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Characterising competitive equilibrium in terms of opportunity

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Characterising competitive equilibrium in terms of opportunity

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1. Introduction

Normative economics has traditionally assumed that individuals have stable and context-independent preferences over all economically relevant outcomes, and that these preferences can be inferred from individuals' decisions. The satisfaction of these assumed preferences has then been used as the criterion against which policy alternatives are evaluated. This methodology is now being challenged by the findings of research in behavioural economics. Behavioural research has uncovered many systematic patterns in individuals' choices which, although explicable in terms of known psychological mechanisms, are not consistent with traditional assumptions about the coherence of revealed preferences. The most common response to this problem has been to model observed deviations from the conventional theory of rational choice as errors induced by imperfections in people's reasoning or self-control, and to try to recover the latent preferences that would have been revealed in the absence of such imperfections. However, this 'preference purification' approach can resolve the problem only if an operational method for identifying latent preferences can be developed, and then only if latent preferences turn out to be coherent.[1] Since to date there has been only limited progress in developing such a methodology, and since it is by no means self-evident that coherent latent preferences exist, it seems prudent to explore other ways of reconciling behavioural and normative economics.

The opportunity-based approach proposed by Sugden (2004) and McQuillin and Sugden (2012) follows a radically different reconciliation strategy. The essential idea is to use opportunity rather than preference-satisfaction as the normative criterion, and to assess the extent of each individual's opportunities without reference to his or her preferences. The fundamental normative intuition is that, whatever individuals' preferences happen to be and whether or not those preferences are stable or context-independent, it is good that individuals have opportunities to make any feasible transactions that might be voluntarily chosen. This intuition is expressed in an 'Opportunity Criterion' which is formulated in slightly different

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[1] Different variants of the preference purification approach have been proposed by Bleichrodt, Pinto-Prades and Wakker (2001), Camerer et al. (2003), Sunstein and Thaler (2003), Kőszegi and Rabin (2007), Salant and Rubinstein (2008), Thaler and Sunstein (2008), Bernheim and Rangel (2009), and Rabin (2013). The common features of this approach are examined by Infante, Lecouteux and Sugden (2015), who argue that the elimination of reasoning errors will not necessarily generate coherent preferences.

ways in the two papers; I will use the formulation from McQuillin and Sugden (2012).[2] Both papers present opportunity-based analogues of the first fundamental theorem of welfare economics, proving that the Opportunity Criterion is satisfied by every competitive equilibrium of an exchange economy (that is, by every profile of market-clearing prices that satisfies the law of one price). McQuillin and Sugden (2012) generalise this result to 'storage economies' in which consumption and exchange take place over a series of time periods and in which individuals' preferences may be dynamically inconsistent. Sugden (2004) shows that competitive equilibrium can be induced by the interaction of profit-seeking arbitrageurs, even if the individuals whose exchanges are being mediated do not act on coherent preferences.

The present paper extends these results in another direction, by asking whether the profile of individual opportunities provided by competitive equilibrium is the *only* way of satisfying the Opportunity Criterion. It is well known that competitive equilibrium is a sufficient but not a necessary condition for Pareto optimality, but that in the limit as the size of an exchange economy increases, the set of core allocations shrinks to the set of competitive equilibria (Edgeworth, 1881/1967, Debreu and Scarf, 1963). I will prove an analogous theorem about the relationship between competitive equilibrium and the Opportunity Criterion. More precisely, I will model alternative 'regimes' (that is, profiles of opportunity sets that are made available to individuals) for an exchange economy. I will define a Strong Opportunity Criterion that implies but is not implied by McQuillin and Sugden's Opportunity Criterion. I will define a 'market-clearing single-price regime' which corresponds closely with the concept of competitive equilibrium. I will show that every market-clearing single-price regime satisfies the Strong Opportunity Criterion. And I will show that, in the limit as an economy increases in size, the set of regimes that satisfy the Strong Opportunity Criterion shrinks to the set of market-clearing single-price regimes. In this sense, competitive equilibrium can be characterised in terms of opportunity.

2. The Opportunity Criterion

I begin by formulating the Opportunity Criterion in relation to the problem of evaluating alternative regimes for an exchange economy.

[2] For details about the differences between the two formulations, and reasons for preferring the later one, see McQuillin and Sugden (2012).

An *exchange economy* is defined by a nonempty set I of *individuals* $i = 1, ..., n$, a set G of two or more infinitely-divisible *goods* $g = 1, ..., m$, and for each individual i, a finite non-negative *endowment* $q_{i,g}$ of *claims* on each good g, such that for each good, the total of all individuals' endowments is strictly positive. The only relevant economic activity is the 'acquisition' and 'disbursement' of claims by individuals, which takes place in a single period.

A claim on a unit of good g confers on its holder both an entitlement and an obligation to consume one unit of that good at the end of the period. There is no general option of free disposal (hence the 'obligation' to consume). 'Consumption' need not be interpreted as something that individuals value positively; it represents whatever opportunities and obligations an individual incurs by virtue of holding a claim at the end of the period. For example, g might be some obsolete type of electronic equipment; 'consumption' might take the form of unwanted storage or costly disposal. However, good 1 (*money*) will be interpreted as a good whose consumption is always valued positively. This property of money has to be treated as a matter of interpretation, because there is no formal concept of preference in the model. Money serves as the medium of exchange and the standard of value in the exchange economy.

In interpreting this model, it is useful to imagine that economic activity is organised by some *trading institution*, distinct from the 'individuals' of the economy. This institution might be thought of as an 'auctioneer' in the sense of Walrasian general equilibrium theory, or as a 'social planner' in the sense of modern welfare economics, or as a set of competing profit-seeking 'traders' who come to the economy from outside (as in the model of Sugden, 2004).[3] The trading institution offers a set of trading opportunities to each individual; these offers constitute the 'regime'.

