Structural Remedies in Merger Regulation in a Cournot Framework

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Abstract

To prevent possible abuse of market power, an antitrust agency can force merging firms to divest some of their assets. The divested assets can be sold *via* auction either to existing competitors or to a new entrant. Divesture of assets extends the range of parameters when a merger satisfies a consumer surplus standard and should be approved. If the agency takes a more active stance toward the selection of a purchaser of the assets (e.g. to exclude an incumbent from the auction), then it could lead to a favourable outcome for consumers and merging firms.

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JEL classification: D43, K21, L51

Introduction

Merger regulation aims at preventing abuses of market power due to a merger. The main concern is that a merger between firms with overlapping markets may lessen the degree of competition, even though the merger may create the potential for efficiencies. In order to prevent expected abuse of market power due to such a merger, an antitrust agency has the power to prohibit a merger. Alternatively, an antitrust agency can choose between behavioural and structural remedies to restore effective competition in relevant markets.

Behavioural remedies set constraints on the merged firms' future behavior such as engagements by the merging parties not to abuse certain assets, force compulsory licensing or access to intellectual property. However, in the case of behavioural remedies the prime difficulty is in overseeing the implementation of the remedies in the post-merger phase. In contrast, structural remedies modify the allocation of property rights and create new firms through entire or partial divesture of assets. In the EU the competition agency prefers to use structural remedies because they are easy to implement and once implemented there is no need to monitor the behaviour of merging firms afterwards.[1] Although there are numerous discussions by antitrust practitioners, academics, and lawyers, to the best of my knowledge the idea of structural remedies as a way to protect consumers is not formally analysed in the literature.

There is an extensive literature on mergers for different market structures and types of competition, which shows that if there are no cost reductions due to a merger, merging firms find it profitable to exercise their market power through a price increase, which decreases consumer surplus.[2] However, if synergies (efficiencies) between merging firms are substantial, then cost reductions outweigh the market power effect and prices might decrease after a merger.[3]

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[1]See Monti (2002); Motta, Polo, and Vasconcelos (2002); and Motta (2004).

[2]Salant, Switzer, and Reynolds (1983); Perry and Porter (1985); Deneckere and Davidson (1985); Kamien and Zang (1990); Farrell and Shapiro (1990).

[3]See Williamson (1968); Werden (1996); Roeller, Stennek, and Verboven (2000); Besanko and

This paper extends the models by Farrell and Shapiro (1990); Perry and Porter (1985); and McAfee and Williams (1992), where the authors conduct an equilibrium analysis of a Cournot market before and after a merger with a focus on profitability and welfare changes. A common feature in those models is the existence of fixed capital (assets) in an industry, whereas the amount of fixed capital in a firm's possession determines its production costs. In this paper, we modify their analyses by allowing partial divesture of assets.

The essential idea of the paper is as follows. In order to prevent possible abuse of market power due to a merger, the agency can force merging firms to divest some of their assets. We can think about landing slots at airports that airlines possess, licences, radio or mobile phone frequencies, electricity generation facilities, etc. The agency can ask to divest a certain number of them as a structural remedy. The divested assets can be sold via auction either to existing competitors or to attract a new entrant into the market. In this paper, we analyse certain outcomes of the second-price sealed-bid auction. The emerged buyer of the divested assets and a new distribution of assets across firms affect equilibrium price in the market. The number of players and the degree of symmetry of the market negatively affect equilibrium price in the market, while marginal costs positively affect the price.

According to EU merger regulations a purchaser of the divested assets must be approved by the agency, i.e. the agency holds veto power over the choice of a purchaser of divested assets[4]. Although the total welfare standard might be a relevant concept to apply in economic analysis, most antitrust agencies extensively apply the consumer welfare standard when making merger approval decisions.[5] As it is stated both in the EU and US merger regulations, the prime objective of the antitrust agency is to protect consumers from price increases after a merger due to increased

Spulder (1993).

[4]The EU Merger Regulation and Competition Commission "Best practice guidelines for divesture commitments" published on: http://europa.eu.int/comm/competition/mergers/legislation.

[5]For theoretical discussions about consumer vs. total surplus approaches in merger regulation see Motta (2004); Neven and Roeller (2000); Besanko and Spulder (1993). Policy relevance of the consumer surplus standard is discussed by Farrell and Shapiro (2001); Roeller et al. (2000); and Yao and Dahdouh (1993)

market power. Note that the agency is to protect consumers from price increases with minimum possible intervention rather than minimisation of prices and protection of competitors, i.e. acting like an industry regulator. Therefore, in the present paper we assume that an antitrust agency applies a consumer surplus standard and wants to approve mergers that decrease prices, while rejecting those that increase prices.

In this paper, we do not address some important practical issues of negotiating and implementing remedies. For example, Lyons and Medvedev (2007) analyses how different negotiation structures affect the outcome of bargaining over remedies in merger regulation. It is also possible that merging firms and the buyer of divested assets might have an anticompetitive incentive to cripple divested assets in order to reduce capacity in an industry (Farrell, 2003).

The structure of the paper is as follows. In Section 2, we describe the basic model. Then we analyse a symmetric cost structure case in a three-firm industry, and proceed with a non-symmetric case, followed by a numerical example and discussion of the results.

