# Hybrid PE-ML Method for nonlinear WSI


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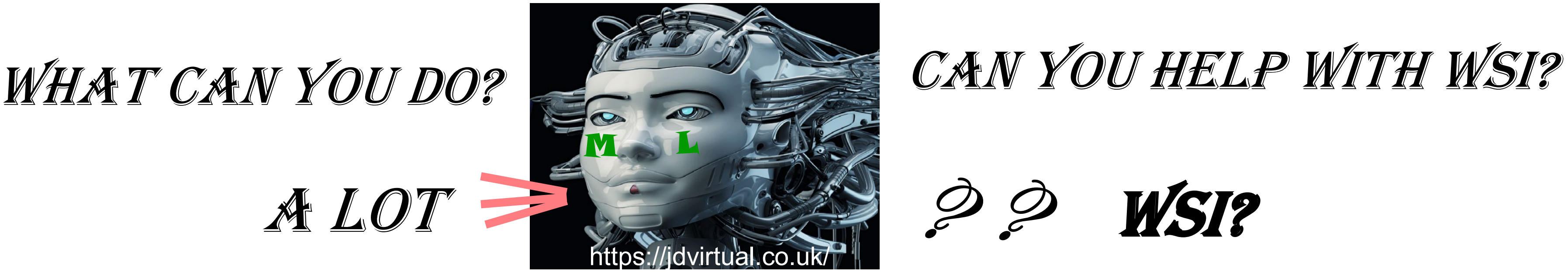


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# Outline

- Numerical models for ocean/offshore engineering
- Machine learning (ML) models
- ML applications to WSI
- Hybrid PE-ML for WSI
- Case study on more complex cases using GNN trained on data of simple cases
- Summary
- Acknowledgements

# Overview of Numerical Modelling

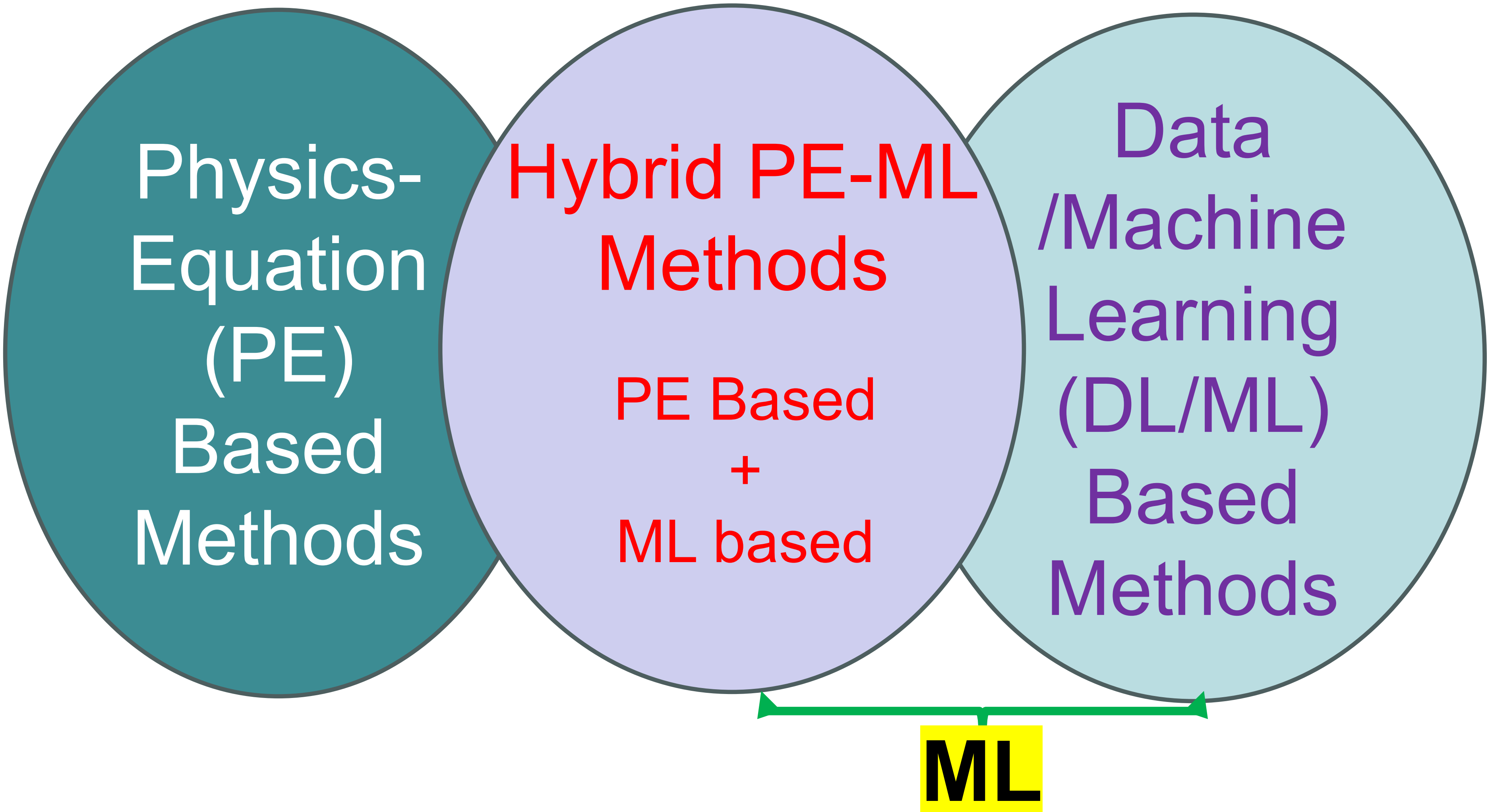
## Need of numerical modelling in ocean/offshore engineering

- Predicting wind/wave/current fields
- Fluid structure interaction
  e.g wind turbine problems; vortex induced vibration
- Wave structure interaction (**WSI**)
  e.g, ships in waves; floating body in waves

***Almost all projects in ocean/offshore need considering WSI***

# Overview of Numerical Methods

Classes of numerical models used in ocean/offshore engineering



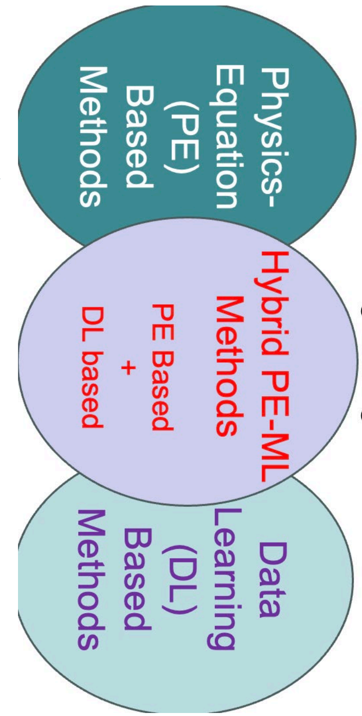
# Overview of Numerical Modelling

## Track records of our team in developing numerical models for ocean/offshore engineering

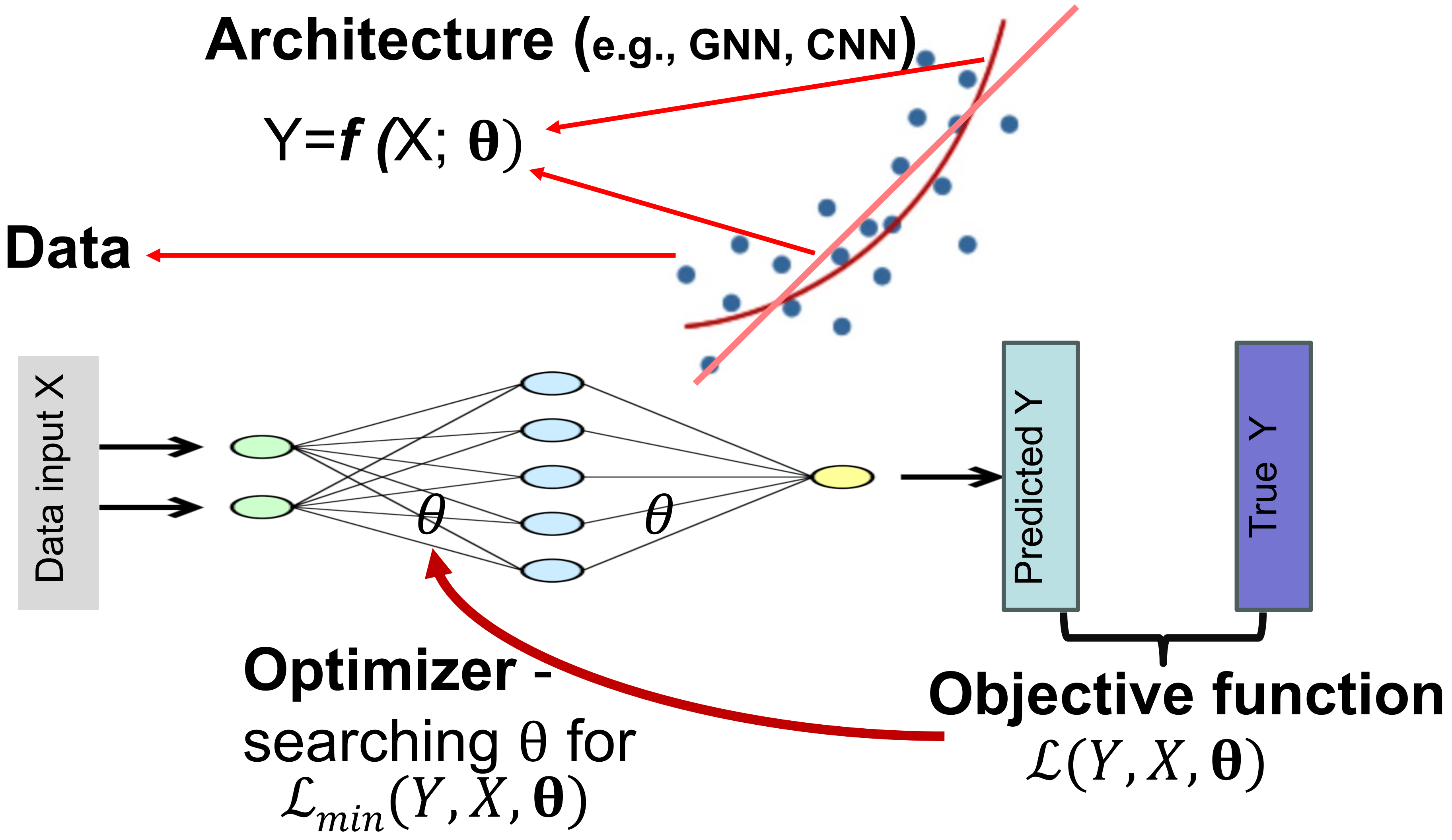
**QALE-FEM**: Fully nonlinear potential method

**qale-FOAM (Hybrid methods)**: combining fully nonlinear potential model with NS model (OpenFOAM)

**Numerical models with embedded ML**: embedding ML in the procedure of conventional numerical modelling to replace a part of numerical module.



