CBESS Discussion Paper 09-13

On modelling vagueness – and on *not* modelling incommensurability

by Robert Sugden*

*University of East Anglia

16 June 2009

Abstract: This paper defines and analyses the concept of a 'ranking problem'. In a ranking problem, a set of objects, each of which possesses some common property *P* to some degree, are ranked by *P*-ness. I argue that every eligible answer to a ranking problem can be expressed as a complete and transitive 'is at least as *P* as' relation. Vagueness is expressed as a multiplicity of eligible rankings. Incommensurability, properly understood, is the absence of a common property *P*. Trying to analyse incommensurability in the same framework as ranking problems causes unnecessary confusion.

Acknowledgements My work has been supported by the Economic and Social Research Council of the UK (award no. RES 051 27 0146). I thank Wlodek Rabinowicz for comments on previous drafts of this paper.

University of East Anglia

CBESS

Centre for Behavioural and Experimental Social Science

CBESS
University of East Anglia
Norwich NR4 7TJ
United Kingdom
www.uea.ac.uk/ssf/cbess

On modelling vagueness – and on *not* modelling incommensurability

Robert Sugden

School of Economics

University of East Anglia

Norwich NR4 7TJ

UK

r.sugden@uea.ac.uk

16 June 2009

Abstract This paper defines and analyses the concept of a 'ranking problem'. In a ranking problem, a set of objects, each of which possesses some common property *P* to some degree, are ranked by *P*-ness. I argue that every eligible answer to a ranking problem can be expressed as a complete and transitive 'is at least as *P* as' relation. Vagueness is expressed as a multiplicity of eligible rankings. Incommensurability, properly understood, is the absence of a common property *P*. Trying to analyse incommensurability in the same framework as ranking problems causes unnecessary confusion.

Acknowledgements My work has been supported by the Economic and Social Research Council of the UK (award no. RES 051 27 0146). I thank Wlodek Rabinowicz for comments on previous drafts of this paper.

As an economist and decision theorist with an interest in philosophy, I have occasionally dipped into philosophical writings on incommensurability and vagueness. I have usually come away with a sense of puzzlement – with a feeling that mountains are being made out of molehills. Since this may reveal my lack of understanding of what philosophers are trying to do, I will begin by explaining how I understand the class of problems to which the concepts of incommensurability and vagueness might seem to apply. I will then offer an analysis of that class of problems, as I have defined it. Formally, this analysis has some similarities with that proposed by Wlodek Rabinowicz (2009), but it has a very different interpretation – and is much simpler.

I start with the following concept of a *ranking problem*. There is some non-empty set S of *objects*; typical objects are denoted w, x, y and z. There is some *ranking property* P, such that each object x in S has this property to some degree (which may be zero), independently of which other members of S it is compared with, and such that, for all x, y in S, the question 'Is x at least as P as y?' is meaningful. This allows us to define a binary relation $\geq_P$ on S, such that $x \geq_P y$ denotes 'x is at least as P as y'. (Formally, $\geq_P$ is a subset of the set of ordered pairs of elements of S.) Let $x =_P y$ denote [$x \geq_P y$ and $y \geq_P x$]; let $x >_P y$ denote [$x \geq_P y$ and not $y \geq_P x$]; and let $x \#_P y$ denote [(not $x \geq_P y$) and (not $y \geq_P x$)]. Deciding what the relation $\geq_P$ is (or ought to be) is a ranking problem. For example, suppose I agree to serve on a committee of the great and the good of British economics, charged with awarding a prize to the best young British economist of the year. We are given a list of young British economists who have been nominated for the prize and are required to select one. This is a ranking problem: S is the set of nominees and P is the property 'good as an economist'.

The theoretical problem to be addressed concerns the formal properties of $\geq_P$. By virtue of our understanding of the concept of 'is at least as ... as', can we impose any general restrictions on $\geq_P$? The two most obvious candidates are *transitivity* and *completeness*. The relation is transitive if, for all x, y, z in S, [$x \geq_P y$ and $y \geq_P z$] implies $x \geq_P z$. It is complete if, for all x, y in S, either $x \geq_P y$ or $y \geq_P x$ (or both). Equivalently, it is complete if there are no x, y in S such that $x \#_P y$. A binary relation that is transitive and complete is an *ordering* (some theorists use the term *complete ordering*). In the context of discussions of incommensurability and vagueness, the crucial question is whether completeness should be imposed. Does it make sense to assert of some x and y in S that it is not the case that x is at least as P as y, but nor is it the case that y is at least as P as x?

Let me now explain how I understand this kind of question. As a decision theorist, I take myself to be constructing *models* that can be used to analyse real problems of judgement and decision. In the case of my example, the real problem is that of the committee charged with awarding the prize. The formal concepts S, P and $\geq_P$ are not part of that reality; they belong to a model of it. I am looking for modelling strategies that are internally consistent, tractable, and usefully applicable to a reasonably wide range of real-world problems. Thus, I shall not ask whether the relation 'is at least as good an economist as', defined on the set of young British economists, *really is* complete. What I shall ask is whether, for the analysis of problems such as that faced by the committee, it is a good modelling strategy to postulate completeness.

For the purposes of this paper, I take it as given that the concern is with *reason-based* models. To use a reason-based framework is to choose a modelling strategy which focuses on the relationship between decisions or judgements and the reasons that are taken to justify them. Within such a framework, only a limited range of decision-relevant attitudes can be taken into account. To expect a model of decision-making to be able to take every potentially relevant factor into account is to fail to see the point of modelling.

1. Some *faux*-ranking problems

My definition of a 'ranking problem' has been deliberately constructed to exclude certain kinds of problem which, at least at first sight, call for what in ordinary language might be called 'ranking'. These *faux-ranking* problems share a number of features which are also common to genuine ranking problems. Each *faux*-ranking problem can be understood as having some set S of objects, some property P, and a corresponding binary relation $\geq_P$ defined on S. In each case, it turns out that this relation can fail to satisfy completeness or transitivity. I shall show that, in these cases, incompleteness or intransitivity is possible because of features of the relevant problem that are inconsistent with my definition of a 'ranking problem'. I shall claim that it is better to exclude these *faux*-ranking problems from the domain of a modelling framework that is intended to deal with ranking problems as I have defined them. To try to adapt such a modelling framework to encompass these problems would introduce unnecessary confusion.

One type of *faux*-ranking problem involves *non-applicability*. Let S be the set of things whose names begin with 'c'. Let x, y and z be Camembert, Cheddar and the chi-squared test. Let P be 'crumbly'. On my definition, this is not a ranking problem. Camembert and Cheddar each have the property of crumbliness to some degree (possibly zero), but the chi-squared test does not. So, to the question 'Is Cheddar crumblier than the chi-squared test?', an appropriate answer is 'Not applicable'. Formally, there would be nothing wrong with defining a relation $\geq_P$ on S and then representing the crumbliness relation between Cheddar and the chi-squared test as $y \#_P z$. Clearly, this relation would violate completeness. But it is hard to imagine a context of decision or judgement in which the 'not applicable' components of such a relation would be useful. So nothing substantial is lost, and a potential source of confusion is removed, if we exclude cases of non-applicability from the concept of a ranking problem.

A second type of *faux*-ranking problem involves *incommensurability* in the literal sense of having no common measure. Let S be defined so that it includes the University of East Anglia (w), the University of York (x), *Wives and Daughters* (y) and *Jane Eyre* (z). Let P be 'good'. There is a sense in which all of these objects have the property of goodness to some degree. The University of East Anglia and the University of York are both good universities (or so I like to believe). *Wives and Daughters* and *Jane Eyre* are both good novels. But to the question 'Is the University of East Anglia better than *Wives and Daughters*?', the obvious answer is again 'not applicable'. The question has no real meaning because 'good as a university' and 'good as a novel' are, for all practical purposes, different concepts. Since there is no common standard of measurement for 'better university than' and 'better novel than', the question is analogous with 'Is 10 decibels greater than 100 grammes?' The concept of 'greater than' can be applied to both sound and mass, but there is no common standard of measurement for 'greater sound than' and 'greater mass than': sound and mass are incommensurable. Problems of this kind are not ranking problems by my definition. Again, there would be nothing formally wrong in defining a non-complete relation $\geq_P$ on S as the union of the ranking of universities by 'good as a university' and the ranking of novels by 'good as a novel', and then representing the betterness relation between the University of East Anglia and *Wives and Daughters* as $w \#_P y$. But I cannot imagine any use for such a relation. If goodness in universities and goodness in novels are different concepts, why amalgamate them in the same ranking?

A third type of *faux*-ranking problem involves *multiple meanings*. Let S be the set of cars on the British roads. Let w, x, y and z be particular cars: w and x are high-performance sports cars; y and z are small, low-emission city cars; w and y are painted green; x and z are painted red. Let P be 'green'. To the question 'Is w greener than z?', the obvious answer is: 'It depends what you mean by "green"'. Although 'green' as a colour and 'green' as 'environmentally friendly' have a common etymological root, these meanings are too far apart to be sensibly combined. This is not a ranking problem by my definition. Formally, we could define a non-complete relation $\geq_P$ on S and stipulate that one car is 'at least as green as' another if and only if it is *both* at least as green in colour *and* at least as environmentally friendly. Then (assuming only two colours and two environmental specifications) we would have $w >_{\mathrm{P}} x$ and $y >_{\mathrm{P}} z$ (by colour), $y >_{\mathrm{P}} w$ and $z >_{\mathrm{P}} x$ (by environmental specification), $y >_{\mathrm{P}} x$ (by both), and $w \#_{\mathrm{P}} z$. But, just as in the case of incommensurability, I can see no use in amalgamating two distinct concepts into a single ranking.