1 Model

Let us consider a market inverse demand function $P = a - bX$, where the total output $X = \sum_{i=1}^{n} x_i$ and x_i is firm i's output. Each firm maximises its profit: $max_{x_i}(a - bX)x_i - C(x_i, s_i)$, where s_i is firm's i assets. The idea of the fixed capital s in the industry is captured through a form of the cost function, $C(x_i, s_i) = \frac{d_i}{s_i}x_i$, such that the more capital a firm possesses, the lower the marginal costs of production. The parameter d_i describes firm's i production technology. Therefore, the market is characterised by constant marginal costs of production $C_{x_i} = \frac{d_i}{s_i}$. This is a simplified form of Perry and Porter's (1985) and McAfee and Williams' (1992) cost function $C(x, s) = sg + dx + \frac{e}{2s}x^2$ with marginal costs $C_x = d + \frac{e}{s}x$ and Shapiro and Farrell's (1990) cost function $C(x, s) = wx^{\frac{1}{b}}s^{-\frac{a}{b}}$ with marginal costs $C_x = \frac{w}{b}x^{\frac{1}{b}-1}s^{-\frac{a}{b}}$, where s is the amount of fixed capital, and a, b, e, d, g, w are constants. The constant

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marginal cost function allows us to capture the main feature of their functions, the inverse relation between assets and marginal costs, while simplifying derivations significantly. However, many results in the paper would be valid for any convex cost function with respect to a fixed capital ($C_s < 0$, $C_{ss} > 0$).

The equilibrium for a Cournot type of competition with constant marginal costs is the following. Each firm maximises its profit function with respect to output: $max_{x_i}(a - bX)x_i - \frac{d_i}{s_i}x_i$ for $i = 1, ..., n$. Given demand and cost functions, the equilibrium output and price before the merger are derived from the system of n first-order conditions:

$a - 2bx_i - b\sum_{j=1, j\neq i}^{n} x_j = \frac{d_i}{s_i}.$

This yields $x_i = \frac{1}{(n+1)b}(a - n\frac{d_i}{s_i} + \sum_{j\neq i}^{n} \frac{d_j}{s_j})$ as the equilibrium output of the i-th firm.[6]

Therefore the total market output is $X_{total} = \frac{1}{(n+1)b}(na - \Sigma_{i=1}^{n} \frac{d_i}{s_i})$.

Hence, the equilibrium price is $P = \frac{1}{n+1}(a + \Sigma_{i=1}^{n} \frac{d_i}{s_i})$.

The profit of the i-th firm is $\Pi_i = \frac{1}{(n+1)^2 b}[a - n\frac{d_i}{s_i} + \sum_{j=1, j\neq i}^{n} \frac{d_j}{s_j}]^2$.

2 Symmetric case

Consider a symmetric case with n=3. Each firm possesses an equal amount of fixed capital s (i.e. $s_1 = s_2 = s_3 = s$) and the same production technology (i.e. $d_1 = d_2 = d_3 = d$). First, we derive a pre-merger equilibrium and then compare it with post-merger cases when there is no agency intervention and when the agency can force divesture, with the divested assets going either to an existing competitor or to a new entrant.

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[6]Restrictions on exogenous parameters are such that they must guarantee non-negative output for all firms: $x_i = \frac{1}{(n+1)b}(a - n\frac{d_i}{s_i} + \sum_{j\neq i}^{n} \frac{d_j}{s_j}) \geq 0$. Further restriction on parameters is $(a - \frac{d_i}{s_i}) > 0$, i.e. a firm with too little capital would always prefer not to produce.

3.1 Pre-merger case

The pre-merger equilibrium output of each firm is $x_1 = x_2 = x_3 = \frac{1}{4b}(a - \frac{d}{s})$;

the total output $X^B = \frac{3}{4b}(a - \frac{d}{s})$;

the equilibrium price before the merger is $P^B = \frac{1}{4}(a + 3\frac{d}{s})$;

the equilibrium profit of each firm is $\Pi_i^B = \frac{1}{16b}(a - \frac{d}{s})^2$.

3.2 Merger to duopoly without agency intervention

When any two firms in the market merge the number of firms decreases and the market becomes more concentrated. However, the merged firm becomes twice the size of its competitor and possesses $(2s)$ assets, which decreases its marginal costs of production. The equilibrium output and price after the merger is derived from the system of two FOCs for the merged firm (M) and the firm-outsider (O):

$$\begin{cases} a - 2bx_M - bx_o = \frac{d}{2s} \\ a - 2bx_o - bx_M = \frac{d}{s} \end{cases} \Leftrightarrow \begin{cases} x_M = \frac{a}{3b} \\ x_o = \frac{1}{3b}(a - \frac{3d}{2s}) \end{cases}.$$

Therefore, the total output after the merger is $X^A = \frac{1}{3b}(2a - \frac{3}{2}\frac{d}{s})$.

Consequently, the equilibrium price is $P^A = a - b\frac{1}{3b}(2a - \frac{3}{2}\frac{d}{s}) = \frac{a}{3} + \frac{d}{2s}$.

Corollary 1: The price after the merger will not increase, $P^A \leq P^B$, if $\frac{a}{3} \leq \frac{d}{s}$.

Proof: $P^A = \frac{a}{3} + \frac{d}{2s} \leq \frac{1}{4}(a + 3\frac{d}{s}) = P^B \quad \Leftrightarrow \quad \frac{a}{3} \leq \frac{d}{s}. \; QED$

The marginal costs of production of the merged firm could decrease substantially because the firm possesses more fixed capital. Lower marginal costs for the merged firm could outweigh the market power effect and be sufficient not to increase market price. This result copies Farrell and Shapiro (1990).[7]

The profit of the merged firm is $\Pi_M = \frac{1}{9b}[a - 2\frac{d}{2s} + \frac{d}{s}]^2 = \frac{a^2}{9b}$.

The profit of the outsider firm is $\Pi_O = \frac{1}{9b}[a - 2\frac{d}{s} + \frac{d}{2s}]^2 = \frac{1}{9b}[a - \frac{3}{2}\frac{d}{s}]^2$.

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[7]Farrell and Shapiro (1990) showed that a merger raises prices if and only if a markup of the would-be merging firms is less than the sum of the pre-merger markups at its constituent firms, where a merged firm produces just as much as its constituent firms did together before the merger.

