

Top Guns May Not Fire: Best-Shot Group Contests with Group-Specific Public Good Prizes^{*}

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Abstract

We analyze a group contest in which n groups compete to win a group-specific public good prize. Group sizes can be different and any individual player may value the prize differently within and across groups. Players expend costly efforts simultaneously and independently. Only the highest effort (the best-shot) within each group represents the group effort and the winning group is determined by a contest success function. We fully characterize the set of equilibria and show that in any equilibrium at most one player in each group exerts strictly positive effort. There always exists an equilibrium in which only the highest value player in each active group expends positive effort and the contest is reduced to an individual contest between individual players. However, there may also be equilibria in which the highest value players completely free ride on others by exerting no effort. We provide conditions under which this can be avoided and discuss contest design implications.

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1. Introduction

Groups often confront each other in order to win a prize. Individual group members contribute costly efforts to the ‘group effort’ which can increase the probability of the group to win. The prize can be of a public-good nature in the sense that every group member of the winning group earns the prize even if some of them do not contribute to the group effort. Examples of this setting include sports events between teams, rent-seeking contest between lobbying groups, electoral confrontation or war between coalitions.

Aside from a few exceptions, most studies on contests between groups assume a *perfect-substitutes group impact function* in which a group’s effort is determined by the sum of individual efforts in that group.¹ In this article we introduce a *best-shot group impact function* where individual group members expend costly efforts simultaneously and independently, but only the highest effort within each group represents the group effort. We characterize the complete set of equilibria, show the required restrictions for equilibria selection and discuss the relevance of the results.

Best-shot impact functions are readily observed in the field, e.g. sporting events, industrial structures and defense mechanism. Among sporting events, racing contests such as cycling team events like *Tour de France* or car race like *Formula 1* have the features of best-shot contest. Although each member of a team participates separately in the contest, if any member of the team finishes first, then it records the win of the whole team. Team archery or shooting contests have similar features in which literally the *best-shot* among the team members determines the fate of the whole team. In industrial organization, the case of competing Research Joint Ventures (RJVs) can be a close example of the aforementioned set up. If a member of the joint venture can make a high quality innovation then it benefits the whole RJV, but other lower quality innovations put forth by members of the same RJV or competing RJVs get obsolete. Similar logic applies to competition for patent pools (e.g. 4G mobiles), where the main patent (best-shot) provides most competing features of a particular patent pool. The best-shot public good structure is also well documented and discussed in the

¹ A function that translates individual group member efforts into the *group effort* is called a *group impact function*. The literature on group contest originated with the work of Katz et al. (1990). Katz et al. use symmetric players within each group, perfect-substitutes group impact function (as in Bergstrom et al. 1986, in a public good game setting), and lottery contest success function (Tullock, 1980). Most studies on group contests use the perfect-substitutes impact function (Baik, 1993; Baik and Shogren, 1998; Baik et al. 2001; Baik, 2008; Münster, 2009). However, recently, the weakest-link impact function, where the minimum effort among the group members represents the group effort, has been analyzed by Lee (2009). Also, Kolmer and Rommeswinkel (2010) analyze the group contest using a Constant Elasticity of Substitution impact function.

defense economics literature. Competing defense coalitions, such as the *NATO* and the *Warsaw Pact*, follow the best-shot technology in which the best performance of the coalition member determines the performance of the whole coalition (Conybeare et al. 1994). The same story holds for inland security coalition comprised of different independent bodies such as *CIA* and *FBI*, or in the context of system reliability (Varian, 2004). The results show that in a defense or system reliability coalition, in the equilibrium the highest valuation group member expends strictly positive effort and the others free-ride by expending zero effort. It is important to emphasize, however, that defense economics studies do not explicitly investigate best-shot contests.

A behavioral background for the best-shot impact function is given in Baik and Shogren (1998). Clark and Konrad (2009) use best-shot impact function as the attack mechanism in the context of terrorism. Topolyan (2011) applies best-shot technology to portray extreme form of free-riding behavior in an all-pay auction between two groups under symmetric valuations. Sheremeta (2011) uses best-shot among other impact functions to experimentally investigate group contests, without general theoretical analysis. Therefore, the current literature on contests does not provide a thorough analysis of the best-shot group contests.

In the current article we characterize the complete set of equilibria for the best-shot group contest and show that in each equilibrium only one player in each group expends strictly positive effort whereas all the other players free-ride by expending zero effort. However, the unique player need not be the highest valuation group member in the group. There are *perverse* equilibria in which a player with value less than the highest value player in a group expends strictly positive effort and the highest value player in that group free-rides by expending zero effort. We rank the possible set of equilibria, show the conditions under which the perverse equilibrium can be avoided and discuss the possible contest design issues.

2. The Model

Consider a contest in which $n \geq 2$ groups compete to win a group-specific public-good prize. Group $g \in N$, where $N = \{1, 2, \dots, n\}$ is a set of groups, consists of $m_g \geq 2$ risk-neutral players who expend costly effort to win the prize.

