p. t.

model Hamiltonian: Ho≈ Ho

Ho & Hc

Ho\$ = & \$ \$ 0

Ho & Hc

 $H_0 = \mathcal{E}_0 = \mathcal{E}_0$ (known e.f.s)

$$H_0 \not=_{\sigma} = \mathcal{E}_{\sigma} \not=_{\sigma}$$
 (known e.f.s)



Ho & Hc

 $H_0 \not=_{\sigma} = \mathcal{E}_{\sigma} \not=_{\sigma}$ (known e.f.s)

 $\Rightarrow H_c$

Ho≈ Hc

 $H_0 = \mathcal{E}_0 = \mathcal{E}_0$ (known e.f.s)

 \Rightarrow $H_c = [\langle \pm_{el} H_c | \pm_{\tau} \rangle]$

Ho≈ Hc

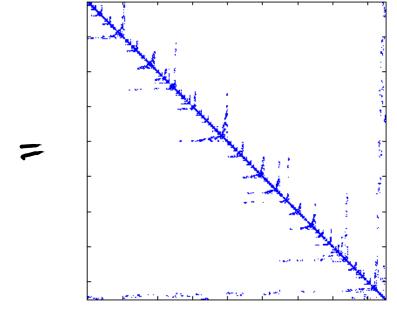
 $H_0 = \mathcal{E}_0 = \mathcal{E}_0$ (known e.f.s)

$$\Rightarrow$$
 $H_c = [\langle \pm_{\sigma} | H_c | \pm_{\tau} \rangle]$

Ho≈ Hc

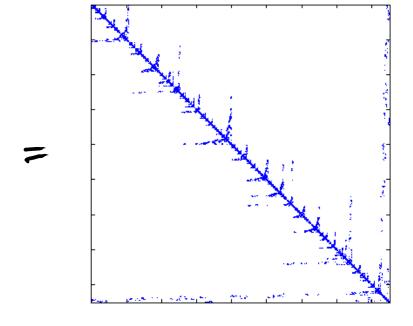
 $H_0 \not= \varepsilon = \varepsilon_0 \not= \sigma$ (known e.f.s)

 \Rightarrow $H_c = [\langle \xi_{el} H_c | \Phi_{\tau} \rangle]$



$$H_0 = \mathcal{E}_0 = \mathcal{E}_0$$
 (known e.f.s)

$$\Rightarrow$$
 $H_c = [\langle \xi_{el} H_c | \Phi_{\tau} \rangle]$

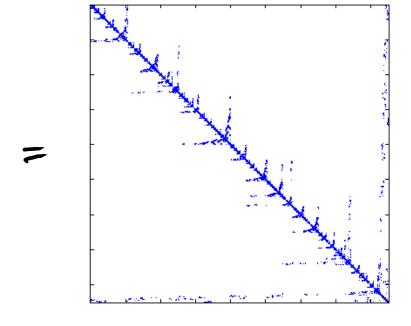


sparse

Ho≈ Hc

 $H_0 = \mathcal{E}_0 = \mathcal{E}_0$ (known e.f.s)

$$\Rightarrow$$
 $H_c = [\langle \xi_0 | H_c | \Phi_\tau \rangle]$



sparse, diagonally dominant

 $(\Phi \approx \Psi)$

$$(\Phi \approx \Psi)$$

$$H_o = f_\rho^\rho \tilde{a}_\rho^\rho$$

$$(\Phi \approx \Psi)$$

$$H_o = f_\rho^r \tilde{a}_\rho^r$$
 works fine

$$(\Phi \approx \Psi)$$

$$H_0 = f_p^r \tilde{a}_p^r$$
 works fine

$$H_{o} = 0.4$$

$$(\Phi \approx \Psi)$$

$$H_0 = f_p^r \tilde{a}_p^r$$
 works fine

H.
$$\Phi_{ij...}^{ab...} = \mathcal{E}_{ij...}^{ab...} \Phi_{ij...}^{ab...}$$

$$(\Phi \approx \Psi)$$

H. $\Phi_{ij}^{ab...} = \mathcal{E}_{ij...}^{ab...} \Phi_{ij...}^{ab...}$

super convenient

Subspace

Projection
operator

Subspace

Projection
operator

"model space"

Subspace

Projection
operator

"model space"
span { \$ \frac{1}{2} \frac{1}{2}

Subspace resortains qualitative approx. for # "model space" span { \$ \$ \$ }

Projection operator

Subspace resortains qualitative approx. for # "model space" span { \$ \$ \$ }

Projection
operator

"orthogonal space"

Subspace resortains qualitative approx. for # "model space" span { \$ \(\P \) }

Projection
operator

"orthogonal space"
span { #= 30 } #= 30 ...

