Innovation Races with the Possibility of Failure

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Abstract

The standard innovation race specification assumes a memoryless exponential distribution for the time to success of an R&D project. This specification implies that a project succeeds, eventually, with probability one. We introduce a positive probability that an R&D project fails. With this modified specification, we compare the non-cooperative and cooperative R&D in terms of innovation effort, consumer surplus, and net social welfare.

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If at first you don't succeed, try try again.

<div align="right">Proverb.</div>

1 Introduction

Adam Smith made productivity growth a central theme of *The Wealth of Nations*, but it was Joseph Schumpeter's diverse views on the economics of innovation that set the broad outlines of the debate that continues to the present day.

In 1934, Schumpeter argued that it was the new firm that carried out innovation (p. 66):

> ...new combinations are, as a rule, embodied, as it were, in new firms which generally do not arise out of the old ones but start producing beside them; ...in general it is not the owner of stage-coaches who builds railways.

In 1943, in contrast, he saw dominant firms as the source of technological advance (p. 82):[1]

> As soon as we go into details and inquire into the individual items in which progress was most conspicuous, the trail leads not to the doors of those firms that work under conditions of comparatively free competition but precisely to the doors of the large concerns ... and a shocking suspicion dawns upon us that big business may have had more to do with creating that standard of life than with keeping it down.

Policymakers' interest in the economics of innovation was reinvigorated by the oil shocks of the 1970s and the decline in productivity growth in industrialized countries in the latter part of the 20th century. Governments, seeking budget-friendly strategies to jump-start their economies, turned to the promotion of innovation. One such approach was to embrace R&D cooperation (Martin and Scott, 2000), a shift away from the market-economy presumption that it is competition that promotes the efficient allocation of resources.[2] One consequence of this policy interest, from the early 1980s,

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[1]Winter (1984) refers to Schumpeter's contrasting views as Schumpeter Mark I (1934) and Schumpeter Mark II (1943).

[2]By competition, we mean (using lay terminology) rivalry, not perfect competition in the classroom sense (although economists would of course take perfect competition if ever they could get it). For an early discussion of economists' use of the term "competition," see McNulty (1968).

is a large literature in economics that investigates the circumstances under which R&D cooperation can be expected to promote innovation.

Uncertainty is an intrinsic characteristic of the innovation process. An R&D project may fail technologically (and this is our focus);[3] an R&D project may succeed from an engineering or scientific point of view but fail commercially.[4] Some approaches to achieve a research goal will be more promising, some less, and, *a priori*, it will be uncertain which is which. It may be uncertain if a research goal can be achieved *at any cost* (Dasgupta and Maskin, 1987, p. 582, fn. 2). Some of this uncertainty may resolve itself over time, as early research results generate knowledge that permits improved assessments of the probability of different states of the world.[5]

Static models of innovation, in the interest of tractability, abstract from uncertainty entirely (d'Aspremont & Jacquemin, 1988; Kamien *et al.*, 1992). Innovation race models admit uncertainty, but in a limited way.[6] The typical innovation race model examines a cost-saving innovation of known magnitude, with an expected time of completion related to R&D expenditures in a known way and with a random time to discovery distributed in such a way that eventually, an R&D project must succeed.

In this paper, we examine the extent to which R&D cooperation can be expected to promote innovation in the presence of uncertainty if one relaxes that aspect of the standard specification which implies that eventual success of an R&D project is certain. We introduce the possibility of project failure by making "completion of the project" a lottery:

- with probability p, an R&D project succeeds, and the aftermath is as in the standard innovation race model;

- with probability $1-p$, an R&D project fails, and the firm has the option of starting a new project.

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[3]Examples include attempts to develop remedies for incurable diseases and attempts to develop low cost renewable fuels, among others.

[4]Richard Maulsby (Director of public affairs for the U.S. Patent & Trademark Office, quoted in Klein, 2005): "There are around 1.5 million patents in effect and in force in this country, and of those, maybe 3,000 are commercially viable." See also Kozinsin *et al.* (2000), Che and Gale (2003), Lee and Park (2006), Scherer (2007, p. 17), and Wilson (2007).

[5]Research results may also reveal the existence of previously unsuspected states of the world. Zeckhauser (2006), although touching only briefly on innovation, is to the point.

[6]Loury (1979); Lee & Wilde (1980); Reinganum (1983). For a survey of the innovation race literature, see Reinganum (1989). Martin (1996a, b) discusses efficiency and welfare aspects of R&D joint ventures; Hinloopen (1997, 2000) highlights policy implications. Chowdhury (2010) models the concept of Loury (1979) as an all-pay auction in which to be 'successful' the highest bid needs to be above a random threshold level.

With this framework, we compare monopoly, duopoly, and R&D joint venture incentives to invest in R&D.

To anticipate our main results, if the probabilities of success of successive research projects are independently and identically distributed, as specified below, eventual success of some project is certain, although any one project may fail. Monopoly innovation effort rises with the probability of success and the magnitude of the reduction in cost that follows from successful innovation. Duopoly R&D efforts are strategic complements, and equilibrium duopoly R&D efforts, like monopoly R&D effort, rise with the probability of success. Equilibrium duopoly R&D effort exceeds equilibrium monopoly R&D effort, all else equal, and equilibrium monopoly R&D effort exceeds equilibrium R&D effort of a R&D joint venture. But a joint venture will find R&D profitable at higher levels of sunk cost per project than will either monopoly or duopoly.

When we extend the basic model to allow multiple R&D projects per firm, monopoly and joint venture R&D intensity per project rises, and the number of R&D projects falls, as the probability of success rises. To state the same result in the opposite way, the lower the probability of success of any one R&D project, the more the monopolist or joint venture pursues success by diversifying the number of R&D projects undertaken, running each project less intensely. In contrast, as the probability of success of an individual R&D project rises, the number of R&D projects per firm in noncooperative duopoly rises, as does R&D intensity per project.

As far as R&D effort is concerned, the qualitative results of the single-R&D project per firm model generalize to the case of multiple R&D projects per firm. Equilibrium R&D effort per firm is least with an R&D joint venture, greatest with noncooperative duopoly R&D.

In numerical examples, monopoly R&D yields the least consumer surplus, a joint venture the most consumer surplus and net social welfare. For the latter result, product market considerations trump the innovation effects emphasized by Schumpeter (1943). A duopoly joint venture slows innovation, but both firms have equal access to the same technology before and after innovation, increasing flow consumer surplus in the post-innovation market. Noncooperative duopoly R&D, which yields the greatest research effort, produces the least net social welfare.

In Section 2 we outline the analytical framework used throughout the paper. In Section 3 we analyze R&D intensity for monopoly, noncooperative duopoly, and a joint venture if each firm, or a joint venture, undertakes at most one R&D project at a time. In Section 4 we make the corresponding comparisons if firms run an endogenous number of parallel R&D projects. In Section 5 we examine market performance from a welfare point of view.

Section 6 concludes. Proofs are in the Appendix.

2   Setup

The initial technology has constant average and marginal cost c_A per unit of output. A firm may undertake a research project to develop a new technology that permits production at average and marginal cost $c_B < c_A$.

If a firm begins an R&D project, it makes a sunk investment S. If the project fails and the firm begins a new research project, the new project again entails a sunk investment S.[7]

An R&D project at effort level h has a flow cost $z(h)$. $z(h)$ has positive first and second derivatives,

$$z'(h) > 0, z''(h) > 0. \tag{1}$$

The time τ to completion of a project has an exponential distribution, with constant success parameter h:

$$\Pr(\tau_i \leq t) = 1 - e^{-ht}. \tag{2}$$

We assume that the times to completion of successive projects are independently and identically distributed.[8]

We modify the standard formulation by making the result of completing a project a lottery. With exogenous probability p a project completes successfully, and it becomes possible to produce at unit cost $c_B < c_A$. With probability $1 - p$, the project completes unsuccessfully. If the project fails, the sunk cost of starting a second research project is S. If it was profitable to make an initial sunk investment S and begin a project at time zero, it will be profitable to make a second sunk investment and begin new project, if the first project should fail. The analysis yields conditions under which it is profitable for a firm to make an initial sunk investment in cost-saving innovation.

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[7]One can show that if successive research projects do not require sunk investment, the innovation race model with the possibility of failure is isomorphic to the standard (no possibility of failure) innovation race model.

[8]These assumptions are standard. Fudenberg *et al.* (1983) and Doraszelski (2003) discuss innovation races without the memoryless property implied by the exponential distribution with constant success parameter.

3 R&D intensity: one R&D project per firm

3.1 Monopoly

3.1.1 Objective function

Let π_i denote the firm's flow rate of profit if it produces at unit cost c_i, for $i = A, B$. A monopolist's present-discounted value is defined by the recursive relationship

$$V^M = -S + \int_{t=0}^{\infty} e^{-(r+h)t} \left\{ \pi_A - z(h) + h \left[p\frac{\pi_B}{r} + (1-p) V^M \right] \right\} dt, \quad (3)$$

where r is the rate of time preference used to discount future payoffs.

