

diagrams

Basic diagram notation:

Basic diagram notation:

$$a_p \equiv \uparrow$$

Basic diagram notation:

$$a_p \equiv \text{stick figure with a dot on its head}$$

$$a_p^+ \equiv \text{stick figure with a dot on its head and a cross on its chest}$$

Basic diagram notation:

$$a_p \equiv \text{diagram of a line with an upward arrow at the bottom}$$

$$a_p^\dagger \equiv \text{diagram of a line with a downward arrow at the bottom}$$

$$a_p^\dagger a_q \equiv \text{diagram of two lines with a downward arrow at the bottom and an upward arrow at the top}$$

Basic diagram notation:

$$a_p \equiv \text{diagram: a vertical line with an incoming arrow from below and an outgoing arrow to a small circle at the top}$$

$$a_p^\dagger \equiv \text{diagram: a vertical line with an incoming arrow from below to a small circle at the bottom and an outgoing arrow to the top}$$

$$a_p^\dagger a_q \equiv \text{diagram: two vertical lines. The left line has an incoming arrow from below to a small circle at the bottom, and an outgoing arrow to a small circle at the top. The right line has an incoming arrow from below and an outgoing arrow to the top. A horizontal line connects the two small circles at the top.$$

$$\overline{a_p a_q^\dagger} \equiv \text{diagram: two vertical lines. The left line has an incoming arrow from below and an outgoing arrow to a small circle at the top. The right line has an incoming arrow from below to a small circle at the bottom, and an outgoing arrow to the top. A horizontal line connects the two small circles at the top.$$

# Basic diagram notation:

$$a_p \equiv \text{diagram: a vertical line with a small circle at the top and an upward-pointing arrow at the bottom}$$

$$a_p^\dagger \equiv \text{diagram: a vertical line with a small circle at the top and a downward-pointing arrow at the bottom}$$

$$a_p^\dagger a_q \equiv \text{diagram: a vertical line with a small circle at the top, a red upward-pointing arrow in the middle, and a downward-pointing arrow at the bottom}$$

$$\overline{a_p a_q^\dagger} \equiv \text{diagram: a vertical line with a small circle at the top, a downward-pointing arrow in the middle, and a small circle at the bottom}$$

Basic diagram notation:

$$a_p \equiv \text{diagram: a vertical line with a small circle at the top and an upward-pointing arrow at the bottom}$$

$$a_p^\dagger \equiv \text{diagram: a vertical line with a small circle at the top and a downward-pointing arrow at the bottom}$$

$$a_p^\dagger a_q \equiv \text{diagram: a vertical line with a small circle in the middle. A red upward-pointing arrow starts from the bottom and ends at the circle. A blue downward-pointing arrow starts from the circle and ends at the bottom.$$

$$\overline{a_p a_q^\dagger} \equiv \text{diagram: a vertical line with a small circle at the top and a small circle at the bottom, connected by a vertical line. A horizontal line connects the two circles, with a small vertical tick mark at the left end of the horizontal line.$$



# Basic diagram notation:

$$a_p \equiv \text{diagram of a vertical line with a dot at the top and an upward arrow at the bottom}$$


$$a_p^\dagger \equiv \text{diagram of a vertical line with a dot at the top and a downward arrow at the bottom}$$


$$a_p^\dagger a_q \equiv \text{diagram of a vertical line with a red dot at the top, a blue dot in the middle, a red upward arrow from the middle dot, and a blue downward arrow from the top dot}$$


$$\overline{a_p a_q^\dagger} \equiv \text{diagram of a vertical line with a red dot at the top, a blue dot in the middle, a red downward arrow from the top dot, and a blue upward arrow from the middle dot}$$

# Basic diagram notation:

$$a_p \equiv \text{diagram}$$


$$a_p^\dagger \equiv \text{diagram}$$


$$a_p^\dagger a_q \equiv \text{diagram}$$


$$\overline{a_p a_q^\dagger} \equiv \text{diagram}$$


Quasiparticles:

# Quasiparticles:

$$b_a \equiv \uparrow$$

# Quasiparticles:

$$b_a \equiv \downarrow \quad b_a^\dagger \equiv \uparrow$$

# Quasiparticles:

$$b_a \equiv \begin{array}{c} \bullet \\ \uparrow \end{array} \quad b_a^\dagger \equiv \begin{array}{c} \uparrow \\ \bullet \end{array} \quad b_a^\dagger b_b \equiv \begin{array}{c} \uparrow \\ \bullet \\ \uparrow \end{array}$$

# Quasiparticles:

$$b_a \equiv \text{down arrow with a dot above it} \quad b_a^\dagger \equiv \text{up arrow with a dot above it} \quad b_a^\dagger b_b \equiv \text{two arrows (one up, one down) with a dot above the up arrow} \quad \overbrace{b_a b_b^\dagger} \equiv \text{two arrows (one up, one down) with a dot above the up arrow}$$

# Quasiparticles:

$$b_a \equiv \text{down arrow with dot} \quad b_a^\dagger \equiv \text{up arrow with dot} \quad b_a^\dagger b_b \equiv \text{up arrow with dot over down arrow with dot} \quad \overbrace{b_a b_b^\dagger} \equiv \text{down arrow with dot over up arrow with dot}$$

$$b_i^\dagger \equiv \text{up arrow with dot}$$



# Quasiparticles:

$$\begin{array}{llll}
 b_a \equiv \text{down arrow with dot} & b_a^\dagger \equiv \text{up arrow with dot} & b_a^\dagger b_b \equiv \text{two up arrows with dot} & \overbrace{b_a b_b}^\text{curly brace} \equiv \text{two down arrows with dot} \\
 b_i^\dagger \equiv \text{up arrow with dot} & b_i \equiv \text{down arrow with dot} & & 
 \end{array}$$

