frequency response functions

$$\langle\langle \tilde{W}(t); \tilde{V}_\beta(t') \rangle\rangle$$

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$$= -i\,\Theta(t-t')\,\langle \Psi_0 | \tilde{W}(t)\tilde{V}_\beta(t') - \tilde{W}(t)\tilde{V}_\beta(t') | \Psi_0 \rangle$$

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$$= -i\,\Theta(t-t') \sum_k \Big(e^{-i\omega_k(t-t')} \langle \Psi_0 | W | \Psi_k \rangle \langle \Psi_k | V_\beta | \Psi_0 \rangle$$

$$- e^{-i\omega_k(t'-t)} \langle \Psi_0 | V_\beta | \Psi_k \rangle \langle \Psi_k | W | \Psi_0 \rangle \Big)$$

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Note that $\langle\langle \tilde{W}(t); \tilde{V}_\beta(t') \rangle\rangle = \langle\langle \tilde{W}(t-t_0); \tilde{V}(t'-t_0) \rangle\rangle$

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$$\langle\langle \tilde{W}(t); \tilde{V}_\beta(t') \rangle\rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega \, \langle\langle W; V_\beta \rangle\rangle_{\omega_\epsilon} \, e^{+i\omega_\epsilon \tau}$$

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(mnemonic:

$$\int_{-\infty}^{\infty} dk \, e^{ikx} = 2\pi \, \delta(x)$$

)

Fourier transform:

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$$= \frac{1}{\omega_\epsilon \mp \omega_k}$$

Consistency check:

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Cauchy:

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Cauchy: $\oint_\gamma dz\, f(z) = 2\pi i \sum_{k=1}^{n} \mathrm{Res}(f, z_k)$

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$\uparrow$

poles

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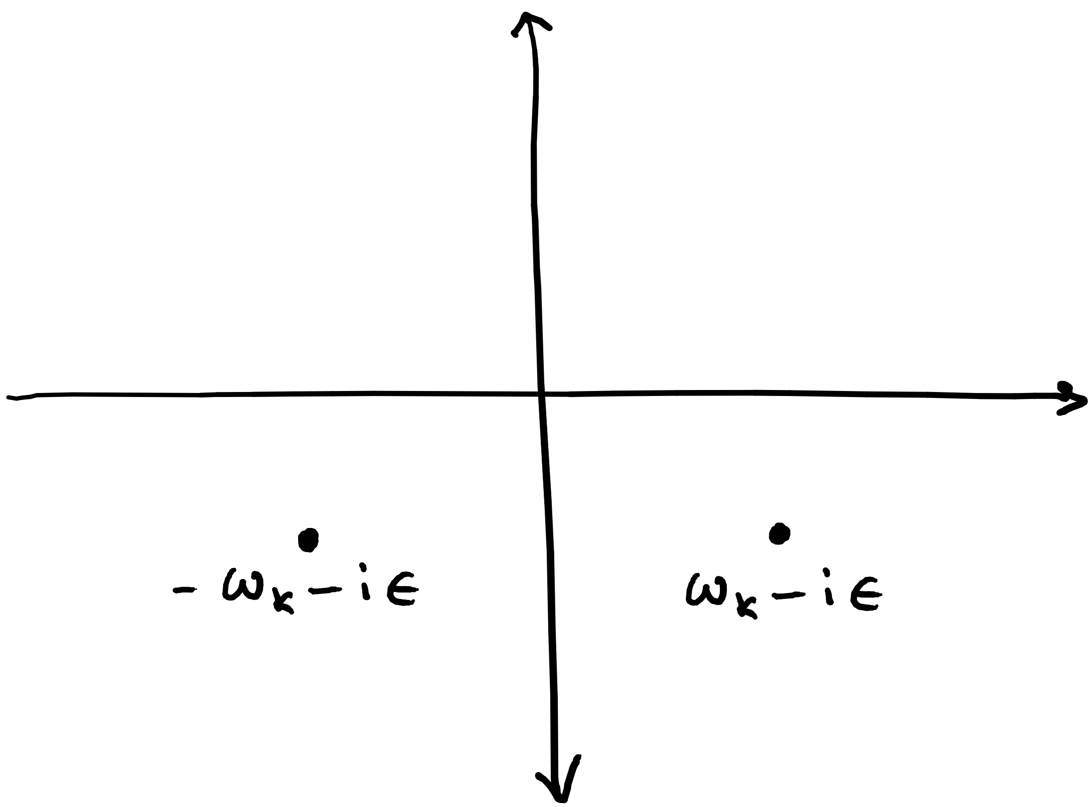
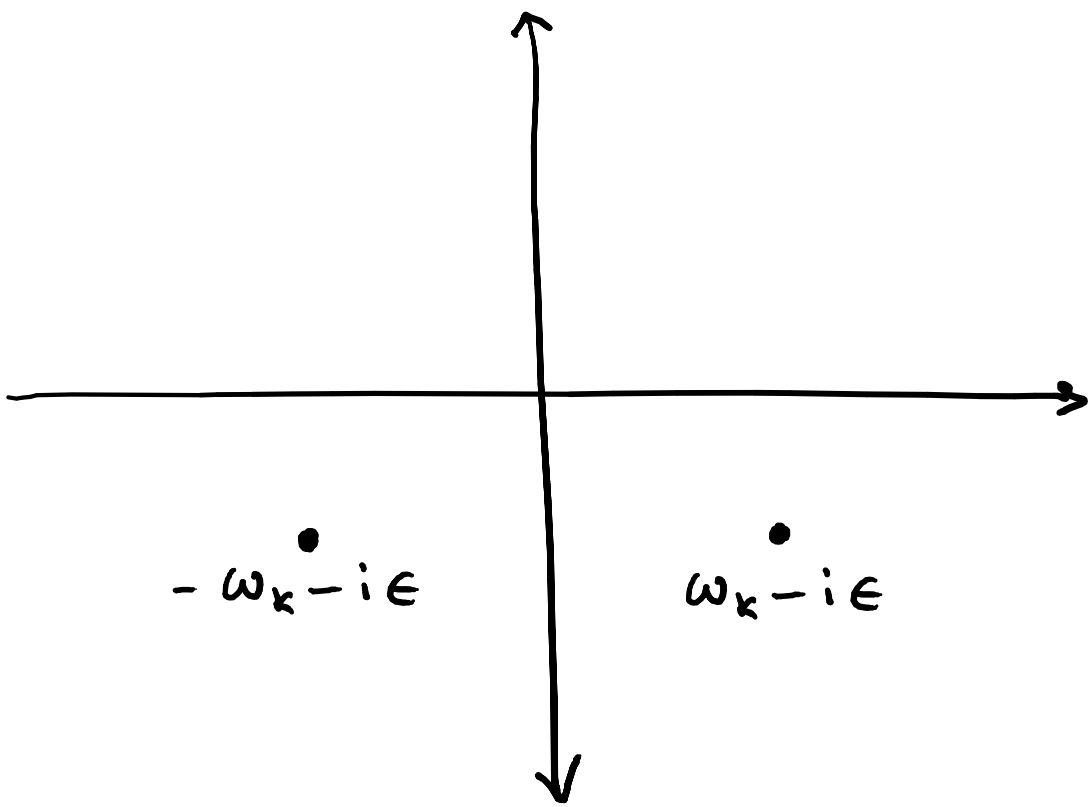


