the quasi-energy formalism

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pt. 1

$V(t)$

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$f_{\beta}(t)$

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$$f_\beta(t) = \int_{-\infty}^{\infty} d\omega \; f_\beta(\omega_\epsilon)\, e^{-i\omega_\epsilon t}$$

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$$f_{\beta}(t) \in \mathbb{R}$$

$$V(t) = \sum_{\beta} V_\beta \, f_\beta(t)$$

$$f_\beta(t) = \int_{-\infty}^{\infty} d\omega \; f_\beta(\omega_\epsilon) \, e^{-i\omega_\epsilon t}$$

$$f_\beta(t) \in \mathbb{R} \quad \Longrightarrow \quad f_\beta^*(\omega_\epsilon) = f_\beta(-\omega_{-\epsilon})$$

$$\langle \Psi(t) | W | \Psi(t) \rangle - \langle \Psi_0 | W | \Psi_0 \rangle$$

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$$\approx \sum_\beta \int_{-\infty}^{\infty} dt' \; f_\beta(t') \, \langle\langle \tilde{W}(t) ; \tilde{V}_\beta(t') \rangle\rangle$$

$$\langle \Psi(t)|W|\Psi(t)\rangle - \langle \Psi_0|W|\Psi_0\rangle$$

$$\approx \sum_\beta \int_{-\infty}^{\infty} dt'\ f_\beta(t') \langle\langle \tilde{W}(t);\tilde{V}_\beta(t')\rangle\rangle$$

$$= \sum_\beta \int_{-\infty}^{\infty} d\omega\ f_\beta(\omega) \langle\langle W;V_\beta\rangle\rangle_\omega e^{-i\omega t}$$

$$\langle \Psi(t) | W | \Psi(t) \rangle - \langle \Psi_0 | W | \Psi_0 \rangle$$

$$\approx \sum_\beta \int_{-\infty}^{\infty} dt' \; f_\beta(t') \langle\langle \tilde{W}(t) ; \tilde{V}_\beta(t') \rangle\rangle$$

$$= \sum_\beta \int_{-\infty}^{\infty} d\omega \; f_\beta(\omega) \langle\langle W ; V_\beta \rangle\rangle_\omega \, e^{-i\omega t}$$

($\epsilon \to 0$ limit)

Store this for later:

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$$\frac{\partial H(t)}{\partial f_\alpha(\omega')}$$

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$$\frac{\partial H(t)}{\partial f_\alpha(\omega')} = V_\alpha \frac{\partial f_\alpha(t)}{\partial f_\alpha(\omega')}$$

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($\epsilon \to 0$ limit)

phase-isolated wfn:

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$$\Psi(t)$$

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$$\Psi(t) = e^{-i\int_0^t dt'\, Q(t')}\, \bar{\Psi}(t)$$

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$$\Psi(t) = e^{-i\int_0^t dt'\, Q(t')}\, \bar{\Psi}(t) \qquad Q(t) = Q^*(t)$$

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Q.E. Schrödinger eq.:

$$\left(H(t) - i\frac{\partial}{\partial t}\right)\bar{\Psi}(t) = Q(t)\,\bar{\Psi}(t)$$

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Q.E. Schrödinger eq.:

$$\left(H(t) - i\frac{\partial}{\partial t}\right)\bar{\Psi}(t) = \underset{\uparrow}{Q(t)}\,\bar{\Psi}(t)$$

"quasi-energy"

phase-isolated wfn:

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$$Q(t) = \langle \bar{\Psi}(t) | H(t) - i\tfrac{\partial}{\partial t} | \bar{\Psi}(t) \rangle$$

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$$\bar{\Psi}^{(0)} = \Psi_0$$

phase-isolated wfn:

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Q.E. Schrödinger eq.:

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$$Q(t) = \langle\bar{\Psi}(t)|\,H(t) - i\tfrac{\partial}{\partial t}\,|\bar{\Psi}(t)\rangle \qquad \text{“quasi-energy”}$$

$$\bar{\Psi}^{(0)} = \Psi_0 \qquad\qquad Q^{(0)} = E_0$$

$$\left(H(t) - i\frac{\partial}{\partial t}\right)\Psi(t) = Q(t)\,\Psi(t)$$

$$\left(H(t) - i\frac{\partial}{\partial t}\right)\hat{\Psi}(t) = Q(t)\,\hat{\Psi}(t)$$

project by $\delta\hat{\Psi}(t)$ and determine real part

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$$2\,\mathrm{Re}\left[\langle \delta\Psi(t) | H(t) | \Psi(t) \rangle\right]$$

$$\left(H(t) - i\frac{\partial}{\partial t}\right)\bar{\Psi}(t) = Q(t)\,\bar{\Psi}(t)$$

project by $\delta\bar{\Psi}(t)$ and determine real part

$$2\,\mathrm{Re}\left[\langle\delta\bar{\Psi}(t)|H(t)|\bar{\Psi}(t)\rangle\right] = \delta\langle\bar{\Psi}(t)|H(t)|\bar{\Psi}(t)\rangle$$

