# p.t. (pt. 2)

From last week:

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$$E^{(m+1)} = \langle \Phi | V_c | \Psi^{(m)} \rangle$$

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$$\Psi(\lambda) = \Phi + R_0 (\lambda V_c - E(\lambda)) \Psi(\lambda)$$

Can show that

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$$\Psi^{(3)} = \frac{1}{3!} \frac{\partial^3 \Psi(\lambda)}{\partial \lambda^3} \bigg|_{\lambda=0}$$

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$$= R_0 V_c R_0 V_c R_0 V_c \Phi - R_0 E_c^{(2)} R_0 V_c \Phi$$

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$$\Rightarrow E^{(4)} = \langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle - \langle V_c R_0 \langle V_c R_0 V_c \rangle R_0 V_c \rangle$$

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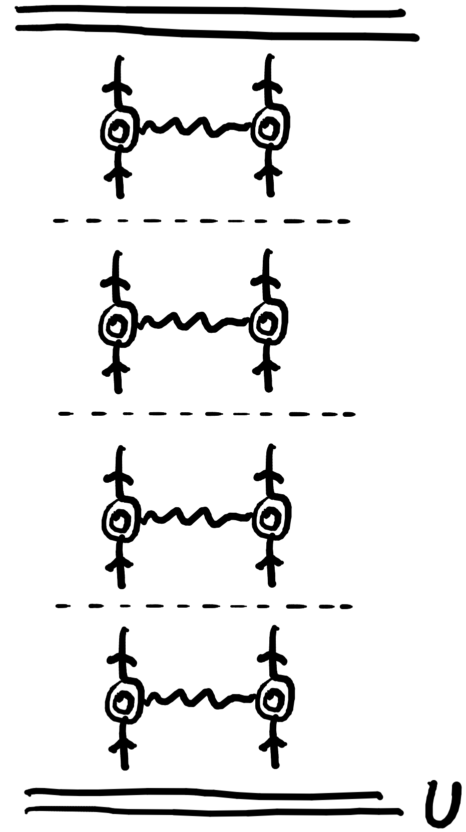
$$\Rightarrow E^{(4)} = \langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle - \langle V_c R_0 \langle V_c R_0 V_c \rangle R_0 V_c \rangle$$

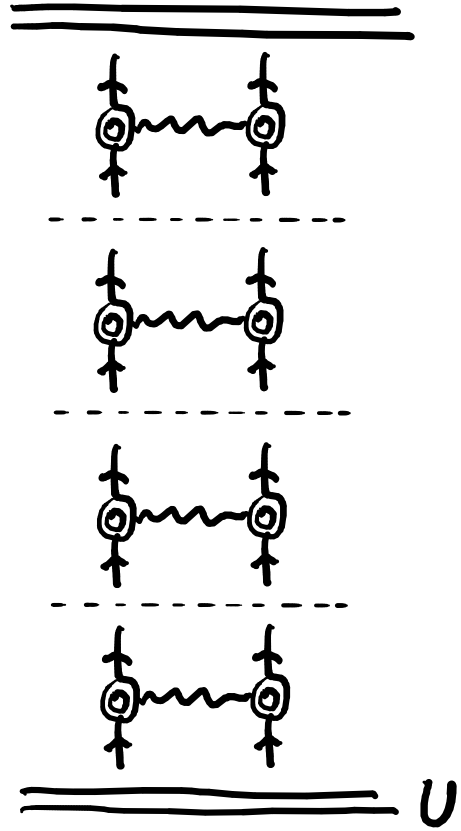
generalizes to the "bracketing thm."

$$\langle V_c R_o V_c R_o V_c R_o V_c \rangle_U$$

$$\langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle_U =$$

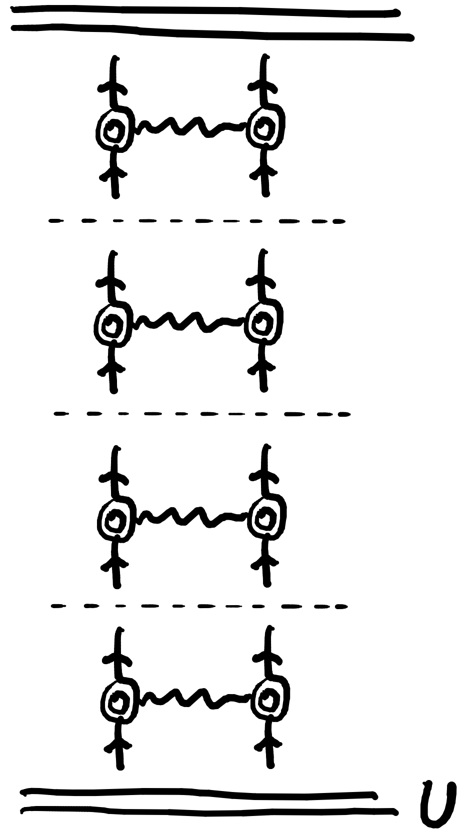
$$\langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle_U =$$ 

