

Increasing Income Inequality: Productivity, Bargaining and Skill-Upgrading

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Abstract

In recent decades most developed countries have experienced an increase in income inequality. In this paper, we use an equilibrium search framework to shed additional light on what is causing income distribution to change. The major benefit of the model is that it is can accommodate shocks to the skill composition in the market, employee bargaining power and productivity. Further, when our model is subjected to skill-upgrading and changes in employee bargaining power, it is capable of predicting the recent changes observed in the Danish income distribution. The model emphasizes that shocks to the employees' relative productivity, i.e., skill-biased technological change, are unlikely to have caused the increase in income inequality.

Keywords: Income inequality, two-sector search model, bargaining power, skill-biased technological change.

JEL codes: J3, J6, M5.

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1 Introduction

In recent decades, most developed countries have experienced increasing income inequality (OECD, 2007). A large body of literature has analyzed the possible causes for the changes in income distribution,¹ and the general picture emerging is that a combination of demand and supply shocks alter both relative income and employment, which in turn drive up income inequality. In this paper, we analyze income distribution using an equilibrium search model because it can accommodate shocks to both sides of the market (and their externalities). Thus, the modeling framework allows for a detailed analysis of what driving forces are causing the income distribution to change. When using our model to predict the recent changes observed in the Danish income distribution, we conclude that the changes are due to general skill-upgrading in the labor market and changes in employee bargaining power. In contrast, shocks to relative productivity, i.e., skill-biased technological change, are unlikely to have caused the changes. We use register based employer employee data from Denmark for the years 1992 to 2003. These data provide information about the structure of the income distribution and the changes occurring in the income distribution over time. Four empirical results are established. First, income inequality is increasing. Second, the action is in the top of the income distribution. Third, skill upgrading in the labor market is highly pronounced. Finally, a negative relation between employment shares and relative wages is established. That is, when an employee subgroup experiences growth in relative income, their employment share drops and vice versa. To better understand what is causing the empirical findings, we build a theoretical model which is based on an equilibrium search framework similar to Pissarides (1994). Our analysis follows Albrecht and Vroman (2002), Gautier (2002), and Dolado, Jansen and Jimeno (2009). Albrecht and Vroman explore an economy with heterogeneous workers and jobs. A key aspect of their model is that the jobs have skill requirements. This implies that low-skilled individuals are unproductive in jobs requiring high skills. Conversely, highly skilled workers can take a job of any type but they are more productive in jobs requiring high skills. Mismatched workers do not search for better jobs. The model in Gautier (2002) is similar to Albrecht and Vroman, but¹ Gautier allows highly educated employees to perform on-the-job searches. A similar approach is taken by Dolado et. al. (2009), but they allow for a

¹For a discussion of income inequality see Katz and Autor (1999). Other seminal papers in the field (the list is by no means exhaustive) are Autor, Katz and Kearney (2008), Bartel and Sicherman (1999), Bound and Johnson (1992), DiNardo, Fortin and Lemieux (1996), Juhn, Murphy and Pierce (1993), Katz and Murphy (1992), Levy and Murnane (1992), Murphy and Welch (1992), Piketty and Saez (2003).

different bargaining setup. Our model can be seen as an extension of Dolado et. al. (2009) in that we allow both low- and highly-educated employees to perform on-the-job searches. This extension is important because we can use it to explain the empirical observation that there is a skill mix in all job types. The equilibrium income distribution derived from the model is non-degenerate because of employee and job heterogeneity. The intuition behind the modeling setup includes two major factors. First, education determines the employee's production capacity in a job. Thus, variation in education implies different levels of productivity and, in turn, different wages. Second, there are two types of jobs in a firm, i.e., management and nonmanagement jobs. We follow the arguments by Lucas (1978) and Rosen (1982) and assume that workers in non-management jobs have limited discretion over resources and hence are less productive than employees in management jobs, who control resources. This implies that productivity and wages increase with employee rank. These assumptions are strongly supported by the data, where both a substantial management and education premium can be established. Further, the two-sided heterogeneity creates a decision problem for the firm which is whether to open vacancies in non-management and management to employees with different education levels. And when doing this, the firm has to anticipate that employees accepting jobs in non-management will continue searching for better jobs in management. Managers have no incentives to search and thus do not perform on-the-job searches. These differences in employee search behavior produce non-trivial wage differentials between management and non-management employees. That is, the management premium arises not only due to productivity differentials but also endogenously from differences in employee search behavior. This finding provides a new perspective on why observationally identical employees (in terms of skills) are paid different wages, which has been central to the search literature for decades (see Mortensen (2003)). The equilibrium income distribution derived from the model shows a critical dependence on productivity, bargaining power and other market conditions, such as the skill composition. This implies that shocks to any of these parameters alter the income distribution by changing relative wages and employment shares. For instance, a detailed analysis shows that productivity shocks altering the relative productivity of a group of employees produce a positive relation between the relative wages of the group and the employment share. Further, shocks improving the bargaining power of a group of employees lead to higher relative wages for the group, but at the detriment of the employment share, i.e., a negative relation exists between the group's relative wages and employment share. Finally, skill-upgrading causes wages to grow for all employees but reduces the relative size of the management sector. Thus, skill-upgrading produces

a negative relation between wages and the employment share in management but a positive relation in non-management. All potential shocks to the economy are capable of increasing income inequality; however, the different consequences of the various shocks on employment and wages allow us, given the theoretical framework, to identify what shocks are more likely to cause income inequality to increase. For example, the Danish economy clearly experienced skill upgrading between 1992 and 2003, which manifested itself as a six percentage point increase in the employment of highly-educated labor over the period. When using our model to examine skill-upgrading, it predicts most of the recent changes in the income distribution. In one respect, however, skill-upgrading cannot account for the empirical findings. The model predicts that skill-upgrading leads to higher wages for all employees. But highly-educated employees in non-management experienced an eight percent decline in real income over the period 1992 to 2003. This implies that additional shocks are required to fully account for the empirical observations. We consider two types of shocks: productivity shocks and shocks to employee bargaining power. A negative shock to the relative productivity of highly-educated non-management employees will reduce both the wages and the employment share for the group. Thus, a shock of this type will counteract the increase in employment share of highly-educated non-managers resulting from skill-upgrading. In contrast, a reduction in the bargaining power of highly-educated non-management employees will result in lower wages but boost the employment share even beyond the positive effect coming from skill-upgrading. Because of the substantial increase in the employment share of highly-educated non-management employees observed over the period, we conclude that a reduction in bargaining power of highly-educated non-management employees is most likely to be the complementary factor to skill-upgrading in explaining the recently observed changes in the Danish income distribution. Our analysis complements previous research on income inequality because it broadens the perspective on what mechanisms can lead to changes in income distribution. For instance, the literature focusing on income inequality in the US during the last decades documents a simultaneous increase in the returns to education and the supply of educated labor. This has led to the conclusion that the recent changes in income distribution are likely caused by skill-biased technological change (see Acemoglu (2002) and Autor and Katz (1999) for surveys). However, historical research proves that income inequality need not be increasing (Piketty and Saez, 2003) and the relation between the education premium and employment shares need not be positive (Autor and Katz 1999). Thus, when times are changing the income distribution may be influenced by factors other than skill-biased technological change, such as changes in bargaining power. Our

model predicts the consequences for the equilibrium income distribution of such alternative shocks to the economy. Further, our findings show that the changes in the income distribution are mainly in the top percentiles, and the model indicates skill-upgrading and changing employee bargaining power as the main causes for this. The fact that the increasing income inequality is driven by activity in the top percentiles has also been established in, for example, the US (see Autor and Katz (1999)). As such, our analysis may help shed some additional light on income dynamics in other countries as well. The outline of the paper is as follows. In the next section, we discuss the changes that have occurred in the Danish income distribution during recent years. In Section 3 we present the theoretical model, and the equilibrium is established in Section 4. In Section 5 we interpret and discuss the results. Finally, Section 6 summarizes and concludes. Proofs and details about the data are relegated to the Appendix.

