

1. Define the Hartree-Fock optimization problem in one sentence.
2. Define canonical Hartree-Fock orbitals in one sentence.
3. Briefly explain in your own words why the Lagrangian approach to constrained optimization works. Draw pictures where necessary.
4. Determine the functional derivatives of the Hartree-Fock Lagrangian, $\frac{\delta \mathcal{L}}{\delta \psi_k^*}$ and $\frac{\delta \mathcal{L}}{\delta \psi_k}$.
5. Derive the following expression for the energy expectation value of a Slater determinant, known as the *first Slater rule*.

$$\langle \Phi | \hat{H}_e | \Phi \rangle = \sum_i^n \langle \psi_i | \hat{h} | \psi_i \rangle + \frac{1}{2} \sum_{ij}^n \langle \psi_i \psi_j | | \psi_i \psi_j \rangle$$