# Quasiparticles:

$$b_a \equiv \text{down arrow with dot} \quad b_a^\dagger \equiv \text{up arrow with dot} \quad b_a^\dagger b_b \equiv \text{up arrow with dot and down arrow with dot} \quad \overbrace{b_a b_b^\dagger} \equiv \text{up arrow with dot and down arrow with dot}$$

$$b_i^\dagger \equiv \text{up arrow with dot} \quad b_i \equiv \text{down arrow with dot} \quad b_i b_j^\dagger \equiv \text{up arrow with dot and down arrow with dot}$$

# Quasiparticles:

$$\begin{array}{cccc}
 b_a \equiv \text{---} \bullet \text{---} \uparrow & b_a^\dagger \equiv \bullet \text{---} \uparrow & b_a^\dagger b_b \equiv \bullet \text{---} \uparrow \text{---} \uparrow & \overbrace{b_a b_b^\dagger} \equiv \bullet \text{---} \uparrow \text{---} \bullet \\
 b_i^\dagger \equiv \bullet \text{---} \downarrow & b_i \equiv \bullet \text{---} \downarrow & b_i b_j^\dagger \equiv \bullet \text{---} \downarrow \text{---} \downarrow & \overbrace{b_i b_j^\dagger} \equiv \bullet \text{---} \downarrow \text{---} \bullet
 \end{array}$$

# Quasiparticles:

$$b_a \equiv \text{dot with up arrow}$$

$$b_a^\dagger \equiv \text{dot with down arrow}$$

$$b_a^\dagger b_b \equiv \text{dot with red up arrow and blue down arrow}$$

$$\overbrace{b_a b_b^\dagger} \equiv \text{vertical line with dot at top and bottom}$$

$$b_i^\dagger \equiv \text{dot with down arrow}$$

$$b_i \equiv \text{dot with up arrow}$$

$$b_i b_j^\dagger \equiv \text{dot with down arrow and up arrow}$$

$$\overbrace{b_i b_j^\dagger} \equiv \text{vertical line with dot at top and bottom}$$

# Quasiparticles:

$$b_a \equiv \text{down arrow with dot}$$

$$b_a^\dagger \equiv \text{up arrow with dot}$$

$$b_a^\dagger b_b \equiv \text{up arrow (red) and down arrow (blue) with dot}$$

$$\overbrace{b_a b_b^\dagger} \equiv \text{down arrow (red) and up arrow (blue) with dot}$$

$$b_i^\dagger \equiv \text{up arrow with dot}$$

$$b_i \equiv \text{down arrow with dot}$$

$$b_i b_j^\dagger \equiv \text{up arrow (blue) and down arrow (red) with dot}$$

$$\overbrace{b_i b_j^\dagger} \equiv \text{down arrow (red) and up arrow (blue) with dot}$$

# Quasiparticles:

$$\begin{array}{llll}
 b_a \equiv \text{down arrow} & b_a^\dagger \equiv \text{up arrow} & b_a^\dagger b_b \equiv \text{up arrow over down arrow} & \overbrace{b_a b_b^\dagger} \equiv \text{down arrow} \\
 b_i^\dagger \equiv \text{up arrow} & b_i \equiv \text{down arrow} & b_i b_j^\dagger \equiv \text{down arrow over up arrow} & \overbrace{b_i b_j^\dagger} \equiv \text{down arrow} \\
 & & b_a^\dagger b_i^\dagger \equiv \text{two arrows meeting at a dot} & 
 \end{array}$$

# Quasiparticles:

$$b_a \equiv \text{down arrow with dot} \quad b_a^\dagger \equiv \text{up arrow with dot} \quad b_a^\dagger b_b \equiv \text{up arrow with dot, down arrow with dot} \quad \overline{b_a b_b^\dagger} \equiv \text{up arrow with dot, down arrow with dot}$$

$$b_i^\dagger \equiv \text{up arrow with dot} \quad b_i \equiv \text{down arrow with dot} \quad b_i b_j^\dagger \equiv \text{down arrow with dot, up arrow with dot} \quad \overline{b_i b_j^\dagger} \equiv \text{down arrow with dot, up arrow with dot}$$

$$b_a^\dagger b_i^\dagger \equiv \text{up arrow with dot, up arrow with dot} \quad b_i b_a \equiv \text{down arrow with dot, down arrow with dot}$$

Excitation operators:



Excitation operators:

$$: a_{q_1}^{p_1} a_{q_2}^{p_2} \dots a_{q_n}^{p_n} :$$

Excitation operators:

$$: a_{q_1}^{p_1} a_{q_2}^{p_2} \dots a_{q_n}^{p_n} : = \begin{array}{c} \uparrow \quad \uparrow \quad \dots \quad \uparrow \\ | \quad | \quad \dots \quad | \\ \circ - \circ - \dots - \circ \\ | \quad | \quad \dots \quad | \\ \uparrow \quad \uparrow \quad \dots \quad \uparrow \end{array}$$