Consistency check:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \, \frac{e^{+i\omega_\epsilon \tau}}{\omega_\epsilon \mp \omega_k} \overset{\tau>0}{=}$$



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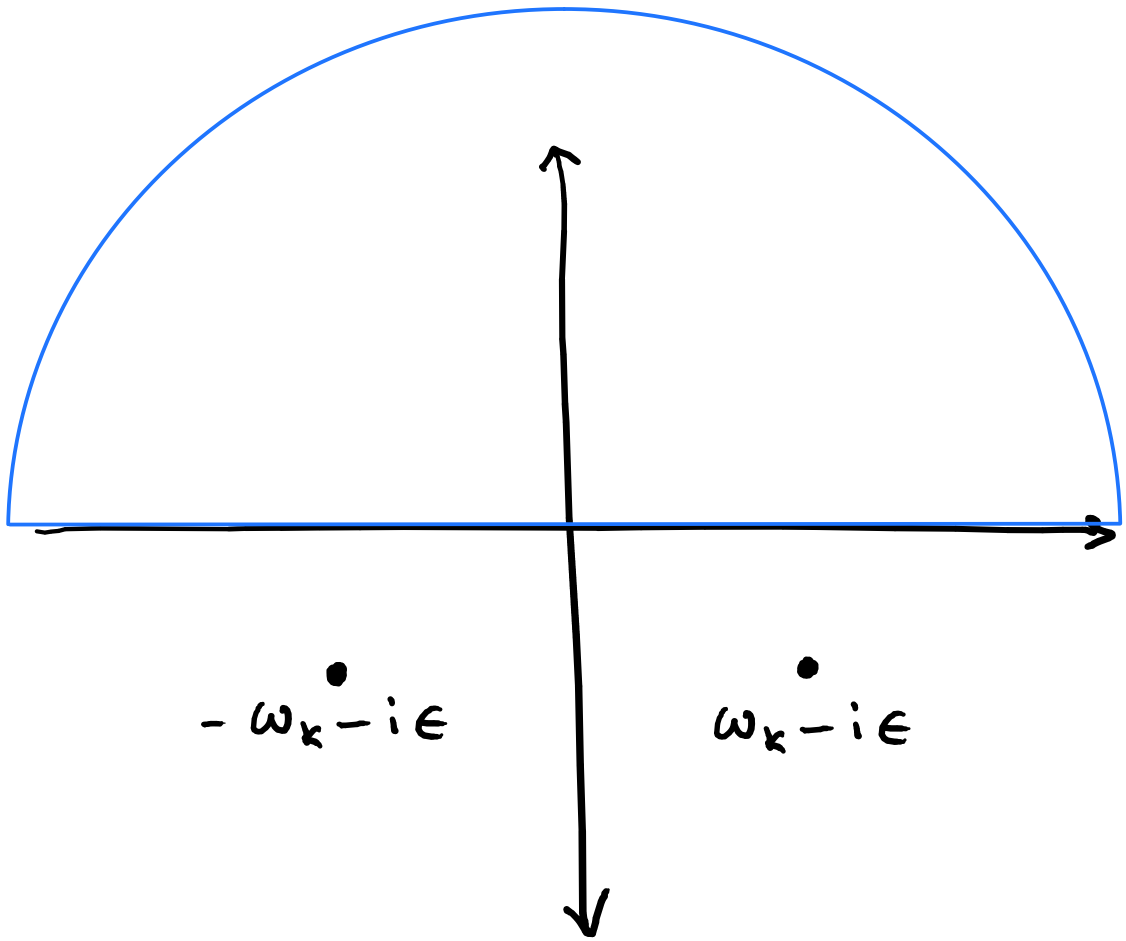

Consistency check:

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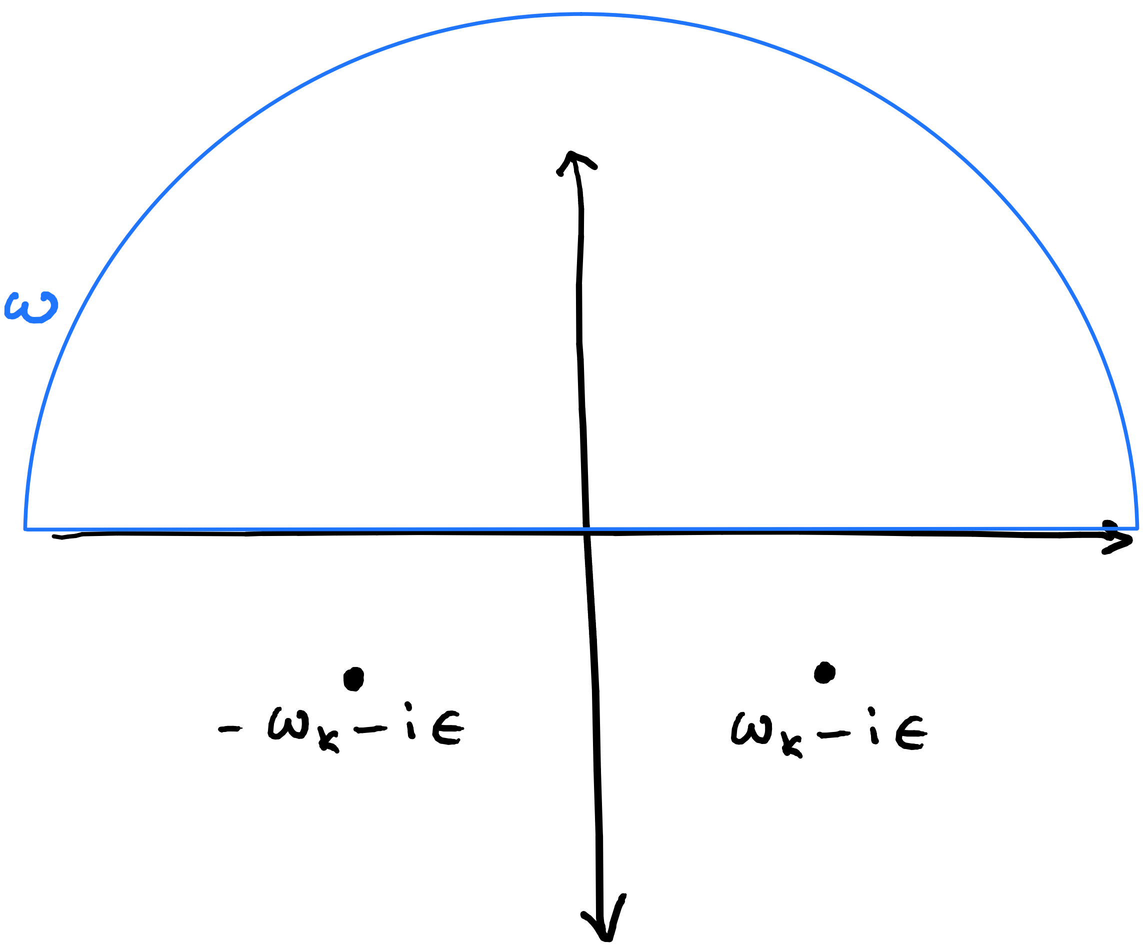
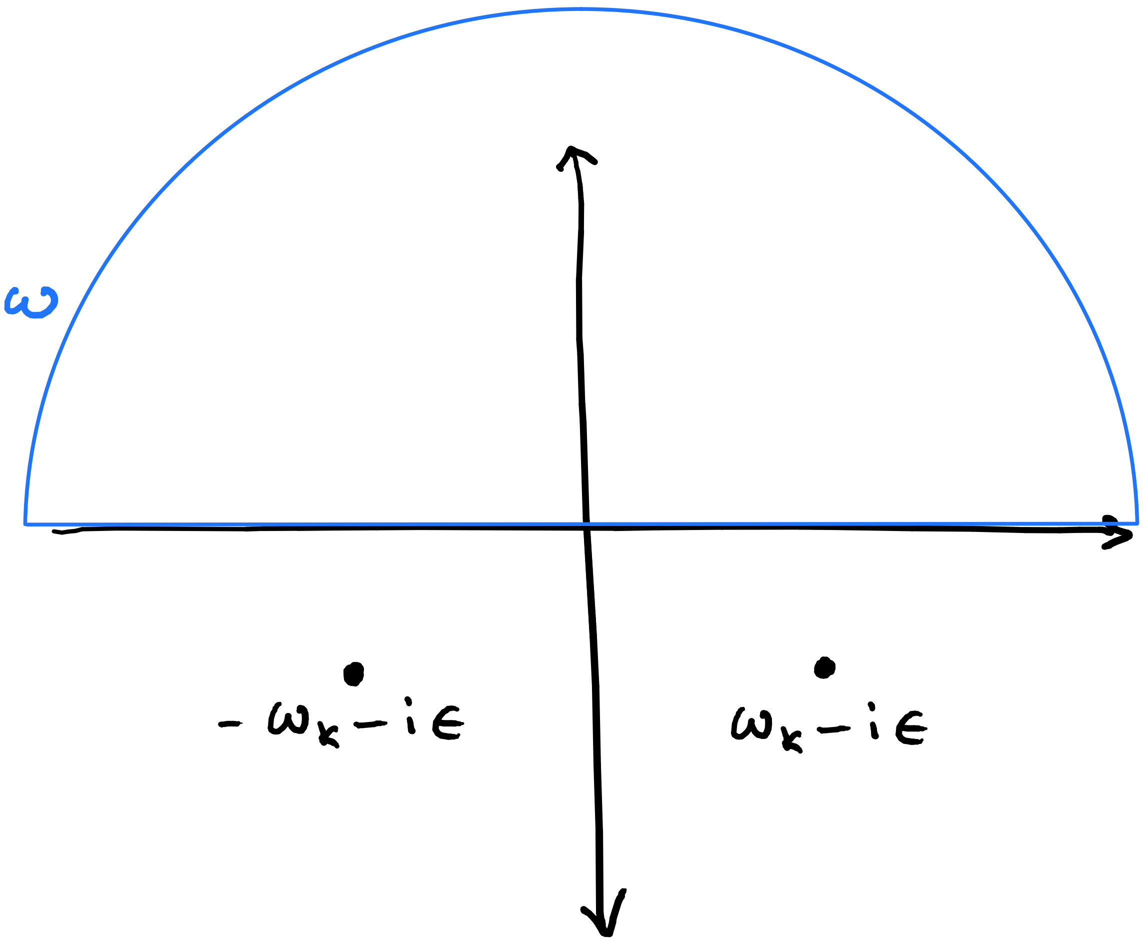
$$+\oint_\gamma d\omega$$



