Bilateral Delegation, Wage Bargaining and Managerial Incentives: Implications for Efficiency and Distribution

Ishita Chatterjee[*]   Bibhas Saha[†]

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Abstract

We develop a model of bilateral delegation in wage and employment bargaining to study efficiency and distributional implications in monopoly and in Cournot duopoly. In both markets delegation causes underproduction, but has contrasting implications for bargaining pie and for its distribution. In monopoly the bargaining pie contracts. In duopoly the bargaining pie expands, sometimes even up to the collusive level suggesting that delegation is conducive to implicit collusion. Surprisingly, a party's payoff can be inversely related to its bargaining power. The well-known duopoly result of overproduction occurs only in unilateral delegations and when the delegating party is sufficiently strong.

JEL Classification: L12, L14, D43. **Key Words:** Managerial incentives, efficient bargaining, bilateral delegation, implicit collusion

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[*]Department of Economics, University of Western Australia, Crawley, WA 6009, Australia; Email: Ishita.Chatterjee@uwa.edu.au

[†]School of Economics, University of East Anglia, Norwich, U.K. NR4 7TJ; E-mail: B.Saha@uea.ac.uk

1　Introduction

The strategic delegation approach developed by Vickers (1985), Fershtman (1985), Fershtman and Judd (1987) and Skilvas (1987) have shown that in oligopoly firms will deviate from profit maximization to gain strategic advantage over their rivals.[1] In Cournot oligopolies this deviation occurs through delegation of output decisions to managers through sales-oriented incentive schemes. The advantage of delegation was first discussed by Schelling (1956); however the context of delegation in Schelling (1956) was bargaining. Vickers (1985) and others showed that delegation can be advantageous in oligopoly as well. However, they did not consider efficiency.

The sales-orientation result of the strategic delegation literature has helped exploring a variety of issues[2]. For example, Basu (1995) offered a new explanation of Stackelberg equilibrium; Zhang and Zhang (1997) studied R&D incentives, Zabojnik (1998) specific human capital, Lambertini and Trombetta (2002) stability of collusion, and Mujumdar and Pal (2007) endogenous market structure in a two-period duopoly. Apart from their reliance on sales-orientation, these articles commonly assume oligopolistic environments (with the exception of Zabojnik(1998)), to justify delegation in the first place.[3] Szymanski (1994) extended the model of Fershtman and Judd (1987) to endogenous cost by introducing a right-to-manage bargaining with labour union. He showed that the strategic delegation result of sales orientation holds only if the union's bargaining power is very low. If the union's bargaining power is sufficiently high, firms would orient the managers to profit maximization. Apart from showing the speciality of sales orientation Szymanski (1994) also showed how delegation can be modelled in bargaining – a step closer to formalizing Schelling's idea. But his delegation was one-sided; that is, only the shareholders could delegate (in each firm), but not the union members. In reality, unions are always represented by union leaders[4].

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[1]There are at least two other approaches to managerial incentive. Hart (1983), and Schmidt (1997), among others, have used agency theory to examine the effects of the product market competition on optimal incentives and managerial effort. The relative performance evaluation literature offers a financial market perspective on the links between managerial compensation, product market competition, and industry performance. See Jensen and Murphy (1990) and more recently Aggarwal and Samwick (1999).

[2]The influence of the strategic delegation approach has been felt even outside the oligopoly literature. See for instance Basu *et al* (1997) for development policy, Ray (1999) for tenancy choice and Das (1997) for trade policy.

[3]Zabojnik (1998) considered a monopoly setup, but assumed that production requires both managerial effort and a specific human capital from worker. A sales-oriented managerial incentive contract helps to overcome the typical problem of underinvestment in specific human capital. Thus, delegation arises even in a non-strategic setup. But, Zabojnik notes, if the specific human capital was not needed for production, there would be no deviation from profit maximization (p. 794, Zabojnik, 1998). Zabojnik did not study oligopoly.

[4]Schelling (1956, p.286) discussed how union leaders employ tactic of self-commitment by agitating the union members.

Thus, delegation is asymmetric in Szymanski's model. Furthermore, his use of right-to-manage bargaining (which is known to be an inefficient protocol) is not helpful in separating efficiency from distribution. The oligopolistic setup provides further hindrance by keeping the bargaining motive inseparable from the strategic motive. To the best of our knowledge, these limitations have remained unresolved in the delegation literature and we aim to overcome them in this paper.

To begin with, we introduce bilateral delegation. That is, the workers offer incentives to their union leader and shareholders to their manager, and bargaining takes place between the union leader(s) and the manager(s). The manager's incentive scheme, as is standard in this literature, is given by a linear combination of profit and sales. The union leader's incentive scheme, in parallel with the manager's, is given by a linear combination of net wage bill (a surplus that the workers' union tries to maximize) and gross wage bill. With respect to bargaining protocol, we adopt efficient bargaining (McDonald and Solow, 1981), as that is best suited to study efficiency and distribution together; that is, both wage and employment are simultaneously negotiated. Then, to separate bargaining motives from strategic motives we study two market scenarios – monopoly and Cournot duopoly. Monopoly, we may add, has not been studied before for delegated bargaining, though this is a natural starting point. Since monopoly is devoid of strategic interactions, delegation is motivated only by bargaining motive. The results of this case can be contrasted with that of duopoly where players also have strategic motives. Last, but not least, we consider fairly general demand and cost conditions; for example, products need not be homogenous if there are two firms, and production technology only need to exhibit non-increasing returns to scale. It is worth noting that most previous contributions (such as Fershtman and Judd (1987) and Szymanski (1994)) considered linear demand and constant returns technology. We have studied these special cases as examples, and relied on them only when the results in the general case appear ambiguous.

Our general formulation not only confirms the existing results as special cases, but also reveals many surprises and offers new insights. In monopoly, where the sole purpose of delegation is achieving bargaining advantage, delegation will result in underproduction regardless of whether both parties delegate or only one party delegates, and also regardless of the bargaining power of the union being high or low.[5] Shareholders will always orient their manager to profit maximization so that he can 'overvalue' the wage bill and try to force a reduction in wage payment. Similarly, the union leader will be oriented to net wage bill maximization, so that he can 'overvalue' the reservation wage and will try to raise the

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[5]In our setup, in the base case of no-delegation employment does not depend on the players' bargaining powers, and thus the bargaining remains invariant.

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minimum acceptable wage. In effect, both sides will end up overvaluing the opportunity cost of labour and hence employment falls below the no-delegation level, which also means that the bargaining pie will shrink in consequence of delegation.

Of course, how much the pie will shrink as well as how it will be distributed depends on the relative bargaining powers of the players. Since the purpose of delegation is seeking bargaining advantage, one sees more benefits from it only when one has less bargaining power. In other words, delegation is a substitute for bargaining power. Thus, the weakest party is going to use the strongest delegation incentive, and the strongest party none. In fact, a very weak player does always better by resorting to delegation. Since delegation (by at least one party) causes the bargaining pie to shrink, there cannot be any situations where both parties can benefit at the same time by resorting to delegation. But as delegation is individually optimal both sides will delegate with the hope of *at best* only one party gaining from it. More curiously, as weakening of bargaining power directly reduces a player's payoff, it also unleashes a countervailing effect by triggering stronger delegation incentives; so potentially a player can experience a *rise* in his payoff after his bargaining power (exogenously) falls – a surprising possibility from the bargaining point of view. All these can be established under general conditions.

Assuming linear demand and constant returns technology, we can sharpen some of these points. First, the conventional bargaining power-payoff (positive) relationship holds only under unilateral delegations (when delegation is artificially restricted to one party). Under bilateral delegation, the countervailing force (via delegation incentives) is so powerful that the direct effect of loss in bargaining power is overturned. In consequence, weaker one gets, higher the payoff one enjoys. Second, unilateral delegation is less inefficient than bilateral delegation. As each side finds delegation either a dominant strategy or an optimal response to its rival's delegation, the equilibrium outcome gives rise to the smallest bargaining pie (compared to the unilateral delegation cases). Third, in the context of the second observation we identify some situations, where both parties are worse off after delegation.

Thus, we see that when bargaining is permitted delegation arises also in monopoly, a point not highlighted in the delegation literature. Moreover, the delegation involves incentive schemes exactly opposite of sales-orientation, assuming of course no uncertainty and asymmetric information. We should recall that Zabojnik (1998) did obtain sales orientation in monopoly, but his model involved moral hazard and no bargaining.[6]

Next, we extend our model to Cournot duopoly with quantity competition. Now apart

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[6]The setup of Zabojnik (1998) is different from ours. Our manager does not provide any effort; nor do our workers supply any specific human capital.

from the bargaining motive, there is a strategic motive for delegation. The strategic motive aims to steal market from the rival firm and expands the bargaining pie, and the bargaining motive aims to increase the share of that pie. But there is a conflict between the two motives. While market stealing through strategic delegation requires overproduction (i.e. being aggressive in the market), bargaining delegation requires underproduction (i.e. being aggressive within the firm). Firms and unions alike will be caught between the two opposing motives, and the force of each motive depends on their relative bargaining power. For example, a weak player will be strongly motivated for bargaining delegation, while not so for strategic delegation, because his marginal gain, as a fraction of the expanded pie, is rather small. Of course, his within-firm opponent (shareholders or union) will feel exactly opposite. The net effect will not only depend on the resolution of the frictions between two bargaining rivals, but also how they are played out in the market place across the two firms. Consequently, the equilibrium incentive schemes and outcomes vary considerably between unilateral delegations and bilateral delegation, even if attention is restricted to symmetric cases.

Consider the case of unilateral delegation by shareholders. Suppose the shareholders have all the bargaining power. Their sole aim will be to steal the market from the rival firm, expand the pie and keep it all for themselves. Hence, they will orient the manager to sales maximization. This is the standard result of strategic delegation; see for instance Fershtman and Judd (1987). But we also know that if both firms overproduce (while each aiming to steal other's business) both will end up with smaller profit. But now reduce the shareholders' bargaining power, but still do not allow the unions to resort to delegation. The shareholders now see that they cannot get to keep the entire pie that they wish to enlarge by stealing the rival firm's business; instead a part will have to be conceded to the workers. Their incremental gain from strategic delegation diminishes with a loss in bargaining power. Gradually, as the shareholders get weaker their bargaining motive outweighs the strategic motive and they end up with profit-orientation; they would like their manager to concentrate on reducing the share of the strong union rather than compete aggressively with the rival manager. This reversal of managerial incentive has already been demonstrated by Szymanski (1994) *albeit* for a different bargaining protocol. Interestingly, profit-orientation helps to cut back production, improve profit and achieve a near-collusive outcome. Hence, Szymnaski (1994, p. 113) observed that "strong unions may be good for the owners (because they indirectly cause profits to rise)"[7]. We confirm that Szymanski's results holds also under efficient bargaining.

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[7]Szymanski also qualifies that it is good for the owners so long as the strong unions care about wages; but it will be bad for the owners, if the strong unions care about employment.

But we also uncover a dual of Szymanski's result. Consider the other case of unilateral delegation – delegation only by the union. If the unions have all the bargaining power, they will orient the union leaders to gross wage bill maximization, and in turn will end up with over-employment and low utility. But if the shareholders' bargaining power increases, unions will be restrained, and eventually induce the union leaders to cut employment in order to fare well in bargaining, which will improve their utility. So a high bargaining power of the shareholders can be good for unions, when unions alone are delegating. Thus, one can say that the bargaining power of the non-delegating party has some disciplining effect on its opponent.

But can we say the same when both parties resort to delegation? Suppose the shareholders are very powerful and driven by strategic motives they orient their managers to sales maximization, which puts upward pressure on employment. But they also know that their unions will take a bigger than a normal bite of the surplus by orienting their leaders to net wage bill maximization, which puts downward pressure on employment. Which pressure will prevail and how will each side fare? The answer is less definitive in the general case, but the following can be said. If the net outcome of combined incentives is a reduction in employment from the no-delegation level, the bargaining pie will expand, allowing at least one party's payoff to improve. But it will not necessarily be the weaker party's payoff. So the observation that stronger union (or firm) can be good for firms (unions) may not necessarily hold.

Considering linear demand and constant returns technology we get a grip on this issue. We see that employment will always be below the no-delegation duopoly level, and hence bargaining pie will always be larger. In fact, when the bargaining powers are more or less equal the bargaining pie will be the largest reaching the collusive level. Each party in these situations will experience payoff improvements relative to their no-delegation level, but not as high as it could under unilateral delegation. What is more intriguing is that bargaining power plays a crucial role in determining the size of the bargaining pie, but not in its distribution. The share of the pie is determined by the incentive schemes; stronger the bargaining motivated incentives, larger the share of the pie. Thus, the payoff of a player is *inversely* related to his bargaining power. Moreover, when a player has the least (most) bargaining power, his payoff is the highest (lowest) among all delegation scenarios.

The breakdown of the conventional positive power-payoff relationship is novel and interesting. It suggests that labour laws and industrial relations regulations which constitute bargaining power may not matter much. By resorting to delegation and by designing clever incentive schemes firms and unions can make up for the lack of institutional support, and

in certain markets such as Cournot oligopolies, can do even better. Empirically it is widely observed that when regulations are favourable to the employers, workers turn to militant union leaders, and when laws favour unions employers hire shrewd negotiators. Such tough bargaining tactics can have unexpected consequences in oligopoly markets, and players may be pleasantly surprised by that. We know policy makers always worry about collusion between firms. But we see that firms do not need to collude; as long as each firm faces a union and if the bargaining with the union is mediated by hired negotiators, it is quite likely that the industry will (nearly) achieve the collusive outcome.

The empirical evidence on sales-oriented managerial scheme is widespread. In comparison the evidence of profit-oriented managerial scheme is limited. Nevertheless, there is some evidence of the importance of profit in managerial incentive schemes. In a US medical industry report Kasinec (2006) writes that managers are discouraged to hire more staff or pay more wages by linking their bonuses mainly to companies' annual profit and annual productivity growth targets. Mujumdar and Pal (2007) mentions that computer manufacturer, Dell, offers profit-oriented incentive schemes. There is also an established literature that studies CEO and managerial compensation in the presence of unions. Generally, managerial compensation in unionized firms is found to be lower than that in non-unionized firms. However, some authors have analyzed union effects on individual components of managerial compensation (such as stock options, pay, bonus etc.). Singh and Agarwal (2002) showed that in the Canadian manufacturing sector union presence is associated with greater CEO pay. Similarly, Gomez and Tzioumis (2006) found positive relationship between union and CEO pay in the context of US firms. On the issue of union facilitating implicit collusion one finds some indirect evidence from Haskel and Martin (1992), who observed in the context of Britain that union membership positively affects the price-cost mark-up via industry concentration. On the union side, evidence of pressing for more new jobs is rather limited in comparison to demanding for higher pay.

In the strategic trade literature some authors have introduced labour unions and examined the firms' incentive to collude or compete (such as reciprocal dumping). For example Straume (2002) has shown that in the presence of labour unions firms' incentive to collude diminishes in international duopolies. The main reason for this is that unions compete against each other to increase their own employment and thus bring down the wages; firms benefit from this wage competition and can sustain higher non-collusive profits. Our result is different from this. Unions here induce implicit collusion by threatening to raise the wage cost, if firms engage in aggressive output competition. In the empirical literature, concentrated industries in Europe are often seen to be unionized, though the direct effect of union

membership on firm profit is generally found to be negative. But unions may have indirect positive effect via firm concentration. For example, Haskel and Martin (1992) showed with UK manufacturing data that the interaction of union and concentration ratio have a positive effect on price-cost markup. Our study suggests that there is need for incorporating both managerial incentives and labour unions in empirical work on market power.

The rest of the paper is organized as follows. Section 2 presents the monopoly case, followed by the case of Cournot duopoly in Section 3. Section 4 concludes. All proofs are in Appendix.

2   The model

We first consider a non-strategic setup, such as monopoly. Labour is the only input, which along with wage is subjected to negotiation.[8] The firm's sales revenue is denoted as $s = p(q)q$, where q is the output, and $p(q)$ is the standard inverse demand curve, $p'(q) < 0, p''(q) \leq 0$. Assuming a concave production function $q = q(l)$, we write $s = s(l)$, $s''(l) < 0$.

The shareholders of the firm hire a manager and offer an incentive scheme z, which is, as in Fershtman and Judd (1987) (henceforth in short FJ), a convex combination of sales s and profit π as follows:[9]

$$
\begin{aligned}
z &= \beta\pi + (1-\beta)s \\
&= s(l) - \beta wl.
\end{aligned}
\tag{1}
$$

Delegation arises (or equivalently deviation from profit maximization occurs) if $\beta \neq 1$, and in that case two types of delegation can arise: sales orientation (i.e. $\beta < 1$), and (stronger) profit orientation (i.e. $\beta > 1$). Shareholders maximize profit $\pi = s - wl$.

The workers are unionized; it consists of N identical workers whose reservation wage is θ and its objective function is $u = (w - \theta)l$. At the worker selection stage, l members are randomly hired and the remaining $(N - l)$ members receive the reservation wage from outside. Workers appoint a union leader who is asked to maximize:

$$
\begin{aligned}
v &= \gamma u(.) + (1-\gamma)wl \\
&= wl - \gamma\theta l.
\end{aligned}
\tag{2}
$$

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[8]If other inputs are considered, they can be regarded as part of fixed cost for simplicity.

[9]Many authors assume a slightly general incentive scheme: $I = A + bz$. For $A = 0$, the assumption of $b \neq 1$ does not change our analysis. For $A > 0$, some of the quantitative results may change, but the qualitative results continue to hold.

Here too delegation is captured by $\gamma \neq 1$. If $\gamma > 1$ the union leader is oriented to net wage bill maximization as opposed to when $\gamma < 1$ he is oriented to gross wage bill maximization.[10] Effectively, the leader is induced to overvalue ($\gamma > 1$) or undervalue ($\gamma < 1$) the opportunity cost of the union.

The firm's wage and employment are an outcome of bargaining between the firm manager and the union leader. This is a scenario of efficient bargaining, except that bargaining takes place between two delegates. The bargaining power of the union leader (and also the union) is exogenously given by α, $0 \leq \alpha \leq 1$, and the manager's (and also the shareholders') bargaining power is $(1 - \alpha)$. The reservation payoffs of all parties are zero.

In stage 1 of this simple game the shareholders choose β and simultaneously, the union chooses γ. Then in stage 2 wage and employment (and consequently output) are determined through generalized Nash bargaining. We first consider the stage 2 problem, which is solved by maximizing $B = [v^\alpha z^{1-\alpha}]$ with respect to (w, l).[11] The solution yields

$$s'(l) = \beta\gamma\theta, \tag{3}$$

$$w = (1 - \alpha)\gamma\theta + \alpha\frac{s(l)}{\beta l}. \tag{4}$$

Eqs. (3) and (4) give the agreed employment and wage respectively. In particular, note that the output choice is not directly affected by the bargaining powers, and l maximizes $(s(l) - \beta\gamma\theta)$. Moreover, when $\beta = \gamma = 1$, equation (3) yields the efficient level of output and equation (4) expresses the wage rate as a weighted average of the marginal revenue product and average revenue productivity of labour.[12]

After substituting (4) and rearranging terms we can write firm's profit and the union's

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[10]It is conceivable that the union leader might be encouraged to negotiate for higher employment. For example, if $v = (w - \theta)l + \gamma l$, then by choosing a positive γ the union leader can be induced to bargain for higher employment. However, the qualitative results will still be the same as the ones obtained here.

[11]The first order condition of this maximization problem can be written as:

$$B_w = [v^{\alpha-1}z^{-\alpha}][\alpha z\frac{\partial v}{\partial w} + (1 - \alpha)v\frac{\partial z}{\partial w}] = 0$$

$$B_l = [v^{\alpha-1}z^{-\alpha}][\alpha z\frac{\partial v}{\partial l} + (1 - \alpha)v\frac{\partial z}{\partial l}] = 0.$$

[12]The output level should be regarded as constrained efficient due to the presence of market power.

utility respectively in the following way:

$$\pi = s(l)\left(1 - \frac{\alpha}{\beta}\right) - (1-\alpha)\theta\gamma l$$

$$= (1-\alpha)[s(l) - \theta l] + \underbrace{\alpha\, s(l)\left[1 - \frac{1}{\beta}\right]}_{\text{Shareholders delegation effect}} - (1-\alpha)\underbrace{\theta l[\gamma - 1]}_{\text{Union delegation effect}}, \tag{5}$$

$$u = \alpha\frac{s(l)}{\beta} - \theta l[1 - (1-\alpha)\gamma]$$

$$= \alpha[s(l) - \theta l] - \underbrace{\alpha\, s(l)\left[1 - \frac{1}{\beta}\right]}_{\text{Shareholders delegation effect}} + (1-\alpha)\underbrace{\theta l[\gamma - 1]}_{\text{Union delegation effect}}. \tag{6}$$

Eq. (5) shows a decomposition of profit. In the absence of any delegation firm's profit is given by the first term, $(1-\alpha)[S(l) - \theta l]$, which is simply the share of the organisational surplus it can claim through bargaining. The next two terms captures the effects of delegation. The second term represents the effects of firm (i.e. shareholders') delegation. It will be non-negative if and only if $\beta \geq 1$. The third term is due to union delegation, and it will be non-positive if and only if $\gamma \geq 1$. Similarly, Eq. (6) shows the effects of delegation on union's utility (or net wage bill). The first term is simply the union's share of the surplus, and the next two terms are due to shareholders' and union's delegation respectively. For its own delegation to be profitable the union must set $\gamma > 1$, and if $\beta > 1$ the union is likely to experience a loss due to firm delegation. Furthermore, what is union's loss is firm's gain, and vice versa.