The merger is possible only if it is profitable for the merging firms themselves $\Pi_M^A >$ $\Pi_1^B + \Pi_2^B$, i.e. the joint profit after the merger is higher than the sum of profits before the merger when they are separate firms. However, this condition holds for a wide range of parameters.

Lemma 1: It is always profitable for the merging firms to merge, if the price after the merger will not increase.

Proof: See Appendix A.

3.3 Merger to duopoly with divesture to an existing competitor

The agency can use structural remedies to "correct" the new market situation and keep prices at least unchanged after the merger. The agency can demand the merged firm to divest some of its obtained assets. The volume of divested assets is denoted by Δ with $\Delta \in [0, s_j]$, where s_j is the amount of fixed assets that belongs to the acquired firm.[8] There is an upper limit on the amount the agency can ask the merged firm to divest because otherwise the merger makes no sense: by asking more, the agency would leave the acquiring firm with fewer assets than before the merger. If two firms merge they control $(2s)$ assets. After the agency asks them to divest (Δ) assets $(2s - \Delta)$ assets remain in their possession. If the divested assets go to an existing competitor, then the outsider to the merger possesses $(s + \Delta)$ assets. Equilibrium output and price are derived from the system of two FOCs for the merged firm (M) and the firm-outsider(O):

$$\begin{cases} a - 2bx_M - bx_o = \frac{d}{2s-\Delta} \\ a - 2bx_o - bx_M = \frac{d}{s+\Delta} \end{cases} \Leftrightarrow \begin{cases} x_M = \frac{1}{3b}[a - \frac{2d}{2s-\Delta} + \frac{d}{s+\Delta}] \\ x_o = \frac{1}{3b}[a + \frac{d}{2s-\Delta} - \frac{2d}{s+\Delta}] \end{cases}.$$

Therefore, the total output is $X^{A1} = \frac{1}{3b}[2a - \frac{d}{(2s-\Delta)} - \frac{d}{(s+\Delta)}]$.

Consequently, the equilibrium price is $P^{A1} = \frac{1}{3}[a + \frac{d}{(2s-\Delta)} + \frac{d}{(s+\Delta)}]$.

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[8]This is a general representation of merging firms' assets (a firm with more assets is acquiring and with fewer assets is acquired) although in this section we consider only the symmetric case in which all firms possess the same amount of assets s. In Section 4, we will look at a non-symmetric case when firms have differing amounts of assets.

The profit of the merged firm is $\Pi_M^{A1} = \frac{1}{9b}[a - 2\frac{d}{2s-\Delta} + \frac{d}{s+\Delta}]^2$ and the profit of the outsider is $\Pi_o^{A1} = \frac{1}{9b}[a - 2\frac{d}{s+\Delta} + \frac{d}{2s-\Delta}]^2$. Like in the previous section, it is possible to show that the profitability constraint $\Pi_M^{A1} > \Pi_1^B + \Pi_2^B$ is not binding for parameters that guarantee non-increased price after the merger.

If we compare this situation with divesture to the one when the agency does not intervene, then the following proposition holds.

Proposition 1: Given a pre-merger symmetric cost structure ($s_1 = s_2 = s_3$ and $d_1 = d_2 = d_3$), any divesture $\Delta \in (0; s)$ leads to lower prices than would prevail without divesture.

Proof: P^{A1} will never exceed P^A: $\frac{1}{3}[a + \frac{d}{(2s-\Delta)} + \frac{d}{(s+\Delta)}] \leq \frac{a}{3} + \frac{d}{2s}$ $\Leftrightarrow$
$\frac{1}{2s-\Delta} + \frac{1}{s+\Delta} \leq \frac{3}{2s}$ $\Leftrightarrow$ $2s^2 \leq (2s - \Delta)(s + \Delta)$ $\Leftrightarrow$ $0 \leq \Delta(s - \Delta)$. *QED*

From the proposition above it is seen that no matter how large the decrease in marginal costs due to more fixed capital for the merged firm, it is always better to divest some assets and restore the symmetry: take assets from the bigger firm and give it to the smaller one. These results from the convexity of the cost function, ($C_x = \frac{d}{s}$): marginal costs are inversely related to the amount of fixed capital a firm possesses. Given firms' identical cost structures and Cournot-type competition the maximum output and the lowest price are achieved when all firms in the market possess an equal amount of fixed capital,[9] i.e. in case of duopoly the best result is achieved when ($\frac{s}{2}$) assets are divested and both firms possess ($\frac{3}{2}s$).

Therefore, we can distinguish three effects that affect equilibrium price in the market: number of players, marginal costs, and the symmetry of the market. The number of players in the market and the degree of symmetry negatively affect the price, while marginal costs positively affect the price.

From Section 3.2 where the agency does not intervene, we know that if $\frac{a}{3} \leq \frac{d}{s}$, there is no need of any divesture (i.e. $\Delta = 0$) to ensure that prices do not increase. Now introducing the possibility to divest certain amount of assets, consider the case $\frac{a}{3} > \frac{d}{s}$.

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[9]The absence of collusive effects is due to the fact that it is a static model rather than a repeated game. However, more symmetry is not to raise concerns in a dynamic setting as long as collusion is an unlikely outcome of the game (for example, when the discount factor is low enough).

Proposition 2: The possibility of divesture of assets from the merging firms to an existing competitor extends the range of parameters that satisfy a consumer surplus standard from $\frac{a}{3} \leq \frac{d}{s}$ (case of a merger without divesture) to $\frac{9a}{33} \leq \frac{d}{s}$. To keep prices at the pre-merger level the agency should ask the merging firms to divest $\Delta = \frac{-s+s\sqrt{\frac{33d-9as}{9d-as}}}{-2}$ assets.

Proof: See Appendix A.