Excitation operators:

$$: a_{q_1}^{p_1} a_{q_2}^{p_2} \dots a_{q_n}^{p_n} : = \begin{array}{c} \uparrow \quad \uparrow \quad \dots \quad \uparrow \\ \circ \text{---} \circ \text{---} \dots \text{---} \circ \\ \uparrow \quad \uparrow \quad \dots \quad \uparrow \end{array}$$

$$: a_{q_1}^{p_1} a_{q_2}^{p_2} \dots a_{q_n}^{p_n} :$$

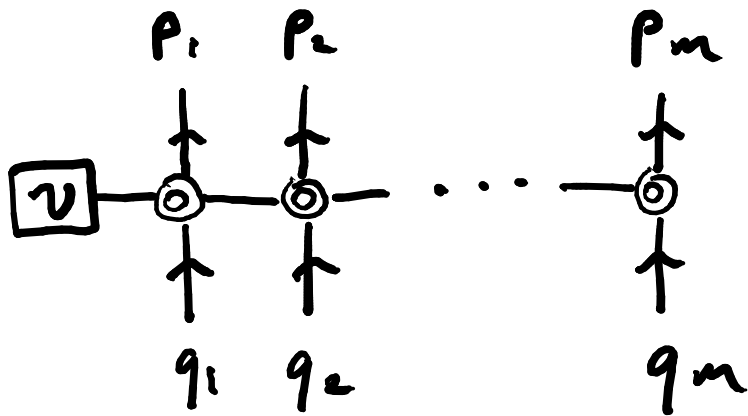
# Excitation operators:

$$: a_{q_1}^{p_1} a_{q_2}^{p_2} \dots a_{q_n}^{p_n} : = \begin{array}{c} \uparrow \quad \uparrow \quad \dots \quad \uparrow \\ \circ \text{---} \circ \text{---} \dots \text{---} \circ \\ \uparrow \quad \uparrow \quad \dots \quad \uparrow \end{array}$$

$$: a_{q_1}^{p_1} a_{q_2}^{p_2} \dots a_{q_n}^{p_n} : = \begin{array}{c} \uparrow \quad \uparrow \quad \dots \quad \uparrow \\ \odot \text{---} \odot \text{---} \dots \text{---} \odot \\ \uparrow \quad \uparrow \quad \dots \quad \uparrow \end{array}$$

$n$ -electron operators:

$m$ -electron operators:



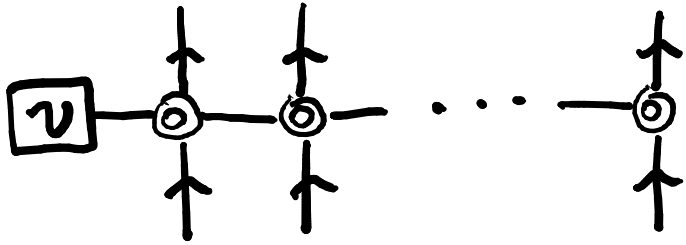
m-electron operators:

$$\boxed{v} - \begin{array}{c} p_1 \\ \uparrow \\ \bigcirc \\ \uparrow \\ q_1 \end{array} - \begin{array}{c} p_2 \\ \uparrow \\ \bigcirc \\ \uparrow \\ q_2 \end{array} - \dots - \begin{array}{c} p_m \\ \uparrow \\ \bigcirc \\ \uparrow \\ q_m \end{array} \equiv \bar{v}_{p_1 \dots p_m}^{q_1 \dots q_m} \sim_{a_{q_1 \dots q_m}}^{p_1 \dots p_m} \quad (\text{no summation})$$

$n$ -electron operators:



$m$ -electron operators:



$m$ -electron operators:

$$\boxed{v} \text{---} \begin{array}{c} \uparrow \\ \bigcirc \\ \uparrow \end{array} \text{---} \begin{array}{c} \uparrow \\ \bigcirc \\ \uparrow \end{array} \cdots \begin{array}{c} \uparrow \\ \bigcirc \\ \uparrow \end{array} \equiv \left(\frac{1}{m!}\right)^2 \sum \boxed{v} \text{---} \begin{array}{c} \uparrow \\ \bigcirc \\ \uparrow \end{array} \text{---} \begin{array}{c} \uparrow \\ \bigcirc \\ \uparrow \end{array} \cdots \begin{array}{c} \uparrow \\ \bigcirc \\ \uparrow \end{array}$$

$m$ -electron operators:

$$\begin{array}{c} \boxed{v} \end{array} \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \cdots \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \equiv \left(\frac{1}{m!}\right)^2 \sum_{\substack{p_1 \cdots p_m \\ q_1 \cdots q_m}} \begin{array}{c} \boxed{v} \end{array} \begin{array}{c} p_1 \\ \uparrow \\ \circ \\ \uparrow \\ q_1 \end{array} \begin{array}{c} p_2 \\ \uparrow \\ \circ \\ \uparrow \\ q_2 \end{array} \cdots \begin{array}{c} p_m \\ \uparrow \\ \circ \\ \uparrow \\ q_m \end{array}$$

$m$ -electron operators:

$$\begin{array}{c} \boxed{v} \end{array} \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \cdots \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \equiv \left(\frac{1}{m!}\right)^2 \sum_{\substack{p_1 \cdots p_m \\ q_1 \cdots q_m}} \begin{array}{c} p_1 \quad p_2 \quad \quad \quad p_m \\ \uparrow \quad \uparrow \quad \quad \quad \uparrow \\ \boxed{v} \end{array} \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \cdots \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \begin{array}{c} q_1 \quad q_2 \quad \quad \quad q_m \\ \uparrow \quad \uparrow \quad \quad \quad \uparrow \end{array}$$