For a given exchange economy, individuals' opportunities are defined in terms of *net acquisition*. For each individual i, for each good g, net acquisition of g by i is denoted $\Delta_{i,g}$. This is to be interpreted as the additional claims on good g taken on by individual i during the period, minus any claims disbursed. Each $\Delta_{i,g}$ is required to be a real number in the interval

[3] It would be possible to close the model by assuming an auctioneer or social planner who is a government employee, any positive or negative surplus from whose trading operations accrues to individuals as lump-sum benefits or taxes. Analogously, one might assume a set of traders employed by firms whose shares are entirely owned by individuals. However, the model is simpler and more transparent if the trading agency is entirely separate from the individuals.

$[-q_{i,g}, \infty)$. Since $q_{i,g} + \Delta_{i,g}$ represents i's consumption of g, this requirement rules out negative consumption. A vector $\Delta_i = (\Delta_{i,1}, \ldots, \Delta_{i,m})$ of net acquisitions (satisfying the condition of non-negative consumption) is a *behaviour* by i. The universal set of such vectors is denoted by $\mathcal{B}_i$. An n-tuple of behaviours $\Delta = (\Delta_i, \ldots, \Delta_n)$ is a *behaviour profile*. The universal set of behaviour profiles (i.e. the Cartesian product $\mathcal{B}_1 \times \ldots \times \mathcal{B}_n$) is denoted $\mathcal{B}$. The *opportunity set* for an individual i, denoted O_i, can be any nonempty subset of $\mathcal{B}_i$; a behaviour Δ_i is *allowable* in O_i if and only if Δ_i is an element of O_i. Each i independently chooses one element from his O_i. A profile $O = (O_1, \ldots, O_n)$ of opportunity sets is a *regime*. A behaviour profile $\Delta = (\Delta_1, \ldots, \Delta_n)$ is *allowable* in regime O if and only if each Δ_i is allowable with respect to O_i. The set of behaviour profiles that are allowable in regime O (i.e. the Cartesian product $O_1 \times \ldots \times O_n$) is denoted $A(O)$.

For any individual i and any behaviour $\Delta_i \in \mathcal{B}_i$, Δ_i is *dominated in O_i* if and only if there is some $\Delta_i' \in O_i$ such that (i) $\Delta'_{i,1} > \Delta_{i,1}$ and (ii) for each $g \geq 2$, $\Delta'_{i,g} = \Delta_{i,g}$. Given the implicit assumption that consumption of money is always valued positively, a dominated behaviour Δ_i is unambiguously less desirable than the behaviour Δ'_i that dominates it. Thus to say that Δ_i is dominated in O_i is to say that, were Δ_i an element of O_i, i would have no reason to choose it.[4]

For each regime O, I assume that the behaviour of each individual is uniquely determined. The *chosen behaviour* of individual i in regime O is denoted by $\chi_i(O)$; the profile $(\chi_1[O], \ldots, \chi_n[O])$ of chosen behaviour for all individuals is denoted by $\chi(O)$. Notice that no assumptions are being made about the mechanism that determines what each individual chooses from her opportunity set. Choices may be rational or irrational: all that is being assumed is that, from the viewpoint of the modeller, they are predictable. In this set-up, the standard account of rational choice would require that each individual's choices depend only on *his own* opportunity set (i.e. that, for all i and for all O and O', $[O_i' = O_i] \Rightarrow [\chi_i(O) = \chi_i(O)]$) and satisfy the weak axiom of revealed preference. I impose neither of these requirements. Thus, for example, an individual's revealed preference between two given behaviours may vary according to the opportunity set in which they appear, as in theories of

[4] In the 'small world' of a partial-equilibrium model, money might be interpreted as potential expenditure on all goods not explicitly represented in that model. On this interpretation, the idea that consumption of money is always valued positively is very close to the idea that increases in opportunity are always desirable.

salience (Bordalo, Gennaioli and Shleifer, 2013) and bad-deal aversion (Isoni, 2011; Weaver and Frederick, 2012).[5]

A behaviour profile Δ is *feasible* if and only if, for each good g, $\sum_{i \in I} \Delta_{i,g} = 0$. The set of feasible behaviour profiles is denoted F (where $F \subseteq \mathcal{B}$). These feasibility constraints represent the resource limitations of the economy, under the assumption that all goods are initially held by individuals as endowments; they are strict equalities because there is no free disposal option. Notice that a behaviour profile can be allowable (and hence can be chosen) even if it is infeasible. However, certain kinds of trading institutions may be able to construct opportunity sets in such a way that, at least in an appropriately-defined equilibrium, the profile of chosen behaviour is feasible. Clearly, this is true of any trading institution that is able to set market-clearing prices at which individuals are allowed to trade freely, as in the model of the Walrasian auctioneer or in a Nash equilibrium of Sugden's (2004) arbitrage model. But there are other kinds of feasible regime. (A trivial example is the regime in which no trade is allowed – that is, in which for each i, O_i is the singleton $\{(q_{i,1}, \ldots, q_{i,m})\}$.) Opportunity-based normative analysis, as pursued in the current paper, is concerned with the evaluation of alternative feasible regimes for a given economy.

To conceive of normative analysis in this way is, loosely speaking, to evaluate the performance of alternative trading institutions in terms of the benefits they provide to individuals. In the opportunity-based approach, these benefits are mediated by opportunity sets. Thus, normative evaluation must be based on properties of opportunity sets that are deemed to be valuable for individuals. At first sight, it might seem obvious that such an evaluation requires, for each individual *i considered separately*, measures or rankings of the value of the opportunities provided by alternative opportunity sets. One might then state as a normative principle that, for any individual i, for any set O_i of allowable behaviours and for any behaviour $\Delta_i \notin O_i$ that is not dominated in O_i, $O_i \cup \{\Delta_i\}$ provides a greater value of opportunity than O_i. Such a principle would be in the spirit of an important strand of literature on the measurement of opportunity (e.g. Kreps, 1979; Jones and Sugden, 1982; Pattanaik and Xu, 1990; Arrow, 1995). Using it in normative analysis would be appropriate *if* the trading institution, in offering a regime O, could guarantee to implement whatever

[5] Because of the simple structure of choice problems in an exchange economy, there are relatively few ways in which revealed preferences can contravene standard coherence assumptions. The storage economies analysed by McQuillin and Sugden (2013) allow 'irrationality' to take a much wider range of forms. An exchange economy is effectively a one-period storage economy.

behaviour each individual i chose from his O_i, irrespective of the choices made by other individuals. But in the present model, the trading institution is to be interpreted merely as an intermediary in the transfer of goods between individuals: it cannot breach the feasibility constraints encoded in F. Normative analysis should therefore be concerned with opportunities for individuals (collectively) to choose *jointly* feasible behaviours.

