Proliferation and Entry Deterrence in Vertically Differentiated Markets

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Abstract

We examine the profitability, entry deterrence and welfare effects of proliferation offered by non-cooperative firms competing in quality and price. In a market of one high quality firm and competitive low quality firms, we find that the established high quality firm will not initiate proliferation but may have an incentive to do so if facing entry threats. The proliferation quality is endogenously determined and the industry profit decreases with such proliferation. Moreover, we show that proliferation increases consumer surplus in the same way as entry does. That is, while proliferation to deter entry is anti-competitive, it is not necessarily welfare-reducing.

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Keywords: Vertical Differentiation; Proliferation; Entry Deterrence; Welfare

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1. Introduction

In vertically differentiated markets with products of the same generic type, firms making quality-price decisions face a trade-off between softening competition and broadening market segments. As a result, proliferation or multi-product strategy (Constantatos and Perrakis, 1997), where firms typically offer a range of products, could have either pro or anti-competitive effect to the market. On the one hand, by filling in the gaps on a quality spectrum, proliferation increases competition since products available become less differentiated. On the other hand, proliferation enables firms to reach different market segments and better discriminate against consumers. Ever since Schmalensee (1978) suggests empirically that an incumbent can preempt entry by introducing new products and restricting the market space available to the entrant, there has been increasing antitrust concerns against excessive proliferation. If one tries to rationalize firms' engagement in proliferation that is not per se profitable, the argument might be that it is optimal if some other considerations are taken into account, e.g., entry deterrence.

Proliferation to deter entry has been under intense investigation in horizontal differentiation settings (Judd, 1985; Choi and Scarpa, 1991; Murooka, 2012), but the same analysis in vertical settings is sparse. Given that excessive proliferation is suspicious, one may tempt to assume that moderate proliferation is more likely to be justified on profitability ground thus seemingly causes no harm. We show that it is possible for proliferation to be completely undesirable in the absence of entry threats, even in the extreme case of introducing only one additional product. Therefore proliferation of any level might involve anticompetitive purpose. More interestingly, when firms optimally conduct proliferation in facing entry threats, such proliferation is not necessarily welfare-reducing. Instead, it could even be more efficient than if entry takes place.

In a vertically differentiated market, suppose that consumers have identical ordinal preferences over product quality and differ only in income levels.[1] Then at equal price, higher quality is preferable. For instance, people would generally agree that a Mason Pearson hairbrush is preferable to a hairbrush from Primark.[2] In a simple duopoly market with no cost, a series of papers (Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982; Donnenfeld and

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[1] Another interpretation of vertical differentiation is by Mussa and Rosen (1978) where consumers differ only in their intensity of preference for quality. The two interpretations may be related; if θ is a parameter of intensity of preference for quality, then wealthier consumers have a higher θ and enjoy quality improvement more that the less wealthy consumers (See Tirole, 1988; Donnenfeld and Weber, 1992).

[2] See http://masonpearson.co.uk/ and http://www.primark.com/en/whats-new.

Weber, 1992) prove the existence of a unique quality-price equilibrium in which the two established firms follow the *Principal of Maximal Differentiation*, that is they first choose distinct qualities then set distinct prices, so as to dampen competition and increase profit.

More often, a single firm offers quality-differentiated products to consumers. For example, for a given size, Mason Pearson usually offers three types of hairbrush, namely Pure Bristle, Bristle/Nylon, and Nylon, where bristle is finer and more expensive than nylon as the component of a hairbrush head. Another form of proliferation, namely brands collaboration, is usually conducted jointly by two firms. Similar to joint venture strategy, brands collaboration is defined by Chun and Niehm (2010) as a strategic and cooperative relationship where brands devote their own competitive advantages to present products under joint names to consumers.

A good example of vertical brands collaboration is the overwhelming high-street (e.g. Gap, H&M) and luxury (e.g. Jimmy Choo, Lanvin) collaboration in the fashion apparel and sportswear industry. The premier case was by Puma and Jil Sander in the late 1990s.[3] Chun and Niehm (2010) suggest that through this collaboration, "the boundaries between genres collapsed." In 2003, Adidas launched its first collaborations with Stella McCartney and Jeremy Scott, then Puma collaborated with Neil Barrett and Alexander McQueen in subsequent years.[4] Nike joined the trend in 2012 and has been collaborating with Liberty.

A more typical case involves the Sweden based clothing retailer Hennes & Mauritz (H&M). In 2004, H&M tied up with Karl Lagerfeld and produced a successful collection. Since then, it has launched 21 collaborations with 16 different designer brands and two top celebrities.[5] The average prices of the items from many H&M collaborations were over £100, with the most expensive collaboration being Maison Martin Margiela for H&M (on average £141.61 per item). But, at the same time, the prices were lower than the typical luxury collections. Despite at much higher prices compared to normal H&M ranges, the collaboration ranges were sold out very quickly. Some popular pieces were sold on eBay for 5 times the original price. As reflected by prices, the perceived quality of collaborated ranges seems to be higher than the normal ranges from the high-street stores, although would not be as high as the original designer luxury collections. As a result, quality configuration expands with proliferation.

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[3] See http://www.jilsander.com/.