2 The Danish Income Distribution

The main goal of this section is to document the changes occurring in the Danish income distribution between 1992 and 2003. The main finding is that income inequality has been rising in Denmark, and this is caused by changes both in employment composition and relative incomes of workers. Most pronounced is the skill-upgrading which occurred in the economy during the period. But relative incomes have also been altered substantially, with income increasing for some employee subgroups and declining for others. In particular, management compensation has been increasing rapidly over the period, while the highly-educated in non-management positions have experienced a decline in real earnings.

2.1 The Data

For an income distribution to rise in a non-degenerate form, some heterogeneity is needed. In our analysis we allow for both employee (education) and job (firm structure) heterogeneity, and the empirical discussion reflects this.

The empirical analysis is conducted using register data from Denmark between 1992 and 2003. The data are collected by Statistics Denmark and contain detailed information about all employers and employees in Denmark. The most important factor for the present analysis is that workers and companies can be matched and this information can be combined with further information on the employees educations, incomes and job assignments. This

provides very detailed knowledge about the income distribution and employment structure of a particular company and, in an aggregated form, about the income distribution in the economy. Companies with at least 50 employees are included in the analysis. The main reason for this size selection is that the structure of the firm (management vs. non-management jobs) has to be well defined. We identify managers in our data using the International Standard Classification of Occupations (ISCO) from the International Labor Organization (ILO). When defining individuals from Major Group One as managers we found that 3.6 percent of our sample are managers. Table 1 contains the descriptive statistics for the sample.² We have divided the sample into four groups: low-educated non-managers, highly-educated non-managers, low-educated managers and highly-educated managers. In order to be considered highly educated the employee must have at least a college degree (or in the Danish context, a Bachelors degree). Low-skilled employees in non-management are clearly the largest group, making up almost 86 percent of the sample, but highly-educated non-managers also constitute a significant proportion of the employees (11 percent). The two management groups are relatively small, as expected, but with proportions exceeding 1 percent². Turning to income, it is clearly the case that compensation increases with education and rank. The figures presented in Table 1 and throughout the article are monthly real income in Danish Kroner and the base year is 2000. Using these data to estimate a standard Mincer wage regressions reveals both an education premium of 38 percent and a management premium of 37 percent³.

2.2 Changes in The Income Distribution

During the period 1992 to 2003 the income distribution is altered significantly. Figure 1 shows the changes in incomes at different percentiles and the change in income inequality as measured by the standard deviation of income. Median real income is remarkably stable across the years, as are the incomes between the 10 and 90 percentiles. Examining the 99 percentile, which is a percentile where most employees are managers, it is clear that high income earners have become relatively better off during the period. This is reflected in the standard deviation of incomes, which is clearly increasing over time.

²More detailed descriptive statistics are presented in the Appendix

³In the regression we regress $\ln(\text{real income})$ on age, age squared, gender, tenure, tenure squared, a set of years dummies together with a dummy for being highly educated, and a management dummy.

In order to get a better understanding of the changes in the income distribution we present the changes by education and job level in Table 2. An interesting observation is that the economy has experienced substantial skill-upgrading between 1992 and 2003. In the early years 10 percent were highly educated, but by 2003 the proportion had increased to 16 percent. This observation is reflected in the education composition of managers, where low skilled employees clearly dominated the management group in 1992, but in later years the education mix has been more or less equalized. Further, skill upgrading has been so significant in non-management that the proportion of highly-educated non-managers has increased from 8.5 percent to 15 percent, causing the relative sizes of all other employee subgroups to decline.

Incomes are also altered during the period. While income progression in non-management has been relatively modest or declining, management compensation has increased substantially (16 percent for highly-educated and 22 percent for low-educated). Overall, real wages have increased by 8.25 percent, or 0.5 percent per year. Comparing the changes in the employment composition to the changes in relative income, a clear picture emerges. When an employee subgroup experiences income progression, their employment share is declining and vice versa. That is, the share of highly educated employees working in non-management positions has almost doubled over the period 1992 to 2003 but at the same time, they have lost 8 percent in real income. In contrast, all other employees have lost in terms of employment shares but have gained substantially in terms of earnings. Summing up, in this section we have shown that the income distribution has been altered significantly over time, which has increased income inequality. We have also documented that this is due to both changes in relative income and employment shares. To better understand what is causing these changes, we will propose in the next section a theoretical model that is capable of encompassing these changes in a general equilibrium framework.

3 The Model

Consider a firm with management (M) and non-management (N) sectors that face a labor force of Ω workers. The workers employed by the firm are members of the internal labor market, denoted by I ; the remaining individuals constitute the external labor force, E . The size of the total labor force is normalized to unity, i.e., $\Omega = I + E = 1$. All workers are distinguished by an observable level of education, where the proportion π of the workers is low-educated and the remaining $1 - \pi$ is highly-educated. Highly-educated workers will be referred to as H-workers while low-educated

workers will be referred to as L-workers. We use the notation $k \in \{H, L\}$ and $\rho \in \{N, M\}$.

When the firm opens up a vacancy, the type is determined ex-ante. Thus, the job search behavior is as follows. First, for reasons explained below, individuals have the highest productivity in management jobs. This implies that employees with the required level of education from both the non-management segment of the internal labor market (i_{kN}) and the external labor market (e_k) apply for management vacancies (v_k). Second, employees currently working in management (i_{kM}) have no incentives to search for a new job. Hence, only individuals in the external labor market search for non-management vacancies (v_N) where there are no skill requirements.

Workers and vacancies meet each other randomly according to a matching function that is increasing in its argument, concave, and homogenous of degree one. The matching processes between jobs and workers in management are represented by:

$$x_{kM} = x_{kM}(v_k, e_k + i_{kN}) = (v_k)^\alpha (e_k + i_{kN})^{1-\alpha},$$

and in non-management by:

$$x_N = x_N(v_N, e_L + e_H) = (v_N)^\alpha (e_L + e_H)^{1-\alpha}.$$

The labor market flows are illustrated in Figure 2. Individuals in the external labor market may get jobs in both non-management and management. If an individual gets a job in the non-management sector of the firm, he has an option to be promoted. Since jobs in management have education requirements, both low- and highly-educated employees are promoted. All employees have an exogenous separation risk; hence, there are flows from both the management and non-management sector back into the external labor market. A workers output is denoted by y_k . We make two assumptions about the workers productivity. First, highly-educated workers are more productive than low-educated workers because schooling results in a higher production capacity. Second, we assume, as do Lucas (1978) and Rosen (1982), that workers in non-management jobs have limited discretion over resources and hence are less productive than employees in management jobs who control resources. Thus, $y_{LN} < y_{HN} < y_{LM} < y_{HM}$. These assumptions match well with the empirical findings presented above. Finally, the probability that a k-type job in management meets a worker of type k is:

$$q_{kM}(\theta_k) = \frac{x_{kM}}{v_k} = \frac{x_{kM}(\theta_k, 1)}{\theta_k},$$

and the probability that a non-management job meets a worker is:

$$q_N(\theta_N) = \frac{x_N}{v} = \frac{x_N(\theta_N, 1)}{\theta_N}.$$

The terms θ_k and θ_N represent labor market tightness and are defined as: $\theta_k = v_k / (e_k + i_{kN})$ and $\theta_N = v_N / (e_L + e_H)$. Similarly, we denote the probability that a worker of type k encounters a vacancy in management of type k by p_{kM} and the rate at which a worker meets a non-management vacancy by p_N . It follows that:

$$p_{kM}(\theta_k) = \frac{x_{kM}}{e_k + i_{kN}} = x_{kM}(\theta_{kM}, 1),$$

and

$$p_N(\theta_N) = \frac{x_N}{e_L + e_H} = x_N(\theta_N, 1).$$

3.1 Payoff Functions and Wage Determination

The firm maximizes the present discounted value (PDV) of expected profits, and the individual maximizes the PDV of the expected income stream. The firm decides if a particular vacancy should be opened or not. The individual assesses if the job offer received is sufficiently attractive, given the alternative options such as other employment or continued job search. When the worker and the firm meet, they bargain over the wage. In the following sections, we describe these processes.

