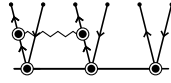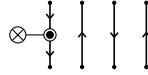1. Interpret the following coefficient graph algebraically, denoting the bare excitation operator by $\tilde{a}^{ijk}_{abc}$.

2. Interpret the following coefficient graph algebraically, denoting the top bare excitation operator by $\tilde{a}^{ij}_{ab}$ and the bottom one by $\tilde{a}^{cd}_{kl}$ and their interaction tensors by $^1\overline{\delta}^{ef}_{mn}$ and $^2\overline{\delta}^{mn}_{ef}$.

Solutions:

1. Axiom 4.1 gives

$$\text{(diagram)} = \frac{1}{2!2!3!} \sum_{\substack{defgh \\ lmn}} \text{(diagram)} = \frac{1}{2!2!3!} \sum_{\substack{defgh \\ lmn}} \overline{\delta}^{def}_{lmn} \overline{g}^{gh}_{de} c^{lmn}_{ghf} \vdots a^{l^{\circ 1}m^{\circ 2}n^{\circ 3}}_{d^{\bullet 1}e^{\bullet 2}f^{\bullet 3}} a^{d^{\bullet 1}e^{\bullet 2}}_{g^{\bullet 4}h^{\bullet 5}} a^{g^{\bullet 4}h^{\bullet 5}f^{\bullet 3}}_{l^{\circ 1}m^{\circ 2}n^{\circ 3}} \vdots$$

since there are two pairs of equivalent lines on the repulsion operator, and the CI triples operator has a set of three equivalent lines. The interaction tensor for the bare excitation operator is $\overline{\delta}^{def}_{lmn} = \hat{P}^{(i/j/k)}_{(a/b/c)} \delta^d_a \delta^e_b \delta^f_c \delta^i_l \delta^j_m \delta^k_n$, so this can be simplified as

$$\sum_{\substack{def \\ lmn}} \overline{\delta}^{def}_{lmn} T^{lmn}_{def} = P^{(i/j/k)}_{(a/b/c)} T^{ijk}_{abc} \qquad \text{where} \quad T^{lmn}_{def} \equiv \frac{1}{2!2!3!} \sum_{gh} \overline{g}^{gh}_{de} c^{lmn}_{ghf} \vdots a^{l^{\circ 1}m^{\circ 2}n^{\circ 3}}_{d^{\bullet 1}e^{\bullet 2}f^{\bullet 3}} a^{d^{\bullet 1}e^{\bullet 2}}_{g^{\bullet 4}h^{\bullet 5}} a^{g^{\bullet 4}h^{\bullet 5}f^{\bullet 3}}_{l^{\circ 1}m^{\circ 2}n^{\circ 3}} \vdots.$$

Using item 3 in Remark 4.3, the contracted operator string evaluates as follows

$$\vdots a^{l^{\circ 1}m^{\circ 2}n^{\circ 3}}_{d^{\bullet 1}e^{\bullet 2}f^{\bullet 3}} a^{d^{\bullet 1}e^{\bullet 2}}_{g^{\bullet 4}h^{\bullet 5}} a^{g^{\bullet 4}h^{\bullet 5}f^{\bullet 3}}_{l^{\circ 1}m^{\circ 2}n^{\circ 3}} \vdots = (-1)^{3+3} = +1$$

since there are three hole lines and three loops in the graph. At this point, we have simplified our interpretation to the following

$$\text{(diagram)} = \frac{1}{2!2!3!} \sum_{de} \hat{P}^{(i/j/k)}_{(a/b/c)} \overline{g}^{de}_{ab} c^{ijk}_{dec}$$

where I have relabeled the summation indices $g \mapsto d$, $h \mapsto e$. Finally, using item 4 under Remark 4.3, we can cancel the degeneracy factors $2!3!$ coming from equivalent coefficient lines by replacing $\hat{P}^{(i/j/k)}_{(a/b/c)}$ with $\hat{P}_{(ab/c)}$. This works because the operand $\overline{g}^{de}_{ab} c^{ijk}_{dec}$ is already antisymmetric with respect to $\{a, b\}$ and $\{i, j, k\}$.

$$\text{(diagram)} = \frac{1}{2!} \sum_{de} \hat{P}_{(ab/c)} \overline{g}^{de}_{ab} c^{ijk}_{dec}$$

2. Axiom 4.1 gives

$$\text{(diagram)} = \frac{1}{2!} \sum_{\substack{ef \\ mno}} \text{(diagram)} = \frac{1}{2!} \sum_{\substack{ef \\ mno}} {}^1\overline{\delta}^{ef}_{mn} f^m_o \, {}^2\overline{\delta}^{on}_{ef} \vdots a^{m^{\circ 1}n^{\circ 2}}_{e^{\bullet 1}f^{\bullet 2}} a^{o^{\circ 3}}_{m^{\circ 1}} a^{e^{\bullet 1}f^{\bullet 2}}_{o^{\circ 3}n^{\circ 2}} \vdots$$

since the two particle lines are equivalent. Using item 3 in Remark 4.3, the operator string evaluates to

$$\vdots a^{m^{\circ 1}n^{\circ 2}}_{e^{\bullet 1}f^{\bullet 2}} a^{o^{\circ 3}}_{m^{\circ 1}} a^{e^{\bullet 1}f^{\bullet 2}}_{o^{\circ 3}n^{\circ 2}} \vdots = (-1)^{3+2} = -1$$

since there are three hole lines and two loops in the diagram. Substituting in the definitions of the interaction tensors, we can simplify the result as follows.

$$\text{(diagram)} = -\frac{1}{2!} \sum_{\substack{ef \\ mno}} {}^1\overline{\delta}^{ef}_{mn} f^m_o \, {}^2\overline{\delta}^{on}_{ef} = -\frac{1}{2!} \sum_{\substack{ef \\ mno}} \left(\hat{P}^{(i/j)}_{(a/b)} \delta^e_a \delta^f_b \delta^i_m \delta^j_n \right) f^m_o \left(\hat{P}^{(c/d)}_{(k/l)} \delta^o_k \delta^n_l \delta^c_e \delta^d_f \right)$$

$$= -\frac{1}{2!} \hat{P}^{(i/j)}_{(a/b)} \hat{P}^{(c/d)}_{(k/l)} f^i_k \delta^j_l \delta^c_a \delta^d_b$$

Finally, using item 4 under Remark 4.3, we can cancel the degeneracy factor $2!$ by replacing $\hat{P}^{(i/j)}_{(a/b)}$ with $\hat{P}^{(i/j)}$.

$$\text{(diagram)} = -\hat{P}^{(i/j|c/d)}_{(k/l)} f^i_k \delta^j_l \delta^c_a \delta^d_b$$