[4] See http://www.stellamccartney.com/gb, http://shop.jeremyscott.com/, http://www.neilbarrett.com/it/, and http://www.alexandermcqueen.com/gb/alexandermcqueen.

[5] Collaborations in the last decade included those with Stella McCartney, Viktor & Rolf, Roberto Cavalli, Comme des Garcons, Marimekko, Matthew Williamson, Jimmy Choo, Sonia Rykiel, Lanvin, Versace, Marni, Anna Dello Russo, Maison Martin Margiela, Isabel Marant and Alexander Wang. See http://about.hm.com/en/About/facts-about-hm/people-and-history/history.html.

It is of importance to study proliferation in vertically differentiated markets since the existing studies relating to it is limited and does not keep up pace with the boom of real life (joint) proliferation cases. Among many strategic incentives, proliferation has mainly been examined for two – to increase profit and to deter entry. While there is literature on the optimality of proliferation, the specific conditions and endogenous proliferation qualities have been largely unaddressed, leaving it difficult to evaluate proliferation decisions even in a simplified context. With the entry deterrence effect of proliferation being rarely examined, the relevant welfare effect remains unclear.[6]

Although fully replicating the real life high-street and luxury collaborations is out of scope of this study, we wish to examine how proliferation works in a context that captures this sort of competition. In absence of entry deterrence, common expectation of firms have on proliferation is to broaden market segments. Whether such expectation could be fulfilled is unclear without the knowledge of where firms wish to locate their additional qualities.

When a firm decides on proliferation, it also faces a menu of options as in how much control it wishes to take. For example, it may wish to take full control over quality-price decisions of any additional product introduced; or any additional product acts as independent unit that sets its own quality and price, while the parent-firm acts as a shareholder collecting profit from it (i.e., divisionalisation: Baye *et al*., 1996). Immediately the message is that an entrant would make the same quality-price decision as the independent unit, precisely because the independent unit maximizes its own profit and not the joint profits of its parent-firm.

In a model of maximal differentiation where one firm specializes in the high end of the quality spectrum and Bertrand competitive firms operate in the low end, we show that the high quality firm would only be able to locate a middle quality product if it allows that product to be an independent unit. The quality level of the middle quality product is endogenously determined and is a convex combination of the existing high and low qualities. To be specific, it is 4/7 of that of the high quality product plus 3/7 of that of the low quality product. The price of the middle quality product is 2/7 of the price of the high quality product.[7] This proliferation is not optimal without entry threats but is optimal with entry threats and a low enough proliferation cost.

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[6] There is a much richer literature on "limit quality" to deter entry (Lane, 1980; Donnenfeld and Weber, 1995; Noh and Moschini, 2006) than proliferation to deter entry in vertical differentiation model.

[7] This is similar to Choi and Shin (1992)'s results in a duopoly model of vertical differentiation with uncovered market. They find that the quality level of the low quality product is 4/7 of that of the high quality product, and the price of the low quality product is 2/7 of the price of the high quality product.

The infeasibility of proliferation in the absence of entry threat proves the stability of the *Principal of Maximal Differentiation* in the market structure of the current paper; firms either have no ability, i.e., the low quality firms, or no incentive, i.e., the high quality firm, to introduce an additional middle quality. Johnson and Myatt (2003) suggest that when established firms strategically expand on the quality spectrum, they usually do so by introducing a lower quality product. They provide an example from the Indian watch market in which a launch of a high-end brand by a low-end firm was unsuccessful and eventually exited the market.

Moving from a two-quality to a three-quality market, if the fixed cost of introducing the third quality is small enough, then the rise in consumer surplus would be more than the fall in the industry profit. Hence, total welfare increases. This increasing welfare could be achieved by the high quality firm through proliferation, or by the entrant if proliferation does not take place. From a social welfare point of view, whether the outcome is achieved more efficiently by the established firm or by the entrant depends on their relative cost of introducing the third product. If the high quality firm benefits from the fact that it is already established hence incur lower cost, then not only proliferation may increase welfare, it also does in a way that is more efficient than entry.

The setting considered in this paper is along the similar lines to Tirole (1988), who provides an intuitive yet simple way to solve the model of vertical differentiation. We allow competitive firms in the low quality segment, which is different from the standard duopoly model. This modification, however, is consistent with the differentiated brands distribution in the fashion apparel industry; there are countless high-street brands and far fewer designer luxury brands. Although it is a simplified model, it delivers a rather complete evaluation of proliferation strategy.

This paper is also close in spirit to Baye *et al*. (1996), who study firms' incentives to add divisions before engaging in Cournot competition. They mention that the profit generated by a new division is offset by the fall in profit of the parent-firm's existing units. Therefore proliferation, however moderate, might fail to fulfil the expectation.

The idea on proliferation location is close to Donnenfeld and Weber (1992), who show in a model of maximal differentiation that a later entrant always selects an intermediate level of quality. They, however, neither endogenously determine the intermediate quality nor consider the situation where the intermediate quality is offered by an incumbent rather than an entrant, which potentially could be a deterrent. Our result on endogenous level of the middle quality is

consistent with the prediction of Choi and Shin (1992), who suggest it to be "just over half that of the established firm".