At time 0, by making a sunk investment S, the firm can begin an R&D project. With probability density e^{-ht}, the project is not completed at time t, and the firm's flow payoff is monopoly profit minus the cost of R&D, $\pi_A - z(h)$. With probability density he^{-ht}, the project is completed at time t. If the project is completed at time t, with probability p the project is successful, and the firm's value from that moment forward is $\frac{\pi_B}{r}$. With probability $1-p$, the project is completed and fails. If the project is completed and fails, the firm's situation at the moment the project fails is identical to its situation at time zero; its value is V^M. The probabilistic payoffs are appropriately discounted.

Carrying out the integration in (3) and rearranging terms gives the monopolist's objective function,

$$V^M = -S + \frac{\pi_A - z(h) + h \left[p\frac{\pi_B}{r} - (1-p) S \right]}{r + hp}. \quad (4)$$

The interpretation of the right-hand side is that to start the first research project, the firm makes sunk investment S. If the project is not complete, the firm's cash flow is the first two terms of the numerator of the fraction on the right. If the project is complete, which happens with probability proportional to h, and is successful, which happens with probability p, the firm's value from that time onward is $\frac{\pi_B}{r}$. If the project is complete and unsuccessful, which happens with probability $1-p$, the firm makes sunk investment S and continues with a new research project.

3.1.2 Expected time to successful discovery

"First successful outcome on the nth project" is a discrete random variable with geometric probability distribution. The probability that the first project

is successful is p. The probability that the first success is with the second project is qp, where $q = 1 - p$ is the probability of failure of a completed project. The probability that the first success is with the nth project is $q^{n-1}p$, and so on.

The probability that discovery occurs on one of the projects is

$$p + qp + ... + q^{n-1}p... = p\left(1 + q + ...\right) = \frac{p}{1-q} = \frac{p}{p} = 1. \tag{5}$$

Although any individual project may fail, if the firm undertakes a long-enough sequence of projects, eventually one of the projects succeeds.

For the geometric distribution, the expected number of trials to successful completion is

$$\frac{1}{p}. \tag{6}$$

With an exponential distribution, the expected time to completion of a single trial is

$$\frac{1}{h}. \tag{7}$$

We assume that the probability of completion and the probability of success, given completion, are independent.[9] The expected time to successful completion is then the product of the means of the two distributions, that is,[10]

$$\frac{1}{p}\frac{1}{h} = \frac{1}{ph}. \tag{9}$$

3.1.3   First-order condition

The first-order condition to maximize (4) is

$$\frac{\partial V^M}{\partial h} = \frac{(r + ph)\left[-z'(h) + p\frac{\pi_B}{r} - (1 - p)S\right] - pNUM^M}{(r + ph)^2} \equiv 0, \tag{10}$$

---

[9]This is a natural simplifying assumption, in view of the fact that we treat p as constant. But see footnote 27.

[10]This may also be shown directly. Assume that the times to completion of successive trials are independently and identically distributed. With probability p, the first project succeeds, and the expected time to successful completion on the first project is $\frac{1}{h}$. With probability qp, the first project fails and the second project succeeds; the expected time to successful completion on the second project is $\frac{2}{h}$. Proceeding in this way, the expected time to successful completion is

$$E(T) = p\frac{1}{h} + qp\left(\frac{2}{h}\right) + q^2p\left(\frac{3}{h}\right) + ... = \frac{p}{h}\left(1 + 2q + 3q^2 + ...\right) = \frac{p}{h}\frac{1}{p\left(1-q\right)} = \frac{1}{ph}. \tag{8}$$

where
$$NUM^M = \pi_A - z(h) + h\left[p\frac{\pi_B}{r} - (1-p)S\right] \qquad (11)$$

is the numerator of the fraction on the right in (4).[11]

Where the first-order condition holds, the monopolist's value is

$$V^M = \frac{\pi_B}{r} - \frac{z'(h^M) + S}{p}, \qquad (12)$$

where h^M is the R&D intensity determined by the first-order condition.

The first-order condition (10) can be rewritten as

$$z(h) - \left(\frac{r}{p} + h\right)z'(h) + x^M - \frac{1-p}{p}rS \equiv 0, \qquad (13)$$

where we write

$$x^M = \pi_B - \pi_A \qquad (14)$$

for the flow increase in profit from adopting the lower-cost technology. (13) implicitly defines the monopolist's profit-maximizing R&D intensity, h^M.

3.1.4 Monopoly comparative statics

Straightforward manipulations show

> **Lemma 1**: (Monopoly equilibrium) (a) the second-order condition for value maximization is satisfied,
>
> $$\left.\frac{\partial^2 V^M}{\partial h^2}\right|^* = -\frac{z''(h^M)}{r + ph^M} < 0, \qquad (15)$$
>
> where the asterisk denotes an equilibrium value and (1) means that the numerator on the right is positive.
>
> (b) Equilibrium monopoly R&D intensity rises with the probability of success,
>
> $$\frac{\partial h^M}{\partial p} = \frac{1}{p}\frac{r}{r + ph^M}\frac{z'(h^M) + S}{z''(h^M)} > 0. \qquad (16)$$
>
> (c) The greater the increase in flow profit that results from successful innovation, the higher the equilibrium monopoly level of innovation effort,
>
> $$\frac{\partial h^M}{\partial x^M} = -\frac{p}{r + ph^M}\frac{1}{z''(h^M)} > 0. \qquad (17)$$

[11]The second-order condition for a maximum, which is satisfied, is considered in the following section.

3.2 Duopoly

3.2.1 Objective functions

Now let there be two firms, 1 and 2. Initially, both firms produce with unit cost c_A and collect noncooperative equilibrium flow profit π_A^D.

The model of innovation is as in the monopoly case. To maintain our focus on the consequences of adding a positive probability of failure of individual R&D projects to the innovation race model, we exclude R&D effort spillovers and incomplete appropriability of R&D output: the first successful completer obtains an infinitely-lived patent that denies its rival the use of the new technology.[12]

Consider firm 1's payoffs in different states of the world. If firm 1 is the first completer, it completes successfully with probability p, and in this case its value is $\frac{\pi_W}{r}$, where π_W is firm 1's flow rate of duopoly profit if it produces with lower unit cost c_B and firm 2 produces with higher unit cost c_A.

With probability $1-p$, firm 1 completes unsuccessfully. If it was profitable to begin the original project, it will be profitable to start again. The value of a new research project is V^{1D}, where V^{1D} is firm 1's value at time zero, since the firm does not need to make a sunk investment in a laboratory. Thus if firm 1 completes first, its expected value is

$$p\frac{\pi_W}{r} + (1 - p) V^{1D}. \tag{18}$$

(18) is analogous to a monopolist's expected value if it completes an R&D project (see the coefficient of h under the integral sign in (3)).

If firm 2 completes its project first, there are again two possibilities for firm 1's value. With probability p, firm 2's project is successful. Firm 1's value from the moment firm 2 successfully completes is $\frac{\pi_L}{r}$, where π_L is firm 1's flow rate of duopoly profit if it produces with higher unit cost c_A and firm 2 produces with lower unit cost c_B.[13] With probability $1 - p$, firm 2's project is unsuccessful. At the moment firm 2 completes unsuccessfully, firm 1's value is

$$V^{1D} + S, \tag{19}$$

---

[12]The assumption that a patent has infinite life is not essential to the qualitative nature of the results that follow. We also assume that the winner of the innovation race does not license use of the new technology to the loser. Admitting this possibility would change the details of post-innovation payoffs but would not alter the general nature of the results. See Martin (2002) for a model of an innovation race without the possibility of failure that includes licensing, R&D input spillovers and imperfect appropriability.

[13]If the high-cost firm would earn negative equilibrium profit in the post-innovation market, it would shut down, making $\pi_L = 0$. For numerical examples, we consider nondrastic innovation ($\pi_L > 0$).

the value of a firm with an ongoing research project, and no need to make any sunk investment. Thus if firm 2 is the first completer, firm i's expected value is

$$p\frac{\pi_L}{r} + (1-p)\left(V^{1D} + S\right). \tag{20}$$

There is no analogue to (20) in the monopoly case.

Weighting the discounted payoffs in different states of the world by the appropriate probability densities and carrying out the integration, firm 1's value V^{1D} satisfies the recursive relationship

$$V^{1D} + S =$$

$$= \frac{\pi_A^D - z(h_1) + h_1\left[p\frac{\pi_W}{r} + (1-p)V^{1D}\right] + h_2\left[p\frac{\pi_L}{r} + (1-p)\left(V^{1D} + S\right)\right]}{(r + h_1 + h_2)}. \tag{21}$$

Combining terms gives firm 1's duopoly objective function,

$$V^{1D} = -S + \frac{\pi_A^D - z(h_1) + h_1\left[p\frac{\pi_W}{r} - (1-p)S\right] + h_2 p\frac{\pi_L}{r}}{r + p(h_1 + h_2)}. \tag{22}$$

Firm 2's objective function can be obtained by appropriately permuting subscripts.

