gain from 'lock-in' exceeds the cost of compensating the buyer for the loss in overall first-period surplus, and the financing constraint is such that the entrant is unable to pay the buyer enough to overcome the difference, exclusion is the unique equilibrium outcome.

The uniqueness is surprising because both exclusionary and efficient outcomes typically co-exist in models in which multiple sellers sell their products to a single buyer. In these cases, the literature often employs a refinement to select among equilibria, usually ruling out the exclusionary ones on the grounds that they are Pareto dominated by the efficient equilibria (see, for example, O'Brien and Shaffer, 1997; and Bernheim and Whinston, 1998). In contrast, in our model, no such selection criterion is needed because Proposition 1 implies that multiple outcomes typically do not exist.

The result is also surprising because it implies that exclusion can arise even though the decision to exclude the entrant is fully internalized by the lone downstream buyer. The Chicago school regards inefficient exclusion as unlikely in this case because it alleges that any benefit to an upstream seller from excluding its rival will typically be outweighed by the concomitant loss to the downstream buyer, and hence, the downstream buyer will not agree to participate in the exclusion. In other words, it alleges that mutually-beneficial exclusion will typically not occur because the amount a seller would have to pay to the buyer to induce exclusion would render such a strategy unprofitable.

The Chicago-school view fails to hold in this case because the entrant's financing constraint and the customer lock-in due to switching costs represent impediments to free and open competition. To see the role of the former, suppose the entrant can rationally borrow in period one up to its maximum possible payoff in period two, $V_E^2 - c$. Substituting this for θ into condition (4), we have

$$V_{II}^2(I) - V_I^2 - c \geq V_{IE}^1 - V_{II}^1 + (V_E^2 - c),$$

or, equivalently,

$$0 \geq V_{IE}^1 - V_{II}^1 + \left(V_{IE}^2(IE) - V_{II}^2(I) \right).$$

But this inequality, which is equivalent to the key condition for exclusion when $\theta = -(V_E^2 - c)$, is never satisfied because by assumption the efficient outcome calls for the buyer to purchase one unit from each seller in each period (recall that we have assumed $V_{IE}^1 > V_{II}^1$ and $V_{IE}^2(IE) > V_{II}^2(I)$). Hence, we can conclude that the entrant's financing constraint is critical for our exclusion result.

Switching costs are also critical because, by linking the two periods, they may cause the entrant's financing constraint to bind, which handicaps the outcome in period one in favor of the incumbent. In contrast, in the absence of switching costs, exclusion would not be profitable for the incumbent.[16] To see this, suppose, to the contrary, that exclusionary equilibria with contracts T_I^{**} and T_E^{**} exist in this case. Then it is optimal for the incumbent to offer $T_I^{**}(1) = \infty$ (which forces the buyer to purchase from either the incumbent, the entrant, or zero), and the entrant can do no better than to offer $T_E^{**}(1) = \theta + c$ (which is the lowest price the entrant can offer and still satisfy its financing constraint). It follows that, given the entrant's best offer, the incumbent cannot charge more than

$$T_I^{**}(2) = V_{II}^1 - V_{EO}^1 + \theta + c$$

because otherwise the buyer would purchase only one unit from the entrant and earn $V_{EO}^1 - \theta - c$. Hence, under $T_I^{**}(2)$, the incumbent's payoff in the hypothesized exclusionary equilibrium is

$$\begin{aligned}
\Pi_I &= T_I^{**}(2) - 2c + \text{ second period payoff} \\
&= V_{II}^1 - V_{EO}^1 + \theta - c + \text{ second period payoff,}
\end{aligned}$$

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[16]In many cases, the extent of customer lock-in will lie between the two polar cases considered in the text. For example, switching may be possible in period two but at some finite cost that a foreclosed seller from period one would have to pay to the buyer/consumer. In these cases, whether or not exclusion is sustainable will depend on the size of the switching costs. In particular, it can be shown that the incumbent will want to induce exclusion in period one if and only if switching in period two is sufficiently costly and the key condition that enables exclusion holds.

where $V_{II}^1 - V_{EO}^1 + \theta - c$ is the incumbent's payoff in the first period. In contrast, the incumbent's payoff if it lowers its price on $T_I^{**}(1)$ to induce the buyer to purchase one unit from each seller is

$$V_{IE}^1 - V_{EO}^1 - c + \text{ second period payoff}, \tag{5}$$

where now $T_I^{**}(1)$ is $V_{IE}^1 - V_{EO}^1$, so that the buyer's payoff is $V_{IE}^1 - T_I^{**}(1) - T_E^{**}(1) = V_{EO}^1 - \theta - c$. Since second-period payoffs are independent of first-period payoffs when switching is costless, it follows that the incumbent's payoff in (5) is strictly higher than in the hypothesized equilibrium.

We can summarize these results as follows:

Proposition 2 *Exclusionary equilibria are more likely to arise the tighter is the entrant's financing constraint and the greater is the second-period gain from lock-in. If the entrant does not have a financing constraint or the buyer does not incur switching costs, exclusionary equilibria do not exist.*

In deciding whether or not to induce exclusion, the incumbent compares the benefit of inducing exclusion to the cost of inducing exclusion, which is what it must pay in the first period to get the buyer to acquiesce. Sometimes the Chicago-school view holds in the sense that the compensation the incumbent must pay the buyer to acquiesce makes exclusion prohibitively costly. However, the Chicago-school view need not hold if the entrant is financially constrained. The reason is that in this case the loss to the buyer if it participates in the exclusion will be constrained, so that the incumbent will only have to compensate the buyer for an amount equal to the value added of the entrant's product plus what the entrant is able to pay out of its own pocket not to be excluded. If the latter is sufficiently constrained, the incumbent may well find it profitable to exclude its rival.

4   Contract Terms and Restrictions

We have thus far discussed when and why exclusion may arise, and we have emphasized the joint role of the entrant's financing constraint and the buyer's switching costs in obtaining this outcome. In this section, we focus on the sellers' equilibrium offers and consider *how* exclusion may arise.

Whether the buyer purchases one or two units from the incumbent depends on the price of the incumbent's second unit relative to the entrant's price. In the following Corollary, we state what must be true of the incumbent's offer in any exclusionary equilibrium and derive its implications.

Corollary 3 *In all exclusionary equilibria, contract T_I^{**} is such that*

$$T_I^{**}(2) - T_I^{**}(1) \ \leq \ c - \left(V_{IE}^1 - V_{II}^1 - \theta \right).$$

It follows that the price of the incumbent's second unit is less than the marginal cost of production, and exclusion requires that the incumbent offer a negative price on its second unit if the marginal cost of production is sufficiently small relative to the discount that is needed to induce exclusion.

The intuition for Corollary 1 is straightforward. We know that in any exclusionary equilibrium, the entrant will be offering to supply the contestable unit at a price of $c+\theta$, as this is the lowest price it can feasibly offer. We also know that the entrant has an advantage in supplying the contestable unit because of the added value that its product offers the buyer given that the buyer is already purchasing from the incumbent. Hence, if the incumbent is to induce the buyer to exclude the entrant, it has to beat the entrant's offer, $c + \theta$, by at least $V_{IE}^1 - V_{II}^1$. Or, in other words, the price of the incumbent's second unit must be less than the marginal cost of production minus the discount that is needed to induce exclusion. The derived implications in Corollary 3 then follow from noting that the discount that is needed to induce exclusion, $V_{IE}^1 - V_{II}^1 - \theta$, is strictly positive.

Corollary 3 implies that exclusionary equilibria can always be supported with contracts that exhibit negative pricing on the incumbent's second unit (this follows because there is no lower bound on $T_I^{**}(2) - T_I^{**}(1)$). Thus, for example, whenever the key condition that enables exclusion is satisfied, there exists an exclusionary equilibrium in which the incumbent refuses to sell just one unit, e.g., offers $T_I^{**}(1) = \infty$, and charges $T_I^{**}(2) = V_{II}^1 - V_{EO}^1 + \theta + c$ (see Proposition 1) if the buyer purchases two units. This would correspond in practice and in the literature to the case of multi-unit bundling, where the individual units are not offered for sale, and it has obvious parallels to the kind of exclusionary bundling that arises in Nalebuff (2004) (albeit here the sellers' offers are made simultaneously) if one re-interprets the incumbent's two units as two distinct products.[17]

The incumbent can also implement a negative price on its second unit by offering a market-share discount, a loyalty rebate, or an all-units discount. As an example of the former, the incumbent can announce an artificially high price per unit if it has 50% of the market (sells one unit) and a substantially lower price per unit if it has 100% of the market (sells two units), where the discount is such that the second unit is effectively offered to the buyer at a negative price. As an example of the latter, the incumbent can implement a negative price on its second unit without explicitly referring to its rival, and without foreclosing the possibility of selling just one unit, by charging a constant price per unit for its product and then offering a loyalty rebate of more than 50% off to

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[17]Alternatively, the incumbent can announce that it will not sell to the buyer if it also purchases from the entrant. As with bundling, this would eliminate from consideration the case in which the buyer purchases from both sellers.

its 'best customers,' or, equivalently, by offering a discount that applies to both units purchased by the buyer if and only if the buyer purchases two units, i.e., by offering it an all-units discount.[18]

Additional insight may be had by noting that exclusionary equilibria *must* be supported by negative pricing if the discount needed to induce exclusion, $V_{IE}^1 - V_{II}^1 - \theta$, is greater than c. This would be the case, for example, if the sellers' marginal cost of production is sufficiently close to zero or if the entrant can finance all of its second-period production costs in the first period, so that $-\theta > c$. If, on the other hand, the marginal production costs are relatively high, the value added by the entrant's product is relatively low, and the entrant's financing constraint is relatively tight, so that $c - \left(V_{IE}^1 - V_{II}^1 - \theta\right) > 0$, Corollary 3 suggests that exclusionary equilibria can sometimes be supported even if the incumbent offers its second unit at a positive price. This is surprising because it suggests that even with 'outlay' schedules that do not exhibit a jump down, it may still be possible, under some circumstances, for an incumbent to exclude an equally-efficient entrant. As we shall see, this has important implications for the efficacy of remedies to improve efficiency.