While Constantatos and Perrakis (1997) also find that proliferation may help an established firm to block entry, their focus is on market coverage. In our model, market coverage is endogenously full with or without proliferation. Proliferation intensifies price competition and reallocate demand; with the appearance of the middle quality product, price of the high quality product decreases and demand increases, whereas demand for the low quality drops significantly.

The paper proceeds as follows. In Section 2 we present the formal model. Section 2.1 provides a simple two-quality market environment where one high quality firm and competitive low quality firms compete in price. We characterize the equilibrium as a benchmark for assessing proliferation incentives later. Section 2.2 evaluates the profitability of proliferation in the existing two-quality market without threat of entry. We determine endogenously the quality choice of proliferation. Section 2.3 examines proliferation as a deterrent when there is threat of entry. Section 3 analyses the change in welfare when moving from two-quality to three-quality market. Section 4 concludes.

2. The Model

2.1 The Two-Quality Environment

In a market of quality-differentiated products of the same generic type. Product i is of quality s_i and is sold at price $p_i, i \in \{h, l\}$, where $s_h > s_l$ and $p_h > p_l$. Products with high quality s_h are provided by a single firm, H, and products with low quality s_l are provided by at least two firms, L_1 and L_2. H and the low quality segments engage in maximal differentiation. Qualities s_h and s_l are fixed at the two ends of the quality spectrum. The difference between the two qualities is denoted as d and is therefore equal to the total length of the spectrum. The unit cost of production c is assumed to be the same for both qualities and is normalized to zero. Assume for now that there is no threat of entry.

There is a continuum of consumers who have identical ordinal preferences over product quality but differ in willingness to pay. One major reason for such differences could be the differences in income levels. Consumers are uniformly distributed over the interval $[0, \overline{\theta}]$ where θ represents their willingness to pay.

Each consumer consumes at most one unit of the products and maximizes the following utility function

$$U = \begin{cases} \theta s_i - p_i, & \text{if consumes,} \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

We obtain $\theta_h = (p_h - p_l)/d$ and $\theta_l = p_l/s_l$, which represent the last consumer with the willingness to buy the high quality product and the low quality product, respectively. That is, consumers with $\theta \in [\theta_h, \bar{\theta}]$ buy the high quality products, consumers with $\theta \in [\theta_l, \theta_h)$ buy the low quality products and consumers with $\theta \in [0, \theta_l)$ buy neither of them.

Lemma 1. *Price competition drives the price of the low quality product p_l down to zero. All the consumers purchase at least one of the products and the market is covered.*

Given *Lemma 1*, profits are zero for all firms in the low quality segment. For H, we may write the optimization problem as

$$\max_{p_h} \pi_H = \max_{p_h} \ p_h(\bar{\theta} - \theta_h), \tag{2}$$

where $\bar{\theta} - \theta_h$ is the demand for high quality product. To solve it, one simply derives the first-order condition with respect to p_h, which leads to *Lemma 2*.

Lemma 2. *In equilibrium, high and low quality products equally share the market; $D_h = D_l = \bar{\theta}d/2$. H sets $p_h = \bar{\theta}d/2$ and obtains $\pi_H = \bar{\theta}^2 d/4$ whereas L_1 and L_2 obtain zero profit.*

Lemma 2 explains firms' price decision in this vertically differentiated market. The price of the high quality product is higher when the distribution of willingness to pay θ is more dispersed, or when quality difference d is more significant. Since we have maximal differentiation in the model, d is maximised.

The equilibrium demand further proves *Lemma 1* that the market is equally and fully covered by the two existing qualities. Unlike the standard non-cooperative duopoly vertical differentiation model where both high and low quality firms enjoy positive surpluses, price competition among products of quality s_l in the current model leaves no surplus for the low quality segment.

2.2. The Proliferation Environment

This section examines H's incentive to proliferate in the existing two-quality market. Since moderate proliferation is more likely to be justified on profitability ground, we allow H to introduce exactly one additional quality, s_m.

Assumption 1. $s_m = \beta s_l + (1 - \beta)s_h, \beta \in (0,1)$.

The above assumption ensures that s_m is of an intermediate level, where β is the location parameter. Given that s_h and s_l are exogenous, β decides s_m. When $\beta > 0.5$, s_m is closer to s_l than to s_h. We exclude the possibility of equality among qualities which is not feasible for H.

Since $s_h > s_m > s_l$, for reputation and technology concerns, we have H, rather than one of the low quality firms, to offer s_m. In fact, if proliferation was initiated by a low quality firm, it would be prevented by H with a side payment. Linking it to real life, the reason that H&M collaborated ranges were sold at much higher prices than normal H&M ranges is precisely the name and reputation of the designer brands. Since in the context of the current paper, proliferation is provided by a single firm, not jointly, it can only be credibly provided by H. H will proliferate by introducing a middle quality to avoid fierce local competition with the existing qualities, as also suggested by Donnenfeld and Weber (1992).