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project by $\delta\Psi(t)$ and determine real part

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$$2\,\mathrm{Re}\left[Q(t) \langle \delta\Psi(t) | \Psi(t) \rangle\right]$$

$$\left(H(t) - i\frac{\partial}{\partial t}\right)\Psi(t) = Q(t)\,\Psi(t)$$

project by $\delta\Psi(t)$ and determine real part

$$2\,\mathrm{Re}\left[\langle \delta\Psi(t)|H(t)|\Psi(t)\rangle\right] = \delta\langle \Psi(t)|H(t)|\Psi(t)\rangle$$

$$2\,\mathrm{Re}\left[Q(t)\langle \delta\Psi(t)|\Psi(t)\rangle\right] = Q(t)\;\delta\langle \Psi(t)|\Psi(t)\rangle$$

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project by $\delta\Psi(t)$ and determine real part

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$$2\,\mathrm{Re}\left[Q(t)\langle\delta\Psi(t)|\Psi(t)\rangle\right] = Q(t)\,\cancel{\delta\langle\Psi(t)|\Psi(t)\rangle}^{\,0}$$

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project by $\delta\Psi(t)$ and determine real part

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$$2\,\mathrm{Re}\left[i\left\langle\delta\Psi(t)\middle|\frac{\partial\Psi(t)}{\partial t}\right\rangle\right]$$

$$\left(H(t) - i\frac{\partial}{\partial t}\right)\bar{\Psi}(t) = Q(t)\,\bar{\Psi}(t)$$

project by $\delta\bar{\Psi}(t)$ and determine real part

$$2\,\mathrm{Re}\left[\langle\delta\bar{\Psi}(t)|H(t)|\bar{\Psi}(t)\rangle\right] = \delta\langle\bar{\Psi}(t)|H(t)|\bar{\Psi}(t)\rangle$$

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$$2\,\mathrm{Re}\left[i\left\langle\delta\bar{\Psi}(t)\middle|\frac{\partial\bar{\Psi}(t)}{\partial t}\right\rangle\right] = i\left\langle\delta\bar{\Psi}(t)\middle|\frac{\partial\bar{\Psi}(t)}{\partial t}\right\rangle - i\left\langle\frac{\partial\bar{\Psi}(t)}{\partial t}\middle|\delta\bar{\Psi}(t)\right\rangle$$

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$$\frac{\partial}{\partial t}\langle\Psi(t)|\delta\Psi(t)\rangle - \langle\Psi(t)|\frac{\partial}{\partial t}\delta\Psi(t)\rangle$$

$$2\,\mathrm{Re}\left[i\left\langle \delta\Psi(t)\middle|\frac{\partial\Psi(t)}{\partial t}\right\rangle\right] = i\left\langle\delta\Psi(t)\middle|\frac{\partial\Psi(t)}{\partial t}\right\rangle - i\,\cancel{\left\langle\frac{\partial\Psi(t)}{\partial t}\middle|\delta\Psi(t)\right\rangle}$$

$$\left(H(t) - i\frac{\partial}{\partial t}\right)\bar{\Psi}(t) = Q(t)\,\bar{\Psi}(t)$$

project by $\delta\bar{\Psi}(t)$ and determine real part

$$2\,\mathrm{Re}\left[\langle \delta\bar{\Psi}(t) | H(t) | \bar{\Psi}(t)\rangle\right] = \delta\langle \bar{\Psi}(t) | H(t) | \bar{\Psi}(t)\rangle$$

$$2\,\mathrm{Re}\left[Q(t)\langle \delta\bar{\Psi}(t) | \bar{\Psi}(t)\rangle\right] = Q(t)\;\cancelto{0}{\delta\langle \bar{\Psi}(t) | \bar{\Psi}(t)\rangle}$$

$$2\,\mathrm{Re}\left[i\left\langle \delta\bar{\Psi}(t) \middle| \frac{\partial\bar{\Psi}(t)}{\partial t}\right\rangle\right] = i\left\langle \delta\bar{\Psi}(t) \middle| \frac{\partial\bar{\Psi}(t)}{\partial t}\right\rangle - i\,\cancelto{\frac{\partial}{\partial t}\langle \bar{\Psi}(t) | \delta\bar{\Psi}(t)\rangle - \langle \bar{\Psi}(t) | \frac{\partial}{\partial t}\delta\bar{\Psi}(t)\rangle}{\left\langle \frac{\partial\bar{\Psi}(t)}{\partial t} \middle| \delta\bar{\Psi}(t)\right\rangle}$$

$$= \delta\left\langle \bar{\Psi}(t) \middle| i\frac{\partial}{\partial t}\bar{\Psi}(t)\right\rangle - i\frac{\partial}{\partial t}\langle \bar{\Psi}(t) | \delta\bar{\Psi}(t)\rangle$$