Several key observations now can be made:

1. If delegation is profitable then the shareholders will set $\beta > 1$ (i.e. profit orientation, rather than sales orientation) and the union will set $\gamma > 1$ (i.e net wage bill orientation rather than gross wage bill orientation).

2. It is also apparent that shareholders delegation will be profitable only if $\alpha > 0$, and the union delegation will be profitable only if $\alpha < 1$. Further, if the firm has all the bargaining power (i.e. $\alpha = 0$) by choosing $\beta \neq 1$ the firm will only reduce the surplus (for any given γ), and hence it must choose $\beta = 1$. A similar reasoning helps us argue that if $\alpha = 1$ the union must choose $\gamma = 1$.

3. Delegation ensures a strictly positive payoff (above its reservation payoff) even in the case of least bargaining power. If $\alpha = 1$ the firm can ensure a strictly positive profit

by choosing $\beta > 1$ and if $\alpha = 0$ the union can ensure a strictly positive net wage bill by setting $\gamma > 1$. Thus, delegation acts as a substitute for bargaining power.

4. Given that delegation will require setting $\beta > 1$ and $\gamma > 1$, the resulting employment will fall below the no-delegation level (evident from Eq. (3)). Hence in the profit equation $\pi = s(l) - wl$, $s(l)$ will be smaller (than the no-delegation level) and therefore the wage bill wl must fall sufficiently to make delegation profitable for the firm. In other words, the wage bill *must* fall with shareholders delegation.[13] By the same logic, the wage rate (or the net wage bill) must rise with union delegation.

To state these observations formally, suppose (β, γ) is arbitrarily given and suppose $l^0 = l^0(\beta, \gamma)$ is the solution to eq. (3). In particular, $l^0(1, \gamma)$ refers to employment corresponding to the case of no-delegation by firm, and $l^0(\beta, 1)$ refers to employment corresponding to the case of no-delegation by union. Further, denote $\beta(\gamma)$ be the optimal β (that maximizes (5) in stage 1) at a given γ, and $\gamma(\beta)$ be the optimal γ (that maximizes (6) in stage 1) at a given β. Accordingly, define $l^*(\gamma) = l^*(\beta(\gamma), \gamma)$ to be employment evaluated at optimal β and arbitrary γ; similarly $l^*(\beta) = l^*(\beta, \gamma(\beta))$. From (3) clearly $l^*(\gamma) < (>)l^0(1, \gamma)$ if $\beta(\gamma) > (<)1$ and $l^*(\beta) < (>)l^0(\beta, 1)$ if $\gamma(\beta) > (<)1$. Let the wage and profit corresponding to $l^0(1, \gamma)$ and $l^*(\gamma)$ be denoted as $(w^0(1, \gamma), \pi^0(1, \gamma))$ and $(w^*(\gamma), \pi^*(\gamma))$ respectively. Similarly, let the wage and utility corresponding to $l^0(\beta, 1)$ and $l^*(\beta)$ be $(w^0(\gamma, 1), u^0(\gamma, 1))$ and $(w^*(\beta), u^*(\beta))$ respectively.

The following claims can be made.

Lemma 1. *(i) If $\pi^*(\gamma) > \pi^0(1, \gamma)$, it must be the case that $\beta(\gamma) > 1$ for any γ and $w^*(\gamma)l^*(\gamma) < w^0(1, \gamma)l^0(1, \gamma)$. (ii) If $u^*(\beta) > u^0(\beta, 1)$, it must be the case that $\gamma(\beta) > 1$ for any β and $w^*(\beta) > w^0(\beta, 1)$.*

We now characterize the optimal incentive schemes. Note that (3) yields $l = l(\beta, \gamma)$ and $\frac{\partial l}{\partial \beta} = \frac{\theta \gamma}{s''(.)} < 0$, $\frac{\partial l}{\partial \gamma} = \frac{\theta \beta}{s''(.)} < 0$. Correspondingly, we have $s(\beta, \gamma) = s(l(\beta, \gamma))$ and $w(\beta, \gamma) = w(l(\beta, \gamma))$. In stage 1, the shareholders and the union perfectly anticipate $l(\beta, \gamma)$ and $w(\beta, \gamma)$ and choose β and γ to maximize $\pi(\beta)$ and $u(\gamma)$ respectively, where $\pi(\beta)$ and $u(\gamma)$ are obtained from Eqs (5) and (6) after substituting $l(\beta, \gamma)$.

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[13]One can further show w will be inversely related to β under certain conditions. In Appendix we show that when $\gamma = 1$ if the sum of the elasticity of total revenue and the elasticity of marginal revenue is greater than 1, w will be inversely related to β. Production functions exhibiting non-increasing returns and linear demand easily satisfy this condition. In such cases, not only the wage bill but also the wage rate can be depressed below the ordinary profit maximizing level.

The first-order condition for profit maximization is $\pi'(\beta) = (1-\alpha)[s'(l)-\theta]\frac{\partial l}{\partial \beta}+\alpha s'(l)\frac{\partial l}{\partial \beta}\{1-\frac{1}{\beta}\} - (1-\alpha)\theta(\gamma-1)\frac{\partial l}{\partial \beta} + \alpha\frac{s(.)}{\beta^2} = 0$. Substituting (3) and multiplying both sides by β^2 we rewrite the first order condition as

$$\pi'(\beta) = \frac{1}{\beta^2}\left[\beta^2(\beta-1)\gamma\theta\frac{\partial l}{\partial \beta} + \alpha s(.)\right] = 0. \tag{7}$$

Similary, the first order condition for the union's utility maximization yields[14]:

$$u'(\gamma) = \theta\left[(\gamma-1)\frac{\partial l}{\partial \gamma} + (1-\alpha)l\right] = 0. \tag{8}$$

We assume $s'''(l) \leq 0$ to ensure $\pi''(\beta) < 0$ and $\frac{\partial \pi'(\beta)}{\partial \gamma} < 0$ (for details see Appendix). It can also be established that $u''(\gamma) < 0$ and $\frac{\partial u'(\gamma)}{\partial \beta} < 0$. Therefore, $\frac{\partial \beta}{\partial \gamma} < 0$ and $\frac{\partial \gamma}{\partial \beta} < 0$. That is, β and γ are strategic substitutes to each other. Let $(\beta^*, \gamma*)$ be the Nash equilibrium incentives in the bilateral delegation game and assume that it is unique and stable.

Proposition 1. (***Delegation for bargaining***) *Suppose $0 < \alpha < 1$ and the Nash equilibrium incentives (β^*, γ^*) is unique. Then $\beta^* > 1, \gamma^* > 1$, and $\beta'(\alpha) > 0$, $\gamma'(\alpha) < 0$. Moreover, at $\alpha = 0$, $\beta^* = 1$, $\gamma^* > 1$, and at $\alpha = 1$, $\gamma^* = 1$, $\beta^* > 1$. Thus, employment will be set below the no-delegation level at all $0 \leq \alpha \leq 1$.*

Now we turn our attention to the bargaining pie, $P = \pi+u = s(l)-\theta l$. The bargaining pie is maximum when $s'(l) = \theta$, which corresponds to the no-delegation case. Under delegation the pie will be smaller. But can we compare the bargaining pies between unilateral delegation and bilateral delegation?

Suppose β_u is the optimal β when only the firm delegates and γ_u is the optimal γ when only the union delegates. Associated bargaining pies are denoted as P_F and P_U respectively. In the bilateral case the same variables are represented by β^*, γ^* and P^*. Since under all cases of delegation, there is underemployment, an expansion of pie must be associated with an increase in employment, which in turn depends on the magnitudes of relevant β and γ. The following proposition shows that bilateral delegation will give rise to the smallest pie, if $\beta\gamma$ is the largest.

Now consider the effect of α on the bargaining pie $P = s(l) - \theta l$.

$$P'(\alpha) = [(\beta^*\gamma^* - 1)\theta]\frac{\partial l^*}{\partial \beta\gamma}\frac{\partial \beta^*\gamma^*}{\partial \alpha}. \tag{9}$$

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[14]$u'(\gamma) = \left[\alpha(s'(l) - \theta) - \alpha(1 - \frac{1}{\beta})s'(l) + (1-\alpha)(\gamma-1)\theta\right]\frac{\partial l}{\partial \gamma} + (1-\alpha)\theta l$. Substituting 3 and simplifying further we obtain Eq. (8).

In Eq. (9) the first term is positive at all α and the second term is negative. But the third term $\partial \beta^* \alpha^* / \partial \alpha$ is ambiguous. Consequently, the effect on the bargaining pie is also ambiguous. However, in the special cases of unilateral delegation we do get unambiguous effects. For example, if only the firm were to delegate we would have $\gamma = 1$ and $\partial \beta^* / \partial \alpha > 0$, the pie would shrink further causing further inefficiency. On the other hand, if only the union were to delegate, clearly $\beta = 1$ and $\partial \gamma^* / \partial \alpha < 0$, and the pie would expand improving inefficiency. If both effects were occurring simultaneously, one would expect a much slower change in the bargaining pie (as one effect would counteract the other), and possibly the size of the pie itself would be smaller e (than any of the unilateral delegation cases). In the trivial case of no delegation (i.e. $\beta = \gamma = 1$) the bargaining pie will remain invariant at the efficient level.

Proposition 2. (*Bargaining pie*)
  (a) $P_U > (\leq) P_F$ if and only if $\gamma_u < (\geq) \beta_u$.
  (b) $P^* \leq \min[P_U, P_F]$ if and only if $\beta^* \gamma^* \geq \max[\gamma_u, \beta_u]$.
  (c) $\partial P_U / \partial \alpha > 0$, $\partial P_F / \partial \alpha < 0$.
  (d) The bargaining pie would increase (decrease) with α if $\partial \beta^* \gamma^* / \partial \alpha < (>)0$.

Now we would like to see how the distribution of the pie changes with the bargaining power of the union. We begin with the investigation of profit. Noting that π will be a function of α in equilibrium and applying the envelope theorem we obtain

$$\pi'(\alpha) \;\; = \;\; - \underbrace{\left[\frac{s(l^*)}{l^*} - \beta^* \gamma^* \theta\right]}_{>0} \frac{l^*}{\beta^*} - \theta \underbrace{\left[(1-\alpha)l^* - \gamma^*(\beta^*-1)\frac{\partial l^*}{\partial \gamma}\right]}_{>0} \underbrace{\frac{\partial \gamma^*}{\partial \alpha}}_{<0.} \qquad (10)$$

The first term of Eq. (10) (inside the bracket) is positive. This can be seen from the fact that $s(l^*)/l^* > s'(l^*) = \beta^* \gamma^* \theta$ due to concavity assumption of $s(l)$. When the negative sign is included, the first term captures the direct effect of the union power on profit. The second term captures the indirect effect occurring through γ; the other indirect effect occurring through β is zero due to the envelope theorem. The second term inside the bracket is also positive due to the fact that $\partial l^* / \partial \gamma < 0$; but because $\partial \gamma^* / \partial \alpha < 0$ the indirect effect of α is positive, and the overall effect is ambiguous. Consequently, we have a curious possibility – profit can increase with the bargaining power of the union. However, if only the firm were to delegate, the ambiguity would disappear; profit would then be inversely related to α as expected.

Similarly, consider the effect of an increase in α on the union's utility:

$$u'(\alpha) = \underbrace{\left[\frac{s(l^*)}{l^*} - \beta^*\gamma^*\theta\right]}_{> 0}\frac{l^*}{\beta^*} - \underbrace{\left[\frac{\alpha s}{\beta^{*2}} - (\gamma^* - 1)\theta\frac{\partial l^*}{\partial \beta}\right]}_{> 0}\underbrace{\frac{\partial \beta^*}{\partial \alpha}}_{> 0}. \tag{11}$$

In Eq. (11) the two terms are of opposite signs. The first term as usual shows the direct effect which goes in favour of the union, but the second term capturing the indirect effect occurring through β hurts the union. The overall effect is once again ambiguous. But if the firm did not delegate (regardless of the union delegates or not) the second term would be zero and we will have $u'(\alpha) > 0$.

Proposition 3. (*Payoffs and the bargaining power*)

    (a) If only the shareholders delegate, $\pi'(\alpha) < 0$ at all α, but $u'(\alpha) > 0$ if $\partial\beta_u/\partial\alpha < \left[1 - \frac{s'(l)}{s(l)}\right]\frac{\beta}{\alpha}$.

    (b) If only the union delegates, $u'(\alpha) > 0$ at all α, but $\pi'(\alpha) < 0$ if $|\frac{\partial\gamma_u}{\partial\alpha}| < \left[1 - \frac{s'(l)}{s(l)}\right]\frac{s(l)}{(1-\alpha)\theta l}$.

    (c) When both delegate, the effects of α on the union's utility and shareholders' profit are ambiguous.

The implication of propositions 1- 3 are that when transactions with input suppliers are subject to negotiation, the firm should delegate the negotiation to a manager by orienting him to profit, rather than sales (or equivalently penalizing sales). Similarly, the union should also delegate its bargaining to a union leader by orientating him more to the net wage bill. The end result is severe inefficiency. In general we would expect that inefficiency would be greater under bilateral delegation, than under any unilateral delegation, though this point is difficult to prove in the general case. This also gives rise to the possibility that stronger union may concede higher profit to the firm. Whether this possibility can occur in a simpler setup such as linear demand and constant returns is worthy of investigation which we intend to do next.

2.1 An example

We assume linear demand and constant returns to scale (CRS) technology. Suppose $p = a - q$ and $q = l$ for simplicity. So $s(l) = al - l^2$ and $z = al - l^2 - \beta wl$ while $u = (w - \theta)l$ and $v = (w - \gamma\theta)l$. Through Nash bargaining in stage 2, we arrive at the following output and

wage choices:

$$l = \frac{a - \beta\gamma\theta}{2}, \quad w = \frac{\alpha(a + \beta\gamma\theta)}{2\beta} + (1 - \alpha)\gamma\theta.$$

It is noteworthy that w is inversely related to β, and indeed in this case the elasticity condition holds.

Next we substitute the above two expressions in the shareholders' profit expression and union's utility to obtain

$$\pi = \left\{ a - \frac{(a - \beta\gamma\theta)}{2} - \alpha\frac{(a + \beta\gamma\theta)}{2\beta} - (1 - \alpha)\theta\gamma \right\} \frac{(a - \beta\gamma\theta)}{2}$$

$$u = \left[\alpha\frac{(a + \beta\gamma\theta)}{2\beta} + (1 - \alpha)\theta\gamma - \theta \right] \frac{(a - \beta\gamma\theta)}{2}.$$

By maximizing this we obtain the optimal β from the following:

$$-\beta^2\gamma^2\theta^2 \left[2(\beta - 1) + \alpha \right] + \alpha a^2 = 0. \tag{12}$$

Similarly, by maximizing u we get the optimal γ as

$$a(1 - \alpha) + \beta\theta - \beta\theta(2 - \alpha)\gamma = 0 \tag{13}$$

$$\text{or,} \quad (1 - \alpha)(a - \beta\theta\gamma) + \beta\theta(1 - \gamma) = 0.$$

Let the solution to (12) and (13) be denoted as β^* and γ^*. From Eq. (12) it is clear that if $\alpha = 0$, $\beta^* = 1$. But if $\alpha > 0$, the second term is unambiguously positive (since $a > \beta\theta$ for positive output), hence optimal β must exceed 1 at any given value of γ (since $a > \beta\gamma\theta$ for positive l). Similarly, from Eq. (13) it is clear that at $\alpha = 1$, $\gamma^* = 1$ and $\alpha = 0$ (when $\beta = 1$), $\gamma = (a + \theta)/2\theta > 1$. It can also be checked that β and γ are strategic substitutes to each other. With these observations we can establish an interesting result, which confirms that the condition we specified in Proposition 2 regarding the bargaining pie is indeed met by this example.

Proposition 4. (**_Linear demand, CRS technology and bargaining pie_**) _Suppose the demand curve is linear and production exhibits constant returns to scale. Then for any $\alpha \in [0, 1]$, the bargaining pie under bilateral delegation, P^*, is bounded above by the smallest of the bargaining pies under unilateral delegation. That is $P^* \leq \min[P_F, P_U]$._

Simulation: While from Eq. (13) we get a unique solution for γ (given any β), Eq. (12) is a cubic equation in β. It turns out that it has only one real root for β, though the

expression is too lengthy to report here. So we consider some values for a and θ to simulate the results of Propositions 1 and 3.

Table 1 shows the simulation results for $a = 2$ and $\theta = 1$. The first part of the table concerns the case of no delegation (Case 0). With the assumption fo $\beta = \gamma = 1$ we obtain the standard results – employment and the bargaining pie $(\pi + u)$ remains invariant to bargaining power α, and profit is inversely related to α. Wage rate increases with α.

Now introduce firm-side delegation (Case 1) only. Optimal β steadily rises from 1 at $\alpha = 0$ to 1.45 at $\alpha = 1$. Wage growth is immediately muted (compare the columns for w between Case 0 and Case 1). But employment now shrinks as α rises, and so does the bargaining pie, reflecting increasing inefficiency. However, profit in this case is much higher than that in the no delegation case, and it is inversely related to α.

Next, consider only union-side delegation (Case 2). Here, the wage rate achieves the maximum (see Case 0 wage at $\alpha = 1$) and remains invariant to α. The invariance is a particular feature of linear demand and constant returns technology.[15] Here, the bargaining pie increases with α, while profit is still inversely related to α. The union's utility is higher than the no delegation case.

Finally, the case of bilateral delegation. In this case employment is lowest, among all four cases and consequently the bargaining pie is also the smallest; but it is increasing in α. So in the case of linear demand and CRS technology, there is no ambiguity, – the pie will be larger with stronger unions. A visual illustration of the relationship between the bargaining pie and the union's bargaining power is given Fig. 1.

But a more dramatic outcome of bilateral delegation is the reversal of relation between π and α. Profit is increasing with the union's bargaining power. That is to say, stronger the union, lower its payoff. We already noted the theoretical possibility of it. But in this example we see that this is systematic over the entire range of α. The intuition is that under bilateral delegation a stronger union unleashes three effects on profit. First, the direct effect which leads to a loss due to a reduction in firm's bargaining power from a given bargaining pie. Second, the indirect effect occurring through β – adjustment in firm's own incentive term. Since β is optimally adjusted (with any change in α) its first order effect on profit is zero. Third, there is another indirect effect occurring through γ which helps to expand the bargaining pie, which leads to a gain for the firm. This second indirect effect outweighs the direct effect in this case resulting in the reversal of profit and union power relationship. For the union, the effect is exactly opposite. For it, the positive direct effect of an increase in α is outweighed by the negative indirect effect occurring through β which tends to depress the bargaining pie. These results are robust to changes in a or θ as shown in Table 2.

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[15]One can verify this by substituting $\beta = 1$ and solving for γ from Eq. (13) and then deriving w.