The individual group members' valuation for the prize may differ within group and across groups. This intra-group asymmetry in values can be a result of player asymmetry, but it can also be interpreted as an exogenous sharing rule of the group-specific prize, in which the

prize-shares among the group members of a group are different. Let $v_{gi} > 0$ represent the valuation for the prize of player i in group g . Without loss of generality assume $v_{g(t-1)} \geq v_{gt}$ for $m_g \geq t > 1$, and $v_{(k-1)1} \geq v_{k1}$ for $n \geq k > 1$. Let $x_{gi} \geq 0$, measured in the same unit as the prize values, represent the effort level expended by player i in group g .

Next we specify the group impact functions as $f_g: \mathbb{R}_+^{m_g} \rightarrow \mathbb{R}_+$. Therefore, the group effort of group g is given by $X_g = f_g(x_{g1}, x_{g2}, \dots, x_{gm_g})$. The following assumption defines the best-shot technology:

Assumption 1. The group effort of group g is represented by the maximum effort level expended by the players in group g , i.e. $X_g = \max\{x_{g1}, x_{g2}, \dots, x_{gm_g}\}$.

To specify the winning probability of group g , denote $p_g(X_1, X_2, \dots, X_n): \mathbb{R}_+^n \rightarrow [0, 1]$ as the contest success function. *Assumption 2* specifies the regularity conditions for p_g .

Assumption 2. $p_g(0, 0, \dots, 0) = 1/n$, $\sum_g p_g = 1$, $\frac{\partial p_g}{\partial X_g} \geq 0$, $\frac{\partial^2 p_g}{\partial X_g^2} \leq 0$, $\frac{\partial p_g}{\partial X_k} \leq 0$, $\frac{\partial^2 p_g}{\partial X_k^2} \geq 0$

where $k, g \in N$ and $k \neq g$. Furthermore, $\frac{\partial p_g}{\partial X_g} > 0$, $\frac{\partial^2 p_g}{\partial X_g^2} < 0$ for some $X_k > 0$, and

$\frac{\partial p_g}{\partial X_k} < 0$, $\frac{\partial^2 p_g}{\partial X_k^2} > 0$ for $X_g > 0$.

We assume all players forgo their efforts and they have a common cost function as described by *Assumption 3*.

Assumption 3. $c: \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is the common cost function of effort with the following properties: $c(0) = 0$, $\frac{\partial c(x_{gi})}{\partial x_{gi}} > 0$, $\frac{\partial^2 c(x_{gi})}{\partial x_{gi}^2} \geq 0$.

Only the members of the winning group receive the prize, while all other players receive nothing. Let π_{gi} represent the payoff for player i in group g . Then, under the above assumptions, the payoff for player i in group g is

$$\pi_{gi} = v_{gi} p_g(X_1, X_2, \dots, X_n) - c(x_{gi}). \quad (1)$$

Equation (1) along with the three assumptions represents the best-shot group contest. To close the structure we assume that all the players in the contest choose their effort levels independently and simultaneously, and that all of the above (including the valuations, group compositions, impact functions, and the contest success function) is common knowledge among the players. We employ Nash equilibrium as our solution concept.

3. The Equilibria of the Game

We use the following definitions throughout our analysis.

Definition 1. If player i in group g exerts strictly positive effort, i.e., $x_{gi} > 0$, then the player is called *active*. Otherwise (when $x_{gi} = 0$) the player is called *inactive*.

Definition 2. If at least one player in group g exerts strictly positive effort, i.e., $X_g > 0$, then group g is called *active*. Otherwise (when $X_g = 0$) then the group is called *inactive*.

Now we state *Lemma 1*.

Lemma 1. *In any equilibrium at least two groups are active.*

Proof: Suppose there exists an equilibrium in which all the groups are inactive. In such a case, from equation (1), the payoff of player i in group g is v_{gi}/n . Now, suppose the player exerts an infinitesimal effort $\varepsilon > 0$ instead of being inactive. Then the payoff becomes $v_{gi} - c(\varepsilon) > v_{gi}/n$. Hence, all groups being inactive can never be an equilibrium. Now, suppose there exists an equilibrium in which only group k is active. In such a case, from equation (1), the payoff of player i in group g is 0. Now suppose the player exerts an effort $x_{gi}^* > 0$ instead of being inactive, where x_{gi}^* is the best response of player i in group g against X_k in the standard two-player individual contest (Tullock, 1980). Consequently, the payoff becomes $p_g v_{gi} - x_{gi}^* > 0$. Hence, only one groups being active cannot be an equilibrium either. Hence in any equilibrium there are at least two active groups. ■

Assumption 1 gives rise to *Lemma 2*.

Lemma 2. *In an equilibrium only one player in each group, if any, will be active.*

Proof: Suppose in an equilibrium more than one player, say players i and j , in group g exert strictly positive efforts with $x_{gi} \geq x_{gj} > 0$. Hence the payoff of player j , under *Assumption 1*, is $\pi'_{gj} = v_{gj} p_g(X_1, \dots, X_{g-1}, x_{gi}, X_{g+1}, \dots, X_n) - c(x_{gj})$. In such a case it is always beneficial for player j to reduce effort to zero and increase payoff to: $\pi''_{gj} = v_{gj} p_g(X_1, \dots, X_{g-1}, x_{gi}, X_{g+1}, \dots, X_n) > \pi'_{gj}$. Hence more than one player in the same group expending strictly positive effort can never be an equilibrium. ■

This result is isomorphic to the best-shot all-pay auction results (Topolyan, 2011) in which only one player in each group exerts zero effort with probability less than 1. Let $x_{gi}^b \in \mathbb{R}_+$ denote the best-response of player i in group g in a situation in which player i in group g is a unique player in the group. *Lemma 3* follows.

