1. Evaluate each of the following using the definition of contraction.

$$
\begin{array}{cccc}
:\overbracket{a_p a_q}: = ? & :\overbracket{a_p^\dagger a_q}: = ? & :\overbracket{a_p a_q^\dagger}: = ? & :\overbracket{a_p^\dagger a_q^\dagger}: = ? \\
\vdots\overbracket{a_a a_b}\vdots = ? & \vdots\overbracket{a_a^\dagger a_b}\vdots = ? & \vdots\overbracket{a_a a_b^\dagger}\vdots = ? & \vdots\overbracket{a_a^\dagger a_b^\dagger}\vdots = ? \\
\vdots\overbracket{a_a a_i}\vdots = ? & \vdots\overbracket{a_a^\dagger a_i}\vdots = ? & \vdots\overbracket{a_a a_i^\dagger}\vdots = ? & \vdots\overbracket{a_a^\dagger a_i^\dagger}\vdots = ? \\
\vdots\overbracket{a_i a_a}\vdots = ? & \vdots\overbracket{a_i^\dagger a_a}\vdots = ? & \vdots\overbracket{a_i a_a^\dagger}\vdots = ? & \vdots\overbracket{a_i^\dagger a_a^\dagger}\vdots = ? \\
\vdots\overbracket{a_i a_j}\vdots = ? & \vdots\overbracket{a_i^\dagger a_j}\vdots = ? & \vdots\overbracket{a_i a_j^\dagger}\vdots = ? & \vdots\overbracket{a_i^\dagger a_j^\dagger}\vdots = ?
\end{array}
$$

2. Assuming Wick's theorem, prove the following corollaries in your own words.
   (a) A product of normal-ordered operators equals the normal-ordering of their product plus all cross-contractions.
   (b) The vacuum expectation value of an operator is the sum of its complete contractions.

3. Visually illustrate the proof of the phase factor for completely contracted products with a non-trivial example – something with more than two line intersections.

4. Derive the expansion of H_e in terms of Φ-normal-ordered excitations.

5. Derive Slater's rules using Wick's theorem.

$$
\langle\Phi|H_e|\Phi\rangle = \sum_i h_{ii} + \tfrac{1}{2}\sum_{ij}\langle ij||ij\rangle \qquad \langle\Phi|H_e|\Phi_i^a\rangle = h_{ia} + \sum_j \langle ij||aj\rangle \qquad \langle\Phi|H_e|\Phi_{ij}^{ab}\rangle = \langle ij||ab\rangle
$$

   Also, explain why the second equation evaluates to zero for canonical Hartree-Fock orbitals.[1] This is known as *Brillouin's theorem*.

6. Derive the CIS matrix elements using Wick's theorem.

[1]Hint: Project the canonical Hartree-Fock equation by another spin-orbital and expand $f_{pq} = \langle\psi_p|\hat{f}\psi_q\rangle$ in terms of one- and two-electron integrals.