Since profit is positively related to α, we see from Fig. 2a that delegation is profitable for the firm (compared to no delegation) only after α exceeds (approximately) 0.55. The downward sloping line is profit under no-delegation and the upward sloping line is profit under bilateral delegation; the latter exceeds the former at $\alpha = 0.55$. But for the union delegation is profitable until α exceeds 0.39 (see Fig. 2b). Therefore, at all $\alpha \in (0.39, 0.55)$ both parties are worse off after delegation. This essentially highlights the inefficiency of delegation in monopoly.
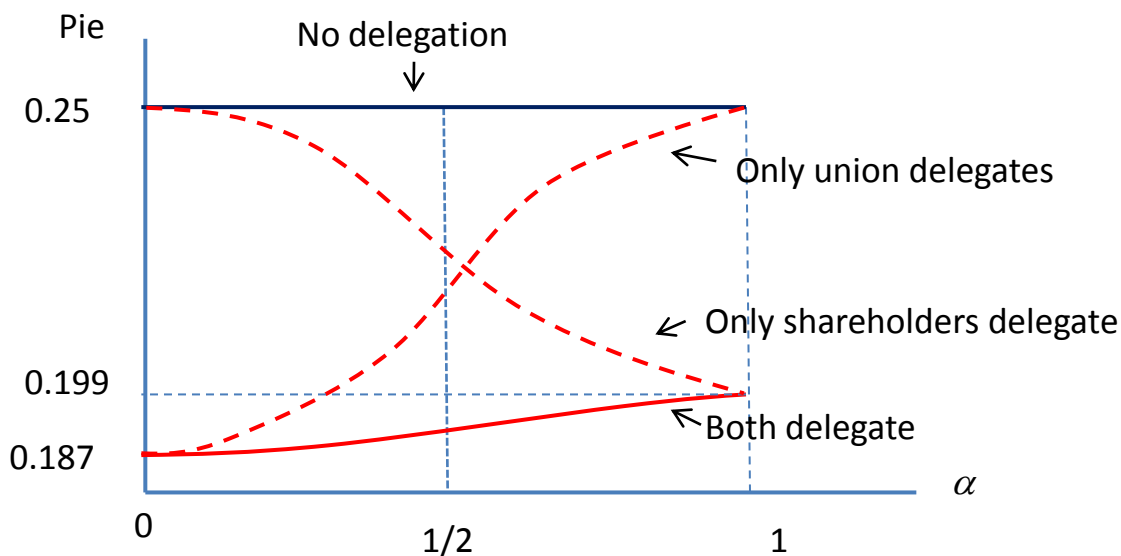

Fig. 1: Bargaining pie

Table 1: Simulation results for monopoly

| $a = 2$	$\theta = 1$						
		Case 0:	No delegation		$(\beta = 1$	$\gamma = 1)$	
α	w	l	u		π	$\pi + u$	
1	1.5	0.5	0.25		0	0.25	
0.75	1.375	0.5	0.187		0.063	0.25	
0.55	1.275	0.5	0.137		0.113	0.25	
0.5	1.25	0.5	0.125		0.125	0.25	
0.39	1.195	0.5	0.097		0.153	0.25	
0.25	1.125	0.5	0.063		0.187	0.25	
0	1	0.5	0		0.25	0.25	
		Case 1:	Only shareholders delegates	$(\gamma = 1)$			
α	β	w	l		u	π	$\pi + u$
1	1.45	1.19	0.275		0.052	0.15	0.199
0.75	1.395	1.16	0.30		0.049	0.16	0.21
0.5	1.322	1.13	0.34		0.043	0.18	0.224
0.25	1.213	1.08	0.39		0.031	0.206	0.24
0	1	1	0.5		0	0.25	0.25
		Case 2:	Only union delegates	$(\beta = 1)$			
α	γ	w	l		u	π	$\pi + u$
1	1	1.5	0.5		0.25	0	0.25
0.75	1.2	1.5	0.4		0.20	0.04	0.24
0.5	1.33	1.5	0.335		0.167	0.05	0.22
0.25	1.42	1.5	0.29		0.145	0.061	0.20
0	1.5	1.5	0.25		0.125	0.062	0.187
		Case 3:	Both	delegate			
α	β^*	γ^*	w	l	u	π	$\pi + u$
1	1.45	1	1.19	0.275	0.052	0.15	0.199
0.75	1.327	1.10	1.25	0.27	0.068	0.13	0.196
0.55	1.235	1.192	1.309	0.264	0.082	0.113	0.195
0.5	1.21	1.217	1.33	0.263	0.086	0.11	0.193
0.39	1.15	1.28	1.369	0.264	0.097	0.096	0.193
0.25	1.10	1.349	1.41	0.256	0.105	0.09	0.191
0	1	1.5	1.5	0.25	0.125	0.062	0.187
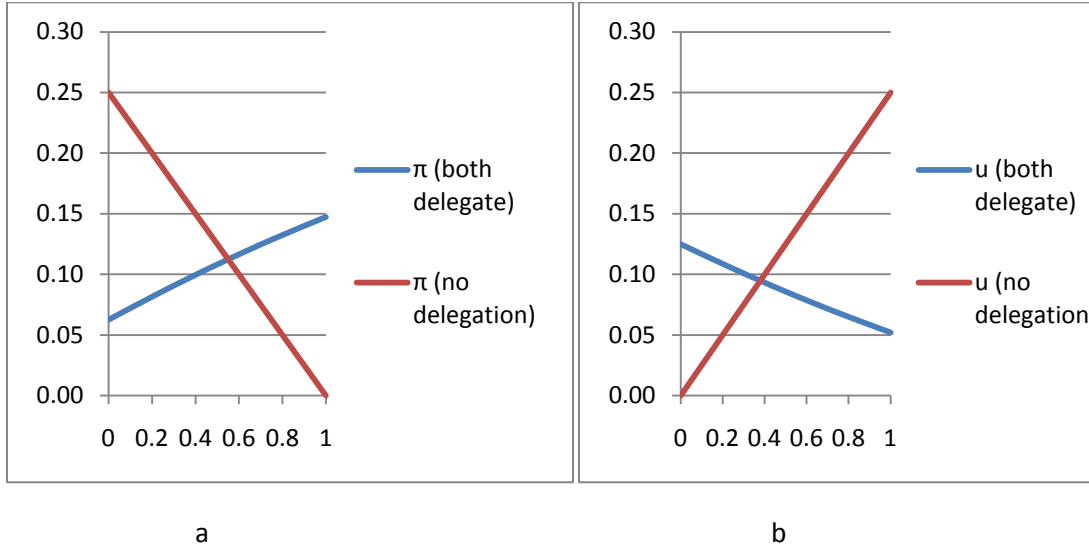
Figure2: Monopoly profit and union utility against union's bargaining power

Table 2: More simulations on bilateral delegation

$a = 3$	$\theta = 1$						
α	β^*	γ^*	w	l	u	π	$\pi + u$
1	1.84	1	1.32	0.58	0.18	0.64	0.82
0.75	1.59	1.18	1.44	0.56	0.25	0.56	0.81
0.5	1.37	1.40	1.60	0.54	0.32	0.47	0.79
0.25	1.17	1.68	1.78	0.52	0.41	0.36	0.77
0	1	2	2	0.5	0.5	0.25	0.75
$a = 3$	$\theta = 2$						
α	β^*	γ^*	w	l	u	π	$\pi + u$
1	1.24	1	2.21	0.263	0.056	0.138	0.194
0.75	1.17	1.05	2.28	0.26	0.072	0.12	0.192
0.5	1.11	1.11	2.35	0.257	0.089	0.101	0.191
0.25	1.06	1.18	2.42	0.253	0.106	0.083	0.189
0	1	1.25	2.5	0.25	0.125	0.062	0.187

3 Cournot duopoly

In a strategic environment the decision to delegate cannot be entirely based on the objective of gaining advantage within the firm. The shareholders and the union members need to take into account the effects their incentive schemes will have on the behaviour of other firms and unions in the industry. For example, consider a Cournot duopoly. If a given firm chooses profit-oriented delegation to keep its wage bill in check, it will concede some strategic advantage to its rival, because by setting $\beta > 1$ it reduces its own output and induces an increase in its rival's output. Alternatively, it can choose a sales-oriented delegation to gain strategic advantage against its rival, but then it will have to concede an increase in the wage bill and even an increase in the wage rate. Thus, in quantity-setting games (and possibly in similar games involving strategic substitutes) there is a dilemma, whether to orient the manager to sales or to profit.[16]

There are two identical firms, 1 and 2. The sales revenue of the i-th firm is $s_i = s_i(l_1, l_2)$, which is strictly concave with the following restrictions:

Assumption 1. $\frac{\partial s_i}{\partial l_j} < 0, \frac{\partial^2 s_i}{\partial l_i^2} < 0, \frac{\partial^2 s_i}{\partial l_i \partial l_j} < 0$ for $i \neq j$.

The first restriction signifies that the two goods are substitutes, the second restriction helps to establish concavity and the third restriction ensures that employments are strategic

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[16]In standard models we see the shareholders' incentive to reward (or penalize) sales is generally unidirectional and it is carried up to a point of leadership advantage. Though, some R&D models allow for leakage to restore sales-orientation via exogenous spill-over (such as Zhang and Zhang, 1997), never do we see countervailing incentives.

substitutes. Profit in firm i is $\pi_i = s_i - w_i l_i$ and the managerial incentive in firm i is $z_i = s_i - \beta_i w_i l_i$. As before each firm confronts a union. Unions are identical in terms of their bargaining power α, reservation wage θ and objective function $u_i = (w_i - \theta) l_i$. Each union delegates the task of bargaining to a union leader whose objective function is $v_i = (w_i - \gamma_i \theta) l_i$.

The game is now modified as follows. In the first stage, shareholders and the unions of the two firms simultaneously choose their incentive schemes by deciding on (β_1, β_2) and (γ_i, γ_2). If delegation is decided, in the second stage, managers and the union leaders in two firms simultaneously (but independently) negotiate over firm-specific wage and employment.

Solving the Nash bargaining problem in stage 2 we get the following:

$$\frac{\partial s_i}{\partial l_i} = \beta_i \gamma_i \theta, \quad i, j = 1, 2 \tag{14}$$

$$w_i = (1 - \alpha) \gamma_i \theta + \alpha \frac{s_i}{\beta_i l_i}, \quad i, j = 1, 2. \tag{15}$$

Solving (14) for firm 1 and 2 simultaneously we obtain the employment functions $l_i = l_i(\beta_1 \gamma_1, \beta_2 \gamma_2), i = 1, 2$ and substituting them in (15) we obtain $w_i = w_i(l_i, l_j)$. Further, it can be shown that $\frac{\partial l_i}{\partial \beta_i} < 0$, $\frac{\partial l_i}{\partial \gamma_i} < 0$ and $\frac{\partial l_i}{\partial \beta_j} > 0$ $\frac{\partial l_i}{\partial \gamma_j} < 0$, $i \neq j$. Note that for employment l_i what matters most is the products $\beta_i \gamma_i$. In stage 1 firms and unions simultaneously choose their respective β_i and γ_i with perfect foresight of the subgame outcome and anticipation of the firm-union pair j's choice.

Conflict between bargaining and strategic motives: We may refer to a firm and its union as a pair just for ease of description as we have moved into a four player game; needless to say, their interactions are non-cooperative. Each player needs to assess the trade-off it faces between its internal bargaining effect and external strategic effects. In order to explain this trade-off we now introduced some new notations.

Given any $\beta_j \gamma_j$ if the firm-union pair i (non-cooperatively) chooses $\beta_i \gamma_i = 1$, we denote the associated employment in firms i and j as $l_i^0 = l_i(1, \beta_j \gamma_j)$ and $l_j^0 = l_j(1, \beta_j \gamma_j)$ and corresponding wages as w_i^0 and w_j^0 respectively. Alternatively, given any $\beta_j \gamma_j$ if pair i chooses some $\beta_i \gamma_i \neq 1$ we denote the consequent employments as l_i^*, l_j^* and wages as w_i^* and w_j^*. Accordingly, we define firm i's sales revenues in the two situations as $s_i^0 = s_i(l_i^0, l_j^0)$, $s_i^* = s_i(l_i^*, l_j^*)$.

We also define a hypothetical level of sales revenue $s_i^{*0} = s_i(l_i^*, l_j^0)$ that can arise if pair j chooses l_j^0 and pair i chooses l_i^*. Clearly, these cannot occur in equilibrium, because these are based on different values of $\beta_i \gamma_i$. But it represents the revenue level, that pair i can achieve by deviating to $\beta_i \gamma_i \neq 1$ when it holds firm j's employment fixed at l_j^0. Put differently, if pair

i thinks that pair j's employment will be unaffected by pair i's incentive choices, then by resorting to delegation pair i will earn revenue s_i^{*0}. At this stage let us note how l_i^* compares with l_i^* and s_i^{*0} with s_i^*.

Lemma 2. *For any given $\beta_j\gamma_j$, if $\beta_i\gamma_i < 1$, $l_i^* > l_i^0$, $l_j^* < l_j^0$ and $s_i^0 < s_i^{*0} < s_i^*$. Conversely, for any given $\beta_j\gamma_j$, if $\beta_i\gamma_i > 1$, $l_i^* < l_i^0$, $l_j^* > l_j^0$ and $s_i^0 > s_i^{*0} > s_i^*$.*

Now we write pair i's payoffs under delegation, $\pi_i^* = s_i^* - w_i^* l_i^*$, and $u^* = w_i^* l_i^*$ as [17]

$$\pi_i^* = \left[(1-\alpha)\left(s_i^{*0} - \theta l_i^*\right) + \alpha s_i^{*0}\left(1 - \frac{1}{\beta_i}\right) - (1-\alpha)\theta l_i^*(\gamma_i - 1) \right]$$
$$+ (s_i^* - s_i^{*0})\left(1 - \frac{\alpha}{\beta_i}\right) \qquad (16)$$

$$u_i^* = \left[\alpha(s_i^{*0} - \theta l_i^*) - \alpha s_i^{*0}\left(1 - \frac{1}{\beta_i}\right) + (1-\alpha)\theta l_i^*(\gamma_i - 1) \right] + (s_i^* - s_i^{*0})\frac{\alpha}{\beta_i}. \qquad (17)$$

Consider first Eq. (16). Suppose given some $\beta_j\gamma_j$, pair i initially chooses $\beta_i\gamma_i = 1$, following which l_i^0 and l_j^0 are chosen. Now if pair i wishes to revise its choice of $\beta_i\gamma_i$, firm i and union i need to assess their individual gains from this revision. In so doing, suppose they ignore firm j's response to revised incentives and assume that firm j's output will not change from l_j^0. Consequently, their revenues will be s_i^{*0}, and the resultant profit is given in the first term (inside the large bracket) of Eq. (16); the first term of Eq. (17) represents the union's net wage bill from at s_i^{*0}. Note that the expressions of these two terms are identical to the monopoly profit and net wage bill as given in Eqs. (5) and (6). that means, these two terms capture the bargaining effect of delegation, *holding firm j's output constant*.

But firm j's output will not remain unchanged. As the manager and union leader of firm j will have the benefit observing $\beta_i\gamma_i$ at the time of choosing employment, they will switch to l_j^*, which will result in the revenue s_i^* for pair i. The second terms in Eqs. (16) and Eq. (17) each capture this 'strategic effect'. From Lemma 2 we know that $s_i^* > s_i^{*0}$ if $\beta_i\gamma_i < 1$. So the <u>strategic effect</u> is positive for union i if $\beta_i\gamma_i < 1$ and it is positive also for the firm i if in addition $\beta_i > \alpha$. So, either β_i or γ_i or both must be less than 1; in particular, for the firm, we must have $\alpha < \beta_i < 1/\gamma_i$.

But we know from the monopoly section that if delegation is to be profitable on the bargaining account, β and γ are both need to be set above 1, which then creates a strategic

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[17]Substituting w_i^* from equation (15) rewrite π_i^* as $\pi_i^* = s_i^* - (1-\alpha)\theta l_i^* - \alpha\frac{s_i^*}{\beta_i} = s_i^*(1 - \frac{\alpha}{\beta_i}) - (1-\alpha)\theta l_i^*$. Now simultaneoulsy add and substract $(1-\alpha)s_i^{*0}$ and $\alpha\frac{s_i^{*0}}{\beta_i}$ to and from π_i^* so that $\pi^* = s_i^*(1 - \frac{\alpha}{\beta_i}) - (1-\alpha)\theta l_i^* + [(1-\alpha)s_i^{*0} - (1-\alpha)s_i^{*0}] + [\alpha\frac{s_i^{*0}}{\beta_i} - \alpha\frac{s_i^{*0}}{\beta_i}]$. Now rearranging terms we can obtain (16). Eq. (17) can be derived similarly.

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disadvantage. Here is the conflict between the bargaining motive and the strategic motive of delegation. In equilibrium this conflict must be held in balance. More importantly, the strategic effect will be commonly shared by the firm and the union, as their bargaining pie will be enlarged at the expense of the rival pair's pie. But in order to achieve the strategic advantage, *at least* one of the parties must give up on the bargaining front.

Proposition 5. *Ignoring the bargaining effect we can say that for delegation to be profitable on strategic count (so that $s_i^* > s_i^*$) firm i must set $\beta_i > \alpha$, given any β_j and (γ_i, γ_j). In particular, when $\alpha = 0$ for delgation to be profitable on strategic count for both firm i and union i it is necessary that $\alpha < \beta_i < 1/\gamma_i$ and $\gamma_i > 1$. But when $\alpha = 1$ delegation on strategic count is jointly profitable if $\beta_i > 1$ and $\gamma_i < 1/\beta_i$.*

Now we explicitly consider the first order condition for profit maximization. Given $(\gamma_i, \beta_j, \gamma_j)$ shareholders in firm i maximize

$$\pi_i = s_i(.)(1 - \frac{\alpha}{\beta_i}) - (1 - \alpha)\theta\gamma_i l_i.$$

The resultant first order condition is (after substituting (14)) for $i = 1, 2$, and $i \neq j$

$$\frac{\partial \pi_i}{\partial \beta_i} = \frac{1}{\beta_i^2} \left[\{\beta_i^2(\beta_i - 1)\gamma_i\theta\} \frac{\partial l_i}{\partial \beta_i} + \alpha s_i \right] + \underbrace{\frac{\partial l_j}{\partial \beta_i} \left[\frac{\partial s_i}{\partial l_j} \left(1 - \frac{\alpha_i}{\beta_i} \right) \right]}_{\text{strategic effect } (< 0)} = 0, \qquad (18)$$

The first term of Eq. (18) captures the bargaining effect of delegation. The second term is the duopoly induced strategic effect. The negative strategic effect encourages the shareholders to lower β while the positive bargaining effect encourages to raise it. If, however, $\frac{\partial l_j}{\partial \beta_i} = 0$ the strategic effect would disappear and optimal β_i would then correspond to pure 'bargaining delegation' as in monopoly.

Similarly, maximize the union's utility for any given $(\beta_i, \beta_j, \gamma_j)$;

$$u_i = \alpha \frac{s_i}{\beta_i} - \theta l_i[1 - (1 - \alpha)\gamma_i].$$

The first order condition is:

$$\frac{\partial u_i}{\partial \gamma_i} = \theta \left[(\gamma_i - 1)\frac{\partial l_i}{\partial \gamma_i} + (1 - \alpha)l_i \right] + \underbrace{\frac{\alpha}{\beta_i} \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \gamma_i}}_{\text{strategic effect } (< 0)} = 0. \qquad (19)$$

The strategic effect is negative also for the union; hence it will also try to lower γ; but on the other hand, the bargaining effect is positive, hence it faces the same type of conflict as shareholders do. However, for the union the strategic effect is directly related to their bargaining power. If their bargaining power weakens, the strategic effect of their delegation will also weaken, and the bargaining effect will dominate.

Since the strategic effects are negative for both shareholders and the union, the respective incentives will be smaller compared to the monopoly case. In other words, the symmetric equilibrium β and γ of duopoly must be smaller than monopoly equilibrium β and γ.

Unilateral delegation by shareholders: Before we address the general case, we consider two special cases of unilateral delegation. Suppose the union in both firms do not delegate ($\gamma_i = \gamma_j = 1$); only the shareholders do. This case is of particular interest for the fact that it turns out as a generalisation of the strategic delegation result popularized by Vickers (1985), Fershtman-Judd (1987), and Skilvas (1987). We will refer to this class of models as 'FJ' models. Our special case not only generalizes the strategic delegation models to wage bargaining, but it also presents an efficient bargaining version of Szymanski (1994).

Set $\gamma_i = 1$ in Eq. (18) and first define β_i for two special cases. Set $\partial l_j / \partial \beta_i = 0$ and denote this soluion of β_i as β_i^B. This β represents delegation for bargaining. Next, set $\alpha = 0$ and denote the solution β_i as β_i^S, which refers to pure strategic delegation (by the firm).

Now consider (β_i^*, β_j^*) satisfying equation (18) for i and j.[18] Assume symmetry: $\beta^* = \beta_i^* = \beta_j^*$. Similarly, $\beta_i^B = \beta_j^B = \beta^B$ and $\beta_i^S = \beta_j^S = \beta^S$. Several properties of β^* can be established unambiguously as shown by the following proposition.

Proposition 6. (*Unilateral delegation by shareholders*) *The symmetric optimal incentive scheme β^* will lie between $\beta^S(< 1)$ and $\beta^B(> 1)$, with $\beta^*(\alpha = 0) = \beta^S < 1$ and $\beta^*(\alpha = 1) = \beta^B > 1$ and $\beta^{*\prime}(\alpha) > 0$. Hence, there exists a unique critical value of α, say $\hat{\alpha} \in (0, 1)$, such that at all $\alpha \in (\hat{\alpha}, 1]$ delegation is profit oriented ($\beta^* > 1$), at all $\alpha \in [0, \hat{\alpha})$ delegation is sales-oriented ($\alpha < \beta^* < 1$), and at $\alpha = \hat{\alpha}$ there is no delegation ($\beta^* = 1$).*

Proposition 6 shows that sales-orientation, which commonly features in strategic delegation models, should not be taken for granted. As Szymanski (1994) had shown earlier with a right-to-manage bargaining model, sales-orientation occurs only when the union has no or little bargaining power. Particularly at $\alpha = 0$ the FJ model is replicated. With stronger unions (firm-side) delegation becomes profit-oriented and the Cournot outputs move in the direction of collusive output. The insight of Szymanski (1994) remains valid with efficient

---

[18]For this solution to be valid we assume that the Cournot stability condition holds along with the second order condition for each firm's profit maximization.

bargaining. But in addition we will be able to see the effect of stronger unions on the overall bargaining pie as well as the distribution of the pie (which we address shortly).

Unilateral delegation by the union: Now assume that only the unions delegate. Set $\beta_i = \beta_j = 1$ in Eq. (19). This can be viewed as the union counterpart of the strategic delegation model. Here too, define γ_i for two special cases. Denote γ_i as γ_i^B when it solves Eq. (19) by setting $\partial l_j / \partial \gamma_i = 0$; this represents delegation for bargaining. Next, set $\alpha = 0$ and denote the solution γ_i as γ_i^S, which refers to pure strategic delegation by the union. Now consider (γ_i^*, γ_j^*) satisfying equation (19) for i and j (assuming that the Nash equilibrium conditions are met). Further, assume β_i^*, β_i^B and β_i^S are symmetric.

Proposition 7. (***Unilateral delegation by unions***) *The symmetric $\gamma^* \in [\gamma^S, \gamma^B]$, where $\gamma^S < 1$ and $\gamma^B > 1$. Further, γ^* decreases with α, with $\gamma^*(\alpha = 1) = \gamma^S$ and $\gamma^*(\alpha = 0) = \gamma^B$. Consequently, there exists a unique critical value of α, say $\tilde{\alpha} \in (0, 1)$, such that at all $\alpha \in (\tilde{\alpha}, 1]$ $\gamma^* < 1$ signifying gross wage bill oriented delegation, and at all $\alpha \in [0, \tilde{\alpha})$ $\gamma^* > 1$ signifying net wage bill orientation. At $\alpha = \tilde{\alpha}$ there is no delegation ($\gamma^* = 1$).*

The nature of delegation for the union is similar to that of shareholders. When the union's bargaing power is high, it delegates mainly for strategic reasons. Consequently we will see strategic overproduction. On the other hand, when the union has very little bargaining power, its delegation is primarily motivated by bargaining objective, and in equilibrium there will be underproduction (due to $\gamma^* > 1$).