Lemma 3. *Given the effort levels of other groups, $x_{g(t-1)}^b \geq x_{gt}^b$ for $m_g \geq t > 1$.*

Proof: The best-response of a player is the non-negative effort level that maximizes the payoff of the player (subject to the participation constraint of non-negative equilibrium payoff), given the effort level of all other players. Hence, $x_{gi}^b = \text{Max} \{ \arg \max_{x_{gi}} \pi_{gi}^b, 0 \}$ subject to $\pi_{gi}^b \geq 0$, where $\pi_{gi}^b = v_{gi} p_g(X_1, \dots, X_{g-1}, x_{gi}, X_{g+1}, \dots, X_n) - c(x_{gi})$. An interior solution that maximizes π_{gi}^b , if exists, will be unique as the payoff function is strictly concave. Hence, from the first-order condition of maximizing π_{gi}^b , we obtain the best response function as the solution of $c'(x_{gi}) / \frac{\partial p_g}{\partial x_{gi}} \geq v_{gi}$. The properties of the contest success function and cost function from *Assumption 2* and *Assumption 3* ensure that given $X_{-g} = (X_1, \dots, X_{g-1}, X_{g+1}, \dots, X_n)$, x_{gi}^b is monotonically increasing in v_{gi} . This observation along with the assumption $v_{g(t-1)} \geq v_{gt}$ for $m_g \geq t > 1$ confirms that $x_{g(t-1)}^b \geq x_{gt}^b$ for all $X_{-g} \geq (0, \dots, 0)$. ■

Lemma 1 and *Lemma 2* transform the problem into a generalized version of asymmetric value individual contest, whereas *Lemma 3* provides restrictions on the participations of the players. A combination of the three lemmas gives *Corollary 1* that we state without proof.

Corollary 1. *An equilibrium in which player 1s in group 1 and 2 are active always exists.*

It follows from *Lemma 3* that $x_{g1}^b > 0$ for some $g \leq k \leq n$ and $x_{g1}^b = 0$ for $g > k$, i.e., given the value distribution, there can be instances in which the best response for every player in a group is to put forth zero effort, as each of their valuation is low enough and expending any positive amount of effort will result in a negative payoff.

The general nature of the current set up restricts us from finding closed form solution for participation and equilibrium effort. To make the problem tractable and attain closed form solutions we make the following restrictive assumptions. First, following the axiomatic foundation of Münster (2009) we apply a logit (Tullock, 1980) form group contest success function. Second, we use linear cost function with unit marginal cost.

Assumption 2'. The probability of winning the prize for group g is

$$p_g(X_1, X_2, \dots, X_n) = \begin{cases} \frac{X_g}{\sum_{k=1}^n X_k} & \text{if } \sum_{k=1}^n X_k > 0 \\ \frac{1}{n} & \text{if } \sum_{k=1}^n X_k = 0 \end{cases}.$$

Assumption 3'. The common cost function is $c(x_{gi}) = x_{gi}$.

Lemma 1, *2* and *3* under these two assumptions convert the best-shot group contest into a generalized asymmetric individual lottery contest in which each group behaves like an

individual contestant, but the valuation of the individual contestant may change depending on which group member within a group is active. We use the results on asymmetric individual contest by Stein (2002). Stein (2002) shows that in an n -player asymmetric contest some players may not be active in an equilibrium. Using *Proposition 1* of Stein (2002) we state the condition in *Lemma 4* for active participation in the contest.

Lemma 4. *Suppose players E_j 's in each group $j (< k \leq n)$ are active in an equilibrium. Then for at least one other player from any other group to be active along with the active players in that equilibrium, the following condition has to be satisfied: $v_{k1} > \frac{(k-2) \prod_{j < k} v_{jE_j}}{\sum_{j < k} \prod_{t \neq j, t < k} v_{tE_t}}$.*

Proof: From *Proposition 1* of Stein (2002) we can observe that when players E_j 's in each group $j (< k < n)$ are active, then in an equilibrium player 1 of group k will be able to earn strictly positive payoff only if the condition $v_{k1} > \frac{(k-2) \prod_{j < k} v_{jE_j}}{\sum_{j < k} \prod_{t \neq j, t < k} v_{tE_t}}$ is satisfied. Moreover, by assumption, $v_{g(t-1)} \geq v_{gt}$ for $m_g \geq t > 1$ and $v_{(k-1)1} \geq v_{k1}$ for $n \geq k > 1$ hold. Hence, if player 1 of group k cannot earn positive payoff, it is impossible for any other players belonging to groups k and above to expend strictly positive effort and earn positive payoff. Consequently, for at least one other player from any other group to be active in addition to players E_j 's in each group j , the condition $v_{k1} > \frac{(k-2) \prod_{j < k} v_{jE_j}}{\sum_{j < k} \prod_{t \neq j, t < k} v_{tE_t}}$ has to be satisfied. ■

Lemma 4 gives us a set of sufficient conditions to exclude one or more group to participate in the best-shot contest. This is important from the contest design point of view as the designer can manipulate the number of active teams and the corresponding rent dissipation from these restrictions. This lemma also gives *Corollary 2* that shows the needed parametric restriction which, in turn, ensures participation of all the groups in an equilibrium.