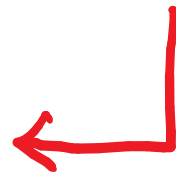
$$= \left(\frac{1}{m!}\right)^2 \sum_{\text{Einstein}} \bar{v}_{p_1 \cdots p_m}^{q_1 \cdots q_m} \sim_{p_1 \cdots p_m} a_{q_1 \cdots q_m}$$

m-electron operators:

$$\boxed{v} \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \cdots \begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} \equiv \left(\frac{1}{m!}\right)^2 \sum_{\substack{p_1 \cdots p_m \\ q_1 \cdots q_m}} \boxed{v} \begin{array}{c} p_1 \\ \uparrow \\ \circ \\ \uparrow \\ q_1 \end{array} \begin{array}{c} p_2 \\ \uparrow \\ \circ \\ \uparrow \\ q_2 \end{array} \cdots \begin{array}{c} p_m \\ \uparrow \\ \circ \\ \uparrow \\ q_m \end{array}$$

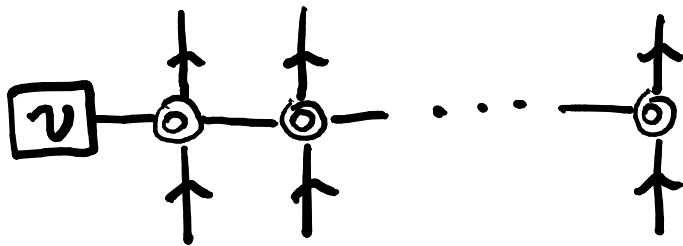
$$= \left(\frac{1}{m!}\right)^2 \sum_{\text{Einstein}} \bar{v}_{p_1 \cdots p_m}^{q_1 \cdots q_m} a_{q_1 \cdots q_m}^{p_1 \cdots p_m}$$

summation and scalar factor  
are built in to the notation



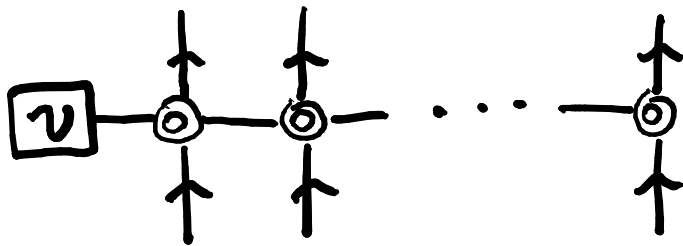
alternative notations:

alternative notations:

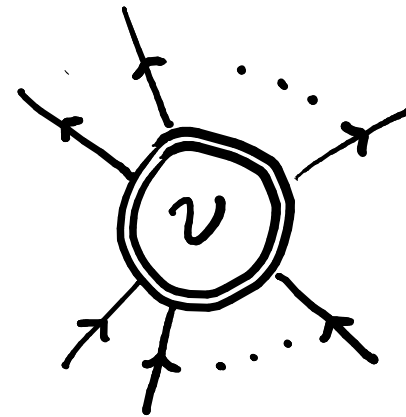


Goldstone

alternative notations:



Goldstone



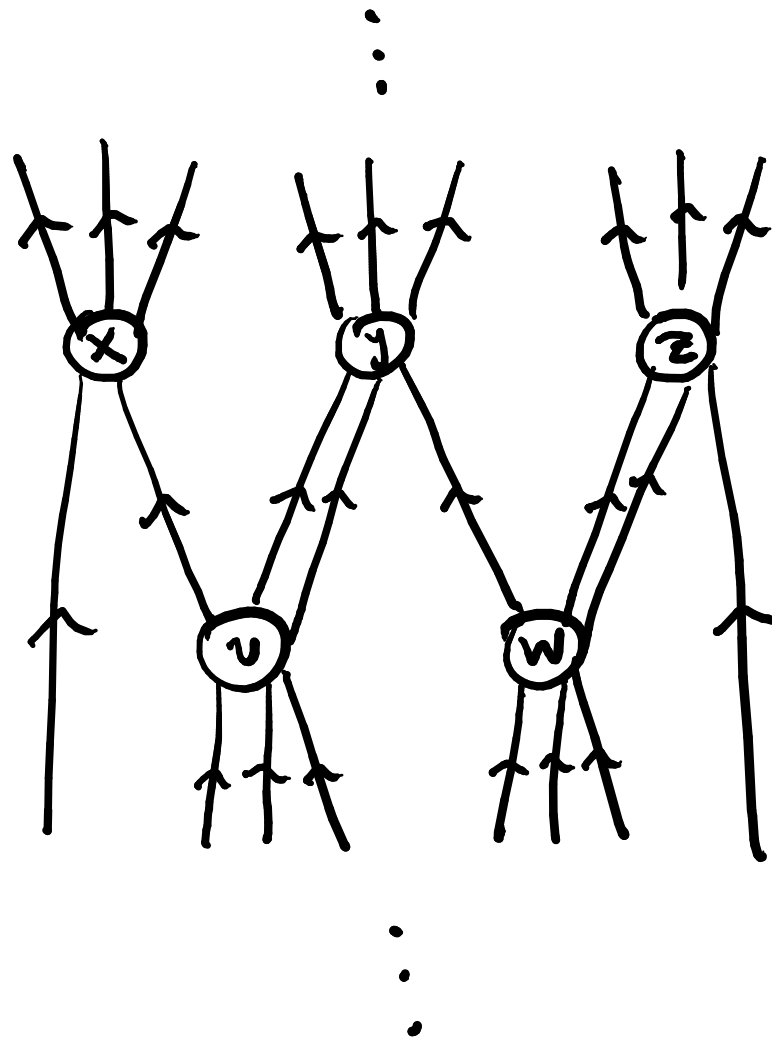
Hugenholtz



graph:

graph: a product of  $m$ -electron ops

graph: a product of  $m$ -electron ops



universal rule for interpreting graphs:

universal rule for interpreting graphs:

graph

universal rule for interpreting graphs:

graph =

universal rule for interpreting graphs:

$$\text{graph} = \frac{1}{\# \text{permutational symmetries}}$$

universal rule for interpreting graphs:

$$\text{graph} = \frac{1}{\# \text{permutational symmetries}}$$

labeled  
graph



universal rule for interpreting graphs:

$$\text{graph} = \frac{1}{\# \text{permutational symmetries}} \sum_{\text{labels}} \text{labeled graph}$$

