Lecture 29:
PCA (cont.) and Latent Dirichlet Allocation

Big Data and Machine Learning for Applied Economics
Econ 4676

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Announcements

- Problem Set 4 is posted
- Course Perception Survey http://bit.ly/encuesta-cursos-uniandes
- Final Exam Date will be Friday Dec 11 8am to Sunday 13 8am
- Remember to turn in your project proposal by December 7

Recap: Text as Data

- Text as Data: Tokenization and Bag of Words Representation
- Tokenization Demo
- Text Regression
- Text Regression: Example
- Topic Models: PCA

Agenda

1. Topic Models
   - PCA
     - Theory
     - Factor Computation
     - Factor Interpretation
   - Latent Dirichlet Allocation
     - LDA: Example
2. Review & Next Steps
3. Further Readings

Topic Models

- Text is super high dimensional
- Some times unsupervized factor model is a popular and useful strategy with text data
- You can first fit a factor model to a giant corpus and use these factors for supervised learning on a subset of labeled documents.
- The unsupervised dimension reduction facilitates the supervised learning

Topic Models: Example

- We have 6166 reviews, with an average length of 90 words per review, `we8there.com`.
- A useful feature of these reviews is that they contain both text and a multidimensional rating on overall experience, atmosphere, food, service, and value.
- For example, one user submitted a glowing review for Waffle House #1258 in Bossier City, Louisiana: *I normally would not revue a Waffle House but this one deserves it. The workers, Amanda, Amy, Cherry, James and J.D. were the most pleasant crew I have seen. While it was only lunch, B.L.T. and chili, it was great. The best thing was the 50' s rock and roll music, not to loud not to soft. This is a rare exception to what you all think a Waffle House is. Keep up the good work.*
  Overall: 5, Atmosphere: 5, Food: 5, Service: 5, Value: 5.

Topic Models: Example

- After cleaning and Porter stemming, we are left with a vocabulary of 2640 bigrams.
- For example, the first review in the document-term matrix has nonzero counts on bigrams indicating a pleasant meal at a rib joint:

```
#load packages
library(textir)
#load data
data(we8there)
x <- we8thereCounts
x[1,x[1,]!=0]
```

```
## even though larg portion  mouth water     red sauc     babi back     back rib chocol mouss
##            1            1            1            1            1            1            1
## veri satisfi
##            1
```

Topic Models: Example

- We can apply PCA to get a factor representation of the review text.
- PC1 looks like it will be big and positive for positive reviews,

```
pca <- prcomp(x, scale=TRUE) # can take a long time

tail(sort(pca$rotation[,1]))
```

```
##     food great     staff veri     excel food high recommend     great food
##    0.007386860    0.007593374    0.007629771    0.007821171    0.008503594
##     food excel
##    0.008736181
```

- while PC4 will be big and negative

```
tail(sort(pca$rotation[,4]))
```

```
##    order got after minut  never came   ask check readi order drink order
##   0.05918712  0.05958572  0.06099509  0.06184512  0.06776281  0.07980788
```

Principal Component Analysis

- Dimensionality via main components

$$X = (x_1, x_2, \ldots, x_K)_{N\times K} \tag{1}$$

- Factor:

$$F = X\delta \;\; \delta \in K \tag{2}$$

- Idea: summarize the K variables in a single (F).
- Vocab: the coefficients of δ are the loadings: how much 'matters' each x s in the factor.
- Dimensionality: summarize the original K variables in a few $q < K$ factors.

Algebra Review

- Let $A_{m \times m}$. It exists
  - a scalar λ such that $Ax = \lambda x$ for a vector $x_{m \times 1}$,
  - if $x \neq 0$, then λ is an eigenvalue of A.
  - and a vector x is an eigenvector of A corresponding to the eigenvalue λ.
- $A_{m \times m}$ with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_m$, then:

$$tr(A) = \sum_{i=1}^{m} \lambda_i \tag{3}$$

$$det(A) = \Pi_{i=1}^{m} \lambda_i \tag{4}$$

- If $A_{m \times m}$ has m different eigenvalues, then the associated eigenvectors are all linearly independent.
- Spectral decomposition: $A = P\Lambda P$, where $\Lambda = diag(\lambda_1, \ldots \lambda_n)$ and P is the matrix whose columns are the corresponding eigenvectors.

Factors via main components

- $x_1, x_2, \ldots, x_K$, K vectors of N observations each.
- Factor: $F = X\delta$
- What is the 'best' linear combination of $x_1, x_2, \ldots, x_K$?
- Best? Maximum variance. Why? The one that best reproduces variability original of all xs

Factors via main components

- Let
  - $X = (x_1, \ldots, x_K)_{N \times K}$,
  - $\Sigma = V(X)$
  - $\delta \in K$
- $F = X\delta$ is a linear combination of X, with $V(X\delta) = \delta'\Sigma\delta$.
- Let's set up the problem as

$$\max_{\delta} \ \delta'\Sigma\delta \tag{5}$$

  - It is obvious that the solution is to bring δ to infinity.

