Homework for Lecture 3.1

1. Show that the number operator N_p is hermitian ($N_p = N_p^\dagger$)
2. Prove the following anticommuntation relations:
   (a) $\{a_p^\dagger, a_q\} = \delta_{pq}$ (Show this for three cases: $p = q$, $p < q$, $p > q$)
   (b) $\{a_p, a_q\} = 0$ (Show this for three cases: $p = q$, $p < q$, $p > q$)
   (c) $\{a_p^\dagger, a_q^\dagger\} = 0$ (Hint: show by forming Hermitian adjoint of 2b)
3. Prove the six properties that are listed in Section 4.4 of the lecture notes using anticommuntation relations
   - *Hint:* If you get stuck, write everything explicitly in terms of creation and annhilation operators and try applying anticommuntation relations to the middle of the expression.
4. Show that $\langle \Phi_{pq} | \Phi_{rs} \rangle = \delta_{pr}\delta_{qs} - \delta_{ps}\delta_{qr}$
   - *Hint 1:* Represent the expectation value in terms of strings of creation and annihilation operators (take care of the order of indices!)
   - *Hint 2:* Apply anticommutation relations while moving annihilation operators to the right
   - Note: Another way to write $\delta_{pr}\delta_{qs} - \delta_{ps}\delta_{qr}$ is $\langle p|r\rangle \langle q|s\rangle - \langle p|s\rangle \langle q|r\rangle$. Note that this is just a Slater determinant of overlaps:
   $$\langle \Phi_{pq} | \Phi_{rs} \rangle = \begin{vmatrix} \langle p|r\rangle & \langle p|s\rangle \\ \langle q|r\rangle & \langle q|r\rangle \end{vmatrix}$$
   In other words, an overlap of Slater determinants can be written as a Slater determinant of overlaps.