The final type of *faux*-ranking I consider involves *comparison relativity*. The following example is borrowed from John Broome (1991, pp. 10–12). Let S be the set of cities in the world. Its elements include Tokyo (w), London (x), New York (y) and Honolulu (z). Let P be 'westerly'. On what I (but not Broome) take to be the most natural interpretation of 'westerly', one city is more westerly than another if the shortest route from the latter to the former involves heading west. Thus London is more westerly than Tokyo, New York than London, Honolulu than New York, and Tokyo than Honolulu: the relation $\geq_P$ is non-transitive. This is not a ranking problem by my definition, because westerliness is not a property that cities possess independently of which other cities they are compared with. Thus, although it is meaningful to ask whether one city is more westerly than another, it is not meaningful to ask which is the most westerly of all cities. According to some theories of choice, including the theory of *regret* proposed by Graham Loomes and me, preference is a comparison-relative concept. In this theory, which of two uncertain acts a_1 and a_2 is chosen in a binary choice problem is influenced by anticipated regret; the possible regret resulting from choosing, say a_1 depends on how its consequences are juxtaposed with those of the non-chosen act a_2. Thus, there need be no inconsistency in a case in which a_1 is chosen from the set $\{a_1, a_2\}$, a_2 from $\{a_2, a_3\}$, and a_3 from $\{a_3, a_1\}$ (Loomes and Sugden, 1982). For a person whose reasoning is represented by regret theory, determining binary preferences is not a ranking problem as I have defined it. Needless to say, I am not denying the usefulness of developing a formal model of regret-based preferences (or, indeed, of westerliness). But the logical structure of a comparison-relative relation is very different from that of $\geq_P$ in a ranking problem.

2. Imprecision

It is now time to consider whether, in a genuine ranking problem, $\geq_P$ can be incomplete. To keep things concrete, I will focus on a particular problem, in which the putative ranking property *P* is vague. I shall not pre-judge whether or not this is a ranking problem; but I shall argue that it can reasonably be understood as such.

During 2002, the BBC ran a series of television programmes on the theme of 'Great Britons'. (My information is taken from BBC websites.) The set of potential 'great Britons' was defined to include 'anyone who was born in the British Isles, including Ireland; or anyone who lived in the British Isles, including Ireland, and who has played a significant part in the life of the British Isles'. The BBC began with a publicity campaign asking people to nominate 'their greatest Briton of all time'; 30,000 people responded. The 100 most-nominated Britons were announced in rank order, except that the relative ranking of the top ten was not revealed. (I regret to report that no economist or philosopher was included in these 100 names.) Then there was a television programme on each of the ten most popular nominees, with a further round of telephone and internet voting to determine the order of those ten. The eventual winner was Winston Churchill.

Is this a ranking problem? There is a reasonably well-defined set *S* of people who were born in, or lived in, the British Isles at any time in history. To focus on issues of incommensurability and vagueness, I shall ignore the problem of gaining factual information about these people and their achievements. There is a property *P* of 'greatness', and a corresponding relation $\geq_P$, 'at least as great as'. To determine whether this is a ranking problem we must ask if every person in this set has the property of greatness to some degree (which may be zero), independently of comparisons with others, and if, for each pair *x*, *y* of such people, the question: 'Is *x* at least as great as *y*?' is meaningful.

One response is to deny that 'greatness' as a property of human beings is a meaningful concept at all. That response seems to me to be coherent but, in the context of a discussion about how to model comparative judgements, it is not particularly interesting. If there is no such thing as greatness, there is no point in worrying about how to model it.

Another response is that there are different kinds of greatness, not comparable with one another. For example, one might recognise greatness as a property of military leaders and greatness as a property of writers, but deny their comparability. One might say that Horatio Nelson (6th in the BBC ranking) is greater than the Duke of Wellington (15th) and Montgomery of Alamein (88th), and that William Shakespeare (5th) is greater than Charles Dickens (41st) and Jane Austen (70th), while denying the meaningfulness of asking whether *any* military leader is greater than *any* writer. At first sight, this position seems to make 'is at least as great as' an incomplete (but perhaps transitive) relation. But that way of expressing non-comparability, although not exactly confused, introduces unnecessary confusion. To say that *all* comparisons of greatness between military leaders and writers are meaningless is to say that the two kinds of greatness are incommensurable, just as goodness of universities is incommensurable with goodness of novels. On this view, the Great Britons problem is a faux-ranking problem. If there are two or more independent and entirely unconnected kinds of greatness, the simplest way to represent that is to define a separate ranking problem for each kind. Incommensurability of different kinds of greatness is then no more of a philosophical problem than the incommensurability of volts and litres.

Nevertheless, the premise of the television series was that comparisons of greatness *can* be made across different categories of human distinction. Presumably the 30,000 people who submitted nominations, and the hundreds of thousands of viewers who voted in the final stage, found the concept of a 'Great Briton' meaningful. From now on I will take that as a working hypothesis.

Even if 'greatness' is meaningful, there is an ambiguity in the concept of 'the greatest Briton of all time': is the criterion greatness *as a Briton* or greatness *per se*, Britishness being used only to delimit the set of candidates? (Analogously, consider the question of whether Arsène Wenger is a great economist. He has a master's degree in economics, and he has shown greatness in football management; but, as far as I know, he has few achievements to his credit in economics.) I conjecture that some respondents to the BBC poll resolved this ambiguity in one way and some in the other. The high rankings given to iconic defenders of Britain and its constituent nations, such as Churchill, Alfred the Great (14th), Owain Glyndwr (23rd), Boudicca (35th), William Wallace (48th) and James Connolly (64th), and to William Blake (38th) and Edward Elgar (60th) – respectively the author of *Jerusalem* and the composer of *Land of Hope and Glory* – suggest that many respondents were thinking about 'greatness as a Briton'. In contrast, Charles Darwin (4th), Isaac Newton (8th) and Alexander Fleming (20th) are perhaps better thought of as great scientists who happen to be British. Fortunately for the success of the television series, there is considerable overlap between the two meanings of ‘great Briton’, particularly from the viewpoint of British judges. For example, Churchill, Shakespeare, Isambard Kingdom Brunel (2nd) and Oliver Cromwell (10th) are great men on any reckoning, but their achievements have special relationships with Britain. In principle, however, this is a case of multiple meanings, analogous with ‘greenness’ as a property of cars.

One way of thinking about this ambiguity is to ask whether the organisers of the poll were at fault for not resolving it. It is reasonable for a respondent to object that the question he is being asked is not adequately specified? In the Great Britons case, I suggest, the answer is ‘Yes’. Analogously, if I were asked to be part of a committee awarding a ‘great economist’ prize, I would feel justified in asking the organisers to make clear whether someone like Wenger counted as a great economist. A reason-based model of judgement or decision cannot be expected to impose order on responses to ill-specified questions.

However, greatness may have a recognisable core meaning while still being *imprecise* (or, which I treat as a synonym, ‘vague’) in ways that are intrinsic to the concept itself. Consider the problem of comparing greatness across categories of achievement. One can recognise the meaningfulness of cross-category comparisons without claiming that such comparisons are precise. For example, one might feel sure that, among physical scientists, Newton is greater than Michael Faraday (22nd) and that Faraday is greater than Stephen Hawking (25th). One might feel equally sure that, among economists, Adam Smith is greater than William Stanley Jevons (both unplaced). But how should these two scales be aligned together? I feel reasonably confident in judging Newton to be greater than Smith and (despite the BBC poll) Smith to be greater than Hawking; so I cannot claim that the idea of aligning the scales is absurd. But it does seem absurd to claim that there are precise equivalents between greatness in economics and greatness in physics.

As another example of imprecision, should we expect a great person to show greatness across a wide range of endeavours, or to be superlative in just one? One might argue that Dickens displays greatness over a wider range of writing than Austen, but that Austen comes closer to perfection in her chosen range. A related question: How much of a person’s work within his or her field of greatness should we require to be great? Dickens wrote a series of great novels, as did Austen. Does it count in favour of Dickens that he wrote more? Does it count against Charlotte Brontë (unplaced) if one judges that she wrote one truly great novel, but that her other books were mediocre?

I suggest that these forms of imprecision are different from ambiguity of meaning. When I struggle to find equivalences between greatness in economics and greatness in physics, or between greatness in breadth and greatness in depth, I do not feel that I need more guidance about how 'greatness' should be interpreted. To the contrary, if the BBC had accompanied its call for nominations with a tariff of equivalences between scientists, artists, political leaders and sporting heroes, I would have found that intrusive and presumptuous. Who is the BBC to be telling me what 'greatness' means?

Analogously, consider the prize for the best young British economist of the year. One issue that might arise for the committee is to find equivalences between candidates from 'mainstream' and 'heterodox' economics. Heterodox economists work with the same basic subject matter as their mainstream counterparts, but they reject the formal mathematical modelling and econometric analysis used in mainstream economists, preferring philosophical and historical modes of analysis. A large part of the content of heterodox economics is a critique of mainstream economics; most mainstream economists either ignore this work altogether or reject it out of hand. Suppose I am on the committee, and the list of nominees for the prize includes a young heterodox economist whose achievements are seen as outstanding by other heterodox economists. I would find the problem of establishing equivalences between the two approaches to economics very difficult. But I would not think that our committee needed more guidance about what counts as good economics. Judging what is good economics, I would assume, is part of our job. In this context, 'good as an economist' is not an ambiguous concept with multiple meanings. We already know all we need to know about what it *means*. If someone was in doubt about its meaning, what more could they do than consult a committee of the great and the good of economics? But (I have assumed) that is exactly what our committee is. Similarly, if someone was in doubt about what 'greatness' means in ordinary English, as used in Britain, what more could they do than research the judgements of greatness made by a representative sample of British people? The problem is that these concepts are *imprecise*.