Factors via main components

- Let's "fix" the problem by normalizing δ

$$\max_{\delta}\ \delta'\Sigma\delta \qquad (6)$$
$$\text{subject to}$$
$$\delta'\delta = 1$$

- Let us call the solution to this problem δ^*.
- $F^* = X\delta^*$ is the 'best' linear combination of X.
- Intuition: X has K columns and $Y = X\delta$ has only one. The factor built with the first principal component is the best way to represent the K variables of X using a single single variable.

Factors via main components

- Result: δ^* is the eigenvector corresponding to the largest eigenvalue of $\Sigma = V(X)$.
- $F^* = X\delta^*$ is the first principal component of X.

Factors via main components

- Solution to the problem of the first principal component
- Problem:

$$\underset{\delta}{max}\ \delta'\Sigma\delta \tag{7}$$

$$\text{subject to}$$

$$\delta'\delta = 1$$

- Seting up the Lagrangian

$$\mathcal{L}(\delta,\lambda) = \delta'\Sigma\delta + \lambda(1-\delta'\delta) \tag{8}$$

- CPO

$$\Sigma\delta = \lambda\delta \tag{9}$$

- At the optimum, δ is the eigenvector corresponding to the eigenvalue λ of Σ.
- Premultiplying by δ and remembering that $\delta'\delta = 1$:

Factors is unsupervised learning

- Recall that
  - In regression we had
$$y = X\beta + u \tag{11}$$
  - We minimized the MSE
$$min \sum (y_i - \hat{y})^2 \tag{12}$$
  - Learning is supervised: the difference between y and $\hat{y}$ "guides' the learning.
- The factor construction problem is unsupervised: we construct an index (the factor) without ever seeing it.
  - We start with
$$X_{n\times k} \tag{13}$$
  - We end with
$$F^*_{n\times 1} = X\delta^* \tag{14}$$

q main components

- The first main component? There's others?
- Let's consider the following problem:

$$\begin{aligned} \underset{\delta_2}{max}\ \delta_2'\Sigma\delta_2 \\ \text{subject to} \\ \delta_2'\delta_2 &= 1 \\ and \\ Cov(\delta_2' X, \delta^{*'} X) &= 0 \end{aligned} \tag{15}$$

- $F_2^* = X\delta_2^*$ is the second principal component : the best linear combination which is orthogonal to the best initial linear combination.
- Recursively, using this logic you can form q main components.
- Note that algebraically we could construct $q = K$ factors.

q main components

- Let $\lambda_1, \ldots, \lambda_K$ be the eigenvalues of $\Sigma = V(X)$, ordered from highest to lowest, and $p_1, \ldots, p_K$ the corresponding eigenvectors. Let us call P the matrix of eigenvectors.
- Result: $\delta_j = p_j$, $\forall j$ ('loadings' of the principal components =ordered eigenvectors of Σ).
- Let $F_j = X\delta_j$, $j = 1, \ldots, K$ be the j-th principal component. It's easy to see that

$$V(F_j) = \delta_j' \Sigma \delta_j = p_j P \Lambda P p_j = \lambda_j \tag{16}$$

(the variance of the j-th principal component is the j-th ordered eigenvalue of Σ).

Relative importance of factors

- The total variance of X is the sum of the variances of x_j , $j = 1, ..., K$, that is $trace(\Sigma)$
- It is easy to show that:

$$trace(\Sigma) = trace(P\Lambda P') = trace(PP'\Lambda) = \sum_{j=1}^{K} \lambda_j = \sum_{j=1}^{K} V(F_j) \tag{17}$$

- Then

$$\frac{\lambda_k}{\sum_{j=1}^{K} \lambda_j} \tag{18}$$

- measures the relative importance of the jth principal component.

Selection of factors

- Look at the importance of the first major components. If the first one explains a lot, there is really only one dimension (one dimension explains almost everything).
- The coefficients of the eigenvectors are weights. See how each of the variables 'participate' in each leading coefficient.
- Beware of differences in scale. Always standardize

Selection of factors

- Let the columns of X be standardized, so that each variable has unit variance.
- In this case:

$$trace(\Sigma) = \sum_{j=1}^{K} V(F_j) = K \tag{19}$$

- and recall $\sum_{j=1}^{K} \lambda_j = \sum_{j=1}^{K} V(F_j)$ then

$$\sum_{j=1}^{K} \lambda_j = K \tag{20}$$

- On average, each factor contributes one unit. When $\lambda_j > 1$, that factor it explains the total variance more than the average. $\rightarrow$ Retain the factors with $\lambda_j > 1$

Useful Tips: Factor Computation

- As a practical aside, note that `prcomp` converts x here from sparse to dense matrix storage.
- For really big text DTMs, which will be very sparse, this will cause you to run out of memory.
- A big data strategy for PCA is to first calculate the covariance matrix for x and then obtain PC rotations as the eigenvalues of this covariance matrix.
- The first step can be done using sparse matrix algebra.
- The rotations are then available as

```
## eigen( xvar, symmetric = TRUE)$vec.
```

- There are also approximate PCA algorithms available for fast factorization on big data. See, for example, the `irlba` package for R.