Consistency check:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \, \frac{e^{+i\omega_E \tau}}{\omega_E \mp \omega_k} \overset{\tau>0}{=}$$

$$+\oint_{\gamma} d\omega$$



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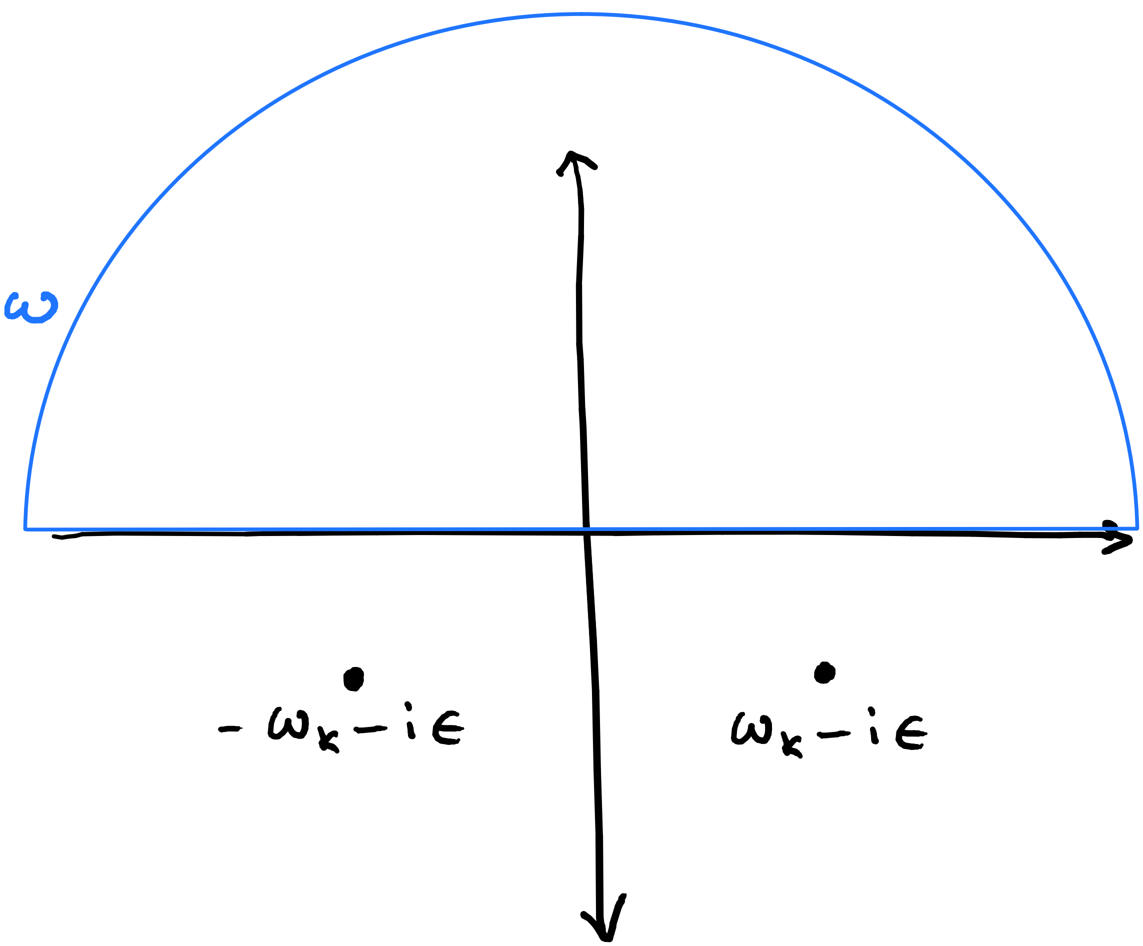

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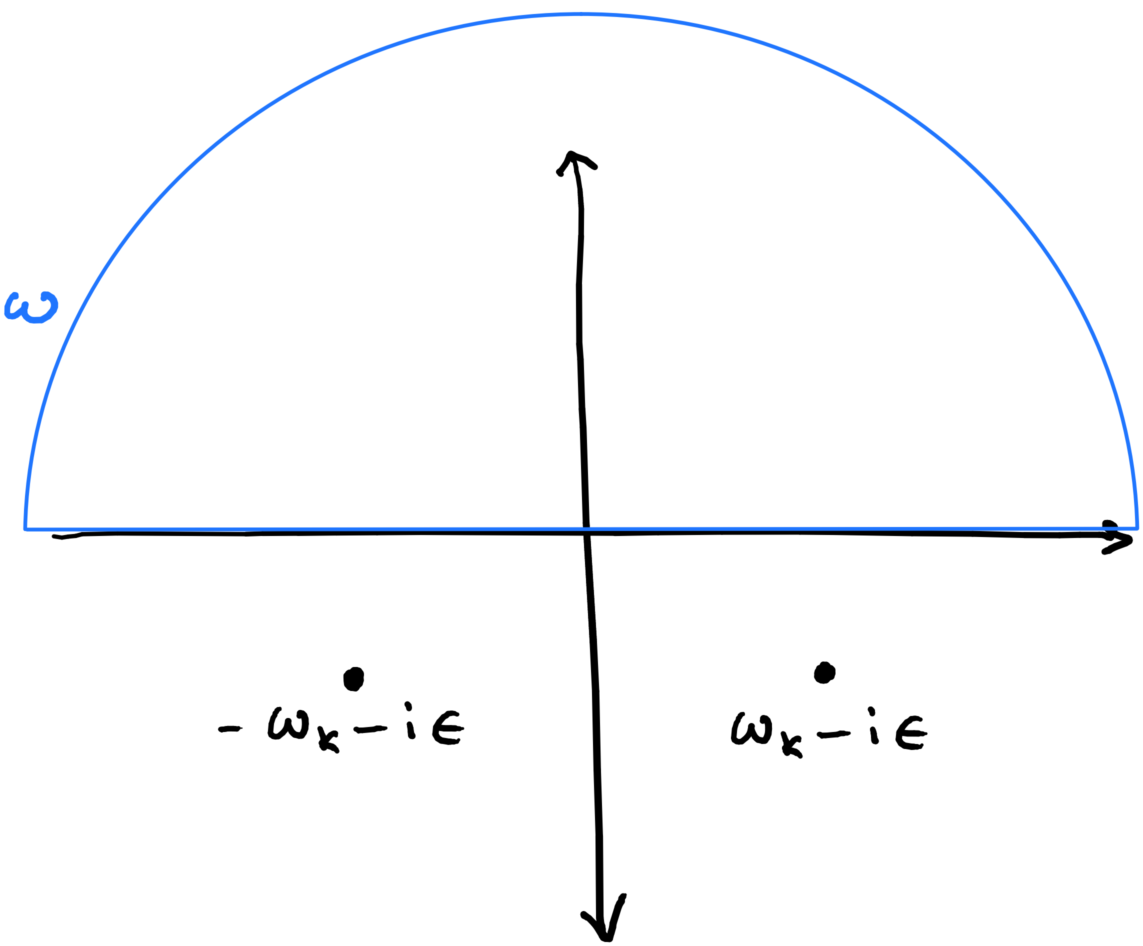