In the examples I have considered so far, the imprecision of the ranking property (greatness, or goodness as an economist) can be analysed as imprecision about how different aspects or dimensions of that property should be integrated. But however finely we subdivide the dimensions of a ranking property, there is no guarantee that we will reach a point at which all legitimate disagreement can be represented as disagreement about how to integrate scores on different sub-dimensions.

To take a concrete example on which I can claim expertise, consider the question of whether Robert Aumann is a greater economist than Thomas Schelling. Aumann and Schelling were the joint winners of the 2005 Nobel Prize in Economics for their very different contributions to game theory. Aumann is a game theorist's game theorist; he has addressed what most game theorists would see as the fundamental problems of their field with great technical rigour. Schelling is a more intuitive and imaginative theorist; he has explored ideas which seem to be highly relevant for the analysis of games, but which have proved resistant to incorporation in the conceptual frameworks used by other theorists. Most economists will agree with what I have said so far, and their respective views about the relative value of technical rigour and loosely-theorised insight will undoubtedly influence their rankings of the two prize-winners. But in order to judge the comparative greatness of Aumann and Schelling as economists, it not enough to take a position on the relative value of these different facets of economic ability. At some point, one has to ask how deep and how significant each person's discoveries have been. And that requires judgement, not a scoring system. My own judgement favours Schelling, but many equally qualified judges will disagree with me.

Although such judgements are subjective, they are not matters of mere taste. In the case of the prize, I may find health a more engaging topic than finance, but it would be improper for me to use that as a reason for favouring a health economist over a financial economist. I am expected to have an understanding of the concept of 'good economist' that is separable from my personal likings. Similarly, if I vote for my greatest Briton of all time, I cannot in good faith favour Smith over Newton on the grounds that I am an economist and not a physicist.

I suggest that the problem faced by voters in the Great Briton contest, and the problem faced by the committee awarding the 'best young economist' prize, can both be understood as genuine ranking problems in which the ranking property *P* (greatness, or goodness as an economist) is imprecise. In each case, for each candidate *x* (each Briton, or each nominated young economist), *P* is a property that *x* has to some degree. In each case, for every ordered pair *x*, *y* of candidates, it is meaningful to ask whether *x* is at least as *P* as *y*. These problems involve imprecision, not because they are ill-specified, but because they admit more than one answer, each of which is in some sense eligible.

'Eligibility' can be construed in different ways. We might take an outsider's perspective, and ask what responses a particular ranking problem would elicit from a community of competent judges. For example: how do British people (or perhaps, reasonably well-informed British people) rank Britons by greatness? Then imprecision is revealed in the fact that different judges report different rankings. Or we might take the perspective of a particular judge. Then imprecision reveals itself in the form of a range of putative judgements, each of which is recognised by the judge as worthy of serious consideration, as a conclusion that could be presented and defended in good faith. As a judge, one might recognise this imprecision, and yet still come down firmly and confidently in favour of some particular conclusion. Or one might reach a settled decision, without feeling confident that it was the right one. Or one might find it difficult to reach any settled decision at all, vacillating between one conclusion and another.

Viewed in the perspective of reason-based modelling, all of these forms of imprecision have a common structure. There is a ranking problem with a set S of objects and a ranking property P. The problem has a set of eligible answers. Each such answer is an 'is at least as P as' relation $\geq_P$. My proposal is that imprecision should be modelled as a family of eligible $\geq_P$ relations.

3. Transitivity and completeness

If we are to use this structure to model imprecision, we need to ask whether there are any formal properties that every eligible relation $\geq_P$ should satisfy. Notice that, in asking this question, we are not treating $\geq_P$ as a final representation of the conclusions of any community of competent judges, or of any particular judge. Nor are we treating it as a representation of any person's state of mind. We are treating it as a putative answer to a well-specified question. The formal structure of a question imposes restrictions on the structure of a legitimate answer.

If the question is a well-specified ranking problem, I submit, every eligible $\geq_P$ should be taken to be complete and transitive – that is, should be taken to be an ordering. Transitivity seems to be a conceptual requirement of any 'is at least as P as' relation where P is a property that each member of S has to some degree, independently of which other members of S it is compared with. (For the reasons I gave in Section 2, transitivity is not a necessary property of $\geq_P$ if P is a comparison-relative property; but that case is excluded by the definition of 'ranking problem'.) In the context of incommensurability and vagueness, completeness is the more significant property. Why must this hold?

By the definition of a ranking problem, every object *x* in *S* has the property *P* to some degree, independently of which other object it is compared with. If this is the case then, for any *x*, *y*, it is surely inescapable that *either x* has more of the property than *y*, *or y* has more of it than *x*, or they both have it to the same degree. One way of thinking about this is to fix any distinct *x* and *y* in *S* and to consider the following *survey question*, framed for a questionnaire or opinion poll:

> Think about the *P*-ness of *x* and *y*, and then tick one of the following:
> (a) *x* is more *P* than *y*
> (b) *y* is more *P* than *x*
> (c) *x* is exactly as *P* as *y*
> (d) don't know/ can't decide

Notice that the implication of the fourth option is that one of (a), (b) or (c) applies, even though the respondent is not sure which of them does. The respondent is not being allowed to say that the question is meaningless, or that something other than (a), (b) or (c) is the case. My claim is that, if we are dealing with a ranking problem, the survey question is well-specified.

Suppose someone is asked this question in relation to the relative greatness of pairs of Britons. (For example: 'Think about the greatness of Shakespeare and Nelson, and then tick one of the following ....') If the respondent thinks that 'greatness' applied to people is a meaningless concept, her reply to any such question should be some expression of incomprehension, such as 'Eh?' She is denying the meaningfulness of the survey question, but she is also denying that the Great Britons exercise is a ranking problem.

Now consider a respondent who thinks that 'greatness as a military leader' and 'greatness as a writer' are both meaningful, but that the two forms of greatness are incommensurable. If this respondent is asked to compare Shakespeare and Nelson, he should express incomprehension. His problem is essentially the same as that of the first respondent: as he sees it, the question is presupposing a non-existent concept, namely 'greatness as a person'. He should *not* reply that he knows, or has decided, that none of (a), (b) or (c) holds. That would imply that it is meaningful to ask whether Shakespeare or Nelson is greater, and that the answer is something other than (a), (b) or (c). For this respondent, the question is not meaningful in the first place. But what if he is asked to compare Nelson and Montgomery? Then, I think, he is entitled to ask for clarification about what 'greatness' is supposed to mean. If the questionnaire designer tells him that it means 'greatness as a person', then he should again deny the meaningfulness of the question. He should not re-interpret it as a question about 'greatness as a military leader'. This respondent, too, is denying that the Great Britons exercise is a ranking problem.

One implication of my argument about completeness is the following. Let us say that a person *P-ranks* x and y if she asserts any one of (a), (b) or (c), as specified in the survey question about x and y. Consider a particular respondent. Suppose she P-ranks w and x – say, she asserts that Shakespeare is greater than Dickens. She also P-ranks y and z – she asserts that Churchill is greater than Peel. Can she then assert that (she knows that, or has decided that) neither (a), (b) nor (c) holds for the comparison between w and y? If P has the same meaning throughout, and if P is not a comparison-relative property, then in my modelling framework the answer has to be 'No'.

Philosophers sometimes argue that population ethics provides counter-examples to the claim I have just made. Rabinowicz seems to be sympathetic to this argument. Consider the following example (adapted from his paper). There is a set S of alternative *worlds*. Each world is specified in terms of the people who exist in it (its *inhabitants*) and their levels of well-being. For ease of exposition, I shall use 'happiness' as a synonym for 'well-being'. In world w, the only inhabitants are Adam and Eve; in worlds x, y and z, the inhabitants are Adam, Eve and Cain. Adam's happiness is the same in all four worlds. So is Eve's. In x, Cain is extremely unhappy. In both y and z, Cain is moderately happy; he is very slightly happier in z than in y. The ranking property P is 'good'.

Broome (1994) expresses confidence that 'we' share certain intuitions about how worlds should be ranked by goodness. One of these – Broome's *principle of personal good* – is that, holding the set of inhabitants constant, worlds in which given individuals are happier are better than worlds in which they are less happy (p. 58). This implies $z >_P y >_P x$. A second is that adding unhappy people to the world makes it worse. Broome expresses this as 'existence at a poor level is not neutral; we are against it' (p. 144). This implies $w >_P x$. Notice that, if we accept these implications, we are P-ranking z and y, and also P-ranking x and w. According to my analysis, and if this is a ranking problem, we cannot then assert $y \#_P w$. But Broome detects a third intuition: 'We think intuitively that adding a person to the world is very often ethically neutral. We do not think that just a single level of wellbeing is neutral ...' (p. 143). The implication is that we should be able to say that y is neither better nor worse than w, and at the same time that z is neither better nor worse than w. If these ideas were represented as $y =_P w$ and $z =_P w$, we would have a violation of transitivity. An alternative representation, which Rabinowicz seems inclined to favour, is as $y \#_P w$ and $z \#_P w$.