Of course, inducing exclusion may sometimes prove to be too costly for the incumbent. In the next Corollary, we state what must be true of the incumbent's offer in any efficient equilibrium.

Corollary 4 *In all efficient equilibria, contract T_I^* is such that*

$$T_I^*(2) - T_I^*(1) \geq c - \left(V_{IE}^1 - V_{II}^1 - \theta\right).$$

$$T_I^*(2) - T_I^*(1) \leq c - \left(V_{II}^2(I) - V_I^2 - c\right).$$

It follows that the price of the incumbent's second unit is less than the marginal cost of production, and efficiency requires that the incumbent offer a negative price on its second unit if the marginal cost of production is sufficiently small relative to the second-period gains from customer lock-in.

Corollary 4 implies that the price of the incumbent's second unit *must* be positive in some cases and negative in other cases. It must be positive in all efficient equilibria if and only if the discount that would have been needed to induce exclusion (if the incumbent had wanted to induce exclusion) is less than the marginal cost of production, a condition that is 'more likely' to be satisfied when the marginal cost of production is relatively high, the value added by the entrant's product to overall

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[18]As we discussed in the introduction, each of these strategies has received attention in the economics literature. We bring them together here in light of a common property they each share (or could share if the per-unit discount is sufficiently large)—namely, they each feature a negative price at the point at which the buyer reaches the threshold.

surplus is relatively low, and the entrant's financing constraint is relatively tight. This implication follows because the price of the incumbent's second unit is bounded below by $c - \left(V_{IE}^1 - V_{II}^1 - \theta\right)$.[19]

The price of the incumbent's second unit must be negative in all efficient equilibria if and only if the second-period gains from lock-in exceed the marginal cost of production, a condition that is 'more likely' to hold when the marginal costs of production are relatively low. This implication follows because the price of the incumbent's second unit is bounded above by $c - \left(V_{II}^2(I) - V_I^2 - c\right)$.[20]

4.1 Restrictions on below-cost pricing

We have thus far focused our attention on whether the price of the incumbent's second unit is positive or negative in equilibrium. In either case, however, we know from Corollaries 3 and 4 that it is always less than c. Whether this is surprising depends on one's perspective. On the one hand, the notion that switching costs may lead to below-cost pricing in a dynamic setting in the period prior to lock-in is well known (see, for example, Klemperer, 1987a and Farrell and Shapiro, 1988). That switching costs might also make it easier to deter entry in some cases is also well-known (see, for example, Klemperer, 1987b). On the other hand, it is surprising because one might be tempted (erroneously) to denounce all such pricing as being inefficient when in fact its existence does not allow one to infer whether the induced equilibria in the model are good or bad for welfare. Below-cost pricing is harmful for welfare if, when competing for the contestable unit, the entrant is not able to match the incumbent's below-cost offer (adjusted for differences in valuations) because of its financing constraint. On the other hand, below-cost pricing is not harmful when the entrant's financing constraint does not bind in equilibrium because then the buyer's choices are efficient and competition to supply the contestable unit simply transfers surplus from the sellers to the buyer. In this latter case, socially-efficient outcomes and below-cost pricing are mutually compatible.

It is not obvious, therefore, whether a ban on below-cost pricing would improve social welfare, as it would have consequences not only for the exclusionary equilibria in the model but also for the model's efficient equilibria. Nevertheless, as we now show, in our model, the consequences for the

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[19]The problem the incumbent would face if it offered a negative price on its second unit when the key condition for exclusion is not satisfied and $c - \left(V_{IE}^1 - V_{II}^1 - \theta\right) > 0$ is that the buyer would then actually buy its second unit, which the incumbent does not want given that its second-period gain from lock-in under these conditions is too small.

[20]But this begs the question why would the incumbent ever offer to sell its second unit at a negative price when the key condition for exclusion does not hold, given that it would lose profit if it actually sold the unit. The reason is that as long as its price is above the price that would induce the buyer to exclude the entrant, the incumbent will not actually be selling the second unit, and so the price that it sets for its second unit will be of no consequence to it.

latter are limited to redistribution effects.[21] If the incumbent is prohibited from pricing at below cost, then exclusionary equilibria do not exist and only a unique, efficient equilibrium remains.

Proposition 5 *When the incumbent is prohibited from offering to sell its second unit for a price that is below its marginal cost, the buyer purchases one unit from each seller and the equilibrium is unique. Contract terms are $T_I^*(1) = V_{IE}^1 - V_{EO}^1$, $T_I^*(2) = V_{IE}^1 - V_{EO}^1 + c$, and $T_E^*(1) = V_{IE}^1 - V_{II}^1 + c$.*

Proof: See appendix B.

In equilibrium, each seller extracts only the value that its product adds to overall surplus. In the case of the incumbent, this is $V_{IE}^1 - V_{EO}^1 - c$. In the case of the entrant, this is $V_{IE}^1 - V_{II}^1$. Notice also that the price of the incumbent's second unit is c. Although the incumbent knows that an additional sale in the first period will generate an additional profit of $V_{II}^2(I) - V_I^2 - c$ in the second period, and hence, that its 'true' opportunity cost of selling another unit in the first period is not c, but rather $c - (V_{II}^2(I) - V_I^2 - c)$, it is constrained from charging a lower price. Hence, the incumbent has no way of compensating the buyer for exclusion when below-cost pricing is banned.

Proposition 5 represents a first-best solution in the sense that if the sellers' costs of production are known to outside parties, and the restriction is costless to enforce, then social welfare is maximized, albeit with a redistribution of surplus from the buyer and the incumbent to the entrant.[22] Redistribution issues aside, however, a potential problem may arise if the sellers' costs are not observable to the parties charged with enforcing the restriction.[23] If this is the case, and if the restriction is enforced based on an erroneous belief that the incumbent's marginal cost is higher than it really is (a type II error), then the effect of the enforcement may be to chill price competition unnecessarily as both sellers will be charging marginal prices that are above their marginal costs (which in the model exacerbates the redistribution of surplus, but, more generally, could lead to a welfare loss if the buyer's demand is downward sloping). Conversely, if the restriction is enforced based on an erroneous belief that the incumbent's marginal cost is lower than it really is (also a type II error), then the enforcement may have little impact and exclusion may result. The possibility of these type II errors lead us next to consider whether there might be another, more easily enforced

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[21]In the model, the sellers' prices do not affect the volume of sales but they do affect the distribution of sales and the distribution of rents. An interesting extension is to examine this result when usage is also sensitive to pricing.

[22]It can be shown that the entrant will always be better off, the incumbent will be weakly worse off, and the buyer will always be worse off, when the incumbent is prohibited from pricing its second unit at below marginal cost.

[23]There is also a wide-range of market settings in which below-cost prices are consistent with 'innocent' profit maximization. See, e.g., Evans and Schmalensee (2006), which discusses pricing in industries with two-sided platforms.

restriction, such as a ban on negative incremental prices, that would not be as susceptible to these errors and which could be imposed on the incumbent's offer terms to improve the market outcome.

4.2 Restrictions on negative incremental prices

One reason for singling out negative incremental prices is that they can be easily identified when the incumbent uses them, and hence, restricting their use does not require knowledge of the incumbent's cost parameters, which may not be verifiable or easy to obtain. Unfortunately, however, the link between 'bad' market outcomes and negative incremental prices is not as transparent as one might have hoped for. As we have seen from Corollaries 3 and 4, there are circumstances in which exclusionary equilibria can be supported even when the incumbent offers its second unit at a positive price, and there are circumstances in which efficient equilibria require that the incumbent charge a negative price. This raises some concerns because it suggests that a ban on negative incremental prices need not always be desirable. One concern is that it may not eliminate exclusionary equilibria in all cases. Another concern is that it may sometimes have adverse consequences for the existence of efficient equilibria. As we now show, however, the latter concern is unfounded, at least in the context of the current model. Although exclusionary equilibria are not eliminated, a ban on negative incremental prices increases the incidence of efficient equilibria and therefore is weakly welfare improving (although it would be a second-best solution if the incumbent's costs were observable).