To explain the terms on the right-hand side, to begin a first research project, firm 1 makes sunk investment S. If neither firm has completed, firm i's flow income is $\pi_A^D - z(h_1)$.

The probability density that firm 1 completes first is proportional to $h_1 e^{-(h_1 + h_2)}$. If firm 1 completes successfully, something that happens with probability p, its value from that point is $\frac{\pi_W}{r}$. If firm 1 completes unsuccessfully, something that happens with probability $1-p$, it makes sunk investment S and begins a new project.

The probability that firm 2 completes first is proportional to $h_2 e^{-(h_1 + h_2)}$. If firm 2 completes successfully, something that happens with probability p, firm 1's value from that point is $\frac{\pi_L}{r}$. If firm 2 completes unsuccessfully, something that happens with probability $1 - p$, firm 1 simply continues its ongoing project.

3.2.2 Expected time to successful discovery

Equilibrium is symmetric. Write h^D for the equilibrium level of R&D intensity. The probability that firm i's project is completed by time t is

$$\Pr\left(\tau_i \leq t\right) = 1 - e^{h^D t}. \tag{23}$$

10

Then the probability that firm i's project is not complete by time t is

$$\Pr(\tau_i \geq t) = e^{h^D t}. \tag{24}$$

We assume the probability of success of different R&D projects is independent. Then the probability that neither firm's project is complete by time t is

$$\Pr(\tau_1 \geq t)\Pr(\tau_2 \geq t) = e^{2h_D t}, \tag{25}$$

and the probability that at least one project is complete by time t is

$$\Pr(\tau_1 \leq t \text{ or } \tau_2 \leq t) = 1 - e^{2h_D t}. \tag{26}$$

By the same kind of argument made in discussion of the expected time to discovery under monopoly, the expected time to first completion of one of the two projects is

$$\frac{1}{2h^D}, \tag{27}$$

and the expected time to successful discovery by one of the two firms is

$$\frac{1}{2ph^D}. \tag{28}$$

3.2.3 First-order conditions

The first-order condition to maximize firm 1's objective function, (22), with respect to h_1 is

$$\frac{\partial V^{1D}}{\partial h_1} = \frac{\left[r + p\left(h_1 + h_2\right)\right]\left[p\frac{\pi_W}{r} - z'(h_1) - (1-p)S\right] - pNUM^{1D}}{\left[r + p\left(h_1 + h_2\right)\right]^2} \equiv 0. \tag{29}$$

where

$$NUM^{1D} = \pi_A^D - z(h_1) + h_1\left[p\frac{\pi_W}{r} - (1-p)S\right] + h_2 p\frac{\pi_L}{r} \tag{30}$$

is the numerator on the right in (22).

Where the first-order condition holds, firm 1's value is

$$V^{1D} = \frac{\pi_W}{r} - \frac{z'(h_1) + S}{p} \tag{31}$$

We discuss the condition that must be satisfied for both firms to engage in R&D in Section 3.4. Here we remark that if it is profitable for both firms to undertake R&D, it must be that

$$V^{1D} > \frac{\pi_L}{r}. \tag{32}$$

11

The right-hand side is the value of the loser in the innovation race, from the moment the rival successfully completes. The left-hand side is an expected value that puts positive weight on the possibility of winning the innovation race, and so must be greater than the losing value.

It follows from (32) that

$$\frac{y^D}{r} - \frac{z'(h^D) + S}{p} > 0$$

or

$$p\frac{y^D}{r} - (1-p)S - z'(h^D) > pS > 0, \tag{33}$$

where

$$y^D = \pi_W - \pi_L, \tag{34}$$

is the difference in flow rates of profit, after discovery, between the winner and the loser.

In (33), $p\frac{y^D}{r} - (1-p)S$ is the expected payoff to first completion of a research project, relative to not undertaking research: incremental value $\frac{y^D}{r}$ if the project is successful, sunk investment S in a subsequent project if the project is not successful. $z'(h^D)$ is the flow marginal cost of a research project. (33) is used in the derivation of comparative static results and to motivate one of the assumptions of Theorem 4.

3.2.4   Duopoly equilibrium

Simplify the first-order condition (29) to obtain

$$z(h_1) - \left(\frac{r}{p} + h_1 + h_2\right) z'(h_1) + ph_2 \left(\frac{y^D}{r} - \frac{1-p}{p}S\right) + x^D - \frac{1-p}{p}rS \equiv 0. \tag{35}$$

for

$$x^D = \pi_W - \pi_A^D, \tag{36}$$

the increase in flow profit from successful completion. (35) implicitly defines firm 1's R&D best response function.

Setting $h_1 = h_2 = h^D$ in (35) gives the equation that characterizes duopoly equilibrium R&D intensity.

$$z(h^D) - \left(\frac{r}{p} + 2h^D\right) z'(h^D) + ph^D \left(\frac{y^D}{r} - \frac{1-p}{p}S\right) + x^D - \frac{1-p}{p}rS \equiv 0. \tag{37}$$

Some of the properties of duopoly equilibrium are given in Lemma 2.

Lemma 2: (Duopoly equilibrium)

(a) R&D best-response curves slope upward in the neighborhood of equilibrium,

$$\left.\frac{\partial h_1}{\partial h_2}\right|_{1's\ brf} > 0, \tag{38}$$

(b) the second-order condition for value maximization is satisfied,

$$\left.\frac{\partial^2 V^{1D}}{\partial h_1^2}\right|^* = -\frac{z''(h^D)}{r + 2ph_D} < 0. \tag{39}$$

(c) Firms' R&D intensities are strategic complements,

$$\frac{1}{p}\left.\frac{\partial^2 V^{1D}}{\partial h_1 \partial h_2}\right|^* > 0. \tag{40}$$

(d) h^D rises with p, x^D, and y^D.

Lemma 2(a) and the first part of Lemma 2(d) are illustrated in Figure 1, which shows duopoly best-responses curves for linear market demand, constant marginal production cost, quadratic R&D cost, and two values of p.[14] R&D best response curves slope upward, and h^D rises as p rises.

3.2.5 Monopoly vs. duopoly research intensity

Our next result outlines conditions under which research intensity per firm is greater in duopoly than in monopoly. Monopolist and duopolist both gain from innovation, and something like the Arrow *replacement effect* is a sufficient condition make duopoly research intensity greater than monopoly research intensity.[15] But independent of the replacement effect, an oligopolist in a technologically progressive market has an incentive to invest in innovation that a monopoly supplier of the same market does not, namely, the flow of profit that is lost is some other firm innovates first.

---

[14]Figure 1 is drawn for $P = 100 - Q$, $c_A = 30$, $c_B = 15$, $\delta = 1000$, $r = 1/20$, and $S = 2500$.

[15]Arrow (1962) noted that the post-innovation profit of a successfully innovating monopolist partially replaces pre-innovation profit, with the result that a firm in a perfectly competitive market stands to gain more from innovation than would a monopolist of the same industry, all else equal.
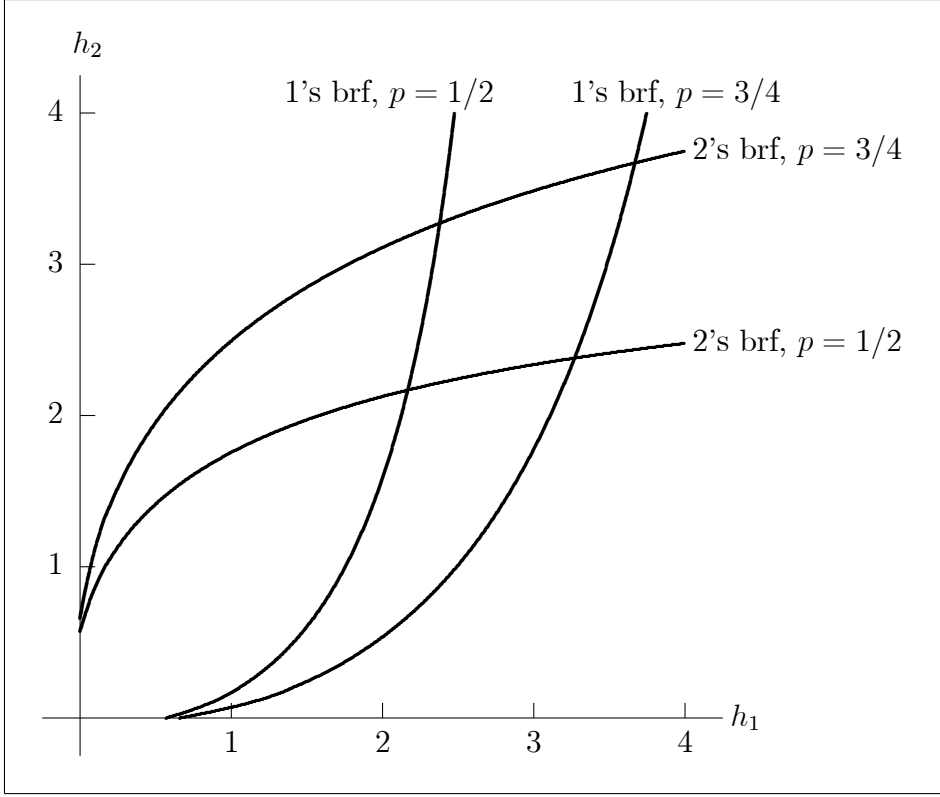
Figure 1: R&D best responses, linear demand, constant marginal production cost, quadratic R&D cost, $p = 1/2$ and $p = 3/4$.

