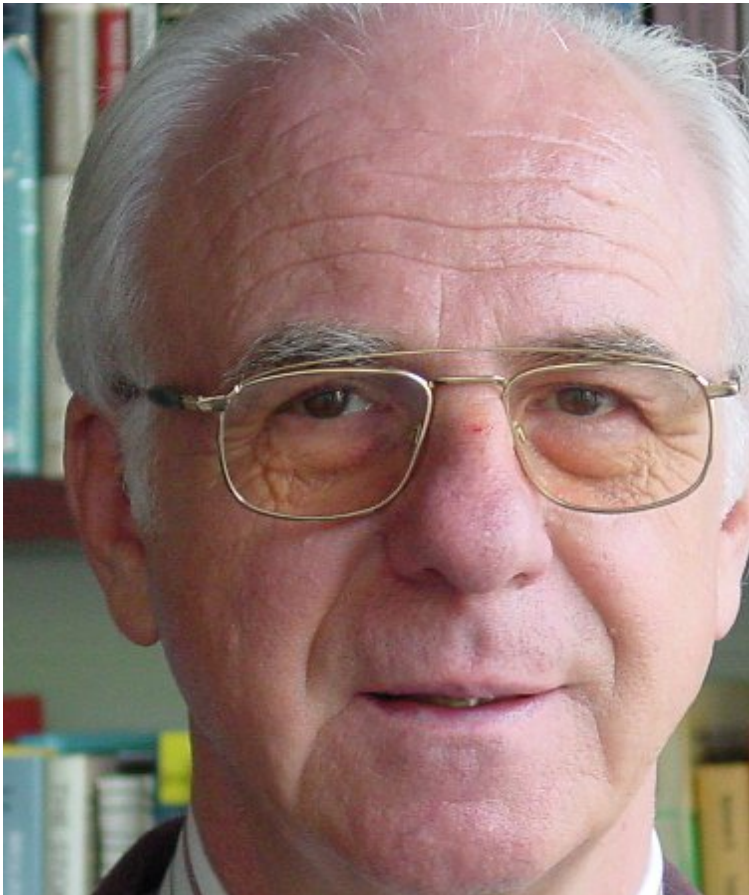


# Kutzelnigg-Mukherjee notation



Einstein notation:

Einstein notation:

$$A B = C$$

Einstein notation:

$$AB = C$$



$$\sum_k A_i^k B_k^j = C_i^j$$

Einstein notation:

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$$\left(\sum_k\right) A_i^k B_k^j = C_i^j$$

Einstein notation:

$$AB = C$$



$$A_i^{\textcolor{red}{k}} B_{\textcolor{red}{k}}^j = C_i^j$$



The  
summation  
is implicit.

Kutzelnigg - Mukherjee notation:

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$$h_{pq} \rightarrow h_p^q$$



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Kutzelnigg - Mukherjee notation:

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$$a_r^\dagger a_q \rightarrow a^p a_q \rightarrow a_q^p$$

$$a_p^\dagger a_q^\dagger a_s a_r \rightarrow a^p a^q a_s a_r$$

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$$a_p^\dagger a_q^\dagger a_s a_r \rightarrow a^p a^q a_s a_r \rightarrow a_{rs}^{pq}$$

$$H_e = h_p^q a_q^p + \frac{1}{4} \bar{g}_{pq}^{rs} a_{rs}^{pq}$$

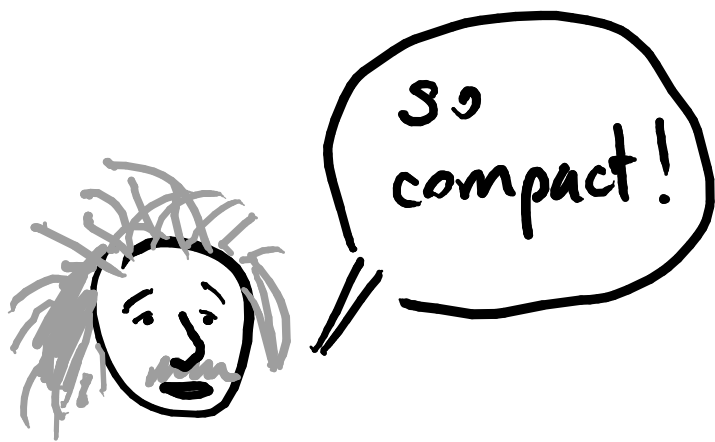
Kutzelnigg-Mukherjee notation:

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$$a_p^\dagger a_q^\dagger a_s a_r \rightarrow a_p^{\dagger} a_q^{\dagger} a_s a_r \rightarrow a_{rs}^{pq}$$



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Wick's theorem:

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$$a^p_q a^r_s$$



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$$a_q^p a_s^r = a_{qs}^{pr} + \delta_q^r a_s^p$$

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$$a_{rs}^{pq} a_u^\dagger$$

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simple rule:

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simple rule: "pair up uncontracted partners"

$\Phi$ -normal ordering:

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$$\gamma_q^p \equiv \langle \Phi | a^p a_q | \Phi \rangle$$

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mind the  
p's and q's!