$$\langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle_U =$$




$$=$$

$$\langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle_U =$$

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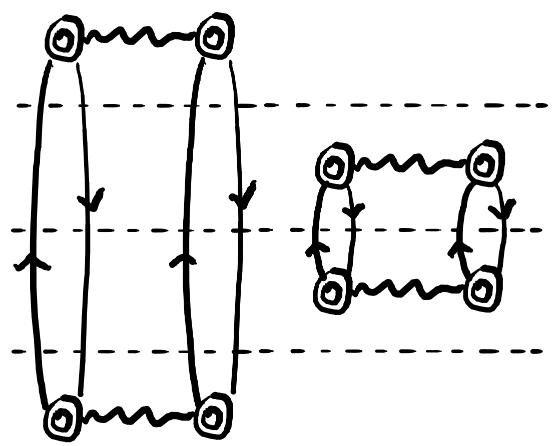
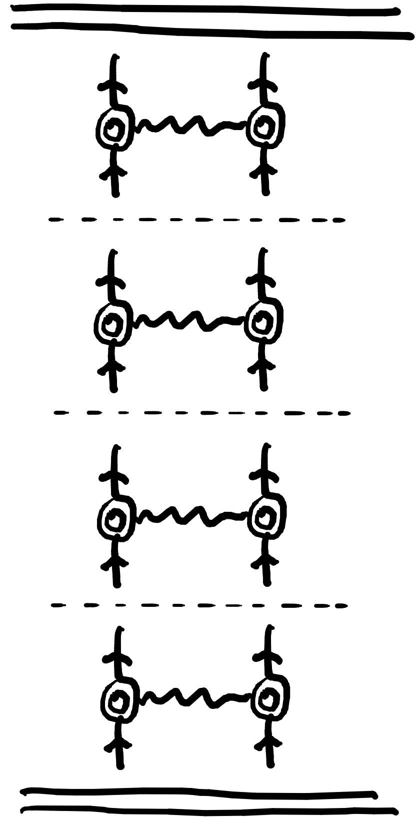
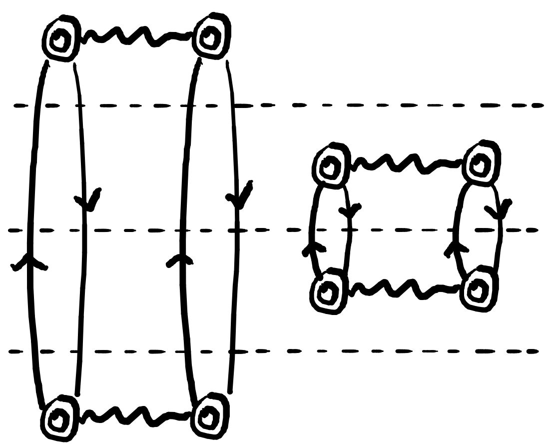


$$\langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle_U =$$



$U$

$$=$$



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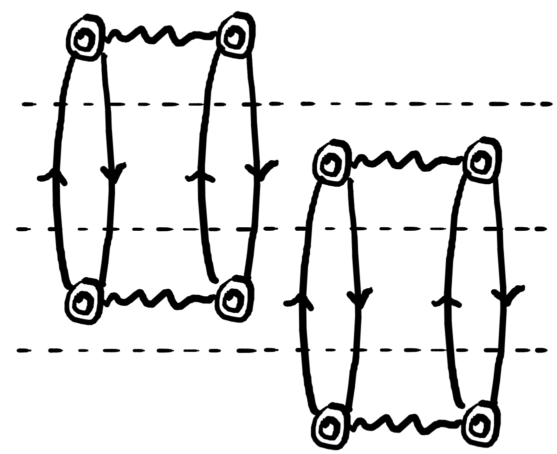

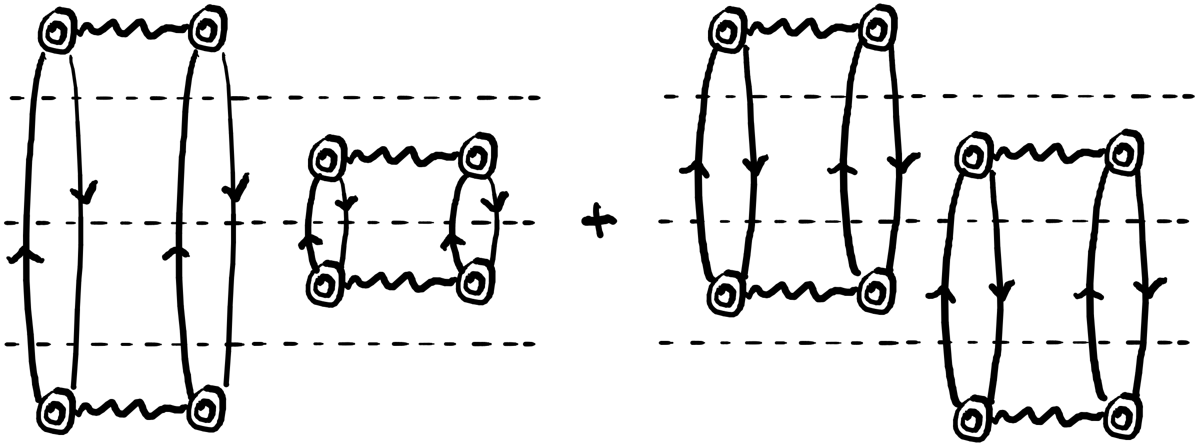

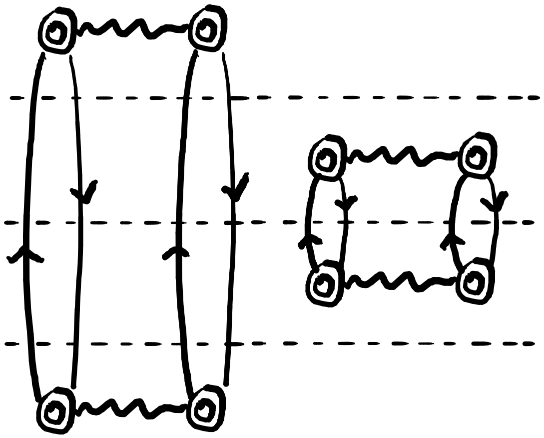

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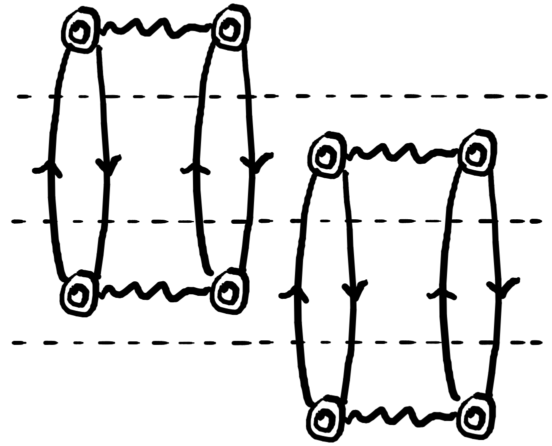

