Homework for Lecture 3.6 a_p and $a_p^\dagger$ in the Particle-Hole Formalism

1. Prove:

$$\overbrace{a_p a_q^\dagger} = \pi(p)\delta_{pq} = \eta_{pq}$$

$$\overbrace{a_p^\dagger a_q^\dagger} = 0$$

2. Put the Hamiltonian in Φ-Normal ordering, define the Fock operator

3. Prove Slater's first rule using the Particle-Hole formalism with a Φ-Normal ordered Hamiltonian

$$\langle \Phi | H | \Phi \rangle = \sum_i h_{ii} + \frac{1}{2} \sum_{ij} \langle ij || ij \rangle$$

4. Prove Slater's second rule using the Particle-Hole formalism with a Φ-Normal ordered Hamiltonian

$$\langle \Phi | H | \Phi_i^a \rangle = h_{ia} + \sum_j \langle ij || aj \rangle$$

5. Prove Slater's third rule using the Particle-Hole formalism with a Φ-Normal ordered Hamiltonian

$$\langle \Phi | H | \Phi_{ij}^{ab} \rangle = \langle ij || ab \rangle$$

6. Practice evaluating the following matrix elements:

$$\langle \Phi_j^b | \Phi_i^a \rangle$$

$$\langle \Phi_j^b | H | \Phi_i^a \rangle$$

$$\langle \Phi_{kl}^{cd} | \Phi_{ij}^{ab} \rangle$$

$$\langle \Phi_{kl}^{cd} | H | \Phi_{ij}^{ab} \rangle$$