Homework for Lecture 3.5 Particle-Hole Formalism

1. Explain what we mean by "particle-hole isomorphism"
2. Prove the anticommutation relations for b_p and $b_p^\dagger$
3. Prove to yourself that the rules listed in Section 5 of Lecture 3.5 are true
4. Prove Slater's first rule using the Particle-Hole formalism

$$\langle \Phi | H | \Phi \rangle = \sum_i h_{ii} + \frac{1}{2} \sum_{ij} \langle ij || ij \rangle$$

5. Prove Slater's second rule using the Particle-Hole formalism

$$\langle \Phi | H | \Phi_i^a \rangle = h_{ia} + \sum_j \langle ij || aj \rangle$$

6. Prove Slater's third rule using the Particle-Hole formalism

$$\langle \Phi | H | \Phi_{ij}^{ab} \rangle = \langle ij || ab \rangle$$