Generally speaking, in all cases of delegation the conflict between the strategic motive and bargaining motive will be severe when the parties' are (nearly) equally powerful. We can also say that over-employment is not exclusive to firm-side delegation, and crucially the overproduction result is specific to the case where the delegating party holds high bargaining power.

Bilateral delegation: We now return to the case of bilatral delegation and consider both Eqs. (18) and (19).

Equilibrium incentives are obtained by solving Eqs. (18) and (19) for firm i and j. We assume that all the second order conditions hold including the Cournot stability condition (See Appendix for details) and a solution exists. Consequently, $\beta_i(\alpha)$ and $\gamma_i(\alpha)$ are continuous. In the general case we can make the following observations.

1. If $\alpha = 0$, in equilibrium $\gamma_i > 1$ and $\beta_i < 1$. Similarly, if $\alpha = 1$, in equilibrium $\gamma_i < 1$ and $\beta_i > 1$. These two results seem to be natural carry-over from one-sided delegation.

It says that the strongest party's delegation will be strategically motivated and the weakest party's delegation will be motivated for bargaining.

2. A straight forward observation from the definition of profit is that when $\beta_i < 1$, it must remain greater than α; otherwise profit will be negative.

3. With some additional assumption (which we show with the help of an example that it can be easily met) we can further establish that strategic underproduction will occur, which will make the bargaining pie expand, a point we address shortly.

4. Unlike in monopoly here β_i and γ_i need not always be strategic substitutes. In fact, with some mild assumption we can ascertain that from the shareholders' perspective γ_i is strategic complement to β_i when α is sufficiently small (i.e. close to zero). Similarly, from the union's perspective β_i is strategic complement to γ_i when α is sufficiently large and close to 1.

To see this, set $\alpha = 0$ in Eq. (18) and rewrite it as

$$\frac{\partial \pi_i}{\partial \beta_i} = \gamma_i \left[(\beta_i - 1)\gamma_i \theta \frac{\partial l_i}{\partial \beta_i \gamma_i} + \frac{\partial l_j}{\partial \beta_i \gamma_i} q_i p'(.)q'_j(.) \right] = 0.$$

Differentiating the above with respect to γ_i we get

$$\frac{\partial^2 \pi_i}{\partial \beta_i \partial \gamma_i} = \gamma_i \left[\underbrace{(\beta_i - 1)\theta \frac{\partial l_i}{\partial \beta_i \gamma_i}}_{> 0} + \underbrace{\frac{\partial l_j}{\partial \beta_i \gamma_i}}_{> 0} \underbrace{\frac{\partial q_i}{\partial \gamma_i}}_{< 0} \underbrace{p'(.)q'_j(.)}_{< 0} > 0 \right].$$

Similarly, set $\alpha = 1$ in Eq. (19) and rewrite it as

$$\frac{\partial u_i}{\partial \gamma_i} = \theta \left[\beta_i(\gamma_i - 1) \frac{\partial l_i}{\partial \beta_i \gamma_i} + \frac{\partial l_j}{\partial \beta_i \gamma_i} q_i p'(.)q'_j(.) \right] = 0.$$

Differentiating it with respect to β_i we obtain

$$\frac{\partial^2 u_i}{\partial \beta_i \partial \gamma_i} = \underbrace{(\gamma_i - 1)\theta \frac{\partial l_i}{\partial \beta_i \gamma_i}}_{> 0} + \underbrace{\frac{\partial l_j}{\partial \beta_i \gamma_i}}_{> 0} \underbrace{\frac{\partial q_i}{\partial \beta_i}}_{< 0} \underbrace{p'(.)q'_j(.)}_{< 0} > 0.$$

Moreover, at $\alpha = 0$ we also get

$$
\begin{aligned}
\frac{\partial^2 u_i}{\partial \beta_i \partial \gamma_i} &= (\gamma_i - 1)\theta \frac{\partial l_i}{\partial \beta_i \gamma_i} + \theta \frac{\partial s_i}{\partial \gamma_i}. \\
&= \theta \underbrace{(\gamma_i(1 + \beta_i) - 1)}_{> 0} \underbrace{\frac{\partial l_i}{\partial \beta_i \gamma_i}}_{< 0} + \underbrace{\frac{\partial s_i}{\partial l_j}}_{< 0} \underbrace{\frac{\partial l_j}{\partial \beta_i \gamma_i}}_{> 0} < 0.
\end{aligned}
$$

Thus, we have an asymmetric situation. At α sufficiently close to zero, unions perceive β to be strategic substitute to γ, but the shareholders perceive γ to be strategic substitute to β. Similarly at α sufficiently close to 1, the union perceives β to be strategic complement to γ while the shareholders perceive the opposite. The shareholders' perception can be established in a similar way.

In proving the above strategic relationships we relied on some assumptions. These are: $\frac{\partial l_i}{\partial \beta_i \gamma_i}$ is constant and $p''(.) = 0$. These assumptions will be justified in an example that we provide later. With similar assumptions we can also establish that β_i and β_j are alway strategic substitutes to each other, and γ_i and γ_j are also strategic substitutes.

Proposition 8. (***Bilateral delegation and strategic underproduction***) *(i) Suppose* $((\beta_i, \gamma_i), (\beta_j, \gamma_j))$ *is a Nash equilibrium. Then for sufficiently small α, $\beta_i < 1$ $\beta_j < 1$ and $\gamma_i > 1$, $\gamma_j > 1$. Conversely, for sufficiently high α we have $\beta_i > 1$ $\beta_j > 1$ and $\gamma_i < 1$, $\gamma_j < 1$.*

(ii) Assume symmetry and denote $\eta_{ij}^s \equiv \frac{\partial s_i}{\partial l_j} \frac{l_j}{s_i}$ and $\eta_{ij}^l \equiv \frac{\partial l_j}{\partial \beta_i} \frac{\beta_i}{l_j}$. Then at any α, $\beta_i \gamma_i > 1$ if and only if

$$
w_i > \frac{s_i}{l_i} |\eta_{ij}^s \eta_{ij}^l|. \tag{20}
$$

What can we say about the comparative static properties of β, γ and $\beta\gamma$? Unfortunately, very little. We know that they are continuous functions of α; but nothing can be said about monotonicity. It turns out that if β is strategic substitute to γ and vice versa then $\beta'(\alpha) > 0$ and $\gamma'(\alpha) < 0$ (see Appendix), though the same signs can be obtained under strategic complementarity. An added complication (which we confirm in our example later) is that β can be strategic complement to γ while γ is startegic substitute to β. In other words, neither strategic substituteness nor strategic complementarity can be uniformly assumed over the entire range of α. Even if we assume monotonicity on individual β and γ, we cannot ensure monotonicity of $\beta\gamma$. However, to get further characterization we make two regularity assumptions, that are often taken for granted. The first one relates to sensitivity

of firm i's employment to firm i-specific variables vis-a-vis firm j-specific variables. The second one relates to monotonicity of $\beta\gamma$.

Assumption 2. $|\frac{\partial l_i}{\partial x_i}| > |\frac{\partial l_i}{\partial x_j}|$ *for $i \neq j$, where x represents a generic variable affecting employment.*

It is reasonable to expect that the impact of firm i's action (represented by x_i) on its own employment choice will be stronger than the impact the irval firm's action will have.

Assumption 3. *The symmetric equilibrium $\beta\gamma$ is monotonic in α – $\partial(\beta\gamma)/\partial\alpha$ is either positive at all α or negative at all α. In addition, $\beta'(alpha) > 0$ and $\gamma'(\alpha) < 0$.*

Bargaining pie: Now we turn our attention to the bargaining pie. We would like to ask: Does delegation enlarge the bargaining pie or reduce it as in monopoly? Intuitively, enlargement of the bargaining pie means approaching a near-collusive outcome, and if the pie is enlarged we can say that delegation helps to achieve implicit collusive, *albeit* through competition.

Consider the firm-level bargaining pie, $P_i = s_i(l_i, l_j) - \theta l_i$ and after substituting the equilibrium employment write $P_i(\alpha) = s_i(l_i(\alpha), l_j(\alpha)) - \theta l_i(\alpha)$. Next, derive

$$
\begin{aligned}
P_i'(\alpha) &= \left(\frac{\partial s_i}{\partial l_i} - \theta \right) \left[\frac{\partial l_i}{\partial(\beta_i\gamma_i)} \frac{\partial(\beta_i\gamma_i)}{\partial\alpha} + \frac{\partial l_i}{\partial(\beta_j\gamma_j)} \frac{\partial(\beta_j\gamma_j)}{\partial\alpha} \right] \\
&\quad + \frac{\partial s_i}{\partial l_j} \left[\frac{\partial l_j}{\partial(\beta_i\gamma_i)} \frac{\partial(\beta_i\gamma_i)}{\partial\alpha} + \frac{\partial l_j}{\partial(\beta_j\gamma_j)} \frac{\partial(\beta_j\gamma_j)}{\partial\alpha} \right] \\
&= \left[(\beta_i\gamma_i - 1)\theta + \frac{\partial s_i}{\partial l_j} \right] \frac{\partial(\beta_i\gamma_i)}{\partial\alpha} \underbrace{\left[\frac{\partial l_i}{\partial(\beta_i\gamma_i)} + \frac{\partial l_i}{\partial(\beta_j\gamma_j)} \right]}_{< 0 \quad \text{by Assumption 2}}. \quad (21)
\end{aligned}
$$

In Eq. (21) the last term is negative by Assumption 2. Therefore, the sign of the overall expression depends on the first two terms. The sign of $\partial(\beta_i\gamma_i)/\partial\alpha$ is hard to determine in the bilateral case; but by Assumption 3 $\partial(\beta_i\gamma_i)/\partial\alpha \neq 0$, which is clearly the case under unilateral delegation. So our attention is then devoted to the first term.

When $P_i(\alpha)$ is maximum we must have the first term zero (given $\partial(\beta_i\gamma_i)/\partial\alpha \neq 0$) for both firms. That is,

$$
\beta_i\gamma_i = 1 - \frac{1}{\theta} \frac{\partial s_i}{\partial l_j} \quad \text{for} \;\; i, j = 1, 2, \; i \neq j. \quad (22)
$$

$(\beta_i \gamma_i, \beta_j \gamma_j)$ that implicitly solve the above equation for $i = 1, 2$ together yield the largest bargaining pie. The second order condition confirms the maximum[19]. Since $\frac{\partial s_i}{\partial l_j} < 0$, $\beta_i \gamma_i$ must be greater than 1.

It can be easily checked that this also coincides with the collusive choice of employment. Substitute Eq. (22) in the duopoly employment equation (14) and obtain

$$\frac{\partial s_i}{\partial l_i} = \theta - \frac{\partial s_i}{\partial l_j}, \quad i, j = 1, 2, \quad i \neq j,$$

which is precisely the first order condition of maximizing the aggregate pie, $P_1 + P_2 = s_1 + s_2 - \theta(l_1 + l_2)$ with respect to l_1 and l_2 after allowing for symmetry $\frac{\partial s_j}{\partial l_i} = \frac{\partial s_i}{\partial l_j}, i \neq j$.

Further write $\frac{\partial s_i}{\partial l_j} = q_i p'(.)q_j'(.)$ and impose symmetry: $\beta_i \gamma_i = \beta_j \gamma_j = \beta\gamma, q_i = q_j = q$. Then we can simplify Eq. (22) as

$$\beta\gamma = 1 - \frac{qp'(.)q'(.)}{\theta}. \tag{23}$$

Let us write $\psi \equiv 1 - \frac{qp'(.)q'(.)}{\theta}$. It is noteworthy that ψ is a function of $\beta\gamma$ (rather than β and γ separately) via $l_i(.)$ because from Eq. (14) it is obvious that β_i and γ_i enter the argument of l_i only as a product term, and not separately. Thus, $\psi = \psi(\beta\gamma)$.

As $\beta\gamma \to 0$, $\psi(.) \to \infty$ (due to $q \to \infty$), and as $\beta\gamma \to \infty$, $\psi(.) \to 1$ (due to $q \to 0$). Further, $\partial s_i / \partial l_j (\equiv qp'(.)q'(.))$ is positively related to $\beta\gamma$ (see Footnote 19). Hence, $\psi(.)$ is a declining function of $\beta\gamma$. On the other hand, $\beta\gamma$ is a 45-degree line. Hence, they must cross exactly once. In other words, there is a unique solution $\beta\gamma$ to Eq. (23). Let this solution be denoted as k; clearly, $k > 1$.

---

[19]We evaluate the second order condition at the maximum. First note that

$$\frac{\partial^2 s_i}{\partial l_j \partial \beta_i \gamma_i} = \underbrace{\left\{ \frac{\partial s_i^2}{\partial l_j \partial l_i} + \frac{\partial s_i^2}{\partial l_j^2} \right\}}_{< 0} \underbrace{\left\{ \frac{\partial l_i}{\partial(\beta_i \gamma_i)} + \frac{\partial l_i}{\partial(\beta_j \gamma_j)} \right\}}_{< 0} > 0, \quad i \neq j, i, \ j = 1, 2.$$

In the above, we have used the assumption of symmetry to write $\frac{\partial(\beta_i \gamma_i)}{\partial \alpha} = \frac{\partial(\beta_j \gamma_j)}{\partial \alpha}$ and $\frac{\partial l_i}{\partial(\beta_i \gamma_i)} = \frac{\partial l_j}{\partial(\beta_j \gamma_j)}$, $\frac{\partial l_j}{\partial(\beta_i \gamma_i)} = \frac{\partial l_i}{\partial(\beta_j \gamma_j)}$. This allows us to derive

$$P_i''(\alpha) = \left[\theta + \underbrace{\frac{\partial^2 s_i}{\partial l_j \partial \beta_i \gamma_i}}_{> 0} \right] \left(\frac{\partial(\beta_i \gamma_i)}{\partial \alpha} \right)^2 \underbrace{\left[\frac{\partial l_i}{\partial(\beta_i \gamma_i)} + \frac{\partial l_i}{\partial(\beta_j \gamma_j)} \right]}_{< 0} < 0.$$

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It is also clear that in all cases of delegation – unilateral or bilateral – the condition for the bargaining pie maximization is same. When only the firm delegates β must be equal to k (with $\gamma = 1$), and when only the union delegates we have $\gamma = k$ with $\beta = 1$. The reason is that the function $\psi(.)$ does not change; only its argument changes from $\beta\gamma$ to β or γ depending on two-sided or one sided delegation.

Now the key question is: Will $\beta\gamma$ (or β or γ in one-sided delegations) be equal to k at some $\alpha \in [0, 1]$? It turns out that there is no clear-cut answer to this question, not in the general case, though the picture is somewhat better in the cases of unilateral delegation. Let us consider the case of only the shareholders opting to delegate. We know at $\alpha = 0$, optimal $\beta < 1$ and hence $\beta < k$. We also know $\beta'(\alpha) > 0$ and at $\alpha = 1$ optimal β must exceed 1. But there is no way of ascertaining that $\beta(\alpha = 1)$ will exceed k. So all we can say that if and only if $\beta(\alpha = 1) > k$, then at some intermediate α the bargaining pie will be maximum. A similar conditional statement can be made about unilateral delegation by the union. But for bilateral delegation, the picture is much more complex. As has already been noted, we cannot ascertain whether $\beta\gamma$ is increasing or decreasing in α; nor can we say whether $\beta\gamma$ will exceed or fall short of 1 at $\alpha = 0$ or $\alpha = 1$. Hence, we cannot say much in the general case. However, if we consider specific examples, as we do later, then we might be able to get some clear results. We summarize our findings in the following two propositions; but before that we need to state a lemma which will be used to map out the bargaining pie over the entire range of α.

Lemma 3. *In a symmetric equilibrium*

$$sign \ \frac{\partial}{\partial \alpha} \left[(\beta_i \gamma_i - 1)\theta + \frac{\partial s_i}{\partial l_j} \right] = sign \ \frac{\partial(\beta_i \gamma_i)}{\partial \alpha}. \tag{24}$$

Proposition 9. (*Bargaining pie under unilateral delegation*)

1. *Suppose only the shareholders delegate, and assume the optimal symmetric $\beta > k$ at $\alpha = 1$. Then there exists a critical α, say $\alpha_0 \in (0, 1)$ such that at all $\alpha \in [0, \alpha_0)$ $P'(\alpha) > 0$, at $\alpha = \alpha_0$ $P'(.) = 0$ and at all $\alpha \in (\alpha_0, 1]$ $P'(\alpha) < 0$.*

2. *Suppose only the union delegates, and assume the symmetric optimal $\gamma > k$ at $\alpha = 0$. Then there exists a critical α, say $\alpha_1 \in (0, 1)$ such that at all $\alpha \in [0, \alpha_1)$ $P'(\alpha) > 0$, at $\alpha = \alpha_1$ $P'(.) = 0$ and at all $\alpha \in (\alpha_1, 1]$ $P'(\alpha) < 0$.*

3. *$\alpha_1 < (>)\alpha_0$ if and only if $\beta(\alpha_1) < (>)\gamma(\alpha_1)$.*

What we establish in the above proposition is that with a mild boundary condition (e.g. $\beta(\alpha = 1) > k$ in the case of firm delegation) it can be ascertained that the bargaining pie will be inverted U-shpaed against α. At some interior value of α the fully collusive pie is achieved. A similar claim can be made also about union-side delegation. If the boundary condition is not met (in any unilateral delegation), then we can say that the bargaining pie will be increasing in α but not achieve the maximum. Unfortunately, to make a similar assertion for bilateral delegation we need to impose far more conditions. In the following proposition we provide a number of suitable conditions to ensure the inverted U-shaped pie curve; later on we support these conditions by considering an example. But one can propose alternative conditions to generate the same.

Proposition 10. (***Bargaining pie under bilateral delegation***) *Suppose the symmetric $\beta\gamma > k$ at $\alpha = 0$ and $\beta\gamma < k$ at $\alpha = 1$, and assume that $\partial(\beta\gamma)/\partial\alpha < 0$ at all α. Then there exists a critical α, say $\hat{\alpha} \in (0, 1)$ such that at all $\alpha \in [0, \hat{\alpha})$ $P'(\alpha) > 0$, at $\alpha = \hat{\alpha}$ $P'(.) = 0$, and at all $\alpha \in (\hat{\alpha}, 1]$ $P'(\alpha) < 0$.*

Alternative way: An alternative way of deriving the same observations as above is to set the bargaining pie at the symmetric collusive level and try to achieve the associated employment via the delegation game. We know that if the firms could collude and no parties resorted to delegation, the aggregate pie for the industry would be at its maximum, which will be the no-delegation monopoly pie, given by $s'(l) = \theta$. Each firm's bargaining pie would be $P^M/2$. It is also expected that under Cournot competition (without any delegation) aggregate employment will exceed the monopoly employment, and the aggregate as well as firm-specific bargaining pie would be much smaller. Since these observations are basic, we do not prove them. We try to see if the collusive pie can be achieved under Cournot competition through delegation. For this purpose let us introduce some definitions.

Definition 1. *Let $P^M = \max[(q_1 + q_2)p(.) - \theta(l_1 + l_2)]$ be the largest pie for the industry and $P^M/2$ be the symmetric largest bargaining pie for a firm.*

Definition 2. *Let l^M be such that $q_i(l^M)p(.) - \theta l^M = \frac{P^M}{2}$; l^M refers to the symmetric firm level employment that supports the largest bargaining pie.*

Definition 3. *Let $k > 1$ be such that $\frac{\partial s_i(l^M, l^M)}{\partial l} = k\theta$ for both firms. That is, $k\theta$ is the effective reservation wage that induces the choice of l^M under Cournot competition.*

It is noteworthy that k is uniquely determined (given l^M) and it is clearly greater than 1. We now wish to examine whether the delegation game, unilateral or bilateral, will ever yield equilibrium incentives equal to k and thus yield the collusive pie. Any incentive term greater or smaller than k will inevitably result in a pie smaller than $P^M/2$.

The next step is to see when $\beta\gamma$ (or β and γ under unilateral delegation) can achieve the value of k. For this we need the conditions stated in Propositions 9 and 10. In light of the inverted U relationship between the bargaining pie and the bargaining power, it is apparent that it is not just the fully collusive pie, but also a range of high profit levels can also be achieved through delegation at some appropriate value of α.