The Opportunity Criterion is designed for this kind of analysis. It is a criterion against which, for a given exchange economy, any regime can be assessed:

> *Opportunity Criterion*. A regime O satisfies the Opportunity Criterion if and only if (i) $\chi(O) \in F$ and (ii) for every feasible behaviour profile $\Delta \notin A(O)$, there is some individual $i \in I$ such that Δ_i is dominated in O_i.

To understand the normative intuition behind the Opportunity Criterion, consider a regime O for which the chosen behaviour profile $\chi(O)$ is feasible. Thus, the opportunities specified by $A(O)$ can be made available to individuals without any breach of feasibility constraints. If, despite this, the Opportunity Criterion is not satisfied, there is some feasible behaviour profile Δ' that is non-dominated for each individual but which has *not* been made available. Clearly, no dominance-based argument can be deployed to show that, had that opportunity been made available in addition to those given by O, some individual would not have wanted to take it up. The implication is that individuals collectively lack the opportunity to make a combination of choices that conceivably they might all want to make and that is compatible with the resource constraints of the economy. The Opportunity Criterion requires that individuals are not deprived in this way.[6]

3. The Market Opportunity Theorem

I now characterise a particular type of regime for an exchange economy – a *single-price regime*. In such a regime, for each non-money good $g = 2, \ldots, m$, there is a *market price* p_g expressed in money units; this price is finite, and may be positive, zero or negative. As a matter of notation, it is convenient to represent the idea that money is the medium of exchange by defining $p_1 = 1$ as the market price of money. Each individual is free to keep his

[6] An equivalent statement of part (ii) of the Opportunity Criterion, used by McQuillin and Sugden (2012), is that it is not possible to relax the constraints that O imposes on individuals (thus expanding the set of allowable outcomes) in such a way that some feasible outcome, not allowable in O, becomes both allowable and non-dominated.

endowments if he chooses, but is also free to exchange claims on non-money goods for claims on money (and vice versa) on terms that are at least as favourable as those implied by market prices, subject to the constraint that his holdings of claims on any good cannot be negative. More formally:

> *Single-price regime*. A regime O is a single-price regime if and only if there exists a finite, real-valued price vector $p = (p_1, \ldots, p_m)$ such that $p_1 = 1$ and, for each individual $i \in I$, every behaviour $\Delta_i \in \mathcal{B}_i$ that satisfies $\Sigma_g\, p_g\, \Delta_{i,g} = 0$ is either allowable or dominated in O_i.

A single-price regime O is *market-clearing* if the chosen behaviour $\Delta(O)$ is feasible.

Notice that the definition of a single-price regime allows the possibility that individuals are free to trade on *more* favourable terms than those implied by market prices. This possibility will turn out to be significant when I consider whether regimes that are *not* single-price and market-clearing can satisfy the Opportunity (or Strong Opportunity) Criterion. However, the following more restrictive definition will sometimes be useful:

> *Strict single-price regime*. A regime O is a strict single-price regime if and only if there exists a finite, real-valued price vector $p = (p_1, \ldots, p_m)$ such that $p_1 = 1$ and, for each individual $i \in I$, $O_i = \{\Delta_i\colon \Delta_i \in \mathcal{B}_i \textit{ and } \Sigma_g\, p_g\, \Delta_{i,g} = 0\}$.

The idea of a single-price market-clearing regime is an equilibrium concept. If prices were fixed arbitrarily, there would be no general reason to expect markets to clear. The implicit assumption is that the trading institution can *set* prices that clear markets. But is there any guarantee that a market-clearing equilibrium exists? Proofs of the existence of competitive equilibrium typically assume that individuals act on coherent preferences, but the conceptual framework that I am using does not allow this. In Appendix 2 I show that, given certain weak assumptions, a market-clearing single-price regime exists for every exchange economy. In the main text of this paper, however, I simply examine the properties of such regimes.

The following theorem identifies an important property of market-clearing single-price regimes:

> *Market Opportunity Theorem*. For every exchange economy, every market-clearing single-price regime satisfies the Opportunity Criterion.

As this is an immediate corollary of a result that will be presented in Section 5, a separate proof would be redundant. The marginally weaker proposition that every *strict* market-clearing single-price regime satisfies the Opportunity Criterion is a corollary of a theorem that applies to all storage economies, proved by McQuillin and Sugden (2012). (An exchange economy is a degenerate case of a storage economy.)

4. The Strong Opportunity Criterion

To say that the Opportunity Criterion is satisfied is to say that individuals collectively are not deprived of opportunities to make combinations of choices that are feasible and non-dominated. But notice that that criterion is framed in terms of behaviour profiles that describe the net acquisitions of *every* individual in the economy. Thus, the phrase 'individuals collectively' means '*all* individuals in the economy, considered together'. One might think that the Opportunity Criterion fails to take account of the presence or absence of opportunities for feasible combinations of choices by sets of individuals that do not contain everyone.

Given any exchange economy (which will be referred to as the *whole* economy) it is possible to define a *sub-economy* for each nonempty set S of individuals (where $S \subseteq I$). This sub-economy is defined by the set S of individuals, the same set G of goods as in the whole economy, and for each individual $i \in S$ and each good $g \in G$, the same endowments $q_{i,g}$ as in the whole economy. Given any regime O for the whole economy, the *opportunity profile for S*, denoted by O_S, is a list of the opportunity sets O_i for all individuals $i \in S$. A *behaviour profile for S*, denoted by Δ_S, is a list of behaviours $\Delta_i \in \mathcal{B}_i$, one for each individual i in S; the universal set of such behaviour profiles is denoted $\mathcal{B}_S$. A behaviour profile for S, Δ_S, is *allowable in* O_S if and only if each of its component behaviours Δ_i is allowable in O_i; the set of profiles that are so allowable is $A_S(O)$. Δ_S is *feasible for S* if and only if, for each good g, $\sum_{i \in S} \Delta_{i,g} = 0$. The set of feasible behaviour profiles for S is denoted F_S. Intuitively, this concept of sub-economy feasibility treats the sub-economy for S as if it were an exchange economy in its own right, with no possibilities for transfers of goods between those individuals who belong to S and those who do not.