Corollary 2. *If $v_{n1} > \frac{(n-2) \prod_{j < n} v_{j1}}{\sum_{j < n} \prod_{t \neq j, t < n} v_{t1}}$ then, in an equilibrium, all n groups are active.*

Proof: From *Lemma 4*, condition $v_{n1} > \frac{(n-2) \prod_{j < n} v_{j1}}{\sum_{j < n} \prod_{t \neq j, t < n} v_{t1}}$ ensures that at least one player (player 1) in group n is always active in an equilibrium when player 1's in all the other groups are active. This, along with the assumption $v_{g(t-1)} \geq v_{gt}$ for $m_g \geq t > 1$, means that at least one player (player 1) in group n is always active in an equilibrium when players other than player 1's from all the other groups are active. Also, if it is participation compatible for player 1 in n , then assumption $v_{(k-1)1} \geq v_{k1}$ for $n \geq k > 1$ and the

properties of harmonic mean imply that it is participation compatible for player 1's in all the other groups. ■

In the following, we assume that at least $n_1 (< n)$ groups can be active in an equilibrium, i.e., $v_{n_1 1} > \frac{(n_1-2) \prod_{j < n_1} v_{jE_j}}{\sum_{j < n_1} \prod_{t \neq j, t < n_1} v_{tE_t}}$. From *Lemma 1, 2, 3 and 4*, one can expect that, under certain restrictions there exists an equilibrium in which player 1 (the highest-valuation player) in each active group exerts strictly positive efforts and the other group members free-ride by exerting zero effort. In such a case each highest-valuation active player exerts its own equilibrium effort in an n_1 -player individual contest with asymmetric values as in Stein (2002). It is easy to show that this constitutes an equilibrium as no player, expending strictly positive or zero effort, has an incentive to deviate from the effort level. This is summarized in *Proposition 1*.

Proposition 1. Suppose $v_{n_1 1} > \frac{(n_1-2) \prod_{j < n_1} v_{j1}}{\sum_{j < n_1} \prod_{t \neq j, t < n_1} v_{t1}}$ and $v_{(n_1+1)1} < \frac{((n_1+1)-2) \prod_{j < (n_1+1)} v_{jm_j}}{\sum_{j < (n_1+1)} \prod_{t \neq j, t < (n_1+1)} v_{tm_t}}$, i.e., only first n_1 groups are active. Define $x_{g1}^* = \frac{n_1-1}{\sum_{k=1}^{n_1} v_{k1}^{-1}} \left(1 - (n_1-1) \frac{v_{g1}^{-1}}{\sum_{k=1}^{n_1} v_{k1}^{-1}} \right)$. Then the profile $\left((x_{11}^*, 0, \dots, 0), (x_{21}^*, 0, \dots, 0), \dots, (x_{n_1 1}^*, 0, \dots, 0), (0, \dots, 0), \dots, (0, \dots, 0) \right)$ is a Nash equilibrium of the best-shot group contest.

Proof: First, note that $x_{g1}^* = \frac{n_1-1}{\sum_{k=1}^{n_1} v_{k1}^{-1}} \left(1 - (n_1-1) \frac{v_{g1}^{-1}}{\sum_{k=1}^{n_1} v_{k1}^{-1}} \right)$ is the equilibrium effort of player 1 in group g when the original contest is reduced to an n_1 -player contest consisting of the highest-valuation players in groups $1, 2, \dots, n_1$. From *Lemma 4*, the conditions $v_{n_1 1} > \frac{(n_1-2) \prod_{j < n_1} v_{j1}}{\sum_{j < n_1} \prod_{t \neq j, t < n_1} v_{t1}}$ and $v_{(n_1+1)1} < \frac{((n_1+1)-2) \prod_{j < (n_1+1)} v_{jm_j}}{\sum_{j < (n_1+1)} \prod_{t \neq j, t < (n_1+1)} v_{tm_t}}$ restrict only the first n_1 groups to be active in equilibrium. Hence, the following profile $\left((x_{11}^*, 0, \dots, 0), (x_{21}^*, 0, \dots, 0), \dots, (x_{n_1 1}^*, 0, \dots, 0), (0, \dots, 0), \dots, (0, \dots, 0) \right)$ constitutes an equilibrium. ■

The implication of *Proposition 1* is crucial. It means that in this equilibrium only one of the players with the highest stake, or the most efficient player, exerts effort on behalf of the whole group that is active. Consequently, the contest becomes equivalent to a group contest with perfect-substitutes impact function as in Baik (1993). The equilibrium strategies are also similar to the unique equilibrium strategies in best-shot public good games (Hirshleifer, 1983).