Proposition 6 *When the incumbent is prohibited from offering to sell its second unit at a negative price, equilibria in which the buyer purchases one unit from each seller exist if and only if $\theta \leq V_{IE}^1 - V_{II}^1 - \min\left\{c, V_{II}^2(I) - V_I^2 - c\right\}$. In all such equilibria, T_I^* and T_E^* are such that*

$$T_E^*(1) \ = \ V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), \quad T_I^*(1) \ = \ V_{IE}^1 - V_{EO}^1,$$

and

$$\max\left\{0, c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\} \ \geq \ T_I^*(2) - T_I^*(1) \ \geq \ \max\left\{0, c - \left(V_{IE}^1 - V_{II}^1 - \theta\right)\right\}.$$

In contrast, equilibria in which the buyer purchases both units from the incumbent exist if and only if $\theta \geq V_{IE}^1 - V_{II}^1 - \min\left\{c, V_{II}^2(I) - V_I^2 - c\right\}$. In all such equilibria, T_I^{**} and T_E^{**} are such that

$$T_E^{**}(1) \ = \ \theta + c, \quad T_I^{**}(2) \ = \ V_{II}^1 - V_{EO}^1 + \theta + c,$$

and

$$c - \left(V_{IE}^1 - V_{II}^1 - \theta\right) \ \geq \ T_I^{**}(2) - T_I^{**}(1) \ \geq \ 0.$$

There do not exist equilibria in which the buyer purchases one unit from one seller or no units.

Proof: See appendix C.

Proposition 6 implies that, except in the special case of $\theta = V_{IE}^1 - V_{II}^1 - \min\left\{c, V_{II}^2(I) - V_I^2 - c\right\}$, the equilibrium outcome is unique. When θ is less than the right-hand side of this expression, only efficient equilibria exist, and conversely, when θ is greater than the right-hand side of this expression only exclusionary equilibria exist. This accords with what we found in Proposition 1, but with one major difference. In Proposition 1, the threshold for the entrant's financing constraint was $\theta = V_{IE}^1 - V_{II}^1 - (V_{II}^2(I) - V_I^2 - c)$. Here, the threshold is weakly higher, and it is strictly higher if and only if $V_{II}^2(I) - V_I^2 - c > c$. This means that efficient equilibria are 'more likely' to arise—and thus exclusionary equilibria are 'less likely' to arise—for a given θ when negative incremental prices are prohibited. Or, to put it another way, for a given set of cost and demand parameters, efficient equilibria exist over a wider range of θ when the incumbent is prohibited from offering negative incremental prices. It follows that such a ban is (weakly) welfare improving—it never makes welfare worse, and in some cases, for example, when $V_{II}^2(I) - V_I^2 - c > c$ and the incumbent would otherwise want to induce exclusion, it may make the difference between the entrant being excluded or not.

The mechanism by which exclusion occurs (if it occurs) in this case is more or less the same as it was in the absence of a prohibition on negative incremental prices: the incumbent's second-period gain from 'lock-in' must exceed the cost of compensating the buyer for the loss in overall first-period surplus, and the entrant's financing constraint must be such that the entrant is unable to 'pay' the buyer enough to overcome the difference. Or, in other words, the discount that is needed to induce exclusion must be less than the incumbent's second-period gain from lock-in. But now, in addition to this, the discount needed to induce exclusion must also be less than the incumbent's marginal cost, so that the incumbent is able to induce the buyer to exclude the entrant while maintaining a positive price on its second unit.[24] It is for this reason that the set of exclusionary equilibria is reduced, and thus it follows that the lower is the incumbent's marginal cost, all else being equal, the less likely it is that exclusion will arise. For example, if the incumbent's marginal cost is zero or sufficiently close to zero, a prohibition of negative incremental prices will be just as effective as the more encompassing policy of prohibiting below-cost pricing in eliminating inefficient exclusion.

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[24]When the condition for exclusion in Proposition 6 is satisfied, the upper bound on the price of the incumbent's second unit, $c - \left(V_{IE}^1 - V_{II}^1 - \theta\right)$ is non-negative, which implies that exclusion only arises in this case when the discount that is needed to induce exclusion is less than both the incumbent's second-period gain from lock-in and c.

5 Extensions

We have thus far considered the case of an entrant who can supply at most one unit (equivalently, a situation in which only one of the buyer's units is contestable). We now relax this assumption and assume the entrant can compete for both units. This is an important extension to consider because it enables us to consider the case in which the entrant can produce the full range of 'products' a la the incumbent. Indeed, it is often alleged that competitive concerns arise only when an incumbent aggregates its discounts across several products (for example, offers bundled discounts), thereby handicapping smaller rivals who offer only one product. According to one prominent antitrust commentator, "One might say that bundled discounts could not exclude an equally efficient firm, if we defined such a firm as one that was an efficient producer of every product that went into the bundled discount" (Hovenkamp, 2005: 173). He then goes on to say that "Bundled discounts exclude precisely because a dominant multiproduct firm is likely to face upstart single-product rivals—or at least, rivals who produce a smaller range of products than the dominant firm does."

This view of bundled discounts is implicit in Nalebuff (2004), who finds that an incumbent's bundled discounts do not lead to exclusion when the rival firm can offer its own bundle. In fact, "bundle-on-bundle" competition is more intense compared to unbundled competition in his model.

As we shall see, however, when the upstart faces a financing constraint, the incumbent's discounts can be exclusionary even when all units are contestable and the entrant is equally efficient. Moreover, the potential concerns in this case are exacerbated—efficient equilibria are no more likely to arise and the threat of exclusion is omnipresent, as exclusionary equilibria now always exist.[25]

Let the buyer's valuation if it purchases both units from the entrant in the first period be V_{EE}^1, and assume that $V_{EE}^1 > V_{EO}^1 + c$ and $V_{IE}^1 > V_{EE}^1$. Let the buyer's valuation if it purchases both units from the entrant in the second period be $V_{EE}^2(k)$, where $k \in \{IE, I, E\}$ now denotes whether the buyer purchased from each seller, only the incumbent, or only the entrant in period one. Assume that $V_{EE}^2(I) = 0$, $V_{EE}^2(E) > V_{EO}^2(E) + c$, $V_{EE}^2(IE) = V_E^2$ and $V_{IE}^2(IE) > V_{EE}^2(E)$. Then, it continues to hold that the efficient outcome is for the buyer to purchase one unit from each seller in period one and, given this, also to purchase one unit from each seller in period two.

We further make the simplifying assumption that the incumbent's product is weakly more

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[25]We abstract here from the impact on final consumers (other than acknowledging that some may be mismatched with the products they prefer). In a more general setting the impact on different final consumers would depend, for example, on the extent to which the intermediary can implement price discriminatory strategies in the retail markets.

valuable to the buyer ex-ante. In particular, we assume that $V_{II}^1 \geq V_{EE}^1$ and $V_{II}^2(I) \geq V_{EE}^2(E)$. Then, we have the following proposition, which proves existence and characterizes the equilibria.

Proposition 7 *Equilibria in which the buyer purchases one unit from each seller exist if and only if $\theta \leq V_{IE}^1 - V_{II}^1 - (V_{II}^2(I) - V_I^2 - c)$. In all such equilibria, contracts T_I^* and T_E^* are such that*

$$T_E^*(1) = V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), \quad T_I^*(1) = V_{IE}^1 - V_{EE}^1 + T_E^*(2) - T_E^*(1),$$

$$c - \left(V_{EE}^2(I) - V_E^2 - c \right) \geq T_E^*(2) - T_E^*(1) \geq c - \left(V_{IE}^1 - V_{EE}^1 \right) - \left(V_I^2 - c \right),$$

and

$$c - \left(V_{II}^2(I) - V_I^2 - c \right) \geq T_I^*(2) - T_I^*(1) \geq c - \left(V_{IE}^1 - V_{II}^1 - \theta \right).$$

In contrast, equilibria in which the incumbent is excluded do not exist, but equilibria in which the entrant is excluded always exist. In all such equilibria, contracts T_I^{**} and T_E^{**} are such that

$$T_E^{**}(1) \geq V_{IE}^1 - V_{II}^1 + c - \left(V_{II}^2(I) - V_I^2 - c \right), \quad T_E^{**}(2) = \theta + 2c,$$

and

$$T_I^{**}(1) \geq V_{IE}^1 - V_{II}^1 + c, \quad T_I^{**}(2) = V_{II}^1 - V_{EE}^1 + \theta + 2c.$$

There do not exist equilibria in which the buyer purchases one unit from one seller or no units.

Proof: See appendix D.

Proposition 7 offers some surprises when compared with Proposition 1. For example, comparing the 'if and only if' conditions for efficient equilibria to arise in these propositions reveal that they do not depend on whether the entrant can compete for both units. That is, efficient equilibria arise under exactly the same set of conditions whether or not one or both units are contestable. This is surprising because one might have thought that the entrant's ability to compete for both units would make the playing field 'more level.' And, in a sense, it does, but the beneficiary is not the entrant but the buyer, who benefits from the more favorable terms offered by the incumbent. To see this, note that the incumbent offers $T_I^*(1) = V_{IE} - V_{EO}$ when only one unit is contestable and $T_I^*(1) = V_{IE}^1 - V_{EE}^1 + T_E^*(2) - T_E^*(1)$ when both units are contestable. Since $T_E^*(2) - T_E^*(1) < c$ and $V_{EE}^1 > V_{EO} + c$, it follows that the incumbent offers better terms when both units are contestable.