The following criterion strengthens the Opportunity Criterion to assess opportunities for all sets of individuals:

Strong Opportunity Criterion. A regime O satisfies the Strong Opportunity Criterion if and only if (i) $\chi(O) \in F$ and (ii) for every nonempty set of individuals $S \subseteq I$, and for every behaviour profile $\Delta_S \in \mathcal{B}_S$ for S such that $\Delta_S \notin A_S(O)$ and $\Delta_S \in F_S$, there is some individual $i \in S$ such that Δ_i is dominated in O_i.

In relation to the set $S = I$, this criterion imposes exactly the same restrictions as the Opportunity Criterion does; so any regime that satisfies the Strong Opportunity Criterion also satisfies the Opportunity Criterion. But the Strong Opportunity Criterion imposes restrictions analogous with those of the Opportunity Criterion for *every* set of individuals. It requires, for each such set S, that the members of S are not deprived of opportunities to make combinations of choices that are non-dominated and that are feasible within the resource constraints imposed by their combined endowments. Notice that, since a sub-economy is defined for each $\{i\}$, the Strong Opportunity Criterion requires that, for each i, the behaviour $\Delta_i = \mathbf{0}$ is allowable in O_i. That is, it requires that each individual has the opportunity to consume exactly the bundle of goods he was endowed with.

5. The Strong Market Opportunity Theorem

The Market Opportunity Theorem can be strengthened to:

Strong Market Opportunity Theorem. For every exchange economy, every market-clearing single-price regime satisfies the Strong Opportunity Criterion.

This theorem can be proved by means of the following lemma, which will also be useful for other purposes:

Lemma 1. Consider any exchange economy and any regime O for that economy. Suppose that, for each individual i, there is some function v_i: $\mathcal{B}_i \rightarrow \mathbb{R}$ satisfying the following three conditions:

(i) for all $\Delta \in \mathcal{B}$, for every nonempty $S \subseteq I$: $\Delta_S \in F_S \Rightarrow \sum_{i \in S} v_i(\Delta_i) = 0$;

(ii) for all $i \in I$, for every $\Delta_i \in \mathcal{B}_i$: *if* $v_i(\Delta_i) \leq 0$, *then either* Δ_i is allowable in O_i *or* Δ_i is dominated in O_i; and

(iii) for every $i \in I$, for every $\Delta_i \in \mathcal{B}_i$: *if* $v_i(\Delta_i) < 0$, *then* Δ_i is dominated in O_i.

Then if $\chi(O) \in F$, O satisfies the Strong Opportunity Criterion.

A proof of this lemma is given in Appendix 1.

Given Lemma 1, the proof of the Strong Opportunity Theorem is straightforward. Suppose that, for some exchange economy, O is a single-price market-clearing regime. Since O is a single-price regime, there exists a finite, real-valued price vector $p = (p_1, \ldots, p_m)$ with $p_1 = 1$ such that, for each individual i, every behaviour Δ_i that satisfies $\Sigma_g\, p_g\, \Delta_{i,g} = 0$ is either allowable or dominated in O_i. Since O is market-clearing, $\chi(O) \in F$. For each $i \in I$, define $v_i(\Delta_i) = \Sigma_g\, p_g\, \Delta_{i,g}$. It is immediate that this definition satisfies conditions (i), (ii) and (iii) of Lemma 1. The Strong Opportunity Criterion is therefore satisfied.

6. Characterising market-clearing single-price regimes

The preceding analysis establishes that, for a regime to satisfy the Strong Opportunity Criterion, it is *sufficient* that that regime is single-price and market-clearing. Is this condition also *necessary*?

In general, the answer is 'No', as the following example shows. Consider a two-person, two-good exchange economy E in which both individuals have non-zero endowments of both goods. Consider a regime O in which there are two different (strictly positive) prices at which good 2 can be traded – a high price p_2^{H} and a low price p_2^{L}. Individual 1 is allowed to buy good 2 at the low price or sell at the high price (but not both), while the opposite is true of individual 2. The market clears at the high price, with individual 2 buying from individual 1 (i.e. $\Delta_{2,2} = -\Delta_{1,2} > 0$). This regime can be described more formally by defining functions $v_i\colon \mathcal{B}_i \to \mathbb{R}$ for $i = 1, 2$ as follows (with $p_2^{\mathrm{L}} < p_2^{\mathrm{H}}$):

(1) $v_i(\Delta_i) = \Delta_{i,1} + p_2^{\mathrm{L}}\, \Delta_{i,2}$ *if either* ($i = 1$ *and* $\Delta_{i,2} \geq 0$) *or* ($i = 2$ *and* $\Delta_{i,2} < 0$) and

(2) $v_i(\Delta_i) = \Delta_{i,1} + p_2^{\mathrm{H}}\, \Delta_{i,2}$ *if either* ($i = 1$ *and* $\Delta_{i,2} < 0$) *or* ($i = 2$ *and* $\Delta_{i,2} \geq 0$).

(Intuitively, $v_i(\Delta_i)$ is the total value of i's net acquisitions, calculated at the prices set for i by the regime.) Then, for each i, the regime is defined by $O_i = \{\Delta_i\colon \Delta_i \in \mathcal{B}_i$ *and* $v_i(\Delta_i) = 0\}$. It is easy to show that these v_i functions satisfy conditions (i), (ii) and (iii) of Lemma 1, and hence that O satisfies the Strong Opportunity Criterion.