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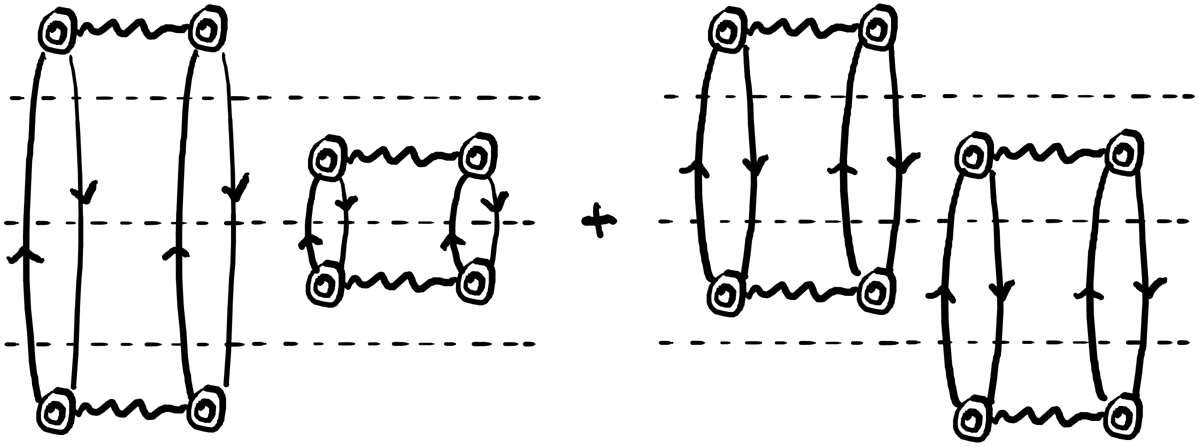
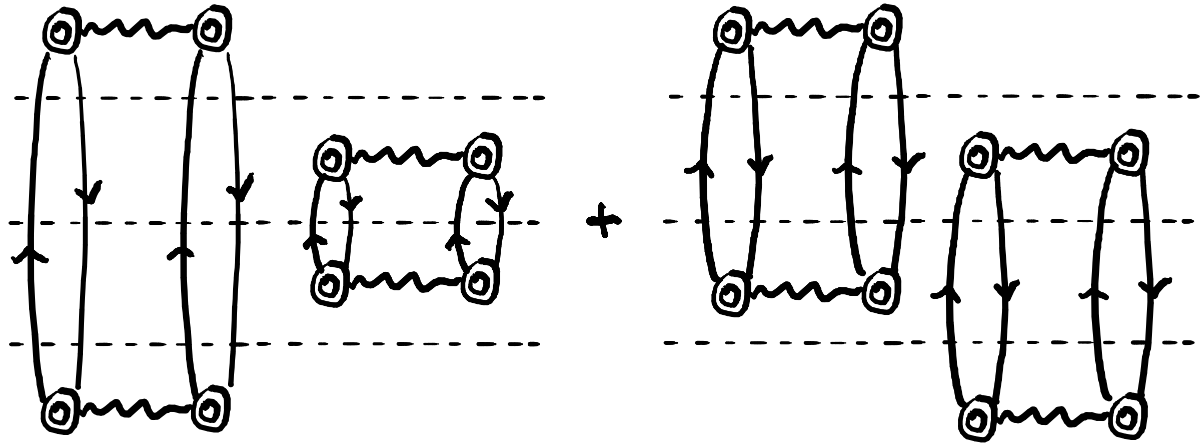
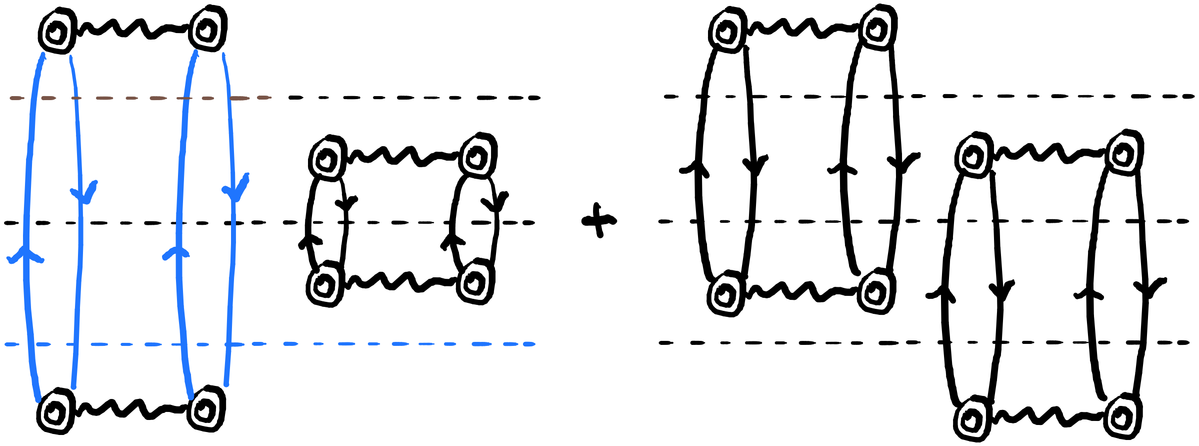


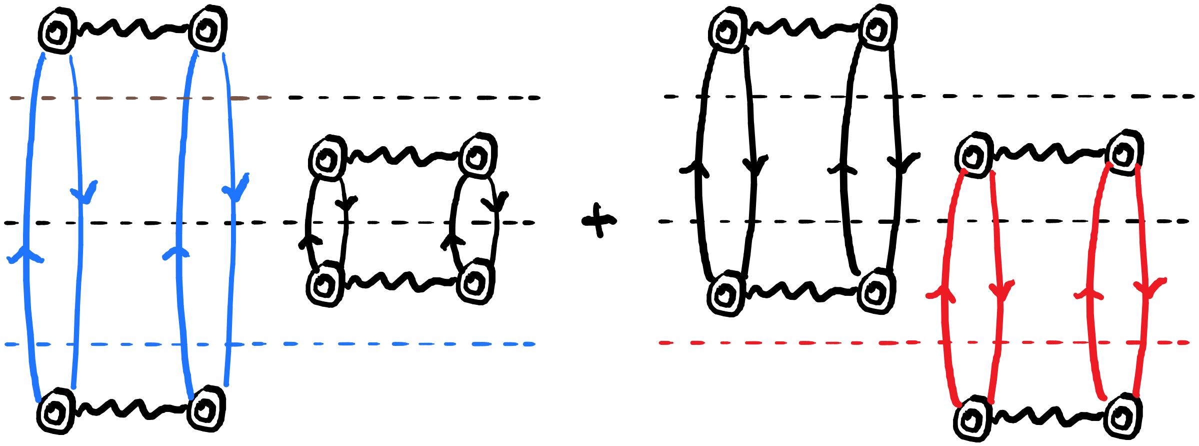
$$= \frac{1}{2^4} \sum_{\substack{abcd \\ ijkl}} \frac{\bar{g}_{ij}^{ab}\, \bar{g}_{ab}^{ij}\, \bar{g}_{kl}^{cd}\, \bar{g}_{cd}^{kl}}{\varepsilon_{ab}^{ij}\, \varepsilon_{abcd}^{ijkl}\, \varepsilon_{ab}^{ij}}$$