Intuitively, it makes sense that the incumbent offers the buyer better terms of trade when both units are contestable because then the buyer's outside option of buying solely from the entrant is

more attractive. But this begs the question why can the entrant not capture any of these gains, or, equivalently, why are efficient equilibria not then more likely? The reason is that the entrant's financing constraint coupled with the assumption that the incumbent's product offers weakly better value implies that the entrant does not have the wherewithal to induce the buyer to exclude the incumbent (and so having two units does not help in that sense), and having two units to offer the buyer does not benefit the entrant if it is only trying to induce the buyer to purchase one unit.

Having the ability to compete for both units not only does not help the entrant, it can also be harmful. Whereas in Proposition 1, exclusionary equilibria were confined to a subset of parameter space, here they always exist. To understand how this can happen, note that having the ability to offer two units expands the entrant's strategy space. Before, the incumbent could threaten to turn the game into an all-or-nothing proposition (effectively forcing the buyer either to purchase two units from it or nothing), and the entrant could only respond by offering an attractive deal on its single unit. Now if the entrant anticipates an all-or-nothing strategy by the incumbent, it can respond with its own attractive offer on two units and compete head to head with the incumbent (either way it knows it is going to lose). However, if it does so, only the buyer ends up gaining. The incumbent succeeds in excluding the entrant but it is costly, and the entrant ends up being excluded, which may not have happened if it could have committed ex-ante to being less aggressive by having the capacity to sell only one unit, as is consistent with the strictures of "judo economics."

6 Conclusion

We have considered a simple model in which two sellers compete to sell their goods to a single buyer. We fully characterized the set of equilibria and showed that, although the efficient outcome calls for the buyer to purchase one unit from each seller, under some plausible conditions, exclusion is the unique outcome. We showed this despite there being no downstream externalities (and thus no coordination difficulties among buyers), complete information about cost and demand parameters, and no economies of scale in production. Moreover, we showed that this result holds even though both firms have the same marginal cost of production and thus are equally efficient, and it holds whether or not the entrant can compete on the full range of product offerings to the buyer (i.e, whether or not the entrant can compete on both units). We found that the key assumptions are (a) the entrant is more financially constrained than the incumbent and (b) the buyer incurs switching costs after its initial round of purchases, which are features of many real-world market settings.

A novel feature of the analysis is that it focused on how the incumbent is able to support the exclusionary outcomes (when the equilibrium calls for exclusion). We found that while exclusionary equilibria were often but not always supported by negative prices on the incumbent's second unit, all exclusionary equilibria were supported by below-cost pricing on this unit. Moreover, the same outcome could not be achieved by simply equating the incumbent's price on each unit sold, i.e., by offering the same overall discount but with a linear price. Instead, we found that the discount had to be structured in a particular way, for example, with a loyalty rebate or an all-units discount.

We considered two potential remedies that a policy maker might adopt to ease competitive concerns. We found that a ban on below-cost pricing was sufficient to eliminate all exclusionary equilibria and, in that sense, was a first-best welfare improvement. However, we also noted that such a prohibition might be difficult to enforce and could lead to Type-two errors unless the incumbent's costs were known and easily verifiable. This led us to consider a second potential remedy, a ban on negative incremental prices (i.e., no decreases in the buyer's total outlay schedule). We found that such a ban was also welfare improving, although in a perfect world it would be a second-best solution in the sense that it would not suffice to eliminate all exclusionary equilibria. Nevertheless, we argued that the latter ban might be preferable in the real-world because of its ease of implementation.[26]

It has been shown in the literature that when an equally efficient entrant is able to produce the full-range of products that matches the range offered by the incumbent, the potential concerns arising from bundling and non-linear price schedules, such as the kind we analyzed here, need not arise. However, we showed in the context of our model that the potential for exclusion persists and may even be enhanced if the entrant is financially constrained, whether or not it is able to offer the full-range of products. In our model, the fact that the entrant can produce an inferior version of the incumbent's preferred product (recall the buyer prefers the latter's first unit to the former's second unit) does not save it from exclusion but does affect the buyer's equilibrium contract terms.

As noted earlier, our results were derived in a stylized model in which discounting emerges as a pro-competitive or anti-competitive strategy but in which there is no role for standard, unilateral reasons for discounting (e.g., to mitigate the effects of double marginalization or to enhance the

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[26]One might argue that a ban on negative incremental prices could also lead to Type-two errors if the discounts are used for purposes other than exclusion. Indeed, one branch of the literature (see the discussion in the introduction) suggests that all-units discounts and loyalty rebates can be used as rent-shifting devices to extract additional surplus, and that these types of contracts can lead to increased competition. Typically, however, the benchmark used for comparison in this other literature is linear pricing. In contrast, if nonlinear pricing is feasible, then, as we have shown, linear pricing is not an appropriate benchmark and more complicated benchmarks need to be considered.

downstream demand of a buyer who may be sensitive to price). It will be important, in future research, to combine these two analytical strands in order to sharpen the policy prescriptions. In these more complex settings, for example, it may be profitable for a retailer to pass on part of the incumbent's discount to final consumers, even when it is exclusionary. Although overall welfare may be lower, the effects on the retailer's final consumers would then need to be tracked as well.

Appendix A

Proof of Proposition 1

Suppose there is an equilibrium in which the buyer purchases one unit from each seller in period one (and hence also in period two), and let T_I^* and T_E^* denote the equilibrium contracts. Then, the payoff to each party, taking into account the subsequent play of the game in period two, is

$$\Pi_I = T_I^*(1) - c + V_I^2 - c, \tag{A.1}$$

$$\Pi_E = T_E^*(1) - c + V_E^2 - c, \tag{A.2}$$

$$\Pi_B = V_{IE}^1 - T_I^*(1) - T_E^*(1), \tag{A.3}$$

where Π_I, Π_E, and Π_B are the overall payoffs of the incumbent, the entrant, and the buyer, respectively, $T_j^*(1) - c$ is seller j's payoff in period one, and $V_j^2 - c$ is seller j's payoff in period two.

It must also be the case that, given contracts T_I^* and T_E^*, the buyer weakly prefers to purchase one unit from each seller (i.e., the buyer's incentive-compatibility constraints must be satisfied):

$$V_{IE}^1 - T_I^*(1) - T_E^*(1) \geq V_{II}^1 - T_I^*(2), \tag{A.4}$$

$$V_{IE}^1 - T_I^*(1) - T_E^*(1) \geq V_{IO}^1 - T_I^*(1), \tag{A.5}$$

$$V_{IE}^1 - T_I^*(1) - T_E^*(1) \geq V_{EO}^1 - T_E^*(1), \tag{A.6}$$

and

$$V_{IE}^1 - T_I^*(1) - T_E^*(1) \geq 0, \tag{A.7}$$

where (A.4) ensures that the buyer does not want to purchase both units from the incumbent, and (A.5) to (A.7) ensure that the buyer does not want to purchase only one unit or no units. These conditions place bounds on $T_E^*(1)$ and $T_I^*(1)$. For example, it follows from (A.4) to (A.6) that

$$T_E^*(1) \leq \min\left\{V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), V_{IE}^1 - V_{IO}^1\right\}, \tag{A.8}$$

and

$$T_I^*(1) \leq V_{IE}^1 - V_{EO}^1. \tag{A.9}$$

Since (A.7) is satisfied whenever $T_E^*(1)$ and $T_I^*(1)$ satisfy (A.8) and (A.9),[27] it follows that (A.8) and (A.9) are necessary and sufficient for the buyer's incentive-compatibility constraints to be satisfied.