3.1.1 The Firm

The firm advertises three different types of jobs: a vacancy in non-management and two vacancies in management. The management jobs require different levels of education, but the non-management vacancy can be filled with a worker of any education level. Hence, in practice the firm employs up to four different employee types: low-educated in non-management, highly-educated in non-management, low-educated in management, and highly-educated in management. These types generate different levels of profits since they differ in their productivities and -as will be shown below- have different costs. We denote the expected PDV of having a vacant non-management job by V_N . The expected PDV of the vacancy depends on the potential workers productivity. For this reason, we denote the PDV of a vacancy filled with an employee of type k by J_{kN} . Further, under perfect competition the PDV of expected profit from a vacant non-management job rV_N (where r is the discount rate) equals the rate of return. Recalling that because the job is

filled by a highly-educated or a low-educated worker with probability q_N , the rate of return can be written as the difference between the vacancy cost of a non-management job, c_N , and the expected return from having the job filled. The expected return from having a low-educated or a highly-educated worker in the job is equal to the sum of the returns generated by low-educated workers and the returns generated by highly-educated workers, weighted by the relative population size. From this it follows that equation (1) will be satisfied in equilibrium:

$$rV_N = q_N \left[\frac{e_L J_{LN} + e_H J_{HN}}{e_L + e_H} - V_N \right] - c_N. \quad (1)$$

Using the same reasoning, the PDV of expected profit for a non-management job filled by a worker of type k , rJ_{kN} , equals its return. In this case, the return is the output produced by the worker, y_{kN} , minus w_{kN} , the wage paid to the worker. In addition to this, the eventual loss of revenue that occur if the worker and the firm separate, which happens with probability s , and the potential loss if the worker finds a job in the management sector must be added. Thus,

$$rJ_{kN} = y_{kN} - w_{kN} + s(V_N - J_{kN}) + p_{kM}(V_N - J_{kN}). \quad (2)$$

Similar expressions can be derived for management. When equating the PDV of expected profit for a vacant management job, rV_{kM} , to the rate of return we obtain:

$$rV_{kM} = q_{kM}(J_{kM} - V_{kM}) - c_{kM}, \quad (3)$$

where c_{kM} are the vacancy costs.

Finally, recalling that management employees do not search it follows that when equating the PDV of expected profit from a filled management job, J_{kM} , to the return we get:

$$rJ_{kM} = y_{kM} - w_{kM} + s(V_{kM} - J_{kM}), \quad (4)$$

where w_{kM} is the wages paid to an employee of type k working in management.

3.1.2 The Workers

A worker of type k earns w_{kN} when employed in non-management and w_{kM} when working in management. For simplicity, we normalize an employee's income in the external labor market to zero.

Let E_k be the PDV of income for a type k worker in the external labor market. The individual may move to a job either in non-management or

in management. The first event occurs with probability p_N . The worker would then earn the PDV of the expected income stream W_{kN} until a job in management arrives or a separation from the firm occurs. If the individual instead gets a job in management (which occurs with probability p_{kM}), the PDV of the expected income stream is W_{kM} until separation. Hence, rE_k is equal to the expected gain when obtaining a job:

$$rE_k = p_N(W_{kN} - E_k) + p_{kM}(W_{kM} - E_k). \quad (5)$$

The payoff to a worker in non-management is given by the wage, w_{kN} , the risk premium against separation, and the option value of being promoted. It follows that:

$$rW_{kN} = w_{kN} + s(E_k - W_{kN}) + p_{kM}(W_{kM} - W_{kN}). \quad (6)$$

Finally, employees already working in management do not search for a new job, but they face a separation risk, s . Thus,

$$rW_{kM} = w_{kM} + s(E_k - W_{kM}). \quad (7)$$

3.1.3 Wage Determination

Wage determination in this model is similar to Pissarides (2002). In equilibrium, occupied jobs in management and non-management generate pure economic rent and the wages paid by the firm to its workers pass along some of the rent. As in Pissarides, we assume that the rent created by a match between a given job and worker is shared according to the Nash bargaining rule. The wage derived from this bargaining process maximizes the weighted product of the workers and the firms net return from the match. In order to form this match and then split the resulting surplus, the worker has to give up the returns obtainable in the external labor market, E_k , in order to receive W_{kN} if employed in non-management and W_{kM} if employed in management. The firm on the other hand, gives up the return from a vacant job (V_N or V_{kM}) and receives $J_{k\rho}$ in exchange. Hence the wage in non-management satisfies:

$$w_{kN} = \operatorname{argmax}(W_{kN} - E_k)^{\beta_{kN}}(J_{kN} - V_N)^{1-\beta_{kN}}. \quad (8)$$

And for management jobs:

$$w_{kM} = \operatorname{argmax}(W_{kM} - E_k)^{\beta_{k,M}}(J_{kM} - V_{kM})^{1-\beta_{k,M}}. \quad (9)$$

where $0 < \beta_{k\rho} < 1$ is the employees bargaining power. This also measures the labor shares of the total surplus generated in the job.

4 Equilibrium

We derive the equilibrium by imposing two assumptions: first, the labor market flows are stable; and second, all profit opportunities in the market are exhausted. That is, $V_N = 0$ and $V_{kM} = 0$. The equilibrium is derived in the Appendix and the equilibrium income distribution is presented in Proposition (1).

Proposition 1 *In equilibrium, for $k = L, H$ and $\rho = N, M$:*

a) The wages in the different sectors of the firms are:

In the non-management sector:

$$w_{kN} = \frac{\beta_{kN}(1 + p_N)y_{kN}}{r + s + p_{kM} + \beta_{kN}p_N}. \quad (10)$$

In the management sector:

$$w_{kM} = \frac{p_N(1 - \beta_{kM})}{(1 - \beta_{kN})}w_{kN} + \beta_{kM}c_{kM}\theta_{kM} + \beta_{kM}y_{kM} \quad (11)$$

b) The employment stocks in the steady-state in different sectors are:

In the non-management sector:

$$i_{kN} = \frac{sp_N\pi_k}{(s + p_{kM} + p_N)(s + p_{kM})} \quad (12)$$

where $\pi_L = \pi$; $\pi_H = 1 - \pi$

In the management sector:

$$i_{kM} = \frac{p_{kM}\pi_k}{s + p_{kM}} \quad (13)$$

Thus, the income distribution is: $w = w_{k\rho}$ with probability $i_{k\rho}$

Proof. See the Appendix. ■

It follows from Proposition 1 that management wages are made up of two parts. The second part ($\beta_{k,M}c_{k,M}\theta_{k,M} + \beta_{k,M}y_{k,M}$) is similar to the expression in the standard Pissarides (2002) model. It shows that workers in management receive a fraction of the vacancy cost $c_{k,M}\theta_{k,M}$ that is saved by the firm when the match is formed, and a share of the surplus created by the match. The first part is more intriguing and can be interpreted as a management premium that is proportional to the non-management wage.