[Figure 1 near here]

To aid intuition about later results, I now show diagrammatically that O does not satisfy the definition of a single-price regime. For each individual $i = 1, 2$, let Ω_i be the set of behaviours $\Delta_i \in \mathcal{B}_i$ that are neither allowable nor dominated in O_i. Figure 1 plots Ω_1 and Ω_2

in a diagram in which the horizontal axis measures net acquisition (by either individual) of good 1, and the vertical axis measures net acquisition (by either individual) of good 2. Ω_1 is the set of points above and to the right of the *dotted* frontier; Ω_2 is the set of points above and to the right of the *dashed* frontier. The horizontal and vertical segments of these frontiers reflect the constraint that each individual's consumption of each good must be non-negative (i.e. for each i and g, $\Delta_{i,g} \geq -q_{i,g}$).

Now (to initiate a proof by contradiction) suppose that O is a single-price regime. By the definition of such a regime, there exists a finite price p_2 such that every behaviour Δ_2 that satisfies $\Delta_{2,1} + p_2\Delta_{2,2} = 0$ is either allowable or dominated in O_2. This is equivalent to saying that the line through $\mathbf{0}$ with gradient $-1/p_2$ does not pass through Ω_2. But it is immediately obvious from the diagram that, whatever the value of p_2, this line *does* pass through Ω_2. So the supposition that O is a single-price regime is false. The implication is that the Strong Opportunity Criterion can be satisfied by regimes that are not single-price.

However, there is a sense in which, for a sufficiently large economy, any regime that satisfies the Strong Opportunity Criterion is 'almost' a single-price regime. The concept of 'largeness' that I will use derives from Edgeworth (1881/ 1967). The intuitive idea is to take some economy, replicate every component of it, and then create a larger economy by combining two or more of these replicas. By increasing the number of replicas that are combined, one can create larger and larger economies which are identical to one another except for scale. The beauty of this method is that it allows one to investigate the effect of changing the scale of an economy while holding other features constant.

Formally, let E be any exchange economy with individuals $i = 1, \ldots, n$, and let O be any regime for that economy. Then for each integer $r \geq 1$, the *r-fold economy* E^r is defined as the exchange economy created by combining r replicas of E. E^r has rn individuals. For each $k = 1, \ldots, n$, each of the individuals $k, k + r, k + 2r, \ldots, k + (n-1)r$ in economy E^r has the same endowment of each of the m goods as does individual k in economy E. The *r-fold regime* O^r is defined similarly. I assume that chosen behaviours are replicated too. Thus, for each $k = 1, \ldots, n$, each of the individuals $k, k + r, k + 2r, \ldots, k + (n-1)r$ in the r-fold economy faces the same opportunity set, and chooses the same behaviour from that set, as does individual k in regime O of economy E. I will use the expression '(E, O) satisfies the Strong Opportunity Criterion' as a synonym for 'in economy E, regime O satisfies the Strong Opportunity Criterion'.

As an illustration of the significance of replication for the Strong Opportunity Criterion, consider the economy E and regime O represented in Figure 1, and the corresponding two-fold economy E^2 and regime O^2. By assumption, O is market-clearing in E. Thus, by virtue of the definition of replication, O^2 is market-clearing in E^2. However, although (E, O) satisfies the Strong Opportunity Criterion, (E^2, O^2) does not. This is because, in the two-fold economy, individuals 2 and 4 (who are replicas of one another) are deprived of opportunities for trade *between themselves* that are feasible *for them*. To see this, consider any price p_2 in the interval $p_2^{\mathrm{H}} > p_2 > p_2^{\mathrm{L}}$. Let x be any strictly positive quantity of good 2 such that (in the two-fold economy) $q_{2,1} = q_{4,1} \geq p_2 x$ and $q_{2,2} = q_{4,2} \geq x$. Consider the behaviours $\Delta_2 = (-p_2 x, x)$ and $\Delta_4 = (p_2 x, -x)$ for individuals 2 and 4 (i.e. individual 2 buys x units of good 2 at price p_2, and individual 4 sells x units at the same price). Clearly, this behaviour profile is feasible for {2, 4}. But, as Figure 1 shows, Δ_2 is not in Ω_2 and (since Ω_4 is identical to Ω_2) Δ_4 is not in Ω_4. In other words, Δ_2 is neither allowable nor dominated in O_2, and Δ_4 is neither allowable nor dominated in O_4. So (E^2, O^2) does not satisfy the Strong Opportunity Criterion.

The intuitive idea is that the larger the scale of an economy, the more difficult it is to find a market-clearing regime that satisfies the Strong Opportunity Criterion but is not single-price. I now present two general results that formalise that idea. (Proofs of these theorems are given in Appendix 1.) The first result establishes that as the scale of an economy increases, the set of regimes that satisfy the Strong Opportunity Criterion shrinks (or at least, does not expand):

> *Shrinkage Theorem*. Let (E, O) be any pair of an exchange economy and a regime for that economy. For any integer $r \geq 1$, let (E^r, O^r) be the corresponding r-fold economy and regime. Then for all $r \geq 1$: *if* the Strong Opportunity Criterion is not satisfied by (E^r, O^r), *then* it is not satisfied by (E^{r+1}, O^{r+1}).

Given the Shrinkage Theorem, the second result shows that, in the limit as the scale of an economy increases, the only regimes that satisfy the Strong Opportunity Criterion are those that are 'almost the same as' market-clearing single-price regimes. To present this result, I need some additional definitions. Consider any pair (E, O) of an exchange economy and a regime for that economy. For any individual i, for any behaviour $\Delta_i \in \mathcal{B}_i$ and any finite real number $\varepsilon > 0$, let $\nu(\Delta_i, \varepsilon)$ be the set of behaviours whose Euclidian distance from Δ_i is no greater than ε. I will say that Δ_i is 'within ε of being allowable', or ε-*allowable,* in O_i if and only if there is some behaviour $\Delta'_i \in \nu(\Delta_i, \varepsilon)$ that is allowable in O_i. I will say that Δ_i is *ε-dominated* in O_i if and only if there is some behaviour $\Delta'_i \in \nu(\Delta_i, \varepsilon)$ that is dominated in O_i. And I will say that O is an *ε-single-price regime* if and only if there exists a finite, real-valued price vector $p = (p_1, \ldots, p_m)$ with $p = 1$ such that, for each individual i, every behaviour Δ_i that satisfies $\Sigma_g\, p_g\, \Delta_{i,g} = 0$ is either ε-allowable in O_i or ε-dominated in O_i. Thus, at sufficiently small values of ε, ε-single-price regimes are 'almost the same as' single-price regimes. The second result can now be stated as:

> *Convergence Theorem*. Let (E, O) be any pair of an exchange economy and a regime for that economy. For any integer $r \geq 1$, let (E^r, O^r) be the corresponding r-fold economy and regime. For any given $\varepsilon > 0$, if for all $r \geq 1$, the Strong Opportunity Criterion is satisfied by (E^r, O^r), then O is an ε-single-price regime.