Distribution of the pie: What can we say about the distribution of the bargaining pie? Will profit be decreasing in α? It turns out that even in the special cases of unilateral delegation the effects are indeterminate. The strategic effects tend to counteract the direct effects of a change in α. To see this let us consider the following (assuming symmetry):

$$
\begin{aligned}
\pi_i'(\alpha) \;=\;& \frac{\partial \pi_i}{\partial \alpha} + \frac{\partial \pi_i}{\partial \gamma_i}\frac{\partial \gamma_i}{\partial \alpha} + \frac{\partial \pi_i}{\partial \gamma_j}\frac{\partial \gamma_j}{\partial \alpha} + \frac{\partial \pi_i}{\partial \beta_j}\frac{\partial \beta_j}{\partial \alpha} \\
=\;& -\left[\frac{s_i}{\beta_i} - \theta\gamma_i l_i\right] + \frac{\partial \gamma_i}{\partial \alpha}\left[\alpha\theta l_i + \left\{(\beta_i - 1)\gamma_i\theta + (1 - \frac{\alpha}{\beta_i})\frac{\partial s_i}{\partial l_j}\right\}\left\{\frac{\partial l_i}{\partial \gamma_i} + \frac{\partial l_i}{\partial \gamma_j}\right\}\right] \\
& + \frac{\partial \beta_j}{\partial \alpha}\left[(\beta_i - 1)\gamma_i\theta\frac{\partial l_i}{\partial \beta_j} + (1 - \frac{\alpha}{\beta_i})\frac{\partial s_i}{\partial l_j}\frac{\partial l_j}{\partial \beta_j}\right].
\end{aligned}
$$

The above expression makes use of symmetry: $\frac{\partial \gamma_i}{\partial \alpha} = \frac{\partial \gamma_j}{\partial \alpha}$, $\frac{\partial \beta_i}{\partial \alpha} = \frac{\partial \beta_j}{\partial \alpha}$, $\frac{\partial s_i}{\partial \beta_j} = \frac{\partial s_j}{\partial \beta_i}$ etc. While the first term is negative, the second and the third terms are signed ambiguously. the first term is negative because $(s_i/l_i) > \beta_i\gamma_i\theta = \frac{\partial s_i}{\partial l_i}$ because of concavity of s_i. In the case of firm's unilateral delegation, the second term is zero; but still the third term can be positive and the whole sign remains ambiguous.

Similarly,

$$
\begin{aligned}
u_i'(\alpha) \;=\;& \frac{\partial u_i}{\partial \alpha} + \frac{\partial u_i}{\partial \gamma_j}\frac{\partial \gamma_j}{\partial \alpha} + \frac{\partial u_i}{\partial \beta_i}\frac{\partial \beta_i}{\partial \alpha} + \frac{\partial u_i}{\partial \beta_j}\frac{\partial \beta_j}{\partial \alpha} \\
=\;& \left[\frac{s_i}{\beta_i} - \theta\gamma_i l_i\right] + \frac{\partial \beta_i}{\partial \alpha}\left[-\alpha\frac{s_i}{\beta_i^2} + \left\{(\gamma_i - 1)\theta + \frac{\alpha}{\beta_i}\frac{\partial s_i}{\partial l_j}\right\}\left\{\frac{\partial l_i}{\partial \beta_i} + \frac{\partial l_i}{\partial \beta_j}\right\}\right] \\
& + \frac{\partial \gamma_j}{\partial \alpha}\left[(\gamma_i - 1)\theta\frac{\partial l_i}{\partial \gamma_j} + \frac{\alpha}{\beta_i}\frac{\partial s_i}{\partial l_j}\frac{\partial l_j}{\partial \gamma_j}\right].
\end{aligned}
$$

Again though the first term is positive, the overall sign remains ambiguous even if we ignore the second term (which will be zero in the special case of unilateral delegation by the

union). Thus, the relationship between profit and union's bargaining power becomes more complex. In monopoly, this relationship was inverse for unilateral delegation; but that cannot be taken for granted in duopoly. In case of bilateral delegation the relationship was unclear also in monopoly, and we showed with an example that the relationship can be reversed. Whether the same can be said about duopoly remains to be seen in the next section.

3.1 An example and simulation

Consider $p = a - q_1 - q_2$ and $q_i = l_i$ as before. Assume that $a < 6\theta$. The second stage duopoly output and wage resulting from efficient bargaining are

$$l_i = \frac{a - 2\beta_i\gamma_i\theta + \beta_j\gamma_j\theta}{3},$$

$$w_i = (1-\alpha)\gamma_i\theta + \alpha\frac{[a + \theta(\beta_i\gamma_i + \beta_j\gamma_j)]}{3\beta_i}, \qquad i \neq j, i, j = 1, 2.$$

We may also note that $p = (a + \theta(\beta_i\gamma_i + \beta_j\gamma_j))/3$ and $\theta\gamma_i < w_i < p_i/\beta_i$.

Now consider the firms' optimal incentives. Profit in firm i is given as,

$$\pi_i = \left[\frac{[a + \theta(\beta_i\gamma_i + \beta_j\gamma_j)]}{3}\frac{(\beta_i - \alpha)}{\beta_i} - (1-\alpha)\theta\gamma_i\right]\frac{(a - 2\beta_i\gamma_i\theta + \beta_j\gamma_j\theta)}{3}, \qquad i \neq j, i, j = 1, 2.$$

Maximizing the above we obtain the following (implicit) reaction functions which can be solved for equilibrium incentives. For $i \neq j, i, j = 1, 2$

$$\frac{\partial\pi_i}{\partial\beta_i} = \frac{1}{9\beta_i^2}\left[4\gamma_i\beta_i^2\theta^2\left\{\frac{6\gamma_i\theta - a - \beta_j\gamma_j\theta}{4\theta} - \beta_i\gamma_i\right\} + \alpha\left\{(a + \beta_j\gamma_j\theta)^2 - 4\beta_i^2\gamma_i^2\theta^2\right\}\right] = 0. \quad (25)$$

Similarly, the union's utility function is

$$u_i = \frac{(a - 2\beta_i\gamma_i\theta + \beta_j\gamma_j\theta)}{3}\left[\frac{\alpha}{3\beta_i}\left\{a + \theta\beta_i\gamma_i + \theta\gamma_j\beta_j\right\} - \theta\{1 - (1-\alpha)\gamma_i\}\right].$$

The first order condition is

$$\frac{\partial u_i}{\partial\gamma_i} = \frac{1}{9}\left[(a + \beta_j\gamma_j\theta)(3 - 4\alpha) + 6\beta_i\theta - 4(3 - 2\alpha)\beta_i\theta\gamma_i\right] = 0. \quad (26)$$

We assume there is a symmetric solution, $\beta_i = \beta_j = \beta$ and $\gamma_i = \gamma_j = \gamma$. The symmetric

solution simplifies Eqs. (25) and (26) as follows.

$$\frac{\partial \pi}{\partial \beta} = \beta^2 \gamma \theta \left[6\gamma\theta - a - 5\beta\gamma\theta \right] + \alpha \left[(a + \beta\gamma\theta)^2 - 4\beta^2\gamma^2\theta^2 \right] = 0, \tag{27}$$

$$\frac{\partial u}{\partial \gamma} = a(3 - 4\alpha) + 6\beta\theta - (9 - 4\alpha)\beta\theta\gamma = 0$$

$$= (a - \beta\gamma\theta)(3 - 4\alpha) + 6\beta\theta(1 - \gamma) = 0. \tag{28}$$

Several observations can be made from Eqs. (27)-(28).

1. Regardless of β the union's best response is $\gamma = 1$ when $\alpha = 3/4$. This is evident from Eq. (28).

2. Suppose only the firm delegates (i.e. $\gamma = 1$). From Eq. (27) we see that at $\alpha = \frac{(a-\theta)\theta}{(a+\theta)^2 - 4\theta^2}$ the firm's best response is $\beta = 1$. Since $\beta'(\alpha) > 0$ under unilateral delegation, we can assert that at $\alpha > (<)\frac{(a-\theta)\theta}{(a+\theta)^2 - 4\theta^2}$ optimal $\beta > (<)1$.

3. From Eq. (28) write the union's reaction function as

$$\gamma = \frac{a(3 - 4\alpha)}{(9 - 4\alpha)\beta\theta} + \frac{6}{9 - 4\alpha}.$$

It is clear that at all $\alpha > (<)3/4$, $\frac{\partial \gamma}{\partial \beta} > (<)0$. That is, above $\alpha = 3/4$, the union perceives β as strategic complement to γ, and below $\alpha = 3/4$, as strategic substitute for γ. Unfortunately, a similar observation cannot be made for the shareholders' reaction function, as it is fairly complex.

Next, in Table 3 we compute the equilibrium incentives and outputs for $a = 2$ and $\theta = 1$, the same parameters considered for monopoly in Table 1. First we consider the no delegation case (Case 0). Bargaining power affects only the distribution of the bargaining pie, but leaves output and the (firm-level) pie unaffected. Note that the aggregate pie (the last column: 2x 0.111= 0.222) is much smaller than the monopoly pie under no delegation 0.25 (see Table 1).

Next, Case 1 presents unilateral delegation by the firm (i.e. the shareholders). At $\alpha = \frac{(a-\theta)\theta}{(a+\theta)^2 - 4\theta^2} = \frac{1}{9-4} = 0.2$ optimal β is exactly 1. Above $\alpha = 0.2$, β is greater than 1 causing underemployment relative to Case 0, and below $\alpha = 0.2$, $\beta < 1$ causing over-employment (relative to Case 0). It is also noteworthy that the firm's profit rises above the no-delegation level only after β exceeds 1. That is, at $\alpha < 0.2$ managers are oriented to sale maximization, and since both firms do the same, the outcome is overproduction and loss

in profit. In other words, in this range of α delegations suffers from a prisoner's dilemma problem; it reduces each firm's profit, but resorting to delegation is a dominant strategy for each firm. On the other hand, at $\alpha > 0.2$ managers are oriented to profit maximization and underproduction helps to improve profit; delegation pays off here.

Case 2 describes unilateral delegation by the union. Above $\alpha = 0.75$, the union leader is oriented to gross wage bill maximization, which leads to overproduction (relative to the no-delegation level) in both firms. Both unions are consequently worse off, a prisoner's dilemma phenomenon similar to the firm-side delegation (Case 1) at $\alpha < 0.2$. Below $\alpha = 0.75$ unions orient their leaders to net wage bill maximization ($\gamma > 1$), which in turn helps to reduce employment below the no-delegation level resulting in higher utility.

Finally, Case 3 concerns bilateral delegation. Here, at $\alpha < 0.22$ the shareholders adopt sales orientation ($\beta^* < 1$) and above $\alpha = 0.22$ they adopt profit orientation. The unions continues to have gross wage bill orientation (as expected) above $\alpha = 0.75$ and net wage bill orientation below $\alpha = 0.75$. The combined impact of these incentives on employment can be seen from the column for $\beta\gamma$ and l. $\beta\gamma$ in this example always remains greater than 1 and it is steadily decreasing in α; consequently employment increases with α. Further, as was seen in monopoly, here too profit increases and union's utility decreases with an increase in α. See Figs. 4a; profit under bilateral delegation is strictly increasing in α; in particular profit exceeds the no-delegation level at $\alpha = 0.43$. On the other hand, Fig. 4b shows that union's utility from delegation is strictly decreasing in α, but it remains above the duopoly level at all $\alpha \leq 0.52$. Thus, there is a range of α, i.e. $(0.43, 0.52)$, over which both parties experience strictly positive gains from delegation.

The fact that both players find delegation mutually beneficial suggests that (bilateral) delegation must have enlarged the pie. Indeed that is the case over the entire range of α. This can be directly confirmed by comparing $\pi + u$ of Case 3 with that of Case 0 in Table 3. Fig. 3 presents a visual illustration. The no-delegation pie is given by the flat line at 0.111, and pie under bilateral delegation is inverted U-shaped. Not only is it strictly greater than 0.111, but it equals the collusive pie at $\alpha = 0.42$. We can also see that under unilateral delegation also the pie exceeds the no-delegation level, and it also reaches the collusive level – at $\alpha = 0.25$ when only the union delegates and at $\alpha = 4/5$ when only the firm delegates. The pie maximization condition Eq. (23) reduces to $\beta\gamma = 1 + \frac{q}{\theta}$ (using $q'(.) = 1$ and $p'(.) = -1$). Since $q = (a - \beta\gamma)/3$, we get $\beta\gamma = (a + 3\theta)/4\theta$. Substituting $a = 2$ and $\theta = 1$ we get the pie-maximizing $\beta\gamma$ or k as 1.25. From Table 3 we can verify that $\beta\gamma = 1.25$ at $\alpha = 0.42$ (Case 3) and the pie is half the monopoly (no-delegation) pie. For the other two cases of delegation as well we can identify the α corresponding β or γ equal to 1.25.

Finally, we comment on the strategic relationship between symmetric β and γ. As observed earlier, we indeed have asymmetric relationship. From the union's perspective β is strategic complement to γ at all $\alpha > 0.75$, and strategic substitute at all $\alpha < 0.75$. From the firm's perspective γ is strategic substitute to β at all $\alpha > 0.2$ and strategic complement at all $\alpha < 0.2$. So only at $\alpha \in (0.2, 0.75)$ they are strategic substitutes to each other. These can be read from Table 3 as well by tracking the movements in β and γ between unilateral delegation and bilateral delegation. But a direct approach of plotting the reaction functions is more helpful. In Figs. 5a-5e, we do that.

To sum up, we can see that unlike in monopoly delegation in duopoly always expands the bargaining pie, regardless of whether one player delegates or both players delegate. Therefore, it is fair to say delegation in bargaining is conducive to implicit collusion.

Decision to delegate or not delegate: If we extend the game backward to add a stage where both sides decide to delegate or not delegate by looking ahead at the symmetric equilibrium payoffs from the ensuing game, then some interesting conclusion emerges. Tables 5a-5c report the payoff matrices at three values of α, 0.8, 0.5 and 0.1 respectively. It turns out that at all $\alpha > 0.75$ the shareholders find delegation a dominant strategy while the union find no-delegation as their dominant strategy. Thus, we have an outcome of asymmetric delegation with only the weaker party delegating. The outcome is reversed when $\alpha < 0.22$, when shareholders being very strong abstain from delegation. At $\alpha \in (0.22, 0.75)$ both parties delegate. Fig. 10 depicts maps the equilibrium delegation decisions. At $\alpha = 0.75$ (alternatively 0.22) the union (alternatively shareholders) is indifferent between delegation and no-delegation. Thus, it seems that the scenario of shareholders unilaterally delegating seen in the strategic delegation literature and elsewhere may not arise after all.

4   Conclusion

In this paper we have developed a model of bilateral delegation in wage and employment bargaining. Shareholders appoint a manager to negotiate on their behalf, and so do the workers by appointing a union leader. We compare the optimal and equilibrium incentive schemes between monopoly and Cournot duopoly. In monopoly, the incentive schemes cause underemployment which in turn reduces the bargaining pie. In duopoly also the combined impact of incentives is underemployment, but here the bargaining pie expands (from the no-delegation level), and thus creates opportunities for mutual gains. But each player's payoff
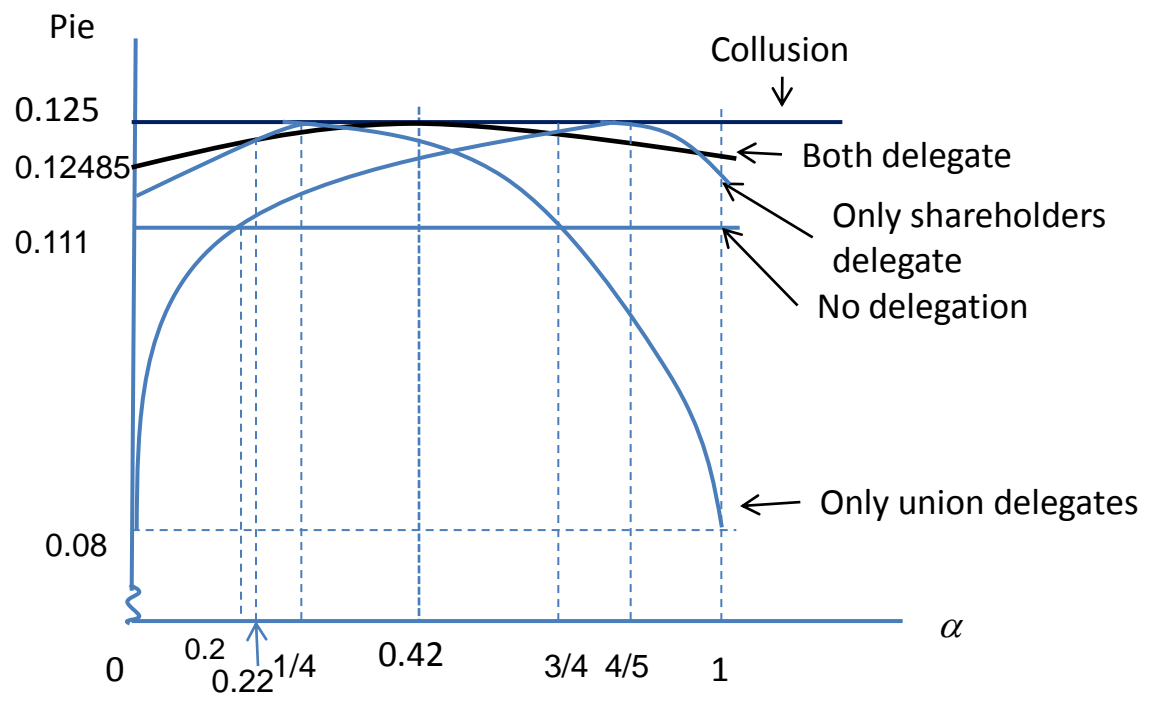
Fig. 3: Bargaining pie for a firm in duopoly

Table 3: Duopoly simulations

a = 2; θ = 1

Case 0: No Delegation ($\beta = 1$; $\gamma = 1$)

α	w	l	u	π	wl	π + u
1	1.333	0.333	0.111	0	0.444	0.111
0.75	1.250	0.333	0.083	0.028	0.417	0.111
0.5	1.167	0.333	0.056	0.056	0.389	0.111
0.42	1.167	0.333	0.056	0.056	0.389	0.111
0.25	1.083	0.333	0.028	0.083	0.361	0.111
0.20	1.067	0.333	0.222	0.89	0.356	0.111
0	1	0.333	0	0.111	0.333	0.111

Case 1: Only shareholders delegates ($\gamma = 1$)

α	$\beta\vert\gamma = 1$	w	l	u	π	wl	π + u
1	1.29	1.182	0.235	0.0427	0.0819	0.278	0.12455
0.95	1.28	1.177	0.239	0.0422	0.0825	0.281	0.12475
0.9	1.27	1.172	0.243	0.0417	0.0832	0.285	0.12490
0.85	1.26	1.167	0.247	0.0412	0.0838	0.288	0.12498
0.8	1.25	1.161	0.251	0.0406	0.0844	0.292	0.12500
0.75	1.23	1.156	0.256	0.0399	0.0851	0.296	0.12493
0.5	1.15	1.123	0.283	0.0349	0.0879	0.318	0.12278
0.25	1.03	1.078	0.323	0.0252	0.0892	0.348	0.11441
0.22	1.01	1.071	0.329	0.0234	0.0890	0.353	0.11241
0.2	1	1.067	0.333	0.0222	0.0889	0.356	0.11111
0.15	0.96	1.054	0.345	0.0186	0.0882	0.364	0.10679
0.1	0.92	1.039	0.360	0.0140	0.0869	0.374	0.10097
0.05	0.87	1.022	0.377	0.0082	0.0846	0.385	0.09274
0	0.80	1	0.4	0	0.08	0.4	0.08

Case 2: Only union delegates ($\beta = 1$)

α	$\gamma\vert\beta = 1$	w	l	u	π	wl	π + u
1	0.8	1.2	0.4	0.08	8.88E-17	0.48	0.08
0.95	0.85	1.212	0.385	0.0814	0.0074	0.466	0.0888
0.9	0.89	1.222	0.370	0.0823	0.0137	0.453	0.0960
0.85	0.93	1.232	0.357	0.0829	0.0191	0.440	0.1020
0.8	0.97	1.241	0.345	0.0832	0.0238	0.428	0.1070
0.75	1	1.250	0.333	0.0833	0.0278	0.417	0.1111
0.5	1.14	1.286	0.286	0.0816	0.0408	0.367	0.1224
0.25	1.25	1.313	0.250	0.0781	0.0469	0.328	0.1250
0.22	1.26	1.315	0.246	0.0776	0.0473	0.324	0.1250
0.2	1.27	1.317	0.244	0.0773	0.0476	0.321	0.1249
0.15	1.29	1.321	0.238	0.0765	0.0482	0.315	0.1247
0.1	1.30	1.326	0.233	0.0757	0.0487	0.308	0.1244
0.05	1.32	1.330	0.227	0.0749	0.0491	0.302	0.1240
0	1.33	1.333	0.222	0.0741	0.0494	0.296	0.1235

Case 3: Both delegate

α	β	γ	βγ	w	l	u	π	wl	π + u
1	1.35	0.904	1.220	1.096	0.2599	0.025	0.100	0.285	0.12480
0.95	1.33	0.922	1.223	1.107	0.2591	0.028	0.097	0.287	0.12483
0.9	1.30	0.940	1.225	1.119	0.2584	0.031	0.094	0.289	0.12486
0.85	1.28	0.960	1.227	1.131	0.2576	0.034	0.091	0.291	0.12489
0.8	1.26	0.980	1.230	1.143	0.2568	0.037	0.088	0.294	0.12491
0.75	1.23	1.0	1.232	1.156	0.2559	0.040	0.085	0.296	0.12493
0.5	1.12	1.112	1.245	1.225	0.2516	0.057	0.068	0.308	0.12500
0.42	1.08	1.12	1.25	1.249	0.2501	0.062	0.063	0.312	0.12500
0.25	1.01	1.244	1.260	1.304	0.2467	0.075	0.050	0.322	0.12498
0.22	1	1.261	1.261	1.315	0.2463	0.078	0.047	0.324	0.12497
0.2	0.99	1.272	1.263	1.322	0.2457	0.079	0.046	0.325	0.12496
0.15	0.97	1.302	1.266	1.340	0.2447	0.083	0.042	0.328	0.12494
0.1	0.95	1.332	1.269	1.358	0.2436	0.087	0.038	0.331	0.12492
0.05	0.93	1.364	1.272	1.377	0.2425	0.091	0.033	0.334	0.12489
0	0.91	1.396	1.276	1.396	0.2414	0.096	0.029	0.337	0.12485

