1. Show why the following relationships hold for antisymmetric functions $\Psi, \Psi' \in \mathcal{F}_n$.

$$\sum_{i=1}^{n} \langle \Psi | \hat{h}(i) \Psi' \rangle = n \langle \Psi | \hat{h}(1) \Psi' \rangle \qquad \sum_{i<j}^{n} \langle \Psi | \hat{g}(i,j) \Psi' \rangle = \tfrac{n(n-1)}{2} \langle \Psi | \hat{g}(1,2) \Psi' \rangle$$

2. Show how to make the following rearrangement.

$$\tfrac{1}{2} \sum_{pqrs} \langle pq|rs \rangle a_p^\dagger a_q^\dagger a_s a_r = \tfrac{1}{4} \sum_{pqrs} \langle pq||rs \rangle a_p^\dagger a_q^\dagger a_s a_r$$

3. For the integral-operator definition of $\hat{a}_p$, show that the following relationships hold.

   (a) $(\hat{a}_p \Phi_{(p_1 \cdots p_n)})(2, \cdots, n) = \begin{cases} (-)^{k-1} \Phi_{(p_1 \cdots \cancel{p_k} \cdots p_n)}(2, \cdots, n) & p = p_k \in (p_1 \cdots p_n) \\ 0 & \text{otherwise} \end{cases}$

   (b) $\hat{a}_p \hat{a}_q = -\hat{a}_q \hat{a}_p$

   (c) $\Psi(1, \cdots, n) = \frac{1}{\sqrt{n}} \sum_{p}^{\infty} \psi_p(1) \left(\hat{a}_p \Psi \right) (2, \cdots, n)$

   (d) $\Psi(1, \cdots, n) = \frac{1}{\sqrt{n(n-1)}} \sum_{pq}^{\infty} \psi_p(1) \psi_q(2) (\hat{a}_q \hat{a}_p \Psi)(3, \cdots, n)$

4. Using your own words, prove that $c_p = a_p^\dagger$. You may use either the determinant formalism or the occupation number formalism.

5. Verify the anticommutator relation $[q, q']_+ = \delta_{q'q^\dagger}$ by proving each of the following cases.

$$[a_p, a_q]_+ = 0 \qquad [a_p^\dagger, a_q^\dagger]_+ = 0 \qquad [a_p, a_q^\dagger]_+ = 0 \;\; (p \neq q) \qquad [a_p, a_p^\dagger]_+ = 1$$

6. Show that the occupation-number definition of a_p and c_p is consistent with the determinant definition.

7. Do Problem 1.4 in Helgaker's big purple book.

8. Derive Slater's rules using second quantization. Where necessary, explain in words why a given term vanishes. (Hint: Use particle-hole isomorphism.)

   (a) $\langle \Phi | H_e | \Phi \rangle = \sum_i h_{ii} + \frac{1}{2} \sum_{ij} \langle ij||ij \rangle$

   (b) $\langle \Phi | H_e | \Phi_i^a \rangle = h_{ia} + \sum_j \langle ij||aj \rangle$

   (c) $\langle \Phi | H_e | \Phi_{ij}^{ab} \rangle = \langle ij||ab \rangle$

   (d) $\langle \Phi | H_e | \Phi_{ijk}^{abc} \rangle = 0$

   For extra credit, show how to derive them without using particle-hole isomorphism.