On the interval $\frac{9a}{33} \leq \frac{d}{s} \leq \frac{a}{3}$, there is a divesture $\Delta \in [0, \frac{s}{2}]$ that would keep the prices after the merger unchanged. If $\frac{9}{33}a = \frac{d}{s}$, then $\Delta = \frac{s}{2}$, i.e. the merged firm should divest exactly half of what it obtained through the merger and, therefore, the two firms in the market would possess an equal amount of fixed capital (as we noted above, this is the divesture that maximises consumer surplus).

3.3.1 Merger specific efficiencies

There could be efficiency gains due to merger-specific synergies between merging firms' assets. In our setting, this is equivalent to a decrease in the parameter d (from d to αd, where $\alpha \in [0,1]$) for the merged firm. The inclusion of an exogenously given "synergy parameter" α implies that the cost function is not continuous in s anymore, and it shifts the marginal cost function. Thus, there will be a trade-off between efficiency gains due to the merger and the amount of divested assets the agency asks the merged firm to sell-off to the competitor to keep prices unchanged after the merger. This trade-off plays an important role in merger regulation after both the EU and US agencies allowed for efficiency defence in merger approvals.

3.4 Merger with divesture to a new entrant

The agency could also enforce divesture of (Δ) assets to a new entrant into the market. Therefore, the merging firms possess ($2s - \Delta$) assets, the old competitor has (s) assets, while their new competitor has (Δ) assets. Equilibrium output and price are derived from the system of three FOCs for the merging firms (M), a firm-outsider (o), and a new entrant to the market (N):

$$
\begin{cases}
a - 2bx_M - bx_o - bx_N = \frac{d}{2s-\Delta} \\
a - 2bx_o - bx_M - bx_N = \frac{d}{s} \\
a - 2bx_N - bx_M - bx_o = \frac{d}{\Delta}
\end{cases}
\Leftrightarrow
\begin{cases}
x_M = \frac{1}{4b}[a - \frac{3d}{2s-\Delta} + \frac{d}{s} + \frac{d}{\Delta}] \\
x_o = \frac{1}{4b}[a + \frac{d}{2s-\Delta} - \frac{3d}{s} + \frac{d}{\Delta}] \\
x_N = \frac{1}{4b}[a + \frac{d}{2s-\Delta} + \frac{d}{s} - \frac{3d}{\Delta}]
\end{cases}.
$$

Therefore, the total output is $X^{A2} = \frac{1}{4b}[3a - \frac{d}{2s-\Delta} - \frac{d}{s} - \frac{d}{\Delta}]$.

Consequently, the equilibrium price is $P^{A2} = \frac{1}{4}[a + \frac{d}{2s-\Delta} + \frac{d}{s} + \frac{d}{\Delta}]$.

Proposition 3: If the pre-merger market is characterised by a symmetric cost structure, and a new entrant possesses the same technology as all other firms ($d_N = d$), then there is no such divesture Δ to a new entrant that would decrease prices after the merger.

Proof: Prices after the merger with a divesture to a new entrant, is always greater than prices before the merger: $P^{A2} = \frac{1}{4}[a + \frac{d}{(2s-\Delta)} + \frac{d}{s} + \frac{d}{\Delta}] \geq \frac{1}{4}[a + 3\frac{d}{s}] = P^B$

$\Leftrightarrow \quad \frac{1}{(2s-\Delta)} + \frac{1}{\Delta} \geq \frac{2}{s} \quad \Leftrightarrow \quad s^2 \geq (2s - \Delta)\Delta \quad \Leftrightarrow \quad (s - \Delta)^2 \geq 0. \quad QED$

The price will never decrease due to the form of the marginal cost function ($\frac{d}{s}$). Under the symmetric cost structure and a Cournot type of competition the maximum output and lowest price are achieved when all firms possess an equal amount of fixed capital (which is the case for the pre-merger situation). After the merger, the number of players stays the same (one firm is eliminated through the merger but a new one is formed) but the cost structure becomes less symmetric.

Remark 1: This proposition is valid for any convex cost function with respect to fixed capital ($C_s < 0$, $C_{ss} > 0$).

Proof: See Appendix A.

From Sections 3.3 and 3.4, it is seen that if the pre-merger market is characterised by a symmetric cost structure, then the agency would never approve a merger with a divesture of assets to a new entrant. Therefore, there is no need to auction the divested assets because an existing competitor is the only potential purchaser of the assets that could be approved by the agency. However, if a market is characterised by a non-symmetric cost structure an auction can lead to different outcomes.

3 Non-symmetric case

Consider a three-firm industry with identical technology parameters for the marginal cost function $d = d_1 = d_2 = d_3$ but unequal amounts of fixed capital: s_1, s_2, and s_3.

Proposition 4: If firms in the market differ only in the amount of fixed capital they possess, then a merger between any two firms with divesture to a new entrant leads to a price increase.
Proof: See Appendix A.

This proposition is valid for any convex cost function with respect to fixed capital (Proof is similar to Remark 1). Like in the symmetric case the agency will never approve divesture of assets to a new entrant. For certain values of parameters it can approve a sale of the divested assets to the existing competitor, or approve the merger without any divesture, or reject the merger. Therefore, this non-symmetric case does not provide us with any new insights on the divesture problem.

As the next step in our analysis consider a three-firm industry with an equal amount of assets across firms $s_1 = s_2 = s_3 = s$ but with different technology parameters d_1, d_2, and d_3. Assume that firms 1 and 2 are merging, and the merged firm would produce at marginal costs which are the lower of the two merging firms.[10] Without loss of generality, we can assume that $d_1 < d_2$.

Then there are again 2 cases:
a) A merger between two firms with divesture to the existing competitor. The equilibrium price in the non-symmetric case with the divesture to an existing competitor is $P^{EC} = \frac{1}{3}[a + \frac{d_1}{2s-\Delta} + \frac{d_3}{s+\Delta}]$.

b) A merger between two firms with divesture to a new entrant. The marginal costs of production for a new entrant are characterised by parameter d_N. This parameter determines the viability of a new entrant, which is a prime concern for antitrust authorities when deciding on the divesture of assets.[11] The equilibrium price in the

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[10]In the literature, this is known as a rationalisation of production, i.e. a shift of output to the production facility with the lower marginal cost (see Farrell and Shapiro 1990).