Table 4: Further duopoly simulations on bilateral delegation

$a = 3;\ \theta = 1$									
α	β	γ	$\beta\gamma$	w	l	u	π	wl	$\pi + u$
1	1.66	0.840	1.398	1.160	0.5340	0.412	0.086	0.620	0.49768
0.95	1.62	0.868	1.405	1.181	0.5318	0.402	0.096	0.628	0.49798
0.9	1.57	0.899	1.412	1.202	0.5294	0.391	0.107	0.637	0.49827
0.85	1.52	0.931	1.419	1.225	0.5270	0.380	0.118	0.645	0.49854
0.8	1.48	0.965	1.426	1.248	0.5246	0.369	0.130	0.655	0.49879
0.75	1.43	1.0	1.434	1.273	0.5220	0.357	0.142	0.664	0.49903
0.5	1.22	1.208	1.476	1.415	0.5079	0.289	0.211	0.719	0.49988
0.25	1.03	1.476	1.525	1.595	0.4916	0.208	0.292	0.784	0.49986
0.22	1	1.535	1.535	1.634	0.4882	0.190	0.309	0.798	0.49972
0.2	1.00	1.538	1.536	1.635	0.4881	0.190	0.310	0.798	0.49972
0.15	0.96	1.603	1.546	1.678	0.4845	0.171	0.328	0.813	0.49952
0.1	0.93	1.671	1.557	1.722	0.4809	0.152	0.347	0.828	0.49927
0.05	0.90	1.742	1.568	1.768	0.4772	0.132	0.367	0.844	0.49896
0	0.87	1.816	1.580	1.816	0.4734	0.112	0.387	0.860	0.49859
$a = 3;\ \theta = 2$									
α	β	γ	$\beta\gamma$	w	l	u	π	wl	$\pi + u$
1	1.18	0.946	1.117	2.108	0.2554	0.097	0.028	0.538	0.12494
0.95	1.17	0.956	1.118	2.120	0.2549	0.094	0.031	0.540	0.12495
0.9	1.16	0.967	1.118	2.132	0.2545	0.091	0.034	0.543	0.12496
0.85	1.14	0.978	1.119	2.144	0.2540	0.088	0.037	0.545	0.12497
0.8	1.13	0.989	1.120	2.157	0.2535	0.085	0.040	0.547	0.12498
0.75	1.12	1.0	1.120	2.169	0.2530	0.082	0.043	0.549	0.12498
0.5	1.06	1.059	1.124	2.236	0.2505	0.066	0.059	0.560	0.12500
0.25	1.00	1.123	1.128	2.308	0.2479	0.049	0.076	0.572	0.12499
0.22	1	1.129	1.129	2.314	0.2476	0.047	0.078	0.573	0.12499
0.2	0.99	1.137	1.129	2.324	0.2473	0.045	0.080	0.575	0.12499
0.15	0.98	1.151	1.130	2.339	0.2467	0.041	0.084	0.577	0.12498
0.1	0.97	1.165	1.131	2.355	0.2462	0.038	0.087	0.580	0.12497
0.05	0.96	1.179	1.132	2.371	0.2456	0.034	0.091	0.582	0.12496
0	0.95	1.194	1.132	2.387	0.2450	0.030	0.095	0.585	0.12495

will be inversely related to his bargaining power. It may not be unfair to say that strong unions may facilitate collusion, or strong firms may benefit weak unions.

Our analysis has abstracted from capital. It is not unreasonable to expect that capital will remain outside the union's negotiation. It remains to be seen whether that will help or hinder the shareholders. There are other issues such as union's objective function. What if the union attaches different weight on employment than on wage? some asymmetric cases might also be interesting - for example absence of union in one firm depicting a contrast between union and non-union sector. We believe that insights developed in this paper will go a long way in exploring such related issues.
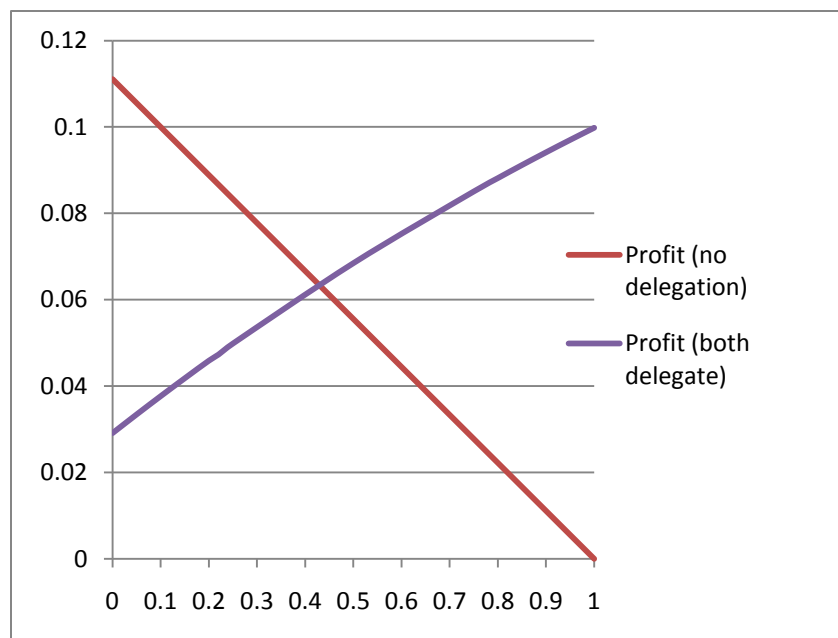
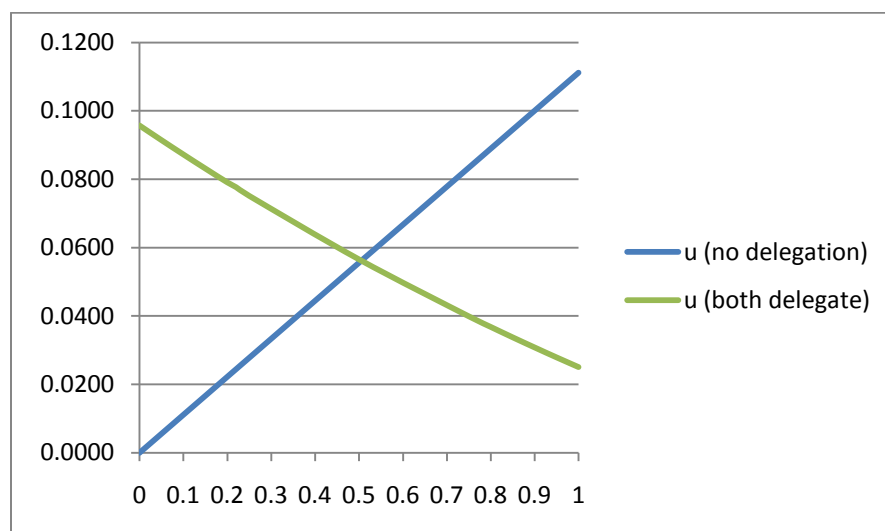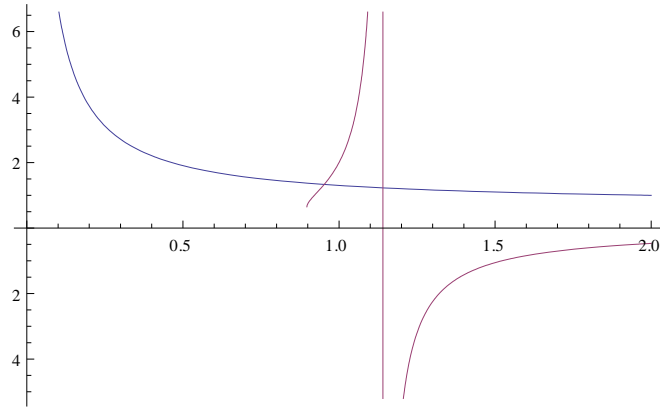Figure 4a: Firm's profit against bargaining power in duopoly

Figure4b: Union's utility against bargaining power in duopoly
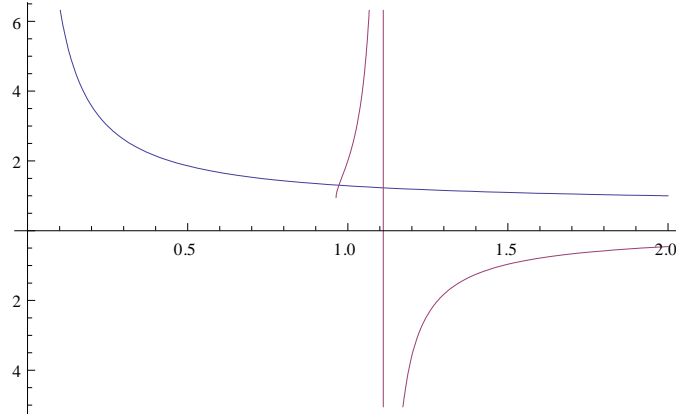
Reaction functions: **Fig 5a**

The red one is Firm's $\gamma(\beta)$ – it has two roots - and the blue is the Union's $\gamma(\beta)$ such that $\beta \in [0,2]$

$$\text{Plot}\left[\left\{U = \frac{2(-3 + 4\alpha - 3\beta)}{(-9 + 4\alpha)\beta}, F = \frac{2\alpha\beta - \beta^2 - \sqrt{-24\alpha\beta^2 + 16\alpha^2\beta^2 + 16\alpha\beta^3 + \beta^4}}{-6\beta^2 + 3\alpha\beta^2 + 5\beta^3}\right\}, \{\alpha = 0.1\}\right]$$
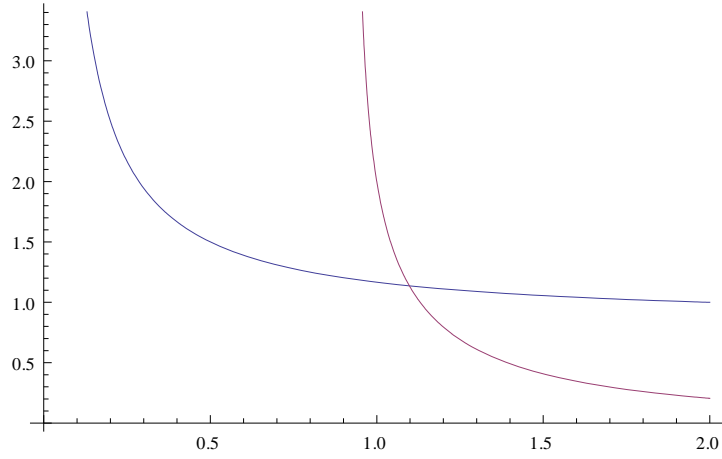

$$\text{Plot}\left[\left\{U = \frac{2(-3 + 4\alpha - 3\beta)}{(-9 + 4\alpha)\beta}, F = \frac{2\alpha\beta - \beta^2 - \sqrt{-24\alpha\beta^2 + 16\alpha^2\beta^2 + 16\alpha\beta^3 + \beta^4}}{-6\beta^2 + 3\alpha\beta^2 + 5\beta^3}\right\}, \{\alpha = 0.15\}\right]$$

Summary: From the firm's perspective γ is strategic complement to β, but from the union's perspective β is strategic substitute to γ at $\alpha < 0.2$

Reaction functions: **Fig. 5b**

At $\alpha = 0.2$, the Firm's reaction function completely lifts off the Union's reaction function and this implies that for $\alpha \geq 0.2$, we use the other root for the firm's reaction function.
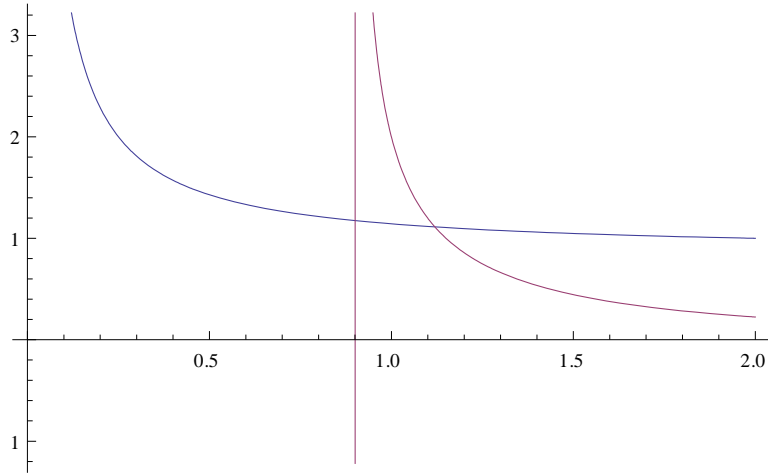
$$\text{Plot}\left[\left\{U = \frac{2(-3 + 4\alpha - 3\beta)}{(-9 + 4\alpha)\beta}, F = \frac{2\alpha\beta - \beta^2 - \sqrt{-24\alpha\beta^2 + 16\alpha^2\beta^2 + 16\alpha\beta^3 + \beta^4}}{-6\beta^2 + 3\alpha\beta^2 + 5\beta^3}\right\}, \{\alpha = 0.2\}\right]$$
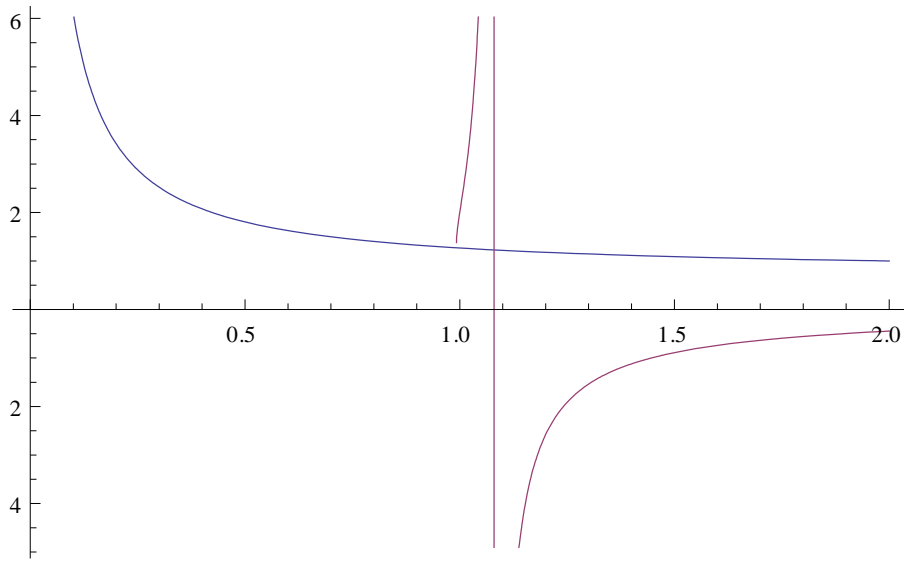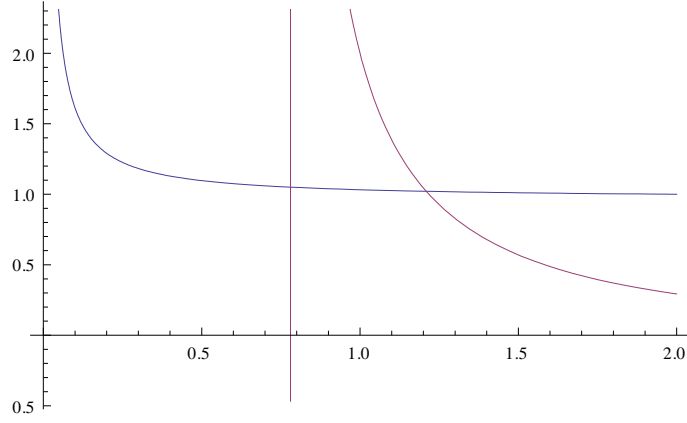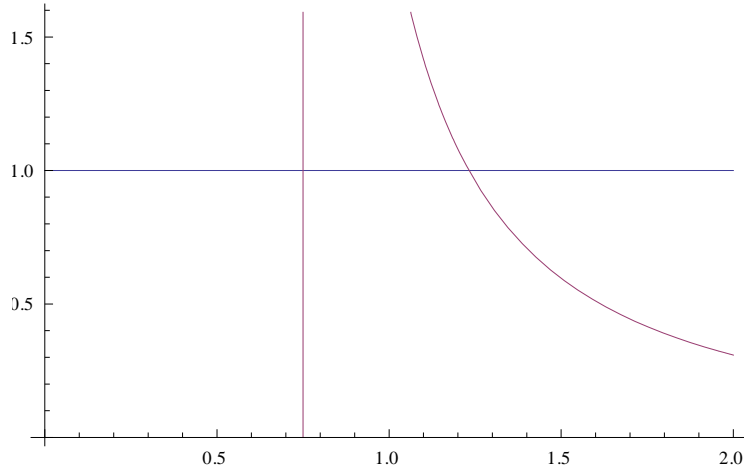

$$\text{Plot}\left[\left\{U = \frac{2(-3 + 4\alpha - 3\beta)}{(-9 + 4\alpha)\beta}, F = \frac{2\alpha\beta - \beta^2 + \sqrt{-24\alpha\beta^2 + 16\alpha^2\beta^2 + 16\alpha\beta^3 + \beta^4}}{-6\beta^2 + 3\alpha\beta^2 + 5\beta^3}\right\}, \{\alpha = 0.25\}\right]$$

Summary: Both γ and β are strategic substitutes.

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Reaction functions: **Fig. 5c**

$$\text{Plot}[\{U = \frac{2(-3+4\alpha-3\beta)}{(-9+4\alpha)\beta}, F = \frac{2\alpha\beta-\beta^2+\sqrt{-24\alpha\beta^2+16\alpha^2\beta^2+16\alpha\beta^3+\beta^4}}{-6\beta^2+3\alpha\beta^2+5\beta^3}\}, \{\alpha = 0.45\}]$$

$$\text{Plot}[\{U = \frac{2(-3+4\alpha-3\beta)}{(-9+4\alpha)\beta}, F = \frac{2\alpha\beta-\beta^2+\sqrt{-24\alpha\beta^2+16\alpha^2\beta^2+16\alpha\beta^3+\beta^4}}{-6\beta^2+3\alpha\beta^2+5\beta^3}\}, \{\alpha = 0.5\}]$$

Summary: Both γ and β are strategic substitutes.

Reaction functions: **Fig. 5d**

$$\text{Plot}\left[\left\{U = \frac{2(-3 + 4\alpha - 3\beta)}{(-9 + 4\alpha)\beta}, F = \frac{2\alpha\beta - \beta^2 + \sqrt{-24\alpha\beta^2 + 16\alpha^2\beta^2 + 16\alpha\beta^3 + \beta^4}}{-6\beta^2 + 3\alpha\beta^2 + 5\beta^3}\right\}, \{\alpha = 0.7\}\right]$$
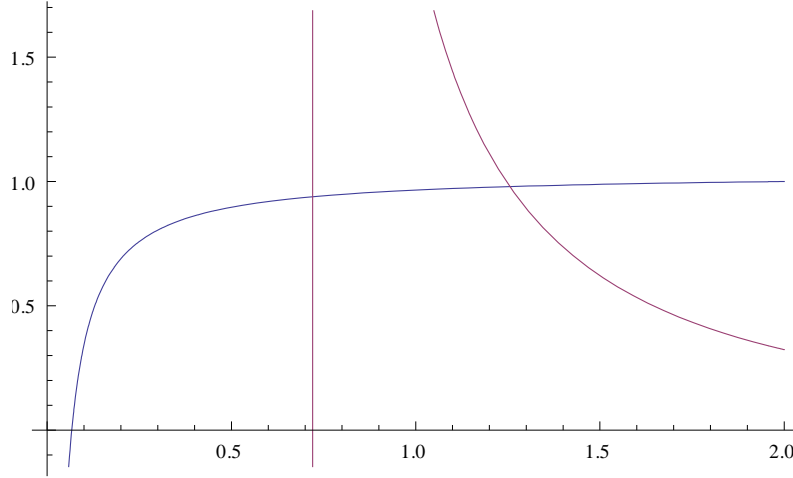

$$\text{Plot}\left[\left\{U = \frac{2(-3 + 4\alpha - 3\beta)}{(-9 + 4\alpha)\beta}, F = \frac{2\alpha\beta - \beta^2 + \sqrt{-24\alpha\beta^2 + 16\alpha^2\beta^2 + 16\alpha\beta^3 + \beta^4}}{-6\beta^2 + 3\alpha\beta^2 + 5\beta^3}\right\}, \{\alpha = 0.75\}\right]$$

Summary: The union's reaction function flattens out at $\gamma = 1$ when $\alpha = 0.75$.
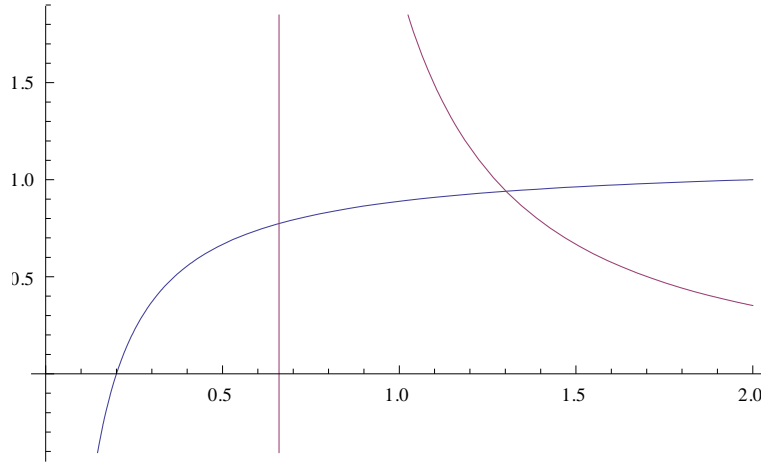
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Reaction functions: **Fig. 5e**

$$\text{Plot}[\{U = \frac{2(-3 + 4\alpha - 3\beta)}{(-9 + 4\alpha)\beta}, F = \frac{2\alpha\beta - \beta^2 + \sqrt{-24\alpha\beta^2 + 16\alpha^2\beta^2 + 16\alpha\beta^3 + \beta^4}}{-6\beta^2 + 3\alpha\beta^2 + 5\beta^3}\}, \{\alpha = 0.8\}]$$

$$\text{Plot}[\{U = \frac{2(-3 + 4\alpha - 3\beta)}{(-9 + 4\alpha)\beta}, F = \frac{2\alpha\beta - \beta^2 + \sqrt{-24\alpha\beta^2 + 16\alpha^2\beta^2 + 16\alpha\beta^3 + \beta^4}}{-6\beta^2 + 3\alpha\beta^2 + 5\beta^3}\}, \{\alpha = 0.9\}]$$

Summary: From the firm's perspective γ is strategic substitute for β, but from the union's perspective β is strategic complement to γ at $\alpha > 0.75$.