Non-management wages are also interesting as they differ from the wage expression in a standard search model. The reason for this is that non-management employees continue searching for management jobs. This implies that the firm must re-advertise non-management jobs more often than

management jobs. Thus, in contrast to the wages paid in a standard model, non-management employees receive a smaller fraction of these costs.

Finally, employment shares are determined by the skill composition in the labor market, recruitment and promotion probabilities as well as the separation rate.

5 Interpretation

The shape of the equilibrium income distribution is determined by productivity, bargaining power and other labor market conditions, such as the skill composition. Thus, a change in any of these parameters alters the income distribution but with varying consequences ⁴. We argue that the model predicts the recent changes in the Danish income distribution when subjected to skill-upgrading and changes in the employees bargaining power. In contrast, shocks to the employees relative productivity, such as skill biased technological change, are unlikely to have caused the empirical findings. This theoretical setup allows us to contrast the effects of technological change that alters the employees relative productivities and changes in the employees bargaining power. First, when a group of employees become relatively more productive, the match value between the worker and the firm increases, which in turn drives up both the wage paid to the group and the groups employment share. In contrast, if the bargaining power of a group of employees increases, a larger proportion of the match value is allocated to the employees, driving up wages, but the group becomes less attractive to the firm, resulting in a declining employment share. A different type of shock influencing the income distribution is skillupgrading. When the proportion of highly educated individuals in the labor market increases, wages grow. The reason is that when the firm recruits from the external labor market to the non-management sector, it is more likely to hire a highly-educated person. Consequently, the average non-management employee is now more productive and the higher average productivity is rewarded with higher wages. Further, because management wages are to some extent dependent on non-management wages, managers also experience an increase in income. Further, as already noted, the firm is more likely to recruit a highly-educated employee when hiring from the external labor market, which implies that the number of highly-educated in non-management increases. In management the situation is different. Management employees benefit from higher wages but they have not become more productive, which makes managers less attractive and leads

⁴The comparative statistics underlying the discussion in this section are presented in the Appendix.

to a reduced employment share. In sum, skill upgrading increases wages and the employment share of non-managers but reduces the employment share of management employees. The different types of shocks are all capable of producing increasing income inequality, but their consequences for the underlying changes in employment shares and wages differ. For this reason some shocks are more likely to have occurred than others. For instance, the economy has experienced substantial skill upgrading in the period from 1992 to 2003, which has manifested itself as a six percentage point increase in the employment of highly educated employees (see Table 2). When our model is subjected to increased skill upgrading, it predicts that wages in general increase, the employment share of highly educated workers in non-management increases and the employment share of management employees decreases. These effects explain to a large extent the empirical findings presented in Table 2. On one important aspect, however, skill-upgrading does not produce the empirical findings; that is, the model predicts that skill upgrading leads to higher wages for all employees. But, empirically highly-educated employees in non-management have experienced a decline in income of eight percent. This suggests that besides skill-upgrading, the economy is hit by additional shocks. We consider two possible shocks which may have caused income for highly-educated non-management employees to fall. First, a fall in relative productivity will reduce wages while at the same time counteracting the positive effect on the employment share from skill upgrading. Second, a reduction in bargaining power will also reduce wages, but in contrast to the productivity shocks, it will boost the employment share. Because the employment share of highly-educated non-management employees has grown substantially over the period (75.11 percent), it is more likely that a reduction in the bargaining power of highly educated employees in non-management has occurred, rather than a decline in their relative productivity. Thus, the discussion allows us to conclude that the model is capable of mimicking the recently observed changes in income distribution related both to the increasing income inequality and the changes in relative income and employment shares. We can further point to the fact that subjecting our model to skill upgrading predicts many of the recently observed changes in the income distribution, but additional shocks are required to fully account for the empirical findings. We point at a reduction in the bargaining power of highly-educated non-management employees as being the complementary explanation for the recent changes in the income distribution.

6 Conclusion

Rising income inequality has been documented in most developed countries during recent decades. In this paper, we propose an equilibrium search model which makes it possible to identify the driving forces behind the changes in the income distribution. In particular, the effects from changes in productivity, bargaining power and other labor market conditions, such as skill upgrading, can be established. For instance, when our model is subjected to skill upgrading and changes in employee bargaining power, it is capable of replicating the recent dynamics in the Danish income distribution. It also allows us to conclude that shocks to relative productivity, such as skill-biased technological change, are unlikely to have occurred. One innovation of our model is that we explicitly model the structure of the firm, i.e., we allow for a non-management and a management sector. Important in this respect is that we endogenously establish a management premium and thus provide one explanation for why observationally identical employees (in terms of education) are paid different wages. Further, the modeling of the firm structure also turns out to be significant in understanding the empirical changes in the income distribution, because the action is in the top percentiles, where most employees are managers. Overall, our paper broadens the perspective of what may cause an income distribution to change. This is of particular importance if we want to understand how, for instance, a financial crisis influences the income distribution, as well as to understand historical data. Further research along the lines of this paper is likely to produce important new insights into why income inequality has increased during recent years.

7 Appendix

7.1 Proof of Proposition 1

We introduce the following notation $y_{k\rho} = \mu_{k\rho}y$, $k = L, H$, and $\rho = N, M$ as well as $\mu_{LN} \equiv 1$. The wages of each type of workers in the different jobs follow from the Nash Bargaining Rule (8), and equations (1),(2), and (5). First,

$$rE_k = p_N \frac{\beta_{kN}(y_{kN} - w_{kN})}{(1 - \beta_{kN})(r + s + p_{kM})} + \frac{\beta_{kM}c_{kM}}{(1 - \beta_{kM})q_{kM}}. \quad (14)$$

Using (14) to substitute for E_k in equations (6) and (7) we get

$$w_{kN} = \frac{\beta_{kN}\mu_{kN}y(1 + p_N)}{r + s + p_{kM} + \beta_{kN}p_N}, \quad (15)$$

$$w_{kM} = p_N \frac{(1 - \beta_{kM})w_{kN}}{1 - \beta_{kN}} + \beta_{kM}c_{kM}\theta_{kM} + \beta_{kM}\mu_{kM}y. \quad (16)$$

7.2 Deriving the Steady State Conditions

We first derive the steady state conditions, which equate the flow of workers into a given job to the flow of workers out of that job. For low-educated workers, the flows in and out of non-management can be expressed as

$$p_N e_k = (s + p_{kM})i_{kN}. \quad (17)$$

For management the condition is

$$p_{kM}(e_k + i_{kN}) = s i_{kN}. \quad (18)$$

Since

$$i_{kN} + i_{kM} + e_k = \pi_k, \quad (19)$$

where $\pi_k = \pi$ if $k = L$, and $\pi_k = 1 - \pi$ if $k = H$, the following equations can be derived from (17), (18) and (19)

$$e_k = \frac{s\pi_k}{(s + p_{kM} + p_N)}, \quad (20)$$

and

$$i_{kM} = \frac{p_{kM}\pi_k}{(s + p_{kM})}, \quad (21)$$

$$i_{kN} = \frac{sp_N\pi_k}{(s + p_{kM} + p_N)(s + p_{kM})}. \quad (22)$$

7.3 Existence of Equilibrium

We first substitute the wage expressions into the equations for $J_{k\rho}$. If we then substitute these expressions into the vacancy conditions (1) and (3) we get:

$$\begin{aligned} c_N \theta_N^{1-\alpha} &= \frac{(1 - \pi)\lambda_{L+1}(\lambda_H - \beta_{HN})\mu_1 y}{(\pi(\theta_{HM}^\alpha - \theta_{LM}^\alpha) + \lambda_{L+1})\lambda_H(\lambda_H + \beta_{HN}\theta_N^\alpha)} \\ &+ \frac{\pi\lambda_{H+1}(\lambda_L - \beta_{LN})y}{\pi(\theta_{HM}^\alpha - \theta_{LM}^\alpha) + \lambda_{L+1})\lambda_L(\lambda_L + \beta_{LN}\theta_N^\alpha)}. \end{aligned} \quad (23)$$