In thinking about this example, it is important to distinguish questions about the relative goodness of different worlds from questions about the rightness and wrongness of individuals' actions. One might maintain that it would be wrong for Adam and Eve to choose to conceive a Cain whom they knew would be extremely unhappy, and yet not assert that a world that includes such a Cain is worse than a world that doesn't. Conversely, one might maintain that it would not be wrong for Adam and Eve to choose to conceive a moderately happy Cain, while accepting that a world that includes such a Cain is marginally worse than one that doesn't. (Speaking for myself, I did not believe that my wife and I had to convince ourselves that we were not making the world worse before choosing to have a child. It was enough that were doing no one any wrong.) It is perhaps significant that, when Broome explains the intuitions that I have represented as $w >_P x$ and $y \#_P w$, he appeals to examples in which a couple is choosing whether to have a child (p. 144).

As used in population ethics, the relevant concept of 'goodness of worlds' seems to be similar to that used in *Genesis* to refer to God's judgements about his creation. We are told that, after creating the whole of the world except for mankind, God looked on the world and 'saw that it was good'. After completing his creation by adding Adam and Eve, 'God saw everything that he had made, and, behold, it was very good'. The implication is that for God, a world with human beings was better than one without. I have to say that I find it difficult to get a mental grip on such a disembodied concept of goodness. At a pinch, I can entertain the idea of a God's-eye viewpoint from which one looks at different possible worlds, with different inhabitants with different levels of well-being, and judges which worlds are better and worse; but I do not have many confident intuitions about what judgements should be made from this viewpoint. Still, if it makes sense at all to pose the problem of how good different worlds are, I cannot see why this is not a ranking problem. If I really can say that there is some common, comparison-independent property of 'goodness' that worlds possess, such that z has more of it than y and w has more of it than x, I cannot see how I can consistently respond 'None of the above' when I am presented with the survey question in relation to y and w. Goodness applied to worlds is just a particularly imprecise concept.

Having considered these objections, I reiterate my original proposal – that in a ranking problem, imprecision should be represented by an *eligible set* of orderings which has more than one element. Imprecision is at its maximum when all logically possible orderings of S are in the eligible set. Conversely, there is complete precision if the eligible set is a singleton. To complete the specification of the framework, we need to ask whether (given that S itself is non-empty) the set of eligible orderings can be empty. I submit that the eligible set should be taken to be *non-empty*. I have already argued that every legitimate answer to a ranking problem takes the form of an ordering $\geq_P$ of S. To recognise the problem as meaningful is to recognise that, in principle, it is capable of being answered. To assert that every logical possible answer was ineligible would be to deny the problem's answerability.

Given an eligible set of orderings of S, we can define an *intersection* relation $\geq^*_P$ such that, for each x, y in S, $x \geq^*_P y$ is true if and only if $x \geq_P y$ holds for all eligible orderings. It is natural to read $x \geq^*_P y$ as *x is indisputably at least as P as y*. Extending a previous notation, we can use $x =^*_P y$ to denote [$x \geq^*_P y$ and $y \geq^*_P x$] or equivalently, that $x =_P y$ holds for all eligible orderings. Similarly, we can use $x >^*_P y$ to denote [$x \geq^*_P y$ and not $y \geq_P^* x$], that is, $x \geq_P y$ holds for all eligible orderings and $x >_P y$ holds for some. And we can use $x \#^*_P y$ to denote [(not $x \geq^*_P y$) and (not $y \geq^*_P x$)], that is, $x >_P y$ holds for some eligible orderings and $y >_P x$ for others. It is immediately clear that, since each eligible $\geq_P$ is complete and transitive, $\geq^*_P$ is transitive but not necessarily complete. Incompleteness of the intersection relation is a manifestation of imprecision. That completes the specification of my proposal.

One merit of the framework I have proposed is its simplicity. Another is that it is isomorphic with the *random preference* approach that some decision theorists have used to represent imprecise preferences. The idea underlying that approach is that, for a given individual, there is a non-empty set of alternative preference orderings; which of these ordering governs the individual's decisions at any particular moment is determined by a random process. Random preference models can be used to organise data from controlled experiments in which individuals face exactly the same decision problems more than once; such experiments typically find a significant degree of apparently random variation in decision-making behaviour (Becker, De Groot and Marschak, 1963; Loomes and Sugden, 1995).

4. Rabinowicz's proposal

Rabinowicz proposes 'a general modelling of value relations' that can accommodate both vagueness and 'incommensurability', the latter term being defined so that two items are incommensurable if neither is better than the other, and they are not equally good. Rabinowicz's starting point is the claim that 'valuable' can be analysed as 'fitting to be favoured'. To say that some object *x* is valuable is to say that some particular 'pro-attitude', appropriate to the relevant value, is in some sense fitting; to say that *x* is more valuable than *y* is to say that some attitude that favours *x* over *y* is similarly called for. 'Better' is a special case of 'more valuable', for which the corresponding attitude is *preference*. Thus, to say that *x* is better than *y* is to say that a preference for *x* over *y* is fitting. Having understood betterness in this way, Rabinowicz proposes an analysis of 'is better than' that is compatible with imprecision and incompleteness in preferences. In Rabinowicz's model, there is a set *K* of eligible preference relations, interpreted as preferential attitudes that are rationally permissible. Each of these relations is required to be transitive but is *not* required to be complete. Rabinowicz proposes to define 'better than' and 'equally good' in terms of the intersection of eligible preference relations. That is, '*x is better than y*' is defined as 'in all eligible preference relations, *x* is strictly preferred to *y*', and '*x and y are equally good*' is defined as 'in all eligible preference relations, *x* is indifferent to *y*'. One implication is that, because there can be *x*, *y* pairs for which no specific preference ranking is rationally required, the relation 'is at least as good as' is not necessarily complete.

I think that this analysis introduces imprecision in the wrong place. For the purposes of the present argument, let us accept the analysis of 'valuable' as 'fitting to be favoured'. Consider the case of greatness among Britons. Greatness is a special kind of value. One might say, following Rabinowicz, that there must be some corresponding pro-attitude that is a fitting response to greatness. Let us say that this is *honouring* – as in erecting statues of a great person, or giving his or her name to major streets and public spaces. It seems perfectly reasonable to use 'honouring' to explicate or even to define 'greatness': if someone was unsure what was meant by a 'great' Briton, examples of statues in public places might well be useful. But this idea, as I have represented it so far, is wholly compatible with my analysis of the Great Britons exercise as a ranking problem. In discussing this problem, I have interpreted the ranking property *P* as ‘greatness’. But we might equally well interpret it as ‘honour-worthiness’: that would make no difference to my analysis.

Consider a case of imprecision. Suppose the problem is to compare the greatness of Newton and Darwin. And suppose that the context of this problem is that a public decision is to be made about which of the two men’s portraits will appear on the British £10 note. One might think (as I do) that it is permissible or reasonable or defensible to judge either of those scientists to be greater than the other, or indeed to judge them to be equally great. Correspondingly, it is permissible to judge either of them to be more honour-worthy, or to judge them equally honour-worthy. As long as we think about eligible *judgements*, we are in the domain of ranking problems – even if the judgements are about properties defined in terms of the fittingness of attitudes.

However, Rabinowicz moves out of this domain by focussing on the eligibility of attitudes themselves, understood as psychological dispositions. For Rabinowicz (and I agree with him about this), a preference is best understood as a disposition to choose one thing rather than another. Analogously, we might think of comparative ‘honouring attitudes’ as dispositions to honour one person rather than another. Let us use the term ‘favour’ as the analogue of preference in the case of honouring. Thus, to say that a person favours Newton over Darwin is to say that she is disposed to honour Newton rather than Darwin. Applying Rabinowicz’s analysis to this case, we might say that the attitude of favouring Newton over Darwin is reasonable. So too is the attitude of favouring Darwin over Newton. So too is the attitude of favouring them equally. And, one might add, it could be reasonable to have none of those attitudes – for example, to have an attitude of indecision or perplexity, or not to care at all about honouring great men and women. Since it is *not rationally required that* Newton is favoured, Rabinowicz’s analysis implies that Newton is not greater than Darwin. Similarly, Darwin is not greater than Newton, nor are the two men equally great. According to Rabinowicz, then, they are incommensurable in greatness.

This seems an odd use of both ‘incommensurable’ and ‘greatness’. If ‘greatness’ is interpreted as ‘honour-worthiness’, it would be more natural to conclude that neither scientist is *indisputably* greater than the other, but that one can reasonably *judge* either to be greater (a judgement that, on Rabinowicz’s analysis, is not disputable but false). And the fact that reasonable people can disagree about which of Newton and Darwin is greater does not imply that there is no common standard of greatness that applies to both men – only that

that standard is imprecise. Incommensurability, I suggest, should not be conflated with imprecision or disputability.

For Rabinowicz, propositions about comparative value are not answers to any ranking problem. When Rabinowicz says that x is 'better than' y, he is not referring to any common property of goodness of which x has more than y. Contrary to the most natural interpretation of the words, he is not treating 'better than' as equivalent to 'more good than' – even if 'good' is interpreted as 'preference-worthy' or 'preferable'. What he is saying might be better expressed by 'x is *indisputably* preferable to y'. Since Rabinowicz's comparative value propositions do not belong to a ranking problem, my analysis does not apply to them. But I hope I have persuaded the reader that, as an aid to clear thinking about comparative value, a model based on ranking problems is sharper and more useful than one based on the intersection of reasonable attitudes. I continue to believe that philosophical discussions of incommensurability and vagueness are creating unnecessary complications.