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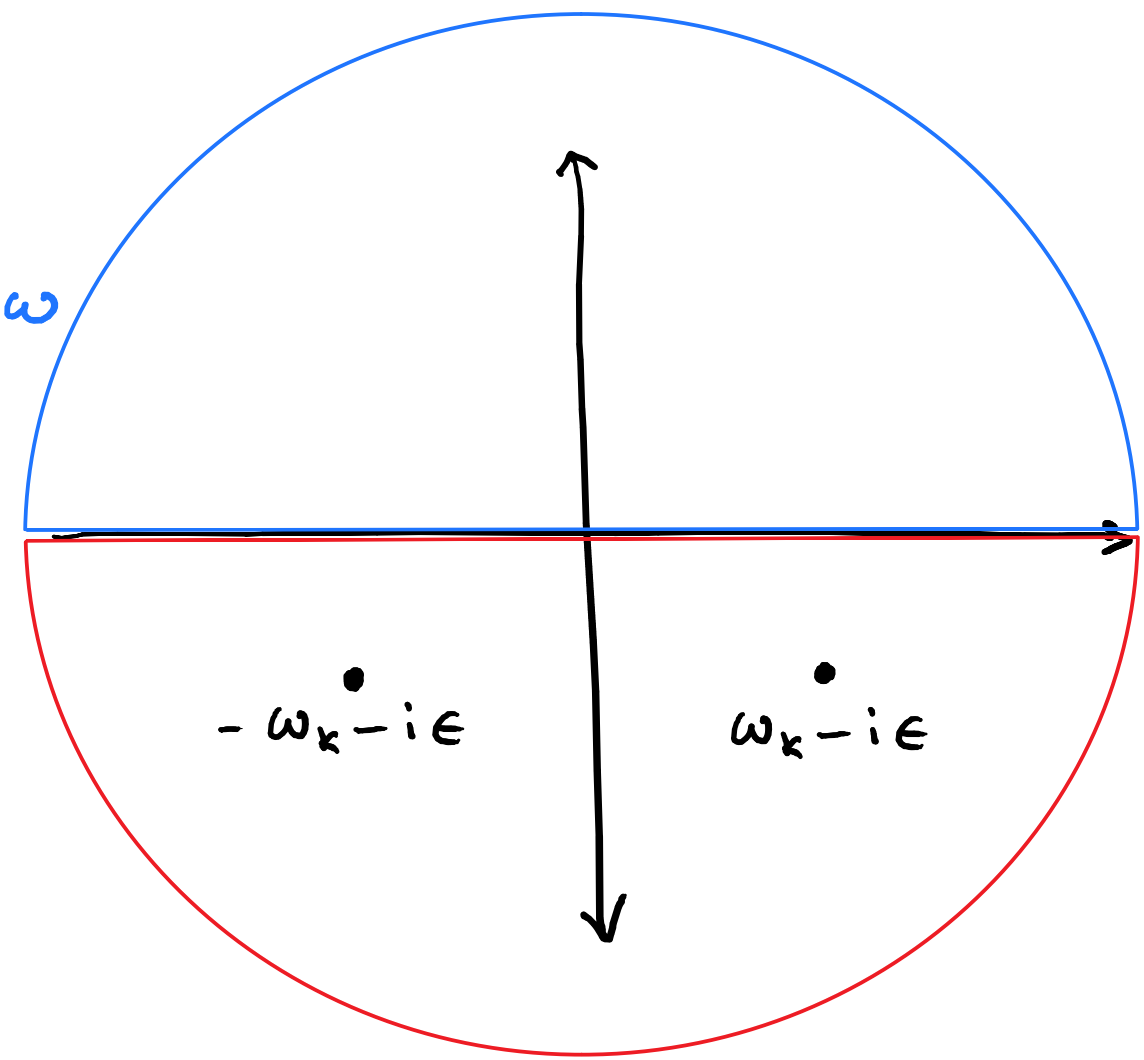