Theorem 3: Let h^α denote the solution of the weighted average of the equation that defines h^M, (13), and the equation that defines h^D, (37):

$$z(h^\alpha) - \left[\frac{r}{p} + (1 + \alpha) h^\alpha\right] z'(h^\alpha)$$

$$+\alpha p h^\alpha \left(\frac{y^D}{r} - \frac{1-p}{p} S\right) + (1 - \alpha) x^M + \alpha x^D - \frac{1-p}{p} r S = 0. \quad (41)$$

Then

$$\frac{\partial h^\alpha}{\partial \alpha} = \frac{h\left[p\frac{y^D}{r} - z'(h) - (1 - p) S\right] + x^D - x^M}{\left[\frac{r}{p} + (1 + \alpha) h\right] z''(h) - \alpha \left[p\frac{y^D}{r} - (1 - p) S - z'(h)\right]}. \quad (42)$$

Assume:

(a) a duopolist gains at least as much from successful innovation than would a monopolist of the same market,

$$x^D - x^M \geq 0; \tag{43}$$

(b) the profitability condition (33),

$$p\frac{y^D}{r} - z'(h^\alpha) - (1-p)S > 0, \tag{44}$$

holds for $0 \leq \alpha \leq 1$; and
(c) the stability condition

$$\left[\frac{r}{p} + (1+\alpha)h^\alpha\right] z''(h) - \alpha\left[p\frac{y^D}{r} - z'(h^\alpha) - (1-p)S\right] > 0, \tag{45}$$

holds for $0 \leq \alpha \leq 1$;
Then duopoly R&D intensity per firm exceeds monopoly R&D intensity.

$$h^D > h^M. \tag{46}$$

Inequality (43) is the condition for an oligopoly version of the Arrow (1962) replacement effect to hold, so that a duopolist gains more from successful innovation than would a monopolist of the same market.

Figure 2 shows equilibrium monopoly and duopoly R&D intensity as functions of p for linear demand, constant marginal production cost, and quadratic R&D cost. Inequality (43), $x^D \geq x^M$, is a sufficient but not a necessary condition for Theorem 3. For the parameter values used to draw Figure 2, $x^D < x^M$. The tendency of $x^M > x^D$ to induce greater monopoly R&D is outweighed by R&D-promoting incentive of y^D, the flow profit lost in oligopoly if a rival is the first to successfully complete an R&D project. Further, $h^D - h^M$ rises with p: a higher value of p means not only that a firm's R&D project is more likely to be successful, but that the rival's R&D project is more likely to be successful, increasing the incentive to invest in R&D.

Considering our results relating R&D intensity and expected time to discovery (Sections 3.1.2 and 3.2.2), it follows from Theorem 3 that expected time to successful discovery is less with duopoly than with monopoly, a result that favors Schumpeter (1934) over Schumpeter (1943).

3.3 R&D joint ventures

There are many taxonomies of R&D joint ventures.[16] Here we examine the implications for technological performance if duopolists form an operating-entity joint venture, each paying half the cost of an R&D project, and each having access to new technology in the post-innovation market, keeping all other aspects of the specification unchanged.[17,18]

3.3.1 Objective function

Assuming noncooperative product-market behavior,[19] the combined value of the two firms that fund an operating-entity joint venture is defined recursively by the equation

$$2V^{JV} = -S+$$

$$\int_{t=0}^{\infty} e^{-(r+h^{JV})t} \left\{ 2\pi_A^D - z(h^{JV}) + h^{JV} \left[p\frac{2\pi_B^D}{r} + (1-p)\left(2V^{JV}\right) \right] \right\} dt. \quad (47)$$

In words, if the joint venture has not completed, flow payoffs are $2\pi_A^D - z(h^{JV})$. If the venture completes successfully, an event that happens with probability density proportional to ph^{JV}, the combined value of the two firms is $\frac{2\pi\pi_B^D}{r}$. If the venture completes unsuccessfully, an event that happens with probability density proportional to $(1-p)\,h^{JV}$, the joint venture makes a sunk investment S and begins a new project; the value of the operating entity joint venture is again $2V^{JV}$.

Carrying out the integration and combining terms gives the objective

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[16]See, for example Ouchi (1989), Kamien *et al.* (1992), and Vonortas (1994). See also Martin (1996a, b) and Hinloopen (1997, 2000).

[17]Hinloopen (2009) examines the consequences if formation of a joint venture increases the probability of success of an R&D project, compared with noncooperative R&D.

[18]One might also consider a secretariat joint venture (each firm conducts its own R&D project, for which it pays. If either project succeeds, both firms have access to the new technology) or hybrid forms of R&D cooperation (simultaneous operation of jointly-financed and individually-financed R&D projects). Results for the secretariat joint venture case are available from the authors on request. For the specific functional forms we use to illustrate our results, an operating-entity joint venture is always more profitable than a secretariat joint venture, and for that reason we limit attention in this paper to operating-entity joint ventures.

[19]Cooperation in R&D may facilitate product-market cooperation (Martin (1996a); Suetens (2008); Goeree and Helland (2010) Duso *et al.* (2010)). If firms collude perfectly in the product market before and after innovation, and form an operating-entity joint venture, the situation of the two firms is that of a monopolist.

function of an operating-entity joint venture:

$$2V^{JV} = -S + \frac{2\pi_A^D - z(h^{JV}) + h^{JV}\left[p\frac{2\pi_B^D}{r} - (1-p)S\right]}{r + ph^{JV}}. \tag{48}$$

3.3.2 First-order condition

The first-order condition to maximize (48) is

$$\frac{\partial\left(2V^{JV}\right)}{\partial h^{JV}} = \frac{\left(r + ph^{JV}\right)\left[p\frac{2\pi_B^D}{r} - z'(h^{JV}) - (1-p)S\right] - pNUM^{JV}}{\left(r + ph^{JV}\right)^2} \equiv 0 \tag{49}$$

where NUM^{JV} is the numerator on the right in (48).

Where first-order condition holds, the joint venture's value is

$$2V^{JV} = \frac{2\pi_B^D}{r} - \frac{z'(h^{JV}) + S}{p}. \tag{50}$$

Simplifying the first-order condition gives the equation that determines equilibrium operating-entity joint venture R&D intensity,

$$z(h^{JV}) - \frac{r + ph^{JV}}{p}z'(h^{JV}) + 2x^{JV} - \frac{1-p}{p}rS \equiv 0, \tag{51}$$

where

$$x^{JV} = \pi_B^D - \pi_A^D \tag{52}$$

is the difference between pre-and post-innovation flow profit rates.

3.3.3 Expected time to discovery

By arguments that parallel those of the monopoly case, if the joint venture runs a research project at intensity h^{JV}, the equilibrium expected time to completion is

$$\frac{1}{h^{JV}}. \tag{53}$$

The equilibrium expected time to successful completion is

$$\frac{1}{ph^{JV}}. \tag{54}$$

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3.3.4 Joint venture vs. monopoly R&D intensity

Theorem 4: If the increase in flow monopoly profit from successful innovation is greater than the increase in flow total duopoly profit from successful innovation,

$$x^M - 2x^{JV} > 0, \tag{55}$$

then joint venture research intensity is less than monopoly research intensity, $h^{JV} < h^M$.

   Proof: see Appendix.

Condition (55) is satisfied for the case of Cournot competition with linear demand and constant marginal cost. Theorem 4 is illustrated in Figure 2 for the linear demand, quadratic R&D cost specification.

When the conditions of Theorems 3 and 4 are satisfied, we have

$$h^D > h^M > h^{JV}, \tag{56}$$

with all three values rising as p rises. Under the conditions of the family of models explored here, if private sector R&D is feasible under all three market structures, then R&D cooperation in the form of an operating-entity joint venture slows the rate of technological progress, all else equal.

3.4 Sunk cost and innovation

But the ranking (56) holds only provided R&D is privately profitable under all three market structures.