Upon proliferation H has to incur some fixed cost f, which may be seen as a product-specific-capital, but the unit cost of production is again normalized to zero. Proliferation works like this: H first decides on β and then all three qualities engage in price competition. As discussed in Section 1 that when a firm decides on proliferation, it faces a menu of options of how much control it wishes to take. Here H has three options of control over quality-price decision of the middle quality product.

a. Full Control

$$\max_{\beta,p_h,p_m} \pi_H{}^{pro} = \max_{\beta,p_h,p_m} [\pi_h + \pi_m - f]. \tag{3}$$

With this option, H takes full control over proliferation. It sets the quality location and price of the middle quality product, as well as the price of the original high quality product, so as to jointly maximize its total proliferation profit, π_H^{pro}.

b. Semi-full Control

$$\max_{\beta} \pi_H{}^{pro} = \max_{\beta} \left[\left(\max_{p_h} \pi_h + \max_{p_m} \pi_m \right) - f \right], \tag{4}$$

where

$$\left.\begin{array}{l} \max_{p_h} \pi_h = \max_{p_h} p_h D_h \\ \max_{p_m} \pi_m = \max_{p_m} p_m D_m \end{array}\right\}. \tag{5}$$

With this option, H sets the location of the middle quality product to maximize total proliferation profit, but does not interfere into price competition. Different qualities act as independent units which maximize unit profit by setting their own prices.

c. Independent Control

$$\pi_H{}^{pro} = \max_{p_h} \pi_h + \max_{\beta, p_m} \pi_m - f. \tag{6}$$

With this option, quality locations and prices are all determined by the market. H simply acts as the parent-firm of its product units and collects profits generated by them.

To determine whether H has an incentive to proliferate and which control option it will choose, we need to compare $\pi_H{}^{pro}$ under options a, b and c to its profit without proliferation, π_H solved in Section 2.1.

That is, H will proliferate if the below is true

$$\pi_H{}^{pro} > \pi_H. \tag{7}$$

Lemma 3. *With Full Control, the demand for the middle quality product, D_m, is zero. $\pi_H{}^{pro} > \pi_H$ does not hold, hence H has no incentive to proliferate. Any positive f, however small, can prevent proliferation from happening.*

Proof: See Appendix A.

By solving optimization problem (3) we find that the market is unaffected by proliferation with Full Control; demand and profit allocation remains the same as in the two-quality environment. β cannot be determined as the game ends at the stage of price competition. Given that $p_m = \bar{\theta}d(1 - \beta)/2$, a possible explanation of *Lemma 3* may be that, when setting quality-price decision jointly, H leaves the middle quality product in a dilemma. H does not wish the middle quality product to compete with the original high quality product, thus locates it further down the quality spectrum but charges a price that is significantly higher than the zero price of the low quality product. As a result, consumers maintain their original purchasing choices and D_m is zero.

Lemma 4. *With Semi-Full Control, the value of β is too high such that p_m is approaching zero. $\pi_H{}^{pro} > \pi_H$ does not hold, H has no incentive to proliferate.*

After independent price competition as presented in (5), H solves optimisation problem (4). Deriving the first-order condition with respect to β, we find the profit maximizing level of β under Semi-Full Control to be $15/11$. This means that the difference between s_m and s_l is infinitesimally small and the middle quality product in fact joins the Bertrand competition with the low quality segment, therefore π_m is approaching zero. Again, any positive f would lead to a decrease in π_H if proliferation happens.

Lemmata 3 and *4* imply that β is large whenever it is set by H, such that the middle quality product is located close to the low quality product to secure surplus generated by the high quality product. Therefore with Independent Control where β is no longer set by H, we expect β to be relatively closer to a half.

Lemma 5. *With Independent Control, $\beta = 3/7$ and $s_m = 3s_l/7 + 4s_h/7$. The middle quality product makes a positive profit.*

β is closer to the high quality product, which would never be the case with the first two controls. It is intuitive that given $p_l = 0$, s_m needs to be sufficiently differentiated from s_l in order for the middle quality product to be attractive. By solving proliferation with different controls we can conclude that only with Independent Control could the proliferated product generate positive profit, but it remains to be checked whether it can increase total profit.

Lemma 6. *With Independent Control, $\pi_H{}^{pro} = \bar{\theta}^2 d/6 - f$. $\pi_H{}^{pro} > \pi_H$ does not hold, hence H has no incentive to proliferate.*

Proof: See Appendix B.

Table 1 puts together the two sets of equilibrium outcomes for comparison.

Environment	Price			Demand			Profit		
	p_l	p_m	p_h	D_l	D_m	D_h	π_l	π_m	π_h
Two-Quality	0	-	$\dfrac{\bar{\theta}d}{2}$	$\dfrac{\bar{\theta}}{2}$	-	$\dfrac{\bar{\theta}}{2}$	0	-	$\dfrac{\bar{\theta}^2 d}{4}$
Proliferation (IC)	0	$\dfrac{\bar{\theta}d}{14}$	$\dfrac{\bar{\theta}d}{4}$	$\dfrac{\bar{\theta}}{8}$	$\dfrac{7\bar{\theta}}{24}$	$\dfrac{7\bar{\theta}}{12}$	0	$\dfrac{\bar{\theta}^2 d}{48}$	$\dfrac{7\bar{\theta}^2 d}{48}$

Table 1. A Comparison of Equilibrium Outcomes from Two-Quality Environment and Proliferation Environment with Independent Control (IC)

Total demand in the proliferation environment is $\bar{\theta}$, so the market is fully covered with and without proliferation. Through the introduction of the middle quality product, H has successfully broadened its market segment by taking away $3\bar{\theta}/8$ out of the total $4\bar{\theta}/8$ demand that the low quality product would have otherwise captured. Proliferation does not reduce the demand for the high quality product; however, it reduces its price. p_h is halved as a result, which contributes to the fall in total profit. Proliferation creates $\pi_m = \bar{\theta}^2 d/48$ but decreases π_h from $12\bar{\theta}^2 d/48$ to $7\bar{\theta}^2 d/48$, therefore is not desirable for H.