# Overview of ML: Main Components



# Overview of ML: Types of ML

## In terms of learning tasks

**Supervised**
Regression

**Unsupervised**
features identification

**Semisupervised**
generative or reinforced

## In terms of the ways ML incorporated

**Fully data-driven (Naïve) ML**
No physics emended

**Physics informed ML**
Embedding physics equations in objective function and/or features

**Hybrid PE-ML**
Embedding ML into the traditional procedure of solving physics equations

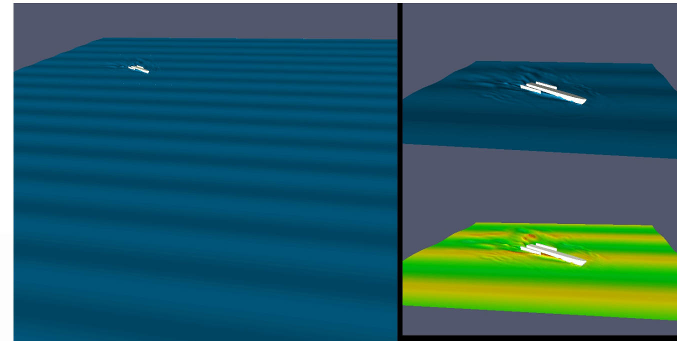
ML Based Models

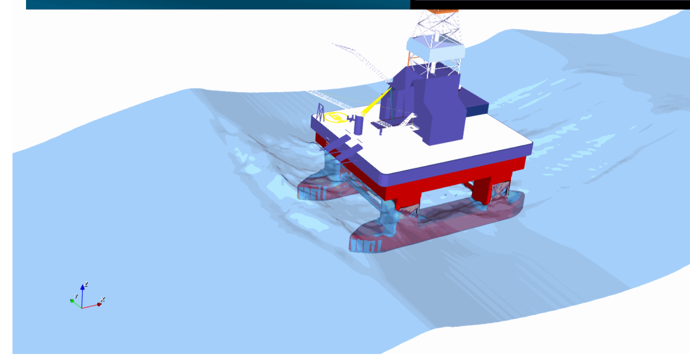
Hybrid PE-ML

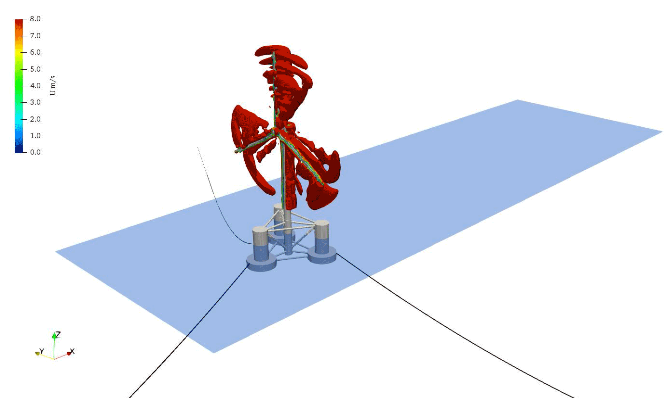
# Overview of ML Applications to WSI: Problems to Be Solved

**Problems to be solved**: Interaction between waves and rigid bodies

- Wave dynamics (single phase)
- Aerodynamics
- Wave loads (hydrodynamics)
- Interface between air and water
- Floating body dynamics (rigid)
- Mooring line dynamics (springs here)
- Conditions on fixed and moving rigid boundaries







# Overview of ML Applications to WSI

| *Physical problems* | *Data-driven* | *Physics-informed* | *Hybrid PE-ML* |
|---|---|---|---|
| Wave parameters | Liu et al (2023) predict short-term wave surface. Many others | Wang et al (2022) simulates wave energy fields | Chang et al (2011) and others predict on the error of numerical models from measured data |
| Parameters in WSI | Tomasz (2020) used ML to predict the added resistance. Guth et al (2024) predicts wave load. | Lang et al (2024) predicts ship speed or engine power. | Not found |
| Responses of structures | ML predicts responses of floating body (Jiang, et al, 2024) | Mentzelopoulos et al (2023) used ML predicts the riser motion. Halder et al (2023) predict the floating box motions | Lee et all (2023) and Eskilsson et al (2023) use ML to predict the nonlinear effects of floating body motions |
| Loading, response and flow fields | Gonzalez et al (2020) and Li et al (2022), predicting flow field of dam breaking, water fall and others | Many for problems without free surfaces | Zhang et al (2023, 2024a.b) uses ML for solving Poisson equation |

# Hybrid PE-ML for WSI: Equations

Governing Equations of fluid:

$$\nabla \cdot \mathbf{u} = 0 \qquad \frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho}\nabla p + \mathbf{g} + \nu \nabla^2 \mathbf{u}$$

Governing Equations of floating body (2D)

$$\boldsymbol{M}\frac{d\mathbf{V}}{dt} = \boldsymbol{F} + \boldsymbol{M}\boldsymbol{g} \qquad \boldsymbol{I}\frac{d\boldsymbol{\Omega}}{dt} = \boldsymbol{T} \qquad \frac{d\mathbf{r}_G}{dt} = \mathbf{V} \qquad \frac{d\boldsymbol{\theta}}{dt} = \boldsymbol{\Omega}$$

Boundary conditions of fluid:

$$\mathbf{n} \cdot \nabla p = \rho\left(\mathbf{n} \cdot \mathbf{g} - \mathbf{n} \cdot \dot{\mathbf{U}}\right)$$

$$\boldsymbol{u_b} = \mathbf{V} + \boldsymbol{\Omega} \times \boldsymbol{R_{b,0}}$$

on moving rigid boundaries

$$\dot{\mathbf{U}} = \frac{d\mathbf{V}}{dt} + \frac{d\boldsymbol{\Omega}}{dt} \times \boldsymbol{R_{b,0}} + \boldsymbol{\Omega} \times \mathrm{V}$$

$$\mathbf{u} \cdot \mathbf{n} = \mathbf{0}$$

on fixed rigid boundaries

$$p = 0 \qquad \tau = 0$$

on free surface

# Hybrid PE-ML for WSI: Numerical Method and Procedure in ISPH

*Incompressible Smoothed Particle Hydrodynamics (**ISPH**) combined with ML*

\- **Hybrid PE-ML**

Fractional step procedure used, needing solve PPE

$$\mathbf{u}^* = \mathbf{u}_t + \Delta\mathbf{u}^*$$
$$\Delta\mathbf{u}^* = (\mathbf{g} + \nu\nabla^2\mathbf{u})\Delta t$$
$$\mathbf{r}^* = \mathbf{r}_t + \mathbf{u}^*\Delta t$$
$$\mathbf{V}^* = \mathbf{V}_t + d\mathbf{V}_t\Delta t$$
$$\boldsymbol{\Omega}^* = \boldsymbol{\Omega}_t + d\boldsymbol{\Omega}_t\Delta t$$
$$\mathbf{u}_b^* = \mathbf{V}^* + \boldsymbol{\Omega}^* \times \mathbf{R}_{b,0}$$

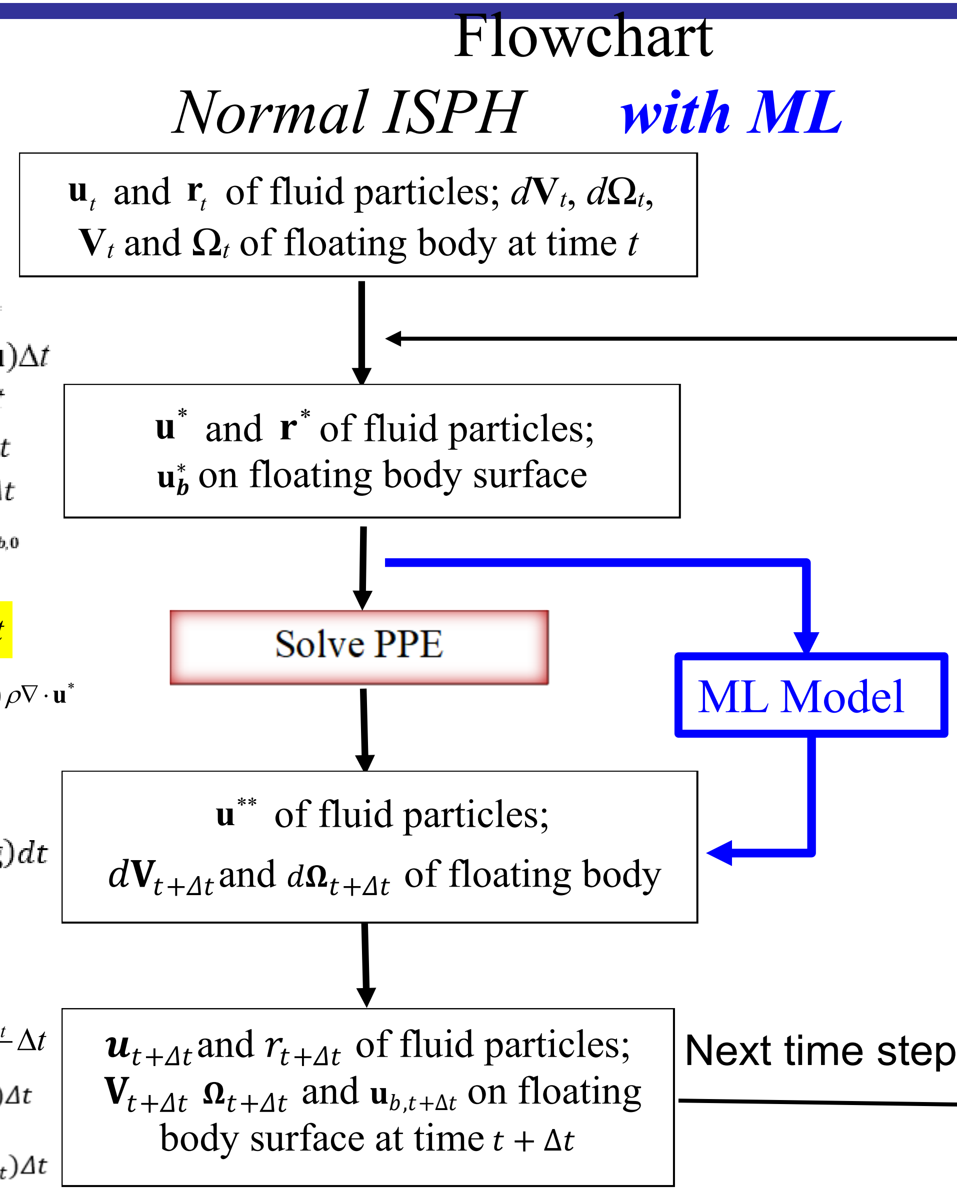
The accuracy and efficiency are largely determined by solving the PPE