This further indicates that in a market coalition such as an RJV or a coalitional situation related to defense, the most efficient player is not better off being a member of the coalition

instead of contesting as an individual. As a result this equilibrium cannot necessarily justify the coalitions under best-shot technology. It can only be possible for the most efficient players to join coalition if there exist other equilibria in which they can free-ride on some other players' efforts. Or, in other words, in those equilibria the 'top guns' do not fire. In *Proposition 2* we show that indeed those equilibria exist, justifying the existence of market, political and international coalitions under best-shot technology.

Proposition 2. Suppose $v_{n_1 1} > \frac{(n_1-2) \prod_{j < n_1} v_{j1}}{\sum_{j < n_1} \prod_{t \neq j, t < n_1} v_{t1}}$ and $v_{(n_1+1)1} < \frac{((n_1+1)-2) \prod_{j < (n_1+1)} v_{jm_j}}{\sum_{j < (n_1+1)} \prod_{t \neq j, t < (n_1+1)} v_{tm_t}}$.

Define $x_{gE_g}^* = \frac{n_1-1}{\sum_{k=1}^{n_1} v_{kE_g}^{-1}} \left(1 - (n_1-1) \frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_g}^{-1}} \right)$ where $E_g > 1 \forall g \in N_1 = \{1, 2, \dots, n_1\}$.

Then the profile $\left((0, \dots, 0, x_{1E_1}^*, 0, \dots, 0), \dots, (0, \dots, 0, x_{n_1E_{n_1}}^*, 0, \dots, 0), \dots, (0, \dots, 0) \right)$ constitutes a Nash equilibrium of the best-shot group contest if

$$\frac{v_{g1}}{v_{gE_g}} \leq \left(1 + \sqrt{1 - (n_1-1) \frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_g}^{-1}}} \right)^2 \quad \forall g \in N_1.$$

Proof: First notice that $x_{gE_g}^* = \frac{n_1-1}{\sum_{k=1}^{n_1} v_{kE_g}^{-1}} \left(1 - (n_1-1) \frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_g}^{-1}} \right)$ is the equilibrium effort of player $E_g > 1$ in group g when the original contest is reduced to the n_1 -player contest consisting of the E_g th highest-valuation players in each group $g \in N_1$. Hence in the profile $\left((0, \dots, 0, x_{1E_1}^*, 0, \dots, 0), \dots, (0, \dots, 0, x_{n_1E_{n_1}}^*, 0, \dots, 0), \dots, (0, \dots, 0) \right)$ player E_g in group g expends strictly positive reduced-form-individual-contest effort and the others in the group expend zero effort.

For this profile to be a Nash equilibrium, no player in the contest should have an incentive to deviate from this profile. It is straightforward to show that any player $i = E_g + 1, E_g + 2, \dots, m_g$ in group g does not have an incentive to deviate according to *Lemma 3*.

Similarly, it is straightforward to show that player 1 in group g has the highest incentive, if any, to deviate from this profile because $x_{g1}^b(X_{-g}) \geq x_{g2}^b(X_{-g}) \geq \dots \geq x_{gE_g}^b(X_{-g})$. This means that if player 1 in group g does not have an incentive to deviate from this profile, then player 2, 3, ..., $(E_g - 1)$ in group g do not have such incentives, either. Therefore, in order to determine if this profile is an equilibrium or not, it is enough to check if player 1 in each group g has any incentive to deviate from this profile or not.

Now we pin down the conditions under which player 1 in group g does not deviate from the profile above. Under the profile, player 1 in group g exerts zero effort, and the winning probability of group g is $p_g = \left(1 - (n_1 - 1) \frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\right)$. Hence, the payoff of player 1 in group g is $\pi_{g1} = v_{g1} \left(1 - (n_1 - 1) \frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}\right)$.

If player 1 in group g deviates from the profile, his optimal effort level is:

$$\begin{aligned} x_{g1}^b(X_{-g}) &= \sqrt{v_{g1} \sum_{k \neq g}^{n_1} X_k} - \sum_{k \neq g}^{n_1} X_k = \sqrt{v_{g1} \sum_{k \neq g}^{n_1} x_{kE_k}^*} - \sum_{k \neq g}^{n_1} x_{kE_k}^* \\ &= \frac{n_1 - 1}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}} \left(\sqrt{\frac{v_{g1}}{v_{gE_g}}} - (n_1 - 1) \frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}} \right). \end{aligned}$$

However, he earns the following expected payoff by deviating from the profile:

$$\pi_{g1}^d = v_{g1} \left(\sqrt{\frac{v_{g1}}{v_{gE_g}}} - (n_1 - 1) \frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}} \right) \left(\sqrt{\frac{v_{gE_g}}{v_{g1}}} - (n_1 - 1) \frac{v_{g1}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}} \right).$$

Then, for player 1 in group g not to deviate from the profile, we need $\pi_{g1} \geq \pi_{g1}^d$, i.e.,

$$\pi_{g1} - \pi_{g1}^d = -\frac{n_1 - 1}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}} \left(\frac{v_{g1}}{v_{gE_g}} - 2 \sqrt{\frac{v_{g1}}{v_{gE_g}}} + (n_1 - 1) \frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}} \right) \geq 0.$$