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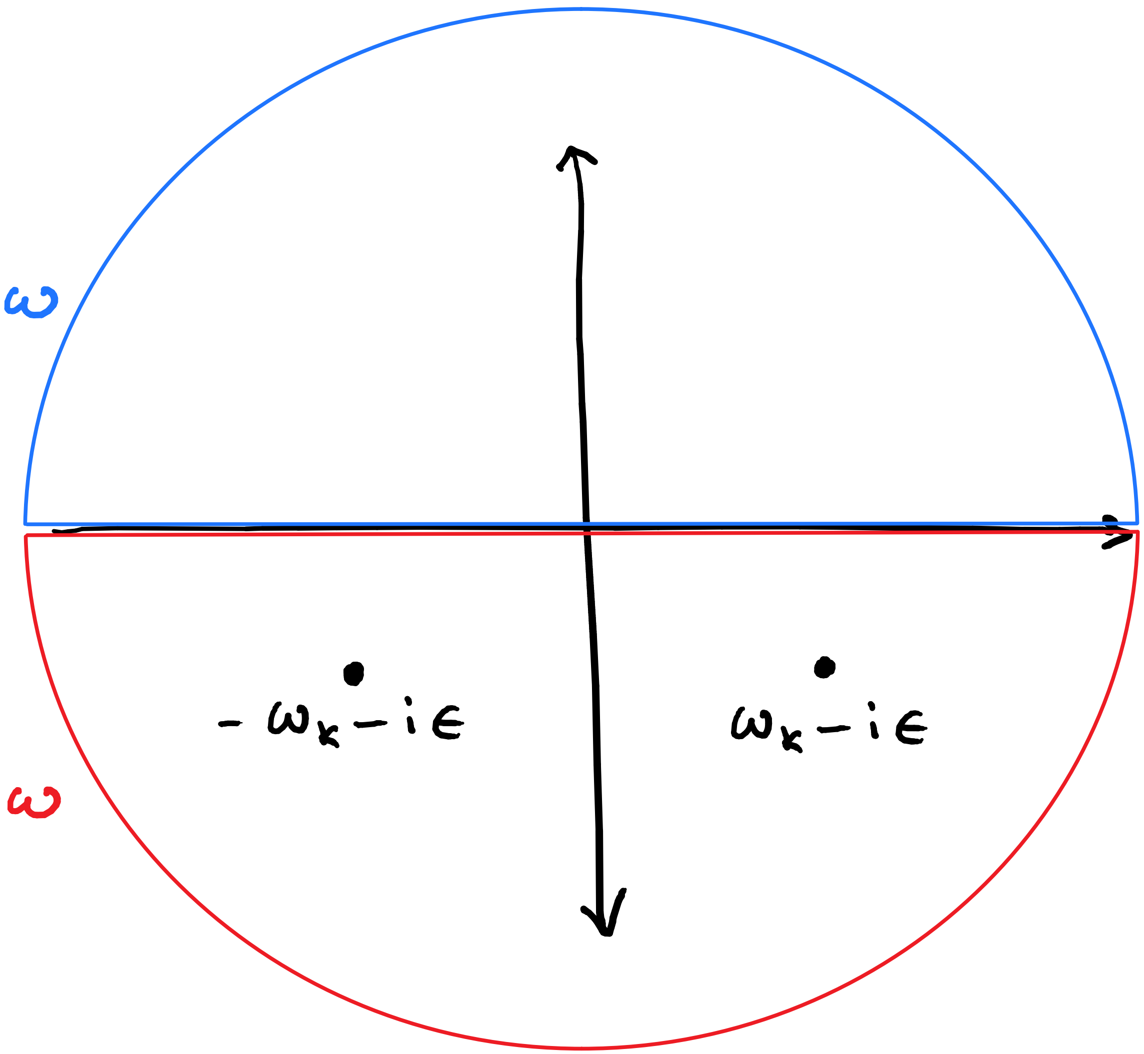

$$\frac{e^{i(\omega_x + i\omega_y)|\tau|}}{\omega_x + i\omega_y} = 0$$

$$-\oint_\gamma d\omega$$

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$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \, \frac{e^{+i\omega_\epsilon \tau}}{\omega_\epsilon \mp \omega_k} \overset{\tau>0}{=} 0$$

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$$+\oint_\gamma d\omega$$

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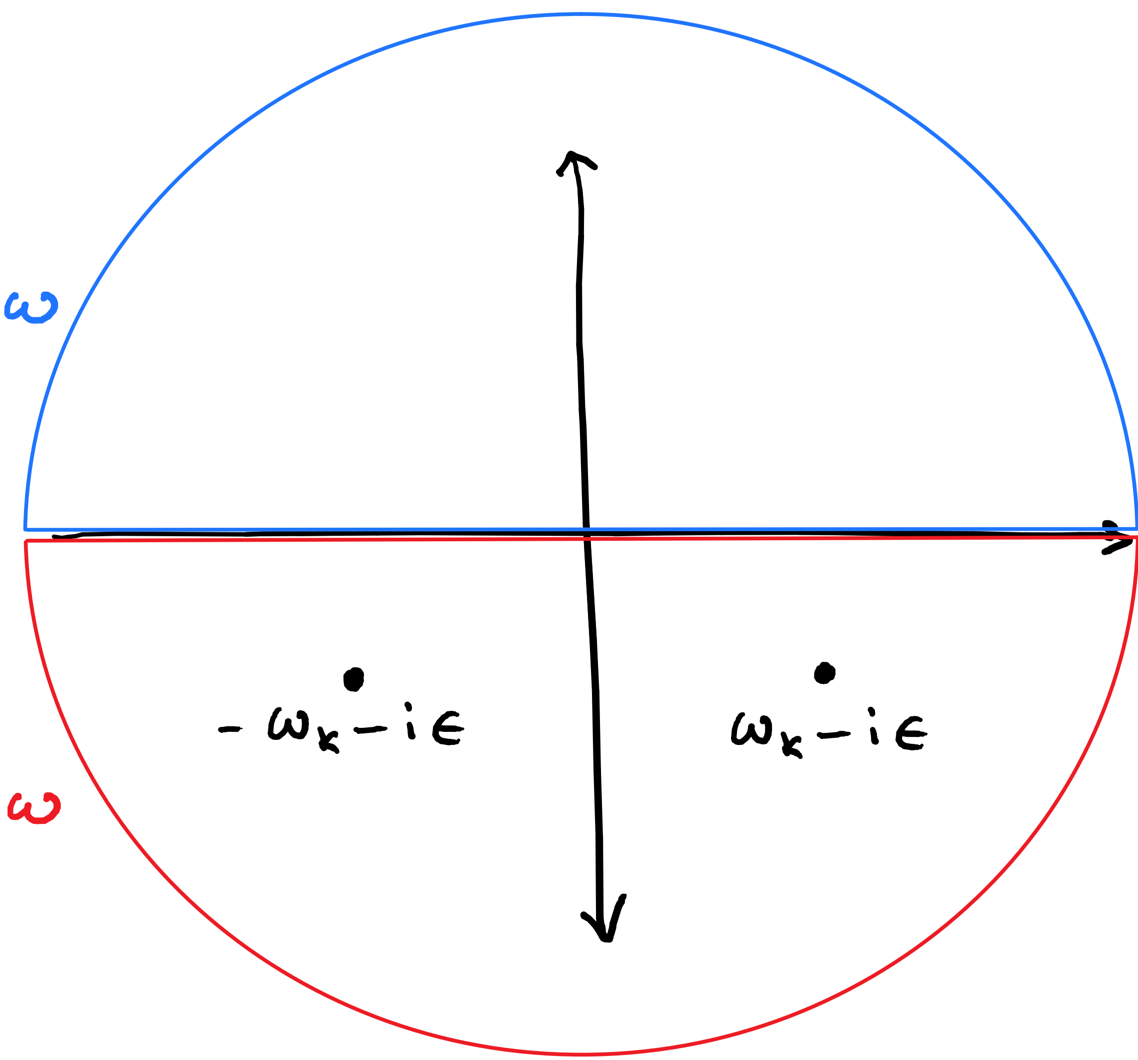