[11]Exogenous parameters should satisfy the condition: $x_N = \frac{1}{4b}(a + \frac{1}{2s-\Delta} + \frac{d_3}{s} - 3\frac{d_N}{\Delta}) > 0$

non-symmetric case with the divesture to a new entrant is $P^{NE} = \frac{1}{4}[a + \frac{d_1}{2s-\Delta} + \frac{d_3}{s} + \frac{d_N}{\Delta}]$.

In merger approval decisions, the agency and the merging firms negotiate the required amount of divested assets Δ and then can decide to auction it. Note that divesture is a needed policy instrument only if the exogenous parameters are such that without any agency intervention the price will increase after the merger. In principle an auction seems a viable mechanism to sell divested assets and often interested parties opt for it.[12] At the auction either an existing competitor or a new entrant could purchase the divested assets, and depending on the winner of the auction the market structure and, consequently, prices are determined. By applying a consumer welfare standard, the agency approves a purchaser only if the price will not increase. At the same time a merger with a divesture is possible only if the merging firms expect a higher joint profit after the merger including the revenue from auctioning the divested assets than the sum of profits before the merger when they are separate firms: $\Pi_1^B(s) + \Pi_2^B(s) < \Pi_M^A(2s - \Delta) + Revenue(\Delta)$.

The analysis proceeds in the following way. First, given exogenous parameters and values of $\Delta \in [0, s]$, we check who has the higher expected profit from the purchase of the divested assets: an existing competitor or a new entrant. The existing competitor (firm 3) compares profits when it purchases divested assets (Δ) and then operates in a duopoly market with the situation in which it stays away from the purchase while a new entrant buys the assets. Therefore, if the inequality $\Pi_3^{buys} - \Pi_3^{away} > \Pi_N$ holds, then the existing competitor bids a higher price than a new entrant. It results from the fact that the increase in its expected profit from buying the divested assets is higher than these assets can generate in profits for a new entrant.[13]

Second, given the amount of divested assets and the winner at the auction, we check whether the merger with a divesture is profitable for the merging firms. As we described above a merger with a divesture is possible only if the merging firms expect a higher joint profit after the merger including the revenue from auctioning the divested assets than the sum of profits before the merger when they are separate

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[12]An alternative to the auction is a direct sale of assets.

[13]The formulas of the profit functions are: $\Pi_3^{buys} = \frac{1}{9b}[a - 2\frac{d_3}{s+\Delta} + \frac{d_1}{2s-\Delta}]^2$,

$\Pi_3^{away} = \frac{1}{16b}[a - 3\frac{d_3}{s} + \frac{d_1}{2s-\Delta} + \frac{d_N}{\Delta}]^2$, and $\Pi_N = \frac{1}{16b}[a - 3\frac{d_N}{\Delta} + \frac{d_1}{2s-\Delta} + \frac{d_3}{s}]^2$.

firms: $\Pi_1^B(s) + \Pi_2^B(s) < \Pi_M^A(2s - \Delta) + Revenue(\Delta)$. Depending on who wins the auction, an existing competitor or a new entrant, the merged firm will operate either in a duopoly or triopoly.[14] In the paper we consider a second-price sealed-bid auction as a mechanism to sell off divested assets, i.e the winner of the auction is the highest bidder but it pays only the value of the second highest bid offered. Respectively, the revenue from the auction will be equal either (Π_N) or $(\Pi_3^{buys} - \Pi_3^{away})$ (with the formulas provided above).

Finally, we answer what will happen to the price, since the agency approves the divesture only if it will not increase the price. Depending who offers the highest bid for the divested assets, we compare the price before the merger, P^B, with the equilibrium price after the merger with the divesture to either an existing competitor, P^{EC}, or a new entrant, P^{NE}.[15]

It is difficult to solve analytically the full system of inequalities above because there are 6 exogenous parameters a, d_1, d_2, d_3, d_N, and s. We know that $d_1 < d_2$ and without loss of generality we can assume that $s = 1$ and $d_1 = 1$, and conditions $a > \frac{d_i}{s_i}$ and $x_i = (a - n\frac{d_i}{s_i} + \sum_{j \neq i}^n \frac{d_j}{s_j}) > 0$ must hold. This, however, is still analytically intractable. Hence, in the next section we conduct a numerical analysis and look at some possible outcomes of structural remedies.

4.1 Possible outcomes of the auction of divested assets: numerical example

The crucial parameters in the model are the firms' marginal costs. The table below reflects the technology parameters d_i of each firm relative to all others. The columns contain a ranking of the two merging firms' parameters (d_1 and d_2) relative to the outsider to the merger (d_3). In the rows is a ranking of a new entrant's parameter (d_N) relative to the three pre-merger firms.[16]

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[14]The formulas of the profit functions are: $\Pi_1^B(s) = \frac{1}{16b}[a - 3\frac{d_1}{s} + \frac{d_2}{s} + \frac{d_3}{s}]^2$, $\Pi_2^B(s) = \frac{1}{16b}[a + \frac{d_1}{s} - 3\frac{d_2}{s} + \frac{d_3}{s}]^2$, $\Pi_M^A(duopoly) = \frac{1}{9b}[a - 2\frac{d_1}{2s-\Delta} + \frac{d_3}{s+\Delta}]^2$, and $\Pi_M^A(triopoly) = \frac{1}{16b}[a - 3\frac{d_1}{2s-\Delta} + \frac{d_3}{s} + \frac{d_N}{\Delta}]^2$.