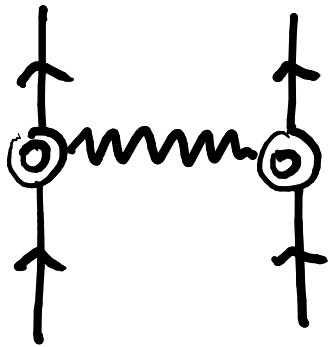
universal rule for interpreting graphs:

$$\text{graph} = \frac{1}{\# \text{permutational symmetries}} \sum_{\text{labels}} \text{labeled graph}$$

directly translates into  
an algebraic expression

Examples:

Examples:



Examples:

$$\begin{array}{c}
 \begin{array}{c} \uparrow \\ | \\ \bigcirc \\ | \\ \uparrow \end{array} \text{---} \begin{array}{c} \uparrow \\ | \\ \bigcirc \\ | \\ \uparrow \end{array} \\
 \end{array} = \sum \begin{array}{c} p \qquad q \\ \begin{array}{c} \uparrow \\ | \\ \bigcirc \\ | \\ \uparrow \end{array} \text{---} \begin{array}{c} \uparrow \\ | \\ \bigcirc \\ | \\ \uparrow \end{array} \\
 r \qquad s
 \end{array} \\
 \\
 = \sum \bar{g}_{pq}^{rs} \tilde{a}_{rs}^{pq}
 \end{array}$$

Examples:

$$\begin{array}{c}
 \begin{array}{c} \uparrow \\ \bigcirc \\ \uparrow \end{array} \text{---} \begin{array}{c} \uparrow \\ \bigcirc \\ \uparrow \end{array} \\
 \end{array} = \sum \begin{array}{c} p \qquad q \\ \uparrow \qquad \uparrow \\ \bigcirc \text{---} \bigcirc \\ \uparrow \qquad \uparrow \\ r \qquad s \end{array} \\
 \\
 = \sum \bar{g}_{p q}^{rs} \tilde{a}_{rs}^{p q}
 \end{array}$$

Examples:

$$\begin{aligned}
 & \text{Diagram 1} = \frac{1}{2} \sum \text{Diagram 2} \\
 & = \frac{1}{2} \sum \bar{g}_{pq}^{rs} \tilde{a}_{rs}^{pq}
 \end{aligned}$$

The diagrams are as follows:

- Diagram 1:** Two vertices (circles with a dot) connected by a wavy line. Each vertex has an incoming vertical line from below and an outgoing vertical line to above. The outgoing lines are red and have red arrows pointing upwards.
- Diagram 2:** Similar to Diagram 1, but the incoming lines are labeled  $r$  and  $s$  at the bottom, and the outgoing lines are labeled  $p$  and  $q$  at the top. The outgoing lines are red and have red arrows pointing upwards.

Examples:

$$\begin{aligned}
 & \begin{array}{c} \uparrow \\ \textcircled{\circ} \\ \uparrow \end{array} \text{---} \begin{array}{c} \uparrow \\ \textcircled{\circ} \\ \uparrow \end{array} = \frac{1}{2} \sum \begin{array}{c} p \\ \uparrow \\ \textcircled{\circ} \\ \uparrow \\ r \end{array} \text{---} \begin{array}{c} q \\ \uparrow \\ \textcircled{\circ} \\ \uparrow \\ s \end{array} \\
 & = \frac{1}{2} \sum \bar{g}_{pq}^{rs} \tilde{a}_{rs}^{pq}
 \end{aligned}$$

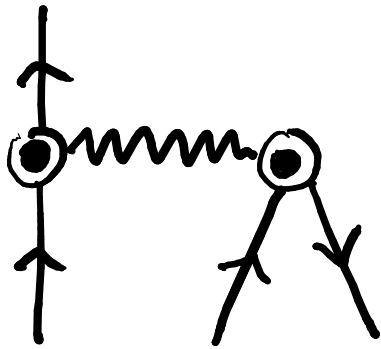


Examples:

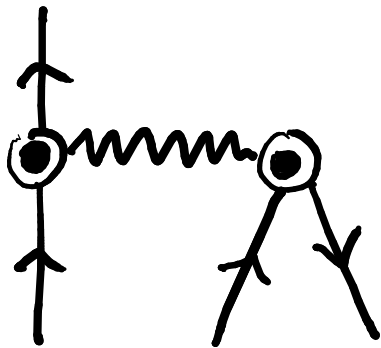
$$\begin{aligned}
 & \begin{array}{c} \uparrow \\ \textcircled{\circ} \\ \uparrow \end{array} \text{---} \begin{array}{c} \uparrow \\ \textcircled{\circ} \\ \uparrow \end{array} = \frac{1}{2 \cdot 2} \sum \begin{array}{c} p \\ \uparrow \\ \textcircled{\circ} \\ \uparrow \\ r \end{array} \text{---} \begin{array}{c} q \\ \uparrow \\ \textcircled{\circ} \\ \uparrow \\ s \end{array} \\
 & = \frac{1}{2 \cdot 2} \sum \bar{g}_{pq}^{rs} \tilde{a}_{rs}^{pq}
 \end{aligned}$$