Lemmata 3, 4 and *6* lead to *Proposition 1*.

Proposition 1. *Proliferation is not profitable for H. Ceteris paribus, H will not initiate proliferation.*

Given that the most moderate level of proliferation of offering only one additional quality cannot be justified on profitability ground, *Proposition 1* also implies the undesirability of excessive proliferation. This is however, conditional on no external forces. Since proliferation is also a candidate for deterrence, it may be optimal for H to do so when the condition is relaxed.

2.3 The Entry Environment

This section examines H's incentive to proliferate in the existing two-quality market, so as to deter a potential entrant E. Upon entry, E makes quality-price decision to ensure positive post-entry surplus. This quality-price decision would be the same as $\{\beta = 3/7 , p_m = \bar{\theta} d/14\}$ obtained under Independent Control, leading to the same surplus of $\bar{\theta}^2 d/48$, shown in *Lemma 5* and Table 1 in Section 2.2. This is because while making decision, the independent unit maximizes its own profit (and not the full profits of its parent-firm) just like what an entrant would do. The only difference is that π_m generated under Independent Control contributes to the total profit of the parent-firm π_H^{pro}, whereas E gets to keep the profit. We assume that the fixed cost upon entry is small enough $k \in [0, \bar{\theta}^2 d/48]$, such that E always has an incentive to enter.

Suppose the distribution of θ is such that H's proliferation of the middle quality product can deter entry, that is, it is not profitable for E to enter with a fourth quality, then H has incentive to proliferate if the below is true

$$\pi_H^{pro} > \pi_H^{E}. \tag{8}$$

Proposition 2. *Whenever $f > \bar{\theta}^2 d/48$, $\pi_H{}^{pro} > \pi_H{}^E$ does not hold, hence proliferation is not profitable for H, with or without threat of entry. Whenever $f \in [0, \bar{\theta}^2 d/48]$, $\pi_H{}^{pro} > \pi_H{}^E$ holds, hence if facing threat of entry, H has incentive to proliferate and achieve a second best profit.*

Proof: when facing entry threats and anticipating the quality-price decision of E, H can choose to deter or to accommodate entry, depending on the profitability of the two strategies. If H proliferates by offering the middle quality product thus restricts the market space available, it could be impossible for E to enter. Table 2 represents firms' profits in different environments.

Environment	Profit of Firm		
	H	**E**	**L_1, L_2**
Two-Quality	$\dfrac{12\bar{\theta}^2 d}{48}$	-	0
Proliferation (IC)	$\dfrac{8\bar{\theta}^2 d}{48} - f$	0	0
Entry	$\dfrac{7\bar{\theta}^2 d}{48}$	$\dfrac{\bar{\theta}^2 d}{48} - k$	0

Table 2. Firms' Profits in Two-Quality, Proliferation (IC) and Entry Environments

Note that the profit of the low quality firms are zero in all three environments due to price competition. H obtains the highest profit in the two-quality environment. If entry takes place, H's profit would be $\pi_H{}^E = 7\bar{\theta}^2 d/48$ and if proliferation deters entry, H's profit would be $\pi_H{}^{pro} = 8\bar{\theta}^2 d/48 - f$. Hence regarding entry deterrence, whether proliferation is profitable depends on the magnitude of proliferation cost. If the cost is small enough, then proliferation strategy will become optimal when its anti-competitive effect is taken into consideration.

Compare to vertically differentiated markets, proliferation cost is likely to be smaller in horizontal differentiation markets where economies of scale and scope seem to work better. Given the same budget, it may be easier to demonstrate that something is different than something is better. This might be part of the reason that proliferation, especially excessive proliferation, is more often exercised in horizontal differentiation markets.

Proposition 2 shows that under certain condition, proliferation of the most moderate level could have anti-competitive effect and be used to achieve such purpose. Note that although credibility is a central issue in entry deterrence (Dixit, 1980; Judd, 1985), it is not the focus of

the current paper. We emphasize firms' incentive to proliferate to deter entry rather than whether entry is deterred as a result.

3. Welfare Analysis

This section examines how consumer surplus and total welfare change when the market has three instead of two qualities. Consumer surplus is generated from the consumption of each quality, recall function (1) of consumer utility.