Letting $\frac{v_{g1}}{v_{gE_g}} \equiv \alpha_g (\geq 1)$, the inequality can be rewritten as following:

$$\begin{aligned} \pi_{g1} - \pi_{g1}^d &= -\frac{n_1 - 1}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}} \times \\ &\left(\sqrt{\alpha_g} - 1 - \sqrt{1 - (n_1 - 1) \frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}} \right) \left(\sqrt{\alpha_g} - 1 + \sqrt{1 - (n_1 - 1) \frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}} \right) \leq 0. \end{aligned}$$

As a result, the profile

$$\left((0, \dots, 0, x_{1E_1}^*, 0, \dots, 0), \dots, (0, \dots, 0, x_{n_1 E_{n_1}}^*, 0, \dots, 0), \dots, (0, \dots, 0) \right)$$

is a Nash equilibrium if, $\forall g \in N_1$, $\alpha_g \leq \left(1 + \sqrt{1 - (n_1 - 1) \frac{v_{gE_g}^{-1}}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}}} \right)^2$. ■

In conclusion, this proposition fully characterizes the set of equilibria for the best-shot group contest and *Proposition 1* is a special case of *Proposition 2* when $E_g = 1 \forall g \in N_1$. There are several important implications of *Proposition 2*. First, this proposition paves way for the possibilities of coalitions in which the most efficient player, as a group member, can

earn higher payoff rather than contesting as an individual. Thus the inclusion of the most efficient players in a coalition is justified. Second, the final payoff crucially depends on the equilibrium selection and the coordination between group members.

The rent dissipation results are also very different in the current analysis compared to the group contests with other contest success functions or the best-shot public good game. We know that in case of perfect-substitutes impact functions, the total rent dissipation is uniquely determined (Baik, 1993) and a coalition proof equilibrium is unique in case of a weakest-link impact function (Lee, 2009). It is not trivial to fully rank the equilibria in the best-shot case in terms of rent dissipation. The following corollary points out the highest and the lowest possible rent dissipation in the best-shot group contest.

Corollary 3. *The highest (lowest) possible equilibrium rent is dissipated for a given number of active groups, when only the highest (lowest) value players in each active group exert strictly positive effort. The rent dissipation is intermediate otherwise.*

Proof. Suppose n_1 groups are active. From *Proposition 2*, the equilibrium rent dissipation

$$\text{is: } \sum_{g=1}^{n_1} x_{gE_g}^* = \sum_{g=1}^{n_1} \left(\frac{n_1-1}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}} - \left[\frac{n_1-1}{\sum_{k=1}^{n_1} v_{kE_k}^{-1}} \right]^2 (v_{gE_g}^{-1}) \right) = (n_1-1) \frac{\prod_{k=1}^{n_1} v_{kE_k}}{\sum_{l=1}^{n_1} \prod_{k \neq l} v_{kE_k}}.$$

Differentiating with respect to the value of an arbitrary active player t we get: $\frac{\partial \sum_{g=1}^{n_1} x_{gE_g}^*}{\partial v_{tE_t}} =$

$$\frac{(n_1-1)}{\left(\sum_{l=1}^{n_1} \prod_{k \neq l} v_{kE_k}\right)^2} \left[\left(\sum_{l=1}^{n_1} \prod_{k \neq l} v_{kE_k}\right) \prod_{k \neq t} v_{kE_k} - \left(\prod_{k=1}^{n_1} v_{kE_k}\right) \sum_{l=1}^{n_1} \prod_{k \neq l, t} v_{kE_k} \right] =$$

$$\frac{(n_1-1) \prod_{k \neq t} v_{kE_k}}{\left(\sum_{l=1}^{n_1} \prod_{k \neq l} v_{kE_k}\right)^2} \left[\left(\sum_{l=1}^{n_1} \prod_{k \neq l} v_{kE_k}\right) - v_{tE_t} \sum_{l=1}^{n_1} \prod_{k \neq l, t} v_{kE_k} \right] = \frac{(n_1-1) \left(\prod_{k \neq t} v_{kE_k}\right)^2}{\left(\sum_{l=1}^{n_1} \prod_{k \neq l} v_{kE_k}\right)^2} > 0, \text{ i.e., the}$$

total rent dissipation is monotonically increasing in the values of the active players. Hence, the equilibrium in which only the highest valuation player in each team exerts positive effort results in the highest rent dissipation among the set of equilibria of the best-shot group contest. Following the same logic, the equilibrium in which only the lowest valuation player in each team exerts positive effort, dissipates the lowest rent and the intermediate cases will result in intermediate rent dissipation and the ranking will depend on the value distribution within and among groups. ■

The best-shot public good structure is included in the discussion of system reliability, or defense coalition. The best-shot public good game with a unique highest valuation player in each group has a unique equilibrium same as the one derived in *Proposition 1*. Moreover, unique equilibrium ensures no equilibrium selection process for a designer. *Proposition 2*,

however, shows that if one models the aforementioned situations in a more realistic contest setting, then the equilibrium results can change. The comparative statics results will also be affected. Moreover, an inclusion of a player of any valuation (even with lower than the highest value) in any group can change the possible set of equilibria, rent dissipation and payoffs. A contest designer would need to find ways for equilibrium selection.²

4. Three (Two-Player) Groups Example

To portray a diagrammatic and easily tractable explanation of the general results in the previous section we consider a 3×2 best-shot group contest where there are three groups, and each group consists of two players each. Here we first show simple conditions for which a group does not participate in the contest and the contest reduces to a 2×2 contest. Then we characterize the set of equilibria for this reduced form contest.