3.4.1 Monopoly

For it to be profitable for the monopolist to undertake R&D, V^M must be at least as great as the monopolist's value if it eschews innovation and simply uses the known technology. Using (12), this condition is

$$\frac{\pi_B}{r} - \frac{z'(h^M) + S}{p} \geq \frac{\pi_A}{r}, \tag{57}$$

or equivalently

$$\frac{x^M}{r} \geq \frac{z'(h^M) + S}{p}. \tag{58}$$

h^M, on the right in (58), is itself a function of S. For the linear demand, quadratic R&D cost specification, (58) can be solved for the maximum value of S consistent with monopoly R&D.
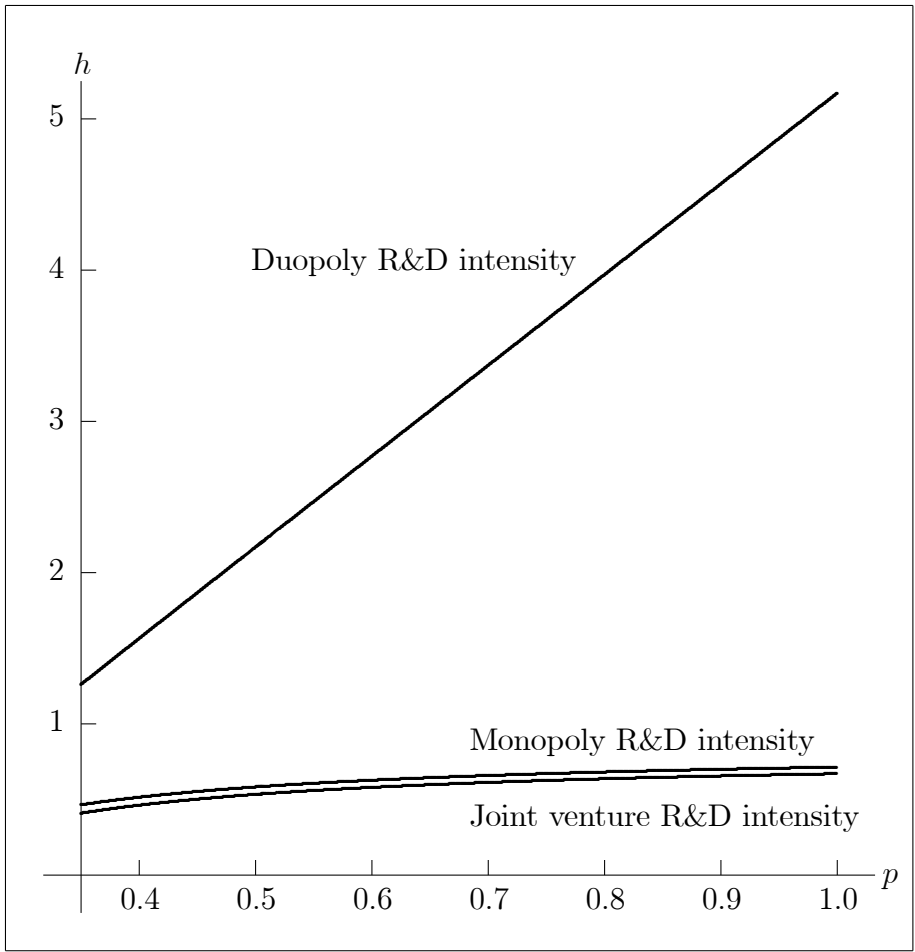
Figure 2: Noncooperative duopoly, monopoly, and joint-venture R&D intensity as functions of p (linear demand, constant marginal production cost, quadratic R&D cost.

3.4.2 Duopoly

The condition for both firms to engage in R&D in symmetric duopoly is that a firm's equilibrium duopoly value exceed its value if its rival does R&D and it does not,

$$V^D = \frac{\pi_W}{r} - \frac{z'(h^D) + S}{p} \geq \frac{\pi_A^D + h^s p \frac{\pi_L}{r}}{r + ph^s} = V_2^{noR\&D}, \qquad (59)$$

where h^s is the equilibrium R&D intensity of the single firm that does R&D.[20]

In (59), h^D and h^S are functions of S. For the linear demand, quadratic R&D cost specification, (59) can be solved numerically for the maximum value of S consistent with both duopolists doing R&D.[21]

3.4.3 Joint Venture

For firms to be willing to form an operating entity joint venture, it must be that

$$2V^{JV} \geq 2V^D,$$

a condition that translates into

$$\frac{2z'(h^D) - z'(h^{JV}) + S}{p} \geq 2\frac{\pi_W - \pi_B^D}{r}. \qquad (60)$$

The left-hand side is the cost saving from forming an operating-entity joint venture, the right-hand side is the value lost by not winning a noncooperative innovation race.

3.4.4 Comparison

For the monopolist, it is the present discounted value of post-innovation monopoly profit that determines the maximum value of S consistent with private investment in innovation. The prospect of lost future profit, (34), if the rival should innovate first is an incentive to noncooperative duopoly R&D, but the post-innovation payoff to successful innovation is less for noncooperative duopoly than for monopoly. Joint duopoly R&D lacks the incentive effect of noncooperative duopoly R&D, and the flow increase in profit in the post-innovation market is less for joint R&D than for monopoly, but joint R&D means each firm bears only half the cost of R&D projects.

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[20]We derive the right-hand side of (59), and give the first-order condition that determines h^s, in the Appendix.

[21]For a further range of sunk cost above this upper limit, there are equilibria in which one of the two firms does R&D.

p	M	D	JV
1/4	2237.3	1911.9	10333
1/2	4829.7	3388.4	11444
3/4	7494.5	4290.4	23497

Table 1: Maximum value of sunk cost consistent with R&D profitability, monopoly, duopoly, joint venture (linear demand, quadratic R&D cost).

Table 1 shows the value of S at which the value of a firm that invests in R&D equals the value of a firm that does not invest in R&D, for the three market structures considered here, and for three different values of p.[22] For each value of p, noncooperative duopoly will support the smallest level of sunk R&D cost, joint duopoly R&D the largest. In this sense, where R&D entails large sunk investment, "slow but steady wins the race."

4   Multiple R&D projects per firm[23]

It can be privately profitable for a firm to run multiple research projects for the same reason that it can be socially beneficial for society to have multiple firms running single research projects in pursuit of the same goal: the probability that one of several projects will succeed is greater than the probability that any one project will succeed. But diversification of research effort comes at a cost — the sunk cost of running additional research projects.

To fix ideas, suppose a monopolist undertakes $n \geq 1$ R&D projects. For analytical convenience, treat n as a continuous variable. Assume that each R&D project requires initial sunk investment S, that the flow cost of an R&D project, $z(h)$, does not depend on n, that the probability of completion of individual R&D projects is independently and identically distributed, that the probability of success given completion, p, is the same for all projects, and that probabilities of completion and probabilities of success, given completion, are independently distributed.

These assumptions imply that research intensity h will be the same for all research projects. If the firm has n research projects, each with exponential distribution of success,

$$\Pr\left(\tau_i \leq t\right) = 1 - e^{-ht},$$

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[22]The figures shown in Table 1 are calculated for the parameter values of footnote 14. See the appendix for discussion.

[23]See Nelson (1961), Dasgupta and Maskin (1987), Scherer (2007). Scott (1993, Chapter 8) models multiple R&D projects per firm in an imperfectly competitive market using the standard innovation race framework.

	Monopoly			Duopoly			Joint Venture		
p	n	h	nh	n	h	nh	n	h	nh
1/4	8.3583	0.2375	1.9847	12.675	1.2676	16.067	8.1184	0.2252	1.8280
1/2	5.4994	0.3060	1.6830	25.650	2.5669	65.84	5.4340	0.2890	1.5705
3/4	4.2695	0.3516	1.5078	30.065	3.8668	116.26	4.2412	0.3331	1.4127

Table 2: Comparative statics with respect to p, multiple research projects model, linear demand, quadratic R&D cost.

then the probability that a single research project is not completed at time t is

$$e^{-ht}.$$

Given independence of completion distributions, the probability that no project has succeeded at time t is

$$e^{-nht}.$$

The probability that at least one of the projects has succeeded by time t is

$$\Pr\left(\text{some } \tau_i \leq t\right) = 1 - e^{-nht}. \tag{61}$$

That is, as is well known, the distribution of time to first completion of one of n independently and identically exponentially distributed random variables is itself exponential, with hazard rate n times the hazard rate of a single random variable. In the multiple-R&D project per firm model, nh can be thought of as the overall or effective R&D intensity of the firm.