Table 5a: Payoff matrix for α=0.8

		Shareholders	
		Not delegate	Delegate
Union	Not delegate	0.0888, 0.0222	0.0406, 0.0844
	Delegate	0.0832, 0.0237	0.0367, 0.881

Outcome: Union does not delegate, Shareholders delegate.

Table 5b: Payoff matrix for α=0.5

		Shareholders	
		Not delegate	Delegate
Union	Not delegate	0.056, 0.056	0.0349, 0.0879
	Delegate	0.0816, 0.0408	0.057, 0.068

Outcome: Both parties delegate.

Table 5c: Payoff matrix for α=0.1

		Shareholders	
		Not delegate	Delegate
Union	Not delegate	0.0111, 0.10	0.0140, 0.0869
	Delegate	0.0757, 0.0487	0.0872, 0.0376

Outcome: Union delegates, Shareholders don't.

Unions delegate, Shareholders don't	Both delegate	Unions do not delegate, Shareholders do

```
_____|_____|_____ α
0          0.22                        0.75         1
```
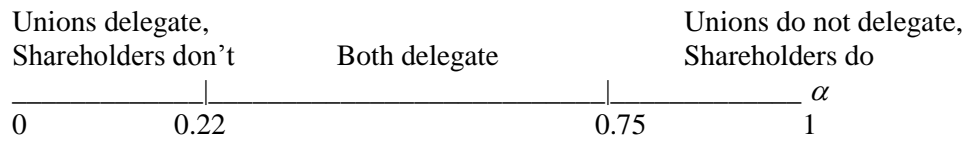
Fig. 10: Decision to delegate

Appendix

1. Proof of Lemma 1.

Following (5) write $\pi^0 = (1-\alpha)[s(l^0) - \theta l^0]$ and $\pi^* = (1-\alpha)[s(l^*) - \theta l^*] + \alpha s(l^*)\left[1 - \frac{1}{\beta}\right]$. By the definition of l^0, $[s(l^0) - \theta l^0] \geq [s(l^*) - \theta l^*]$. Therefore, for $\pi^* > \pi^0$ we must have $\beta > 1$. Next, write $\pi^* = s(l^*) - w^* l^*$ and $\pi^0 = s(l^0) - w^0 l^0$. Since we must have $\beta > 1$, $l^* < l^0$ and $s(l^*) < s(l^0)$. Therefore, if $\pi^* > \pi^0$ we must also have $w^* l^* < w^0 l^0$. ||

2. Proof of Proposition 1.

First we derive the second order conditions for stability of the Nash equilibrium.

Consider $\pi''(\beta)$. Write $\pi''(\beta) = \frac{1}{\beta^2}\left[\beta^2(\beta-1)\theta\gamma\frac{\partial^2 l}{\partial\beta^2} + [\theta\gamma\beta(3\beta-2) + \alpha s'(l)]\frac{\partial l}{\partial\beta}\right]$. Substituting (3) this can be further reduced to $\frac{\theta\gamma}{\beta^2}\left[\beta(\beta-1)\frac{\partial^2 l}{\partial\beta^2} + [(3\beta-2) + \alpha]\frac{\partial l}{\partial\beta}\right]$. Consider the bracketed term. Since $\beta > 1$ and $\frac{\partial l}{\partial\beta} < 0$, the second term is clearly negative. So we need either $\frac{\partial^2 l}{\partial\beta^2}$ sufficiently small, or non-positive. It is straight-forward to derive $\frac{\partial^2 l}{\partial\beta^2} = -\theta\frac{\partial l}{\partial\beta}\frac{s'''(l)}{(s'')^2}$ which is non-positive if $s'''(l) \leq 0$.

Next, we show $\frac{\partial\pi'(\beta)}{\partial\gamma} < 0$. $\frac{\partial\pi'(\beta)}{\partial\gamma} = \beta^2(\beta-1)\theta\frac{\partial l}{\partial\beta} + \beta^2(\beta-1)\theta\gamma\left[\frac{\theta}{s''(l)} - \theta\gamma\frac{s'''(l)}{(s''(l))^2}\frac{\partial l}{\partial\gamma}\right]$. Since $s'''(l) \leq 0$, this expression is negative. Hence $\frac{\partial\beta}{\partial\gamma} < 0$.

Now consider $u''(\gamma)$ and $\frac{\partial u'(\gamma)}{\partial\beta}$. $u''(\gamma) = \theta\left[(\gamma-1)\frac{\partial^2 l}{\partial\gamma^2} + (2-\alpha)\frac{\partial l}{\partial\gamma}\right]$. Following the same procedure as above we can establish that $\frac{\partial^2 l}{\partial\gamma^2} \leq 0$. Hence $u''(\gamma) < 0$. Finally, using the fact $\frac{\partial l}{\partial\gamma} = \frac{\partial l}{\partial\beta}\frac{\beta}{\gamma}$ we write $\frac{\partial u'(\gamma)}{\partial\beta} = \theta\beta\frac{(\gamma-1)}{\gamma}\frac{\partial^2 l}{\partial\beta^2} + \theta\frac{\partial l}{\partial\beta}\left[\frac{(\gamma-1)}{\gamma} + (1-\alpha)\right]$. Since we have already established that $\frac{\partial^2 l}{\partial\beta^2} \leq 0$, it is clear that $\frac{\partial u'(\gamma)}{\partial\beta} < 0$. Therefore, $\frac{\partial\gamma}{\partial\beta} < 0$.

Further we assume that the stability condition for the Nash equilibrium holds: $\Delta = \pi''(\beta)u''(\gamma) - \frac{\partial\pi'(.)}{\partial\gamma}\frac{\partial u'(.)}{\partial\beta} > 0$. Let $(\beta^*, \gamma*)$ be the Nash equilibrium incentives in the bilateral delegation game and assume that it is unique.

We now investigate how the incentive terms vary with the bargaining power of the

union. Differentiating Eqs. (7) and (8) we obtain[20]

$$\frac{\partial \beta}{\partial \alpha} = \frac{1}{\Delta}\left[-\frac{\partial \pi'(\beta)}{\partial \alpha}u''(\gamma) + \frac{\partial \pi'(\beta)}{\partial \gamma}\frac{\partial u'(\gamma)}{\partial \alpha}\right]$$

$$= -\frac{1}{\Delta}\left[\frac{s(l)}{\beta^2}u''(\gamma) + \theta l\frac{\partial \pi'(\beta)}{\partial \gamma}\right] > 0, \tag{29}$$

$$\frac{\partial \gamma}{\partial \alpha} = \frac{1}{\Delta}\left[-\frac{\partial u'(\gamma)}{\partial \alpha}\pi''(\beta) + \frac{\partial \pi'(\beta)}{\partial \alpha}\frac{\partial u'(\gamma)}{\partial \beta}\right]$$

$$= \frac{1}{\Delta}\left[\theta l\pi''(\beta) + \frac{s(l)}{\beta^2}\frac{\partial u'(\gamma)}{\partial \beta}\right] < 0, \tag{30}$$

Now set $\alpha = 0$ in Eq. (7). This yields $\beta = 1$. Alternatively set $\alpha = 1$ in Eq. (8). Clearly $\gamma = 1$. **Q.E.D.**

3. **The elasticity condition for the inverse relationship between w and β under monopoly.**

Write the first order condition for β as $\pi'(\beta) = [s'(l) - w]l'(\beta) - lw'(\beta) = 0$, which after substituting (3) reduces to:

$$[\beta\theta - w]l'(\beta - lw'(\beta) = 0.$$

From the above, one can obtain some bounds on the optimal β depending on the sign of $w'(\beta)$. That is, $\beta > \frac{w}{\theta} > 1$ if $w'(\beta) < 0$, and $1 < \beta < \frac{w}{\theta}$ if $w'(\beta) > 0$, and finally if $w'(\beta) = 0$, we have $\beta = \frac{w}{\theta} > 1$. However, it can be shown that under commonly used demand and production functions only the first possibility arises.

From (4) we derive

$$\frac{\partial w}{\partial \beta} = \frac{\alpha}{\beta^2}\frac{s}{l}\left[-1 + \{\frac{s'(l)}{s}l - 1\}\{\frac{\partial l}{\partial \beta\theta}\frac{\beta\theta}{l}\}\right],$$

$$= \frac{\alpha}{\beta^2}\frac{s}{l}\left[-1 + (1 - \epsilon)|\eta|\right],$$

where $\epsilon = \frac{s'(l)}{s}l$ is the employment elasticity of revenue, and $\eta = \frac{\partial l}{\partial(\beta\theta)}\frac{\beta\theta}{l}$ is the wage elasticity of labour demand. By the assumption of concavity of $s(.)$, $0 < \epsilon < 1$. As for η, we know it is negative but its magnitude can be greater or smaller than 1. Therefore, $w'(\beta) < 0$, if and only if $(1 - \epsilon)|\eta| < 1$.

---

[20]Note that $\frac{\partial \pi'(\beta)}{\partial \alpha} = \frac{s(l)}{\beta^2} > 0$ and $\frac{\partial u'(\gamma)}{\partial \alpha} = -\theta l < 0$.

The labour demand elasticity η can be expressed also in terms of the revenue function by substituting $\beta\theta = s'(.)$ and $\frac{\partial l}{\partial(\beta\theta)} = \frac{1}{s''(l)}$, which then yields $\eta = \frac{s'(l)}{s''(.)l}$, or the inverse of the elasticity of the marginal revenue. Thus, our critical condition for $w'(\beta) < 0$ reduces to

$$\frac{s'(l)l}{s(l)} + \frac{|s''(l)|l}{s(l)} > 1.$$

That is the sum of the elasticity of total revenue and the elasticity of marginal revenue must exceed 1.

It is easy to check that for standard demand (such as linear demand) and production functions (such as homogenous and concave) this condition is easily satisfied. For example, consider $p = a - bq$, and $q = kl^\gamma$ with $\gamma \le 1$. Here, $s(l) = k(al^\gamma - bl^{2\gamma})$ from which one obtains $s'(l) = k\gamma l^{\gamma-1}(a - 2bl^\gamma)$, $s''(l) = -k\gamma l^{\gamma-1}\left[(1-\gamma)\frac{(a-2bl^\gamma)}{l} + 2bl^{\gamma-1}\right]$. Utilizing these expressions write $\frac{s'(l)l}{s} = \gamma\frac{(a-2bl^\gamma)}{a-bl^\gamma}$, or $1 - \frac{s'(.)l}{s} = 1 - \gamma + \gamma\frac{bl^\gamma}{a-bl^\gamma}$, and $\frac{s''(.)l}{s'(l)} = -\left[1 - \gamma + \frac{2b\gamma l^\gamma}{a-2bl^\gamma}\right]$. Subsequently, $\frac{s'(.)l}{s} + \frac{|s''(.)|l}{s'(l)} > 1$ implies $\frac{|s''(.)|l}{s'(l)} > 1 - \frac{s'(.)l}{s}$, or equivalently $\gamma\frac{2bl^\gamma}{a-2bl^\gamma} > \gamma\frac{bl^\gamma}{a-bl^\gamma}$ which is indeed true. ||

4. **Proof of Proposition 2.** Since the proof is straight forward we discuss only the sufficiency part. The necessity part is omitted for economy.

1. Let the solution to the equation $s'(l) = \theta$ be written as $l(\theta)$. Given strict concavity of $s(l)$, the bargaining pie $P = s(l) - \theta l$ increases in l at all $l < l(\theta)$, and decreases in l at all $l > l(\theta)$. Now consider the cases of unilateral delegation. When only the firm delegates, employment is given by $s'(l) = \beta_u\theta$, and when only the union delegates employment is given by $s'(l) = \gamma_u\theta$ resulting in bargaining pies P_F and P_U respectively. If $\beta_u > (\le)\gamma_u$, we have $l(\beta_u\theta) < (\ge)l(\gamma_u\theta)$. Consequently $P_F < (\ge)P_U$.

2. In the bilateral case, employment is given by $s'(l) = \beta^*\gamma^*\theta$, and the pie is P^*. If $\beta^*\gamma^* \ge \max[\beta_u, \gamma_u]$, then $l(\beta^*\gamma^*\theta) \le \min[\beta_u, \gamma_u]$, and hence $P^* \le \min[P_F, P_U]$. **Q.E.D.**

5. **Proof of Proposition 4.** Suppose $\alpha \in (0, 1)$. From Eq. (12) we obtain

$$\beta^*\gamma^*\theta = a\sqrt{\frac{\alpha}{2(\beta^* - 1) + \alpha}}.$$

Contrast this with the case of unilateral delegation by the firm (where $\gamma = 1$, and we write $\beta = \beta_u$):

$$\beta_u\theta = a\sqrt{\frac{\alpha}{2(\beta_u - 1) + \alpha}}.$$

β_u is the optimal response to $\gamma = 1$, and β^* is the optimal response to $\gamma^* > 1$. Since, β and γ are strategic substitutes, it must be that $\beta_u > \beta^*$. Hence, $\beta^*\gamma^*\theta > \beta_u\theta$.

Now consider Eq. (13). Rewrite it as

$$\beta^*\gamma^*\theta = \frac{(1-\alpha)a + \beta^*\theta}{2-\alpha}.$$

Contrast this with the case of unilateral delegation by the union. Set $\beta = 1$ and $\gamma = \gamma_u$ in the above and it is clearly smaller. Hence, we can say $\beta^*\gamma^*\theta > \gamma_u\theta$. This allows us to conclude that for any $\alpha \in (0, 1)$, $\beta^*\gamma^* > \max[\beta_u, \gamma_u]$ and hence, $P^* < \min[P_U, P_F]$.

Finally consider $\alpha = 0$ and $\alpha = 1$. At $\alpha = 0$, $\beta^* = \beta_u = 1$ and $\gamma^* = \gamma_u > 1$. Therefore, $P^* = P_U$. Alternatively, when $\alpha = 1$, $\beta_u > 1$ and $\gamma^* = \gamma_u = 1$. Hence, $P^* = P_F$.

Combining all these observations we write, $P^* \leq \min[P_F, P_U]$. **Q.E.D.**

6. **Proof of Lemma 2.**

Since l_i is inversely related to $\beta_i\gamma_i$ and positively related to $\beta_j\gamma_j$ the inequalities on l_i and l_j immediately follow. For other inequalities, note when $\beta_i\gamma_i < 1$, $s_i^{*0} = s_i(l_i^*, l_j^0) > s_i^0 = s_i(l_i^0, l_j^0)$ because $l_i^* > l_i^0$ and $s_i(.)$ is increasing in l_i by the implication of (14). Next, $s_i^* = s_i(l_i^*, l_j^*) > s_i^{*0} = s_i(l_i^*, l_j^0)$, because $l_j^* < l_j^0$ and $s_i(.)$ is inversely related to l_j. The converse part analogously follows. **Q.E.D.**

7. **Proof of Proposition 5.**

Suppose $\beta_j \geq 0$ and (γ_i, γ_j) is given. Recall $\pi_i^0 = s_i^0 - w_i^0 l_i^0$ which can be rewritten utilizing (15) as $\pi_i^0 = (1-\alpha)(s_i^0 - \theta l_i^0)$. Now consider π_i^*. In the expression of π_i^* as given in (16) suppose $\beta_i < \alpha$. Since $\alpha < 1$ we will have $s_i^* > s_i^0$, but $(1 - \frac{\alpha}{\beta_i}) < 0$. So the second term in (16) will be negative. Now consider the first term. There are two terms inside the bracket. The second term inside the bracket is clearly negative. So consider the first term (inside the bracket): $[s_i^{*0} - \theta l_i^*] = [s_i(l_i^*, l_j^0) - \theta l_i^*]$. This must exceed π^0, if $\pi_i^* > \pi_i^0$ is to hold. But recall that $\pi^0 = [s_i(l_i^0, l_j^0) - \theta l_i^0]$ is the maximum value of $[s_i(l_i, l_j^0) - \theta l_i]$ by the definition of l_i^0 and therefore $[s_i^{*0} - \theta l_i^*] = [s_i(l_i^*, l_j^0) - \theta l_i^*] < [s_i(l_i^0, l_j^0) - \theta l_i^0]$. Therefore, if $\beta_i < \alpha$, π_i^* will be strictly less than π_i^0 for any given $\beta_j \geq 0$. It can be easily checked that with $\beta_i > \alpha$, $\pi_i^* > \pi_i^0$ can hold at $\beta_i > 1$ or $\beta_i < 1$.

Now ignore the bargaining effect and restrict our attention only to strategic effects. Consider two special cases: $\alpha = 0$ and $\alpha = 1$. First consider $\alpha = 0$. From Eq. (17) we see that for $u_i^* > 0$ γ_i must exceed 1. From Eq. (16) we see that $\pi_i^0 = s_i^0 - \theta l_i^0$

and $\pi_i^* = [s_i^{*0} - \theta l_i^*] - \theta l_i^*(\gamma_i - 1) + [s_i^* - s_i^{*0}]$. But since by definition l_i^0 maximizes $[s_i(l_i, l_j^0) - \theta l_i]$, $[s_i^{*0} - \theta l_i^*] = [s_i(l_i^*, l_j^0) - \theta l_i^*] < [s_i(l_i, l_j^0) - \theta l_i] = \pi_i^0$. Therefore, for $\pi_i^* > \pi_i^0$ we must have $s_i^* > s_i^{*o}$. By Lemma (2) this is possible if $\beta_i \gamma_i < 1$, or $\beta_i < 1/\gamma_i$.

Similarly, consider $\alpha = 1$. From Eq. (16) we obtain $\pi_i^* = s_i^*(1 - \frac{1}{\beta})$, which is strictly positive if and only if $\beta_i > 1$, and since $\pi_i^0 = 0$ at $\alpha = 1$, $\pi_i^* > \pi_i^0$ if and only if $\beta_i > 1$. From Eq. (17) we see that $u_i^* = [\frac{s_i^{*0}}{\beta_i} - \theta l_i^*] + (s_i^* - s_i^{*0})\frac{1}{\beta}$. The union can improve its payoff by setting $\gamma_i < 1/\beta_i$ and making $s_i^* > s_i^{*0}$. **Q.E.D.**

8. Proof of Proposition 6.

By definition β^* is the symmetric subgame perfect equilibrium incentive scheme, which is obvious from the preceding discussion. To see that $\beta^S < \beta^*$ consider (18). Set $\alpha = 0$ and solve for β; this gives β^S. Clearly as $\frac{\partial s_i}{\partial l_j} < 0, \frac{\partial l_j}{\partial \beta_i} > 0$ and $\frac{\partial l_i}{\partial \beta_i} < 0$, we must have $\beta^S < 1$. Now plug back β^S in (18), but revert back to $\alpha > 0$. The first (bracketed) term is clearly zero at β^S, but the second (bracketed) term is strictly positive. Hence, for equation (18) to hold, β must be increased above β^S. Since by definition β^* satisfies (18), $\beta^* > \beta^S$.

Now for the upper bound on β^* set $\frac{\partial l_j}{\partial \beta_i} = 0$ in equation (18) so that there is no strategic effect, and solve for β. This gives β^B. As $\alpha s_i > 0$ and $\frac{\partial l_i}{\partial \beta_i} < 0$, β^B must exceed 1. Now reconsider (18) reverting back to $\frac{\partial l_j}{\partial \beta_i} > 0$ and substitute $\beta = \beta^B$. The first (bracketed) term is then zero. The second term is negative due to the facts that $\frac{\partial s_i}{\partial l_j} <$ and $\beta^B > \alpha$. Hence, for Eq. (18) to hold β must be reduced below β^B. Since β^* solves (18), we must have $\beta^* < \beta^B$.

