tdpt.

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(Schwinger-Tomonaga equation)

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$$+ i^2 \int_{-\infty}^{t} dt' \, \tilde{V}(t') \int_{-\infty}^{t'} dt'' \, \tilde{V}(t'') \, \tilde{\Psi}(t'')$$

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$$+ \dots$$

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interaction w/ E-field

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$$+ \frac{1}{2} \sum_{\beta\gamma} \int_{-\infty}^{\infty} dt' \int_{-\infty}^{\infty} dt'' \, f_\beta(t') \, f_\gamma(t'') \left. \frac{d^2 X(t)}{df_\beta(t') \, df_\gamma(t'')} \right|_{f=0}$$

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$$= -i \, \Theta(t-t') \, \tilde{V}_\beta(t') \, \tilde{\Psi}^{(0)}$$

$$\langle\langle \tilde{W}(t) ; \tilde{V}_\beta(t') \rangle\rangle$$

$$\langle\!\langle \tilde{W}(t) ; \tilde{V}_\beta(t') \rangle\!\rangle = \frac{d}{df_\beta(t')} \langle \tilde{\Psi}(t) | \tilde{W}(t) | \tilde{\Psi}(t) \rangle \Big|_0$$

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$$= -i\,\Theta(t-t') \langle \tilde{\Psi}^{(0)} | [\tilde{W}(t), \tilde{V}_\beta(t')] | \tilde{\Psi}^{(0)} \rangle$$

the end.