$$\nabla^2 p_{t+\Delta t} = \Psi/\Delta t$$
$$\Psi = \alpha\frac{\rho - \rho^*}{\Delta t} + (1-\alpha)\rho\nabla\cdot\mathbf{u}^*$$

$$\mathbf{u}^{**} = -\frac{\Delta t}{\rho}\nabla p_{t+\Delta t}$$
$$d\mathbf{V}_{t+\Delta t} = (M^{-1}\mathbf{F} + \mathbf{g})dt$$
$$d\boldsymbol{\Omega}_{t+\Delta t} = \mathbf{I}^{-1}\mathbf{T}dt$$

PPE is of the most time-consuming

$$\mathbf{u}_{t+\Delta t} = \mathbf{u}^* + \mathbf{u}^{**}$$
$$\mathbf{r}_{t+\Delta t} = \mathbf{r}_t + \frac{\mathbf{u}_t + \mathbf{u}_{t+\Delta t}}{2}\Delta t$$
$$\mathbf{V}_{t+\Delta t} = \mathbf{V}_t + \frac{1}{2}(d\mathbf{V}_t + d\mathbf{V}_{t+\Delta t})\Delta t$$
$$\boldsymbol{\Omega}_{t+\Delta t} = \boldsymbol{\Omega}_t + \frac{1}{2}(d\boldsymbol{\Omega}_t + d\boldsymbol{\Omega}_{t+\Delta t})\Delta t$$
$$\mathbf{u}_{b,t+\Delta t} = \mathbf{V}_{t+\Delta t} + \boldsymbol{\Omega}_{t+\Delta t} \times \mathbf{R}_{b,0}$$



# Hybrid PE-ML for WSI: Numerical Method and Procedure in qaleFOAM

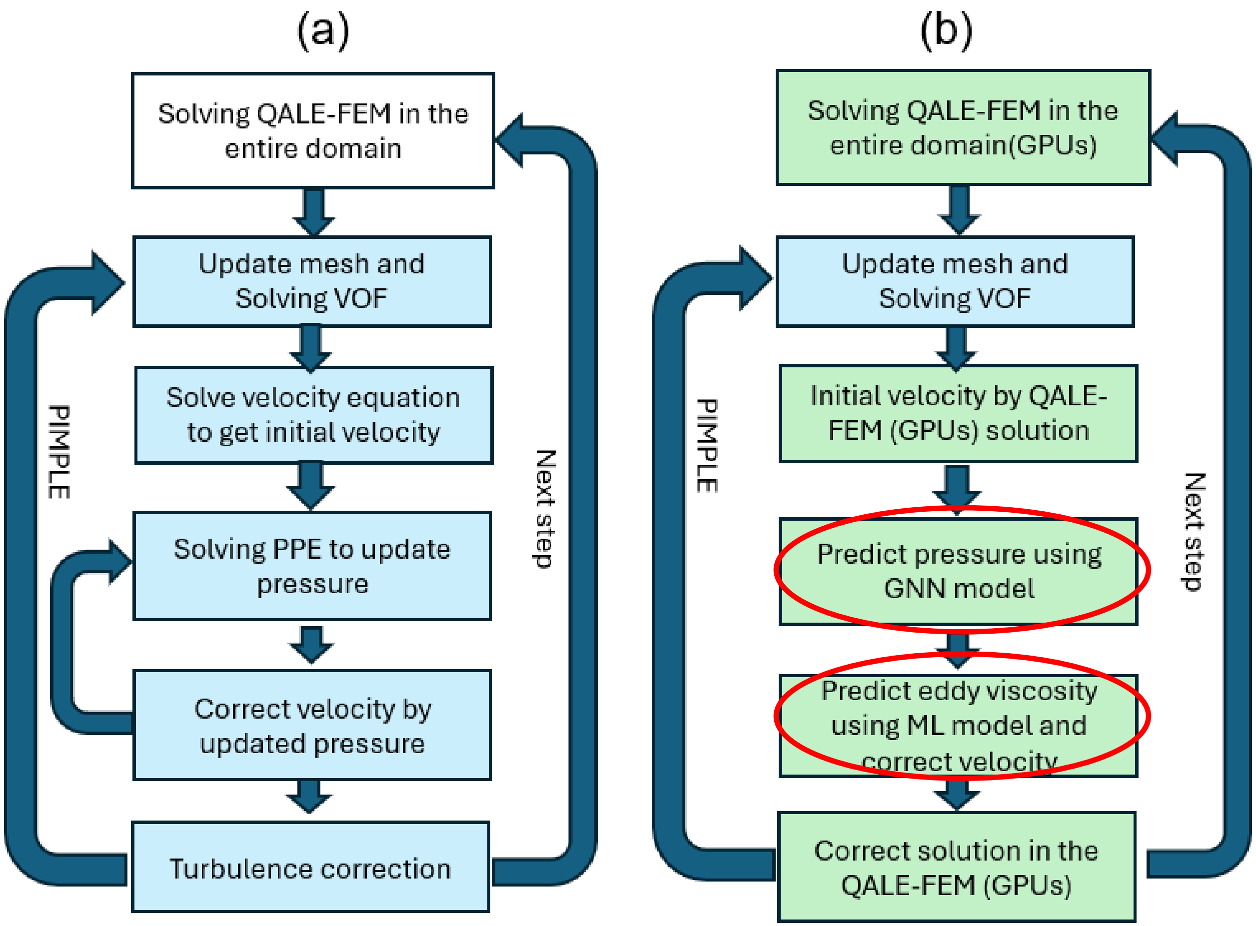



Fig.1 flowchart of (a) current CPU-based qaleFOAM and (b) proposed GPU-based qaleFOAM

Colour code: CPU: QALE-FEM | CPU: OpenFOAM | Single or multi-GPU development

ML estimate pressure

ML estimate eddy viscosity

# Hybrid PE-ML for WSI: Challenges Met and Tackled

- How to consider the boundary conditions when using ML to evaluate the pressure?
- Which part of the pressure evaluated using ML
- How to formulate the objective functions?
- What are inputs to ML?
- Which ML model is used?
- Can we use the data of simpler cases for training?

# Hybrid PE-ML for WSI: Implementing Boundary Conditions

- Two kinds of BC for WSI: free surface and rigid boundaries
- Satisfying these BC is important
- Difficult for ML to accurately satisfy these BC due to nature of training
- ***Our approach:*** ML trained and evaluating pressure excluding the points or particle on BC
- Pressure on BC evaluated by formulation satisfying BC

*On free surface of single phase:* $p=0$

*On rigid boundary: determine pressure using boundary conditions*

# Hybrid PE-ML for WSI: Evaluation of Pressure

Evaluation of the pressure plays a decisive role in correctly simulate WSI problem.

Total pressure include dynamic pressure and pressure related to vertical coordinate z. Dynamic pressure is more dominant in WSI.

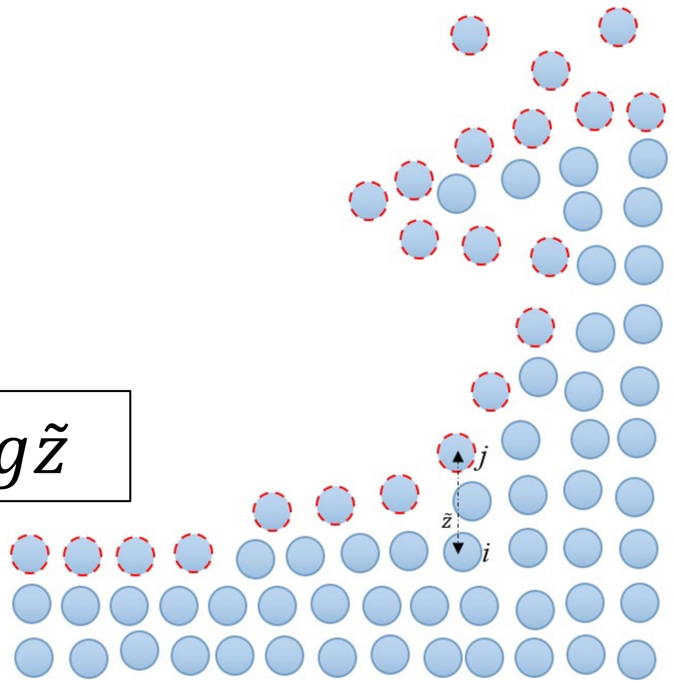
ML may lead large error in region where dynamic pressure is relatively smaller if it is used for total pressure.