Lastly, it must be the case that neither the incumbent nor the entrant can profitably deviate given the other's contract. Among other things, this means that (A.8) and (A.9) must hold with equality (otherwise, one or both sellers could increase their asking price for one unit without causing the buyer to cease buying from them). It also means that each seller must earn non-negative payoff

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[27]To see this, substitute (A.9) into the right-hand side of (A.5), and note that $V_{IO}^1 + V_{EO}^1 \geq V_{IE}^1$ from (1).

under the proposed contracts, and in the case of the entrant, it means that the entrant must earn at least θ in period one. And, finally, it means that there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$, such that, given T_E^*, the incumbent can make itself and the buyer better off by inducing exclusion.[28]

The requirement that the incumbent earn non-negative payoff is satisfied when (A.9) holds with equality because then the incumbent's payoff simplifies to $\Pi_I = V_{IE}^1 - V_{EO}^1 - c + V_I^2 - c$, which is positive given (1) and (3). The requirement that the entrant earn non-negative payoff and at least θ in period one is satisfied when $T_E^*(1)$ equals the second term on the right-hand side of (A.8) because then the entrant's first and second period payoff simplify to $V_{IE}^1 - V_{IO}^1 - c > 0$ and $V_E^2 - c > 0$. However, if $T_E^*(1)$ equals the first term on the right-hand side of (A.8), then the entrant's payoff is

$$\Pi_E = V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1) - c + V_E^2 - c,$$

which, since $V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1) - c$ is its first period payoff, implies that the entrant earns non-negative payoff and at least θ in period one if and only if $T_I^*(2) - T_I^*(1)$ is bounded below by

$$T_I^*(2) - T_I^*(1) \geq c - \left(V_{IE}^1 - V_{II}^1 - \theta\right). \tag{A.10}$$

The requirement that the incumbent not find it profitable to exclude the entrant implies that for T_I^* and T_E^* to arise in an efficient equilibrium, there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that

$$\tilde{T}_I(2) - 2c + V_{II}^2(I) - 2c > T_I^*(1) - c + V_I^2 - c, \tag{A.11}$$

$$V_{II}^1 - \tilde{T}_I(2) > V_{IE}^1 - \tilde{T}_I(1) - T_E^*(1), \tag{A.12}$$

$$V_{II}^1 - \tilde{T}_I(2) > V_{IO}^1 - \tilde{T}_I(1), \tag{A.13}$$

$$V_{II}^1 - \tilde{T}_I(2) > V_{EO}^1 - T_E^*(1), \tag{A.14}$$

and

$$V_{II}^1 - \tilde{T}_I(2) \geq 0, \tag{A.15}$$

where (A.11) ensures that the incumbent is better off excluding the entrant, (A.12) to (A.14) ensure that the buyer is better off excluding the entrant, and (A.15) ensures that the buyer earns non-negative payoff. Since (A.9) holds with equality, we can replace the right-hand side of (A.14) with $V_{IE}^1 - T_I^*(1) - T_E^*(1)$, and since $\tilde{T}_I(1)$ can be arbitrarily large, we know that (A.12) and (A.13) can be satisfied. Hence, we can reduce the set of conditions (A.11) to (A.15) to the equivalent set[29]

$$\tilde{T}_I(2) - 2c + V_{II}^2(I) - 2c > T_I^*(1) - c + V_I^2 - c, \tag{A.16}$$

and

$$V_{II}^1 - \tilde{T}_I(2) > V_{IE}^1 - T_I^*(1) - T_E^*(1). \tag{A.17}$$

[28]Note that the buyer must also be made better off because it must agree to go along with the exclusion.

[29]We do not need to include (A.15) in the set because we know that the right-hand side of (A.17) is weakly positive.

It follows that for T_I^* and T_E^* to arise in an efficient equilibrium, it must be that

$$V_{II}^1 - 2c + V_{II}^2(I) - 2c \;\le\; V_{IE}^1 - T_E^*(1) - c + V_I^2 - c. \qquad (A.18)$$

Once again, we have an upper bound on $T_E^*(1)$ if T_I^* and T_E^* are to arise in an efficient equilibrium:

$$T_E^*(1) \;\le\; V_{IE}^1 - V_{II}^1 + c - \left(V_{II}^2(I) - V_I^2 - c\right). \qquad (A.19)$$

Comparing (A.19) and the fact that (A.8) holds with equality in an efficient equilibrium implies

$$T_I^*(2) - T_I^*(1) \;\le\; c - \left(V_{II}^2(I) - V_I^2 - c\right). \qquad (A.20)$$

Hence, (A.10) and (A.20) imply the following upper and lower bound on $T_I^*(2) - T_I^*(1)$:

$$c - \left(V_{II}^2(I) - V_I^2 - c\right) \;\ge\; T_I^*(2) - T_I^*(1) \;\ge\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right), \qquad (A.21)$$

which can be satisfied if and only if

$$\theta \;\le\; V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c\right). \qquad (A.22)$$

To summarize, we have shown that, in any efficient equilibrium, (A.8) and (A.9) hold with equality, and (A.21) and (A.22) hold. Thus, in any efficient equilibrium, we have that

$$T_E^*(1) \;=\; V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), \quad T_I^*(1) \;=\; V_{IE}^1 - V_{EO}^1,$$

$$c - \left(V_{II}^2(I) - V_I^2 - c\right) \;\ge\; T_I^*(2) - T_I^*(1) \;\ge\; c - \left(V_{IE}^1 - V_{II}^1 - \theta\right),$$

and

$$\theta \;\le\; V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c\right). \qquad (A.23)$$

The necessity of (A.23) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (A.23) follows because when it holds, the incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

Exclusionary Equilibria

Now suppose there is an equilibrium in which the buyer purchases both units from the incumbent in period one (and hence also in period two), and let T_I^{**} and T_E^{**} denote the equilibrium contracts. Then, the payoff to each party, taking into account the subsequent play of the game, is

$$\Pi_I \;=\; T_I^{**}(2) - 2c + V_{II}^2(I) - 2c, \qquad (A.24)$$

$$\Pi_E \;=\; 0, \qquad (A.25)$$

$$\Pi_B \;=\; V_{II}^1 - T_I^{**}(2), \qquad (A.26)$$

where $T_I^{**}(2) - 2c$ and $V_{II}^2(I) - 2c$ are the incumbent's payoffs in periods one and two, respectively.

It must also be the case that, given contracts T_I^{**} and T_E^{**}, the buyer weakly prefers to purchase both units from the incumbent. In this case, the buyer's incentive-compatibility constraints are:

$$V_{II}^1 - T_I^{**}(2) \geq V_{IE}^1 - T_I^{**}(1) - T_E^{**}(1), \tag{A.27}$$

$$V_{II}^1 - T_I^{**}(2) \geq V_{IO}^1 - T_I^{**}(1), \tag{A.28}$$

$$V_{II}^1 - T_I^{**}(2) \geq V_{EO}^1 - T_E^{**}(1), \tag{A.29}$$

and

$$V_{II}^1 - T_I^{**}(2) \geq 0, \tag{A.30}$$

where (A.27) ensures that the buyer does not want to purchase one unit from each seller, and (A.28) to (A.30) ensure that the buyer does not want to purchase only one unit or no units. These conditions place bounds on the terms of T_I^{**}. For example, it follows from (A.27) to (A.29) that

$$T_I^{**}(2) - T_I^{**}(1) \leq \min\left\{V_{II}^1 - V_{IE}^1 + T_E^{**}(1), \ V_{II}^1 - V_{IO}^1\right\}, \tag{A.31}$$

and

$$T_I^{**}(2) \leq V_{II}^1 - V_{EO}^1 + T_E^{**}(1). \tag{A.32}$$

Since (A.30) is satisfied whenever (A.31) and (A.32) are satisfied, provided that $T_E^{**}(1) \leq V_{EO}^1$ (as it will be in any equilibrium in which the entrant is excluded), it follows that if $T_E^{**}(1) \leq V_{EO}^1$ then (A.31) and (A.32) are necessary and sufficient for the buyer's incentive-compatibility conditions to be satisfied in any equilibrium in which the buyer purchases both units from the incumbent.

Lastly, it must be the case that neither the incumbent nor the entrant can profitably deviate given the other's contract. For the entrant, this means that there must not exist a profitable deviation in which it induces the buyer to purchase from it. For the incumbent, this means that there must not exist a profitable deviation in which it continues to sell both units or only one unit to the buyer. It also means that its payoff under the proposed contracts must be non-negative.

Consider the entrant's situation. For T_I^{**} and T_E^{**} to arise in an exclusionary equilibrium, there must not be a deviation such that the entrant can make itself and the buyer better off. That is, for T_I^{**} and T_E^{**} to arise in an exclusionary equilibrium, there must not exist $\tilde{T}_E(1)$ such that

$$\tilde{T}_E(1) \geq \theta + c, \tag{A.33}$$

$$\tilde{T}_E(1) - c + V_E^2 - c > 0, \tag{A.34}$$

$$\max\left\{V_{IE}^1 - T_I^{**}(1) - \tilde{T}_E(1), V_{EO}^1 - \tilde{T}_E(1)\right\} > V_{II}^1 - T_I^{**}(2), \tag{A.35}$$

$$\max\left\{V_{IE}^1 - T_I^{**}(1) - \tilde{T}_E(1), V_{EO}^1 - \tilde{T}_E(1)\right\} > V_{IO}^1 - T_I^{**}(1), \tag{A.36}$$

and

$$\max\left\{V_{IE}^1 - T_I^{**}(1) - \tilde{T}_E(1), V_{EO}^1 - \tilde{T}_E(1)\right\} \geq 0, \tag{A.37}$$

where (A.33) ensures that the entrant earns at least θ in period one, (A.34) ensures that the entrant is better off under the deviation, (A.35) and (A.36) ensure that the buyer is better off under the deviation, and (A.37) ensures that the buyer earns non-negative payoff. Since (A.35) implies (A.36) and (A.37), and (A.33) and (A.34) are satisfied if $\tilde{T}_E(1) > \theta + c$, it follows that $T_I^{**}(1)$ and $T_I^{**}(2)$ must be such that (A.35) does not hold at $\tilde{T}_E(1) = \theta + c$ if they are to arise in equilibrium:

$$T_I^{**}(2) - T_I^{**}(1) \leq V_{II}^1 - V_{IE}^1 + \theta + c, \tag{A.38}$$

and

$$T_I^{**}(2) \leq V_{II}^1 - V_{EO}^1 + \theta + c. \tag{A.39}$$

The interpretation is that the terms of T_I^{**} must be bounded above by (A.38) and (A.39) in any exclusionary equilibrium because if (A.38) or (A.39) were not satisfied, there would exist $\tilde{T}_E(1) > \theta + c$ such that the entrant would be able to profitably induce the buyer to purchase from it.