[15]The formulas for equilibrium prices are:

$P^B = \frac{1}{4}[a + \frac{d_1}{s} + \frac{d_2}{s} + \frac{d_3}{s}]$, $P^{EC} = \frac{1}{3}[a + \frac{d_1}{2s-\Delta} + \frac{d_3}{s+\Delta}]$, and $P^{NE} = \frac{1}{4}[a + \frac{d_1}{2s-\Delta} + \frac{d_3}{s} + \frac{d_N}{\Delta}]$.

[16]There are 4 parameters (d_1, d_2, d_3, and d_N) and $(4!) = 24$ combinations when the order matters.

Table 1. Firms' ranking according to the marginal costs

	Merging firms d_1 and d_2 wrt d_3		
New entrant d_N	Two lowest	Two highest	Lowest and Highest
Lowest	Case 1	Case 2	Case 3
b/w first and second	Case 4	Case 5	Case 6
b/w second and third	Case 7	Case 8	Case 9
Highest	Case 10	Case 11	Case 12

A numerical analysis is conducted using MATLAB software (see a sample of the code in Appendix B). Without loss of generality, it is assumed that $a = 4$ (a is a parameter of the inverse demand function). We check for different types of equilibria for $\Delta \in (0; 1)$ (grid is 100) and parameters $d_2, d_3, d_N \in (0; 4)$ (grid is 100). Given inequalities in Section 4, the following results are obtained.

The viability of a new entrant that is captured by the ratio $\frac{d_N}{\Delta}$ is crucial for the firm's competitiveness. If Δ is small or d_N is high, then the condition $x_N = \frac{1}{4b}(a + \frac{1}{2-\Delta} + d_3 - 3\frac{d_N}{\Delta}) > 0$ does not hold, i.e. a new entrant cannot have a positive output level. It is seen that the higher d_3, marginal costs of the existing competitor, and the more efficient a new entrant relative to the merging firms (which is normalised to one), the more likely the condition is to hold. If the condition does not hold, then the existing competitor would always win the auction whenever it is profitable for him and for the merging firms (see Section 4.2); otherwise, the merger would not happen. Depending on the exogenous parameters the price will increase or decrease, and hence, the merger will be rejected or approved by the agency, respectively.

There are parameters when a new entrant wins the auction, the price decreases, and it is profitable for the merging firms to proceed with the merger and divesture. From the numerical analysis, we can say that such a situation can emerge in Cases 1, 2, and 3 (it appears most frequently in Case 1). In all these cases, a new entrant is the most efficient firm in the market (d_N is the lowest among all firms). Although in many industries it is difficult to imagine that a firm which is a newcomer to a market

Assuming $d_1 < d_2$, there are only 12 possible cases left.

could possess the most advanced technology, there are cases where it could be true. For example, there is tough competition to operate flights from Heathrow Airport in London. Landing slots are the necessary assets to do business. Incumbents (British Airways, United, and others) do not allow other airlines to buy or lease landing slots at the airport so as to keep competitors away from the lucrative transatlantic flight business. However, it is possible that a low-cost carrier could enter the market[17] by buying divested assets and turn out to be the most efficient player (efficient enough to reduce the price in the market). Probably a new efficient entrant needs few assets to start a profitable business: if a new entrant is efficient, d_N is small, then divesture Δ could be small. At the same time 'little' divesture keeps the merger profitable for the merging firms. In this case, the results of the auction are beneficial for all parties involved: consumers, merging firms, and a new entrant.

Another situation occurs when a new entrant wins the auction and the price decreases, but this new market structure is unprofitable for the merging firms. This is possible in Cases 1, 2, 3, and 5 (it appears most frequently in Case 1). In this situation we can use the same example of Heathrow Airport as above with the only difference that the merging firms would not allow the divesture to a new entrant to happen. Appearance of a new more efficient competitor will decrease their future profits. Therefore, the merging firms prefer to abandon the merger.

It is possible that if a new entrant wins the auction, the price would decrease and it would be profitable for the merging firms. However, the existing competitor bids a higher price for the assets, and a new market structure either leads to higher prices or makes the merger unprofitable for the merging firms.

The situation which leads to higher prices can emerge in Cases 1, 2, 3, 5, and 8 (it appears most frequently in Case 2), i.e. when either a new entrant is the most efficient firm or the merging firms are the least efficient firms in the pre-merger market (firms 1 and 2 have the highest marginal costs). The existing competitor does not allow the merger to happen simply by overbidding a new entrant and causing the price to increase. As a result, the agency rejects the purchaser of divested assets and the

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[17]Here the definition of the market is flights from Heathrow.

merger, the merging firms have to abandon the merger and forego expected profits in the future, and consumers have to stay with the pre-merger price, which could have decreased due to the merger.

The situation in which divesture to the existing competitor makes the merger unprofitable can emerge in Cases 1, 3, 4, 6, 8, and 9 (it appears most frequently in Case 4). As a result, the merging firms have to abandon the merger and forego expected profits in the future, and consumers have to stay with the pre-merger price, which could have decreased due to the merger with divesture to a new entrant.

In both cases, the agency (and consumers) would be better off if a new entrant wins the auction. Therefore, by excluding the existing competitor (incumbent) from the auction, the agency can enhance consumer and the merging firms' welfare. Furthermore, this policy is easy to implement.

Whenever a new entrant has the highest marginal costs among all firms (Case 10, 11, and 12) and the assets are divested to it, the price will always increase and the agency will reject the divesture and, consequently, the merger. The intuition is that the number of firms in the market stays the same (three) and given the convexity of the marginal cost function it is unreasonable to divest assets from an 'efficient' to a less efficient new entrant because in the model the market price depends on the sum of marginal costs across all firms.

Under a wide range of parameters, the existing competitor wins the auction, the price decreases, and such a market structure is profitable for the merging firms to proceed with the merger. However, the price could also increase in some cases or a new market structure could be unprofitable for the merging firms. These results are similar to those discussed in the symmetric case.