***Our approach***:

$$p = p_d + p_s$$

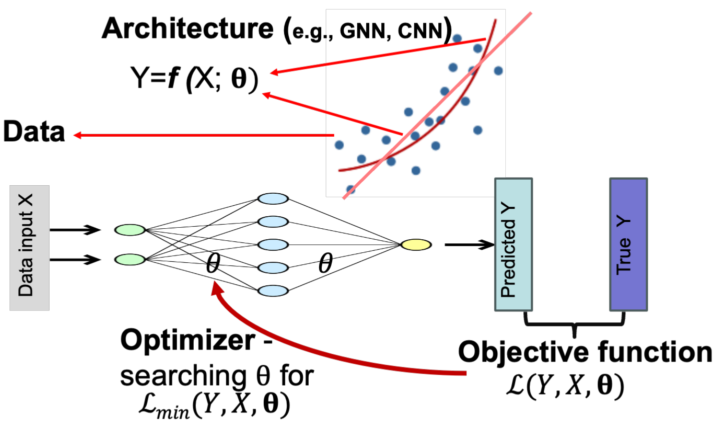
$$p_s = \rho g \tilde{z}$$

$$ML$$



# Hybrid PE-ML for WSI: Weights in Objective Function

During ML training, learnable variables are found by minimising the objective function, which is sum of errors of all points. Due to the nature of WSI, we desire the pressure errors near rigid boundary as smaller as possible. ***Our approach***:




$$f_{obj} = \sum_{i}^{N} s_i \left(\hat{p}_{d,i} - p_{d,i}\right)^2$$

$$s_i = \begin{cases} 3 & d_b \leq 3.0 \cdot dx \\ 2 & 3.0 \cdot dx < d_b \leq 6.0 \cdot dx \\ 1 & d_b > 6.0 \cdot dx \\ 0 & \text{free surface} \end{cases}$$

# Hybrid PE-ML for WSI: Inputs

ML used to evaluate the pressure governed by PPE

$$\nabla^2 p_{t+\Delta t} = \Psi / \Delta t$$

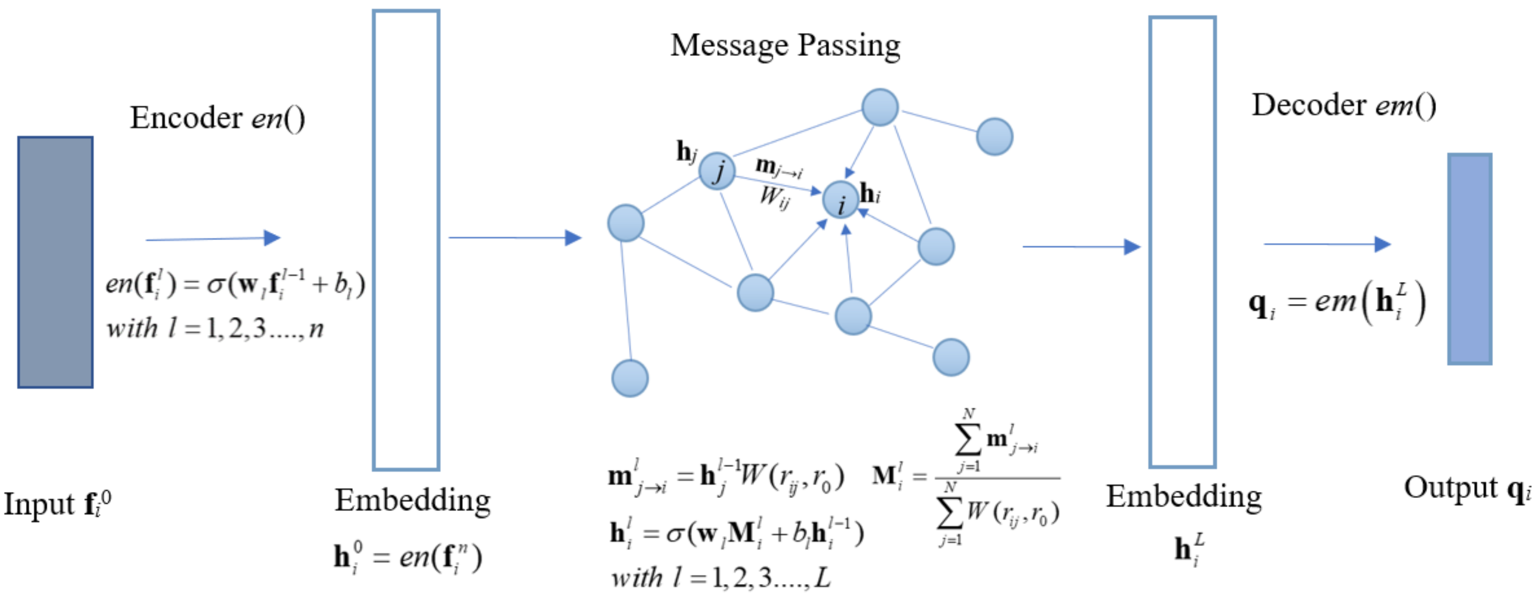
$$\Psi = \alpha \frac{\rho - \rho^*}{\Delta t} + (1-\alpha)\rho \nabla \cdot \mathbf{u}^*$$

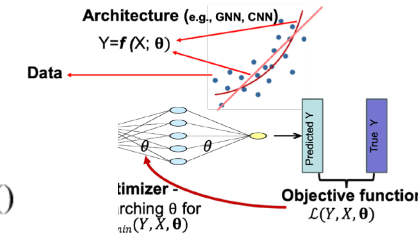
***Our approach*** using ML is to establish the relationship below

$$\hat{p}_{d,t+\Delta t} = f\left(\Psi, \mathbf{u}^*, p_{d,t}, c_p\right)$$

# Hybrid PE-ML for WSI: ML Models -GNN

## GNN (graphic neural network)



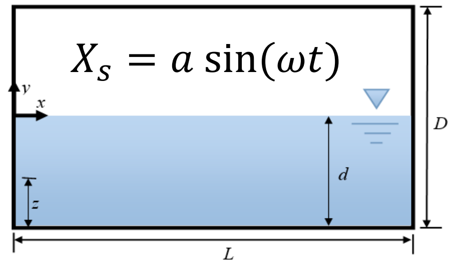

**Encoder:**
Convert the input data into network embeddings with learnable weights

**Message Passing:**
Establish the relation between the initial embeddings and final embeddings with learnable weights

**Decoder:**
Establish the relation between final embeddings and output with learnable weights

# Case Study on More Complex Cases Using GNN Trained on data of Simpler Cases- single phase

## SPH Case A: sloshing for training



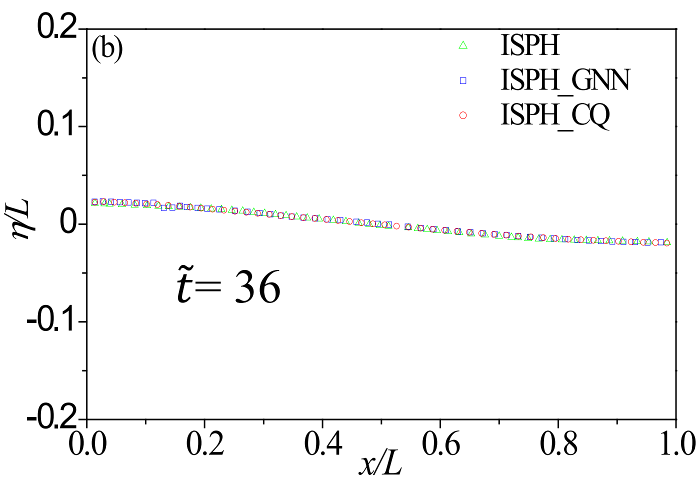

For generating the training/testing data *without baffle*: $L/d = 2$ $D/L = 1$; $a = 0.3L \sim 0.7L$, $\omega = 0.6\omega_1$, $\omega = 0.8\omega_1$ and $\omega = 0.9\omega_1$

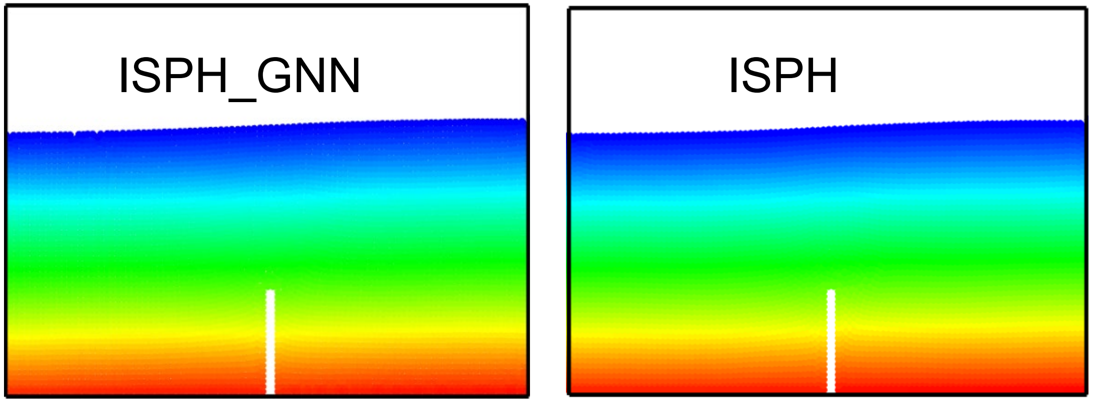
## Applied case *with a baffle* using GNN:

$a = 0.01L$ and $\omega = 0.8\omega_1$; Height and width of the baffle are $0.2L$ and $0.04L$


Applied case *with a baffle* using GNN:

$a = 0.01L$ and $\omega = 0.8\omega_1$; Height and width of the baffle are $0.2L$ and $0.04L$

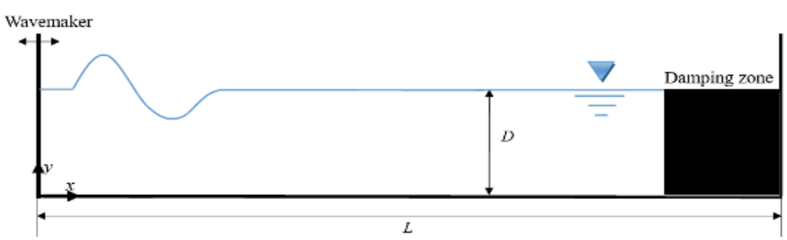

$\tilde{t} = 18.79$

# Case Study on More Complex Cases Using GNN Trained on data of Simpler Cases- single phase

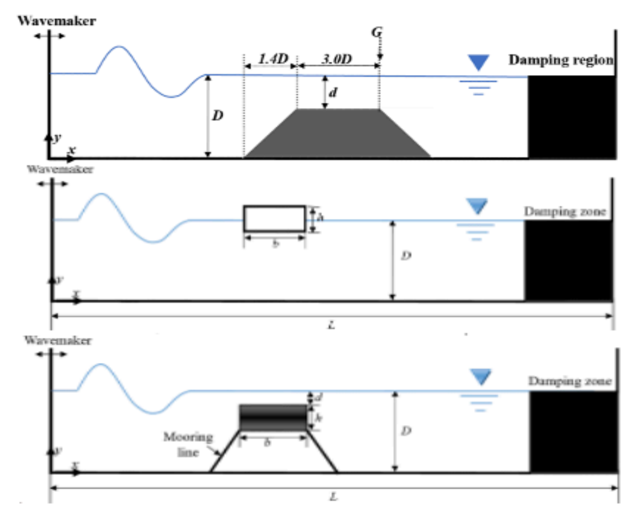
SPH Case B : Datasets generated for regular waves and solitary waves *without any object* and trained GNN applied to cases *with various structures*