In the two-quality market, consumer surplus consists of CS_h from consumers with $\theta \in [\theta_h, \bar{\theta}]$ and CS_l from consumers with $\theta \in [\theta_l, \theta_h)$, where

$$\left. \begin{array}{l} CS_h = \int_{\theta_h}^{\bar{\theta}} (\theta s_h - p_h)\, d\theta \\ CS_l = \int_{\theta_l}^{\theta_h} (\theta s_l - p_l)\, d\theta \end{array} \right\}. \tag{9}$$

In the three-quality market, consumer surplus consists of CS_h from consumers with $\theta \in [\theta_h, \bar{\theta}]$, CS_m from consumers with $\theta \in [\theta_m, \theta_h)$ and CS_l from consumers with $\theta \in [\theta_l, \theta_m)$, where

$$\left. \begin{array}{l} CS_h = \int_{\theta_h}^{\bar{\theta}} (\theta s_h - p_h)\, d\theta \\ CS_m = \int_{\theta_m}^{\theta_h} (\theta s_m - p_m)\, d\theta \\ CS_l = \int_{\theta_l}^{\theta_m} (\theta s_l - p_l)\, d\theta \end{array} \right\}. \tag{10}$$

Calculation: See Appendix C.

Summing up consumer surplus and firms' profits, we are able to compare total welfare in the following markets, as shown in Table 3.

Market	Welfare Comparison		
	Consumer surplus	Producer surplus	Total welfare
Two-quality	$\frac{1}{8}\bar{\theta}^2 s_h + \frac{3}{8}\bar{\theta}^2 s_l$	$\frac{1}{4}\bar{\theta}^2 d$	$\frac{3}{8}\bar{\theta}^2 s_h + \frac{1}{8}\bar{\theta}^2 s_l$
Three-quality (Proliferation)	$\frac{119}{288}\bar{\theta}^2 s_h + \frac{91}{1152}\bar{\theta}^2 s_m$ $+\frac{1}{128}\bar{\theta}^2 s_l - \frac{1}{6}\bar{\theta}^2 d$	$\frac{1}{6}\bar{\theta}^2 d - f$	$\frac{119}{288}\bar{\theta}^2 s_h + \frac{91}{1152}\bar{\theta}^2 s_m$ $+\frac{1}{128}\bar{\theta}^2 s_l - f$
Three-quality (Entry)	$\frac{119}{288}\bar{\theta}^2 s_h + \frac{91}{1152}\bar{\theta}^2 s_m$ $+\frac{1}{128}\bar{\theta}^2 s_l - \frac{1}{6}\bar{\theta}^2 d$	$\frac{1}{6}\bar{\theta}^2 d - k$	$\frac{119}{288}\bar{\theta}^2 s_h + \frac{91}{1152}\bar{\theta}^2 s_m$ $+\frac{1}{128}\bar{\theta}^2 s_l - k$

Table 3. Welfare Comparison

Since the third quality could be offered by proliferation or by the entrant, we include both of them here. Proliferation and entry achieve the same consumer surplus, which is greater than the consumer surplus in the two-quality market, whereas they decrease the industry profit.

Lemma 7. *Moving from two-quality to three-quality market, consumer surplus strictly increases and producer surplus strictly decreases, regardless of the values of f and k.*

Proof: See Appendix D.

As shown in Table 3, proliferation and entry affect welfare in the identical way if $f = k$. When the fixed cost is sufficiently small, e.g., $f = k = 0$, the rise in consumer surplus outweighs the fall in producer surplus, having the middle quality product is welfare-increasing. When the fixed cost is larger than a critical point, total welfare decreases as a result of large fall in producer surplus.

Proposition 3. *Whether total welfare increases or decreases with the third quality in the proliferation market and entry market depends on f and k, respectively. Hence proliferation is preferable from the social welfare point of view if and only if, $f < k$, despite its anti-competitive nature.*

Proof: See Appendix D.

From a social welfare point of view, whether the third quality should be offered by H or E depends on their relative cost of introducing it. If H benefits from the fact that it is already established therefore incur lower cost, then not only proliferation may increase welfare, it also does in a way that is more efficient than entry. This leads to an interesting result that, despite being anti-competitive, proliferation is not necessarily welfare-reducing.

4. Conclusion

Proliferation in horizontal differentiation markets is well understood. Since proliferation is also popular in vertical differentiation markets, and many brands, especially those in the fashion apparel industry, have been repeatedly engaging in joint-proliferations, it seems important to investigate proliferation under vertically differentiated setting. This paper provides some initial steps towards this goal.

We offer a rather complete evaluation of proliferation by assessing its effects on profitability, entry deterrence and welfare in a simplified yet intuitive model. We highlight the trade-off between softening competition and broadening market segments brought about by the proliferation and show that even the most moderate level of proliferation may be of anticompetitive concern. Nonetheless when proliferation is carried out as a division that competes independently with parent-firm, it is not necessarily welfare-reducing and could even be more efficient than entry.

In addition, we have extended the literature on entrant choice in duopoly markets following the *Principal of Maximal Differentiation* by endogenously determining the quality level of an additional product. The framework in the current paper can be positively taken forward by considering m high quality firms and n low quality firms ($1 < m < n$) in the market and allow proliferation to be jointly conducted by high and low quality firms and capturing the bargaining among firms. Given strategic interactions, proliferation may be justified as purely a strategy to increase profit.

References

Baye, M., K. Crocker and J. Ju (1996) "Divisionalization, Franchising, and Divestiture Incentives in Oligopoly," *American Economic Review*, Vol. 86, p.223-236.