$$\frac{e^{i(\omega_x + i\omega_y)|\tau|}}{\omega_x + i\omega_y} = 0$$

$$\frac{e^{-i(\omega_x + i\omega_y)|\tau|}}{\omega_x + i\omega_y} = 0$$

Consistency check:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \, \frac{e^{+i\omega_\epsilon \tau}}{\omega_\epsilon \mp \omega_k} \overset{\tau>0}{=} 0$$

$$\overset{\tau<0}{=} -2\pi i \, \mathrm{Res}(\pm\omega_k)$$

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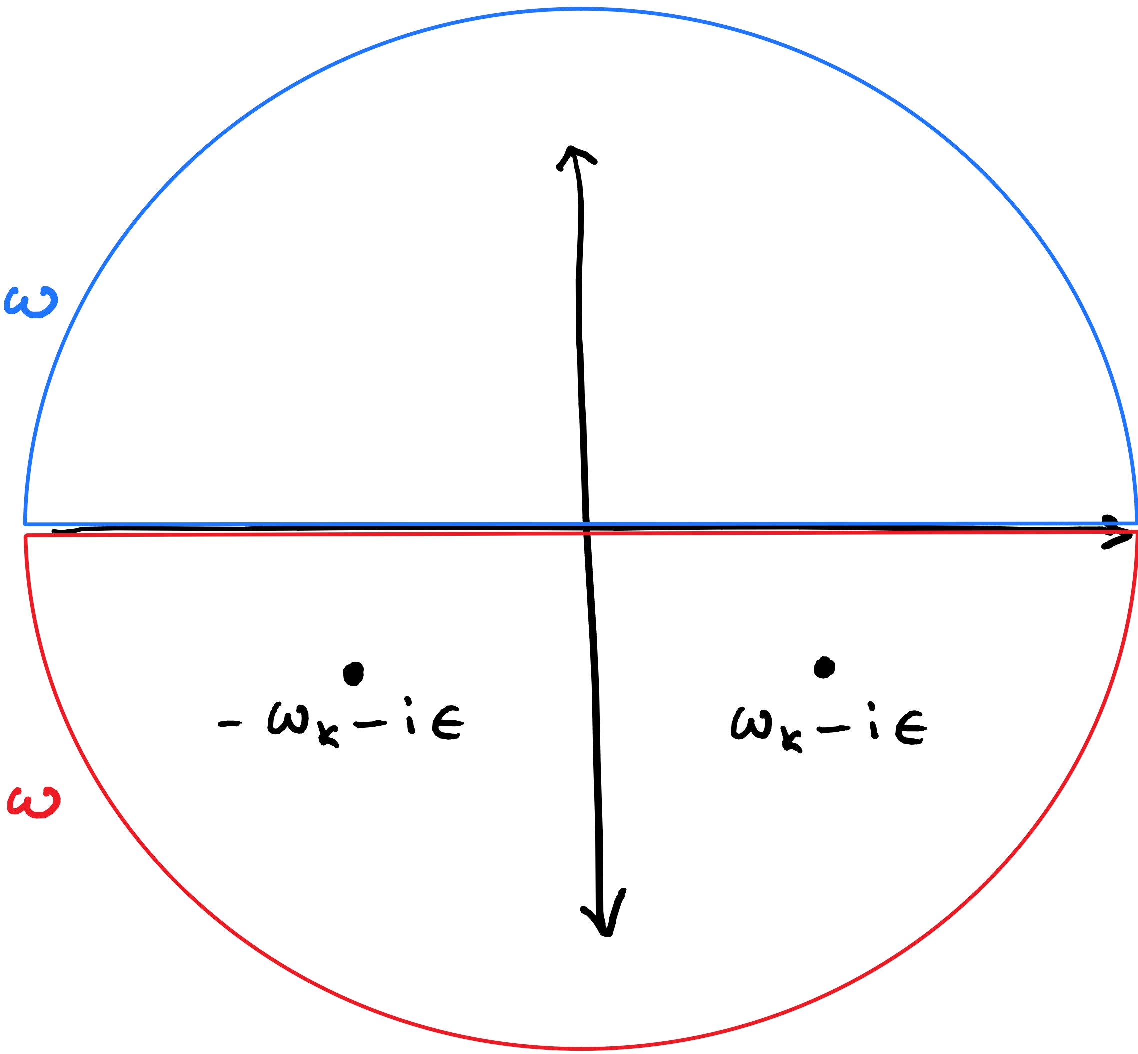
$$\frac{e^{i(\omega_x + i\omega_y)|\tau|}}{\omega_x + i\omega_y} = 0$$



