#4) PROVE that CIS···m IS CORRECT TO ORDER $\lfloor m/2 \rfloor$ IN THE WAVEFUNCTION.

PROOF: $\Psi_{CIS\cdots m} = (1 + C_1 + C_2 + \cdots + C_m)\Phi$

BY QUESTION #2, C_k has leading order $\lceil k/2 \rceil$

SO WE KNOW THAT THE CIS···m APPROXIMATION INTRODUCES ERROR AT ORDER $\left\lceil \frac{(m+1)}{2} \right\rceil$, OR IN OTHER WORDS IS CORRECT TO $\left\lceil \frac{(m+1)}{2} \right\rceil - 1$ IN THE WAVEFUNCTION

m EVEN: $\left\lceil \frac{m+1}{2} \right\rceil - 1 = \left\lceil \frac{m}{2} + \frac{1}{2} \right\rceil = \left(\frac{m}{2} + 1\right) - 1 = \frac{m}{2}$

m ODD: $\left\lceil \frac{(m+1)}{2} \right\rceil - 1 = \frac{(m+1)}{2} - 1 = \frac{m}{2} - \frac{1}{2}$ ⟶ $= \lfloor m/2 \rfloor$ ∎.

PROVE that CIS···m IS CORRECT TO ORDER $2\lfloor m/2 \rfloor + 1$ IN THE ENERGY.

PROOF: $E_{0,CIS\cdots m} = \dfrac{\langle \Psi | H_0 | \Psi \rangle}{\langle \Psi | \Psi \rangle} = \dfrac{\langle \Phi | H_0 C_1 | \Phi \rangle + \cdots + \langle \Phi | C_m^\dagger H_0 C_m | \Phi \rangle}{\langle \Phi | \Phi \rangle + \cdots + \langle \Phi | C_m^\dagger C_m | \Phi \rangle}$

→ SO THE LEADING ORDER OF THE ERROR IS EQUAL TO THE ERROR OF THE TERM $\langle \Phi | C_{m+1}^\dagger H_0 C_{m+1} | \Phi \rangle$
$\left(\left\lceil \frac{(m+1)}{2} \right\rceil + \right)$ $(0+)$ $\left(\left\lceil \frac{(m+1)}{2} \right\rceil + \right)$

WHICH IS $2\left\lceil \frac{(m+1)}{2} \right\rceil$. THEREFORE THE ENERGY IS CORRECT TO $2\left\lceil \frac{(m+1)}{2} \right\rceil - 1$.

m EVEN: $2\left\lceil \frac{(m+1)}{2} \right\rceil - 1 = 2\left(\frac{m+1}{2} + \frac{1}{2}\right) - 1 = 2\left(\frac{m}{2} + 1\right) - 1 = 2\left(\frac{m}{2}\right) + 1$

m ODD: $2\left\lceil \frac{(m+1)}{2} \right\rceil - 1 = 2\left(\frac{m+1}{2}\right) - 1 = 2\left(\frac{m}{2}\right) + 1 - 1 = 2\left(\frac{m}{2} - \frac{1}{2}\right) + 1$

$= 2\lfloor m/2 \rfloor + 1$ ∎.