Training data: 30 regular wave cases and 40 solitary wave cases

- For each of regular wave cases, $H$ randomly selected in $0.05D$ to $0.25D$ and $T$ in 1.0 s to 1.2 s ($D = 0.4$ m and $L = 25D$);
- For each of the solitary wave cases, $H$ randomly selected in $0.2D$ to $0.4D$ with $D$ is 0.25 m, 0.275 m or 0.3 m, $L = 40D$




Training

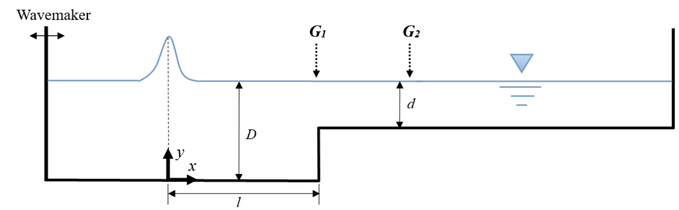


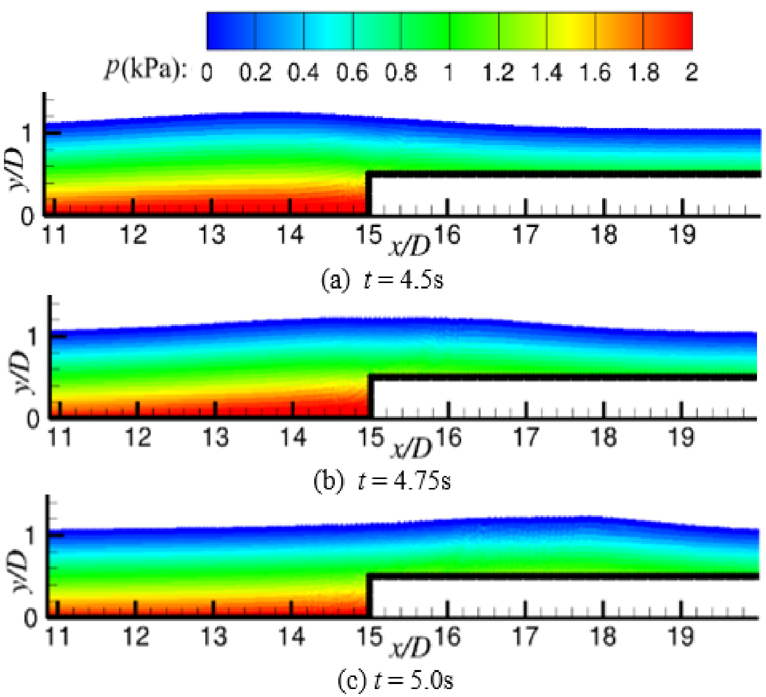

Applied

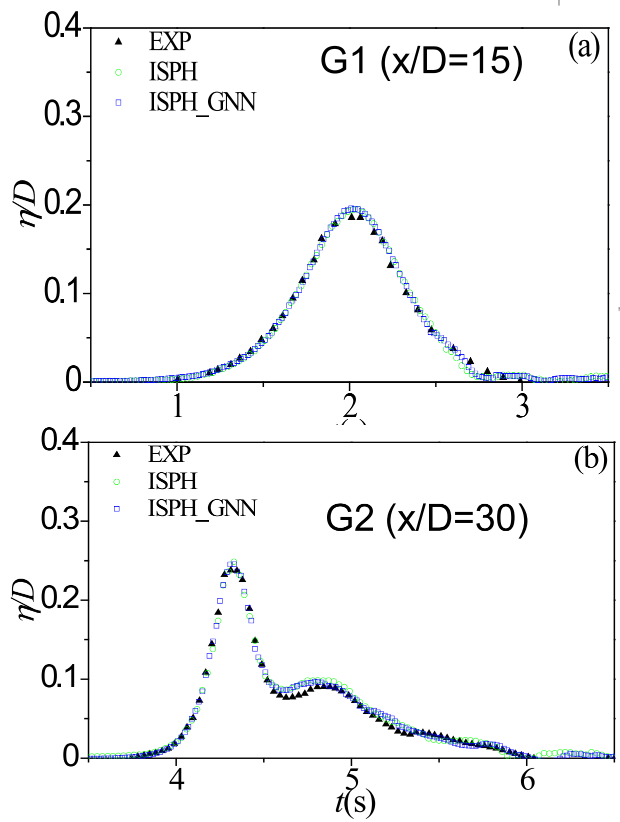
# Case Study on More Complex Cases Using GNN Trained on data of Simpler Cases- single phase

## Applied SPH Case B1: *Solitary wave overtopping over a step*

***Wave***: *H= 0.1825D* ***Tank***: *l/D*=15, *d/D*=0.3, *length=75D, D=0.2m*



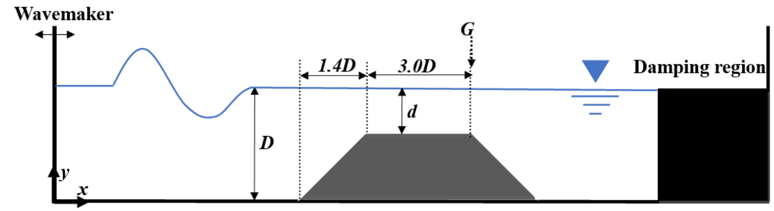

Mean error from experimental data:

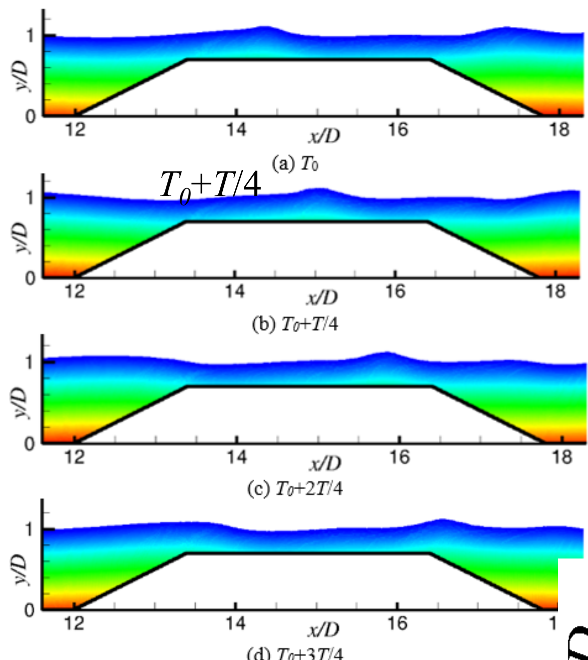
| Position | ISPH | ISPH_GNN |
|---|---|---|
| G1 | 5.9% | 5.2% |
| G2 | 7.3% | 5.8% |

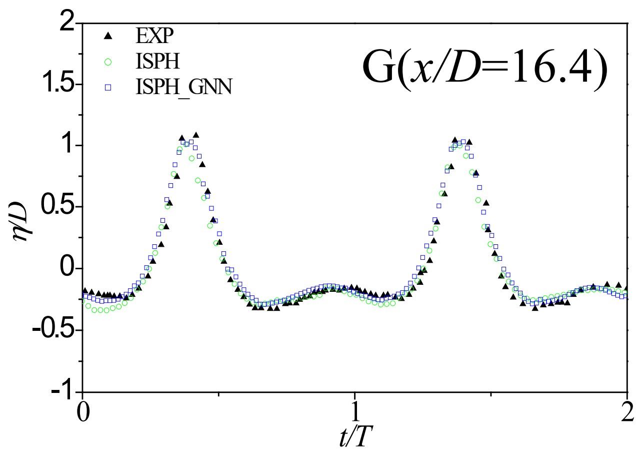
# Case Study on More Complex Cases Using GNN Trained on data of Simpler Cases- single phase

Applied SPH Case B2: *Regular wave passing over a submerged fixed structure*

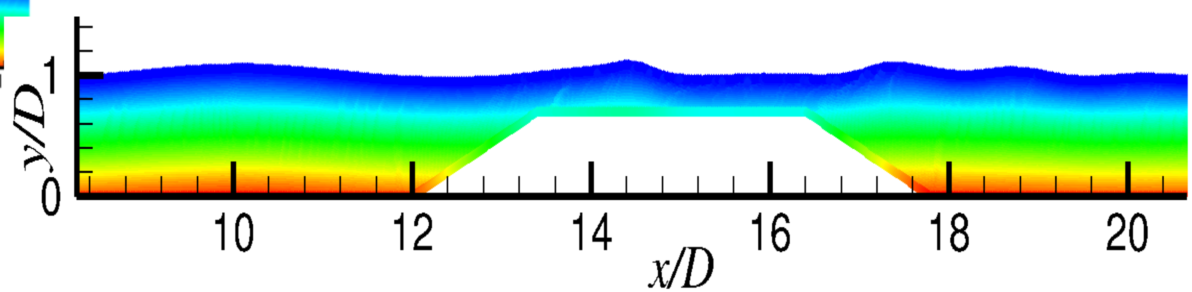
***Wave***: $T$=1.34s, $H$=0.3$D$, ***Tank***: $d$=0.3$D$, $D$=0.5m, Length=40$D$