Now consider the incumbent's situation. For T_I^{**} and T_E^{**} to arise in an exclusionary equilibrium, there must not exist a profitable deviation in which the incumbent sells both units to the buyer. It follows from this that (A.32) must hold with equality (because otherwise, the incumbent could increase $T_I^{**}(2)$ and still sell both units to the buyer), and therefore, using this result, (A.33), and (A.39), it follows that in any exclusionary equilibrium the entrant's offer must be such that:

$$T_E^{**}(1) = \theta + c, \tag{A.40}$$

and hence the incumbent's offer must be such that

$$T_I^{**}(2) = V_{II}^1 - V_{EO}^1 + \theta + c, \tag{A.41}$$

and

$$T_I^{**}(2) - T_I^{**}(1) \leq V_{II}^1 - V_{IE}^1 + \theta + c,$$

or, equivalently,

$$T_I^{**}(2) - T_I^{**}(1) \leq c - \left(V_{IE}^1 - V_{II}^1 - \theta\right). \tag{A.42}$$

There must also not exist a profitable deviation in which the incumbent sells only one unit to the buyer. It follows from this that there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that

$$\tilde{T}_I(1) - c + V_I^2 - c > T_I^{**}(2) - 2c + V_{II}^2(I) - 2c, \tag{A.43}$$

$$\max\left\{V_{IE}^1 - \tilde{T}_I(1) - T_E^{**}(1), V_{IO}^1 - \tilde{T}_I(1)\right\} > V_{II}^1 - \tilde{T}_I(2), \tag{A.44}$$

$$\max\left\{V_{IE}^1 - \tilde{T}_I(1) - T_E^{**}(1), V_{IO}^1 - \tilde{T}_I(1)\right\} > V_{EO}^1 - T_E^{**}(1), \tag{A.45}$$

and

$$\max\left\{V_{IE}^1 - \tilde{T}_I(1) - T_E^{**}(1), V_{IO}^1 - \tilde{T}_I(1)\right\} \geq 0, \tag{A.46}$$

where (A.43) ensures that the incumbent is better off under the deviation, (A.44) and (A.45) ensure that (whether or not it is better off under the deviation) the buyer is induced to purchase only one unit from the incumbent, and (A.46) ensures that the buyer earns non-negative payoff. Since (A.45) and (A.40) imply (A.46), and since (A.44) can always be satisfied by choosing $\tilde{T}_I(2)$ to be arbitrarily large, we can reduce the set of constraints in (A.43) to (A.46) to the equivalent set

$$\tilde{T}_I(1) - c + V_I^2 - c \;>\; V_{II}^1 - V_{EO}^1 + \theta + c - 2c + V_{II}^2(I) - 2c, \tag{A.47}$$

and

$$V_{IE}^1 - \tilde{T}_I(1) - \theta - c \;>\; V_{EO}^1 - \theta - c, \tag{A.48}$$

where we have substituted (A.40) and (A.41) into (A.43) and (A.45), and simplified the left-hand side of (A.45). It follows that if T_I^{**} and T_E^{**} are to arise in an exclusionary equilibrium, then

$$\theta + c \;\geq\; V_{IE}^1 - V_{II}^1 - V_{II}^2(I) + V_I^2 + 2c,$$

or, equivalently,

$$\theta \;\geq\; V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c \right). \tag{A.49}$$

Finally, it must be that the incumbent earns non-negative payoff under the proposed equilibrium contracts. Substituting (A.41) into Π_I, we have that the incumbent's payoff is

$$\Pi_I \;=\; V_{II}^1 - V_{EO}^1 + \theta + c - 2c + V_{II}^2(I) - 2c,$$

which, from (A.49), weakly exceeds $V_{IE}^1 - V_{EO}^1 - c + V_I^2 - c$, which is positive given (1) and (3).

To summarize, we have shown that, in any exclusionary equilibrium, contracts are such that (A.40), (A.41), (A.42), and (A.49) hold. Thus, in any exclusionary equilibrium, we have that

$$T_E^{**}(1) \;=\; \theta + c, \quad T_I^{**}(2) \;=\; V_{II}^1 - V_{EO}^1 + \theta + c,$$

$$T_I^{**}(2) - T_I^{**}(1) \;\leq\; c - \left(V_{IE}^1 - V_{II}^1 - \theta \right),$$

and

$$\theta \;\geq\; V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c \right). \tag{A.50}$$

The necessity of (A.50) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (A.50) follows because when it holds, the incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

It remains only to show that there do not exist equilibria in which the buyer purchases one unit from one seller or no units. Suppose there is an equilibrium in which the buyer purchases only from the entrant or no units. Then the incumbent can profitably deviate by offering $T_1(1) = c + \epsilon$, for ϵ sufficiently small but positive, a contradiction. Similarly, suppose there is an equilibrium in which the buyer purchases only one unit from the incumbent or no units. Then the entrant can profitably deviate by offering $T_E(1) = c + \epsilon$, for ϵ sufficiently small but positive, a contradiction.   Q.E.D.

Appendix B

Proof of Proposition 5

For a complete proof of Proposition 5, see Appendix B in the online version of this paper available at www.simon.rochester.edu/fac/shaffer/Published/two-units-fullproof.pdf.

The idea is that, as in the proof of Proposition 1, (A.8) and (A.9) must hold with equality in any equilibrium in which the buyer purchases one unit from each seller in period one (and hence also in period two). Given they hold with equality, the equilibrium contracts follow straightforwardly from noting in addition that the prohibition on pricing below marginal cost implies $T_I^*(2) - T_I^*(1) = c$.

Moreover, to show that exclusionary equilibria do not exist under the prohibition, note that, as in Proposition 1, (A.38) and (A.39) must be satisfied in any exclusionary equilibrium. But $V_{IE}^1 > V_{II}^1$ and $\theta \leq 0$ imply that (A.38) is satisfied only if $T_I^{**}(2) - T_I^{**}(1)$ is less than c, which violates the prohibition. It follows that there is no equilibrium in which the entrant is excluded.

Appendix C

Proof of Proposition 6

For a complete proof of Proposition 6, see Appendix C in the online version of this paper available at www.simon.rochester.edu/fac/shaffer/Published/two-units-fullproof.pdf.

The idea is that, as in the proof of Proposition 1, the requirement that the incumbent not find it profitable to exclude the entrant implies that for T_I^* and T_E^* to arise in an efficient equilibrium, there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that conditions (A.11), (A.12), (A.13), (A.14), and (A.15) hold. The prohibition on negative prices implies in addition that $\tilde{T}_I(1) \geq 0$ and $\tilde{T}_I(2) - \tilde{T}_I(1) \geq 0$.

Since (A.9) holds with equality, we can replace (A.14) with (A.17), and since $\tilde{T}_I(1)$ has an upper bound of $\tilde{T}_I(2)$, we can replace (A.12) and (A.13) with this upper bound. It follows that we can reduce the set of conditions (A.11) to (A.15), $\tilde{T}_I(1) \geq 0$, and $\tilde{T}_I(2) \geq \tilde{T}_I(1)$ to the equivalent set

$$\tilde{T}_I(2) - 2c + V_{II}^2(I) - 2c \quad > \quad T_I^*(1) - c + V_I^2 - c, \tag{C.1}$$

$$V_{II}^1 - \tilde{T}_I(2) \quad > \quad V_{IE}^1 - T_I^*(1) - T_E^*(1), \tag{C.2}$$

$$V_{II}^1 \quad > \quad V_{IE}^1 - T_E^*(1), \tag{C.3}$$

and

$$\tilde{T}_I(2) \quad \geq \quad 0. \tag{C.4}$$

It follows that for T_I^* and T_E^* to arise in an efficient equilibrium, it must be that

$$V_{II}^1 - 2c + V_{II}^2(I) - 2c \quad \leq \quad V_{IE}^1 - T_E^*(1) - c + V_I^2 - c, \tag{C.5}$$

or

$$V_{II}^1 \leq V_{EO}^1 - T_E^*(1), \tag{C.6}$$

or

$$V_{II}^1 \leq V_{IE}^1 - T_E^*(1), \tag{C.7}$$

where condition (C.5) comes from summing the left and right-hand sides of (C.1) and (C.2), condition (C.6) comes from (C.2) by evaluating $\tilde{T}_I(2)$ at 0, and condition (C.7) comes from (C.3). Once again, we have an upper bound on $T_E^*(1)$ if T_I^* and T_E^* are to arise in an efficient equilibrium:

$$T_E^*(1) \leq \max\left\{V_{IE}^1 - V_{II}^1, V_{IE}^1 - V_{II}^1 + c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\}. \tag{C.8}$$

