#2) Prove that the leading contribution to the k-tuples operator in CI (C_k) has order $\lceil k/2 \rceil$.

$C_k = C_k^{(0)} + C_k^{(1)} + C_k^{(2)} + \ldots$] we want to show that the 1st nonzero term is $C_k^{(m)}$, $m = \lceil k/2 \rceil$.

Start with C_1. C_k has excitation level k, so it can only contribute to parts of the wave function with excitation level 1.

$\psi = \Phi + \psi^{(1)} + \psi^{(2)} + \ldots$] lots of these could have pieces with excitation level 1, but the contribution will only be leading if it's the lowest order with that property.

excitation level 0

So consider $\psi^{(1)} = R_0 V_c \Phi =$

excitation levels

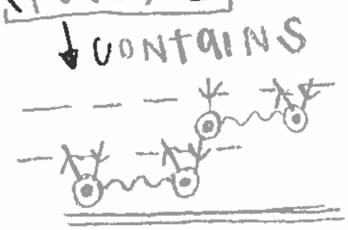
Must be part of C_1 Must be part of C_2

$\Leftarrow$ so C_1 and C_2 have leading order $1 = \lceil 1/2 \rceil = \lceil 2/2 \rceil$ ✓.

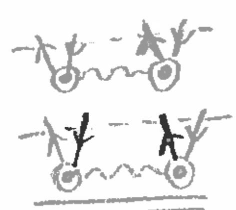
Now we will proceed by induction on (odd, even) pairs $(k, k+1)$, and show that C_k and C_{k+1} both have leading order $\frac{k+1}{2}$. [For $k \geq 3$]

Base case: $\overset{k=3}{(3,4)}$. C_3 and C_4 have excitation levels 3 and 4, so we have already shown above that they cannot contribute at orders 0 or 1. So if they contribute at order 2, that's their leading order.

$\psi^{(2)} = \underbrace{(R_0 V_c)^2 \Phi}_{\text{contains}} + \langle\ \rangle$ (Bracketing theorem)



and



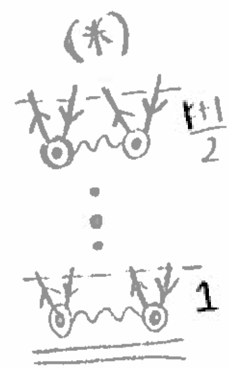
} excitation levels 3+4

So C_3 and C_4 have leading order $2 = \frac{k+1}{2}$ ✓

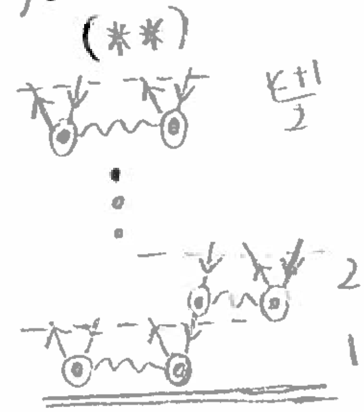
Induction step: suppose that U_k and U_{k+1} both have leading orders $\frac{k+1}{2}$ $(k \geq 3)$.

$\psi^{(\frac{k+1}{2})} = (R_0 V_c)^{(\frac{k+1}{2})} + \langle\ \rangle \rightarrow$ which contains:

(*)



(**)



So the max. and max.-1 excitation levels in $\psi^{(\frac{k+1}{2})}$ are $k+1$ and k. Thus U_k and U_{k+1} have leading contributions of order $(\frac{k+1}{2})$.

Excitation level: $2(\frac{k+1}{2}) = \boxed{k+1}$

excitation level: $1 + 2(\frac{k+1}{2} - 1) = 1 + k + 1 - 2 = \boxed{k}$

Now consider U_{k+2} and U_{k+3}. We have already shown that the max. excitation level of $\psi^{(\frac{k+1}{2})}$ is $k+1$, and so the lowest possible leading order is $\psi^{(\frac{k+3}{2})} = (R_0 V_c)^{(\frac{k+3}{2})} + \langle\ \rangle$

$=$  $(R_0 V_c)^{\frac{k+1}{2}}$

We can easily show that $\psi^{(\frac{k+3}{2})}$ contains terms of excitation level $k+3$, $k+2$ by adding one of these:  to the top of each diagrams (*) and (**).

Thus U_{k+2} and U_{k+3} have leading order $\frac{(k+3)}{2}$ ∎.