1. Derive the spectral resolution of the resolvent operator by expanding Q in $R_0 = -(H_0)^{-1}Q$.
2. Show that $E(\lambda) = \langle\Phi|\lambda V_{\mathrm{c}}|\Psi(\lambda)\rangle$.
3. Derive the following by induction

$$\frac{\partial^{m+1}}{\partial\lambda^{m+1}}\langle\Phi|\lambda V_{\mathrm{c}}|\Psi(\lambda)\rangle = (m+1)\langle\Phi|V_{\mathrm{c}}|\frac{\partial^{m}}{\partial\lambda^{m}}\Psi(\lambda)\rangle + \langle\Phi|\lambda V_{\mathrm{c}}|\frac{\partial^{m+1}}{\partial\lambda^{m+1}}\Psi(\lambda)\rangle \tag{1}$$

   and then use it to prove $E^{(m+1)} = \langle\Phi|V_{\mathrm{c}}|\Psi^{(m)}\rangle$.

4. Derive the recursive equation for $\Psi(\lambda)$.

$$\Psi(\lambda) = \Phi + R_0(\lambda V_{\mathrm{c}} - E(\lambda))\Psi(\lambda) \tag{2}$$

5. Derive all wavefunction and energy contributions up to second and third order, respectively.[1]
6. Determine the first- and second-order components of the CI coefficients.
7. Show that ${}^{(2)}C_4 = \frac{1}{2}{}^{(1)}C_2^2$.
8. Prove the bracketing theorem and the linked diagram theorem for the fourth-order energy.

$$E^{(4)} = \langle V_{\mathrm{c}}R_0V_{\mathrm{c}}R_0V_{\mathrm{c}}R_0V_{\mathrm{c}}\rangle - \langle V_{\mathrm{c}}R_0\langle V_{\mathrm{c}}R_0V_{\mathrm{c}}\rangle R_0V_{\mathrm{c}}\rangle = \langle V_{\mathrm{c}}R_0V_{\mathrm{c}}R_0V_{\mathrm{c}}R_0V_{\mathrm{c}}\rangle_{\mathrm{L}}$$

[1] You may assume Brillouin's theorem for this problem and the ones that follow.