Examples:

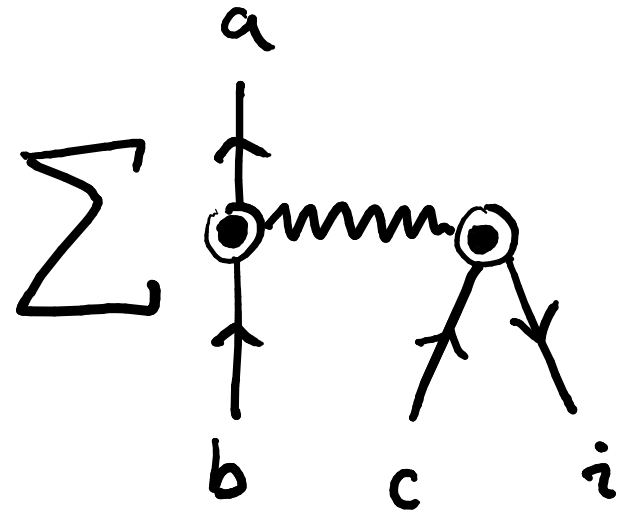
Examples:



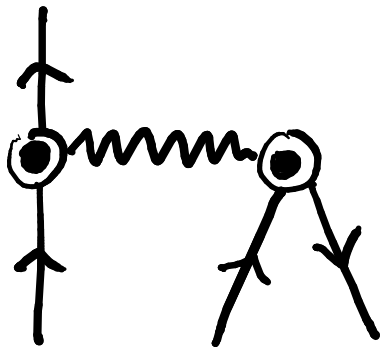
Examples:



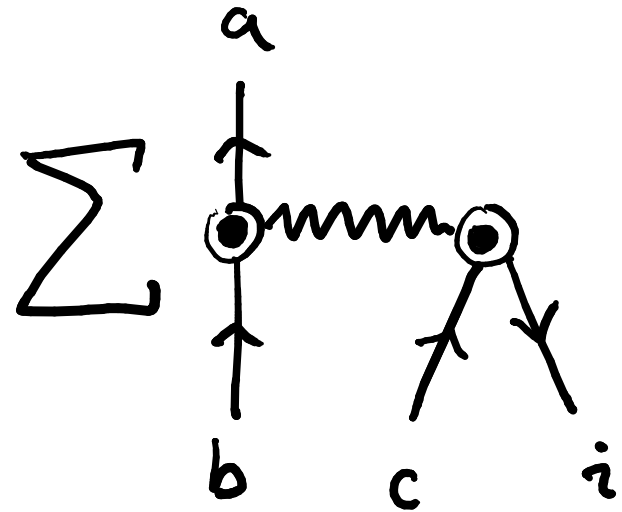
=



Examples:



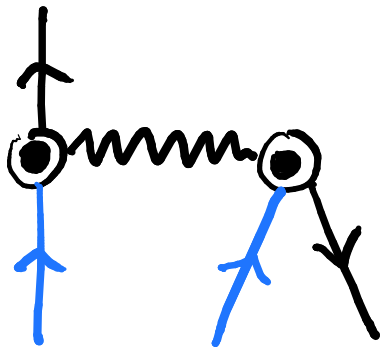
=



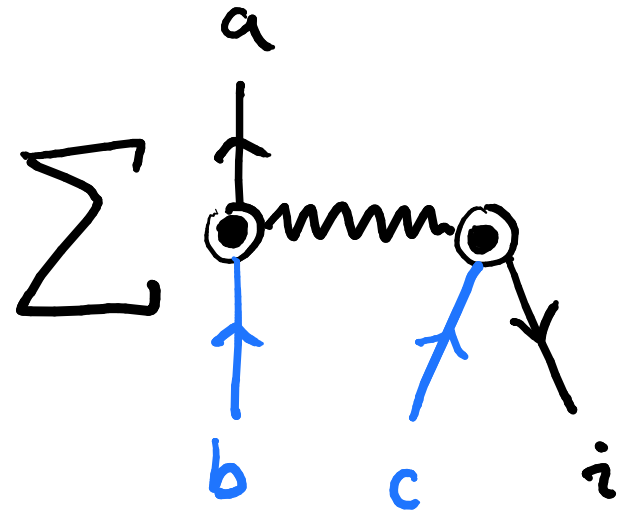
=

$$\sum \bar{g}_{ai}^{bc} a_{bc}^i$$

Examples:



=



=

$$\sum \bar{g}_{ai}^{bc} a_{bc}^{ai}$$

Examples:

$$\begin{aligned}
 & \text{Diagram 1} = \frac{1}{2} \sum \text{Diagram 2} \\
 & = \frac{1}{2} \sum \bar{g}_{ai}^{bc} a_{bc}^{ai}
 \end{aligned}$$

The diagram on the left shows two vertices connected by a wavy line. The left vertex has an incoming blue line from below and an outgoing black line from above. The right vertex has an incoming blue line from below and an outgoing black line from above.

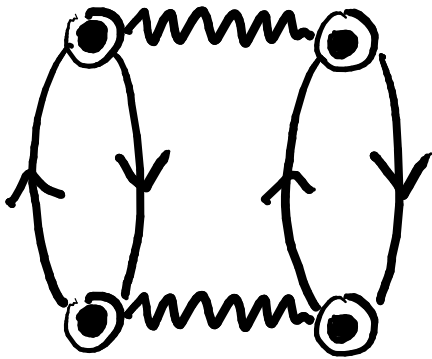
The diagram in the middle shows the same structure, but with an additional index 'a' on the outgoing black line of the left vertex and an additional index 'i' on the outgoing black line of the right vertex. The incoming blue lines are labeled 'b' and 'c' respectively.