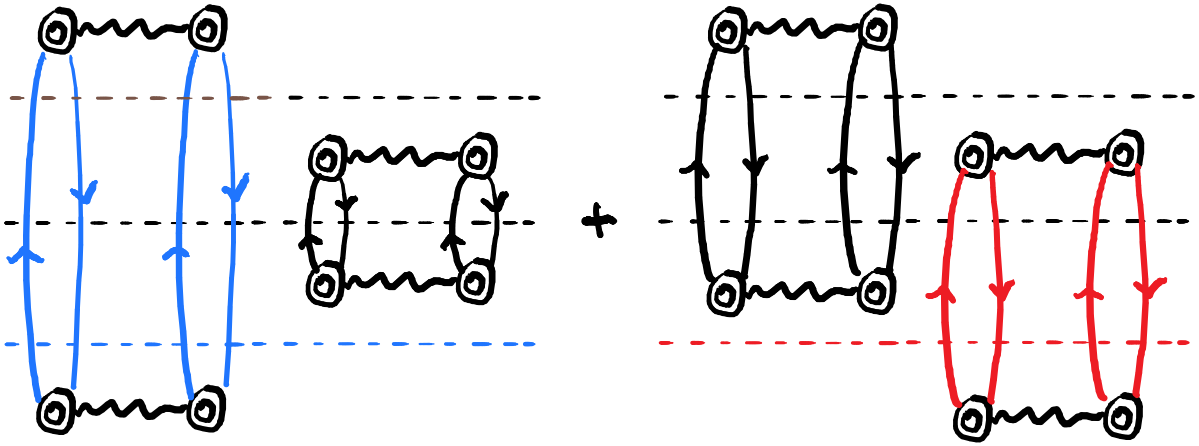
$$= \frac{1}{2^4} \sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\, \bar{g}_{ab}^{ij}\, \bar{g}_{kl}^{cd}\, \bar{g}_{cd}^{kl}}{\varepsilon_{ab}^{ij}\, \varepsilon_{abcd}^{ijkl}\, \varepsilon_{ab}^{ij}} + \frac{1}{2^4} \sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\, \bar{g}_{ab}^{ij}\, \bar{g}_{kl}^{cd}\, \bar{g}_{cd}^{kl}}{\varepsilon_{ab}^{ij}\, \varepsilon_{abcd}^{ijkl}\, \varepsilon_{cd}^{kl}}$$



$$= \frac{1}{2^4}\sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\,\bar{g}_{ab}^{ij}\,\bar{g}_{kl}^{cd}\,\bar{g}_{cd}^{kl}}{\varepsilon_{ab}^{ij}\,\varepsilon_{abcd}^{ijkl}\,\varepsilon_{ab}^{ij}} + \frac{1}{2^4}\sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\,\bar{g}_{ab}^{ij}\,\bar{g}_{kl}^{cd}\,\bar{g}_{cd}^{kl}}{\varepsilon_{ab}^{ij}\,\varepsilon_{abcd}^{ijkl}\,\varepsilon_{cd}^{kl}}$$

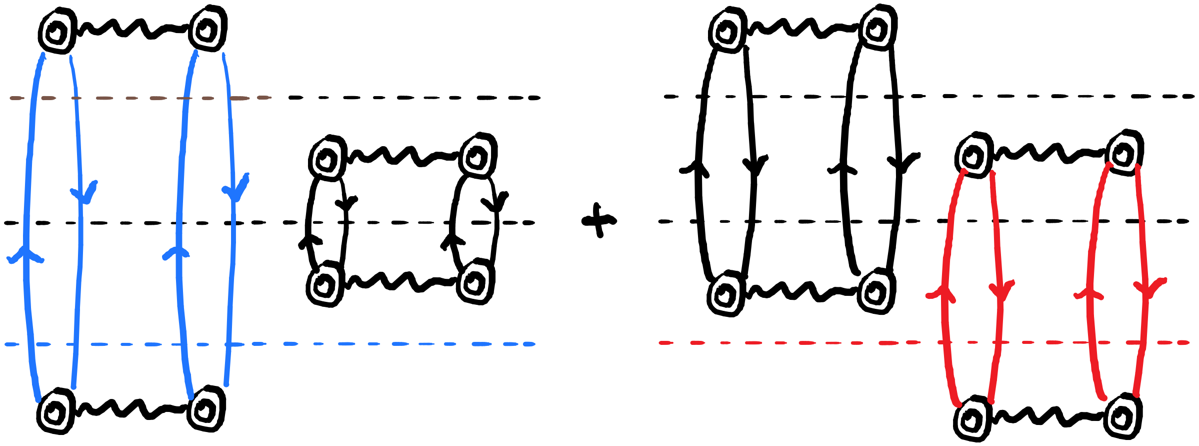


$$= \frac{1}{2^4} \sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\, \bar{g}_{ab}^{ij}\, \bar{g}_{kl}^{cd}\, \bar{g}_{cd}^{kl}}{\varepsilon_{ab}^{ij}\, \varepsilon_{abcd}^{ijkl}\, \varepsilon_{ab}^{ij}} + \frac{1}{2^4} \sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\, \bar{g}_{ab}^{ij}\, \bar{g}_{kl}^{cd}\, \bar{g}_{cd}^{kl}}{\varepsilon_{ab}^{ij}\, \varepsilon_{abcd}^{ijkl}\, \varepsilon_{cd}^{kl}}$$