4 Conclusion

The model presented here introduces a simple theoretical framework to analyse structural remedies in merger regulation. It captures all the main issues that are at stake

in merger approval decisions: efficiency defence and consumer welfare; amount of divesture and auction design; viability of a new entrant; and the rationalisation of output between merging firms.

Under the current merger guidelines, the merging firms can sell divested assets through an auction, while a purchaser of the assets must be approved by the agency. Evidently the merging firms choose a purchaser, which is the most profitable for them based on future profit from a new market structure. The agency only checks whether the price will decrease or increase after the purchase and, subsequently, will approve or reject a purchaser and the merger.

The analysis of the symmetric case shows that divesture allows for the extension of the range of parameters when a merger should be approved. The non-symmetric case shows the importance of the rationalisation of production between the merging firms and the viability of a new entrant.

From the results of the numerical analysis, we can suggest making the agency more active in the selection of a potential purchaser of divested assets. The agency can stipulate in the merger guidelines that first they want to look for a new entrant (a viable one) and only if one is not found or not desirable, to consider existing competitors. For some parameters the agency is better off to exclude the existing competitor (incumbent) from the auction. As was shown in the paper, this could lead to a more favorable outcome for consumers and merging firms.

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Appendix A

Lemma 1: It is always profitable for the merging firms to merge, if the price after the merger will not increase.

Proof: Consider first a case of a symmetric 3-firm industry. Plug in formulas of profit functions in the inequality $\Pi_M^A > \Pi_1^B + \Pi_2^B : \frac{a^2}{9b} > 2\frac{1}{16b}(a - \frac{d}{s})^2 \iff \frac{a}{3}(3 - \sqrt{8}) < \frac{d}{s}$. Given that $\frac{a}{3}(3 - \sqrt{8}) < \frac{a}{3}$, the inequality $\Pi_M^A > \Pi_1^B + \Pi_2^B$ will definitely hold if the condition $\frac{a}{3} \le \frac{d}{s}$ is satisfied. *QED*

Proposition 2: The possibility of divesture of assets from the merging firms to an existing competitor extends the range of parameters that satisfy a consumer surplus standard from $\frac{a}{3} \le \frac{d}{s}$ (case of a merger without divesture) to $\frac{9a}{33} \le \frac{d}{s}$. To keep prices at the pre-merger level the agency should ask the merging firms to divest $\Delta = \frac{-s + s\sqrt{\frac{33d - 9as}{9d - as}}}{-2}$ assets.

Proof: $\frac{1}{3}[a + \frac{d}{(2s - \Delta)} + \frac{d}{(s + \Delta)}] \le \frac{1}{4}(a + 3\frac{d}{s}) \iff -\Delta^2 + s\Delta + 2s^2\frac{3d - as}{9d - as} \ge 0$.

The solution to this quadratic expression is the following:

$\Delta = \frac{-s \pm s\sqrt{1 + 8\frac{3d - as}{9d - as}}}{-2} = \frac{-s \pm s\sqrt{\frac{33d - 9as}{9d - as}}}{-2}$.

The expression under the square root should be positive but less than one, $0 \le \frac{33d - 9as}{9d - as} \le 1$, to ensure that the divesture is less than s assets (otherwise the agency asks to divest more than the acquiring firm obtained due to the merger). Thus the price will stay unchanged after the merger when:

$\Delta_1 = \frac{-s + s\sqrt{\frac{33d - 9as}{9d - as}}}{-2} \le \frac{s}{2}$ or $\Delta_2 = \frac{-s - s\sqrt{\frac{33d - 9as}{9d - as}}}{-2} \ge \frac{s}{2}$,

and the following inequalities hold: $0 \le \frac{33d - 9as}{9d - as} \le 1 \iff \frac{9a}{33} \le \frac{d}{s} \le \frac{a}{3}$.

There are two solutions to the quadratic equation, but the one with the least divesture, Δ_1, should be the focus of the analysis. Both solutions lead to the same result (keeping the price unchanged) and the agency should choose the one with minimum possible intervention into the market. *QED*

Remark 1: Convex function

If $f(s)$ is a marginal cost function with assets (s), then the price is $P = \frac{1}{n+1}(a + \sum_{i=1}^{n} f(s_i))$. For the symmetric case of $n = 3$ we should compare P^B and P^A, where the latter is the price after merger between two firms with a divesture to a new entrant, and all firms in the market, including the newcomer, possess the same technology d. Then the price after the merger will increase if:

$\frac{1}{4}(a + 3f(s)) < \frac{1}{4}(a + f(2s - \Delta) + f(\Delta) + f(s))$

$2f(s) < f(2s - \Delta) + f(\Delta)$

$f(s) < \frac{1}{2}(f(2s - \Delta) + f(\Delta))$

$f(\frac{1}{2}(2s - \Delta) + \frac{1}{2}(\Delta)) < \frac{1}{2}f(2s - \Delta) + \frac{1}{2}f(\Delta).$

This is an exact definition of any convex function. *QED*

Proposition 4: If firms in the market differ only in the amount of fixed capital they possess, then a merger between any two firms with divesture to a new entrant leads to a price increase.