Also,

$$\begin{aligned} c_{kM} \theta_{kM}^{1-\alpha} &= \frac{(1 - \beta_{kM})\mu_{kM}y - \beta_{kM}c_{kM}\theta_{kM}}{(r + s)} \\ &- \frac{\theta_N^\alpha(1 - \beta_{kM})(\lambda_k - \beta_{kN})\mu_{kN}y\beta_{kN}}{(r + s)(1 - \beta_{kN})\lambda_k(\lambda_k + \beta_{kN}\theta_N^\alpha)}. \end{aligned} \quad (24)$$

We denote (23) by F_N , and (24) by F_{kM} where $k = L, H$, and where

$$\begin{aligned}\lambda_k &= r + s + \theta_{kM}^\alpha, \\ \lambda_{k+1} &= \theta_{kM}^\alpha + s + \theta_N^\alpha.\end{aligned}$$

We assume that $\beta_{kM} < r + s + \theta_{kM}^\alpha < 2\beta_{kM}$.

The existence proof is standard, i.e., we need to show that the Jacobian matrix of equations (23) and (24) is non-zero in the steady state. Most derivatives are straightforward to sign, i.e, we have $F_{kM, \theta_N} > 0, k = L, H$, and $F_{k, \theta_{jM}} = 0, j \neq k$, as well as $F_{kM, \theta_{kM}} > 0, k = L, H$. For the remaining derivatives we derive sufficient conditions that allow for existence.

It follows that $F_{N, \theta_N} > 0$ if $\pi > 1/2$, and $\mu_{HN} > \underline{\mu}_{HN}$, where

$$\underline{\mu}_{HN} = \frac{(\lambda_L - \beta_{LN})\lambda_H(\lambda_H + \theta_N^\alpha\beta_{HN})}{\lambda_L(\lambda_L + \theta_N^\alpha\beta_{LN})(\lambda_H - \beta_{HN})}.$$

Moreover, $F_{N, \theta_{LM}} < 0$ if $\beta_{LM} < r + s + \theta_{LM}^\alpha < 2\beta_{LM}$ and $\underline{\mu}_{HN} < \mu_{HN}$.

Finally, a sufficient condition for $F_{N, \theta_{HM}} < 0$ is $\beta_{HN}(3 + \beta_{HN}\theta_N^\alpha) > 2\lambda_H$.

It follows that in the steady state

$$\begin{aligned}Det J &= F_{N, \theta_N}(F_{LM, \theta_{LM}}F_{HM, \theta_{HM}}) - F_{N, \theta_{LM}}(F_{LM, \theta_N}F_{HM, \theta_{HM}}) \\ &\quad + F_{N, \theta_{HM}}(-F_{LM, \theta_{LM}}F_{HM, \theta_N}) \\ &> 0.\end{aligned}$$

This proves existence.

7.4 Comparative Statics with respect to $\mu_{k\rho}$ for $k = L, H$ and $\rho = N, M$

7.4.1 Comparative Statics on Steady State Probabilities

We first obtain

$$\begin{aligned}F_{N, \mu_{HN}} &= -\frac{(1 - \pi)\lambda_{L+1}y(\lambda_H - \beta_{HN})}{\lambda_H(\lambda_{L+1} + \pi(\theta_{HM}^\alpha - \theta_{LM}^\alpha))(\lambda_H + \theta_N^\alpha\beta_{HN})} < 0, \\ F_{N, \mu_{LM}} &= 0, \\ F_{N, \mu_{HM}} &= 0.\end{aligned}$$

Second,

$$\begin{aligned}F_{LM, \mu_{HN}} &= 0, \\ F_{LM, \mu_{LM}} &= \frac{-y(1 - \beta_{LM})}{r + s} < 0, \\ F_{LM, \mu_{HM}} &= 0.\end{aligned}$$

Moreover,

$$\begin{aligned} F_{HM,\mu_{HN}} &= \frac{\theta_N^\alpha y(\lambda_H - \beta_{HN})(1 - \beta_{HM})}{(r + s)\lambda_H(1 - \beta_{HN})(\lambda_H + \theta_N^\alpha \beta_{HN})} > 0, \\ F_{HM,\mu_{LM}} &= 0, \\ F_{HM,\mu_{HM}} &= \frac{-y(1 - \beta_{HM})}{r + s} < 0. \end{aligned}$$

Using Cramer's rule we obtain

$$\begin{aligned} \frac{\partial \theta_N}{\partial \mu_{LM}} &> 0, \frac{\partial \theta_N}{\partial \mu_{HM}} > 0, \\ \frac{\partial \theta_{LM}}{\partial \mu_{LM}} &> 0, \frac{\partial \theta_{LM}}{\partial \mu_{HM}} < 0, \\ \frac{\partial \theta_{HM}}{\partial \mu_{HN}} &< 0, \frac{\partial \theta_{HM}}{\partial \mu_{LM}} < 0, \\ \frac{\partial \theta_{HM}}{\partial \mu_{HM}} &> 0. \end{aligned}$$

The sign of the remaining two derivatives depends on the sign of $-F_{N,\mu_{HN}}F_{HM,\theta_{HM}} + F_{N,\theta_{HM}}F_{HM,\mu_{HN}}$, which is ambiguous. It can be shown (full details are available on request) that $\frac{F_{N,\theta_{HM}}}{F_{N,\mu_{HN}}} - \frac{F_{HM,\theta_{HM}}}{F_{HM,\mu_{HN}}} < 0$, implying that $-F_{N,\mu_{HN}}F_{HM,\theta_{HM}} + F_{N,\theta_{HM}}F_{HM,\mu_{HN}} > 0$. It follows that $\frac{\partial \theta_N}{\partial \mu_{HN}} > 0$, $\frac{\partial \theta_{LM}}{\partial \mu_{HN}} < 0$.

7.4.2 Comparative Statics on Wages

Looking at (15) we derive the following expression for $j, k = L, H$ and $\rho = N, M$:

$$\begin{aligned} \frac{\partial w_{kN}}{\partial \mu_{j\rho}} &= \frac{\beta_{kN}\mu_{jN}y(\lambda_k - \beta_{kN})}{(\lambda_k + \beta_{kN}\theta_N^\alpha)^2} \frac{\partial p_N}{\partial \mu_{j\rho}} - \frac{\beta_{kN}\mu_{kN}y(1 + \theta_N^\alpha)}{(\lambda_k + \beta_{kN}\theta_N^\alpha)^2} \frac{\partial p_{kM}}{\partial \mu_{j\rho}} \\ &+ \frac{\beta_{kN}y(1 + \theta_N^\alpha)}{(\lambda_k + \beta_{kN}\theta_N^\alpha)^2} \frac{\partial \mu_{kN}}{\partial \mu_{j\rho}}, \end{aligned} \quad (25)$$

where $\frac{\partial \mu_{kN}}{\partial \mu_{j\rho}} = 0$ for $j \neq k$ and $\rho \neq N$. From (25) it follows that:

$$\begin{aligned} \frac{\partial w_{LN}}{\partial \mu_{HN}} &> 0, \frac{\partial w_{HN}}{\partial \mu_{HN}} > 0, \\ \frac{\partial w_{HN}}{\partial \mu_{LN}} &= 0, \frac{\partial w_{LN}}{\partial \mu_{LN}} = 0, \\ \text{and } \frac{\partial w_{LN}}{\partial \mu_{HM}} &> 0, \frac{\partial w_{HN}}{\partial \mu_{LM}} > 0. \end{aligned}$$