Factor Interpretation

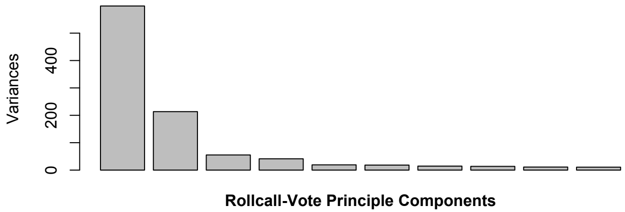
- $F_s = X\delta_s$: 'loadings' often suggest that a factor works as a 'index' of a group of variables.
- Idea: look at the 'loadings'
- Caution: factors via principal components are orthogonal recursively.

Factor Interpretation: Example

- **Congress and Roll Call Voting**
  - Votes in which names and positions are recorded are called 'roll calls'.
  - The site `voteview.com` archives vote records and the R package `pscl` has tools for this data.
  - 445 members in the last US House (the 111^{th})
  - 1647 votes: nea = -1, yea=+1, missing = 0.
  - This leads to a large matrix of observations that can probably be reduced to simple factors (party).

Factor Interpretation

- Vote components in the **111**th house
- Each PC is $F_s = X\delta_s$



- Huge drop in variance from 1^{st} to 2^{nd} and 2^{nd} to 3^{rd} PC.
- Poli-Sci holds that PC1 is usually enough to explain congress.
  2nd component has been important twice: 1860's and 1960's.

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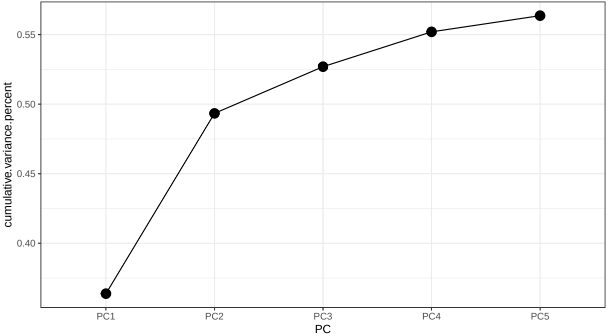

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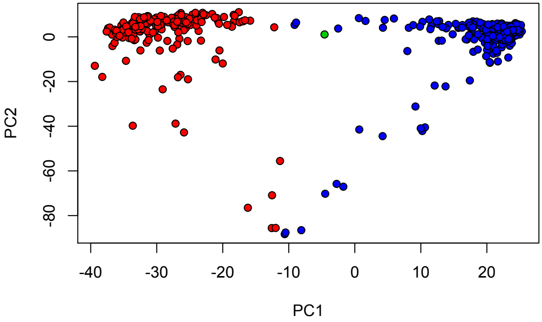
Factor Interpretation

- Top two PC directions in the **111**th house



- Republicans in red and Democrats in blue:
  - Clear separation on the first principal component.
  - The second component looks orthogonal to party.

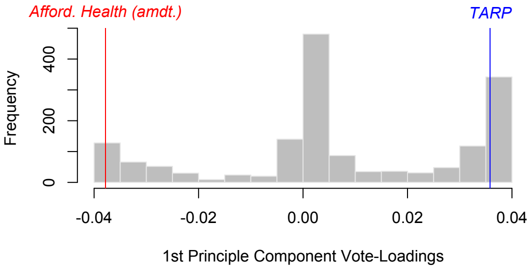
Factor Interpretation

```
## Far right (very conservative)
> sort(votepc[,1])
    BROUN (R GA-10)       FLAKE (R AZ-6)   HENSARLIN (R TX-5)
        -39.3739409          -38.2506713          -37.5870597

## Far left (very liberal)
> sort(votepc[,1], decreasing=TRUE)
   EDWARDS (D MD-4)   PRICE (D NC-4)    MATSUI (D CA-5)
        25.2915083        25.1591151         25.1248117

## social issues?  immigration?  no clear pattern
> sort(votepc[,2])
    SOLIS (D CA-32) GILLIBRAND (D NY-20)      PELOSI (D CA-8)
       -88.31350926         -87.58871687         -86.53585568
   STUTZMAN (R IN-3)       REED (R NY-29)      GRAVES (R GA-9)
       -85.59217310         -85.53636319         -76.49658108
```

- PC1 is easy to read, PC2 is ambiguous (is it even meaningful?)

