1. Derive the expansion of H_e in terms of Φ-normal excitations using KM notation with index antisymmetrizers.

2. Evaluate the following expectation values by Wick's theorem, using KM notation.

$$\begin{array}{cccc} \langle\Phi|a^p_q|\Phi\rangle = \ ? & \langle\Phi|a^p_q a^a_i|\Phi\rangle = \ ? & \langle\Phi|a^p_q a^{ab}_{ij}|\Phi\rangle = \ ? & \langle\Phi|a^p_q a^{abc}_{ijk}|\Phi\rangle = \ ? \\ \langle\Phi|a^{pq}_{rs}|\Phi\rangle = \ ? & \langle\Phi|a^{pq}_{rs} a^a_i|\Phi\rangle = \ ? & \langle\Phi|a^{pq}_{rs} a^{ab}_{ij}|\Phi\rangle = \ ? & \langle\Phi|a^{pq}_{rs} a^{abc}_{ijk}|\Phi\rangle = \ ? \end{array}$$

   Substitute your results into $\langle\Phi|H_e|\Phi\rangle$, $\langle\Phi|H_e|\Phi^a_i\rangle$, $\langle\Phi|H_e|\Phi^{ab}_{ij}\rangle$, and $\langle\Phi|H_e|\Phi^{abc}_{ijk}\rangle$ to get the Slater rules.

3. Show that

$$\tfrac{1}{m!} v^{q_1\cdots q_m}_{p_1\cdots p_m} a^{p_1\cdots p_m}_{q_1\cdots q_m} = (\tfrac{1}{m!})^2\, \overline{v}^{q_1\cdots q_m}_{p_1\cdots p_m} a^{p_1\cdots p_m}_{q_1\cdots q_m}$$

   which proves that any m-electron operator can be represented with an antisymmetrized interaction tensor. This generalizes the expression for electron repulsion in terms of antisymmetrized two-electron integrals.

4. Prove the following identities.

$$\tilde{a}^{p_1\cdots p_m}_{q_1\cdots q_m} = \vdots a^{p_1}_{q_1}\cdots a^{p_m}_{q_m}\vdots \qquad\qquad \vdots a^{p_1\cdots p_m}_{q_1\cdots q_m} a^{r_1\cdots r_n}_{s_1\cdots s_n}\vdots = \vdots a^{p_1}_{q_1}\cdots a^{p_m}_{q_m} a^{r_1}_{s_1}\cdots a^{r_n}_{s_n}\vdots = \tilde{a}^{p_1\cdots p_m r_1\cdots r_n}_{q_1\cdots q_m s_1\cdots s_n}$$

   Furthermore, explain how the presence of a contraction line restricts which rearrangements are possible, and how this is remedied by the use of dot notation.

5. Show algebraically that $\hat{P}_{(p/q/r)} = \hat{P}_{(p/qr)}\hat{P}_{(q/r)} = \hat{P}_{(pq/r)}\hat{P}_{(p/q)}$ and explain why these identities follow from the definition of the index antisymmetrizers.

6. Show that the following identity holds for any four-index tensor t^{pq}_{rs}, whether antisymmetric or not.

$$\overline{g}^{rs}_{pq}\, \hat{P}^{(p/q)}_{(r/s)}\, t^{pq}_{rs} = 4\, \overline{g}^{rs}_{pq}\, t^{pq}_{rs}$$

7. Show that the following identity holds for any w^{pqr} if v_{pqr} is antisymmetric.

$$v_{pqr}\hat{P}^{(p/qr)}w^{pqr} = 3\, v_{pqr}\, w^{pqr}$$

8. Derive the Wick expansion of a^{pqr}_{stu} in terms of Φ-normal excitations, using index antisymmetrizers to generate the full expansion from the unique contraction patterns.

9. Derive the CIS equations in KM notation.

10. Derive the CID equations in KM notation.