Comparing (C.8) and the fact that (A.8) holds with equality in an efficient equilibrium implies

$$T_I^*(2) - T_I^*(1) \leq \max\left\{0, c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\}. \tag{C.9}$$

Hence, (A.10), (C.9), and the ban on negative marginal pricing, which implies $T_I^*(2) \geq T_I^*(1)$, imply the following upper and lower bound on $T_I^*(2) - T_I^*(1)$:

$$\max\left\{0, c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\} \geq T_I^*(2) - T_I^*(1) \geq \max\left\{0, c - \left(V_{IE}^1 - V_{II}^1 - \theta\right)\right\}, \tag{C.10}$$

which can be satisfied if and only if

$$\theta + c \leq V_{IE}^1 - V_{II}^1 + \max\left\{0, c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\},$$

or, equivalently,

$$\theta \leq V_{IE}^1 - V_{II}^1 - \min\left\{c, V_{II}^2(I) - V_I^2 - c\right\}. \tag{C.11}$$

To summarize, we have shown that, in any efficient equilibrium, (A.8) and (A.10) hold with equality, and (C.10) and (C.11) hold. Thus, in any efficient equilibrium, we have that

$$T_E^*(1) = V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), \quad T_I^*(1) = V_{IE}^1 - V_{EO}^1,$$

$$\max\left\{0, c - \left(V_{II}^2(I) - V_I^2 - c\right)\right\} \geq T_I^*(2) - T_I^*(1) \geq c - \max\left\{0, c - \left(V_{IE}^1 - V_{II}^1 - \theta\right)\right\},$$

and

$$\theta \leq V_{IE}^1 - V_{II}^1 - \min\left\{c, V_{II}^2(I) - V_I^2 - c\right\}. \tag{C.12}$$

The necessity of (C.12) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (C.12) follows because when it holds, the buyer's incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

Necessary and sufficient conditions for exclusionary equilibria to exist are derived similarly.

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Appendix D

Proof of Proposition 7

For a complete proof of Proposition 7, see Appendix D in the online version of this paper available at www.simon.rochester.edu/fac/shaffer/Published/two-units-fullproof.pdf.

The idea is that, in any efficient equilibrium, the payoffs to all parties are given by (A.1), (A.2), and (A.3), and the incentive compatibility constraints are given by (A.4), (A.5), (A.6), (A.7) and the requirement that the buyer does not want to purchase both units from the entrant:

$$V_{IE}^1 - T_I^*(1) - T_E^*(1) \geq V_{EE}^1 - T_E^*(2). \tag{D.1}$$

It follows from (A.4), (A.5), (A.6) and (D.1) that

$$T_E^*(1) \leq \min\left\{V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), V_{IE}^1 - V_{IO}^1\right\}, \tag{D.2}$$

and

$$T_I^*(1) \leq \min\left\{V_{IE}^1 - V_{EE}^1 + T_E^*(2) - T_E^*(1), V_{IE}^1 - V_{EO}^1\right\}. \tag{D.3}$$

Since (A.7) is satisfied whenever $T_E^*(1)$ and $T_I^*(1)$ satisfy (D.2) and (D.3), it follows that (D.2) and (D.3) are necessary and sufficient for the buyer's incentive-compatibility constraints to be satisfied.

It must be the case that neither seller can profitably deviate given the other's contract. This means that (D.2) and (D.3) must hold with equality. It also means that each seller must earn non-negative payoff under the proposed contracts, and in the case of the entrant, it means that the entrant must earn at least θ in period one. And, finally, it means that there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that the incumbent can make itself and the buyer better off by inducing exclusion, and similarly there must not exist a profitable deviation for the entrant that induces exclusion.

The requirement that the incumbent earn non-negative payoff implies that

$$T_E^*(2) - T_E^*(1) \geq c - \left(V_{IE}^1 - V_{EE}^1\right) - \left(V_I^2 - c\right). \tag{D.4}$$

The requirement that the entrant earn non-negative payoff implies that

$$T_I^*(2) - T_I^*(1) \geq c - \left(V_{IE}^1 - V_{II}^1 - \theta\right). \tag{D.5}$$

The requirement that the incumbent not be able to make itself and the buyer better off by excluding the entrant implies that for T_I^* and T_E^* to arise in an efficient equilibrium, there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that conditions (A.11), (A.12), (A.13), (A.14), and (A.15) hold and

$$V_{II}^1 - \tilde{T}_I(2) > V_{EE}^1 - T_E^*(2), \tag{D.6}$$

where (D.6) ensures that the buyer does not want to purchase both units from the entrant. Since (D.3) holds with equality, either (A.14) is equal to (A.17) and (D.6) is satisfied, or vice-versa, and

since $\tilde{T}_I(1)$ can be chosen arbitrarily large, we know that (A.12) and (A.13) can be satisfied. It follows that we can reduce the set of conditions (A.11) to (A.15) and (D.6) to (A.16) and (A.17).

Following the proof of Proposition 1, we thus have the following bounds on $T_I^*(2) - T_I^*(1)$:

$$c - \left(V_{II}^2(I) - V_I^2 - c\right) \ \geq \ T_I^*(2) - T_I^*(1) \ \geq \ c - \left(V_{IE}^1 - V_{II}^1 - \theta\right), \tag{D.7}$$

which can be satisfied if and only if

$$\theta \ \leq \ V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c\right). \tag{D.8}$$

The requirement that the entrant not be able to make itself and the buyer better off by excluding the incumbent implies that for T_I^* and T_E^* to arise in an efficient equilibrium, there must not exist $\tilde{T}_E(1)$ and $\tilde{T}_E(2)$ such that the analogues of conditions (A.11), (A.12), (A.13), (A.14), (A.15) and (D.6) hold. Following our earlier reasoning, these conditions can be reduced to an equivalent set of conditions, i.e., the analogues of (A.16) and (A.17), and thus ultimately to the analogue of (D.7):

$$c - \left(V_{EE}^2(I) - V_E^2 - c\right) \ \geq \ T_E^*(2) - T_E^*(1) \ \geq \ c - \left(V_{IE}^1 - V_{EE}^1\right) - \left(V_I^2 - c\right). \tag{D.9}$$

To summarize, we have shown that, in any efficient equilibrium, (D.2) and (D.3) hold with equality, and (D.7), (D.8) and (D.9) hold. Thus, in any efficient equilibrium, we have that

$$T_E^*(1) \ = \ V_{IE}^1 - V_{II}^1 + T_I^*(2) - T_I^*(1), \quad T_I^*(1) \ = \ V_{IE}^1 - V_{EE}^1 + T_E^*(2) - T_E^*(1),$$

$$c - \left(V_{EE}^2(I) - V_E^2 - c\right) \ \geq \ T_E^*(2) - T_E^*(1) \ \geq \ c - \left(V_{IE}^1 - V_{EE}^1\right) - \left(V_I^2 - c\right).$$

$$c - \left(V_{II}^2(I) - V_I^2 - c\right) \ \geq \ T_I^*(2) - T_I^*(1) \ \geq \ c - \left(V_{IE}^1 - V_{II}^1 - \theta\right),$$

and

$$\theta \ \leq \ V_{IE}^1 - V_{II}^1 - \left(V_{II}^2(I) - V_I^2 - c\right). \tag{D.10}$$

The necessity of (D.10) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (D.10) follows because when it holds, the buyer's incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

Exclusionary Equilibria

In any exclusionary equilibrium, the payoffs to all three parties are given by (A.24) – (A.26), and the buyer's incentive-compatibility constraints are given by (A.27) – (A.30). In addition, we have the requirement that the buyer does not want to purchase both units from the entrant:

$$V_{II}^1 - T_I^{**}(2) \ \geq \ V_{EE}^1 - T_E^*(2). \tag{D.11}$$

It follows from (A.27), (A.28), (A.29) and (D.11) that

$$T_I^{**}(2) - T_I^{**}(1) \ \leq \ \min\left\{V_{II}^1 - V_{IE}^1 + T_E^{**}(1), \ V_{II}^1 - V_{IO}^1\right\}, \tag{D.12}$$

and

$$T_I^{**}(2) \leq \min\left\{V_{II}^1 - V_{EO}^1 + T_E^{**}(1), \ V_{II}^1 - V_{EE}^1 + T_E^{**}(2)\right\} \tag{D.13}$$

Since (A.30) is satisfied if (D.12) and (D.13) are satisfied and $T_E^{**}(2) \leq V_{EE}^1$ (as it will be in any equilibrium in which the entrant is excluded), it follows that if $T_E^{**}(2) \leq V_{EE}^1$ then (D.12) and (D.13) are necessary and sufficient for the buyer's incentive-compatibility constraints to be satisfied.

It must be the case that neither the incumbent nor the entrant can profitably deviate given the other's contract. For the entrant, this means that there must not exist a profitable deviation in which it induces the buyer to purchase from it. For the incumbent, this means that there must not exist a deviation in which it increases its payoff while continuing to sell both units or only one unit to the buyer. It also means that its payoff under the proposed contracts must be non-negative.