The signs of $\frac{\partial w_{kN}}{\partial \mu_{kM}}$ are ambiguous, since $\frac{\partial p_N}{\partial \mu_{kM}} > 0$ and $\frac{\partial p_{kM}}{\partial \mu_{kM}} > 0$. If we assume that

$$s > p_{kM} > \beta_{kN}, \quad (26)$$

$$r > \beta_{kN}, \quad (27)$$

$$1/2 > p_N > 1/3, \quad (28)$$

$$\beta_{kN} > 1/4, \quad (29)$$

$$(30)$$

then we obtain

$$\frac{\partial w_{kN}}{\partial \mu_{kM}} > \frac{2\beta_{kN}\mu_{kN}yp_{kM}}{(\lambda_k + \beta_{kN}\theta_N^\alpha)^2\mu_2} \left\{ \frac{\mu_{kM}}{p_N} \frac{\partial p_N}{\partial \mu_{kM}} - 2 \frac{\mu_{kM}}{p_{kM}} \frac{\partial p_{kM}}{\partial \mu_{kM}} \right\}.$$

A sufficient condition for $\frac{\partial w_{kN}}{\partial \mu_{kM}} > 0$ is then

$$|\epsilon_{\mu_{kM}, p_N}| > 2|\epsilon_{\mu_{kM}, p_{kM}}|,$$

where $\epsilon_{\mu_{kM}, p_N} = -\frac{\mu_{kM}}{p_N} \frac{\partial p_N}{\partial \mu_{kM}}$. Similarly, for $j, k = L, H$ and $\rho = N, M$, we compute

$$\frac{\partial w_{kM}}{\partial \mu_{j\rho}} = \frac{(1 - \beta_{kM})}{(1 - \beta_{kN})} w_{kN} \frac{\partial p_N}{\partial \mu_{j\rho}} + \frac{p_N(1 - \beta_{kM})}{(1 - \beta_{kN})} \left(\frac{\beta_{kN}y(1 + \theta_N^\alpha)}{(\lambda_k + \beta_{kN}\theta_N^\alpha)^2} \frac{\partial \mu_{kN}}{\partial \mu_{j\rho}} \right) \quad (31)$$

$$+ \frac{\beta_{kN}\mu_{kN}y(\lambda_k - \beta_{kN})}{(\lambda_k + \beta_{kN}\theta_N^\alpha)^2} \frac{\partial p_N}{\partial \mu_{j\rho}} + \left\{ -\frac{\beta_{kN}\mu_{kN}y(1 + \theta_N^\alpha)}{(\lambda_k + \beta_{kN}\theta_N^\alpha)^2} \alpha \theta_{kM}^{\alpha-1} + \beta_{kM}c_{kM} \right\} * \\ * \frac{\partial \theta_{kM}}{\partial \mu_{j\rho}} + \beta_{kM}y \frac{\partial \mu_{kM}}{\partial \mu_{j\rho}}, \quad (32)$$

where $\frac{\partial \mu_{k\rho}}{\partial \mu_{j\nu}} = 0$, for $j \neq k$, and $\rho \neq \nu$. Assume that

$$p_N > p_{kM}, \quad (33)$$

$$1 > \lambda_k + \beta_{kN}\theta_N^\alpha. \quad (34)$$

We see from (31) that a sufficient condition for $\frac{\partial w_{kM}}{\partial \mu_{j\rho}} > 0, j \neq k$, is

$$\mu_{kN}y > \frac{(1 - \beta_{kN})\beta_{kM}c_{kM}}{\alpha(1 - \beta_{kM})\beta_{kN}}. \quad (35)$$

However, looking at (31) the signs of the derivatives $\frac{\partial w_{kM}}{\partial \mu_{kM}}$ are ambiguous. We have, under condition (26), that $(1 + p_N)\lambda_k + p_N(\lambda_k + \beta_{kN}p_N) > p_{kM}(1 + p_N)$, and since $p_N(1 + p_N) < 2p_N(1 + p_N)$, this implies that

$$\frac{\partial w_{kM}}{\partial \mu_{kM}} > \frac{(1 - \beta_{kM})\beta_{kN}\mu_{kN}yp_{kM}p_N(1 + p_N)}{(1 - \beta_{kN})(\lambda_k + \beta_{kN}\theta_N^\alpha)^2\mu_{kM}} \left\{ \frac{\mu_{kM}}{p_N} \frac{\partial p_N}{\partial \mu_{kM}} - 2 \frac{\mu_{kM}}{p_{kM}} \frac{\partial p_{kM}}{\partial \mu_{kM}} \right\} \\ + \beta_{kM}c_{kM} \frac{\partial \theta_{kM}}{\partial \mu_{kM}} + \beta_{kM}y.$$

A sufficient condition for $\frac{\partial w_{kM}}{\partial \mu_{kM}} > 0$ is then $|\epsilon_{\mu_{kM}, p_N}| > 2|\epsilon_{\mu_{kM}, p_{kM}}|$.

7.4.3 Comparative Statics on Steady State Stocks

We first compute

$$\frac{\partial i_{kN}}{\partial \mu_{jv}} = \frac{s\pi_k}{\lambda_{k+1}^2} \frac{\partial p_N}{\partial \mu_{jv}} - \frac{s\pi_K p_N (2s + 2p_{kM} + p_N)}{\lambda_{k+1}^2 (s + p_{kM})^2} \frac{\partial p_{kM}}{\partial \mu_{jv}}, \quad (36)$$

and

$$\frac{\partial i_{kM}}{\partial \mu_{jv}} = \frac{s\pi_k}{(s + p_{kM})^2} \frac{\partial p_{kM}}{\partial \mu_{jv}}. \quad (37)$$

It follows from (36) and (37) that

$$\frac{\partial i_{LN}}{\partial \mu_{HN}} > 0, \frac{\partial i_{LM}}{\partial \mu_{HN}} < 0,$$

$$\frac{\partial i_{HN}}{\partial \mu_{HN}} > 0, \frac{\partial i_{HM}}{\partial \mu_{HN}} < 0,$$

$$\text{as well as } \frac{\partial i_{LM}}{\partial \mu_{LM}} > 0, \frac{\partial i_{HN}}{\partial \mu_{LM}} > 0,$$

$$\text{and } \frac{\partial i_{HM}}{\partial \mu_{LM}} < 0, \frac{\partial i_{LN}}{\partial \mu_{HM}} > 0.$$

$$\text{Finally, } \frac{\partial i_{LM}}{\partial \mu_{HM}} < 0, \text{ and } \frac{\partial i_{HM}}{\partial \mu_{HM}} > 0.$$

The signs of $\frac{\partial i_{kN}}{\partial \mu_{kM}}$, $k = L, H$ are both ambiguous.

Under conditions (26), (28), and (29) we have

$$(s + p_{kM})^2 > 2p_{kM}(s + p_{kM}) > 4p_{kM}^2 > 4\beta_{kN}p_{kM}, \quad (38)$$

$$p_N(2s + 2p_{kM} + p_N) < p_N(1 + p_N) < 8\beta_{kN}p_N. \quad (39)$$

It then follows that

$$\frac{\partial i_{kN}}{\partial \mu_{kM}} > \frac{4sp_{kM}p_{LN}\pi}{\lambda_k^2(s + p_{kM})^2} \left\{ \frac{\beta_{kN}}{p_N} \frac{\partial p_N}{\partial \mu_{kM}} - 2 \frac{\beta_{kN}}{p_{kM}} \frac{\partial p_{kM}}{\partial \mu_{kM}} \right\} > 0,$$

where the last inequality holds if $|\epsilon_{\mu_{kM}, p_N}| > 2|\epsilon_{\mu_{kM}, p_{LM}}|$.