As shown in *Lemma 1* and 2, in any equilibrium at least two groups will be active and only one player in each group will be active. One can show that there can be 20 equilibria in which one player in each group is active and at least two groups are active. Now we impose condition $v_{31} < \frac{v_{12}v_{22}}{v_{12}+v_{22}}$. From *Lemma 4*, this means that both members of group 3 have relatively low valuations of the prize and participating in the contest ensures loss to them. Hence, both of them expend zero effort and group 3 is always inactive. As a result the set of equilibria reduces to only 4. Let us denote the equilibrium in which the player i from group 1 and the player j from group 2 are active as N_{ij} . Then the four possible equilibria are N_{11} , N_{12} , N_{21} , and N_{22} . Define $\alpha_1 \equiv \frac{v_{11}}{v_{12}} (\geq 1)$ and $\alpha_2 \equiv \frac{v_{21}}{v_{22}} (\geq 1)$, the results are summarized in Table 1 and are shown diagrammatically in Figure 1.

[Table 1 about here]

As stated in *Corollary 3*, the rent dissipation is highest for N_{11} , lowest for N_{22} and intermediate for the other two equilibria. This gives the flexibility to a contest designer not only to select equilibrium in terms of deciding upon the active players, but also on the possible rent dissipation in the equilibrium, if he can distribute the value among the players. Once we know the ranking of the equilibria in terms of rent dissipation, we can describe the equilibrium condition in terms of the values. Depending on value distribution one may obtain

² Researchers are often interested in equilibrium selection in best-shot public good games under network settings (see Galeotti et al., 2010; Dall'Asta et al., 2011). Our findings also reinforce the need for further research on equilibrium selection in contest games with best-shot impact functions.

one of several equilibria. Whereas N_{11} always exists, one may obtain a combination of other equilibria for different value distribution. Figure 2 summarizes the required equilibrium conditions for N_{11} , N_{12} , N_{21} , and N_{22} equilibrium in $\alpha_1 - \alpha_2$ graph.

[Figure 1 about here]

[Figure 2 about here]

5. Discussion

In this article we construct and analyze a best-shot group contest. We find that depending on the value distribution there can be multiple equilibria, but in each equilibrium only one player from each group expends strictly positive effort. This result is robust to the number of groups, the number of players in each group, and the valuation of the players. However, we also show that the equilibrium strategy and rent dissipation are not easily determined and not robust.

This research has implications in coalition formation under best-shot structure. It also enables a designer to design strategy according to the designer objective. In our analysis we pin down the conditions on value distribution that can give rise to different equilibria, show needed conditions to determine the number of active groups and manipulate strategies to select equilibria. Hence, if the value distribution is portrayed as the within-group prize sharing rule, then a designer is able to choose the rule in a way such that the most efficient players do not exert zero effort. Finally, a designer can also achieve higher or lower rent dissipation by imposing appropriate prize sharing rules for the groups.

In the context of public goods and defense economics literature (Hirshleifer 1983, 1985; Bliss and Nelbuff, 1984; Harrison and Hirshleifer, 1989; Cornes, 1993) it has been well recognized that using best-shot technology leads to equilibria in which the most able players contribute to the public good. The current study shows that, if one uses public good scenario, then the corresponding result is only a special case of the best-shot group contest. In general, some groups might not participate and most efficient players may also be inactive in some equilibria if we model the public good scenario as a best-shot contest with group-specific public good prizes.

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Tables and Figures

Table 1: Equilibrium Effort and Corresponding Condition (Two Active Groups)

Equilibrium	x_{11}	x_{12}	x_{21}	x_{22}	Equilibrium Condition
N_{11}	$\frac{v_{11}^2 v_{21}}{(v_{11} + v_{21})^2}$	0	$\frac{v_{11} v_{21}^2}{(v_{11} + v_{21})^2}$	0	No condition required
N_{12}	$\frac{v_{11}^2 v_{22}}{(v_{11} + v_{22})^2}$	0	0	$\frac{v_{11} v_{22}^2}{(v_{11} + v_{22})^2}$	$\alpha_2 \leq \left(1 + \sqrt{\frac{v_{22}}{\alpha_1 v_{12} + v_{22}}}\right)^2$
N_{21}	0	$\frac{v_{12}^2 v_{21}}{(v_{12} + v_{21})^2}$	$\frac{v_{12} v_{21}^2}{(v_{12} + v_{21})^2}$	0	$\alpha_1 \leq \left(1 + \sqrt{\frac{v_{12}}{v_{12} + \alpha_2 v_{22}}}\right)^2$
N_{22}	0	$\frac{v_{12}^2 v_{22}}{(v_{12} + v_{22})^2}$	0	$\frac{v_{12} v_{22}^2}{(v_{12} + v_{22})^2}$	$\alpha_1 \leq \left(1 + \sqrt{\frac{v_{12}}{v_{12} + v_{22}}}\right)^2,$ $\alpha_2 \leq \left(1 + \sqrt{\frac{v_{22}}{v_{12} + v_{22}}}\right)^2$

