

- Expand the electronic Hamiltonian  $H$  in terms of  $\Phi$ -normal-ordered operators using Wick's theorem for graphs, writing the core Hamiltonian and electron repulsion operators as  $\boxtimes \begin{array}{c} \uparrow \\ \circ \\ \downarrow \end{array} \equiv h_p^q a_q^p$  and  $\begin{array}{c} \uparrow \\ \circ \end{array} \text{---} \begin{array}{c} \uparrow \\ \circ \\ \downarrow \end{array} \equiv \frac{1}{4} \bar{g}_{pq}^{rs} a_{rs}^{pq}$ .

$$H = \boxtimes \begin{array}{c} \uparrow \\ \circ \\ \downarrow \end{array} + \begin{array}{c} \uparrow \\ \circ \end{array} \text{---} \begin{array}{c} \uparrow \\ \circ \\ \downarrow \end{array} = ?$$

2. Evaluate the following using Wick's theorem for graphs. Fully simplify your answer assuming the indices refer to a basis of canonical Hartree-Fock spin-orbitals.

$$\langle \Phi_{ijk}^{abc} | F_c C_3 | \Phi \rangle = ?$$

$$F_c \equiv f_p^q \tilde{a}_q^p$$

$$C_3 \equiv \left(\frac{1}{3!}\right)^2 c_{def}^{lmn} \tilde{a}_{lmn}^{def}$$

3. (a) Explain how to get from the projected CCD Schrödinger equation

$$E_c t_{ab}^{ij} = \langle \Phi_{ij}^{ab} | H_c \exp(T_2) | \Phi \rangle \quad H_c = F_c + V_c \quad \begin{aligned} F_c &\equiv f_p^q \tilde{a}_q^p \\ V_c &\equiv \frac{1}{4} \bar{g}_{pq}^{rs} \tilde{a}_{rs}^{pq} \end{aligned} \quad (1)$$

to the working equation for CCD amplitudes

$$t_{ab}^{ij} = (\mathcal{E}_{ab}^{ij})^{-1} \langle \Phi_{ij}^{ab} | V_c \exp(T_2) | \Phi \rangle_L \quad \mathcal{E}_{ab}^{ij} \equiv \epsilon_i + \epsilon_j - \epsilon_a - \epsilon_b \quad (2)$$

assuming a canonical Hartree-Fock reference.<sup>1</sup>

- (b) Write out an algorithm to numerically solve equation 2.

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<sup>1</sup>Hint: You only need to evaluate three diagrams to answer this question.

**Extra Credit:** Derive the following interpretation rule in your own words:

Each open cycle in a graph contributes  $(-)^{h_i} a_q^p$  to the normal-ordered product of operators, where  $p$  and  $q$  label the free ends and  $h_i$  is the number of hole contractions in the cycle. A closed cycle (loop) contributes  $(-)^{h_i+1}$ .