Choi, C. and C. Scarpa (1991) "Credible Spatial Preemption through Reputation Extension," *International Journal of Industrial Organization*, Vol. 10, p. 439-447.

Choi, C. and S. Shin (1992) "A Comment on a Model of Vertical Product Differentiation," *Journal of Industrial Economics*, Vol. 60, p. 229-231.

Constantatos, C. and S. Perrakis (1997) "Vertical Differentiation: Entry and Market Coverage with Multiproduct Firms," *International Journal of Industrial Organization*, Vol. 16, p. 81-103.

Chun, E. and L. Niehm (2010) "Collaboration Strategies of Fashion Companies and Customer Attitudes," *Journal of Global Academy of Marketing Science*, Vol. 20-1, p. 4-14.

Dixit, A. (1980) "The Role of Investment in Entry deterrence,"*Economic Journal*, Vol. 90, p. 95-106.

Donnenfeld, S. and S. Weber (1992) "Vertical Product Differentiation with Entry," *International Journal of Industrial Organization*, Vol. 10, p. 449-472.

Donnenfeld, S. and S. Weber (1995) "Limit Qualities and Entry Deterrence," *RAND Journal of Economics*, Vol. 26, p. 113-130.

Gabszewicz, J. and J.-F. Thisse (1979) "Price Competition, Quality and Income Disparities," *Journal of Economic Theory*, Vol. 20, p. 340-359.

Judd, L. (1985) "Credible Spatial Preemption," *RAND Journal of Economics*, Vol. 16, p. 153-166.

Lane, W. (1980) "Product Differentiation in a Market with Sequential Entry," *Bell Journal of Economics*, Vol. 11, p. 237-260.

Murooka, T. (2012) "A Note on Credible Spatial Preemption in an Entry-Exit Game," *Economics Letters*, Vol. 118, p. 26-28.

Mussa, M. and S. Rosen (1978) "Monopoly and Product Quality," *Journal of Economic Theory*, Vol. 18, p. 301-317.

Noh, Y-H. and G. Moschini (2006) "Vertical Product Differentiation, Entry deterrence Strategies, and Entry Qualities," *Review of Industrial Organization*, Vol. 29, p. 227-252.

Shaked, A. and J. Sutton (1982) "Relax Price Competition through Product Differentiation," *Review of Economic Studies*, Vol. 49, p. 3-13.

Schmalensee, R. (1978) "Entry Deterrence in the Ready-to-Eat Breakfast Cereal Industry," *Bell Journal of Economics*, Vol. 9, p.305-328.

Tirole, J. (1988) *Theory of Industrial Organization*, Cambridge: The MIT Press.

Appendix

A. Proof of *Lemma 3* (Section 2.2)

Expand *H*'s optimisation problem (3)

$$\pi_H^{pro} = p_h \left(\overline{\theta} - \frac{p_h - p_m}{\beta d} \right) + p_m \left[\frac{p_h - p_m}{\beta d} - \frac{p_m}{(1-\beta)d} \right] - f.$$

The first-order conditions with respect to p_h and p_m are $\frac{\partial \pi_h}{\partial p_h} = \bar{\theta} - \frac{2p_h}{\beta d} + \frac{2p_m}{\beta d} = 0$; $\frac{\partial \pi_h}{\partial p_m} = \frac{2p_h}{\beta d} - \frac{2p_m}{\beta d} - \frac{2p_m}{(1-\beta)d} = 0$. The second-order conditions are fulfilled. Prices and demands in

terms of β and $\bar{\theta}$ are therefore $\begin{cases} p_h = \frac{\bar{\theta}d}{2} \\ p_m = \frac{\bar{\theta}d(1-\beta)}{2} \end{cases}$; $\begin{cases} D_h = \frac{\bar{\theta}}{2} \\ D_m = 0 \\ D_l = \frac{\bar{\theta}}{2} \end{cases}$. With Full Control, despite

positive p_m, demand allocation stays the same as in the two-quality environment. □

B. Proofs of Lemmata 4, 5 and 6 (Section 2.2)

The two quality units first engage in independent price competition as in (5). Derive the first-order conditions respectively and solve for prices, demands and profits in terms of β

and $\bar{\theta}$: $\begin{cases} p_h = \frac{2\bar{\theta}\beta d}{3+\beta} \\ p_m = \frac{\bar{\theta}\beta d(1-\beta)}{3+\beta} \end{cases}$, $\begin{cases} D_h = \frac{2\bar{\theta}}{3+\beta} \\ D_m = \frac{\bar{\theta}}{3+\beta} \\ D_l = \frac{\bar{\theta}\beta}{3+\beta} \end{cases}$ and $\begin{cases} \pi_h = \frac{4\bar{\theta}^2\beta d}{(3+\beta)^2} \\ \pi_m = \frac{\bar{\theta}^2\beta d(1-\beta)}{(3+\beta)^2} \\ \pi_l = 0 \end{cases}$. Since $\frac{\partial p_h}{\partial \beta} > 0$, p_h is higher if

s_m is located closer to s_l. When β is jointly set, for a high p_h, H has incentives to set a large β which in turn makes the middle quality product unattractive, therefore with Semi-Full Control, proliferation is not feasible.