7. Discussion

Taken together, the Strong Market Opportunity Theorem, the Shrinkage Theorem and the Convergence Theorem characterise a market-clearing single-price regime for a sufficiently large exchange economy. In the limit as the size of the economy tends to infinity, such a regime, and only such a regime, satisfies the Strong Opportunity Criterion. I conclude by considering the significance of this formal result for a normative appraisal of competitive markets.

McQuillin and Sugden (2012) have shown that every competitive equilibrium – that is, every market-clearing *strict* single-price regime – of an exchange economy satisfies the Opportunity Criterion. The objective of the analysis reported in the present paper was to investigate whether that criterion, or some closely related condition, *characterised* competitive equilibrium. In order to arrive at the characterisation stated in the previous paragraph, it was necessary to revise McQuillin and Sugden's theoretical framework in three ways. First, the *Strong* Opportunity Criterion was used. In contrast to the criterion used by McQuillin and Sugden, the Strong Opportunity Criterion takes account of the distribution of endowments between individuals. In particular, it requires (as McQuillin and Sugden's criterion does not) that each individual's opportunity set contains the option of consuming exactly his endowments. Since this is a property of opportunity sets in every competitive equilibrium, some such revision is unavoidable in a characterisation of competitive equilibrium. Second, the characterisation refers to single-price regimes, without requiring strictness. To see why some such revision is necessary, consider any exchange economy E and any market-clearing *strict* single-price regime O for that economy that satisfies the Strong Opportunity Criterion. Now consider the regime O' that differs from O in only one respect, namely that for some individual i, $O'_i = O_i \cup \{\Delta'_i\}$ where Δ'_i is a behaviour that is neither allowable nor dominated in O_i. O' is a *non-strict* single-price regime. But if this expansion does not affect the chosen behaviour of any individual, O' must satisfy the Strong Opportunity Criterion.[7] Third, my characterisation result applies only to large economies. As was shown by the example presented at the beginning of Section 6, the Strong Opportunity Criterion can be satisfied by non-single-price regimes in small economies.

I submit that, despite these qualifications, the characterisation presented in this paper says something important about the opportunity-enhancing properties of competitive equilibrium that does not depend on any assumptions about the coherence of individuals' preferences. Intuitively, the Strong Opportunity Criterion requires that, for any set of individuals in an economy, every transaction that those individuals might reasonably want to make and that is feasible, given their endowments, is available to them in their respective opportunity sets. Let us say that a person is *willing to pay for* something if he is willing to give up enough of his endowments to make others willing to play their parts in supplying it to him. A regime that satisfies the Strong Opportunity Criterion, one can then say, allows every individual to get whatever he wants and is willing to pay for. That every competitive equilibrium has this property, and that every regime that has this property is fundamentally similar to competitive equilibrium, are normatively significant statements.

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[7] The definition of O' implies that Δ'_i has a strictly positive value at the market prices of regime O. In O', the chosen net acquisitions of individuals other than i must have zero value at these prices. Thus it would be inconsistent with feasibility constraints for i to *choose* Δ'_i. But there is no inconsistency in including Δ'_i in i's opportunity set.

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Appendix 1: Proofs of results in main text

A1.1 Proof of Lemma 1

Consider any exchange economy, any regime O for that economy such that $\chi(O) \in F$, and any list $v_1, \ldots, v_n$ of functions satisfying conditions (i), (ii) and (iii). Consider any nonempty set $S \subseteq I$ of individuals, and any behaviour profile $\Delta_S \in \mathcal{B}_S$ for S such that $\Delta_S \in F_S$ and $\Delta_S \notin A_S(O)$. By (i), $\sum_{i \in S} v_i(\Delta_i) = 0$. Thus, *either* $v_i(\Delta_i) = 0$ for all $i \in S$ (Case 1) *or* $v_j(\Delta_j) < 0$ for some $j \in S$ (Case 2). In Case 1, by (ii), for each $i \in S$, Δ_i is either allowable in O_i or dominated in O_i. Since $\Delta_S \notin A_S(O)$, there must be some $i \in S$ such that Δ_i is dominated in O_i. In Case 2, by (iii), there is some $i \in S$ such that Δ_i is dominated in O_i. Since these results hold for every nonempty $S \subseteq I$, the Strong Opportunity Criterion is satisfied. □

A1.2 Proof of the Shrinkage Theorem

Let (E, O) be any pair of an exchange economy and a regime for that economy and let n be the number of individuals in that economy. For any integer $r \geq 1$, let (E^r, O^r) be the corresponding r-fold economy and regime, and let $I^r = \{1, \ldots, rn\}$ be the set of individuals in that economy, indexed so that for each $k = 1, \ldots, n$, the individuals in the set $\{k, k + r, k + 2r, \ldots, k + (n-1)r\}$ are replicas of one another. Suppose that (E^r, O^r) does not satisfy the Strong Opportunity Criterion. First, suppose $\chi(O^r) \notin F^r$, where F^r is the set of feasible behaviour profiles for economy E^r. Then it follows from the definition of replication that $\chi(O^{r+1}) \notin F^{r+1}$, and hence that (E^{r+1}, O^{r+1}) does not satisfy the Strong Opportunity Criterion. Now suppose $\chi(O^r) \in F^r$. Then, by the definition of the Strong Opportunity Criterion, there is some non-empty set of individuals $S \subseteq I^r$ and some behaviour profile Δ'_S that is feasible for S in economy E^r, such that (i) Δ'_S is not allowable in $O^r{}_S$ and (ii) for every individual $i \in S$, Δ'_i is not dominated in $O^r{}_i$. Then it follows from the definition of replication that $S \subseteq I^{r+1}$ and that each member of S has the same endowments and the same opportunity set in (E^{r+1}, O^{r+1}) as in (E^r, O^r). Thus Δ'_S is feasible for S in economy E^{r+1}, Δ'_S is not allowable in $O^{r+1}{}_S$, and for every individual $i \in S$, Δ'_i is not dominated in $O^{r+1}{}_i$. Hence (E^{r+1}, O^{r+1}) does not satisfy the Strong Opportunity Criterion. □