Subspace respondent qualitative approx. for 14 model space span { 4 }

contains last little bit of the to eek out quant. accuracy "orthogonal space"

span {\(\pm\)\ \Pi\)\ \Pi\)\ \Pi\)\ \Span \{\pm\}\ \Pi\)\ \Pi\)\ \Span \{\pm\}\ \Pi\)\ \Pi\)\ \Span \(\pm\)\ \Pi\)\ \Span \{\pm\}\ \Pi\)\ \Pi\)\ \Span \{\pm\}\ \Pi\)\ \Pi\)\ \Span \{\pm\}\ \Pi\}\ \Span \{\pm\}\ \Pi\)\ \Span \{\pm\}\ \Pi\}\ \Span \{\pm\}\ \Pi\)\ \Span \{\pm\}\ \Pi\}\ \Pi\

Projection
operator

Subspace respondent qualitative approx. for 14 model space span { 4 }

P= |重 > < 垂 |

"orthogonal space"

span { \(\pm_{i}^{\alpha} \) \(\pm_{ij}^{\alpha} \) \(\

Subspace resontains qualitative approx. for # model space span { # }

P+Q=1

$$P+Q=1$$
 $P^2=P$

$$P+Q=1$$
 $P^2=P$ $Q^2=Q$

$$P+Q=1$$
 $P^2=P$ $Q^2=Q$ $PQ=QP=0$

$$P+Q=1$$
 $P^2=P$ $Q^2=Q$ $PQ=QP=D$

with intermediate normalization:

$$P+Q=1$$
 $P^2=P$ $Q^2=Q$ $PQ=QP=0$

with intermediate normalization:

$$\rho^2 = P$$

$$Q^2 = Q$$

$$P+Q=1$$
 $P^2=P$ $Q^2=Q$ $PQ=QP=0$

with intermediate normalization:



$$Q = \sum_{k} \left(\frac{1}{k!}\right)^{2} \sum_{\substack{a_{1} \dots a_{k} \\ i_{1} \dots i_{k}}} \left| \underbrace{\pm_{i_{1} \dots i_{k}}^{a_{1} \dots a_{k}}} \right\rangle \left\langle \underbrace{\pm_{i_{1} \dots i_{k}}^{a_{1} \dots a_{k}}} \right|$$

$$Q = \sum_{i} |\underline{\mathbf{z}}_{i}\rangle\langle\underline{\mathbf{z}}_{i}|$$

+ ...

$$R_o = -H_o^{-1}$$

$$R_o = -H_o^{-1}$$
 ortho. space

$$R_0 = -H_0^{-1}$$
 ortho. space Why?

$$R_0 = -H_0^{-1}$$

Why?

 $H_0 \mid_{model space} = 0$

$$R_0 = -H_0^{-1}$$
 orthorspace

Why?

 $H_0 \mid_{\text{model space}} = 0$
 $singular \iff \text{non-invertible}$

 R_{o}

$$R_o = -H_o^{-1}Q$$

$$R_o = -H_o'Q$$

$$=\sum_{\mathbf{k}}\left(\frac{1}{\mathbf{k}!}\right)^{2}\sum_{\substack{\mathbf{q}_{1}\cdots\mathbf{q}_{\mathbf{k}}\\\mathbf{i}_{1}\cdots\mathbf{i}_{\mathbf{k}}}}\left(-H_{0}\right)^{-1}\left|\underline{\mathbf{F}}_{\mathbf{i}_{1}\cdots\mathbf{i}_{\mathbf{k}}}^{\mathbf{q}_{1}\cdots\mathbf{q}_{\mathbf{k}}}\right|$$