Contractions:

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$$a_p \cdot a^q \equiv \overbrace{a_p a^q}$$

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$$a^{p\cdot} a_{q\cdot} \equiv \overbrace{a^p a_q}$$

Contractions:

$$a_p \cdot a^q \equiv \overbrace{a_p a^q}$$

$$a^{p^0} a_{q^0} \equiv \overbrace{a^{p^0} a_{q^0}}$$

particle contraction

Contractions:

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particle contraction

$$a^p \cdot a_q \equiv \overbrace{a^p a_q}$$

hole contraction



Contractions:

$$a_{p\cdot} a^{q\cdot} \equiv \overbrace{a_p a^q}^{\text{particle contraction}}$$

particle contraction

$$a^{p\circ} a_{q\circ} \equiv \overbrace{a^p a_q}^{\text{hole contraction}}$$

hole contraction

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hole contraction

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$$\Rightarrow a_q^p = -\eta_q^p$$

Contractions:

$$a_p \cdot a^q \cdot \equiv \overbrace{a_p a^q}$$

particle contraction

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$$a^q \cdot a_p \cdot \equiv - \overbrace{a_p a^q}$$

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$$a_q^p \circ = \gamma_q^p$$

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Wick's theorem:

$$\tilde{a}_q^p \tilde{a}_s^r = \tilde{a}_{rs}^{pq} + \tilde{a}_{q \bullet s}^{p \bullet r}$$

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Wick's theorem:

$$\begin{aligned}\tilde{a}_q^p \tilde{a}_s^r &= \tilde{a}_{rs}^{pq} + \tilde{a}_{q^0 s}^{p r^0} + \tilde{a}_q^{p^0 r} s^0 + \tilde{a}_{q^0 s^0}^{p^0 r^0} \\ &= \tilde{a}_{rs}^{pq} + \eta_q^r \tilde{a}_s^p - \gamma_s^p \tilde{a}_q^r\end{aligned}$$

Wick's theorem:

$$\begin{aligned}\tilde{a}_q^p \tilde{a}_s^r &= \tilde{a}_{rs}^{pq} + \tilde{a}_{q \bullet s}^{p \bullet r} + \tilde{a}_q^{p \bullet} \tilde{a}_s^{\bullet r} + \tilde{a}_{q \bullet}^{p \bullet} \tilde{a}_s^{\bullet r} \\ &= \tilde{a}_{rs}^{pq} + \eta_q^r \tilde{a}_s^p - \gamma_s^p \tilde{a}_q^r + \gamma_s^p \eta_q^r\end{aligned}$$



Index antisymmetrizers:

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$$\hat{P}_{(p/q)} \tau_{pq} \equiv \tau_{pq} - \tau_{qp}$$

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Example:

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$a_{rs}^{pq}$

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$$a_{rs}^{pq} = \tilde{a}_{rs}^{pq} + \tilde{a}_{r^0s}^{p^0q} + \tilde{a}_r^{p^0q} s^0 + \tilde{a}_{r^0s}^{pq^0} + \tilde{a}_{rs^0}^{pq^0} + \tilde{a}_{r^0s^{00}}^{p^0q^{00}} + \tilde{a}_{r^{00}s^0}^{p^0q^{00}}$$



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=

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$$= \tilde{a}_{rs}^{pq}$$

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 a_{rs}^{pq} &= \tilde{a}_{rs}^{pq} + \underbrace{\tilde{a}_{r^0s}^{p^0q} + \tilde{a}_r^{p^0q^0s} + \tilde{a}_{r^0s}^{pq^0} + \tilde{a}_{rs^0}^{pq^0} + \tilde{a}_{r^0s^{00}}^{p^0q^{00}} + \tilde{a}_{r^{00}s^0}^{p^0q^{00}}}_{\hat{p}_{(r/s)}^{(p/q)} \tilde{a}_{r^0s}^{p^0q}} \\
 &= \tilde{a}_{rs}^{pq} + \hat{p}_{(r/s)}^{(p/q)} \tilde{a}_{r^0s}^{p^0q}
 \end{aligned}$$