Proof: This proposition follows from the convexity of the cost function. Assume the price after the merger is lower than before the merger:

$P^{Before} = \frac{1}{4}[a + \frac{d}{s_1} + \frac{d}{s_2} + \frac{d}{s_3}] > \frac{1}{4}[a + \frac{d}{(s_1+s_2-\Delta)} + \frac{d}{s_3} + \frac{d}{\Delta}] = P^{NonSym}$

$\frac{d}{s_1} + \frac{d}{s_2} > \frac{d}{s_1+s_2-\Delta} + \frac{d}{\Delta} \quad \Leftrightarrow \quad \frac{1}{s_1} + \frac{1}{s_2} > \frac{1}{s_1+s_2-\Delta} + \frac{1}{\Delta} \quad \Leftrightarrow \quad s_1 s_2 < (s_1 + s_2)\Delta - \Delta^2.$

By solving the quadratic equation we can see that the inequality holds if $\Delta \in (s_2, s_1)$, which requires to divest more than was acquired through the merger. This is impossible due to our initial assumption. *QED*

Appendix B:

MATLAB code for the situation in which an existing competitor wins the auction but the price will increase. However, if a new entrant won the auction, the price would decrease, and it would be profitable for the merging firms.

```
clear; tic;
a=4;
N=100;
K=100;
delta=linspace(.0001,1,N);
d2=linspace(.0001,4,K);
d3=linspace(.0001,4,K);
dN=linspace(.0001,4,K);

X1=0;  X2=0; X3=0; X4=0; X5=0; X6=0; X7=0;  X8=0; X9=0; X10=0; X11=0; X12=0;

for i1=1:N
 for i2=1:K
  for i3=1:K
   for i4=1:K

        deltai=delta(i1);
        d2i=d2(i2);
        d3i=d3(i3);
        dNi=dN(i4);
```

```
I1=(a-1-3*d2i+d3i)>0;                        % without divesture the price will increase
I211=(a-3+d2i+d3i)>0;                        % positive output for Firm 1 before
I212=(a-3/(2-deltai)+d3i+dNi/deltai)>0;      % positive output for Firm 1 after with new entrant
I213=(a-3/(2-deltai)+d3i/(1+deltai))>0;      % positive output for Firm 1 after with existing competitor
I22=(a+1-3*d2i+d3i)>0;                        % positive output for Firm 2 before
I231=(a+1+d2i-3*d3i)>0;                       % positive output for Firm 3 before
I232=(a-3*d3i+1/(2-deltai)+dNi/deltai)>0;    % positive output for Firm 3 after with new entrant
I233=(a-3*d3i/(1+deltai)+1/(2-deltai))>0;    % positive output for Firm 3 after with existing competitor
I2N=(a-3*dNi/deltai+1/(2-deltai)+d3i)>0;     % positive output for the new entrant with Delta assets
```

```
% Existing competitor bids higher price at an auction
I3=((16/9)*(a-2*d3i/(1+deltai)+1/(2-deltai))^2-(a-3*d3i+1/(2-deltai)+dNi/deltai)^2-
(a-3*dNi/deltai+1/(2-deltai)+d3i)^2)>0;
```

```
% it is profitable for the merging firms 1 and 2 if a new entrant buys assets
I41=((a-3/(2-deltai)+d3i+dNi/deltai)^2-(a-3+d2i+d3i)^2-(a+1-3*d2i+d3i)^2)+((16/9)*(a-2*d3i/(1+deltai)+1/(2-
deltai))^2-(a-3*d3i+1/(2-deltai)+dNi/deltai)^2))>0;
```

```
% it is profitable for the merging firms 1 and 2 if an existing competitor (EC) buys assets
I42=((16/9)*(a-2/(2-deltai)+d3i/(1+deltai))^2-(a-3+d2i+d3i)^2-(a+1-3*d2i+d3i)^2)+(a-3*dNi/deltai+1/(2-
deltai)+d3i)^2)>0;
```

```
% price does NOT increase if a new entrant (N) buys assets
I51=(1+d2i-1/(2-deltai)-dNi/deltai)>0;
```

```
% price INCREASES if an existing competitor (EC) buys assets
I52=(a-3+4/(2-deltai)-3*d2i-3*d3i+4*d3i/(1+deltai))>0;
```

```
I=I1*I211*I212*I213*I22*I231*I232*I233*I2N*I3*I41*I42*I51*I52;
% if all conditions hold then it is 1, if at least one doesn't then 0
```

```
    Case1=(dNi<1)&(1<d2i)&(d2i<d3i);          % All possible ordered combinations of d1,d2,d3,dN
    Case2=(dNi<d3i)&(d3i<1)&(1<d2i);          % given that d2>d1=1
    Case3=(dNi<1)&(1<d3i)&(d3i<d2i);
    Case4=(1<dNi)&(dNi<d2i)&(d2i<d3i);        % If the conditions are satisfied then 1, otherwise 0
    Case5=(d3i<dNi)&(dNi<1)&(1<d2i);
    Case6=(1<dNi)&(dNi<d3i)&(d3i<d2i);
    Case7=(1<d2i)&(d2i<dNi)&(dNi<d3i);
    Case8=(d3i<1)&(1<dNi)&(dNi<d2i);
    Case9=(1<d3i)&(d3i<dNi)&(dNi<d2i);
    Case10=(1<d2i)&(d2i<d3i)&(d3i<dNi);
    Case11=(d3i<1)&(1<d2i)&(d2i<dNi);
    Case12=(1<d3i)&(d3i<d2i)&(d2i<dNi);

if Case1*I==1
    X1=X1+1;            % Count number of situations that fall in Case 1 and satisfy inequalities in I
elseif Case2*I==1
    X2=X2+1;
elseif Case3*I==1
    X3=X3+1;
elseif Case4*I==1
    X4=X4+1;
elseif Case5*I==1
    X5=X5+1;
elseif Case6*I==1
    X6=X6+1;
elseif Case7*I==1
    X7=X7+1;
elseif Case8*I==1
    X8=X8+1;
elseif Case9*I==1
    X9=X9+1;
elseif Case10*I==1
    X10=X10+1;
elseif Case11*I==1
    X11=X11+1;
elseif Case12*I==1
    X12=X12+1;
end
end
end
end
end
toc;

X1      % Display the total number of situations that satisfied Case 1 and all specified inequalities
X2      % matters whether it is 0 or not
X3
X4
X5
X6
X7
X8
X9
X10
X11
X12
```