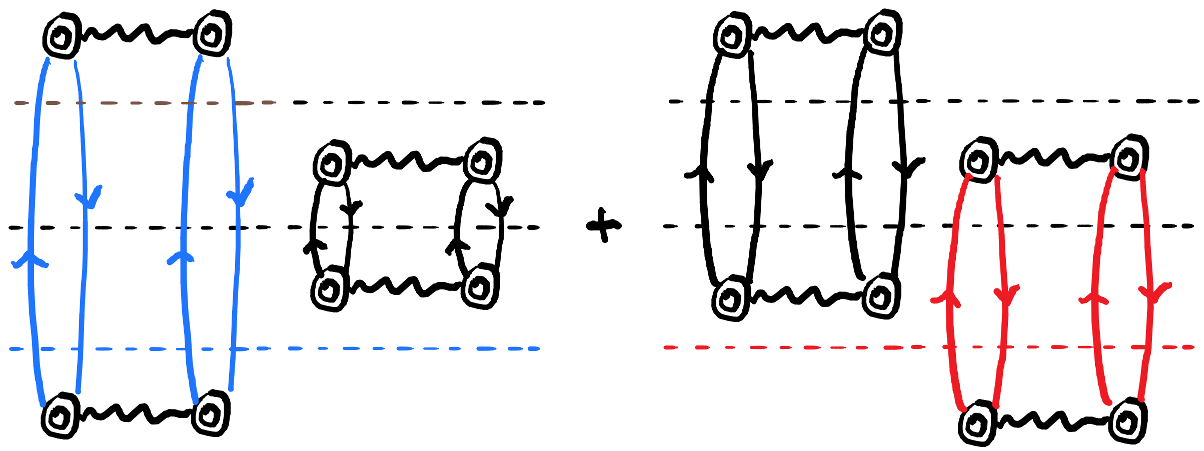
$$= \frac{1}{2^4} \sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\,\bar{g}_{ab}^{ij}\,\bar{g}_{kl}^{cd}\,\bar{g}_{cd}^{kl}}{\varepsilon_{ab}^{ij}\,\varepsilon_{abcd}^{ijkl}\,\varepsilon_{ab}^{ij}} + \frac{1}{2^4} \sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\,\bar{g}_{ab}^{ij}\,\bar{g}_{kl}^{cd}\,\bar{g}_{cd}^{kl}}{\varepsilon_{ab}^{ij}\,\varepsilon_{abcd}^{ijkl}\,\varepsilon_{cd}^{kl}}$$

$$= \frac{1}{2^4} \sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\,\bar{g}_{ab}^{ij}\,\bar{g}_{kl}^{cd}\,\bar{g}_{cd}^{kl}\left(\varepsilon_{cd}^{kl} + \varepsilon_{ab}^{ij}\right)}{\varepsilon_{ab}^{ij}\,\varepsilon_{abcd}^{ijkl}\,\varepsilon_{ab}^{ij}\,\varepsilon_{cd}^{kl}}$$



$$= \frac{1}{2^4} \sum_{\substack{abcd\\ijkl}} \frac{\bar{g}^{ab}_{ij}\, \bar{g}^{ij}_{ab}\, \bar{g}^{cd}_{kl}\, \bar{g}^{kl}_{cd}}{\varepsilon^{ij}_{ab}\, \varepsilon^{ijkl}_{abcd}\, \varepsilon^{ij}_{ab}} + \frac{1}{2^4} \sum_{\substack{abcd\\ijkl}} \frac{\bar{g}^{ab}_{ij}\, \bar{g}^{ij}_{ab}\, \bar{g}^{cd}_{kl}\, \bar{g}^{kl}_{cd}}{\varepsilon^{ij}_{ab}\, \varepsilon^{ijkl}_{abcd}\, \varepsilon^{kl}_{cd}}$$

$$= \frac{1}{2^4} \sum_{\substack{abcd\\ijkl}} \frac{\bar{g}^{ab}_{ij}\, \bar{g}^{ij}_{ab}\, \bar{g}^{cd}_{kl}\, \bar{g}^{kl}_{cd} \cancel{\left( \varepsilon^{kl}_{cd} + \varepsilon^{ij}_{ab} \right)}}{\varepsilon^{ij}_{ab}\, \cancel{\varepsilon^{ijkl}_{abcd}}\, \varepsilon^{ij}_{ab}\, \varepsilon^{kl}_{cd}}$$



$$= \frac{1}{2^4} \sum_{\substack{abcd \\ ijkl}} \frac{\bar{g}^{ab}_{ij}\, \bar{g}^{ij}_{ab}\, \bar{g}^{cd}_{kl}\, \bar{g}^{kl}_{cd}}{\varepsilon^{ij}_{ab}\, \varepsilon^{ijkl}_{abcd}\, \varepsilon^{ij}_{ab}} + \frac{1}{2^4} \sum_{\substack{abcd \\ ijkl}} \frac{\bar{g}^{ab}_{ij}\, \bar{g}^{ij}_{ab}\, \bar{g}^{cd}_{kl}\, \bar{g}^{kl}_{cd}}{\varepsilon^{ij}_{ab}\, \varepsilon^{ijkl}_{abcd}\, \varepsilon^{kl}_{cd}}$$

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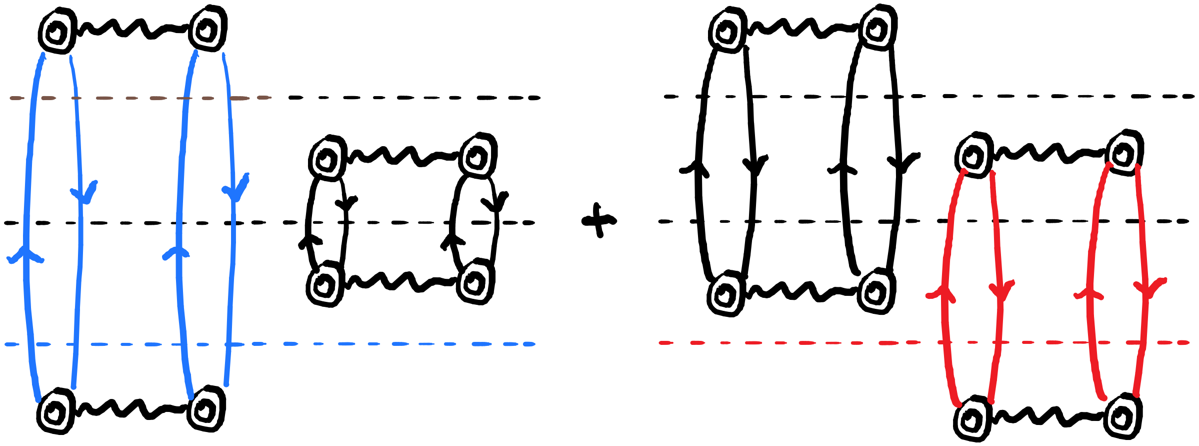
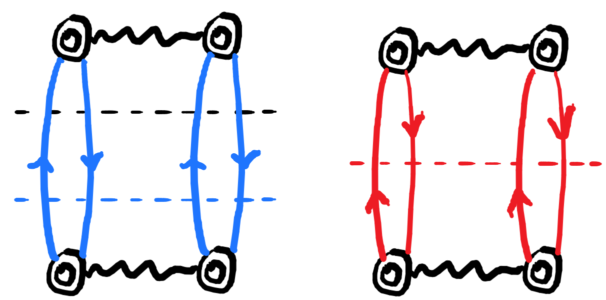
$$= \frac{1}{2^2} \sum_{\substack{ab \\ ij}} \frac{\bar{g}^{ab}_{ij}\, \bar{g}^{ij}_{ab}}{\varepsilon^{ij}_{ab}\, \varepsilon^{ij}_{ab}} \cdot \frac{1}{2^2} \sum_{\substack{cd \\ kl}} \frac{\bar{g}^{cd}_{kl}\, \bar{g}^{kl}_{cd}}{\varepsilon^{kl}_{cd}}$$