$$R_o = -H_o'Q$$

$$=\sum_{\mathbf{k}}\left(\frac{1}{\mathbf{k}!}\right)^{2}\sum_{\substack{q_{1}\cdots q_{k}\\i_{1}\cdots i_{k}}}\left(-\mathcal{E}_{i_{1}\cdots i_{k}}^{q_{1}\cdots q_{k}}\right)\left|\underline{\Phi}_{i_{1}\cdots i_{k}}^{q_{1}\cdots q_{k}}\right\rangle\left\langle\underline{\Phi}_{i_{1}\cdots i_{k}}^{q_{1}\cdots q_{k}}\right|$$

$$R_o = -H_o'Q$$

$$=\sum_{k}\left(\frac{1}{k!}\right)^{2}\sum_{\substack{q_{1}\cdots q_{k}\\i_{1}\cdots i_{k}}}\left(+\mathcal{E}_{q_{1}\cdots q_{k}}^{i_{1}\cdots i_{k}}\right)\left|\mathcal{E}_{i_{1}\cdots i_{k}}^{q_{1}\cdots q_{k}}\right\rangle\left\langle \Phi_{i_{1}\cdots i_{k}}^{q_{1}\cdots q_{k}}\right|$$

$$R_{o} = -H_{o}^{-1}Q$$

$$= \sum_{i} \frac{|\underline{\mathbf{E}}_{i}^{a}\rangle\langle\underline{\mathbf{\Phi}}_{i}^{a}|}{|\underline{\mathbf{E}}_{i}^{a}\rangle\langle\underline{\mathbf{E}}_{i}^{ab}\rangle\langle\underline{\mathbf{E}}_{ijk}^{abc}|}$$

$$+ (\frac{1}{3!})^{2} \sum_{\substack{abc \\ ijk}} \frac{|\underline{\mathbf{E}}_{ijk}^{abc}\rangle\langle\underline{\mathbf{E}}_{ijk}^{abc}|}{|\underline{\mathbf{E}}_{abc}^{ijk}\rangle\langle\underline{\mathbf{E}}_{abc}^{ijk}|}$$

$$+ (\frac{1}{3!})^{2} \sum_{\substack{abc \\ ijk}} \frac{|\underline{\mathbf{E}}_{ijk}^{abc}\rangle\langle\underline{\mathbf{E}}_{ijk}^{abc}|}{|\underline{\mathbf{E}}_{abc}^{ijk}\rangle\langle\underline{\mathbf{E}}_{abc}^{ijk}|}$$

+ ...

R. Dij ...

$$R_o \Phi_{ij}^{ab\cdots} = (\mathcal{E}_{ab\cdots}^{ij\cdots})^T \Phi_{ij\cdots}^{ab\cdots}$$

$$R_o \Phi_{ij}^{ab\cdots} = (\mathcal{E}_{ab\cdots}^{ij\cdots})^T \Phi_{ij\cdots}^{ab\cdots}$$

and

$$R_o \Phi_{ij}^{ab\cdots} = (\mathcal{E}_{ab\cdots}^{ij\cdots})^T \Phi_{ij\cdots}^{ab\cdots}$$

and

$$R_o \Phi_{ij}^{ab\cdots} = (\mathcal{E}_{ab\cdots}^{ij\cdots})^T \Phi_{ij\cdots}^{ab\cdots}$$

and

 $R_o X_1 \cdots X_n | \Phi \rangle$

$$R_o X_1 \cdots X_n | \Phi \rangle$$

$$=R_o:\overline{X_1\cdots X_n}:|\overline{\pm}\rangle$$

$$R_{o}X_{1}\cdots X_{n}|\Phi\rangle$$

$$=R_o:\overline{X_1\cdots X_n}:|\Phi\rangle$$

observations:

$$R_o X_1 \cdots X_n | \Phi \rangle$$

$$=R_o:\overline{X_1\cdots X_n}:|\Phi\rangle$$

observations:

1. complete contractions vanish

$$R_0X_1\cdots X_n/\Phi$$

$$=R_o:\overline{X_1\cdots X_n}:|\Phi\rangle$$

observations:

such terms will have the form

such terms will have the form const x $a_{j:"jm}$ | \pm)

such terms will have the form

const x $a_{j:"jn}^{b_{j:"bn}} | \Phi \rangle$ = const x $| \Phi_{j:"jn}^{b_{j:"bn}} \rangle$

such terms will have the form

const
$$\times a_{j:"jm}^{b,"bm} | \Phi \rangle$$

$$= const \times | \Phi_{j:"jm}^{b,"bm} \rangle$$



such terms will have the form

const
$$\times a_{j:"jm}^{b,"bm} | \Phi$$

$$= const \times | \Phi_{j:"jm}^{b,...bm} \rangle$$

$$\Rightarrow$$
 $R_o \times const \times | E_{j_1 \cdots j_m}^{b_1 \cdots b_m} \rangle$

such terms will have the form

const
$$\times a_{j:"jm}^{b,"bm} | \Phi \rangle$$

$$= const \times | \Phi_{j:"jm}^{b,"bm} \rangle$$

$$\Rightarrow$$
 $R_o \times const \times | E_{j_1 \cdots j_m}^{b_1 \cdots b_m} \rangle$

$$= const \times \frac{|\mathcal{I}_{j_1\cdots j_m}^{b_1\cdots b_m}\rangle}{\mathcal{E}_{j_1\cdots j_m}^{b_1\cdots b_m}}$$

RoY, ... RoYn-2 RoYn-1 RoYn ()

 $R_{0}Y_{1}\cdots R_{0}Y_{n-2}R_{0}Y_{n-1}R_{0}Y_{n}|\overline{\pm}\rangle$ $=R_{0}Y_{1}\cdots R_{0}Y_{n-2}R_{0}:Y_{n-1}Y_{n}:|\overline{\pm}\rangle$

Roy, ... Royn-2 Royn-1 Royn (#)

= RoY, ... RoYn-2 Ro: Yn-1 Yn:1 E)

= RoY, ... Ro: Yn-2 Yn-1 Yn:1 E>

RoY, ... RoYn-2 RoYn-1 RoYn ()

= RoY, ... RoYn-2 Ro: Yn-1 Yn:1 E>

= RoY, ... Ro: Yn-2 Yn-1 Yn:1 E>

= ...

$$R_{0}Y_{1} \cdots R_{0}Y_{n-2}R_{0}Y_{n-1}R_{0}Y_{n}|\underline{\pm}\rangle$$

$$= R_{0}Y_{1} \cdots R_{0}Y_{n-2}R_{0}:Y_{n-1}|Y_{n}:|\underline{\pm}\rangle$$

$$= R_{0}Y_{1} \cdots R_{0}:Y_{n-2}|Y_{n-1}|Y_{n}:|\underline{\pm}\rangle$$

$$=R_{0}:Y_{1}\cdots Y_{n-2}Y_{n-1}Y_{n}:|E\rangle$$

analyze dependence of Ψ or $0 = \langle \Psi | \hat{0} | \Psi \rangle$ on $V_c = H_c - H_o$

analyze dependence of 4 or $0 = (4)\hat{0}(4)$ on $V_c = H_c - H_o$

"fluctuation potential"

 $H(\lambda) \equiv H_o + \lambda V_c$

Perturbation Theory: $H(\lambda) \equiv H_0 + \lambda V_c$

Perturbation Theory: $I(\lambda) \equiv H_0 + \lambda V_c$ $I(\lambda) = H_0 + \lambda V_c$

Perturbation Theory: $P(x) = H_0 + \lambda V_c$ $\lambda = 1$ "on"

Perturbation theory: $H(\lambda) \equiv H_0 + \lambda V_c$ $\lambda = 1$ "on" $\lambda = 0$

 Perturbation theory: $H(\lambda) \equiv H_0 + \lambda V_c \qquad \lambda = 1 \text{ "on"}$ $\lambda = 0 \text{ "off"}$ $H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$

Perturbation theory: $H(\chi) \equiv H_0 + \lambda V_c$ $\lambda = 1$ "on" $\lambda = 0$ "off"

 $H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$

 $\Psi(\lambda) = \sum_{n=0}^{\infty} \lambda^n \Psi^{(n)}$

Perturbation Meory: on/off switch

 $H(\lambda) \equiv H_o + \lambda V_c$ $\lambda = 1$ "on" $\lambda = 0$ "off"

