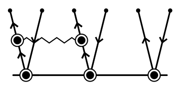
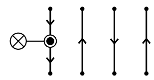
1. Interpret the following coefficient graph algebraically, denoting the bare excitation operator by $\tilde{a}^{ijk}_{abc}$.

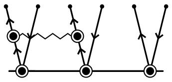


2. Interpret the following coefficient graph algebraically, denoting the top bare excitation operator by $\tilde{a}^{ij}_{ab}$ and the bottom one by $\tilde{a}^{cd}_{kl}$ and their interaction tensors by ${}^{1}\overline{\delta}^{ef}_{mn}$ and ${}^{2}\overline{\delta}^{mn}_{ef}$.

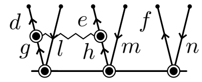


Solutions:

1. Axiom 4.1 gives



$$= \frac{1}{2!2!3!} \sum_{\substack{defgh\\lmn}}$$



$$= \frac{1}{2!2!3!} \sum_{\substack{defgh\\lmn}} \overline{\delta}^{def}_{lmn} \overline{g}^{gh}_{de}\, c^{lmn}_{ghf} \,⁝ a^{l^{\circ 1} m^{\circ 2} n^{\circ 3}}_{d^{\bullet 1} e^{\bullet 2} f^{\bullet 3}}\, a^{d^{\bullet 1} e^{\bullet 2}}_{g^{\bullet 4} h^{\bullet 5}}\, a^{g^{\bullet 4} h^{\bullet 5} f^{\bullet 3}}_{l^{\circ 1} m^{\circ 2} n^{\circ 3}} ⁝$$

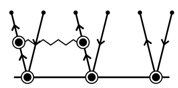
since there are two pairs of equivalent lines on the repulsion operator, and the CI triples operator has a set of three equivalent lines. The interaction tensor for the bare excitation operator is $\overline{\delta}^{def}_{lmn} = \hat{P}^{(i/j/k)}_{(a/b/c)} \delta^d_a \delta^e_b \delta^f_c \delta^i_l \delta^m_j \delta^n_k$, so this can be simplified as

$$\sum_{\substack{def\\lmn}} \overline{\delta}^{def}_{lmn} T^{lmn}_{def} = P^{(i/j/k)}_{(a/b/c)} T^{ijk}_{abc} \qquad \text{where} \quad T^{lmn}_{def} \equiv \frac{1}{2!2!3!} \sum_{gh} \overline{g}^{gh}_{de}\, c^{lmn}_{ghf} \,⁝ a^{l^{\circ 1} m^{\circ 2} n^{\circ 3}}_{d^{\bullet 1} e^{\bullet 2} f^{\bullet 3}}\, a^{d^{\bullet 1} e^{\bullet 2}}_{g^{\bullet 4} h^{\bullet 5}}\, a^{g^{\bullet 4} h^{\bullet 5} f^{\bullet 3}}_{l^{\circ 1} m^{\circ 2} n^{\circ 3}} ⁝ .$$

Using item 3 in Remark 4.3, the contracted operator string evaluates as follows

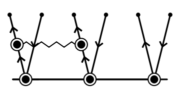
$$⁝ a^{l^{\circ 1} m^{\circ 2} n^{\circ 3}}_{d^{\bullet 1} e^{\bullet 2} f^{\bullet 3}}\, a^{d^{\bullet 1} e^{\bullet 2}}_{g^{\bullet 4} h^{\bullet 5}}\, a^{g^{\bullet 4} h^{\bullet 5} f^{\bullet 3}}_{l^{\circ 1} m^{\circ 2} n^{\circ 3}} ⁝ = (-1)^{3+3} = +1$$

since there are three hole lines and three loops in the graph. At this point, we have simplified our interpretation to the following



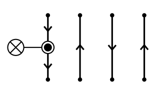
$$= \frac{1}{2!2!3!} \sum_{de} \hat{P}^{(i/j/k)}_{(a/b/c)} \overline{g}^{de}_{ab}\, c^{ijk}_{dec}$$

where I have relabeled the summation indices $g \mapsto d$, $h \mapsto e$. Finally, using item 4 under Remark 4.3, we can cancel the degeneracy factors coming from inequivalent coefficient lines by replacing $\hat{P}^{(i/j/k)}_{(a/b/c)}$ with $2!3!\hat{P}_{(ab/c)}$. This works because the operand $\overline{g}^{de}_{ab} c^{ijk}_{dec}$ is already antisymmetric with respect to $\{a, b\}$ and $\{i, j, k\}$.

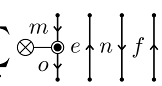


$$= \frac{1}{2!} \sum_{de} \hat{P}_{(ab/c)} \overline{g}^{de}_{ab}\, c^{ijk}_{dec}$$

2. Axiom 4.1 gives



$$= \frac{1}{2!} \sum_{\substack{ef\\mno}}$$

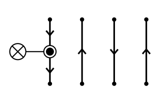


$$= \frac{1}{2!} \sum_{\substack{ef\\mno}} {}^1\overline{\delta}^{ef}_{mn} f^m_o\, {}^2\overline{\delta}^{on}_{ef} \,⁝ a^{m^{\circ 1} n^{\circ 2}}_{e^{\bullet 1} f^{\bullet 2}}\, a^{o^{\circ 3}}_{m^{\circ 1}}\, a^{e^{\bullet 1} f^{\bullet 2}}_{o^{\circ 3} n^{\circ 2}} ⁝$$

since the two particle lines are equivalent. Using item 3 in Remark 4.3, the operator string evaluates to

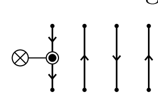
$$⁝ a^{m^{\circ 1} n^{\circ 2}}_{e^{\bullet 1} f^{\bullet 2}}\, a^{o^{\circ 3}}_{m^{\circ 1}}\, a^{e^{\bullet 1} f^{\bullet 2}}_{o^{\circ 3} n^{\circ 2}} ⁝ = (-1)^{3+2} = -1$$

since there are three hole lines and two loops in the diagram. Substituting in the definitions of the interaction tensors, we can simplify the result as follows.



$$\begin{aligned} = -\frac{1}{2!} \sum_{\substack{ef\\mno}} {}^1\overline{\delta}^{ef}_{mn} f^m_o\, {}^2\overline{\delta}^{on}_{ef} &= -\frac{1}{2!} \sum_{\substack{ef\\mno}} \left(\hat{P}^{(i/j)}_{(a/b)} \delta^e_a \delta^f_b \delta^i_m \delta^j_n \right) f^m_o \left(\hat{P}^{(c/d)}_{(k/l)} \delta^o_k \delta^n_l \delta^c_e \delta^d_f \right) \\ &= -\frac{1}{2!} \hat{P}^{(i/j)}_{(a/b)} \hat{P}^{(c/d)}_{(k/l)} f^i_k \delta^j_l \delta^c_a \delta^d_b \end{aligned}$$

Finally, using item 4 under Remark 4.3, we can cancel the degeneracy factor by replacing $\hat{P}^{(i/j)}_{(a/b)}$ with $2!\hat{P}^{(i/j)}$, canceling the degeneracy factor arising from the two particle lines.



$$= -\hat{P}^{(i/j|c/d)}_{(k/l)} f^i_k \delta^j_l \delta^c_a \delta^d_b$$