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 a_{rs}^{pq} &= \tilde{a}_{rs}^{pq} + \underbrace{\tilde{a}_{r^0s}^{p^0q} + \tilde{a}_r^{p^0q^0s} + \tilde{a}_{r^0s}^{pq^0} + \tilde{a}_{rs^0}^{pq^0}}_{\hat{P}_{(r/s)}^{(p/q)} \tilde{a}_{r^0s}^{p^0q}} + \underbrace{\tilde{a}_{r^0s^{00}}^{p^0q^{00}} + \tilde{a}_{r^{00}s^0}^{p^0q^{00}}}_{\hat{P}_{(r/s)} \tilde{a}_{r^0s^{00}}^{p^0q^{00}}} \\
 &= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \tilde{a}_{r^0s}^{p^0q} + \hat{P}_{(r/s)} \tilde{a}_{r^0s^{00}}^{p^0q^{00}}
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 &= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \tilde{a}_{r^0s}^{p^0q} + \hat{P}_{(r/s)} \tilde{a}_{r^0s^{00}}^{p^0q^{00}} \\
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 &= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \tilde{a}_{r^0s}^{p^0q} + \hat{P}_{(r/s)} \tilde{a}_{r^0s^{00}}^{p^0q^{00}} \\
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$$= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \tilde{a}_{r^0 s}^{p^0 q} + \hat{P}_{(r/s)} \tilde{a}_{r^0 s^{00}}^{p^0 q^{00}}$$

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 &= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \tilde{a}_{r^0s}^{p^0q} + \hat{P}_{(r/s)} \tilde{a}_{r^0s^{00}}^{p^0q^{00}} \\
 &= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \gamma_r^p \tilde{a}_s^q + \hat{P}_{(r/s)} \gamma_r^p \gamma_s^q
 \end{aligned}$$



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$$\bar{g}_{pq}^{rs} a_{rs}^{pq}$$

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$$\bar{g}_{pq}^{rs} a_{rs}^{pq} = \bar{g}_{pq}^{rs} \tilde{a}_{rs}^{pq} + \bar{g}_{pq}^{rs} \cdot 4 \cdot \gamma_r^p \tilde{a}_s^q + \bar{g}_{pq}^{rs} \hat{P}_{(r/s)} \gamma_r^p \gamma_s^q$$

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$$a_{rs}^{pq} = \tilde{a}_{rs}^{pq} + \underbrace{\tilde{a}_{r^0 s}^{p^0 q} + \tilde{a}_r^{p^0 q^0 s} + \tilde{a}_{r^0 s}^{p^0 q^0} + \tilde{a}_{rs^0}^{p^0 q^0}}_{\text{}} + \underbrace{\tilde{a}_{r^0 s^0}^{p^0 q^0} + \tilde{a}_{r^0 s^0}^{p^0 q^0}}_{\text{}}$$

$$= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \tilde{a}_{r^0 s}^{p^0 q} + \hat{P}_{(r/s)} \tilde{a}_{r^0 s^0}^{p^0 q^0}$$

$$= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \gamma_r^p \tilde{a}_s^q + \hat{P}_{(r/s)} \gamma_r^p \gamma_s^q$$

$$\bar{g}_{pq}^{rs} a_{rs}^{pq} = \bar{g}_{pq}^{rs} \tilde{a}_{rs}^{pq} + \bar{g}_{pq}^{rs} \cdot 4 \cdot \gamma_r^p \tilde{a}_s^q + \bar{g}_{pq}^{rs} \cdot 2 \cdot \gamma_r^p \gamma_s^q$$

→ Someone show it on the board!

Example:

$$\begin{aligned}
 a_{rs}^{pq} &= \tilde{a}_{rs}^{pq} + \underbrace{\tilde{a}_{r^0 s}^{p^0 q} + \tilde{a}_r^{p^0 q^0 s} + \tilde{a}_{r^0 s}^{p^0 q^0} + \tilde{a}_{rs^0}^{p^0 q^0}}_{\hat{P}_{(r/s)}^{(p/q)} \tilde{a}_{r^0 s}^{p^0 q}} + \underbrace{\tilde{a}_{r^0 s^0}^{p^0 q^0} + \tilde{a}_{r^0 s^0}^{p^0 q^0}}_{\hat{P}_{(r/s)} \tilde{a}_{r^0 s^0}^{p^0 q^0}} \\
 &= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \tilde{a}_{r^0 s}^{p^0 q} + \hat{P}_{(r/s)} \tilde{a}_{r^0 s^0}^{p^0 q^0} \\
 &= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \gamma_r^p \tilde{a}_s^q + \hat{P}_{(r/s)} \gamma_r^p \gamma_s^q
 \end{aligned}$$

$$\bar{g}_{pq}^{rs} a_{rs}^{pq} = \bar{g}_{pq}^{rs} \tilde{a}_{rs}^{pq} + \bar{g}_{pq}^{rs} \cdot 4 \cdot \gamma_r^p \tilde{a}_s^q + \bar{g}_{pq}^{rs} \cdot 2 \cdot \gamma_r^p \gamma_s^q$$

→ Someone show it on the board!

→ Use this to derive  $\mathbb{E}$ -normal  $H_e$



Now try  $\langle \Phi_i^a | H_e | \Phi_j^b \rangle$ .

the end.