Numerical results for the three regimes we consider are reported in Table 2.[24] If firms carry out multiple R&D projects, effective R&D intensities in the three regimes stand in the same relation as (56):

$$(nh)^D > (nh)^M > (nh)^{JV}. \tag{62}$$

Monopoly and joint venture research intensity h rises, and n falls, as p rises.[25] As the probability of success rises, firms immune from the pressure of rivalry carry out fewer R&D projects, and make a greater effort for each project. Effective monopoly and joint venture R&D intensity nh both fall as p rises.

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[24] The figures in Table 2 are obtained using the parameter values of footnote 14, except that we set $S = 100$ to find solutions with several research projects. Details of the multiple R&D projects models are contained in an appendix that is available on request from the authors.

[25] This is a general result (that is, not dependent on the assumptions of linear demand and quadratic R&D cost) for monopoly.

	h	Firm Value	Consumer Surplus	Net Social Welfare
Monopoly	0.58283	28793	17211	46004
Duopoly	2.1680	2×8550	26583	43683
Joint Venture	0.53377	24976	30482	55458

Table 3: Welfare results, single R&D project per firm, linear demand, quadratic R&D cost, three regimes.

Due to the rivalry inherent in noncooperative duopoly, a higher p means not only that any one of a firm's R&D projects is more likely to succeed, but also that the other firm's projects are more likely to succeed. Duopolists increase both the number of projects and the intensity of each project as p rises. With multiple R&D projects per firm, duopoly effective research intensity rises with the probability of success of individual research projects.

5 Welfare

The policy literature on R&D cooperation focuses on technological progress, and this may justify our focus on equilibrium research intensity under alternative market structures and cooperation regimes. But economists ought not to be interested in the rate of technological for its own sake, but rather for its implications for market performance. With this in mind, we present typical welfare results for the cases we consider in Table 3.[26]

From Adam Smith onward, economists' rebuttable presumption is to favor competition as a resource allocation mechanism. The central question of the R&D cooperation literature is whether or not this presumption should be set aside for innovation. The results of Table 3 favor rivalry as a means of generating technological progress and rivalry as a product market resource allocation mechanism. They also depict a conflict between the determinants of static and dynamic market performance, although the conflict is not that highlighted by Schumpeter (1943) in the extract quoted in our introduction.

For the parameters used to generate Table 3, and more generally, duopoly R&D yields the most private-sector investment in innovation. It is precisely the rivalry inherent in noncooperative R&D that distinguishes duopoly R&D from the alternatives — each duopolist invests more in R&D, all else equal, because of the future profit lost if the rival innovates first. Yet duopoly R&D

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[26] "Consumer surplus" is the expected present discounted value of flow consumer surplus. "Net social welfare" is the sum of firm value(s) and consumer surplus. The results of Table 3 are generated for the parameter values given in footnote 14, and for $p = 1/2$.

yields less net social welfare than the other two regimes.

R&D cooperation yields the least private investment in R&D — the slowest discovery, in an expected value sense — but the greatest consumer surplus and net social welfare. Discovery comes more slowly if firms cooperate, but when it arrives, both firms have access to the new technology on the same terms. Symmetric duopoly competition in quantity-setting markets is far from perfect competition, but dominates (in a welfare sense) monopoly and high-cost-firm, low-cost-firm duopoly.

6   Conclusion

Where R&D projects may fail, monopoly and duopoly R&D intensity rise with the probability of success. The profit that would be lost if a rival succeeds first induces greater R&D effort (per project and number of projects) under duopoly than under monopoly. If a successful duopolist gains more profit than would a monopolist, that effect operates in the same direction. Joint R&D, the success of which brings less profit than successful noncooperative R&D, is least intense, and offers the worst technological performance, of the regimes considered.

R&D cooperation worsens technological performance, reducing investment in R&D and delaying the expected time of discovery. It may nonetheless yield the best market performance, since it ensures product-market rivalry on equal terms in the post-innovation market.

Schumpeter's 1943 vision was that product market power would improve market performance, on balance, despite static distortions, because it would enable rapid technological progress. In the framework developed here, the trade-off is in some sense reversed — R&D cooperation *slows* technological progress, but improves market performance, because it strengthens post-innovation product market rivalry. As we have emphasized in Section 3.4, its cost-sharing aspect strengthens the case for joint R&D in sectors where the sunk cost of R&D is great: when it comes to the economics of innovation, different policy prescriptions are appropriate for different sectors of the economy (Nelson and Winter, 1977).

Uncertainty impinges on innovation in ways that we have noted above, but not treated formally. p can be treated as endogenous.[27] The case for parallel R&D is likely to be strengthened if early R&D generates information that

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[27]If the flow cost of R&D is made $z\left(h, p\right)$, with $z_1, z_{11} > 0$ and $z_2, z_{22} > 0$, results will hinge on the sign and magnitude of z_{12}. In such as extension, one might wish to examine the case in which the probability of completion and the probability of success are jointly distributed random variables.

permits the profitable redirecting of R&D effort. It is possible to endogenize the number of firms in an market, as well as the number of R&D projects per firm. Much has been done to understand the impact of uncertainty on the financing of R&D (Hall and Lerner, 2010), but much remains to be done, and this impact will affect market structure as well as market performance. These are all promising areas for future research.

7 Appendix

This appendix contains terse versions of proofs. Detailed statements of the arguments are available on request from the authors, as are Maple programs to evaluate solutions for the linear demand, quadratic R&D cost specification.

7.1 Monopoly

The first derivative of the monopolist's objective function is

$$\frac{\partial V^M}{\partial h} = p \frac{z(h) - \left(\frac{r}{p} + h\right) z'(h) + x^M - \frac{1-p}{p} rS}{(r + ph)^2}, \tag{63}$$

The second derivative is

$$\frac{\partial V^M}{\partial h} =$$

$$p \frac{(r + ph)^2 \left[z'(h) - \left(\frac{r}{p} + h\right) z''(h) - z'(h)\right] - 2(r + ph) NUM^M}{(r + ph)^4}, \tag{64}$$

where NUM^M, the numerator on the right in (13), is given by (11).

Differentiating the first-order condition (13) with respect to p gives (16). Differentiating (13) with respect to x^M gives (17).

The condition for the monopolist to do R&D is that its value if it does R&D be at least as great as its value if it simply produces with the known technology; using (12),

$$V^M = \frac{\pi_B}{r} - \frac{z'(h^M) + S}{p} \geq \frac{\pi_A}{r} \tag{65}$$

or

$$rS \leq px^M - rz'(h^M). \tag{66}$$

h^M is function of S; (66) implicitly defines the maximum value of S consistent with monopoly R&D, given the other parameters of the model.

For quadratic R&D cost $z(h) = \delta h^2$, the upper limit of S is

$$\overline{S}^M = \frac{p}{r}x^M + 2\delta r - 2\sqrt{\delta\left(px^M + \delta r^2\right)}. \tag{67}$$

7.2 Duopoly

7.2.1 Objective function

The integral that defines firm 1's objective function is

$$V^{1D} + S =$$

$$\int_{t=0}^{\infty} e^{-(r+h_1+h_2)t} \left\{ \begin{array}{c} \pi_A^D - z(h_1) + h_1\left[p\frac{\pi_W}{r} + (1-p)V^{1D}\right] \\ +h_2\left[p\frac{\pi_L}{r} + (1-p)\left(V^{1D} + S\right)\right] \end{array} \right\} dt, \tag{68}$$

and this leads to (21).

7.2.2 Condition for equilibrium duopoly R&D

If one firm invests in R&D and the other firm does not, the value of the firm that does not invest in R&D is the expected present discounted value of duopoly profit if both firms produce at unit cost c_A in the pre-innovation period (π_A^D) plus its expected present discounted value as a high-cost firm in the post-innovation period. The condition for both firms to engage in R&D, if one firm does, that a firm's duopoly value if it does R&D be at least as great as its value if the other firm invests in R&D and it does not:

$$V^D = \frac{\pi_W}{r} - \frac{z'(h^D) + S}{p} \geq \frac{\pi_A^D + h_1^s p\frac{\pi_L}{r}}{r + ph_1^s} = V_2^{noR\&D}. \tag{69}$$

h^D and h_1^s (the value-maximizing R&D intensity of the single firm that does R&D) both depend on S. For quadratic R&D cost, (69) can be solved numerically for the maximum value of S for which both firms will invest in R&D.

7.2.3 Second-order condition

The first derivative of V^{1D} satisfies

$$\frac{1}{p}\frac{\partial V^{1D}}{\partial h_1} =$$

$$\frac{z(h_1) - \left(\frac{r}{p} + h_1 + h_2\right)z'(h_1) + h_2\left[p\frac{y^D}{r} - (1-p)S\right] + x^D - r\frac{1-p}{p}S}{\left[r + p\left(h_1 + h_2\right)\right]^2}. \tag{70}$$

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The second derivative satisfies

$$\frac{1}{p}\frac{\partial^2 V^{1D}}{\partial h_1^2} =$$

$$\frac{[r+p(h_1+h_2)]^2 \left\{ z'(h_1) - \left(\frac{r}{p}+h_1+h_2\right) z''(h_1) - z'(h_1) \right\} - 2p[r+p(h_1+h_2)] NUM^{D2}}{[r+p(h_1+h_2)]^4}. \tag{71}$$

where NUM^{D2} is the numerator on the right in (70).