7.5 Comparative Statics with respect to π

7.5.1 Comparative Statics on Steady State Probabilities

We compute

$$F_{N,\pi} = \frac{\lambda_L y \lambda_H (\lambda_H - \beta_{HN})}{(\lambda_H + p_N \beta_{HN})(\lambda_{L+1} + \pi(p_{HM} - p_{LM}))^2} (\mu_{HN} - \underline{\mu}_{HN}) > 0,$$

where the sign follows from the assumption that $\underline{\mu}_{HN} < \mu_{HN}$. We also have

$$F_{LM,\pi} = F_{HM,\pi} = 0.$$

Hence using Cramer's rule we obtain

$$\frac{\partial \theta_N}{\partial \pi} < 0, \frac{\partial \theta_{LM}}{\partial \pi} > 0, \text{ and } \frac{\partial \theta_{HM}}{\partial \pi} > 0.$$

7.5.2 Comparative Statics on Wages

If we compute the effect of a change of π on the wages in non-management we get

$$\frac{\partial w_{kN}}{\partial \pi} = \frac{\beta_{kN} \mu_{kN} y (\lambda_k - \beta_{kN})}{(\lambda_k + \beta_{kN} p_N)^2} \frac{\partial p_N}{\partial \pi} - \frac{\beta_{kN} \mu_{kN} y (1 + p_N)}{(\lambda_k + \beta_{kN} p_N)^2} \frac{\partial p_{kM}}{\partial \pi} < 0.$$

For the wages in management

$$\begin{aligned} \frac{\partial w_{kM}}{\partial \pi} &= \frac{p_N (1 - \beta_{kM})}{(1 - \beta_{kN})} \frac{\beta_{kN} \mu_{kN} y (\lambda_k - \beta_{kN})}{(\lambda_k + \beta_{kN} p_N)^2} \frac{\partial p_N}{\partial \pi} \\ &+ \frac{(1 - \beta_{kM})}{(1 - \beta_{kN})} \frac{\beta_{kN} \mu_{kN} y (1 + p_N)}{(\lambda_k + \beta_{kN} p_N)} \frac{\partial p_N}{\partial \pi} \\ &+ \frac{\partial \theta_{kM}}{\partial \pi} \left\{ - \frac{p_N (1 - \beta_{kM})}{(1 - \beta_{kN})} \frac{\beta_{kN} \mu_{kN} y (1 + p_N)}{(\lambda_k + \beta_{kN} p_N)} \alpha \theta_{kM}^{\alpha-1} + \beta_{kM} c_{kM} \right\}. \end{aligned}$$

It can be shown that the condition derived in (35) is a sufficient condition for $\frac{\partial w_{kM}}{\partial \pi} < 0$.

7.5.3 Comparative Statics on Steady State Stocks

Moving on to the effect of a change of π on the steady state stocks we get

$$\frac{\partial i_{kN}}{\partial \pi} = \frac{\delta_k s p_N}{\lambda_{k+1} (s + p_{kM})} + \frac{s \pi_k}{\lambda_{k+1}^2} \frac{\partial p_N}{\partial \pi} - \frac{s \pi p_N (2s + 2p_{kM} + p_N)}{\lambda_{k+1}^2 (s + p_{kM})^2} \frac{\partial p_{kM}}{\partial \pi}, \quad (40)$$

where $\delta_L = 1$ and $\delta_H = -1$. It follows from (40) that $\frac{\partial i_{HN}}{\partial \pi} < 0$. However the sign of $\frac{\partial i_{LN}}{\partial \pi}$ is ambiguous.

We will show that $\frac{\partial i_{LN}}{\partial \pi} > 0$ if conditions (38) and (39) are satisfied. (40) can be rewritten as

$$\frac{\partial i_{LN}}{\partial \pi} > \frac{4s \beta_{LN} p_N p_{LM}}{\lambda_{L+1}^2 (s + p_{LM})^2} \left\{ 1 + \frac{\pi}{p_N} \frac{\partial p_N}{\partial \pi} - 2 \frac{\pi}{p_{LM}} \frac{\partial p_{LM}}{\partial \pi} \right\}.$$

This is positive if

$$1 > -\frac{\pi}{p_N} \frac{\partial p_N}{\partial \pi} + 2 \frac{\pi}{p_{LM}} \frac{\partial p_{LM}}{\partial \pi},$$

i.e if $1 > |\epsilon_{\pi, p_N}| + 2|\epsilon_{\pi, p_{LM}}|$.

We also have

$$\frac{\partial i_{LM}}{\partial \pi} = \frac{p_{LM}}{s + p_{LM}} + \frac{s \pi}{(s + p_{LM})^2} \frac{\partial p_{LM}}{\partial \pi} > 0.$$

In a similar fashion,

$$\frac{\partial i_{HM}}{\partial \pi} = -\frac{p_{HM}}{s + p_{HM}} + \frac{s\pi}{(s + p_{HM})^2} \frac{\partial p_{HM}}{\partial \pi}.$$

The sign of this derivative is ambiguous. We will show that $\frac{\partial i_{HM}}{\partial \pi} > 0$ if $|\epsilon_{\pi, p_{HM}}| > \frac{\pi}{p_{HM}}$. To do so assume that $p_{HM} < s(1 - \pi)$ and that $(s + p_{HM})^2 < 4s^2 < 2\pi p_{HM} < (s + p_{HM})$. Then

$$\frac{\partial i_{HM}}{\partial \pi} > \frac{s(1 - \pi)}{2\pi^2} \left\{ \frac{\pi}{p_{HM}} \frac{\partial p_{HM}}{\partial \pi} - \frac{\pi}{p_{HM}} \right\}.$$

This is positive if $|\epsilon_{\pi, p_{HM}}| > \frac{\pi}{p_{HM}}$.

7.6 Comparative Statics with respect to $\beta_{j\rho}, j, k = L, H$ and $\nu, \rho = N, M$

7.6.1 Comparative Statics on Steady State Probabilities

Computing the derivatives of F_1 with respect to the different $\beta_{k\rho}$ we get

$$\begin{aligned} F_{N, \beta_{kN}} &= \frac{\pi_k(1 + \theta_N^\alpha) \mu_{kN} y \lambda_{k+1}}{(\lambda_k + \pi(\theta_{HM}^\alpha - \theta_{LM}^\alpha))(\lambda_k + \theta_N^\alpha \beta_{kN})^2} > 0 \\ F_{N, \beta_{LM}} &= F_{1, \beta_{HM}} = 0 \end{aligned}$$

Computing the partial derivatives of $F_{kM}; j, k = L, M$ and $\rho = N, M$

$$\begin{aligned} F_{k, \beta_{kN}} &= -\frac{y\left(\frac{1}{\lambda_k} - \frac{(1 + \theta_N^\alpha)(\lambda_k + \theta_N^\alpha \beta_{kN}^2)}{(\lambda_k + \theta_N^\alpha \beta_{kN})^2}\right)(1 - \beta_{kM})}{(r + s)(1 - \beta_{kN})^2} \\ F_{k, \beta_{j\rho}} &= 0, j \neq k \\ F_{k, \beta_{kM}} &= \frac{c_{kM} \theta_{kM} - \frac{\theta_N^\alpha y (\lambda_k - \beta_{kN}) \beta_{kN}}{\lambda_k (1 - \beta_{kN}) (\lambda_k + \theta_N^\alpha \beta_{kN})} + y \mu_{kM}}{r + s} > 0. \end{aligned}$$

It can be shown that $F_{kM}, \beta_{kN}, k = L, H > 0$ if $\lambda_k + 2\theta_N^\alpha \beta_{kN} < 1$. Then a sufficient condition for $F_{kM, \beta_{kN}} > 0$ is $1 < \beta_{kN}(1 + \theta_N^\alpha)$. A sufficient condition for $F_{kM, \theta_{kM}} > 0; k = L, H$ is $(1 - \beta_{kN}) \mu_{kM} > 1$.

