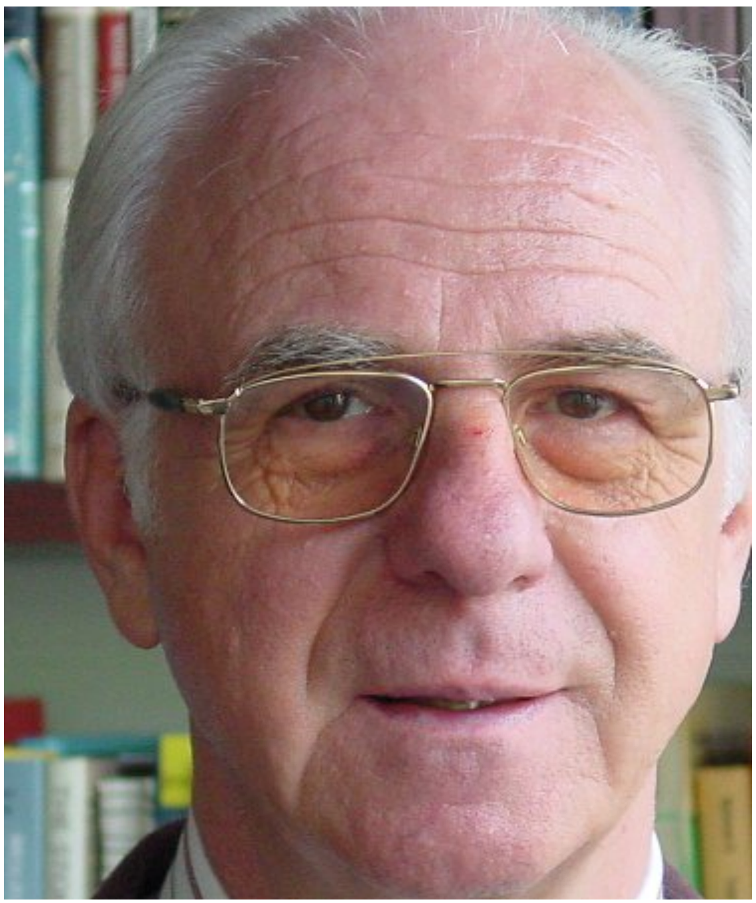
Kutzelnigg-Mukherjee notation



Einstein notation:

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$$\mathbb{A}\mathbb{B} = \mathbb{C}$$

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$$\downarrow$$

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$$a_p^\dagger a_q \longrightarrow a^p a_q$$

Kutzelnigg-Mukherjee notation:

$$h_{pq} \longrightarrow h^{q}_{p}$$

$$\langle pq \| rs \rangle \longrightarrow \bar{g}^{rs}_{pq}$$

$$a^{\dagger}_{p} a_{q} \longrightarrow a^{p} a_{q} \longrightarrow a^{p}_{q}$$

$$a^{\dagger}_{p} a^{\dagger}_{q} a_{s} a_{r} \longrightarrow a^{p} a^{q} a_{s} a_{r}$$

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$$a_p^\dagger a_q^\dagger a_s a_r \longrightarrow a^p a^q a_s a_r \longrightarrow a_{rs}^{pq}$$

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$$a_p^\dagger a_q^\dagger a_s a_r \longrightarrow a^p a^q a_s a_r \longrightarrow a_{rs}^{pq}$$

$$H_e = h_p^q a_q^p + \frac{1}{4} \bar{g}_{pq}^{rs} a_{rs}^{pq}$$

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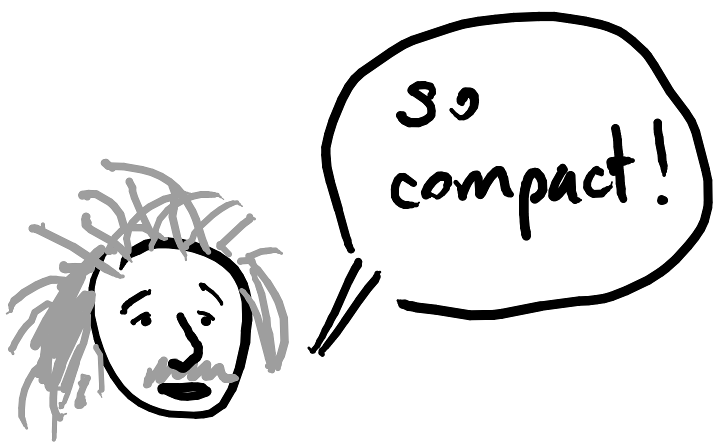