Where the first-order condition holds, $NUM^{D2} = 0$. The result is (39).

7.2.4 Equilibrium cross-derivative

Differentiate (70) with respect to h_2 to obtain

$$\frac{1}{p}\frac{\partial^2 V^{1D}}{\partial h_1 \partial h_2} =$$

$$\frac{[r+p(h_1+h_2)]^2 \left[p\frac{y^D}{r} - z'(h_1) - (1-p) S \right] - 2p[r+p(h_1+h_2)] NUM^{D2}}{[r+p(h_1+h_2)]^4}.$$

In equilibrium, $NUM^{D2} = 0$ and the cross-derivative is

$$\frac{1}{p}\left.\frac{\partial^2 V^{1D}}{\partial h_1 \partial h_2}\right|^* = \frac{p\frac{y^D}{r} - z'(h^D) - (1-p) S}{\left(r+2ph^D\right)^2} > 0, \tag{72}$$

where the numerator on the right is positive by (33). This is (40).

7.2.5 Slope of the best-response line

From (70), firm 1's first-order condition can be written

$$z(h_1) - \left(\frac{r}{p}+h_1+h_2\right) z'(h_1) + h_2 \left[p\frac{y^D}{r} - (1-p) S \right] + x^D - r\frac{1-p}{p} S \equiv 0. \tag{73}$$

Differentiate (73) with respect to h_2:

$$z'(h_1)\frac{\partial h_1}{\partial h_2} - \left(\frac{r}{p}+h_1+h_2\right) z''(h_1)\frac{\partial h_1}{\partial h_2} - \left(\frac{\partial h_1}{\partial h_2}+1\right) z'(h_1) + p\frac{y^D}{r} - (1-p) S = 0.$$

$$-\left(\frac{r}{p}+h_1+h_2\right) z''(h_1)\frac{\partial h_1}{\partial h_2} + p\frac{y^D}{r} - z'(h_1) - (1-p) S = 0.$$

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$$\left.\frac{\partial h_1}{\partial h_2}\right|_{1\text{'s brf}} = \frac{p\frac{y^D}{r} - z'(h_1) - (1-p)S}{\left(\frac{r}{p} + h_1 + h_2\right)z''(h_1)}. \tag{74}$$

In equilibrium this becomes

$$\left.\frac{\partial h_1}{\partial h_2}\right|_{1\text{'s brf}} = \frac{p\frac{y^D}{r} - z'(h^D) - (1-p)S}{\left(\frac{r}{p} + 2h^D\right)z''(h^D)} > 0, \tag{75}$$

where the expression in brackets in the numerator on the right is positive by (33) and the denominator is positive by (1). R&D effort best-response lines slope upward in the neighborhood of equilibrium.

7.2.6 Duopoly comparative statics

Preliminaries Some parts of working out comparative static derivatives are common to the derivations that follow. Carry out this first part here, then use it in later discussions.

Impose symmetry on the first-order condition:

$$\frac{\partial V^{1D}\left(h^D, h^D\right)}{\partial h_1} \equiv 0. \tag{76}$$

Differentiate with respect to any parameter z (we will be interested in $z = p$, x^D, and y^D).

$$\frac{\partial^2 V^{1D}\left(h^D, h^D\right)}{\partial^2 h_1}\frac{\partial h^D}{\partial z} + \frac{\partial^2 V^{1D}\left(h^D, h^D\right)}{\partial h_1 \partial h_2}\frac{\partial h^D}{\partial z} + \frac{\partial^2 V^{1D}\left(h^D, h^D\right)}{\partial z \partial h_1} = 0.$$

$$\left[\frac{\partial^2 V^{1D}\left(h^D, h^D\right)}{\partial^2 h_1} + \frac{\partial^2 V^{1D}\left(h^D, h^D\right)}{\partial h_1 \partial h_2}\right]\frac{\partial h^D}{\partial z} = -\frac{\partial^2 V^{1D}\left(h^D, h^D\right)}{\partial z \partial h_1}.$$

$$\frac{\partial h^D}{\partial z} = -\frac{\frac{\partial^2 V^{1D}\left(h^D, h^D\right)}{\partial z \partial h_1}}{\frac{\partial^2 V^{1D}\left(h^D, h^D\right)}{\partial^2 h_1} + \frac{\partial^2 V^{1D}\left(h^D, h^D\right)}{\partial h_1 \partial h_2}}. \tag{77}$$

Stability implies that the denominator is negative. Hence $\frac{\partial h^D}{\partial z}$ and $\frac{\partial^2 V^{1D}\left(h^D, h^D\right)}{\partial z \partial h_1}$ have the same sign.

With respect to p Let $z = p$. Impose symmetry on (70) to obtain

$$\left.\frac{\partial V^{1D}}{\partial h_1}\right|_{h_1 = h_2 = h^D} = p\frac{NUM^{D3}}{(r + 2ph^D)^2}, \tag{78}$$

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where
$$NUM^{D3} =$$
$$z(h^D) - \left(\frac{r}{p} + 2h^D\right) z'(h^D) + h^D \left[p\frac{y^D}{r} - (1-p)\,S\right] + x^D - r\frac{1-p}{p}S. \quad (79)$$

Differentiate (78) with respect to p:

$$\left.\frac{\partial^2 V^{1D}(h_1, h_2)}{\partial p \partial h_1}\right|_{h_1=h_2=h^D} =$$

$$\frac{NUM^{D3}}{(r + 2ph^D)^2} + p\frac{(r + 2ph^D)^2 \frac{\partial NUM^{D3}}{\partial p} - 2\left(r + 2ph^D\right) NUM^{D3}}{(r + 2ph^D)^4}$$

$$= \frac{p}{(r + 2ph^D)^2}\frac{\partial NUM^{D3}}{\partial p}, \quad (80)$$

since $NUM^{D3} = 0$ in equilibrium.

Evaluate $\frac{\partial NUM^{D3}}{\partial p}$:

$$\frac{\partial NUM^{D3}}{\partial p} = \frac{r}{p^2}z'(h^D) + h^D\left(\frac{y^D}{r} + S\right) + \frac{r}{p^2}S > 0. \quad (81)$$

Hence

$$\left.\frac{\partial^2 V^{1D}(h_1, h_2)}{\partial p \partial h_1}\right|_{h_1=h_2} = p\frac{\frac{r}{p^2}z'(h^D) + h^D\left(\frac{y^D}{r} + S\right) + \frac{r}{p^2}S}{(r + 2ph^D)^2} > 0. \quad (82)$$

and from (77), $\frac{\partial h^D}{\partial p} > 0$.

With respect to $x^D = \pi_W - \pi_A^D$ and $y^D = \pi_W - \pi_L$ Differentiate (78) with respect to x^D:

$$\left.\frac{\partial^2 V^{1D}(h_1, h_2)}{\partial x^D \partial h_1}\right|_{h_1=h_2=h^D} = \frac{p}{(r + 2ph^D)^2}\left.\frac{\partial NUM^{D3}}{\partial x^D}\right|_{h_1=h_2=h^D} =$$

$$\frac{p}{(r + 2ph^D)^2} > 0. \quad (83)$$

Hence

$$\frac{\partial h^D}{\partial x^D} > 0. \quad (84)$$

In the same way

$$\left.\frac{\partial^2 V^{1D}(h_1, h_2)}{\partial y^D \partial h_1}\right|_{h_1=h_2=h^D} = \frac{p}{(r + 2ph^D)^2}\left.\frac{\partial NUM^{D3}}{\partial y^D}\right|_{h_1=h_2=h^D} =$$

$$= \frac{p^2}{(r + 2ph^D)^2}\frac{h^D}{r} > 0. \quad (85)$$

7.2.7 Theorem 3

h^α is the solution of (41), the weighted average of the equation that defines h^M and the equation that defines h^D. $h^\alpha = h^M$ for $\alpha = 0$; $h^\alpha = h^D$ for $\alpha = 1$.

Differentiating (41) with respect to α gives

$$\frac{\partial h}{\partial \alpha} = \frac{h\left[p\frac{y^D}{r} - z'(h) - (1-p)S\right] + x^D - x^M}{\left[\frac{r}{p} + (1+\alpha)h\right]z''(h) - \alpha\left[p\frac{y^D}{r} - z'(h) - (1-p)S\right]}. \tag{86}$$

The assumptions stated in the theorem imply

$$\frac{\partial h^\alpha}{\partial \alpha} > 0. \tag{87}$$

Then $h^0 = h^M$, $h^1 = h^D$, and h^α rises as α increases from 0 to 1. This establishes the result.