References

Becker, Gordon, Morris DeGroot, and Jacob Marschak (1963). Stochastic models of choice behaviour. *Behavioral Science* 8: 41–55.

Broome, John (1991). *Weighing Goods*. Oxford: Blackwell.

Broome, John (1994). *Weighing Lives*. Oxford: Oxford University Press.

Loomes, Graham and Robert Sugden (1982). Regret theory: an alternative theory of rational choice under uncertainty. *Economic Journal* 92: 805–824.

Loomes, Graham and Robert Sugden (1995). Incorporating a stochastic element into decision theories. *European Economic Review* 39: 641–648.

Rabinowicz, Wlodek (2009). Incommensurability and vagueness.

On modelling vagueness – and on *not* modelling incommensurability

Robert Sugden

School of Economics

University of East Anglia

Norwich NR4 7TJ

UK

r.sugden@uea.ac.uk

16 June 2009

Abstract   This paper defines and analyses the concept of a 'ranking problem'. In a ranking problem, a set of objects, each of which possesses some common property *P* to some degree, are ranked by *P*-ness. I argue that every eligible answer to a ranking problem can be expressed as a complete and transitive 'is at least as *P* as' relation. Vagueness is expressed as a multiplicity of eligible rankings. Incommensurability, properly understood, is the absence of a common property *P*. Trying to analyse incommensurability in the same framework as ranking problems causes unnecessary confusion.

Acknowledgements My work has been supported by the Economic and Social Research Council of the UK (award no. RES 051 27 0146). I thank Wlodek Rabinowicz for comments on previous drafts of this paper.

As an economist and decision theorist with an interest in philosophy, I have occasionally dipped into philosophical writings on incommensurability and vagueness. I have usually come away with a sense of puzzlement – with a feeling that mountains are being made out of molehills. Since this may reveal my lack of understanding of what philosophers are trying to do, I will begin by explaining how I understand the class of problems to which the concepts of incommensurability and vagueness might seem to apply. I will then offer an analysis of that class of problems, as I have defined it. Formally, this analysis has some similarities with that proposed by Wlodek Rabinowicz (2009), but it has a very different interpretation – and is much simpler.

I start with the following concept of a *ranking problem*. There is some non-empty set S of *objects*; typical objects are denoted w, x, y and z. There is some *ranking property* P, such that each object x in S has this property to some degree (which may be zero), independently of which other members of S it is compared with, and such that, for all x, y in S, the question 'Is x at least as P as y?' is meaningful. This allows us to define a binary relation $\geq_P$ on S, such that $x \geq_P y$ denotes 'x is at least as P as y'. (Formally, $\geq_P$ is a subset of the set of ordered pairs of elements of S.) Let $x =_P y$ denote [$x \geq_P y$ and $y \geq_P x$]; let $x >_P y$ denote [$x \geq_P y$ and not $y \geq_P x$]; and let $x \#_P y$ denote [(not $x \geq_P y$) and (not $y \geq_P x$)]. Deciding what the relation $\geq_P$ is (or ought to be) is a ranking problem. For example, suppose I agree to serve on a committee of the great and the good of British economics, charged with awarding a prize to the best young British economist of the year. We are given a list of young British economists who have been nominated for the prize and are required to select one. This is a ranking problem: S is the set of nominees and P is the property 'good as an economist'.

The theoretical problem to be addressed concerns the formal properties of $\geq_P$. By virtue of our understanding of the concept of 'is at least as ... as', can we impose any general restrictions on $\geq_P$? The two most obvious candidates are *transitivity* and *completeness*. The relation is transitive if, for all x, y, z in S, [$x \geq_P y$ and $y \geq_P z$] implies $x \geq_P z$. It is complete if, for all x, y in S, either $x \geq_P y$ or $y \geq_P x$ (or both). Equivalently, it is complete if there are no x, y in S such that $x \#_P y$. A binary relation that is transitive and complete is an *ordering* (some theorists use the term *complete ordering*). In the context of discussions of incommensurability and vagueness, the crucial question is whether completeness should be imposed. Does it make sense to assert of some x and y in S that it is not the case that x is at least as P as y, but nor is it the case that y is at least as P as x?

Let me now explain how I understand this kind of question. As a decision theorist, I take myself to be constructing *models* that can be used to analyse real problems of judgement and decision. In the case of my example, the real problem is that of the committee charged with awarding the prize. The formal concepts S, P and $\geq_P$ are not part of that reality; they belong to a model of it. I am looking for modelling strategies that are internally consistent, tractable, and usefully applicable to a reasonably wide range of real-world problems. Thus, I shall not ask whether the relation 'is at least as good an economist as', defined on the set of young British economists, *really is* complete. What I shall ask is whether, for the analysis of problems such as that faced by the committee, it is a good modelling strategy to postulate completeness.

For the purposes of this paper, I take it as given that the concern is with *reason-based* models. To use a reason-based framework is to choose a modelling strategy which focuses on the relationship between decisions or judgements and the reasons that are taken to justify them. Within such a framework, only a limited range of decision-relevant attitudes can be taken into account. To expect a model of decision-making to be able to take every potentially relevant factor into account is to fail to see the point of modelling.

1. Some *faux*-ranking problems

My definition of a 'ranking problem' has been deliberately constructed to exclude certain kinds of problem which, at least at first sight, call for what in ordinary language might be called 'ranking'. These *faux-ranking* problems share a number of features which are also common to genuine ranking problems. Each *faux*-ranking problem can be understood as having some set S of objects, some property P, and a corresponding binary relation $\geq_P$ defined on S. In each case, it turns out that this relation can fail to satisfy completeness or transitivity. I shall show that, in these cases, incompleteness or intransitivity is possible because of features of the relevant problem that are inconsistent with my definition of a 'ranking problem'. I shall claim that it is better to exclude these *faux*-ranking problems from the domain of a modelling framework that is intended to deal with ranking problems as I have defined them. To try to adapt such a modelling framework to encompass these problems would introduce unnecessary confusion.

One type of *faux*-ranking problem involves *non-applicability*. Let S be the set of things whose names begin with 'c'. Let x, y and z be Camembert, Cheddar and the chi-squared test. Let P be 'crumbly'. On my definition, this is not a ranking problem. Camembert and Cheddar each have the property of crumbliness to some degree (possibly zero), but the chi-squared test does not. So, to the question 'Is Cheddar crumblier than the chi-squared test?', an appropriate answer is 'Not applicable'. Formally, there would be nothing wrong with defining a relation $\geq_P$ on S and then representing the crumbliness relation between Cheddar and the chi-squared test as $y \#_P z$. Clearly, this relation would violate completeness. But it is hard to imagine a context of decision or judgement in which the 'not applicable' components of such a relation would be useful. So nothing substantial is lost, and a potential source of confusion is removed, if we exclude cases of non-applicability from the concept of a ranking problem.

A second type of *faux*-ranking problem involves *incommensurability* in the literal sense of having no common measure. Let S be defined so that it includes the University of East Anglia (w), the University of York (x), *Wives and Daughters* (y) and *Jane Eyre* (z). Let P be 'good'. There is a sense in which all of these objects have the property of goodness to some degree. The University of East Anglia and the University of York are both good universities (or so I like to believe). *Wives and Daughters* and *Jane Eyre* are both good novels. But to the question 'Is the University of East Anglia better than *Wives and Daughters*?', the obvious answer is again 'not applicable'. The question has no real meaning because 'good as a university' and 'good as a novel' are, for all practical purposes, different concepts. Since there is no common standard of measurement for 'better university than' and 'better novel than', the question is analogous with 'Is 10 decibels greater than 100 grammes?' The concept of 'greater than' can be applied to both sound and mass, but there is no common standard of measurement for 'greater sound than' and 'greater mass than': sound and mass are incommensurable. Problems of this kind are not ranking problems by my definition. Again, there would be nothing formally wrong in defining a non-complete relation $\geq_P$ on S as the union of the ranking of universities by 'good as a university' and the ranking of novels by 'good as a novel', and then representing the betterness relation between the University of East Anglia and *Wives and Daughters* as $w \#_P y$. But I cannot imagine any use for such a relation. If goodness in universities and goodness in novels are different concepts, why amalgamate them in the same ranking?

A third type of *faux*-ranking problem involves *multiple meanings*. Let S be the set of cars on the British roads. Let w, x, y and z be particular cars: w and x are high-performance sports cars; y and z are small, low-emission city cars; w and y are painted green; x and z are painted red. Let P be 'green'. To the question 'Is w greener than z?', the obvious answer is: 'It depends what you mean by "green"'. Although 'green' as a colour and 'green' as 'environmentally friendly' have a common etymological root, these meanings are too far apart to be sensibly combined. This is not a ranking problem by my definition. Formally, we could define a non-complete relation $\geq_P$ on S and stipulate that one car is 'at least as green as' another if and only if it is *both* at least as green in colour *and* at least as environmentally friendly. Then (assuming only two colours and two environmental specifications) we would have $w >_P x$ and $y >_P z$ (by colour), $y >_P w$ and $z >_P x$ (by environmental specification), $y >_P x$ (by both), and $w \#_P z$. But, just as in the case of incommensurability, I can see no use in amalgamating two distinct concepts into a single ranking.