A1.3 Proof of the Convergence Theorem

Let (E, O) be any pair of an exchange economy and a regime for that economy and let $I = \{1, \ldots, n\}$ be the set of individuals in that economy. For each $r \geq 1$, let (E^r, O^r) be the corresponding r-fold economy and regime, with the set of individuals $I^r = \{1, \ldots, rn\}$. Suppose that the Strong Opportunity Criterion holds for every such r-fold economy. Fix any finite $\varepsilon > 0$.

For each individual i, let Ω_i be the set of behaviours for i that are neither allowable nor dominated in O_i. Each of these sets can be plotted in a common space $\mathbb{R}^m$ where m is the number of goods in the economy. (Each dimension $g = 1, \ldots, m$ of this space measures increments of good g, but without referring to any particular individual. Compare Figure 1.) Let Ω be the convex hull of the sets Ω_i for $i = 1, \ldots, n$. Let $\Omega^*_i(\varepsilon)$ be the set of behaviours for i that are neither ε-allowable nor ε-dominated in O_i. Let $\Omega^*(\varepsilon)$ be the convex hull of the sets $\Omega^*_i(\varepsilon)$ for $i = 1, \ldots, n$. By the definitions of ε-allowability and ε-dominance, for each i, every element of $\Omega^*_i(\varepsilon)$ is strictly in the interior of Ω_i.

Consider any $\omega \in \Omega^*(\varepsilon)$. By the definition of a convex hull, ω can be constructed as a convex combination of some finite set of vectors $\{\omega_1, \ldots, \omega_N\}$ where each ω_j is an element of $\Omega^*_i(\varepsilon)$ for some individual $i \in I$ and each ω_j has a real-valued weight α_j in this combination, where $0 < \alpha_j \leq 1$ and $\Sigma_j \alpha_j = 1$. I will say that i is the *actor* for ω_j, and write this as $i = a(j)$. The α_j weights need not be rational numbers. However, because the rational numbers form a dense subset of the reals, we can find vectors $\omega'_1, \ldots, \omega'_N$, such that each ω'_j is sufficiently close to the corresponding ω_j that it is an element of $\Omega_{a(j)}$, and such that ω is a convex combination of $\omega'_1, \ldots, \omega'_N$ in which each of the weights $\alpha'_1, \ldots, \alpha'_N$ is a strictly positive *rational* number.

Now consider, for some $r \geq 1$, the r-fold economy and regime (E^r, O^r) that corresponds with (E, O). The set of individuals in this economy is $I^r = \{1, \ldots, rn\}$. By virtue of the results established in the previous paragraph, if r is sufficiently large, we can construct a non-empty set $S \subseteq I^r$ of individuals, and a behaviour profile Δ_S for S, such that S can be partitioned into non-empty subsets $S_1, \ldots, S_N$ and the following properties hold for each $j = 1, \ldots, N$: (i) the ratio between the number of individuals in S_j and the number of individuals in S is α'_j; (ii) each individual in S_j is a replica of the individual $a(j)$; and (iii) the behaviour Δ_i for each individual i in S_j is ω'_j. (Note that, because ω'_j is an element of $\Omega_{a(j)}$, $\omega'_j \in \mathcal{B}_{a(j)}$. Thus it is legitimate to treat the vector ω'_j as a *behaviour* for any replica of $a(j)$.) Given these properties, Δ_S is feasible for S if and only if $\omega = \mathbf{0}$.

Now suppose that $\mathbf{0} \in \Omega^*(\varepsilon)$. Recall that, for each $i = 1, \ldots, n$, $\Omega^*_i(\varepsilon) \subset \Omega_{i'}$. Applying the conclusions of the previous paragraph to the case $\omega = \mathbf{0}$, if r is sufficiently large, there exists a non-empty set $S \subseteq I^r$ of individuals, and a feasible behaviour profile Δ_S for S, such that, for each $i \in S$, $\Delta_i \in \Omega_i$ (i.e. Δ_i is neither allowable nor dominated in $O^r{}_i$). This implies that (E^r, O^r) does not satisfy the Strong Opportunity Criterion, contradicting an initial supposition of the proof. Thus, $\mathbf{0} \notin \Omega^*(\varepsilon)$.

Since $\Omega^*(\varepsilon)$ is a convex set by construction, $\mathbf{0} \notin \Omega^*(\varepsilon)$ implies that there is some hyperplane through $\mathbf{0}$ that does not intersect $\Omega^*(\varepsilon)$, and hence does not intersect any $\Omega^*_i(\varepsilon)$. Thus, by the definition of $\Omega_i^*(\varepsilon)$, for each individual $i = 1, \ldots, rn$, every behaviour $\Delta_i \in \mathcal{B}_i$ on this hyperplane is either ε-allowable or ε-dominated in $O^r{}_i$. Equivalently, there exists a finite, real-valued price vector $p = (p_1, \ldots, p_m)$ with $p_1 = 1$ such that, for each individual i, every behaviour Δ_i that satisfies $\Sigma_{\mathrm{g}}\, p_g\, \Delta_{i,g} = 0$ is either ε-allowable or ε-dominated in $O^r{}_i$, i.e., O^r is an ε-single-price regime. □