7.3 Operating-entity joint venture

7.3.1 Second-order condition

From (49) and (51), the first derivative of the operating-entity joint venture value function is

$$\frac{\partial\left(2V^{JV}\right)}{\partial h^{JV}} = p\frac{z(h^{JV}) - \left(\frac{r}{p} + h^{JV}\right)z'(h^{JV}) + 2x^{JV} - \frac{1-p}{p}rS}{(r + ph^{JV})^2}. \tag{88}$$

Differentiate with respect to h^{JV} (and evaluate the result in equilibrium, using the fact that the numerator is then zero):

$$\frac{\partial\left(2V^{JV}\right)^2}{\partial^2 h^{JV}} = \frac{pz'(h^{JV}) - \left(r + ph^{JV}\right)z''(h^{JV}) - pz'(h^{JV})}{(r + ph^{JV})^2}$$

$$\left.\frac{\partial\left(2V^{JV}\right)^2}{\partial^2 h^{JV}}\right|^* = -\frac{z''(h^{JV})}{(r + ph^{JV})} < 0. \tag{89}$$

The second-order condition is satisfied.

7.3.2 Theorem 4

Consider the equation that is the weighted average of (13), the equation that defines h^M, and (51), the equation that defines h^{JV}:

$$\beta NUM^M(h) + (1 - \beta) NUM^{JV}(h) =$$

$$z(h) - \left(\frac{r}{p} + h\right) z'(h) + \beta x^M + (1 - \beta)\left(2x^{JV}\right) - \frac{1-p}{p} rS = 0, \qquad (90)$$

The solution to (90) is h^{JV} for $\beta = 0$, h^M for $\beta = 1$.
Differentiate (90) with respect to β:

$$z'(h)\frac{\partial h}{\partial \beta} - \left(\frac{r}{p} + h\right) z''(h)\frac{\partial h}{\partial \beta} - z'(h)\frac{\partial h}{\partial \beta} + x^M - 2x^{JV} = 0$$

$$- \left(\frac{r}{p} + h\right) z''(h)\frac{\partial h}{\partial \beta} + \left(x^M - 2x^{JV}\right) = 0$$

$$\frac{\partial h}{\partial \beta} = \frac{x^M - 2x^{JV}}{\left(\frac{r}{p} + h\right) z''(h)}. \qquad (91)$$

$\frac{\partial h}{\partial \beta}$ has the same sign as $x^M - 2x^{JV}$.
Rewrite $x^M - 2x^{JV}$ as

$$\pi_B - \pi_A - 2\left[\pi_B^D - \pi_A^D\right] =$$

$$\pi_B - 2\pi_B^D - \left[\pi_A - 2\pi_A^D\right]. \qquad (92)$$

For arbitrary marginal cost c, output $Q^m(c)$ makes marginal revenue equal to c,

$$\frac{dP(Q)Q}{dQ} = c. \qquad (93)$$

For arbitrary marginal cost c, industry output Q^D makes each firm's marginal revenue on its residual demand curve equal to c:

$$\left.\frac{dP\left(q_1 + \frac{1}{2}Q^D\right)q_1}{dq_1}\right|_{q_1 = \frac{1}{2}Q^D} = c. \qquad (94)$$

Monopoly profit with constant marginal cost c is

$$\pi^m = \int_0^{Q^m}\left[\frac{dP(Q)Q}{dQ} - c\right] dQ. \qquad (95)$$

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Duopoly profit of one firm with constant marginal cost c is

$$\int_0^{\frac{1}{2}Q^D} \left[\frac{dP\left(q + \frac{1}{2}Q^D\right) q}{dq} - c \right] dq, \tag{96}$$

and total duopoly profit is twice this.

$$\int_0^{\frac{1}{2}Q^D} \left[P\left(q + \frac{1}{2}Q^D\right) + q \frac{dP\left(q + \frac{1}{2}Q^D\right)}{dq} - c \right] dq.$$

Then

$$\pi^m - 2\pi_N(c) =$$

$$\int_0^{Q^m} \left[\frac{dP\left(Q\right)Q}{dQ} - c \right] dQ - 2 \int_0^{\frac{1}{2}Q^D} \left[\frac{dP\left(q + \frac{1}{2}Q^D\right) q}{dq} - c \right] dq. \tag{97}$$

Differentiate with respect to c. This requires use of Leibnitz' Rule. Take each term in turn.

$$\frac{d}{dc} \int_0^{Q^m} \left[\frac{dP\left(Q\right)Q}{dQ} - c \right] dQ =$$

$$\left[\frac{dP\left(Q\right)Q}{dQ} \bigg|_{Q=Q^m} - c \right] \frac{dQ^m}{dc} + \int_0^{Q^m} (-1)\, dQ =$$

$$-Q^m < 0. \tag{98}$$

The next term:

$$\frac{d}{dc} \int_0^{\frac{1}{2}Q^D} \left[\frac{dP\left(q + \frac{1}{2}Q^D\right) q}{dq} - c \right] dq =$$

$$\left[\frac{dP\left(q + \frac{1}{2}Q^D\right) q}{dq} \bigg|_{q=\frac{1}{2}Q^D} - c \right] \frac{d\left(\frac{1}{2}Q^D\right)}{dc}$$

$$+ \int_0^{\frac{1}{2}Q^D} \left[P'\left(q + \frac{1}{2}Q^D\right) \frac{d\left(\frac{1}{2}Q^D\right)}{dc} + q \frac{d^2 P\left(q + \frac{1}{2}Q^D\right)}{dq^2} \frac{d\left(\frac{1}{2}Q^D\right)}{dc} - 1 \right] dq =$$

$$\frac{d\left(\frac{1}{2}Q^D\right)}{dc} \int_0^{\frac{1}{2}Q^D} \left[P'\left(q + \frac{1}{2}Q^D\right) + q \frac{d^2 P\left(q + \frac{1}{2}Q^D\right)}{dq^2} \right] dq - \frac{1}{2}Q^D. \tag{99}$$

Then

$$\frac{\partial \left[\pi^m - 2\pi_N(c) \right]}{\partial c} =$$

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$$-Q^m + Q^D - 2\frac{d\left(\frac{1}{2}Q^D\right)}{dc} \int_0^{\frac{1}{2}Q^D} \left[P'\left(q + \frac{1}{2}Q^D\right) + q\frac{d^2 P\left(q + \frac{1}{2}Q^D\right)}{dq^2} \right] dq =$$

$$Q^D - Q^m - 2\frac{d\left(\frac{1}{2}Q^D\right)}{dc} \int_0^{\frac{1}{2}Q^D} \left[P'\left(q + \frac{1}{2}Q^D\right) + q\frac{d^2 P\left(q + \frac{1}{2}Q^D\right)}{dq^2} \right] dq.$$

If

$$Q^D - Q^m - 2\frac{d\left(\frac{1}{2}Q^D\right)}{dc} \int_0^{\frac{1}{2}Q^D} \left[P'\left(q + \frac{1}{2}Q^D\right) + q\frac{d^2 P\left(q + \frac{1}{2}Q^D\right)}{dq^2} \right] dq < 0,$$

which it is for the linear demand case, then

$$\frac{\partial \left[\pi^m - 2\pi_N(c)\right]}{\partial c} < 0$$

and as c falls from c_A to c_B, $\pi^m - 2\pi_N(c)$ rises from $\pi^m - 2\pi_A^D$ to $\pi^m - 2\pi_B^D$. Then $x^M - 2x^{JV} > 0$, which completes the proof.

7.3.3 Condition for joint-venture R&D

For firms to be willing to form an operating entity joint venture, it must be that value with a joint venture is at least as great as value doing noncooperative R&D,

$$2V^{JV} \geq 2V^D, \tag{100}$$

For a joint venture to do R&D, $2V^{JV}$ and h^{JV} must also be nonnegative. Numerical analysis suggests that (100) is always satisfied for the linear demand, quadratic R&D cost model. For high values of p, the upper limit of S is the value that makes $2V^{JV} = 0$:

$$\overline{S}^{JV} = 2\left(r\delta + \frac{p}{r}\pi_B^D\right) - 2\sqrt{(r\delta)^2 - 2\delta(2-p)\pi_B^D - \left(\frac{p}{r}\pi_B^D\right)^2 + 2\delta x^{JV}}. \tag{101}$$

For low values of p, the upper limit of S is the value that makes $h^{JV} = 0$:

$$S = \frac{2p}{1-p}\frac{x^{JV}}{r}. \tag{102}$$

7.4 Background for numerical examples

All figures and tables assume linear demand

$$P = 100 - Q,$$

initial marginal cost $c_A = 30$, post-innovation marginal cost $c_B = 15$, and interest rate $r = 1/20$.

We assume quadratic R&D cost

$$z(h) = 1000h^2.$$

For Figures 1 and 2 and Table 3, $S = 2500$. For Table 2, $S = 100$.

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