The final type of *faux*-ranking I consider involves *comparison relativity*. The following example is borrowed from John Broome (1991, pp. 10–12). Let S be the set of cities in the world. Its elements include Tokyo (w), London (x), New York (y) and Honolulu (z). Let P be 'westerly'. On what I (but not Broome) take to be the most natural interpretation of 'westerly', one city is more westerly than another if the shortest route from the latter to the former involves heading west. Thus London is more westerly than Tokyo, New York than London, Honolulu than New York, and Tokyo than Honolulu: the relation $\geq_P$ is non-transitive. This is not a ranking problem by my definition, because westerliness is not a property that cities possess independently of which other cities they are compared with. Thus, although it is meaningful to ask whether one city is more westerly than another, it is not meaningful to ask which is the most westerly of all cities. According to some theories of choice, including the theory of *regret* proposed by Graham Loomes and me, preference is a comparison-relative concept. In this theory, which of two uncertain acts a_1 and a_2 is chosen in a binary choice problem is influenced by anticipated regret; the possible regret resulting from choosing, say a_1 depends on how its consequences are juxtaposed with those of the non-chosen act a_2. Thus, there need be no inconsistency in a case in which a_1 is chosen from the set $\{a_1, a_2\}$, a_2 from $\{a_2, a_3\}$, and a_3 from $\{a_3, a_1\}$ (Loomes and Sugden, 1982). For a person whose reasoning is represented by regret theory, determining binary preferences is not a ranking problem as I have defined it. Needless to say, I am not denying the usefulness of developing a formal model of regret-based preferences (or, indeed, of westerliness). But the logical structure of a comparison-relative relation is very different from that of $\geq_P$ in a ranking problem.

2. Imprecision

It is now time to consider whether, in a genuine ranking problem, $\geq_P$ can be incomplete. To keep things concrete, I will focus on a particular problem, in which the putative ranking property *P* is vague. I shall not pre-judge whether or not this is a ranking problem; but I shall argue that it can reasonably be understood as such.

During 2002, the BBC ran a series of television programmes on the theme of 'Great Britons'. (My information is taken from BBC websites.) The set of potential 'great Britons' was defined to include 'anyone who was born in the British Isles, including Ireland; or anyone who lived in the British Isles, including Ireland, and who has played a significant part in the life of the British Isles'. The BBC began with a publicity campaign asking people to nominate 'their greatest Briton of all time'; 30,000 people responded. The 100 most-nominated Britons were announced in rank order, except that the relative ranking of the top ten was not revealed. (I regret to report that no economist or philosopher was included in these 100 names.) Then there was a television programme on each of the ten most popular nominees, with a further round of telephone and internet voting to determine the order of those ten. The eventual winner was Winston Churchill.

Is this a ranking problem? There is a reasonably well-defined set *S* of people who were born in, or lived in, the British Isles at any time in history. To focus on issues of incommensurability and vagueness, I shall ignore the problem of gaining factual information about these people and their achievements. There is a property *P* of 'greatness', and a corresponding relation $\geq_P$, 'at least as great as'. To determine whether this is a ranking problem we must ask if every person in this set has the property of greatness to some degree (which may be zero), independently of comparisons with others, and if, for each pair *x*, *y* of such people, the question: 'Is *x* at least as great as *y*?' is meaningful.

One response is to deny that 'greatness' as a property of human beings is a meaningful concept at all. That response seems to me to be coherent but, in the context of a discussion about how to model comparative judgements, it is not particularly interesting. If there is no such thing as greatness, there is no point in worrying about how to model it.

Another response is that there are different kinds of greatness, not comparable with one another. For example, one might recognise greatness as a property of military leaders and greatness as a property of writers, but deny their comparability. One might say that Horatio Nelson (6th in the BBC ranking) is greater than the Duke of Wellington (15th) and Montgomery of Alamein (88th), and that William Shakespeare (5th) is greater than Charles Dickens (41st) and Jane Austen (70th), while denying the meaningfulness of asking whether *any* military leader is greater than *any* writer. At first sight, this position seems to make 'is at least as great as' an incomplete (but perhaps transitive) relation. But that way of expressing non-comparability, although not exactly confused, introduces unnecessary confusion. To say that *all* comparisons of greatness between military leaders and writers are meaningless is to say that the two kinds of greatness are incommensurable, just as goodness of universities is incommensurable with goodness of novels. On this view, the Great Britons problem is a faux-ranking problem. If there are two or more independent and entirely unconnected kinds of greatness, the simplest way to represent that is to define a separate ranking problem for each kind. Incommensurability of different kinds of greatness is then no more of a philosophical problem than the incommensurability of volts and litres.

Nevertheless, the premise of the television series was that comparisons of greatness *can* be made across different categories of human distinction. Presumably the 30,000 people who submitted nominations, and the hundreds of thousands of viewers who voted in the final stage, found the concept of a 'Great Briton' meaningful. From now on I will take that as a working hypothesis.

Even if 'greatness' is meaningful, there is an ambiguity in the concept of 'the greatest Briton of all time': is the criterion greatness *as a Briton* or greatness *per se*, Britishness being used only to delimit the set of candidates? (Analogously, consider the question of whether Arsène Wenger is a great economist. He has a master's degree in economics, and he has shown greatness in football management; but, as far as I know, he has few achievements to his credit in economics.) I conjecture that some respondents to the BBC poll resolved this ambiguity in one way and some in the other. The high rankings given to iconic defenders of Britain and its constituent nations, such as Churchill, Alfred the Great (14th), Owain Glyndwr (23rd), Boudicca (35th), William Wallace (48th) and James Connolly (64th), and to William Blake (38th) and Edward Elgar (60th) – respectively the author of *Jerusalem* and the composer of *Land of Hope and Glory* – suggest that many respondents were thinking about 'greatness as a Briton'. In contrast, Charles Darwin (4th), Isaac Newton (8th) and Alexander Fleming (20th) are perhaps better thought of as great scientists who happen to be British. Fortunately for the success of the television series, there is considerable overlap between the two meanings of ‘great Briton’, particularly from the viewpoint of British judges. For example, Churchill, Shakespeare, Isambard Kingdom Brunel (2nd) and Oliver Cromwell (10th) are great men on any reckoning, but their achievements have special relationships with Britain. In principle, however, this is a case of multiple meanings, analogous with ‘greenness’ as a property of cars.

One way of thinking about this ambiguity is to ask whether the organisers of the poll were at fault for not resolving it. It is reasonable for a respondent to object that the question he is being asked is not adequately specified? In the Great Britons case, I suggest, the answer is ‘Yes’. Analogously, if I were asked to be part of a committee awarding a ‘great economist’ prize, I would feel justified in asking the organisers to make clear whether someone like Wenger counted as a great economist. A reason-based model of judgement or decision cannot be expected to impose order on responses to ill-specified questions.

However, greatness may have a recognisable core meaning while still being *imprecise* (or, which I treat as a synonym, ‘vague’) in ways that are intrinsic to the concept itself. Consider the problem of comparing greatness across categories of achievement. One can recognise the meaningfulness of cross-category comparisons without claiming that such comparisons are precise. For example, one might feel sure that, among physical scientists, Newton is greater than Michael Faraday (22nd) and that Faraday is greater than Stephen Hawking (25th). One might feel equally sure that, among economists, Adam Smith is greater than William Stanley Jevons (both unplaced). But how should these two scales be aligned together? I feel reasonably confident in judging Newton to be greater than Smith and (despite the BBC poll) Smith to be greater than Hawking; so I cannot claim that the idea of aligning the scales is absurd. But it does seem absurd to claim that there are precise equivalents between greatness in economics and greatness in physics.

As another example of imprecision, should we expect a great person to show greatness across a wide range of endeavours, or to be superlative in just one? One might argue that Dickens displays greatness over a wider range of writing than Austen, but that Austen comes closer to perfection in her chosen range. A related question: How much of a person’s work within his or her field of greatness should we require to be great? Dickens wrote a series of great novels, as did Austen. Does it count in favour of Dickens that he wrote more? Does it count against Charlotte Brontë (unplaced) if one judges that she wrote one truly great novel, but that her other books were mediocre?

I suggest that these forms of imprecision are different from ambiguity of meaning. When I struggle to find equivalences between greatness in economics and greatness in physics, or between greatness in breadth and greatness in depth, I do not feel that I need more guidance about how 'greatness' should be interpreted. To the contrary, if the BBC had accompanied its call for nominations with a tariff of equivalences between scientists, artists, political leaders and sporting heroes, I would have found that intrusive and presumptuous. Who is the BBC to be telling me what 'greatness' means?

Analogously, consider the prize for the best young British economist of the year. One issue that might arise for the committee is to find equivalences between candidates from 'mainstream' and 'heterodox' economics. Heterodox economists work with the same basic subject matter as their mainstream counterparts, but they reject the formal mathematical modelling and econometric analysis used in mainstream economists, preferring philosophical and historical modes of analysis. A large part of the content of heterodox economics is a critique of mainstream economics; most mainstream economists either ignore this work altogether or reject it out of hand. Suppose I am on the committee, and the list of nominees for the prize includes a young heterodox economist whose achievements are seen as outstanding by other heterodox economists. I would find the problem of establishing equivalences between the two approaches to economics very difficult. But I would not think that our committee needed more guidance about what counts as good economics. Judging what is good economics, I would assume, is part of our job. In this context, 'good as an economist' is not an ambiguous concept with multiple meanings. We already know all we need to know about what it *means*. If someone was in doubt about its meaning, what more could they do than consult a committee of the great and the good of economics? But (I have assumed) that is exactly what our committee is. Similarly, if someone was in doubt about what 'greatness' means in ordinary English, as used in Britain, what more could they do than research the judgements of greatness made by a representative sample of British people? The problem is that these concepts are *imprecise*.