Appendix 2: Existence of a market-clearing single-price regime

Consider any exchange economy. For every finite price vector $p = (p_1, \ldots, p_m)$ with $p_1 = 1$, there is a corresponding strict single-price regime O, defined (for each individual i) by $O_i = \{\Delta_i\colon \Delta_i \in \mathcal{B}_i \textit{ and } \Sigma_{\mathrm{g}}\, p_g\, \Delta_{i,g} = 0\}$. Thus, considering only strict single-price regimes, the chosen behaviour Δ_i of each individual i can be expressed as a function of p. And so, for each good $g = 1, \ldots, m$, we can write *net excess demand* for g (i.e. the sum of the chosen values of $\Delta_{i,g}$ for all individuals i) as a function $x_g(p)$. For any given price vector p, the corresponding strict single-price regime is market-clearing if $x_g(p) = 0$ for every good g. Notice that if $x_g(p) = 0$ holds for all *non-money* goods $g = 2, \ldots, m$, it necessarily holds for good 1 too. More generally, *the value of* net excess demand, expressed in money units by using the price vector p and summed over all individuals and all goods (including money), is identically equal to zero, irrespective of whether markets clear. That is, $\Sigma_i\, \Sigma_g\, p_g\, x_g(p) = 0$. This identity (a version of *Walras's Law*) is an implication of the assumption that each individual's chosen behaviour is in his opportunity set; it does not depend on any assumptions about preferences.

Now consider the following two additional assumptions:

Continuity. For each non-money good $g = 2, \ldots, m$, $x_g(p)$ is a continuous function.

Intrinsic Value of Money. For each non-money good $g = 2, \ldots, m$, there is an *upper limit* price $p_g^U > 0$ and a *lower limit* price $p_g^L < 0$, such that, for all price vectors p, $p_g \geq p_g^H$ implies $x_1(p) > 0$, and $p_g \leq p_g^L$ implies $x_1(p) > 0$.

Since I am not assuming that individuals act on coherent preferences, I cannot follow the neoclassical strategy of deriving Continuity as a property of the demand functions of rational individuals whose preferences are 'well-behaved'. However, I suggest that Continuity is a plausible assumption about *aggregate* behaviour in a large economy. Intrinsic Value of Money expresses the idea that money is always perceived as a desirable consumption good, and that this desire is never satiated. Intuitively, if the price of some non-money good g is sufficiently high, individuals who have positive endowments of g will want to take advantage of the opportunity to acquire large amounts of money by giving up small amounts of g, and so money will be in excess demand. Similarly, if the price of some non-money good (or, in this case, bad) g is sufficiently negative, individuals will want to take advantage of the opportunity to acquire large amounts of money by taking on small amounts of g, and so again money will be in excess demand.

The following theorem can be proved:

Existence Theorem. For any exchange economy, if Continuity and Intrinsic Value of Money are satisfied, there exists a market-clearing strict single-price regime.

Proof. Consider any exchange economy. Assume that Continuity and Intrinsic Value of Money are satisfied. Let P be the set of price vectors p that satisfy the condition that, for each non-money good $g = 2, \ldots, m$, $p_g^H \geq p_g \geq p_g^L$. For any such price vector p, for each good g = $1, \ldots, m$, let $x_g(p)$ be the net excess demand for good g that would occur if p was the price vector in a strict single-price regime. I now define a *tâtonnement function* $f: P \to P$. For the purposes of the proof, this is merely a mathematical construction.

As a first step, I define $\phi(z) = \alpha(1 - e^{-z})/(1 + e^{-z})$ for all real numbers z, where e is Euler's number and α is some constant satisfying $1 \geq \alpha > 0$. Notice that $\phi(.)$ is a continuous and monotonically increasing function with $\phi(0) = 0$; $\phi(z) \to -\alpha$ as $z \to -\infty$, and $\phi(z) \to \alpha$ as $z \to \infty$. Writing $f(p)$ as $[f_1(p), \ldots, f_G(p)]$, I define $f_g(p)$ for $g = 2, \ldots, m$ by:

(A1) $f_g(p) = (1 - \phi[x_g(p)])p_g + \phi[x_g(p)]p_g^H$ *if* $x_g(p) \geq 0$; *and*

(A2) $f_g(p) = (1 - \phi[x_g(p)])p_g + \phi[x_g(p)]p_g^L$ *if* $x_g(p) \leq 0$.

Because of Walras's Law, these equations also define $f_1(p)$. Because $\phi(.)$ is a continuous function, and because (by Continuity) each $x_g(.)$ is a continuous function, $f(.)$ is a continuous function from P to P. By construction, P is a closed, bounded, convex set. Thus, by Brouwer's Theorem, there is a fixed point $p^* \in P$ such that $f(p^*) = p^*$.

Consider any such p^*. First, suppose there is some non-money good g such that *either* $p^*_g = p_g^{\mathrm{H}}$ *or* $p^*_g = p_g^{\mathrm{L}}$. So, by Intrinsic Value of Money, $x_1(p^*) > 0$. ByWalras's Law, there must be some non-money good h (which may or may not be g) for which the value of net excess demand is strictly negative, i.e. $p^*_h\, x_h(p^*) < 0$. By the definition of p^*, $f_h(p^*) = p^*_h$. Thus, *either* $p^*_h > 0$ and $x_h(p^*) < 0$ (Case 1), *or* $p^*_h < 0$ and $x_h(p^*) > 0$ (Case 2). Suppose Case 1 holds. By (1b), $[f_h(p^*) = p^*_h$ and $x_h(p^*) < 0]$ implies $p^*_h = p_h^{\mathrm{L}} < 0$, a contradiction. Suppose Case 2 holds. By (1a), $[f_h(p^*) = p^*_h$ and $x_h(p^*) > 0]$ implies $p^*_h = p_h^{\mathrm{H}} > 0$, a contradiction. So the original supposition is false. That is, for every non-money good g, $p_g^{\mathrm{H}} > p^*_g > p_g^{\mathrm{L}}$.

It then follows from (A1) and (A2) that, for each non-money good g, $f_g(p^*) = p^*_g$ implies $x_g(p^*) = 0$. Thus, there exists a market-clearing strict single-price regime, namely the strict single-price regime in which the price vector is p^*. □

Figure 1: Non-allowable, non-dominated behaviours in the counter-example regime