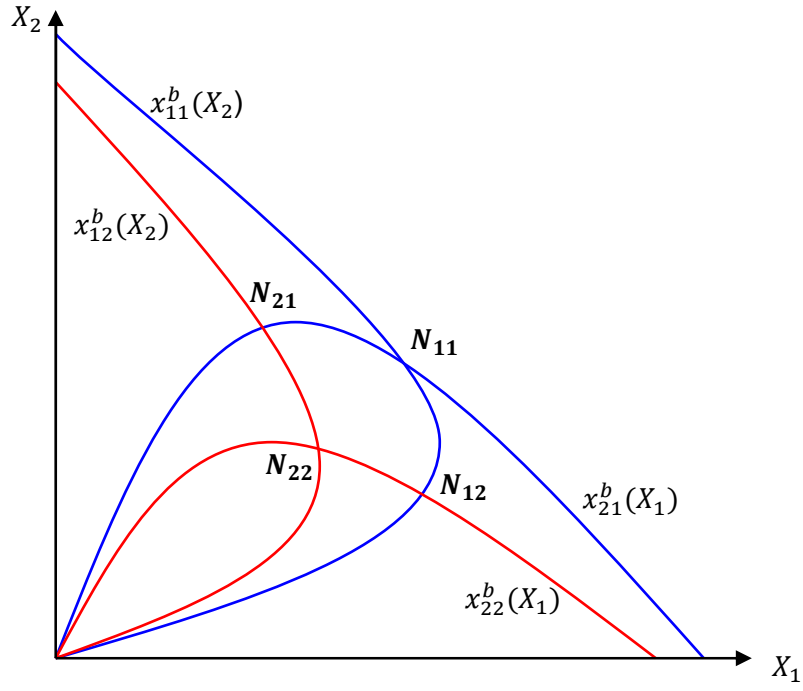


Figure 1: The Equilibria

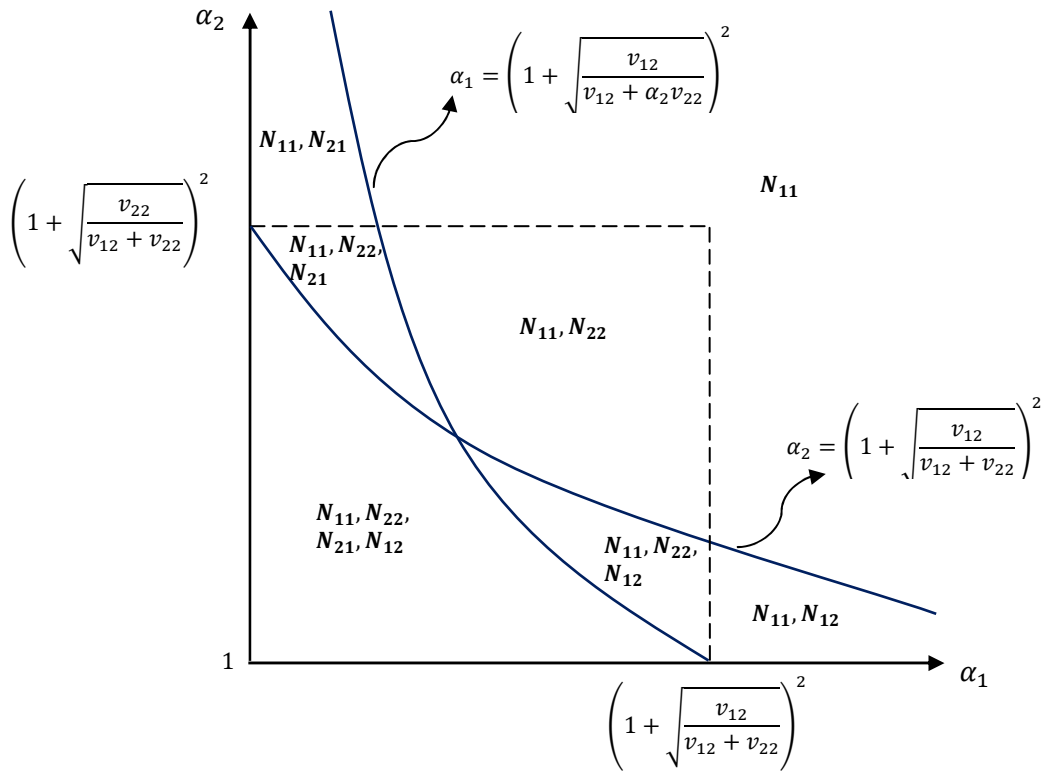


Figure 2: The Equilibria Ranges and Conditions