In the examples I have considered so far, the imprecision of the ranking property (greatness, or goodness as an economist) can be analysed as imprecision about how different aspects or dimensions of that property should be integrated. But however finely we subdivide the dimensions of a ranking property, there is no guarantee that we will reach a point at which all legitimate disagreement can be represented as disagreement about how to integrate scores on different sub-dimensions.

To take a concrete example on which I can claim expertise, consider the question of whether Robert Aumann is a greater economist than Thomas Schelling. Aumann and Schelling were the joint winners of the 2005 Nobel Prize in Economics for their very different contributions to game theory. Aumann is a game theorist's game theorist; he has addressed what most game theorists would see as the fundamental problems of their field with great technical rigour. Schelling is a more intuitive and imaginative theorist; he has explored ideas which seem to be highly relevant for the analysis of games, but which have proved resistant to incorporation in the conceptual frameworks used by other theorists. Most economists will agree with what I have said so far, and their respective views about the relative value of technical rigour and loosely-theorised insight will undoubtedly influence their rankings of the two prize-winners. But in order to judge the comparative greatness of Aumann and Schelling as economists, it not enough to take a position on the relative value of these different facets of economic ability. At some point, one has to ask how deep and how significant each person's discoveries have been. And that requires judgement, not a scoring system. My own judgement favours Schelling, but many equally qualified judges will disagree with me.

Although such judgements are subjective, they are not matters of mere taste. In the case of the prize, I may find health a more engaging topic than finance, but it would be improper for me to use that as a reason for favouring a health economist over a financial economist. I am expected to have an understanding of the concept of 'good economist' that is separable from my personal likings. Similarly, if I vote for my greatest Briton of all time, I cannot in good faith favour Smith over Newton on the grounds that I am an economist and not a physicist.

I suggest that the problem faced by voters in the Great Briton contest, and the problem faced by the committee awarding the 'best young economist' prize, can both be understood as genuine ranking problems in which the ranking property P (greatness, or goodness as an economist) is imprecise. In each case, for each candidate x (each Briton, or each nominated young economist), P is a property that x has to some degree. In each case, for every ordered pair x, y of candidates, it is meaningful to ask whether x is at least as P as y. These problems involve imprecision, not because they are ill-specified, but because they admit more than one answer, each of which is in some sense eligible.

‘Eligibility’ can be construed in different ways. We might take an outsider’s perspective, and ask what responses a particular ranking problem would elicit from a community of competent judges. For example: how do British people (or perhaps, reasonably well-informed British people) rank Britons by greatness? Then imprecision is revealed in the fact that different judges report different rankings. Or we might take the perspective of a particular judge. Then imprecision reveals itself in the form of a range of putative judgements, each of which is recognised by the judge as worthy of serious consideration, as a conclusion that could be presented and defended in good faith. As a judge, one might recognise this imprecision, and yet still come down firmly and confidently in favour of some particular conclusion. Or one might reach a settled decision, without feeling confident that it was the right one. Or one might find it difficult to reach any settled decision at all, vacillating between one conclusion and another.

Viewed in the perspective of reason-based modelling, all of these forms of imprecision have a common structure. There is a ranking problem with a set S of objects and a ranking property P. The problem has a set of eligible answers. Each such answer is an ‘is at least as P as’ relation $\geq_P$. My proposal is that imprecision should be modelled as a family of eligible $\geq_P$ relations.

3. Transitivity and completeness

If we are to use this structure to model imprecision, we need to ask whether there are any formal properties that every eligible relation $\geq_P$ should satisfy. Notice that, in asking this question, we are not treating $\geq_P$ as a final representation of the conclusions of any community of competent judges, or of any particular judge. Nor are we treating it as a representation of any person’s state of mind. We are treating it as a putative answer to a well-specified question. The formal structure of a question imposes restrictions on the structure of a legitimate answer.

If the question is a well-specified ranking problem, I submit, every eligible $\geq_P$ should be taken to be complete and transitive – that is, should be taken to be an ordering. Transitivity seems to be a conceptual requirement of any ‘is at least as P as’ relation where P is a property that each member of S has to some degree, independently of which other members of S it is compared with. (For the reasons I gave in Section 2, transitivity is not a necessary property of $\geq_P$ if P is a comparison-relative property; but that case is excluded by the definition of 'ranking problem'.) In the context of incommensurability and vagueness, completeness is the more significant property. Why must this hold?

By the definition of a ranking problem, every object *x* in *S* has the property *P* to some degree, independently of which other object it is compared with. If this is the case then, for any *x*, *y*, it is surely inescapable that *either x* has more of the property than *y*, *or y* has more of it than *x*, or they both have it to the same degree. One way of thinking about this is to fix any distinct *x* and *y* in *S* and to consider the following *survey question*, framed for a questionnaire or opinion poll:

> Think about the *P*-ness of *x* and *y*, and then tick one of the following:
> (a) *x* is more *P* than *y*
> (b) *y* is more *P* than *x*
> (c) *x* is exactly as *P* as *y*
> (d) don't know/ can't decide

Notice that the implication of the fourth option is that one of (a), (b) or (c) applies, even though the respondent is not sure which of them does. The respondent is not being allowed to say that the question is meaningless, or that something other than (a), (b) or (c) is the case. My claim is that, if we are dealing with a ranking problem, the survey question is well-specified.

Suppose someone is asked this question in relation to the relative greatness of pairs of Britons. (For example: 'Think about the greatness of Shakespeare and Nelson, and then tick one of the following ....') If the respondent thinks that 'greatness' applied to people is a meaningless concept, her reply to any such question should be some expression of incomprehension, such as 'Eh?' She is denying the meaningfulness of the survey question, but she is also denying that the Great Britons exercise is a ranking problem.

Now consider a respondent who thinks that 'greatness as a military leader' and 'greatness as a writer' are both meaningful, but that the two forms of greatness are incommensurable. If this respondent is asked to compare Shakespeare and Nelson, he should express incomprehension. His problem is essentially the same as that of the first respondent: as he sees it, the question is presupposing a non-existent concept, namely 'greatness as a person'. He should *not* reply that he knows, or has decided, that none of (a), (b) or (c) holds. That would imply that it is meaningful to ask whether Shakespeare or Nelson is greater, and that the answer is something other than (a), (b) or (c). For this respondent, the question is not meaningful in the first place. But what if he is asked to compare Nelson and Montgomery? Then, I think, he is entitled to ask for clarification about what 'greatness' is supposed to mean. If the questionnaire designer tells him that it means 'greatness as a person', then he should again deny the meaningfulness of the question. He should not re-interpret it as a question about 'greatness as a military leader'. This respondent, too, is denying that the Great Britons exercise is a ranking problem.

One implication of my argument about completeness is the following. Let us say that a person *P-ranks* x and y if she asserts any one of (a), (b) or (c), as specified in the survey question about x and y. Consider a particular respondent. Suppose she P-ranks w and x – say, she asserts that Shakespeare is greater than Dickens. She also P-ranks y and z – she asserts that Churchill is greater than Peel. Can she then assert that (she knows that, or has decided that) neither (a), (b) nor (c) holds for the comparison between w and y? If P has the same meaning throughout, and if P is not a comparison-relative property, then in my modelling framework the answer has to be 'No'.

Philosophers sometimes argue that population ethics provides counter-examples to the claim I have just made. Rabinowicz seems to be sympathetic to this argument. Consider the following example (adapted from his paper). There is a set S of alternative *worlds*. Each world is specified in terms of the people who exist in it (its *inhabitants*) and their levels of well-being. For ease of exposition, I shall use 'happiness' as a synonym for 'well-being'. In world w, the only inhabitants are Adam and Eve; in worlds x, y and z, the inhabitants are Adam, Eve and Cain. Adam's happiness is the same in all four worlds. So is Eve's. In x, Cain is extremely unhappy. In both y and z, Cain is moderately happy; he is very slightly happier in z than in y. The ranking property P is 'good'.

Broome (1994) expresses confidence that 'we' share certain intuitions about how worlds should be ranked by goodness. One of these – Broome's *principle of personal good* – is that, holding the set of inhabitants constant, worlds in which given individuals are happier are better than worlds in which they are less happy (p. 58). This implies $z >_P y >_P x$. A second is that adding unhappy people to the world makes it worse. Broome expresses this as 'existence at a poor level is not neutral; we are against it' (p. 144). This implies $w >_P x$. Notice that, if we accept these implications, we are P-ranking z and y, and also P-ranking x and w. According to my analysis, and if this is a ranking problem, we cannot then assert $y \#_P w$. But Broome detects a third intuition: 'We think intuitively that adding a person to the world is very often ethically neutral. We do not think that just a single level of wellbeing is neutral ...' (p. 143). The implication is that we should be able to say that y is neither better nor worse than w, and at the same time that z is neither better nor worse than w. If these ideas were represented as $y =_P w$ and $z =_P w$, we would have a violation of transitivity. An alternative representation, which Rabinowicz seems inclined to favour, is as $y \#_P w$ and $z \#_P w$.

In thinking about this example, it is important to distinguish questions about the relative goodness of different worlds from questions about the rightness and wrongness of individuals' actions. One might maintain that it would be wrong for Adam and Eve to choose to conceive a Cain whom they knew would be extremely unhappy, and yet not assert that a world that includes such a Cain is worse than a world that doesn't. Conversely, one might maintain that it would not be wrong for Adam and Eve to choose to conceive a moderately happy Cain, while accepting that a world that includes such a Cain is marginally worse than one that doesn't. (Speaking for myself, I did not believe that my wife and I had to convince ourselves that we were not making the world worse before choosing to have a child. It was enough that were doing no one any wrong.) It is perhaps significant that, when Broome explains the intuitions that I have represented as $w >_P x$ and $y \#_P w$, he appeals to examples in which a couple is choosing whether to have a child (p. 144).