The diagram on the right is a mathematical expression representing the sum over all possible indices 'a' and 'i' of the product of the coupling  $\bar{g}_{ai}^{bc}$  and the amplitude  $a_{bc}^{ai}$ , multiplied by a factor of  $\frac{1}{2}$ .

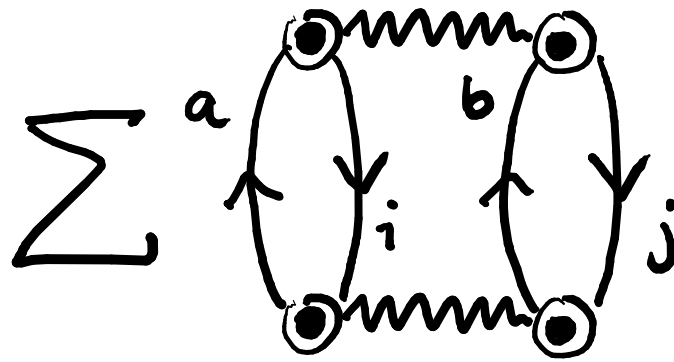
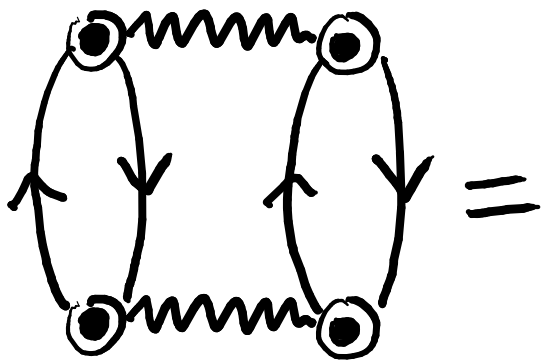
Examples:



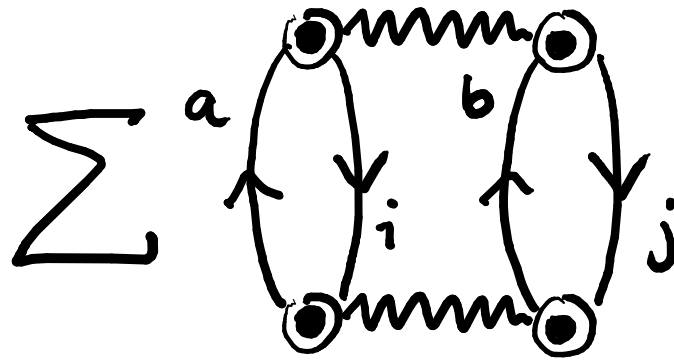
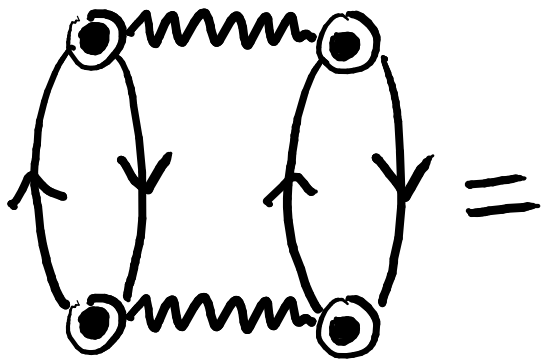
Examples:



Examples:

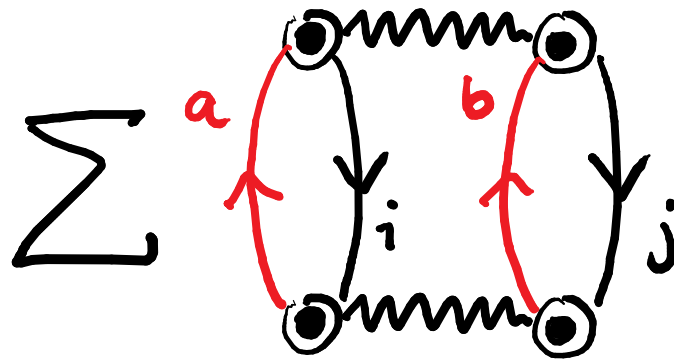
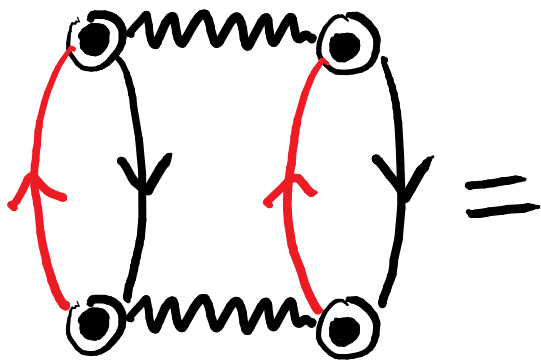


Examples:



$$= \sum \bar{g}_{ij}^{ab} \bar{g}_{ab}^{ij} : a_a^{i\circ} b^{j\circ\circ} a_{i\circ}^{a\circ} b_{j\circ\circ}^{b\circ} :$$

Examples:



$$= \sum \bar{g}_{ij}^{\bar{ab}} \bar{g}_{ab}^{ij} : a_{a^{\bullet} b^{\bullet\bullet}}^{i^{\circ} j^{\circ\circ}} a_{i^{\circ} j^{\circ\circ}}^{a^{\bullet} b^{\bullet\bullet}} :$$