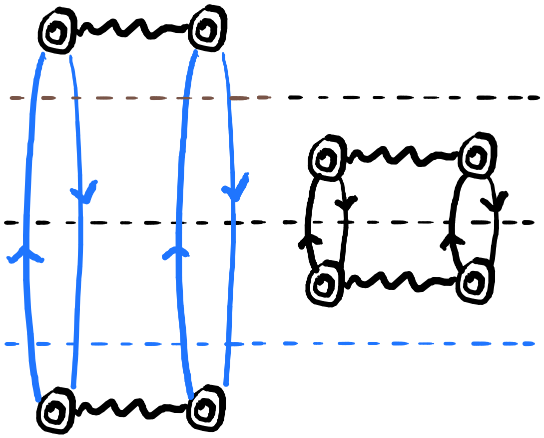


$$= \frac{1}{2^4}\sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\,\bar{g}_{ab}^{ij}\,\bar{g}_{kl}^{cd}\,\bar{g}_{cd}^{kl}}{\varepsilon_{ab}^{ij}\,\varepsilon_{abcd}^{ijkl}\,\varepsilon_{ab}^{ij}} + \frac{1}{2^4}\sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\,\bar{g}_{ab}^{ij}\,\bar{g}_{kl}^{cd}\,\bar{g}_{cd}^{kl}}{\varepsilon_{ab}^{ij}\,\varepsilon_{abcd}^{ijkl}\,\varepsilon_{cd}^{kl}}$$

$$= \frac{1}{2^4}\sum_{\substack{abcd\\ijkl}} \frac{\bar{g}_{ij}^{ab}\,\bar{g}_{ab}^{ij}\,\bar{g}_{kl}^{cd}\,\bar{g}_{cd}^{kl}\,\cancel{\left(\varepsilon_{cd}^{kl} + \varepsilon_{ab}^{ij}\right)}}{\varepsilon_{ab}^{ij}\,\cancel{\varepsilon_{abcd}^{ijkl}}\,\varepsilon_{ab}^{ij}\,\varepsilon_{cd}^{kl}}$$

$$= \frac{1}{2^2}\sum_{\substack{ab\\ij}} \frac{\bar{g}_{ij}^{ab}\,\bar{g}_{ab}^{ij}}{\varepsilon_{ab}^{ij}\,\varepsilon_{ab}^{ij}} \cdot \frac{1}{2^2}\sum_{\substack{cd\\kl}} \frac{\bar{g}_{kl}^{cd}\,\bar{g}_{cd}^{kl}}{\varepsilon_{cd}^{kl}} =$$





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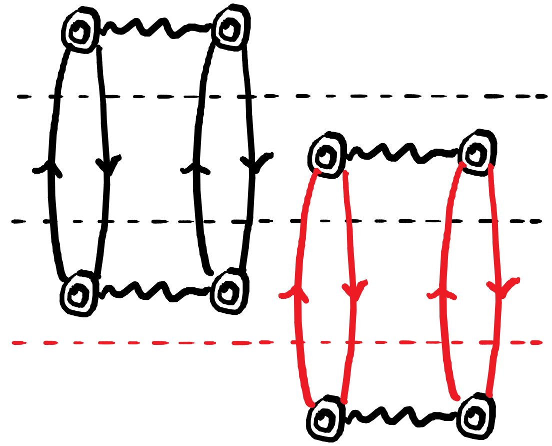
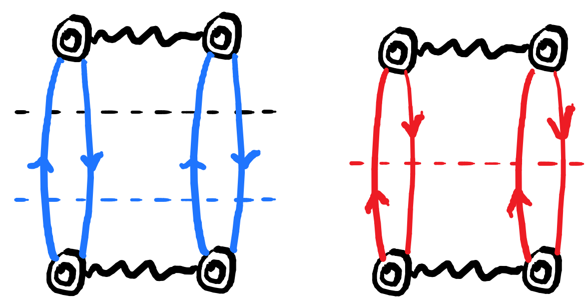


$$= \frac{1}{2^4} \sum_{\substack{abcd \\ ijkl}} \frac{\bar{g}^{ab}_{ij}\, \bar{g}^{ij}_{ab}\, \bar{g}^{cd}_{kl}\, \bar{g}^{kl}_{cd}}{\varepsilon^{ij}_{ab}\, \varepsilon^{ijkl}_{abcd}\, \varepsilon^{ij}_{ab}} + \frac{1}{2^4} \sum_{\substack{abcd \\ ijkl}} \frac{\bar{g}^{ab}_{ij}\, \bar{g}^{ij}_{ab}\, \bar{g}^{cd}_{kl}\, \bar{g}^{kl}_{cd}}{\varepsilon^{ij}_{ab}\, \varepsilon^{ijkl}_{abcd}\, \varepsilon^{kl}_{cd}}$$

$$= \frac{1}{2^4} \sum_{\substack{abcd \\ ijkl}} \frac{\bar{g}^{ab}_{ij}\, \bar{g}^{ij}_{ab}\, \bar{g}^{cd}_{kl}\, \bar{g}^{kl}_{cd}\, \cancel{\left(\varepsilon^{kl}_{cd} + \varepsilon^{ij}_{ab}\right)}}{\varepsilon^{ij}_{ab}\, \cancel{\varepsilon^{ijkl}_{abcd}}\, \varepsilon^{ij}_{ab}\, \varepsilon^{kl}_{cd}}$$