 $H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$

$$\Psi(\lambda) = \sum_{n=0}^{\infty} \lambda^n \Psi^{(n)}$$

 $E(\lambda) = \sum_{n=0}^{\infty} \lambda^n E_n^{(n)}$

Perturbation theory:

$$H(\lambda) \equiv H_0 + \lambda V_c$$
 $\lambda = 0$ "off"

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$\Psi(\lambda) = \sum_{n=0}^{\infty} \lambda^n \Psi(n) \qquad \text{if } \frac{\partial^n \Psi(\lambda)}{\partial \lambda^n} \Big|_{0}$$

$$E(\lambda) = \sum_{n=0}^{\infty} \lambda^n E_c^{(n)}$$

Perturbation Theory:

$$H(\lambda) \equiv H_0 + \lambda V_c \qquad \lambda = 1 \text{ "on"}$$

$$\lambda = 0 \text{ "off"}$$

$$L(\lambda) W(\lambda) = E(\lambda) Y(\lambda)$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$\Psi(\lambda) = \sum_{n=0}^{\infty} \lambda^n \Psi(n) \qquad \text{if } \frac{\partial \lambda^n}{\partial \lambda^n} \Big|_{0}$$

$$E(\lambda) = \sum_{n=0}^{\infty} \lambda^n E_{(n)} \qquad \text{if } \frac{\partial \lambda^n}{\partial \lambda^n} \Big|_{0}$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

project by Φ :

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$



$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$\implies \langle \mathcal{I} | V_c | \mathcal{Y}^{(n)} \rangle = E^{(n+1)}$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$R_oH_oY(\lambda) + R_o\lambda V_cY(\lambda) = E(\lambda)R_oY(\lambda)$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$R_{o}H_{o}\Upsilon(\lambda) + R_{o}\lambda V_{c}\Upsilon(\lambda) = E(\lambda)R_{o}\Upsilon(\lambda)$$

$$-Q\Upsilon(\lambda)$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$R_{o}H_{o}\Psi(\lambda) + R_{o}\lambda V_{c}\Psi(\lambda) = E(\lambda)R_{o}\Psi(\lambda)$$

$$-Q\Psi(\lambda) = -\Psi(\lambda) + \Phi$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$R_0H_0Y(\lambda) + R_0\lambda V_cY(\lambda) = E(\lambda)R_0Y(\lambda)$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$R_{o}H_{o}Y(\lambda) + R_{o}\lambda V_{c}Y(\lambda) = E(\lambda)R_{o}Y(\lambda)$$

$$-GA(y) = -A(y) + \overline{\Phi}$$

$$\Rightarrow \Upsilon(\lambda)$$

$$H(\lambda) \Psi(\lambda) = E(\lambda) \Psi(\lambda)$$

$$R_{o}H_{o}\Psi(\lambda) + R_{o}\lambda V_{c}\Psi(\lambda) = E(\lambda)R_{o}\Psi(\lambda)$$

$$-Q\Psi(\lambda) = -\Psi(\lambda) + \Phi$$

$$\Rightarrow \Psi(\lambda) = \Phi + R_o(\lambda V_c - E(\lambda)) \Psi(\lambda)$$

$$\Upsilon(\lambda) = \Xi + R_o(\lambda V_c - E(\lambda))\Upsilon(\lambda)$$

$$E^{(n+1)} = \langle \Xi | V_c | \Upsilon^{(n)} \rangle$$

$$\Upsilon(\lambda) = \bar{\Psi} + R_o(\lambda V_c - E(\lambda))\Upsilon(\lambda)$$

$$E^{(n+1)} = \langle \bar{\Psi} | V_c | \Upsilon^{(n)} \rangle$$

$$\Upsilon(\lambda) = \Xi + R_o(\lambda V_c - E(\lambda))\Upsilon(\lambda)$$

$$E^{(n+1)} = \langle \Xi | V_c | \Upsilon^{(n)} \rangle$$

$$\Upsilon(\lambda) = \Xi + R_o(\lambda V_c - E(\lambda))\Upsilon(\lambda)$$

$$E^{(n+1)} = \langle \Xi | V_c | \Upsilon^{(n)} \rangle$$

$$\Psi(\lambda) = \Phi + R_o(\lambda V_c - E(\lambda)) \Psi(\lambda)$$

$$E^{(n+1)} = \langle \Phi | V_c | \Psi^{(n)} \rangle$$

end.