Examples:

The diagram shows a vacuum loop with two wavy lines (top and bottom) and two vertices (left and right). The left vertex has two red curved arrows forming a loop. The right vertex has two black curved arrows forming a loop. This is equal to  $\frac{1}{2}$  times a sum over indices  $i$  and  $j$  of a similar loop, but with the left vertex labeled  $a$  and the right vertex labeled  $j$ .

$$= \frac{1}{2} \sum$$

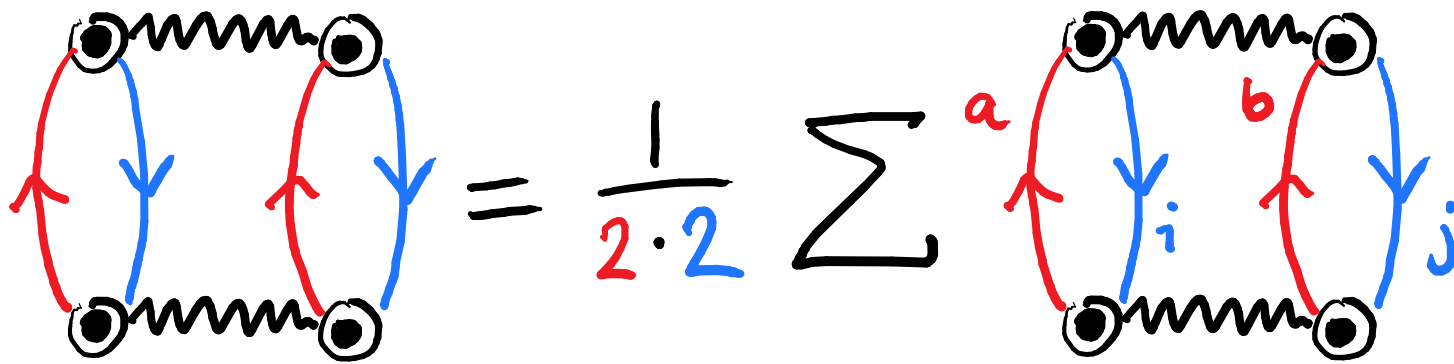
$$= \frac{1}{2} \sum \bar{g}_{ij}^{ab} \bar{g}_{ab}^{ij} : a_{a^{\bullet} b^{\bullet\bullet}}^{i^{\circ} j^{\circ\circ}} a_{i^{\circ} j^{\circ\circ}}^{a^{\bullet} b^{\bullet\bullet}} :$$

Examples:

$$\begin{array}{c} \bullet \\ \circlearrowleft \\ \bullet \end{array} \begin{array}{c} \bullet \\ \circlearrowright \\ \bullet \end{array} = \frac{1}{2} \sum \begin{array}{c} \bullet \\ \circlearrowleft^a \\ \bullet \end{array} \begin{array}{c} \bullet \\ \circlearrowright^b \\ \bullet \end{array}$$

$$= \frac{1}{2} \sum \bar{g}_{ij}^{ab} \bar{g}_{ab}^{ij} : a_{a^{\bullet} b^{\bullet}}^{i^{\circ} j^{\circ}} a_{i^{\circ} j^{\circ}}^{a^{\bullet} b^{\bullet}} :$$

Examples:



$$= \frac{1}{2 \cdot 2} \sum$$

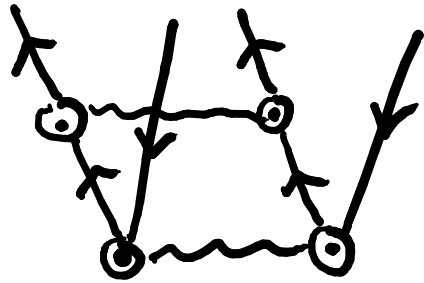
$$= \frac{1}{2 \cdot 2} \sum \bar{g}_{ij}^{ab} \bar{g}_{ab}^{ij} : a_{a'}^{i'} b_{b'}^{j'} a_{i''}^{j''} a_{j''}^{i''} :$$

Now you try!

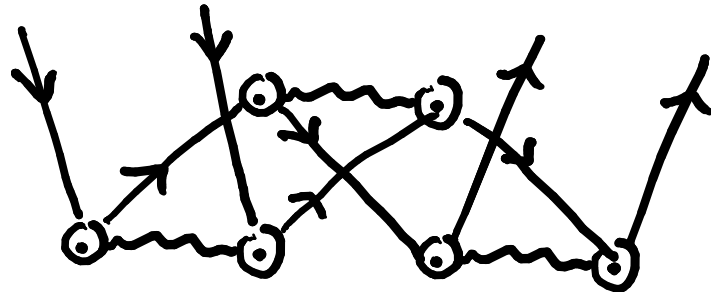


Now you try!

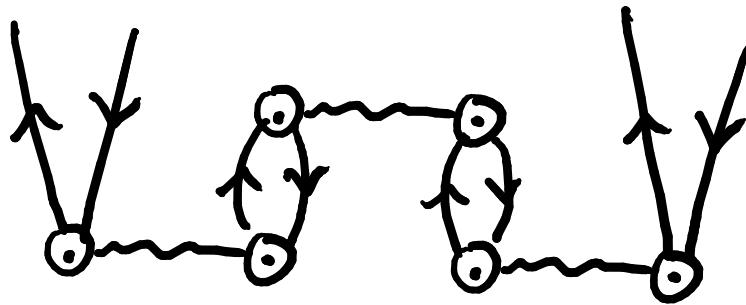
1.



2.



3.



the end.