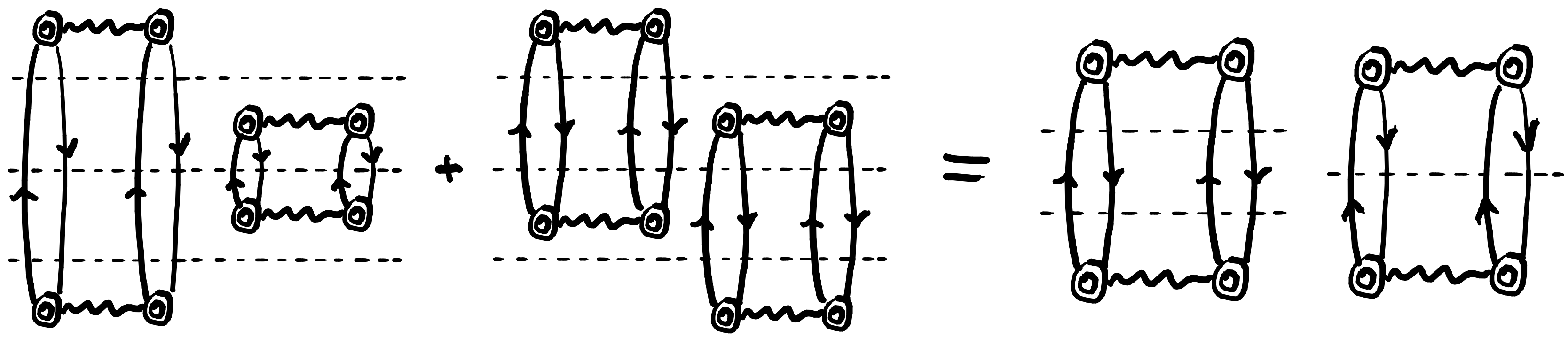
"Frantz-Mills factorization thm."

$$= \frac{1}{2^2} \sum_{\substack{ab \\ ij}} \frac{\bar{g}^{ab}_{ij}\, \bar{g}^{ij}_{ab}}{\varepsilon^{ij}_{ab}\, \varepsilon^{ij}_{ab}} \cdot \frac{1}{2^2} \sum_{\substack{cd \\ kl}} \frac{\bar{g}^{cd}_{kl}\, \bar{g}^{kl}_{cd}}{\varepsilon^{kl}_{cd}} =$$

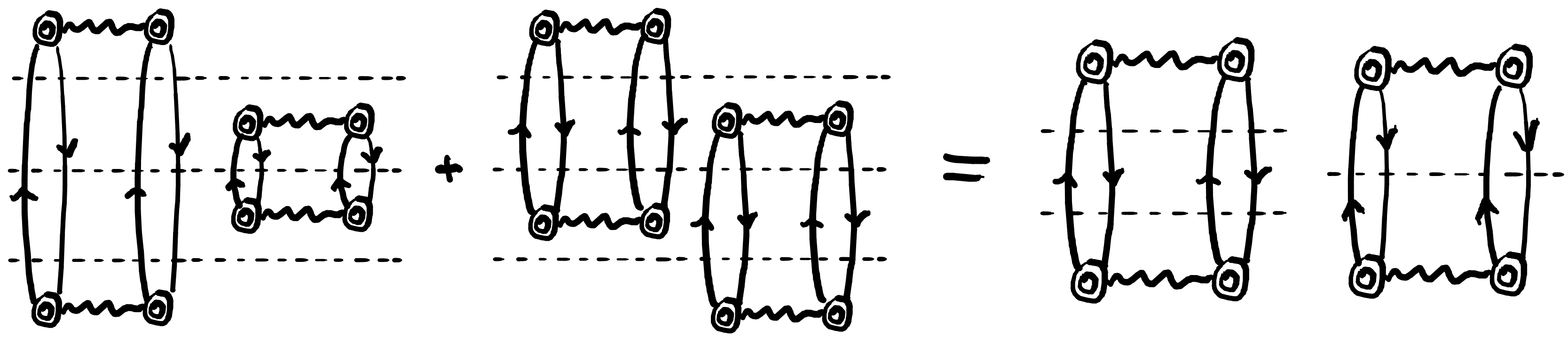


Conclusion:

Conclusion:



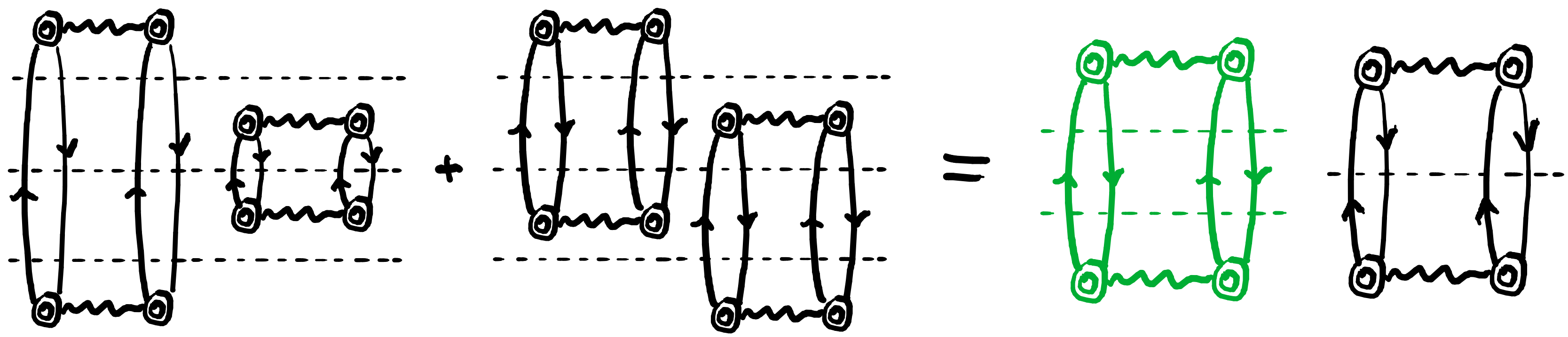
# Conclusion:



$$=$$

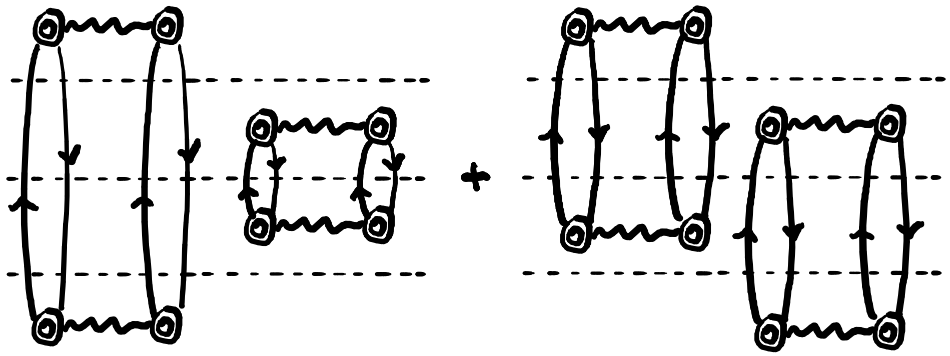
$$\langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle_U$$