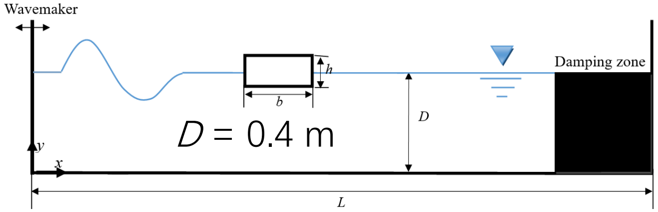

Mean error from experimental data:
ISPH: 15.6%,
ISPH_GNN: 14.9%

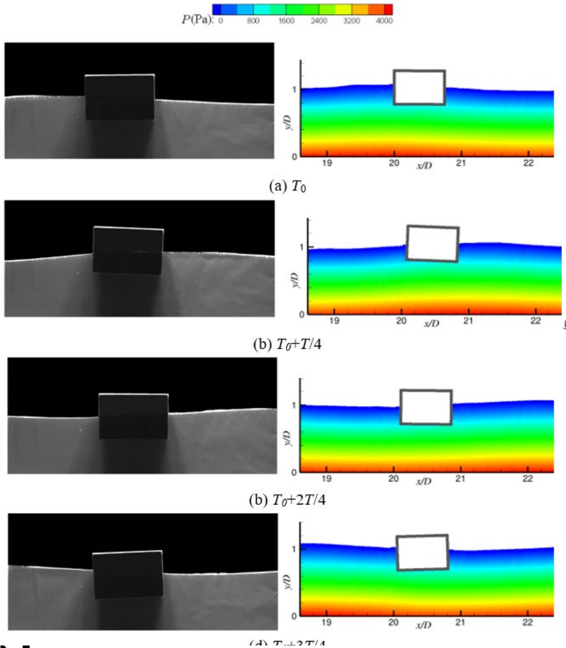


# Case Study on More Complex Cases Using GNN Trained on data of Simpler Cases- single phase

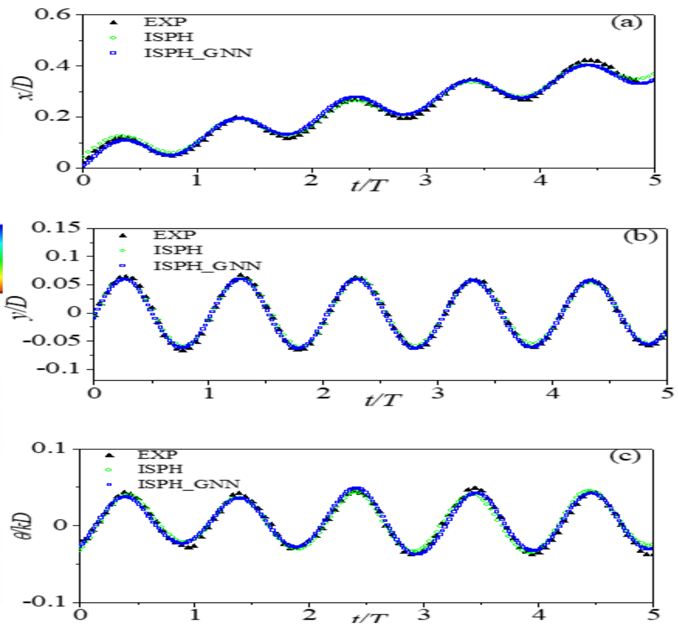
## Applied SPH Case B3: *Motions of a floating body in regular waves*

***Wave**: H=0.1D, T=1.2s;* ***Body**:* $b = 0.75D$, $h = 0.5D$, uniform mass: 500 kg/m$^3$ , draft=0.25$D$, *CoG: (*20.0$D$, 1.0$D$*);* ***Tank**: L= 50D, D=0.4*



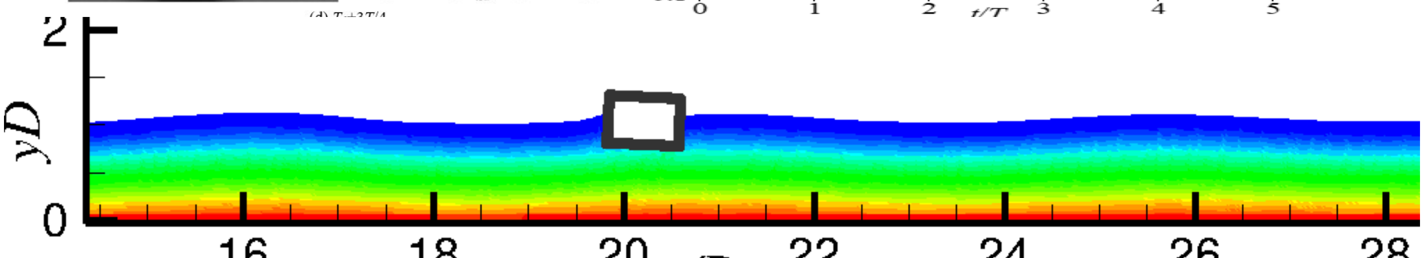



(a) $T_0$
(b) $T_0+T/4$
(b) $T_0+2T/4$




Mean error from experimental data:

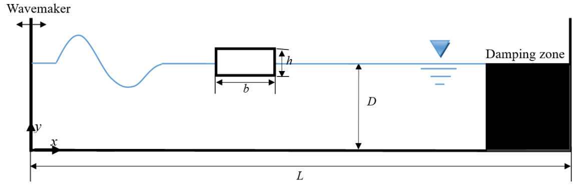
| Motion | ISPH | ISPH_GNN |
|---|---|---|
| Sway | 4.1% | 4.4% |
| Heave | 8.9% | 7.7% |
| Roll | 15.2% | 13.8% |

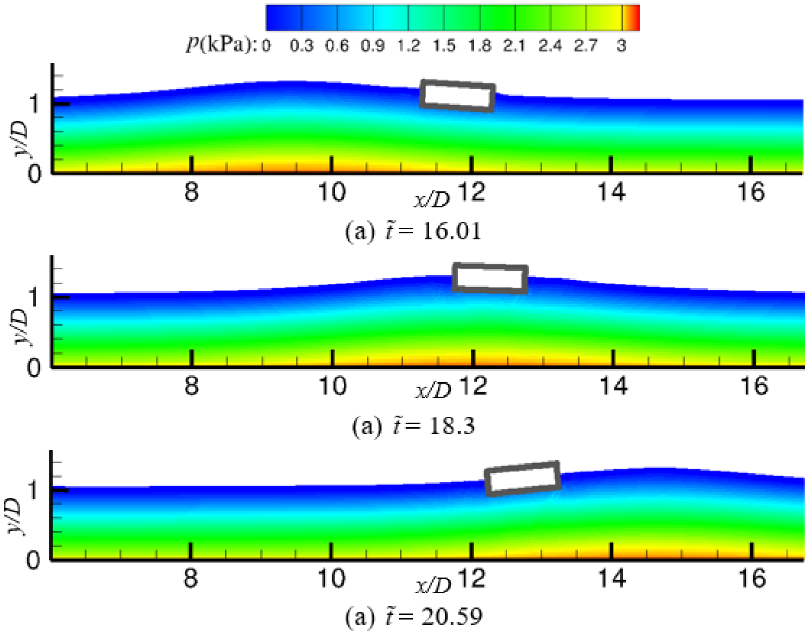


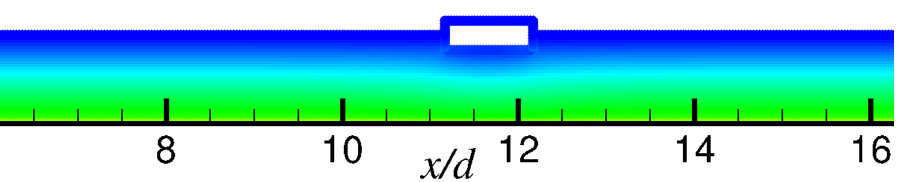
# Case Study on More Complex Cases Using GNN Trained on data of Simpler Cases- single phase

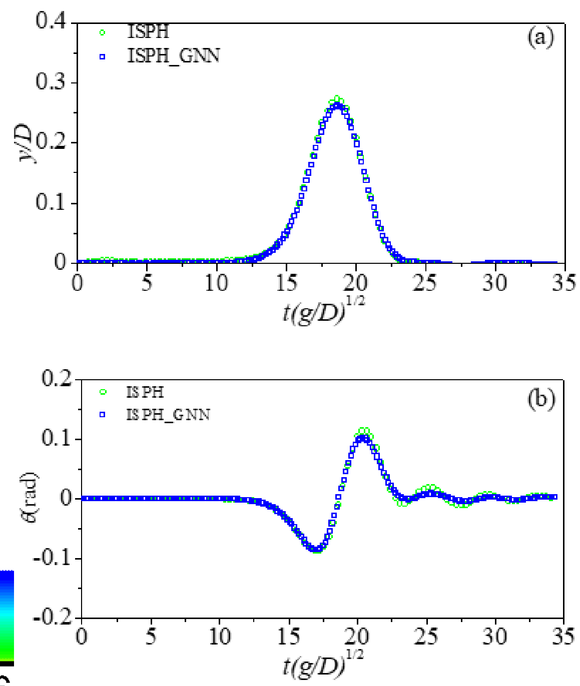
## Applied SPH Case B4: *Motions of a floating body in solitary waves*

***Wave***: $H = 0.27D$; ***Body***: $b = 1.0D$, $h = 0.33D$, uniform mass: 500 kg/m$^3$ , draft=$0.25D$, *CoG*: $(11.67D, 1.0D)$; ***Tank***: $L=33D$, $D = 0.3$ m;



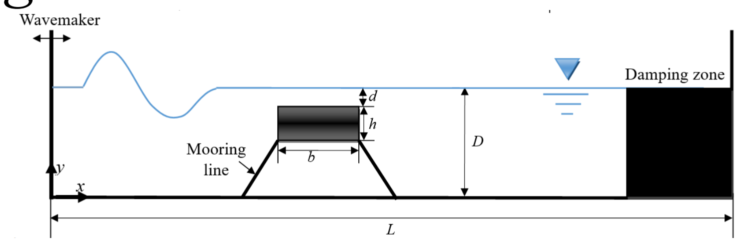

Mean Difference between results of ISPH and ISPH-GNN

| Motion | Difference |
|---|---|
| Heave | 3.7% |
| Roll | 11.6% |

# Case Study on More Complex Cases Using GNN Trained on data of Simpler Cases- single phase

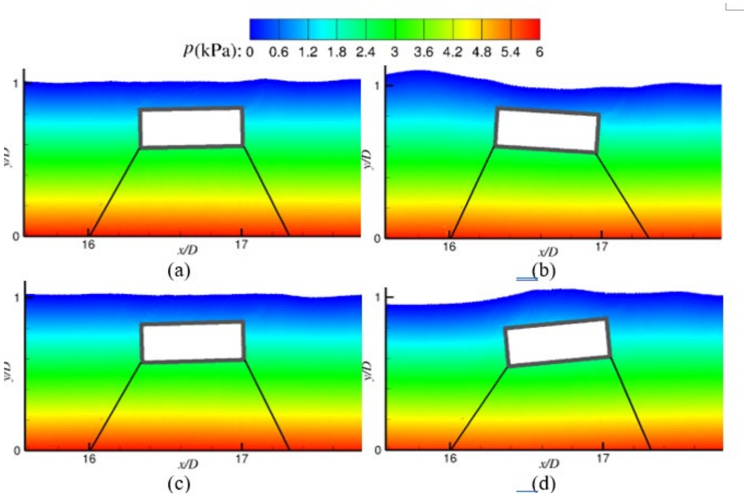
## Applied SPH Case B5: *a moored floating breakwater*