Finally, for the comparative statics with respect to α one derives from the first order condition of firm i and j,

$$\frac{\partial^2 \pi_i}{\partial \beta_i^2}\beta_i^{*'}(\alpha) + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j}\beta_j^{*'}(\alpha) + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \alpha} = 0, \tag{31}$$

$$\frac{\partial^2 \pi_j}{\partial \beta_j \partial \beta_i}\beta_i^{*'}(\alpha) + \frac{\partial^2 \pi_j}{\partial \beta_j^2}\beta_j^{*'}(\alpha) + \frac{\partial^2 \pi_i}{\partial \beta_j \partial \alpha} = 0.$$

From (18) derive $\frac{\partial^2 \pi_i}{\partial \beta_i \partial \alpha} = \left[s_i - \frac{\partial s_i}{\partial l_j}\frac{\partial l_j}{\partial \beta_i}\beta_i\right] > 0$ for $i, j, i \neq j$.

By the second order condition for profit maximization $\frac{\partial^2 \pi_i}{\partial \beta_i^2} < 0$ for both i, j. By the Cournot stability condition $\frac{\partial^2 \pi_i}{\partial \beta_i^2}\frac{\partial^2 \pi_j}{\partial \beta_j^2} > \frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j}\frac{\partial^2 \pi_j}{\partial \beta_j \partial \beta_i}$. By symmetry $\frac{\partial^2 \pi_i}{\partial \beta_i^2} = \frac{\partial^2 \pi_j}{\partial \beta_j^2}$ and $\frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j} = \frac{\partial^2 \pi_j}{\partial \beta_j \partial \beta_i}$. Therefore, the stability condition implies $|\frac{\partial^2 \pi_i}{\partial \beta_i^2}| > |\frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j}|$, which in turn yields $\frac{\partial^2 \pi_i}{\partial \beta_i^2} + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j} < 0$.

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Now consider (31). By symmetry $\beta_i^{*\prime}(\alpha) = \beta_j^{*\prime}(\alpha) = \beta^{*\prime}(\alpha)$, which can be obtained from (31) as

$$\beta^{*\prime}(\alpha) = -\frac{\frac{\partial^2 \pi_i}{\partial \beta_i \partial \alpha}}{\frac{\partial^2 \pi_i}{\partial \beta_i^2} + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j}} = -\frac{\left[\frac{s_i}{\beta_i^2} - \frac{\partial s_i}{\partial l_j}\frac{\partial l_j}{\partial \beta_i}\frac{1}{\beta_i}\right]}{\frac{\partial^2 \pi_i}{\partial \beta_i^2} + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j}} > 0.$$

At $\alpha = 0$, $\beta^* = \beta^S < 1$. At $\alpha = 1$, $\beta^* = \beta^B > 1$. Therefore, as β^* is continuous and increasing in α, there exists a unique α, namely $\hat\alpha$, at which $\beta^* = 1$. At all $\alpha > \hat\alpha$ it is obvious that $\beta^* > 1$ and at all $\alpha < \hat\alpha$, $\beta^* < 1$ due to the monotonicity of β^*. **Q.E.D.**

9. **Proof of Proposition 7.** The proof is analogous to the proof of Proposition 6 and it involves the use of Eq. (19). Fix $\beta_i = 1$ in (19) for $i = 1, 2$. Then by setting $\alpha = 0$ and $\alpha = 1$ obatin $\gamma^B > 1$ and $\gamma^S < 1$ respectively. To show that $\gamma^S \le \gamma^* \le \gamma^B$ apply the same reasoning as in the proof of Proposition 6. Next, by the continuity of γ there must exist a critical α $(\tilde\alpha)$ such that $\gamma^*(\tilde\alpha) = 0$. That $\tilde\alpha$ is unique follows from the fact that $\partial \gamma^* / \partial \alpha < 0$, which we establish below.

Using symmetry we derive from Eq. (19) for $i = 1, 2$ and $i \ne j$,

$$\gamma^{*\prime}(\alpha) = -\frac{\frac{\partial^2 u_i}{\partial \gamma_i \partial \alpha}}{\frac{\partial^2 u_i}{\partial \gamma_i^2} + \frac{\partial^2 u_i}{\partial \gamma_i \partial \gamma_j}} = \frac{\left[\theta l_i - \frac{\partial s_i}{\partial l_j}\frac{\partial l_j}{\partial \gamma_i}\frac{1}{\beta_i}\right]}{\frac{\partial^2 u_i}{\partial \gamma_i^2} + \frac{\partial^2 u_i}{\partial \gamma_i \partial \gamma_j}} < 0.$$

Since $\partial \gamma^* / \partial \alpha < 0$ and $\gamma^*(\alpha = 0) = \gamma^B > 1$ and $\gamma^*(\alpha = 1) = \gamma^S < 1$, $\tilde\gamma$ must be unique, and at all $\gamma < (>)\tilde\gamma$, $\gamma^* > (<)1$. **Q.E.D.**

10. **Second order condition for bilateral delegation under duopoly.**

The second order condition involves

$$\frac{\partial^2 \pi_i}{\partial \beta_i^2} < 0, \quad \frac{\partial^2 u_i}{\partial \gamma_i^2} < 0, \qquad \text{for all} i = 1, 2;$$

and the following matrix is negative definite.

$$\begin{bmatrix} \frac{\partial^2 \pi_1}{\partial \beta_1^2} & \frac{\partial^2 \pi_1}{\partial \beta_1 \partial \beta_2} & \frac{\partial^2 \pi_1}{\partial \beta_1 \partial \gamma_1} & \frac{\partial^2 \pi_1}{\partial \beta_1 \partial \gamma_2} \\ \frac{\partial^2 \pi_2}{\partial \beta_2 \partial \beta_1} & \frac{\partial^2 \pi_2}{\partial \beta_2^2} & \frac{\partial^2 \pi_2}{\partial \beta_2 \partial \gamma_1} & \frac{\partial^2 \pi_2}{\partial \beta_2 \partial \gamma_2} \\ \frac{\partial^2 u_1}{\partial \gamma_1 \partial \beta_1} & \frac{\partial^2 u_1}{\partial \gamma_1 \partial \beta_2} & \frac{\partial^2 u_1}{\partial \gamma_1^2} & \frac{\partial^2 u_1}{\partial \gamma_1 \partial \gamma_2} \\ \frac{\partial^2 u_2}{\partial \gamma_2 \partial \beta_1} & \frac{\partial^2 u_2}{\partial \gamma_2 \partial \beta_2} & \frac{\partial^2 u_2}{\partial \gamma_2 \partial \gamma_1} & \frac{\partial^2 u_2}{\partial \gamma_2^2} \end{bmatrix}$$

11. **Proof of Proposition 8.**

(i) Suppose $((\beta_1, \gamma_1), (\beta_2, \gamma_2))$ is a Nash equilibrium and β_i and γ_i are continuous functions of α; hence they satisfy Eqs. (18) and (19) for $i = 1, 2$. Set $\alpha = 0$ in (18) and (19). For (18) to hold we must have $\beta_i < 1$ (since $\partial l_i / \partial \beta_i < 0$) and for (19) to hold we must also have $\gamma_i > 1$ since $\partial l_i / \partial \gamma_i < 0$. Then by continuity of β_i and γ_i, at all sufficiently small α we will also have $\beta_i < 1$ and $\gamma_i > 1$.

Next, set $\alpha = 1$ in Eq. (19) from which we get $\gamma_i < 1$. From the expression of profit it becomes clear that $\pi_i = s_i(.)(1 - \frac{1}{\beta_i}) > 0$ only if $\beta_i > 1$. Again by continuity the same result will hold at sufficiently high values of α close to 1.

(ii) Consider

$$\pi_i = P_i - u_i = (s_i - \theta l_i) - \left[\frac{\alpha s_i}{\beta_i} - (1 - (1 - \alpha)\gamma_i)\theta l_i \right].$$

Maximize π_i with respect to β_i to obtain

$$\frac{\partial \pi_i}{\partial \beta_i} = (\beta_i \gamma_i - 1)\theta \frac{\partial l_i}{\partial \beta_i} + \frac{\alpha s_i}{\beta_i^2} + \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial (\beta_i \gamma_i)} \gamma_i \left(1 - \frac{\alpha}{\beta_i} \right) - \theta \gamma_i (\gamma_i - 1) \frac{\partial l_i}{\partial (\beta_i \gamma_i)} = 0. \quad (32)$$

Now noting that $\frac{\partial l_i}{\partial \gamma_i} = \frac{\partial l_i}{\partial (\beta_i \gamma_i)} \beta_i$ we write out the first order condition of the union's optimization condition (19) as:

$$\frac{\partial u_i}{\partial \gamma_i} = \theta \beta_i (\gamma_i - 1) \frac{\partial l_i}{\partial (\beta_i \gamma_i)} + (1 - \alpha)\theta l_i + \alpha \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial (\beta_i \gamma_i)} = 0$$

$$\text{or} \qquad \theta(\gamma_i - 1) \frac{\partial l_i}{\partial (\beta_i \gamma_i)} = -\frac{(1 - \alpha)}{\beta_i} \theta l_i - \frac{\alpha}{\beta_i} \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial (\beta_i \gamma_i)} \qquad (33)$$

Now substitute (33) into (32) and simplify the expression to obtain

$$\begin{aligned} \frac{\partial \pi_i}{\partial \beta_i} &= (\beta_i \gamma_i - 1)\theta \frac{\partial l_i}{\partial \beta_i} + \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i} + \frac{1}{\beta_i} \left[\frac{\alpha}{\beta_i} s_i + (1 - \alpha)\theta \gamma_i l_i \right] = 0 \\ &= (\beta_i \gamma_i - 1)\theta \frac{\partial l_i}{\partial \beta_i} + \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i} + \frac{w_i l_i}{\beta_i} \\ &= (\beta_i \gamma_i - 1)\theta \frac{\partial l_i}{\partial \beta_i} + \frac{l_i}{\beta_i} \left[\left(\frac{\partial s_i}{\partial l_j} \frac{l_j}{s_i} \right) \left(\frac{\partial l_j}{\partial \beta_i} \frac{\beta_i}{l_j} \right) \frac{s_i}{l_i} + w_i \right] \\ &= (\beta_i \gamma_i - 1)\theta \frac{\partial l_i}{\partial \beta_i} + \frac{l_i}{\beta_i} \left[\eta_{ij}^s \eta_{ij}^l \frac{s_i}{l_i} + w_i \right] = 0. \quad (34) \end{aligned}$$

Since $\eta_{ij}^s < 0$, $\eta_{ij}^l > 0$ $\frac{\partial l_i}{\partial \beta_i} < 0$, it is clear that $\beta_i \gamma_i > 1$ if and only if $w_i > |\eta_{ij}^s \eta_{ij}^l| \frac{s_i}{l_i}$.

Q.E.D.

12. **Derivation of $\beta'(\alpha)$ and $\gamma'(\alpha)$.**

Assume symmetry: $\beta_1 = \beta_2 = \beta$ and $\gamma_1 = \gamma_2 = \gamma$. Symmetry implies the following equalities:

$$\frac{\partial^2 \pi_1}{\partial \beta_1^2} = \frac{\partial^2 \pi_2}{\partial \beta_2^2} \quad \frac{\partial^2 \pi_1}{\partial \beta_1 \partial \beta_2} = \frac{\partial^2 \pi_2}{\partial \beta_2 \partial \beta_1}$$

$$\frac{\partial^2 \pi_1}{\partial \beta_1 \partial \gamma_1} = \frac{\partial^2 \pi_2}{\partial \beta_2 \partial \gamma_2} \quad \frac{\partial^2 \pi_1}{\partial \beta_1 \partial \gamma_2} = \frac{\partial^2 \pi_2}{\partial \beta_2 \partial \gamma_1}$$

$$\frac{\partial^2 u_1}{\partial \gamma_1 \partial \beta_1} = \frac{\partial^2 u_2}{\partial \gamma_2 \partial \beta_2} \quad \frac{\partial^2 u_1}{\partial \gamma_1 \partial \beta_2} = \frac{\partial^2 u_2}{\partial \gamma_2 \partial \beta_1}$$

$$\frac{\partial^2 u_1}{\partial \gamma_1^2} = \frac{\partial^2 u_2}{\partial \gamma_2^2} \quad \frac{\partial^2 u_1}{\partial \gamma_1 \partial \gamma_2} = \frac{\partial^2 u_2}{\partial \gamma_2 \partial \gamma_1}$$

With these symmetric relations we can derive $\beta'(\alpha)$ and $\gamma'(\alpha)$ from the following two equations (which are obtained by totally differentiating Eqs. (18) and (19) for $i = 1, 2$):

$$A_1 \beta'(\alpha) + A_2 \gamma'(\alpha) = -\frac{\partial^2 \pi_i}{\partial \beta_i \partial \alpha}$$

$$A_3 \beta'(\alpha) + A_4 \gamma'(\alpha) = -\frac{\partial^2 u_i}{\partial \gamma_i \partial \alpha}$$

where for $i \neq j$ and $i = 1, 2$,

$$A_1 = \frac{\partial^2 \pi_i}{\partial \beta_i^2} + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \beta_j} \quad A_2 = \frac{\partial^2 \pi_i}{\partial \beta_i \partial \gamma_i} + \frac{\partial^2 \pi_i}{\partial \beta_i \partial \gamma_j}$$

$$A_3 = \frac{\partial^2 u_i}{\partial \gamma_i \partial \beta_i} + \frac{\partial^2 u_i}{\partial \gamma_i \partial \beta_j} \quad A_4 = \frac{\partial^2 u_i}{\partial \gamma_i^2} + \frac{\partial^2 u_i}{\partial \gamma_i \partial \gamma_j}.$$

By the second order condition we have $A_1 < 0, A_4 < 0, A_1 A_4 - A_2 A_3 > 0$. Further, if γ is strategic substitute (complement) to β, we will have $A_2 < 0(> 0)$, and if β is strategic substitute (complement) to γ, we will have $A_3 < 0(> 0)$.

Also note that

$$\frac{\partial^2 \pi_i}{\partial \beta_i \partial \alpha} = \frac{s_i}{\beta_i^2} - \frac{1}{\beta_i} \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \beta_i} > 0$$

$$\frac{\partial^2 u_i}{\partial \gamma_i \partial \alpha} = -\theta l_i + \frac{1}{\beta_i} \frac{\partial s_i}{\partial l_j} \frac{\partial l_j}{\partial \gamma_i} < 0.$$

So we get

$$\beta'(\alpha) = \frac{1}{A_1 A_4 - A_2 A_3}\left[-A_4\frac{\partial^2 \pi_i}{\partial \beta_i \partial \alpha} + A_2\frac{\partial^2 u_i}{\partial \gamma_i \partial \alpha}\right]$$

$$\gamma'(\alpha) = \frac{1}{A_1 A_4 - A_2 A_3}\left[-A_1\frac{\partial^2 u_i}{\partial \gamma_i \partial \alpha} + A_3\frac{\partial^2 \pi_i}{\partial \beta_i \partial \alpha}+\right].$$

So if $A_2 < 0 A_3 < 0$, we have unambiguously $\beta'(\alpha) > 0$ and $\gamma'(\alpha) < 0$; however the same signs can be obtained even if $A_2 > 0$ and/or $A_3 > 0$.

13. **Proof of Lemma 3.**

Write

$$y = \left[(\beta_i\gamma_i - 1)\theta + \frac{\partial s_i}{\partial l_j}\right]$$

$$y'(\alpha) = \frac{\partial(\beta_i\gamma_i)}{\partial\alpha}\left[\theta + \underbrace{\left\{\frac{\partial s_i^2}{\partial l_j l_i} + \frac{\partial s_i^2}{\partial l_j^2}\right\}}_{<0}\underbrace{\left\{\frac{\partial l_i}{\partial(\beta_i\gamma_i)} + \frac{\partial l_i}{\partial(\beta_j\gamma_j)}\right\}}_{<0}\right].$$

In the above, we have used the assumption of symmetry to write $\frac{\partial(\beta_i\gamma_i)}{\partial\alpha} = \frac{\partial(\beta_i\gamma_i)}{\partial\alpha}$ and $\frac{\partial l_i}{\partial(\beta_i\gamma_i)} = \frac{\partial l_j}{\partial(\beta_j\gamma_j)}, \frac{\partial l_j}{\partial(\beta_i\gamma_i)} = \frac{\partial l_i}{\partial(\beta_j\gamma_j)}$.

Since the bracketed term is positive, the sign of $y'(\alpha)$ is given by the sign of $\partial(\beta_i\gamma_i)/\partial\alpha$.

Q.E.D.

14. **Proof of Proposition 9.**

1. Set $\gamma_i = 1$ in Eq. (21) and write $y_\beta = \left[(\beta_i - 1)\theta + \frac{\partial s_i}{\partial l_j}\right]$. From Proposition 6 we know that $\partial\beta_i/\partial\alpha > 0$. Further, as $\beta_i(\alpha = 0) < 1$ (already established in Proposition 6) $y_\beta(\alpha = 0) < 0$. Since by assumption $y_\beta(\alpha = 1) > 0$ and by Lemma 3 $y'_\beta(\alpha) > 0$, by the intermediate value theorem , there must exist a critical α, say α_0, at which $y_\beta(.) = 0$ and $y_\beta(\alpha) < (>)0$ for $\alpha < (>)\alpha_0$. The uniqueness of α_0 follows from the fact that β_i is monotonic in α and thus $l_i(\beta_i)$ is also monotonic in α.

2. The reasoning is analogous to (i) except that set $\beta_i = 1$ in Eq. (21) and analogously define $y_\gamma = \left[(\gamma_i - 1)\theta + \frac{\partial s_i}{\partial l_j}\right]$ one needs to use $\partial\gamma_i/\partial\alpha < 0$ from Proposition 7.

3. *Only if:* Suppose $\alpha_1 < \alpha_0$. By definition $y_\gamma = 0$ at $\alpha = \alpha_1$, and $y_\beta = 0$ at $\alpha = \alpha_0$. From Lemma 3 we know that the sign of $y'_\beta(\alpha)$ is given by the sign of $\beta'(\alpha)$ which has

been shown to be positive in Proposition 7. Therefore, $y'_\beta(\alpha) > 0$. Hence, we must have $y_\beta(\alpha_1) < 0$.

Now consider y_γ first: $y_\gamma = (\gamma_i - 1)\theta + \frac{\partial s_i}{\partial l_j}$. Let us denote $\frac{\partial s_i}{\partial l_j} \equiv -\phi(l(\gamma_i))$. Because of symmetry and Assumption 1, we have $\frac{\partial^2 s_i}{\partial l_i \partial l_j} < 0$, which implies $\phi(l(.))$ must be an increasing function of $l(.)$. Thus, $y_\gamma(\alpha_1) = (\gamma_i - 1)\theta - \phi(l(\gamma_i)) = 0$.

Similarly, we write $y_\beta(\alpha_1) = (\beta_i - 1)\theta - \phi(l(\beta_i)) < 0$. From $y_\beta(\alpha_1) < y_\gamma(\alpha_1)$ we obtain $(\beta_i - \gamma_i)\theta < \phi(l(\beta_i)) - \phi(l(\gamma_i))$.

Suppose the right hand side of this inequality is positive, i.e. $\phi(l(\beta_i)) > \phi(l(\gamma_i))$ which implies $l(\beta_i) > l(\gamma_i)$. But this is possible only if $\beta_i < \gamma_i$ at $\alpha = \alpha_1$, which is precisely the condition we have specified. Alternatively, if the right hand side is negative, i.e. $\phi(l(\beta_i)) < \phi(l(\gamma_i))$, we get $l(\beta_i) < l(\gamma_i)$ which is possible only if $\beta_i > \gamma_i$. Then in the above inequality, the left hand side is positive, but the right hand side is negative, a contradiction to the fact $y_\beta(\alpha_1) < 0 = y_\gamma(\alpha_1)$, and in turn a contradiction to our premise $\alpha_1 < \alpha_0$.

If: Suppose at α_1 we have $\beta_i < \gamma_i$, which then implies $l(\beta_i) > l(\gamma_i)$ and in turn $\phi(l(\beta_i)) > \phi(l(\gamma_i))$, or equivalently $-\phi(l(\beta_i)) < -\phi(l(\gamma_i))$. Since $\beta_i < \gamma_i$ we can add $(\beta_i - 1)\theta$ to the left hand side and $(\gamma_i - 1)\theta$ to the right hand side and maintain the strict inequality as $(\beta_i - 1)\theta - \phi(l(\beta_i)) < (\gamma_i - 1)\theta - \phi(l(\gamma_i))$. That is to say, $y_\beta(\alpha_1) < y_\gamma(\alpha_1)$. Since $y_{\gamma_1}(\alpha_1) = 0$ we must have $y_\beta(\alpha_1) < 0$. Further, by Lemma 3 and Proposition 7 (which we have already noted above) y_β is strictly increasing in α; hence, $y_\beta = 0$ at some $\alpha > \alpha_1$, and that is what precisely α_0 is. **Q.E.D.**

15. **Proof of Proposition 10.** Suppose $\beta\gamma(\alpha = 0) > k$ and $\beta\gamma(\alpha = 1) < k$. Then by intermediate value theorem (since $\beta\gamma(\alpha)$ is continuous in α) there must exist at least one value of $\alpha \in (0, 1)$, say $\hat{\alpha}$, at which $\beta\gamma = k$. Further, if $\partial(\beta\gamma)/\partial\alpha < 0$, $\hat{\alpha}$ must be unique. Further, from Footnote 19 we know that when $P'_i(\alpha) = 0$, $P''_i(\alpha) < 0$. Hence, at $\hat{\alpha}$ we have a maxium of $P(\alpha)$. Then it immediately follows that at all $\alpha < \hat{\alpha}$, $P'(\alpha) > 0$ and at all $\alpha > \hat{\alpha}$, $P'(\alpha) < 0$. **Q.E.D.**

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