# Conclusion:



$$\langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle_U \qquad\qquad \langle V_c R_0 \langle V_c R_0 V_c \rangle R_0 V_c \rangle$$

# Conclusion:



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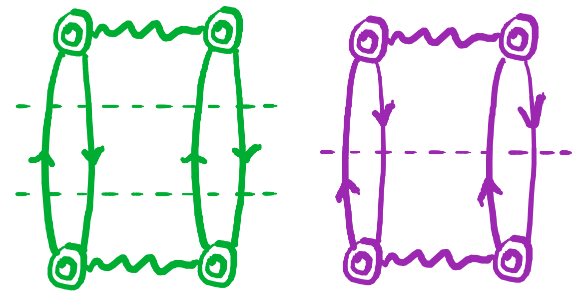
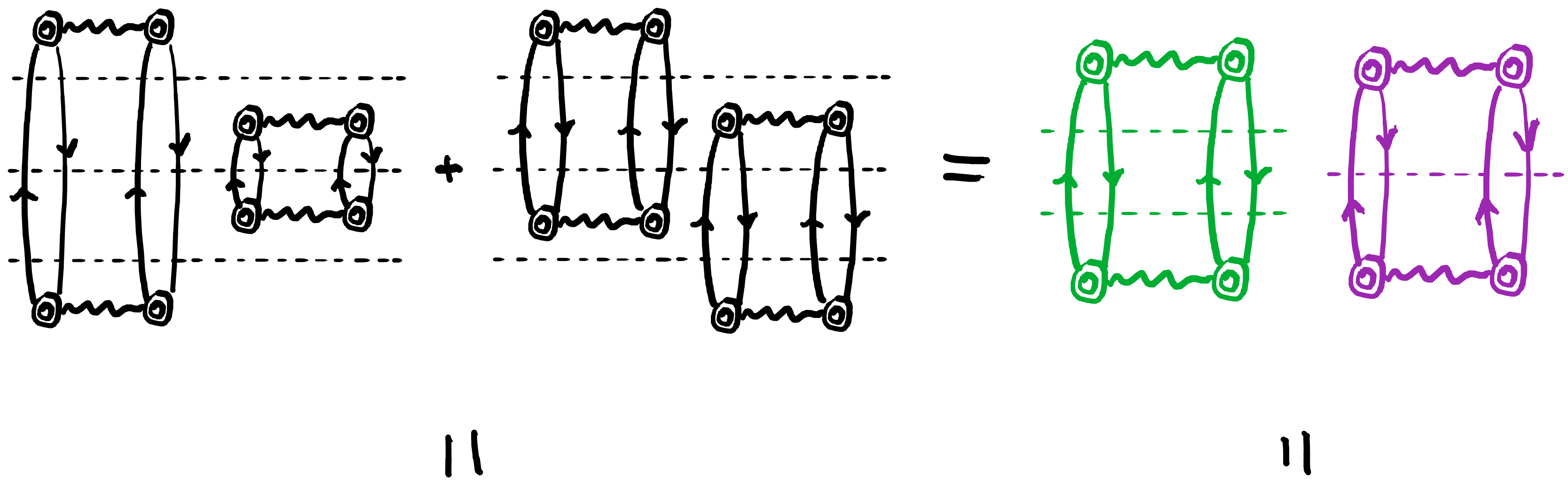


$$\langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle_U = \langle V_c R_0 \langle V_c R_0 V_c \rangle R_0 V_c \rangle$$

# Conclusion:



$$\langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle_U = \langle V_c R_0 \langle V_c R_0 V_c \rangle R_0 V_c \rangle$$

Conclusion:

Conclusion:

$E^{(4)}$

Conclusion:

$$E^{(4)} = \langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle - \langle V_c R_0 \langle V_c R_0 V_c \rangle R_0 V_c \rangle$$

Conclusion:

$$E^{(4)} = \langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle - \langle V_c R_0 \langle V_c R_0 V_c \rangle R_0 V_c \rangle$$

$$= \langle V_c R_0 V_c R_0 V_c R_0 V_c \rangle_L$$

In general:

In general:

$\Psi^{(m)}$

In general:

$$\Psi^{(m)} = R_0 V_c \cdots R_0 V_c \Phi$$

In general:

$$\Psi^{(m)} = \underbrace{R_0 V_c \cdots R_0 V_c}_{m \text{ times}} \Phi$$

In general:

$$\Psi^{(m)} = \underbrace{R_0 V_c \cdots R_0 V_c}_{m \text{ times}} \Phi \;+\; \text{all possible bracketings weighted by } (-)^{\#\,\text{brackets}}$$

In general:

$$- (R_0 V_c \cdots R_0 V_c)_U \, \Phi$$

$$\Psi^{(m)} = \underbrace{R_0 V_c \cdots R_0 V_c}_{m \text{ times}} \Phi \; + \; \text{all possible bracketings weighted by } (-)^{\#\,\text{brackets}}$$

In general:

$$- (R_0 V_c \cdots R_0 V_c)_U \Phi$$

$$\Psi^{(m)} = \underbrace{R_0 V_c \cdots R_0 V_c}_{m \text{ times}} \Phi \; + \; \text{all possible bracketings weighted by } (-)^{\# \text{ brackets}}$$

$$= (R_0 V_c \cdots R_0 V_c)_L \Phi$$

In general:

$$- (R_0 V_c \cdots R_0 V_c)_U \Phi$$

$$\Psi^{(m)} = \underbrace{R_0 V_c \cdots R_0 V_c}_{m \text{ times}} \Phi + \text{all possible bracketings weighted by } (-)^{\#\text{ brackets}}$$

$$= (R_0 V_c \cdots R_0 V_c)_L \Phi$$

The Linked Diagram Theorem