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Wick's theorem:

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$$a^{pq}_{rs} a^{+}_u$$

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$$a^{pq}_{rs} a^t_u = a^{pqt}_{rsu} + \delta^t_r a^{pq}_{us} + \delta^t_s a^{pq}_{ru}$$

Wick's theorem:

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simple rule:

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simple rule: "pair up uncontracted partners"

Φ-normal ordering:

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$$\tilde{a}^{p}_{q} \equiv \vdots\, a^{p} a_{q} \,\vdots$$

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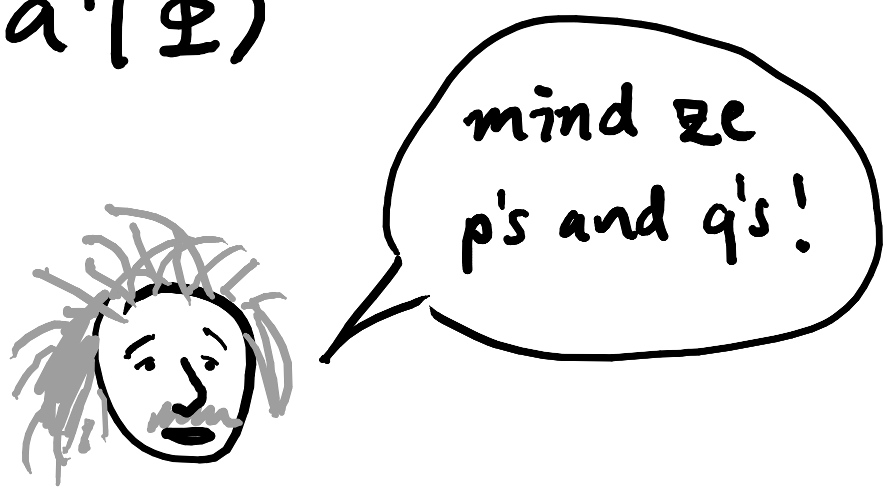

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$$a_{p\bullet}\, a^{q\bullet} \equiv \overbracket{a_p\, a^q} \qquad \text{particle contraction}$$

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$$\Longrightarrow \quad a^{p\bullet}_{q\bullet} = -\eta^p_q$$

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$$a^{q^\bullet} a_{p^\bullet} \equiv - a_p a^q$$

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$$\Longrightarrow \quad a^{p^\bullet}_{q^\bullet} = -\eta^p_q \qquad a^{p^\circ}_{q^\circ} = \gamma^p_q$$

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$$= \tilde{a}^{pr}_{qs} + \eta^{r}_{q}\,\tilde{a}^{p}_{s} - \gamma^{p}_{s}\,\tilde{a}^{r}_{q}$$

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Example:

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a^{pq}_{rs}

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$$a^{pq}_{rs} = \tilde{a}^{pq}_{rs} + \tilde{a}^{p^\circ q}_{r^\circ s} + \tilde{a}^{p^\circ q}_{r\, s^\circ} + \tilde{a}^{p\, q^\circ}_{r^\circ s} + \tilde{a}^{p q^\circ}_{r s^\circ} + \tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}} + \tilde{a}^{p^\circ q^{\circ\circ}}_{r^{\circ\circ} s^\circ}$$

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$$= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)} \tilde{a}^{p^\circ q}_{r^\circ s} + \hat{P}_{(r/s)} \tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}}$$

$$= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)} \gamma^{p}_{r} \tilde{a}^{q}_{s}$$