As used in population ethics, the relevant concept of 'goodness of worlds' seems to be similar to that used in *Genesis* to refer to God's judgements about his creation. We are told that, after creating the whole of the world except for mankind, God looked on the world and 'saw that it was good'. After completing his creation by adding Adam and Eve, 'God saw everything that he had made, and, behold, it was very good'. The implication is that for God, a world with human beings was better than one without. I have to say that I find it difficult to get a mental grip on such a disembodied concept of goodness. At a pinch, I can entertain the idea of a God's-eye viewpoint from which one looks at different possible worlds, with different inhabitants with different levels of well-being, and judges which worlds are better and worse; but I do not have many confident intuitions about what judgements should be made from this viewpoint. Still, if it makes sense at all to pose the problem of how good different worlds are, I cannot see why this is not a ranking problem. If I really can say that there is some common, comparison-independent property of 'goodness' that worlds possess, such that z has more of it than y and w has more of it than x, I cannot see how I can consistently respond 'None of the above' when I am presented with the survey question in relation to y and w. Goodness applied to worlds is just a particularly imprecise concept.

Having considered these objections, I reiterate my original proposal – that in a ranking problem, imprecision should be represented by an *eligible set* of orderings which has more than one element. Imprecision is at its maximum when all logically possible orderings of S are in the eligible set. Conversely, there is complete precision if the eligible set is a singleton. To complete the specification of the framework, we need to ask whether (given that S itself is non-empty) the set of eligible orderings can be empty. I submit that the eligible set should be taken to be *non-empty*. I have already argued that every legitimate answer to a ranking problem takes the form of an ordering $\geq_P$ of S. To recognise the problem as meaningful is to recognise that, in principle, it is capable of being answered. To assert that every logical possible answer was ineligible would be to deny the problem's answerability.

Given an eligible set of orderings of S, we can define an *intersection* relation $\geq^*_P$ such that, for each x, y in S, $x \geq^*_P y$ is true if and only if $x \geq_P y$ holds for all eligible orderings. It is natural to read $x \geq^*_P y$ as *x is indisputably at least as P as y*. Extending a previous notation, we can use $x =^*_P y$ to denote [$x \geq^*_P y$ and $y \geq^*_P x$] or equivalently, that $x =_P y$ holds for all eligible orderings. Similarly, we can use $x >^*_P y$ to denote [$x \geq^*_P y$ and not $y \geq_P^* x$], that is, $x \geq_P y$ holds for all eligible orderings and $x >_P y$ holds for some. And we can use $x \#^*_P y$ to denote [(not $x \geq^*_P y$) and (not $y \geq^*_P x$)], that is, $x >_P y$ holds for some eligible orderings and $y >_P x$ for others. It is immediately clear that, since each eligible $\geq_P$ is complete and transitive, $\geq^*_P$ is transitive but not necessarily complete. Incompleteness of the intersection relation is a manifestation of imprecision. That completes the specification of my proposal.

One merit of the framework I have proposed is its simplicity. Another is that it is isomorphic with the *random preference* approach that some decision theorists have used to represent imprecise preferences. The idea underlying that approach is that, for a given individual, there is a non-empty set of alternative preference orderings; which of these ordering governs the individual's decisions at any particular moment is determined by a random process. Random preference models can be used to organise data from controlled experiments in which individuals face exactly the same decision problems more than once; such experiments typically find a significant degree of apparently random variation in decision-making behaviour (Becker, De Groot and Marschak, 1963; Loomes and Sugden, 1995).

4. Rabinowicz's proposal

Rabinowicz proposes 'a general modelling of value relations' that can accommodate both vagueness and 'incommensurability', the latter term being defined so that two items are incommensurable if neither is better than the other, and they are not equally good. Rabinowicz's starting point is the claim that 'valuable' can be analysed as 'fitting to be favoured'. To say that some object *x* is valuable is to say that some particular 'pro-attitude', appropriate to the relevant value, is in some sense fitting; to say that *x* is more valuable than *y* is to say that some attitude that favours *x* over *y* is similarly called for. 'Better' is a special case of 'more valuable', for which the corresponding attitude is *preference*. Thus, to say that *x* is better than *y* is to say that a preference for *x* over *y* is fitting. Having understood betterness in this way, Rabinowicz proposes an analysis of 'is better than' that is compatible with imprecision and incompleteness in preferences. In Rabinowicz's model, there is a set *K* of eligible preference relations, interpreted as preferential attitudes that are rationally permissible. Each of these relations is required to be transitive but is *not* required to be complete. Rabinowicz proposes to define 'better than' and 'equally good' in terms of the intersection of eligible preference relations. That is, '*x is better than y*' is defined as 'in all eligible preference relations, *x* is strictly preferred to *y*', and '*x and y are equally good*' is defined as 'in all eligible preference relations, *x* is indifferent to *y*'. One implication is that, because there can be *x*, *y* pairs for which no specific preference ranking is rationally required, the relation 'is at least as good as' is not necessarily complete.

I think that this analysis introduces imprecision in the wrong place. For the purposes of the present argument, let us accept the analysis of 'valuable' as 'fitting to be favoured'. Consider the case of greatness among Britons. Greatness is a special kind of value. One might say, following Rabinowicz, that there must be some corresponding pro-attitude that is a fitting response to greatness. Let us say that this is *honouring* – as in erecting statues of a great person, or giving his or her name to major streets and public spaces. It seems perfectly reasonable to use 'honouring' to explicate or even to define 'greatness': if someone was unsure what was meant by a 'great' Briton, examples of statues in public places might well be useful. But this idea, as I have represented it so far, is wholly compatible with my analysis of the Great Britons exercise as a ranking problem. In discussing this problem, I have interpreted the ranking property *P* as 'greatness'. But we might equally well interpret it as 'honour-worthiness': that would make no difference to my analysis.

Consider a case of imprecision. Suppose the problem is to compare the greatness of Newton and Darwin. And suppose that the context of this problem is that a public decision is to be made about which of the two men's portraits will appear on the British £10 note. One might think (as I do) that it is permissible or reasonable or defensible to judge either of those scientists to be greater than the other, or indeed to judge them to be equally great. Correspondingly, it is permissible to judge either of them to be more honour-worthy, or to judge them equally honour-worthy. As long as we think about eligible *judgements*, we are in the domain of ranking problems – even if the judgements are about properties defined in terms of the fittingness of attitudes.

However, Rabinowicz moves out of this domain by focussing on the eligibility of attitudes themselves, understood as psychological dispositions. For Rabinowicz (and I agree with him about this), a preference is best understood as a disposition to choose one thing rather than another. Analogously, we might think of comparative 'honouring attitudes' as dispositions to honour one person rather than another. Let us use the term 'favour' as the analogue of preference in the case of honouring. Thus, to say that a person favours Newton over Darwin is to say that she is disposed to honour Newton rather than Darwin. Applying Rabinowicz's analysis to this case, we might say that the attitude of favouring Newton over Darwin is reasonable. So too is the attitude of favouring Darwin over Newton. So too is the attitude of favouring them equally. And, one might add, it could be reasonable to have none of those attitudes – for example, to have an attitude of indecision or perplexity, or not to care at all about honouring great men and women. Since it is *not rationally required that* Newton is favoured, Rabinowicz's analysis implies that Newton is not greater than Darwin. Similarly, Darwin is not greater than Newton, nor are the two men equally great. According to Rabinowicz, then, they are incommensurable in greatness.

This seems an odd use of both 'incommensurable' and 'greatness'. If 'greatness' is interpreted as 'honour-worthiness', it would be more natural to conclude that neither scientist is *indisputably* greater than the other, but that one can reasonably *judge* either to be greater (a judgement that, on Rabinowicz's analysis, is not disputable but false). And the fact that reasonable people can disagree about which of Newton and Darwin is greater does not imply that there is no common standard of greatness that applies to both men – only that

that standard is imprecise. Incommensurability, I suggest, should not be conflated with imprecision or disputability.

For Rabinowicz, propositions about comparative value are not answers to any ranking problem. When Rabinowicz says that *x* is 'better than' *y*, he is not referring to any common property of goodness of which *x* has more than *y*. Contrary to the most natural interpretation of the words, he is not treating 'better than' as equivalent to 'more good than' – even if 'good' is interpreted as 'preference-worthy' or 'preferable'. What he is saying might be better expressed by '*x* is *indisputably* preferable to *y*'. Since Rabinowicz's comparative value propositions do not belong to a ranking problem, my analysis does not apply to them. But I hope I have persuaded the reader that, as an aid to clear thinking about comparative value, a model based on ranking problems is sharper and more useful than one based on the intersection of reasonable attitudes. I continue to believe that philosophical discussions of incommensurability and vagueness are creating unnecessary complications.

References

Becker, Gordon, Morris DeGroot, and Jacob Marschak (1963). Stochastic models of choice behaviour. *Behavioral Science* 8: 41–55.

Broome, John (1991). *Weighing Goods*. Oxford: Blackwell.

Broome, John (1994). *Weighing Lives*. Oxford: Oxford University Press.

Loomes, Graham and Robert Sugden (1982). Regret theory: an alternative theory of rational choice under uncertainty. *Economic Journal* 92: 805–824.

Loomes, Graham and Robert Sugden (1995). Incorporating a stochastic element into decision theories. *European Economic Review* 39: 641–648.

Rabinowicz, Wlodek (2009). Incommensurability and vagueness.