Homework for Lecture 5: Perturbation Theory

1. Show that the bracketing function
$$f(\kappa) = \langle \Phi_0 | H \Omega_\kappa | \Phi_0 \rangle$$
becomes
$$f(\kappa) = E_0^{(0)} + \langle \Phi_0 | V \Omega_\kappa | \Phi_0 \rangle$$
when you substitute $H = H_0 + V$. How does this relate to our final result for the perturbation expansion of E_0?

2. Derive the analytic form the reduced resolvent starting from the iterative expression
$$\mathcal{R}_\kappa = \mathcal{R}_0 + \mathcal{R}_0 V' \mathcal{R}_\kappa$$

3. What is the order of the following terms according to perturbation theory?
$$(\mathcal{R}_0 V)^{10} |\Phi_0\rangle \qquad (\mathcal{R}_0 V)^5 E_0^{(3)} \mathcal{R}_0 V |\Phi_0\rangle \qquad E_0^{(5)} E_0^{(3)} E_0^{(1)} \mathcal{R}_0 \mathcal{R}_0 \mathcal{R}_0 V |\Phi_0\rangle$$

4. Derive an expression for $\Phi_0^{(3)}$ by explicitly pulling out terms from the perturbation expansion. Explain you steps.

5. Derive $\Phi_0^{(4)}$ using the energy substitution trick.