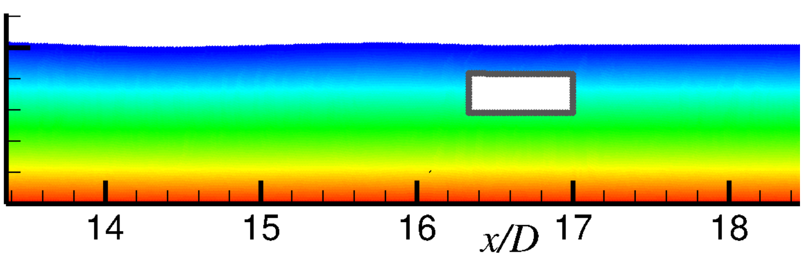
***Wave***: $H$=0.0767$D$, $T$=1.0s; ***Body***: $b$ = 0.6$D$, $h$ = 0.25$D$, $M$=42kg, $I$=0.64 kg.m$^2$, $d$=0.17$D$, *CoG*: (20.0$D$, 0.705$D$); ***Tank***: $L$= 35$D$; $D$=0.6m;
***Mooring lines:*** *linear spring with* $k$ = 1.2*10$^5$ N/m

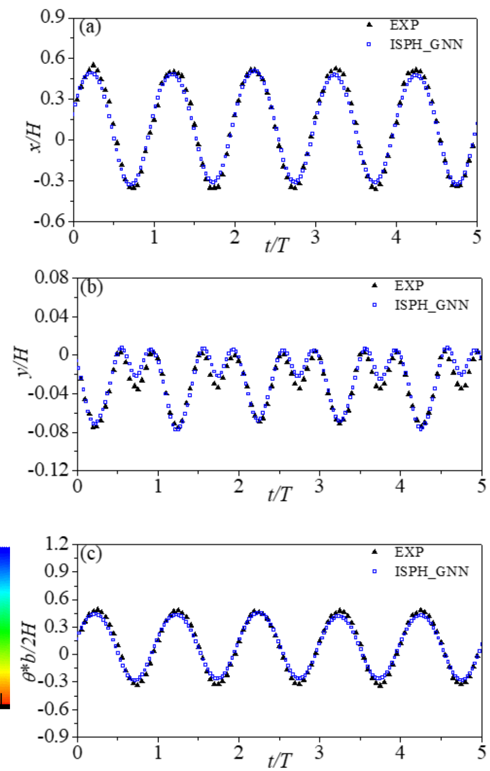


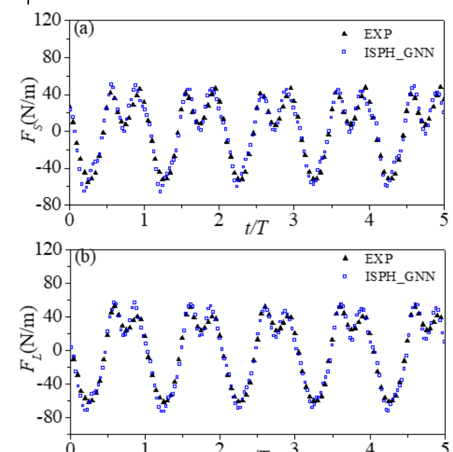

Mean error from experimental data:

| Motion/ mooring forces | ISPH_GNN |
|---|---|
| Sway | 11.5% |
| Heave | 16.2% |
| Roll | 13.8% |
| $F_S$ | 19.3% |
| $F_L$ | 17.2% |



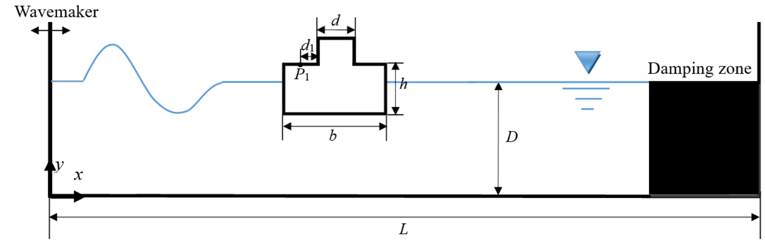



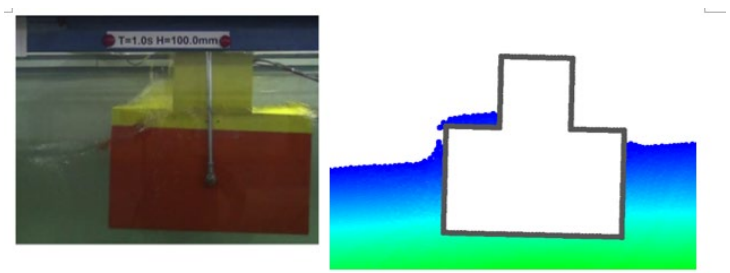



# Case Study on More Complex Cases Using GNN Trained on data of Simpler Cases- single phase

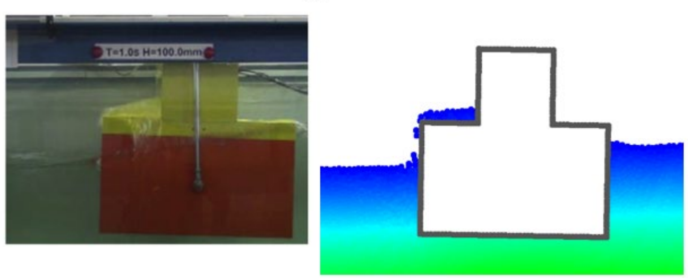
## Applied SPH Case B6: *Green water impact on a floating deck*

***Wave***: *H=0.143D, T=1.2s;* ***Body***: *b = 0.714D, h = 0.429D, M=?kg, I=6.531kg.m², draft=0.357D, d=0.286D, d1 = 0.0714D, CoG: (14.3D, 0.83D);* ***Tank***: *L= 28D; D=0.7m*

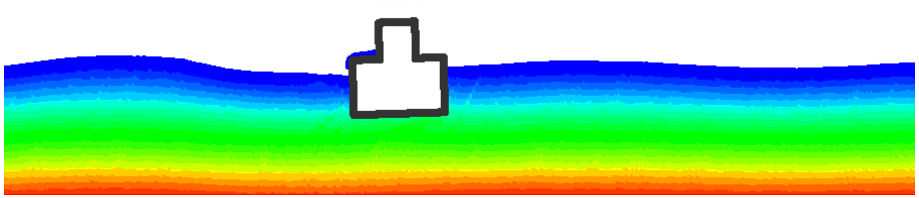


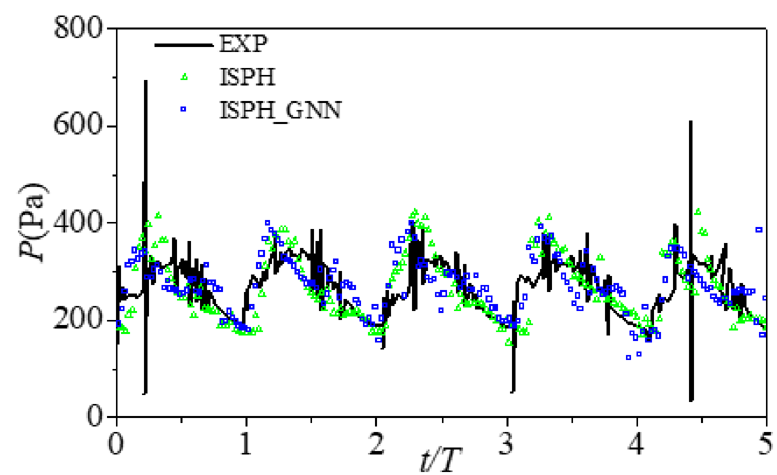

(c) $T_0$+2$T$/4




(d) $T_0$+3$T$/4



Mean error from experimental data:
18.7% for ISPH_GNN; 19.5% for ISPH



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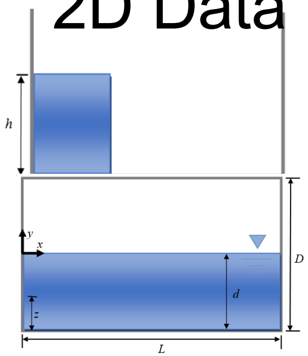
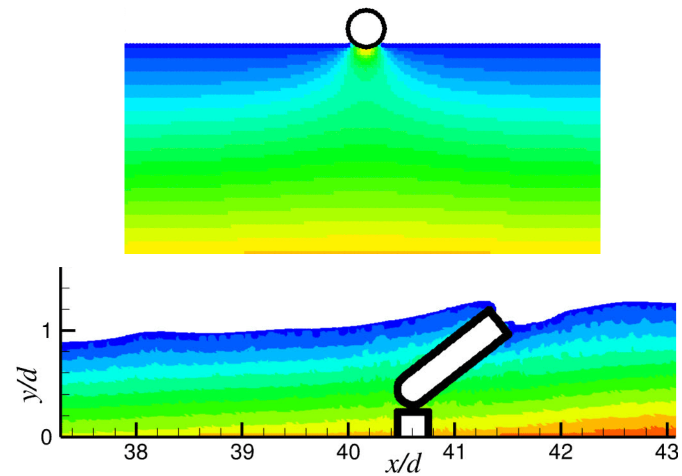
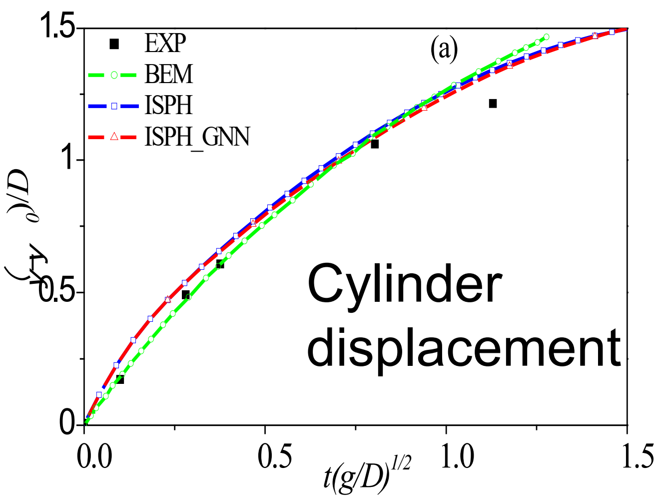
SPH Case B7: data from dam breaking and sloshing to model object dropping and oyster-like WECs

