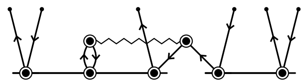
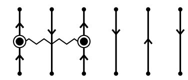
1. Explain why each of the following terms vanishes.

   (a) $\frac{1}{5!}\langle\Phi_{ijklm}^{abcde}|V_{\mathrm{c}}T_1^5|\Phi\rangle_{\mathrm{C}}$ (b) $\langle\Phi_{ij}^{ab}|V_{\mathrm{c}}T_2T_3|\Phi\rangle_{\mathrm{C}}$ (c) $\frac{1}{2!}\langle\Phi_{ijkl}^{abcd}|V_{\mathrm{c}}T_1^2|\Phi\rangle_{\mathrm{C}}$ (d) $\frac{1}{2!}\langle\Phi_{ijk}^{abc}|V_{\mathrm{c}}T_1^2|\Phi\rangle_{\mathrm{C}}$

2. Interpret the following graph and fully simplify your answer.



3. Interpret the following graph and fully simplify it the "long way." That is, you may use Rules 1-3 but you must start from Axiom 1 and show each step to get to your final answer.



Extra Credit. Prove Rule 3 for a closed graph with a single bare excitation operator of the following form.

$$\tilde{a}^{i_1\cdots i_m}_{a_1\cdots a_m} = \left(\tfrac{1}{m!}\right)^2 \bar{\delta}^{b_1\cdots b_m}_{j_1\cdots j_m}\,\tilde{a}^{j_1\cdots j_m}_{b_1\cdots b_m} \qquad\qquad \bar{\delta}^{b_1\cdots b_m}_{j_1\cdots j_m} \equiv \hat{P}^{(i_1/\cdots/i_m)}_{(a_1/\cdots/a_m)}\delta^{i_1}_{j_1}\cdots\delta^{i_m}_{j_m}\delta^{b_1}_{a_1}\cdots\delta^{b_m}_{a_m}$$

Appendix.

Axiom 1. The algebraic of a graph G is obtained from a corresponding summand graph $\Sigma(G)$ as follows.

$$G = \frac{1}{\mathrm{dg}(G)}\sum_{\text{labels}}\Sigma(G)$$

Rule 1. Each set of k equivalent lines or equivalent subgraphs contributes a factor of $k!$ to the degeneracy.
Rule 2. The overall sign of a closed graph is $(-)^{h+l}$, where h and l denote the total number of hole lines and loops.
Rule 3. For bare excitation operators, cancel the degeneracy factors from their equivalent coefficient lines by replacing the full antisymmetrizer, $P^{(p_1/\cdots/p_m)}_{(q_1/\cdots/q_m)}$, with a reduced antisymmetrizer over inequivalent coefficient lines, $\hat{P}^{(P_1/\cdots/P_h)}_{(Q_1/\cdots/Q_k)}$.[1] [2]

[1]Here $\{p_1,\ldots,p_m\} = P_1\cup\cdots\cup P_h$ and $\{q_1,\ldots,q_m\} = Q_1\cup\cdots\cup Q_k$ are the upper and lower indices on the bare excitation operator $\tilde{a}^{p_1\cdots p_m}_{q_1\cdots q_m}$, and the P_i's and Q_i's label subsets of equivalent coefficient lines.

[2]For equivalent lines connecting two bare excitation operators, this cancellation can only be performed once.