Example:

$$
\begin{aligned}
a^{pq}_{rs} &= \tilde{a}^{pq}_{rs} + \underbrace{\tilde{a}^{p^\circ q}_{r^\circ s} + \tilde{a}^{p^\circ q}_{r\, s^\circ} + \tilde{a}^{p\, q^\circ}_{r^\circ s} + \tilde{a}^{p q^\circ}_{r s^\circ}} + \underbrace{\tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}} + \tilde{a}^{p^\circ q^{\circ\circ}}_{r^{\circ\circ} s^\circ}} \\
&= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)}\, \tilde{a}^{p^\circ q}_{r^\circ s} + \hat{P}_{(r/s)}\, \tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}} \\
&= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)}\, \gamma^{p}_{r}\, \tilde{a}^{q}_{s} + \hat{P}_{(r/s)}\, \gamma^{p}_{r}\, \gamma^{q}_{s}
\end{aligned}
$$

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&= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)}\, \tilde{a}^{p^\circ q}_{r^\circ s} + \hat{P}_{(r/s)}\, \tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}} \\
&= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)}\, \gamma^p_r\, \tilde{a}^q_s + \hat{P}_{(r/s)}\, \gamma^p_r \gamma^q_s
\end{aligned}
$$

$$\bar{g}^{rs}_{pq}\, a^{pq}_{rs}$$

Example:

$$a^{pq}_{rs} = \tilde{a}^{pq}_{rs} + \underbrace{\tilde{a}^{p^\circ q}_{r^\circ s} + \tilde{a}^{p^\circ q}_{r\, s^\circ} + \tilde{a}^{p\, q^\circ}_{r^\circ s} + \tilde{a}^{p q^\circ}_{r\, s^\circ}}_{} + \underbrace{\tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}} + \tilde{a}^{p^\circ q^{\circ\circ}}_{r^{\circ\circ} s^\circ}}_{}$$

$$= \tilde{a}^{pq}_{rs} \quad + \quad \hat{P}^{(p/q)}_{(r/s)}\, \tilde{a}^{p^\circ q}_{r^\circ s} \quad + \quad \hat{P}_{(r/s)}\, \tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}}$$

$$= \tilde{a}^{pq}_{rs} \quad + \quad \hat{P}^{(p/q)}_{(r/s)}\, \gamma^p_r\, \tilde{a}^q_s \quad + \quad \hat{P}_{(r/s)}\, \gamma^p_r\, \gamma^q_s$$

$$\bar{g}^{rs}_{pq}\, a^{pq}_{rs} = \bar{g}^{rs}_{pq}\, \tilde{a}^{pq}_{rs} + \bar{g}^{rs}_{pq}\, \hat{P}^{(p/q)}_{(r/s)} \gamma^p_r\, \tilde{a}^q_s + \bar{g}^{rs}_{pq}\, \hat{P}_{(r/s)} \gamma^p_r\, \gamma^q_s$$

Example:

$$a_{rs}^{pq} = \tilde{a}_{rs}^{pq} + \underbrace{\tilde{a}_{r^\circ s}^{p^\circ q} + \tilde{a}_{r s^\circ}^{p^\circ q} + \tilde{a}_{r^\circ s}^{p q^\circ} + \tilde{a}_{r s^\circ}^{p q^\circ}} + \underbrace{\tilde{a}_{r^\circ s^{\circ\circ}}^{p^\circ q^{\circ\circ}} + \tilde{a}_{r^{\circ\circ} s^\circ}^{p^\circ q^{\circ\circ}}}$$

$$= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \tilde{a}_{r^\circ s}^{p^\circ q} + \hat{P}_{(r/s)} \tilde{a}_{r^\circ s^{\circ\circ}}^{p^\circ q^{\circ\circ}}$$

$$= \tilde{a}_{rs}^{pq} + \hat{P}_{(r/s)}^{(p/q)} \gamma_r^p \tilde{a}_s^q + \hat{P}_{(r/s)} \gamma_r^p \gamma_s^q$$

$$\bar{g}_{pq}^{rs} a_{rs}^{pq} = \bar{g}_{pq}^{rs} \tilde{a}_{rs}^{pq} + \bar{g}_{pq}^{rs} {\color{red}\hat{P}_{(r/s)}^{(p/q)}} \gamma_r^p \tilde{a}_s^q + \bar{g}_{pq}^{rs} \hat{P}_{(r/s)} \gamma_r^p \gamma_s^q$$