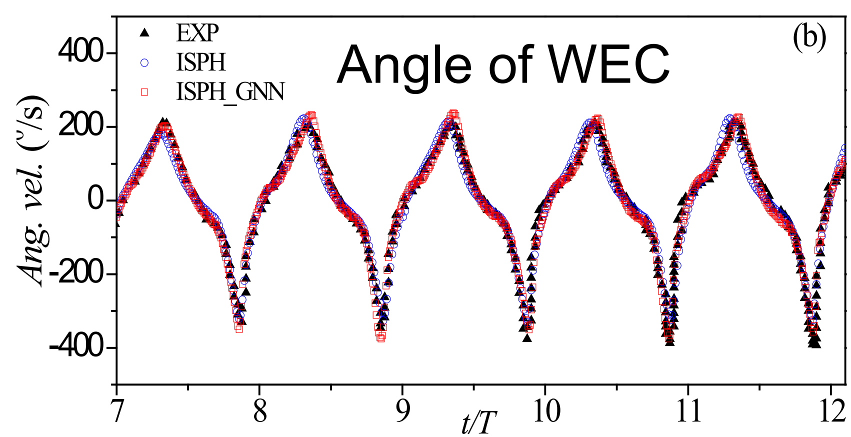
2D Data




Train ML



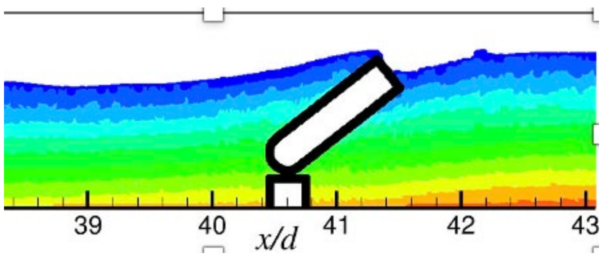



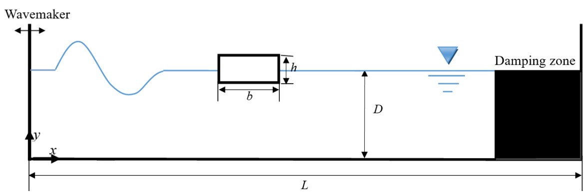



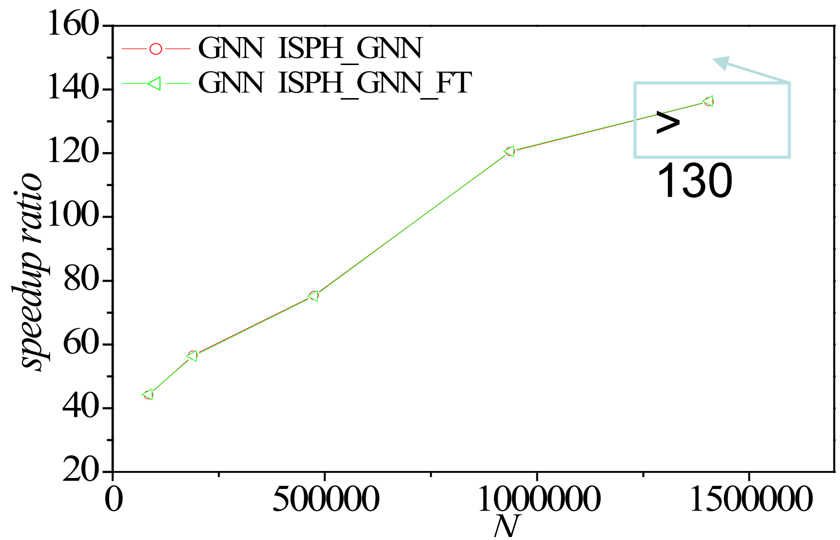
# Case Study on More Complex Cases Using GNN Trained on data of Simple Cases

## Computational Efficiency:

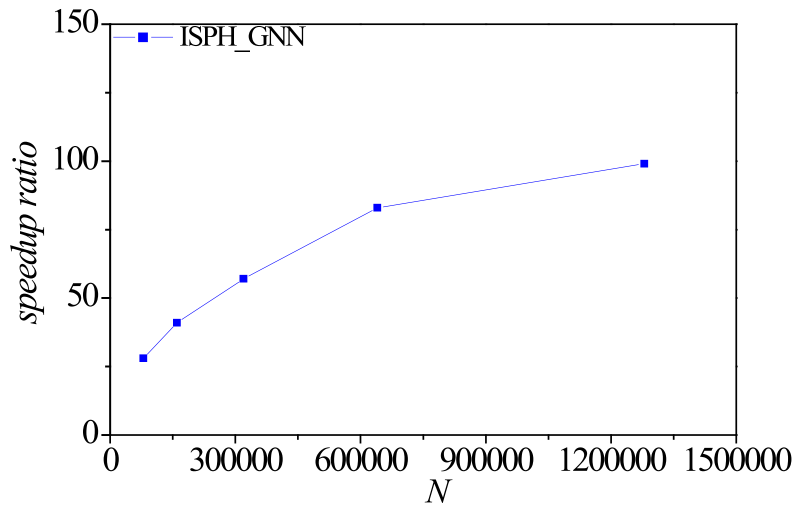
Applied SPH Case B3 and B7:



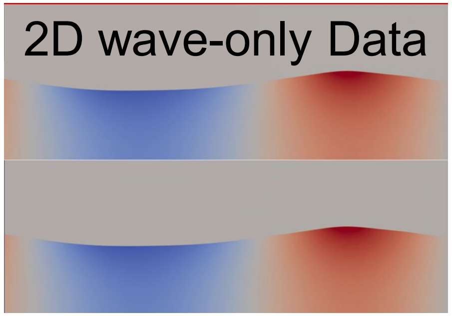

Averaged speedup ratio for pressure evaluation per step



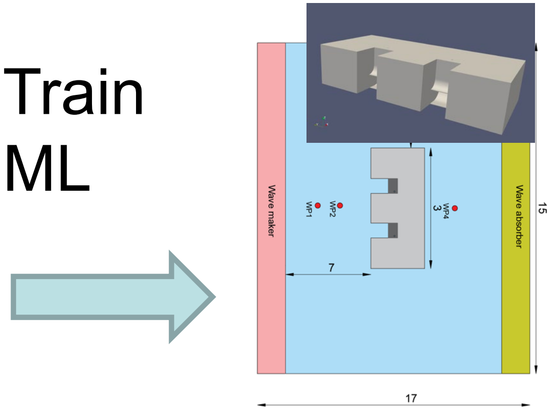

Averaged speedup ratio for pressure evaluation per step

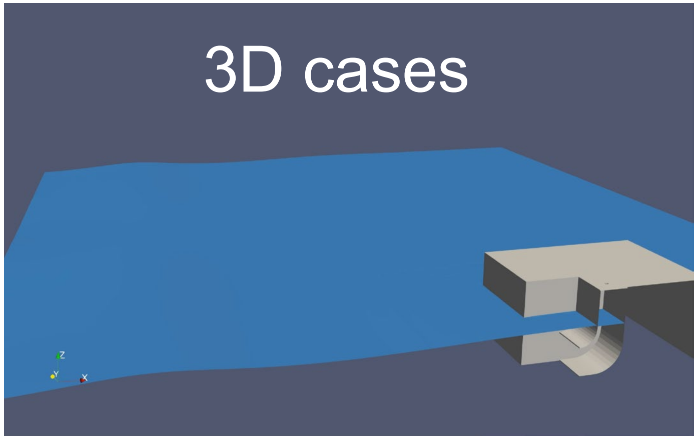
# Case Study on More Complex Cases Using GNN Trained on data of Simple Cases - multiphase

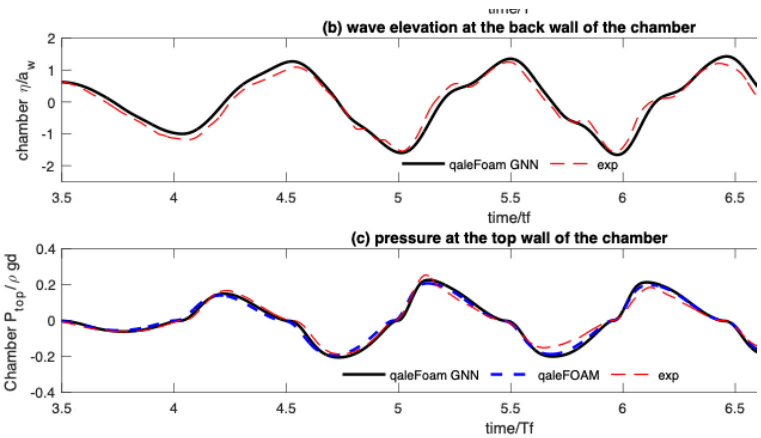
qaleFOAM Case 1




Train ML




6.54 M cells, to achieve the results at 10.5 s With ML for pressure: 55 hours (laptops or workstation
Without ML: 333 hours (laptops or workstation

T = 1.8 s, H = 0.08 m, d = 0.35 m; OpenFoam domain ( 6m * 14m * 1m ) , QALE-FEM domain (70 m * 0.35 m)

# Summary

**Hybrid PE-ML** methods: solving PPE is replaced by ML or solving turbulent models by ML. It has been applied to model single/multi-phase WSI problems, related to four type of WECs. Good agreement with experimental results

**Our experience shows that**

(1) Not using ML to evaluating the pressure on boundaries;

(2) Not using ML to evaluate total pressure;

(3) Carefully select objective function, with considering effects of boundaries;

(4) GNN is better than CNN for particle-based or irregular grid methods.

**Our results demonstrate that**

(1) Evaluating pressure using ML is much faster than directly solving PPE;

(2) ML trained on simpler cases can be applied to more complex cases. This is particularly interesting as generating data on simpler cases costs less

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- Imperial College team participates in discussions on some contents during project meetings
- Dr. Qian Li contributes to development of ML model for eddy viscosity.
- Mr Pai Liu participates in the tests related to CorPower model.

We are very grateful to all the contributors

# References Cited


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