Consider the entrant's situation. For T_I^{**} and T_E^{**} to arise in equilibrium, there must not be a profitable deviation for the entrant in which it sells only one unit. That is, there must not exist $\tilde{T}_E(1)$, $\tilde{T}_E(2)$ such that the following conditions hold: (A.33), (A.34), (A.35), (A.36), (A.37) and

$$\max\left\{V_{IE}^1 - T_I^{**}(1) - \tilde{T}_E(1), V_{EO}^1 - \tilde{T}_E(1)\right\} > V_{EE}^1 - \tilde{T}_E(2), \tag{D.14}$$

Since (A.35) implies (A.36) and (A.37), (D.14) can easily be satisfied by choosing $\tilde{T}_E(2)$ to be arbitrarily large, and (A.33) and (A.34) are satisfied if $\tilde{T}_E(1) > \theta + c$, it follows that $T_I^{**}(1)$ and $T_I^{**}(2)$ must be such that (A.38) and (A.39) hold if they are to arise in an exclusionary equilibrium.

For T_I^{**} and T_E^{**} to arise in equilibrium, there also must not be a profitable deviation for the entrant in which it sells both units. That is, there must not exist $\tilde{T}_E(1)$ and $\tilde{T}_E(2)$ such that

$$\tilde{T}_E(2) \geq \theta + 2c, \tag{D.15}$$

$$\tilde{T}_E(2) - 2c + V_{EE}^2(E) - 2c > 0, \tag{D.16}$$

$$V_{EE}^1 - \tilde{T}_E(2) > V_{II}^1 - T_I^{**}(2), \tag{D.17}$$

$$V_{EE}^1 - \tilde{T}_E(2) > V_{IO}^1 - T_I^{**}(1), \tag{D.18}$$

$$V_{EE}^1 - \tilde{T}_E(2) > V_{IE}^1 - T_I^{**}(1) - \tilde{T}_E(1), \tag{D.19}$$

$$V_{EE}^1 - \tilde{T}_E(2) > V_{EO}^1 - \tilde{T}_E(1), \tag{D.20}$$

and

$$V_{EE}^1 - \tilde{T}_E(2) \geq 0, \tag{D.21}$$

where (D.15) ensures that the entrant earns at least θ in period one, (D.16) ensures that the entrant is better off under the deviation, (D.17) – (D.20) ensure that the buyer would prefer to purchase both units from the entrant, and (D.21) ensures that the buyer earns non-negative payoff. Since (D.17) implies (D.18) and (D.21), (D.19) and (D.20) can easily be satisfied by choosing $\tilde{T}_E(1)$ to be

arbitrarily large, and (D.15) and (D.16) are satisfied if $\tilde{T}_E(2) > \theta + 2c$, it follows that $T_I^{**}(1)$ and $T_I^{**}(2)$ must be such that (D.17) does not hold at $\tilde{T}_E(2) = \theta + 2c$ if they are to arise in equilibrium:

$$T_I^{**}(2) \leq V_{II}^1 - V_{EE}^1 + \theta + 2c. \tag{D.22}$$

Now consider the incumbent's situation. For T_I^{**} and T_E^{**} to arise in equilibrium, there must not be a profitable deviation in which the incumbent sells both units to the buyer. It follows that (D.13) must hold with equality (because otherwise, the incumbent could increase $T_I^{**}(2)$ and still sell both units to the buyer), and therefore, using this result, (A.33), (A.39), (D.15), (D.22), and noting that (D.22) implies (A.39), it follows that the entrant's equilibrium offer must be such that

$$T_E^{**}(2) = \theta + 2c, \tag{D.23}$$

and hence the incumbent's offer must be such that

$$T_I^{**}(2) = V_{II}^1 - V_{EE}^1 + \theta + 2c, \tag{D.24}$$

and

$$T_I^{**}(2) - T_I^{**}(1) \leq c - \left(V_{IE}^1 - V_{II}^1 - \theta\right). \tag{D.25}$$

For T_I^{**} and T_E^{**} to arise in equilibrium, there must also not exist a profitable deviation for the incumbent in which it sells only one unit to the buyer. That is, there must not exist $\tilde{T}_I(1)$ and $\tilde{T}_I(2)$ such that the following conditions hold: (A.43), (A.44), (A.45), (A.46), and

$$\max\left\{V_{IE}^1 - \tilde{T}_I(1) - T_E^{**}(1), V_{IO}^1 - \tilde{T}_I(1)\right\} > V_{EE}^1 - T_E^{**}(2), \tag{D.26}$$

where (D.26) ensures that the buyer does not want to purchase both units from the entrant. Since (D.26) and (D.23) imply (A.45) and (A.46), and (A.44) can be satisfied by choosing $\tilde{T}_I(2)$ arbitrarily large, we can reduce the set of constraints in (A.43) to (A.46) and (D.26) to the equivalent set

$$\tilde{T}_I(1) - c + V_I^2 - c > V_{II}^1 - V_{EE}^1 + \theta + V_{II}^2(I) - 2c, \tag{D.27}$$

and

$$\max\left\{V_{IE}^1 - \tilde{T}_I(1) - T_E^{**}(1), V_{IO}^1 - \tilde{T}_I(1)\right\} > V_{EE}^1 - \theta - 2c, \tag{D.28}$$

where we have substituted (D.24) into (A.43) to get (D.27) and (D.23) into (D.26) to get (D.28). By summing the left and right-hand sides of (D.27) and (D.28) using the first argument in the left-hand side of (D.28) and then doing it again using the second argument in the left-hand side of (D.28), we can further reduce the constraints in (A.43) to (A.46) and (D.26) to the equivalent set

$$V_{IE}^1 + V_I^2 - T_E^{**}(1) > V_{II}^1 + V_{II}^2(I) - 2c, \tag{D.29}$$

or

$$V_{IO}^1 + V_I^2 \quad > \quad V_{II}^1 + V_{II}^2(I) - 2c. \tag{D.30}$$

Since $V_{II}^1 - V_{IO}^1 > c$ and $V_{II}^2(I) - V_I^2 > c$ (using (1), (??), and (3)), (D.30) fails to hold, and thus, it follows that the set of constraints in (A.43) to (A.46) and (D.26) hold if and only if (D.29) holds.

It follows that if T_I^{**} and T_E^{**} are to arise in equilibrium, then

$$T_E^{**}(1) \quad \geq \quad V_{IE}^1 - V_{II}^1 + 2c - \left(V_{II}^2(I) - V_I^2\right). \tag{D.31}$$

Finally, it must be that the incumbent earns non-negative payoff under the proposed equilibrium contracts. Substituting (D.24) into (A.24), we have that the incumbent's payoff is

$$\Pi_I \quad = \quad V_{II}^1 - V_{EE}^1 + \theta + V_{II}^2(I) - 2c, \tag{D.32}$$

which, given our assumption that $V_{II}^1 \geq V_{EE}^1$ and $V_{II}^2(I) \geq V_{EE}^2(E)$, implies that Π_I is nonnegative.

To summarize, we have shown that in any equilibrium in which the entrant is excluded, contracts are such that (D.23), (D.24), (D.25), and (D.31) hold. Thus, in any such equilibrium, we have

$$T_E^{**}(1) \quad \geq \quad V_{IE}^1 - V_{II}^1 + c - \left(V_{II}^2(I) - V_I^2 - c\right),$$

$$T_E^{**}(2) \quad = \quad \theta + 2c,$$

$$T_I^{**}(2) \quad = \quad V_{II}^1 - V_{EE}^1 + \theta + 2c,$$

and

$$T_I^{**}(2) - T_I^{**}(1) \quad \leq \quad c - \left(V_{IE}^1 - V_{II}^1 - \theta\right). \tag{D.33}$$

The necessity of (D.33) follows because otherwise profitable deviations for one or more parties exist, and the sufficiency of (D.33) follows because when it holds, the incentive-compatibility conditions are satisfied and neither seller can profitably deviate given the other seller's contract.

Now suppose there is an equilibrium in which the buyer purchases both units from the entrant in period one (and hence also in period two). Then, using the reasoning above, we obtain the analogue to (D.33): in any equilibrium in which the incumbent is excluded, the entrant offers

$$T_E^{**}(2) \quad = \quad V_{EE}^1 - V_{II}^1 + \theta_I + 2c, \tag{D.34}$$

where $\theta_I = -\left(V_{II}^2(I) - 2c\right)$ is the maximum amount that an unconstrained incumbent would be willing to pay in period one to avoid being excluded. Given our assumption that $V_{II}^1 \geq V_{EE}^1$ and $V_{II}^2(I) \geq V_{EE}^2(E))$, it follows that $T_E^{**}(2) \leq \theta_I + 2c$, which violates our assumption that the entrant has a financing constraint. Hence, there is no equilibrium in which the incumbent is excluded.

It remains only to show that there do not exist equilibria in which the buyer purchases one unit from one seller or no units. The proof of this is given in the last paragraph of Appendix A.   Q.E.D.

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