Combine results:

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$$\delta \left\langle \Psi(t) \middle| H(t) - i\frac{\partial}{\partial t} \middle| \Psi(t) \right\rangle + i\frac{\partial}{\partial t} \left\langle \Psi(t) \middle| \delta\Psi(t) \right\rangle = 0$$

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$$\delta \underbrace{\left\langle \Psi(t) \middle| H(t) - i\frac{\partial}{\partial t} \middle| \Psi(t) \right\rangle}_{Q(t)} + i\frac{\partial}{\partial t}\left\langle \Psi(t) \middle| \delta\Psi(t) \right\rangle = 0$$

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<u>almost</u> a variational condition

Assume periodic

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$$H(t) = H(t+T) = U(T)\,H(t)\,U^{\dagger}(T)$$

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$$\left(H(t) - i\frac{\partial}{\partial t}\right)U^{\dagger}(T)\Psi(t) = Q(t)U^{\dagger}(T)\Psi(t)$$

$$\Longrightarrow \Psi(t)$$

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$$\left(H(t) - i\frac{\partial}{\partial t}\right)U^{\dagger}(T)\Psi(t) = Q(t)\,U^{\dagger}(T)\Psi(t)$$

$$\Longrightarrow \Psi(t) = U^{\dagger}(T)\,\Psi(t)$$

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$$\Big(H(t) - i\frac{\partial}{\partial t}\Big)U^{\dagger}(T)\Psi(t) = Q(t)\,U^{\dagger}(T)\Psi(t)$$

$$\Longrightarrow \Psi(t) = U^{\dagger}(T)\Psi(t) = \Psi(t-T)$$

$$\delta \left\langle \Psi(t) \middle| H(t) - i\frac{\partial}{\partial t} \middle| \Psi(t) \right\rangle + i\frac{\partial}{\partial t} \left\langle \Psi(t) \middle| \delta \Psi(t) \right\rangle = 0$$

$$\delta \left\langle \Psi(t) \middle| H(t) - i\frac{\partial}{\partial t} \middle| \Psi(t) \right\rangle + i\frac{\partial}{\partial t}\left\langle \Psi(t) \middle| \delta \Psi(t) \right\rangle = 0$$

$$\Downarrow$$

$$\delta \left\langle \Psi(t) \middle| H(t) - i\frac{\partial}{\partial t} \middle| \Psi(t) \right\rangle + i\frac{\partial}{\partial t} \left\langle \Psi(t) \middle| \delta\Psi(t) \right\rangle = 0$$

$$\Downarrow$$

$$\int_0^T dt' \, \delta \left\langle \Psi(t') \middle| H - i\frac{\partial}{\partial t} \middle| \Psi(t') \right\rangle$$

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$$\delta \langle \Psi(t) | H(t) - i\frac{\partial}{\partial t} | \Psi(t) \rangle + i \frac{\partial}{\partial t} \langle \Psi(t) | \delta \Psi(t) \rangle = 0$$

$$\Downarrow \qquad \text{(FTC)}$$

$$\int_0^T dt' \, \delta \langle \Psi(t') | H - i\frac{\partial}{\partial t} | \Psi(t') \rangle + i \langle \Psi(t') | \delta \Psi(t') \rangle \Big|_0^T$$

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(FTC)

$$\int_0^T dt' \, \delta \langle \Psi(t') | H - i \frac{\partial}{\partial t} | \Psi(t') \rangle + i \cancelto{0}{\langle \Psi(t') | \delta \Psi(t') \rangle \Big|_0^T} = 0$$

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$$\Longrightarrow$$

$$\delta \langle \Psi(t) | H(t) - i \frac{\partial}{\partial t} | \Psi(t) \rangle + i \frac{\partial}{\partial t} \langle \Psi(t) | \delta \Psi(t) \rangle = 0$$

$$\Downarrow \qquad \text{(FTC)}$$

$$\int_0^T dt' \, \delta \langle \Psi(t') | H - i \frac{\partial}{\partial t} | \Psi(t') \rangle + i \cancelto{0}{\langle \Psi(t') | \delta \Psi(t') \rangle \Big|_0^T} = 0$$

$$\Longrightarrow \qquad \delta \frac{1}{T} \int_0^T dt' \, Q(t') = 0$$

$$\delta \langle \Psi(t) | H(t) - i\frac{\partial}{\partial t} | \Psi(t) \rangle + i\frac{\partial}{\partial t} \langle \Psi(t) | \delta \Psi(t) \rangle = 0$$

$$\Downarrow$$

(FTC)

$$\int_0^T dt' \, \delta \langle \Psi(t') | H - i\frac{\partial}{\partial t} | \Psi(t') \rangle + i \, \xcancel{\langle \Psi(t') | \delta \Psi(t') \rangle \Big|_0^T}^{\,0} = 0$$

$$\Longrightarrow \qquad \delta \underbrace{\frac{1}{T}\int_0^T dt' \, Q(t')}_{\mathcal{Q}} = 0$$

time-avgd. Q.E.:

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$$\frac{d\mathcal{Q}}{d\xi}$$

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Hellmann-Feynman:

$$\frac{d\mathcal{Q}}{d\xi} = \left\{ \langle \Psi(t) | \frac{\partial H(t)}{\partial \xi} | \Psi(t) \rangle \right\}_T$$

$$\frac{d\mathcal{Q}}{df_\alpha(\omega')}$$

$$\frac{dQ}{df_\alpha(\omega')} = \left\{ \langle \Psi(t) | \frac{\partial H(t)}{\partial f_\alpha(\omega')} | \Psi(t) \rangle \right\}_T$$

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$$= \left\{ \langle \Psi(t) | V_\alpha | \Psi(t) \rangle \, e^{-i\omega' t} \right\}_T$$

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$$\left. \frac{d^2 Q}{df_\alpha(\omega')\, df_\beta(\omega)} \right|_{f=0}$$

$$\frac{dQ}{df_\alpha(\omega')} = \left\{ \langle \Psi(t) | \frac{\partial H(t)}{\partial f_\alpha(\omega')} | \Psi(t) \rangle \right\}_T$$

$$= \left\{ \langle \Psi(t) | V_\alpha | \Psi(t) \rangle e^{-i\omega' t} \right\}_T$$

$$\left. \frac{d^2 Q}{df_\alpha(\omega')\, df_\beta(\omega)} \right|_{f=0} = \langle\langle V_\alpha ; V_\beta \rangle\rangle_\omega$$

$$\frac{dQ}{df_\alpha(\omega')} = \left\{ \langle \Psi(t) | \frac{\partial H(t)}{\partial f_\alpha(\omega')} | \Psi(t) \rangle \right\}_T$$

$$= \left\{ \langle \Psi(t) | V_\alpha | \Psi(t) \rangle \, e^{-i\omega' t} \right\}_T$$

$$\left. \frac{d^2 Q}{df_\alpha(\omega')\, df_\beta(\omega)} \right|_{f=0} = \langle\langle V_\alpha ; V_\beta \rangle\rangle_\omega \left\{ e^{-i(\omega'+\omega)t} \right\}_T$$

$$\frac{d\mathcal{Q}}{df_\alpha(\omega')} = \left\{ \langle \Psi(t) | \frac{\partial H(t)}{\partial f_\alpha(\omega')} | \Psi(t) \rangle \right\}_T$$

$$= \left\{ \langle \Psi(t) | V_\alpha | \Psi(t) \rangle \, e^{-i\omega' t} \right\}_T$$

$$\left. \frac{d^2 \mathcal{Q}}{df_\alpha(\omega')\, df_\beta(\omega)} \right|_{f=0} = \langle\langle V_\alpha ; V_\beta \rangle\rangle_\omega \left\{ e^{-i(\omega' + \omega)t} \right\}_T$$

$\downarrow$

freqs have period T, so we need $\omega' + \omega = 0$

$$\langle\!\langle V_\alpha ; V_\beta \rangle\!\rangle_\omega$$

$$\langle\langle V_\alpha ; V_\beta \rangle\rangle_\omega = \left. \frac{d^2 Q}{d f_\alpha(\omega') \, d f_\beta(\omega)} \right|_{f=0}^{\omega'=-\omega}$$

$$\langle\langle V_\alpha ; V_\beta \rangle\rangle_\omega = \left. \frac{d^2 \mathcal{Q}}{d f_\alpha(\omega') \, d f_\beta(\omega)} \right|_{f=0}^{\omega' = -\omega}$$

$$= \left. \frac{d^2 \mathcal{Q}}{d f_\alpha^*(\omega) \, d f_\beta(\omega)} \right|_{f=0}$$

$$\langle\langle V_\alpha ; V_\beta \rangle\rangle_\omega = \left. \frac{d^2 Q}{d f_\alpha(\omega') \, d f_\beta(\omega)} \right|_{f=0}^{\omega'=-\omega}$$

$$= \left. \frac{d^2 Q}{d f_\alpha^*(\omega) \, d f_\beta(\omega)} \right|_{f=0}$$