When β is set independently to maximise unit profit not total profit

$$\frac{\partial \pi_m}{\partial \beta} = \frac{(3+\beta)^2(\bar{\theta}^2 d - 2\bar{\theta}^2\beta d) - 2\bar{\theta}^2\beta d(1-\beta)(3+\beta)}{(3+\beta)^4} = 0.$$

Solve for the above equation and $\beta = \frac{3}{7}$. All second-order conditions are fulfilled. The equilibrium outcomes are presented in Table 1 in Section 2.2. □

C. Calculations for Equations 9 and 10 (Section 3)

Expand and solve for (9), in the two-quality market

$$CS_h = \frac{1}{2}\bar{\theta}^2 s_h - \bar{\theta}p_h - \frac{1}{2}\theta_h^2 s_h + \theta_h s_h = \frac{3}{8}\bar{\theta}^2 s_h - \frac{1}{4}\bar{\theta}^2 d,$$

$$CS_l = \frac{1}{2}\theta_h^2 s_l = \frac{1}{8}\bar{\theta}^2 s_l,$$

$$CS = CS_l + CS_h = \frac{1}{8}\bar{\theta}^2 s_h + \frac{3}{8}\bar{\theta}^2 s_l.$$

Expand and solve for (10), in the three-quality market

$$CS_h = \frac{1}{2}\bar{\theta}^2 s_h - \bar{\theta}p_h - \frac{1}{2}\theta_h{}^2 s_h + \theta_h s_h = \frac{119}{288}\bar{\theta}^2 s_h - \frac{7}{48}\bar{\theta}^2 d,$$

$$CS_m = \frac{1}{2}\theta_h{}^2 s_m - \theta_h p_m - \frac{1}{2}\theta_m{}^2 s_m + \theta_m s_m = \frac{91}{1152}\bar{\theta}^2 s_m - \frac{1}{48}\bar{\theta}^2 d,$$

$$CS_l = \frac{1}{2}\theta_m{}^2 s_l = \frac{1}{128}\bar{\theta}^2 s_l,$$

$$CS = CS_l + CS_m + CS_h = \frac{119}{288}\bar{\theta}^2 s_h + \frac{91}{1152}\bar{\theta}^2 s_m + \frac{1}{128}\bar{\theta}^2 s_l - \frac{1}{6}\bar{\theta}^2 d. \qquad \Box$$

D. Proofs of *Lemma 7* and *Proposition 3* (Section 3)

Moving from two-quality to three-quality market, consumer surplus strictly increases if

$$\frac{119}{288}\bar{\theta}^2 s_h + \frac{91}{1152}\bar{\theta}^2 s_m + \frac{1}{128}\bar{\theta}^2 s_l - \frac{1}{6}\bar{\theta}^2 d > \frac{1}{8}\bar{\theta}^2 s_h + \frac{3}{8}\bar{\theta}^2 s_l.$$

That is, if

$$\frac{140}{1152}s_h + \frac{91}{1152}s_m > \frac{231}{1152}s_l,$$

which always holds since $\frac{140}{1152}s_h + \frac{91}{1152}s_m > \frac{140}{1152}s_m + \frac{91}{1152}s_m$ and $\frac{231}{1152}s_m > \frac{231}{1152}s_l$.

Moving from two-quality to three-quality market, producer surplus strictly decreases if $\frac{1}{4}\bar{\theta}^2 d > \frac{1}{6}\bar{\theta}^2 d - f$ and $\frac{1}{4}\bar{\theta}^2 d > \frac{1}{6}\bar{\theta}^2 d - k$, which always holds since $\frac{1}{4} > \frac{1}{6}$.

Moving from two-quality to three-quality market with proliferation, total welfare strictly increases if

$$\frac{119}{288}\bar{\theta}^2 s_h + \frac{91}{1152}\bar{\theta}^2 s_m + \frac{1}{128}\bar{\theta}^2 s_l - f > \frac{3}{8}\bar{\theta}^2 s_h + \frac{1}{8}\bar{\theta}^2 s_l.$$

That is, if

$$\frac{44}{1152}\bar{\theta}^2 s_h + \frac{91}{1152}\bar{\theta}^2 s_m - \frac{135}{1152}\bar{\theta}^2 s_l > f.$$

The left hand side of the above inequation is strictly positive since $\frac{44}{1152} s_h + \frac{91}{1152} s_m >$ $\frac{44}{1152} s_m + \frac{91}{1152} s_m$ and $\frac{135}{1152} s_m > \frac{135}{1152} s_l$. Whether the inequation holds depends on the value of f. When $f < \frac{44}{1152} \overline{\theta}^2 s_h + \frac{91}{1152} \overline{\theta}^2 s_m - \frac{135}{1152} \overline{\theta}^2 s_l$, total welfare is higher in the three-quality market with proliferation than in the two-quality market.

Similarly, when $k < \frac{44}{1152} \overline{\theta}^2 s_h + \frac{91}{1152} \overline{\theta}^2 s_m - \frac{135}{1152} \overline{\theta}^2 s_l$, total welfare is higher in the three-quality market with entry than in the two-quality market. Compare the total welfare in the market with proliferation and the market with entry, it is higher in the market with proliferation if and only if $f < k$. □