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&= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)}\, \gamma^{p}_{r}\, \tilde{a}^{q}_{s} + \hat{P}_{(r/s)}\, \gamma^{p}_{r}\, \gamma^{q}_{s}
\end{aligned}
$$

$$
\bar{g}^{rs}_{pq}\, a^{pq}_{rs} = \bar{g}^{rs}_{pq}\, \tilde{a}^{pq}_{rs} + \bar{g}^{rs}_{pq} \cdot 4 \cdot \gamma^{p}_{r}\, \tilde{a}^{q}_{s} + \bar{g}^{rs}_{pq}\, \hat{P}_{(r/s)}\, \gamma^{p}_{r}\, \gamma^{q}_{s}
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$$= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)}\, \tilde{a}^{p^\circ q}_{r^\circ s} + \hat{P}_{(r/s)}\, \tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}}$$

$$= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)}\, \gamma^p_r\, \tilde{a}^q_s + \hat{P}_{(r/s)}\, \gamma^p_r \gamma^q_s$$

$$\bar{g}^{rs}_{pq}\, a^{pq}_{rs} = \bar{g}^{rs}_{pq}\, \tilde{a}^{pq}_{rs} + \bar{g}^{rs}_{pq} \cdot 4 \cdot \gamma^p_r\, \tilde{a}^q_s + \bar{g}^{rs}_{pq} \cdot 2 \cdot \gamma^p_r \gamma^q_s$$

Example:

$$a^{pq}_{rs} = \tilde{a}^{pq}_{rs} + \underbrace{\tilde{a}^{p^\circ q}_{r^\circ s} + \tilde{a}^{p^\circ q}_{r s^\circ} + \tilde{a}^{p q^\circ}_{r^\circ s} + \tilde{a}^{p q^\circ}_{r s^\circ}} + \underbrace{\tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}} + \tilde{a}^{p^\circ q^{\circ\circ}}_{r^{\circ\circ} s^\circ}}$$

$$= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)} \tilde{a}^{p^\circ q}_{r^\circ s} + \hat{P}_{(r/s)} \tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}}$$

$$= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)} \gamma^p_r \tilde{a}^q_s + \hat{P}_{(r/s)} \gamma^p_r \gamma^q_s$$

$$\bar{g}^{rs}_{pq} a^{pq}_{rs} = \bar{g}^{rs}_{pq} \tilde{a}^{pq}_{rs} + \bar{g}^{rs}_{pq} \cdot 4 \cdot \gamma^p_r \tilde{a}^q_s + \bar{g}^{rs}_{pq} \cdot 2 \cdot \gamma^p_r \gamma^q_s$$

Someone show it on the board!

Example:

$$a^{pq}_{rs} = \tilde{a}^{pq}_{rs} + \underbrace{\tilde{a}^{p^\circ q}_{r^\circ s} + \tilde{a}^{p^\circ q}_{r\, s^\circ} + \tilde{a}^{p\, q^\circ}_{r^\circ s} + \tilde{a}^{p q^\circ}_{r s^\circ}} + \underbrace{\tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}} + \tilde{a}^{p^\circ q^{\circ\circ}}_{r^{\circ\circ} s^\circ}}$$

$$= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)}\, \tilde{a}^{p^\circ q}_{r^\circ s} + \hat{P}_{(r/s)}\, \tilde{a}^{p^\circ q^{\circ\circ}}_{r^\circ s^{\circ\circ}}$$

$$= \tilde{a}^{pq}_{rs} + \hat{P}^{(p/q)}_{(r/s)}\, \gamma^p_r\, \tilde{a}^q_s + \hat{P}_{(r/s)}\, \gamma^p_r \gamma^q_s$$

$$\bar{g}^{rs}_{pq}\, a^{pq}_{rs} = \bar{g}^{rs}_{pq}\, \tilde{a}^{pq}_{rs} + \bar{g}^{rs}_{pq} \cdot 4 \cdot \gamma^p_r\, \tilde{a}^q_s + \bar{g}^{rs}_{pq} \cdot 2 \cdot \gamma^p_r \gamma^q_s$$

Someone show it on the board!

Use this to derive Φ-normal H_e

Now try $\langle \Phi_i^a | H_e | \Phi_j^b \rangle$.

the end.