Using Cramer's rule we compute the sign of the following derivatives $\frac{\partial \theta_N}{\partial \beta_{kN}} < 0, \frac{\partial \theta_{kM}}{\partial \beta_{jN}} > 0, j \neq k, \frac{\partial \theta_N}{\partial \beta_{kM}} < 0, \frac{\partial \theta_{kM}}{\partial \beta_{kM}} < 0, \frac{\partial \theta_{kM}}{\partial \beta_{jM}} > 0, j \neq k$.

However the signs of $\frac{\partial \theta_{kM}}{\partial \beta_{kN}}, k = L, H$ are ambiguous. We derive sufficient conditions for $\frac{\partial \theta_{kM}}{\partial \beta_{kN}} < 0$.

This inequality is obtained if

$$\begin{aligned}(s + \theta_N^\alpha)\beta_{kN} + (1 + \beta_{kN})\theta_{kM}^\alpha &> \theta_{kM}^\alpha \\ (1 + \beta_{kN})\theta_{kM}^\alpha &> r.\end{aligned}$$

We now derive the comparative statics of the different wages with respect to the different $\beta_{j\rho}$

7.6.2 Comparative Statics on Wages

$$\begin{aligned}\frac{\partial w_{kN}}{\partial \beta_{j\rho}} &= \frac{\beta_{LN}\mu_{kN}y(\lambda_k - \beta_{kN})}{(\lambda_k + \beta_{kN}\theta_N^\alpha)^2} \frac{\partial p_N}{\partial \beta_{j\rho}} - \frac{\beta_{kN}\mu_{kN}y(1 + p_N)}{(\lambda_k + \beta_{kN}\theta_N^\alpha)^2} \frac{\partial p_{kM}}{\partial \beta_{j\rho}} \\ &+ \frac{\mu_{kN}y(1 + p_N p_N)\lambda_k}{(\lambda_k + \beta_{kN}\theta_N^\alpha)^2} \frac{\partial \beta_{kN}}{\partial \beta_{j\rho}},\end{aligned}\quad (41)$$

where $\frac{\partial \beta_{kN}}{\partial \beta_{j\rho}} = 0$ for $j \neq k$ and $\rho \neq N$. Similarly,

$$\begin{aligned}\frac{\partial w_{kM}}{\partial \beta_{j\rho}} &= \frac{(1 - \beta_{kM})}{(1 - \beta_{kN})} \left\{ w_{kN} \frac{\partial p_N}{\partial \beta_{j\rho}} + p_N \frac{\partial w_{kN}}{\partial \beta_{j\rho}} \right\} + \beta_{kM} c_{kM} \frac{\partial \theta_{LM}}{\partial \beta_{j\rho}} \\ &+ \frac{(1 - \beta_{kM})w_{kN}}{(1 - \beta_{kN})^2} \frac{\partial \beta_{kN}}{\partial \beta_{j\rho}} + (c_{kM}\theta_{LM} + \mu_{kM}y - \frac{p_N w_{kN}}{(1 - \beta_{LN})}) \frac{\partial \beta_{kM}}{\partial \beta_{j\rho}}.\end{aligned}\quad (42)$$

We see from (41) and (42) that $\frac{\partial w_{k\nu}}{\partial \beta_{j\rho}} = 0$ for $j \neq k$, and $\nu \neq \rho$.

From these derivatives we see that $\frac{\partial w_{kN}}{\partial \beta_{jN}} < 0$ for $j \neq k$, and $\frac{\partial w_{kN}}{\partial \beta_{jM}} < 0$, for $j \neq k$. All the signs of the other partial derivatives are ambiguous. In a proof available upon request we show that that $\frac{\partial w_{kN}}{\partial \beta_{kN}} > 0$ if $|\epsilon_{\beta_{kN}, p_N}| < 1$. Similarly we show that $\frac{\partial w_{kN}}{\partial \beta_{kM}} < 0$ if conditions (26) to (29) are satisfied. In that case a sufficient condition for $\frac{\partial w_{kN}}{\partial \beta_{kM}} < 0$ is then

$$|\epsilon_{\beta_{kM}, p_N}| > 2|\epsilon_{\beta_{kM}, p_{kM}}|.$$

It can also be shown that if condition (28) are satisfied then a sufficient condition for $\frac{\partial w_{kM}}{\partial \beta_{kN}} > 0$ is $|\epsilon_{\beta_{LN}, p_N}| < 1$.

It is straightforward to show that $\frac{\partial w_{kM}}{\partial \beta_{jM}} < 0$ if (35) holds.

Finally, one can show that $\frac{\partial w_{kM}}{\partial \beta_{kM}} > 0$ if (35) holds, and if the following inequalities are satisfied:

$$\begin{aligned}2\beta_{kM} &> 1, \\ \mu_{kM}(1 - \beta_{kN}) &> (1 + 2p_N)\beta_{kN}, \\ |\epsilon_{\mu_{kM}, p_N}| &< 1.\end{aligned}$$

7.6.3 Comparative Statics on Steady State Stocks

Computing the comparative statics with respect to the steady state stocks we get

$$\frac{\partial i_{kN}}{\partial \beta_{j\rho}} = \frac{s\pi_k(s + p_{kM})^2}{\lambda_{k+1}^2(s + p_{kM})^2} \frac{\partial p_N}{\partial \beta_{j\rho}} - \frac{s\pi_k p_N(2s + 2p_{kM} + p_N)}{\lambda_{k+1}^2(s + p_{kM})^2} \frac{\partial p_{LM}}{\partial \beta_{j\rho}},$$

and

$$\frac{\partial i_{kM}}{\partial \beta_{j\rho}} = \frac{s\pi_k}{(s + p_{kM})^2} \frac{\partial p_{kM}}{\partial \beta_{j\rho}}.$$

From this we see that $\frac{\partial i_{kN}}{\partial \beta_{j\rho}} < 0$ for $j \neq k$, $\frac{\partial i_{kM}}{\partial \beta_{j\rho}} > 0$ for $j \neq k$, and $\frac{\partial i_{kM}}{\partial \beta_{k\rho}} < 0$.

It can also be shown that $\frac{\partial i_{kN}}{\partial \beta_{k\rho}} < 0$

Under conditions (26), (27), (28), and (29), we showed that conditions (38) and (39) are satisfied implying that

$$\frac{\partial i_{kN}}{\partial \beta_{k\rho}} < \frac{4sp_{kM}p_{LN}\pi}{\lambda_{k+1}^2(s + p_{kM})^2} \left\{ \frac{\beta_{kN}}{p_N} \frac{\partial p_N}{\partial \beta_{k\rho}} - 2 \frac{\beta_{kN}}{p_{kM}} \frac{\partial p_{kM}}{\partial \beta_{k\rho}} \right\} < 0,$$

where the last inequality holds if $|\epsilon_{\beta_{kN}, p_N}| > 2|\epsilon_{\beta_{kN}, p_{kM}}|$.

7.7 Additional Descriptive Statistics

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