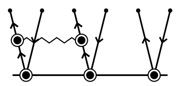
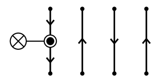
1. Interpret the following coefficient graph algebraically, denoting the bare excitation operator by $\tilde{a}^{ijk}_{abc}$.

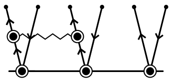


2. Interpret the following coefficient graph algebraically, denoting the top bare excitation operator by $\tilde{a}^{ij}_{ab}$ and the bottom one by $\tilde{a}^{cd}_{kl}$ and their interaction tensors by ${}^{1}\overline{\delta}^{ef}_{mn}$ and ${}^{2}\overline{\delta}^{mn}_{ef}$.

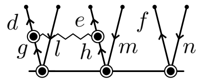


Solutions:

1. Axiom 4.1 gives



$$= \frac{1}{2!2!3!}\sum_{\substack{defgh\\lmn}}$$



$$= \frac{1}{2!2!3!}\sum_{\substack{defgh\\lmn}} \overline{\delta}^{def}_{lmn}\overline{g}^{gh}_{de}\, c^{lmn}_{ghf} \vdots a^{l^{\circ 1}m^{\circ 2}n^{\circ 3}}_{d^{\bullet 1}e^{\bullet 2}f^{\bullet 3}}\, a^{d^{\bullet 1}e^{\bullet 2}}_{g^{\bullet 4}h^{\bullet 5}} a^{g^{\bullet 4}h^{\bullet 5}f^{\bullet 3}}_{l^{\circ 1}m^{\circ 2}n^{\circ 3}} \vdots$$

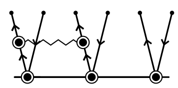
since there are two pairs of equivalent lines on the repulsion operator, and the CI triples operator has a set of three equivalent lines. The interaction tensor for the bare excitation operator is $\overline{\delta}^{def}_{lmn} = \hat{P}^{(i/j/k)}_{(a/b/c)}\delta^d_a\delta^e_b\delta^f_c\delta^i_l\delta^j_m\delta^k_n$, so this can be simplified as

$$\sum_{\substack{def\\lmn}}\overline{\delta}^{def}_{lmn}T^{lmn}_{def} = P^{(i/j/k)}_{(a/b/c)}T^{ijk}_{abc} \qquad \text{where} \quad T^{lmn}_{def} \equiv \frac{1}{2!2!3!}\sum_{gh}\overline{g}^{gh}_{de}\, c^{lmn}_{ghf} \vdots a^{l^{\circ 1}m^{\circ 2}n^{\circ 3}}_{d^{\bullet 1}e^{\bullet 2}f^{\bullet 3}}\, a^{d^{\bullet 1}e^{\bullet 2}}_{g^{\bullet 4}h^{\bullet 5}} a^{g^{\bullet 4}h^{\bullet 5}f^{\bullet 3}}_{l^{\circ 1}m^{\circ 2}n^{\circ 3}} \vdots\,.$$

Using item 3 in Remark 4.3, the contracted operator string evaluates as follows

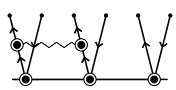
$$\vdots a^{l^{\circ 1}m^{\circ 2}n^{\circ 3}}_{d^{\bullet 1}e^{\bullet 2}f^{\bullet 3}}\, a^{d^{\bullet 1}e^{\bullet 2}}_{g^{\bullet 4}h^{\bullet 5}} a^{g^{\bullet 4}h^{\bullet 5}f^{\bullet 3}}_{l^{\circ 1}m^{\circ 2}n^{\circ 3}} \vdots = (-1)^{3+3} = +1$$

since there are three hole lines and three loops in the graph. At this point, we have simplified our interpretation to the following



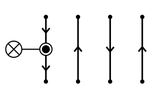
$$= \frac{1}{2!2!3!}\sum_{de}\hat{P}^{(i/j/k)}_{(a/b/c)}\overline{g}^{de}_{ab}\, c^{ijk}_{dec}$$

where I have relabeled the summation indices $g \mapsto d$, $h \mapsto e$. Finally, using item 4 under Remark 4.3, we can cancel the degeneracy factors 2!3! coming from equivalent coefficient lines by replacing $\hat{P}^{(i/j/k)}_{(a/b/c)}$ with $\hat{P}_{(ab/c)}$. This works because the operand $\overline{g}^{de}_{ab}c^{ijk}_{dec}$ is already antisymmetric with respect to $\{a, b\}$ and $\{i, j, k\}$.

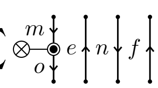


$$= \frac{1}{2!}\sum_{de}\hat{P}_{(ab/c)}\overline{g}^{de}_{ab}\, c^{ijk}_{dec}$$

2. Axiom 4.1 gives



$$= \frac{1}{2!}\sum_{\substack{ef\\mno}}$$

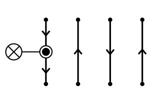


$$= \frac{1}{2!}\sum_{\substack{ef\\mno}} {}^1\overline{\delta}^{ef}_{mn} f^m_o\, {}^2\overline{\delta}^{on}_{ef} \vdots a^{m^{\circ 1}n^{\circ 2}}_{e^{\bullet 1}f^{\bullet 2}}\, a^{o^{\circ 3}}_{m^{\circ 1}} a^{e^{\bullet 1}f^{\bullet 2}}_{o^{\circ 3}n^{\circ 2}} \vdots$$

since the two particle lines are equivalent. Using item 3 in Remark 4.3, the operator string evaluates to

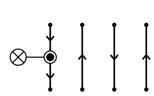
$$\vdots a^{m^{\circ 1}n^{\circ 2}}_{e^{\bullet 1}f^{\bullet 2}}\, a^{o^{\circ 3}}_{m^{\circ 1}} a^{e^{\bullet 1}f^{\bullet 2}}_{o^{\circ 3}n^{\circ 2}} \vdots = (-1)^{3+2} = -1$$

since there are three hole lines and two loops in the diagram. Substituting in the definitions of the interaction tensors, we can simplify the result as follows.



$$\begin{aligned} &= -\frac{1}{2!}\sum_{\substack{ef\\mno}} {}^1\overline{\delta}^{ef}_{mn} f^m_o\, {}^2\overline{\delta}^{on}_{ef} = -\frac{1}{2!}\sum_{\substack{ef\\mno}} \left(\hat{P}^{(i/j)}_{(a/b)}\delta^e_a\delta^f_b\delta^i_m\delta^j_n\right) f^m_o \left(\hat{P}^{(c/d)}_{(k/l)}\delta^o_k\delta^n_l\delta^c_e\delta^d_f\right) \\ &= -\frac{1}{2!}\hat{P}^{(i/j)}_{(a/b)}\hat{P}^{(c/d)}_{(k/l)} f^i_k\delta^j_l\delta^c_a\delta^d_b \end{aligned}$$

Finally, using item 4 under Remark 4.3, we can cancel the degeneracy factor 2! by replacing $\hat{P}^{(i/j)}_{(a/b)}$ with $\hat{P}^{(i/j)}$.



$$= -\hat{P}^{(i/j|c/d)}_{(k/l)} f^i_k\delta^j_l\delta^c_a\delta^d_b$$