

1. Show why the following relationships hold for antisymmetric functions $\Psi, \Psi' \in \mathcal{F}_n$.

$$\sum_{i=1}^n \langle \Psi | \hat{h}(i) | \Psi' \rangle = n \langle \Psi | \hat{h}(1) | \Psi' \rangle \qquad \sum_{i < j}^n \langle \Psi | \hat{g}(i, j) | \Psi' \rangle = \frac{n(n-1)}{2} \langle \Psi | \hat{g}(1, 2) | \Psi' \rangle$$

2. Show how to make the following rearrangement.

$$\frac{1}{2} \sum_{pqrs} \langle pq | rs \rangle a_p^\dagger a_q^\dagger a_s a_r = \frac{1}{4} \sum_{pqrs} \langle pq || rs \rangle a_p^\dagger a_q^\dagger a_s a_r$$

3. For the integral-operator definition of \hat{a}_p , show that the following relationships hold.

$$(a) \quad (\hat{a}_p \Phi_{(p_1 \dots p_n)})(2, \dots, n) = \begin{cases} (-1)^{k-1} \Phi_{(p_1 \dots \cancel{p_k} \dots p_n)}(2, \dots, n) & p = p_k \in (p_1 \dots p_n) \\ 0 & \text{otherwise} \end{cases}$$

$$(b) \quad \hat{a}_p \hat{a}_q = -\hat{a}_q \hat{a}_p$$

$$(c) \quad \Psi(1, \dots, n) = \frac{1}{\sqrt{n}} \sum_p \psi_p(1) (\hat{a}_p \Psi)(2, \dots, n)$$

$$(d) \quad \Psi(1, \dots, n) = \frac{1}{\sqrt{n(n-1)}} \sum_{pq} \psi_p(1) \psi_q(2) (\hat{a}_q \hat{a}_p \Psi)(3, \dots, n)$$

4. Using your own words, prove that $c_p = a_p^\dagger$. You may use either the determinant formalism or the occupation number formalism.

5. Verify the anticommutator relation $[q, q']_+ = \delta_{q'q^\dagger}$ by proving each of the following cases.

$$[a_p, a_q]_+ = 0 \qquad [a_p^\dagger, a_q^\dagger]_+ = 0 \qquad [a_p, a_q^\dagger]_+ = 0 \quad (p \neq q) \qquad [a_p, a_p^\dagger]_+ = 1$$

6. Show that the occupation-number definition of a_p and c_p is consistent with the determinant definition.

7. Do Problem 1.4 in Helgaker's big purple book: Show that

$$[\hat{v}, \hat{w}] = \sum_{pq} (\mathbf{v}\mathbf{w} - \mathbf{w}\mathbf{v})_{pq} a_p^\dagger a_q \qquad \text{where} \qquad \begin{aligned} \hat{v} &\equiv \sum_{pq} v_{pq} a_p^\dagger a_q, & \mathbf{v} &= [v_{pq}] \\ \hat{w} &\equiv \sum_{pq} w_{pq} a_p^\dagger a_q, & \mathbf{w} &= [w_{pq}]. \end{aligned}$$

8. Derive Slater's rules using second quantization. Where necessary, explain in words why a given term vanishes. (Hint: Use particle-hole isomorphism.)

$$(a) \quad \langle \Phi | H | \Phi \rangle = \sum_i h_{ii} + \frac{1}{2} \sum_{ij} \langle ij || ij \rangle$$

$$(b) \quad \langle \Phi | H | \Phi_i^a \rangle = h_{ia} + \sum_j \langle ij || aj \rangle$$

$$(c) \quad \langle \Phi | H | \Phi_{ij}^{ab} \rangle = \langle ij || ab \rangle$$

$$(d) \quad \langle \Phi | H | \Phi_{ijk}^{abc} \rangle = 0$$

For extra credit, show how to derive them without using particle-hole isomorphism.