$$-\oint_\gamma d\omega$$

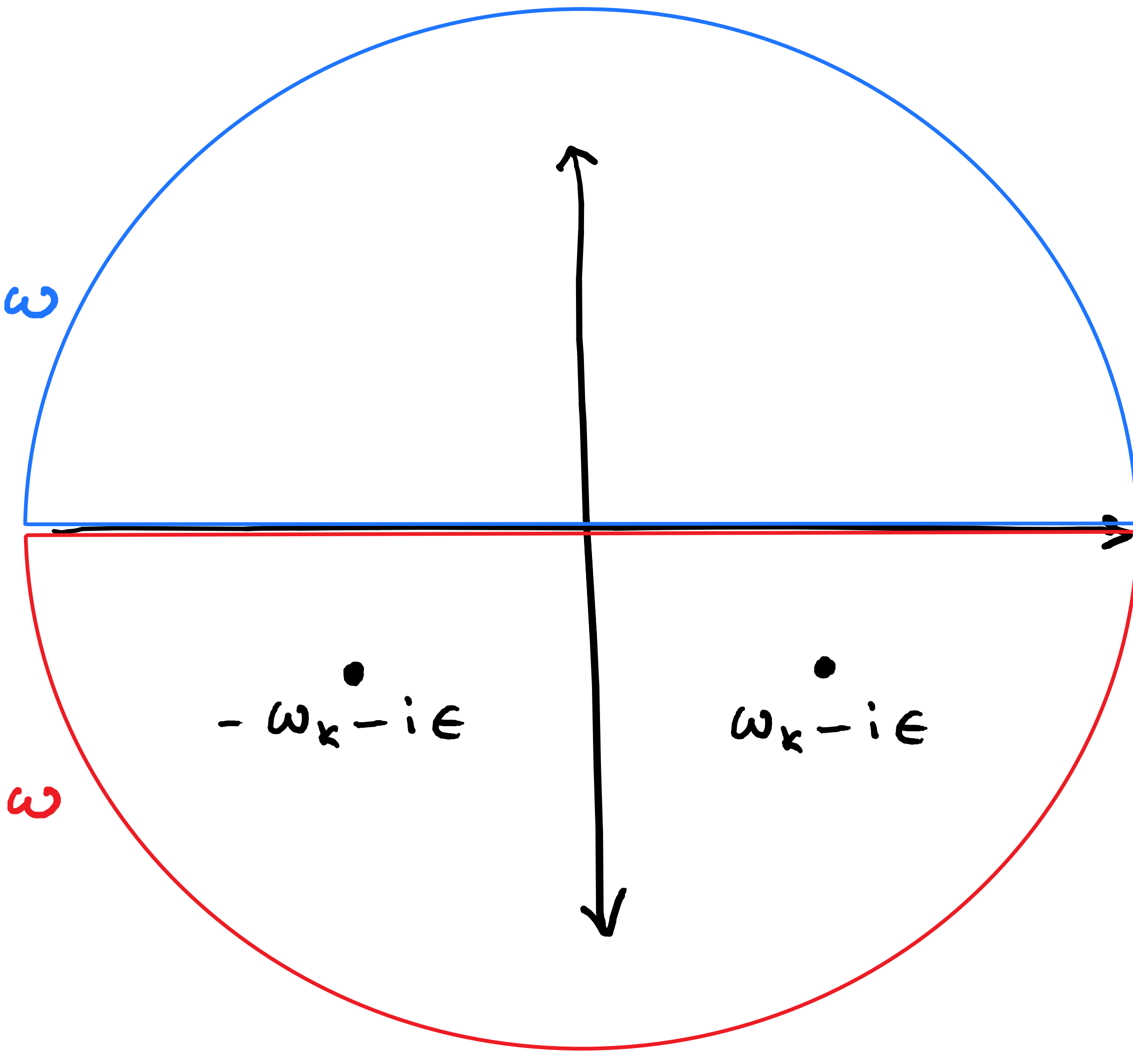
$$\frac{e^{-i(\omega_x + i\omega_y)|\tau|}}{\omega_x + i\omega_y} = 0$$

Consistency check:

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega \, \frac{e^{+i\omega_\epsilon \tau}}{\omega_\epsilon \mp \omega_k} \overset{\tau>0}{=} 0$$

$$\overset{\tau<0}{=} -2\pi i \, \mathrm{Res}(\pm\omega_k) = -i \, e^{\pm \omega_k \tau}$$

$$+\oint_\gamma d\omega \qquad \frac{e^{i(\omega_x + i\omega_y)|\tau|}}{\omega_x + i\omega_y} = 0$$



$$-\oint_\gamma d\omega \qquad \frac{e^{-i(\omega_x + i\omega_y)|\tau|}}{\omega_x + i\omega_y} = 0$$

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$$\langle\langle \tilde{W}(t); \tilde{V}_\beta(t') \rangle\rangle = \sum_{k=0}^{\infty} \Big(g_k^+(\tau) \langle \Psi_0 | W | \Psi_k \rangle \langle \Psi_k | V_\beta | \Psi_0 \rangle$$

$$- g_k^-(\tau) \langle \Psi_0 | V_\beta | \Psi_k \rangle \langle \Psi_k | W | \Psi_0 \rangle \Big)$$

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$$\langle\langle W ; V_\beta \rangle\rangle_{\omega_\epsilon}$$

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$$\langle\langle W ; V_\beta \rangle\rangle_{\omega_\epsilon} = \sum_{k=0}^{\infty} \Big(g_k^+(\omega_\epsilon) \, \langle \Psi_0 | W | \Psi_k \rangle \langle \Psi_k | V_\beta | \Psi_0 \rangle - g_k^-(\omega_\epsilon) \, \langle \Psi_0 | V_\beta | \Psi_k \rangle \langle \Psi_k | W | \Psi_0 \rangle \Big)$$

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