Factor Interpretation

- **High PC1-loading votes are ideological battles.**
- These tend to have informative voting across party lines.



- A vote for Repulican amendments to 'Affordable Health Care for America' strongly indicates a negative PC1 (more conservative), while
  a vote for Troubled Asset Relief Program (TARP) indicates a positive PC1 (more progressive).

Factor Interpretation

- Look at the largest loadings in δ_2 to discern an interpretation.

```
> loadings[order(abs(loadings[,2]), decreasing=TRUE)[1:5],2]
 Vote.1146   Vote.658  Vote.1090  Vote.1104  Vote.1149
0.05605862 0.05461947 0.05300806 0.05168382 0.05155729
```

- These votes all correspond to near-unanimous symbolic action.
- For example, 429 legislators voted for resolution 1146:
  'Supporting the goals and ideals of a Cold War Veterans Day'
  If you didn't vote for this, you weren't in the house.
- Mystery Solved: the second PC is just attendance!

```
> sort(rowSums(votes==0), decreasing=TRUE)
     SOLIS (D CA-32) GILLIBRAND (D NY-20)       REED (R NY-29)
                1628                 1619                 1562
   STUTZMAN (R IN-3)       PELOSI (D CA-8)      GRAVES (R GA-9)
                1557                 1541                 1340
```

Factor Interpretation

PCR: Principal Component Regression

The concept is very simple: instead of regressing onto $\mathbf{x}$, use a lower dimension set of principal components $\mathbf{z}$ as covariates.

This works well for a few reasons:

- PCA reduces dimension, which is always good.
- Higher variance covariates are good in regression, and we choose the top PCs to have highest variance.
- The PCs are independent: no multicollinearity.

The 2-stage algorithm is straightforward. For example,

```
mypca = prcomp(X, scale=TRUE)
z = predict(mypca)[,1:K]
reg = glm(y~., data=as.data.frame(z))
```

Latent Dirichlet Allocation

- The approach of using PCA to factorize text was common before the 2000s.
- Versions of this algorithm were referred to under the label latent semantic analysis.
- However, this changed with the introduction of topic modeling, also known as Latent Dirichlet Allocation (LDA), by Blei et al. in 2003.
- These authors pointed out that the squared error loss (i.e., Gaussian model) implied by PCA is inappropriate for analysis of sparse word-count data.
- Instead, they proposed you take the bag-of-words representation seriously and model token counts as realizations from a multinomial distribution.

Latent Dirichlet Allocation

- That is, they proposed topic models as a multinomial factor model.
- Topic models are built on a simple document generation process:
  - For each word, pick a "topic" k. This topic is defined through a probability vector over words, say, θ_k with probability θ_{kj} for each word j.
  - Then draw the word according to the probabilities encoded in θ_k .
- After doing this over and over for each word in the document, you have proportion ω_{i1} from topic 1, ω_{i2} from topic 2, and so on.

Latent Dirichlet Allocation

- This basic generation process implies that the full vector of word counts, x_i, has a multinomial distribution:

$$x_i \sim MN(\omega_{i1}\theta_1 + \cdots + \omega_{iK}\theta_K, m_i) \tag{21}$$

- where $m_i = \sum_j x_{ij}$ is the total document length and, for example,
- the probability of word j in document i will be $\sum_k \omega_{ik}\theta_{kj}$

Latent Dirichlet Allocation vs PCA

- Recall our PC model:

$$E(x_i) = \delta_{i1}F_1 + \cdots + \delta_{iK}F_K \tag{22}$$

- The analogous topic model representation, implied by the above equation, is

$$E(x_i) = \omega_{i1}\theta_1 + \cdots + \omega_{iK}\theta_K \tag{23}$$

- such that topic score ω_{ik} is like PC score δ_{ik} and
- θ_k topic probabilities are like rotations F_k.
- The distinction is that the multinomial in implies a different loss function (from a multinomial) rather than the sums of squared errors that PCA minimizes.
- Note that we condition on document length here so that topics are driven by relative rather than absolute term usage.

LDA: Example

```r
library(textir)

library(maptpx) # for the topics function

data(we8there)

# you need to convert from a Matrix to a `slam' simple_triplet_matrix
x <- as.simple_triplet_matrix(we8thereCounts)

# to fit, just give it the counts, number of `topics' K, and any other args
tpc <- topics(x,K=10)
```

```
##
## Estimating on a 6166 document collection.
## Fitting the 10 topic model.
## log posterior increase: 4441.8, 461.4, 101.5, 57.4, 51, 19.2, 26.2, 15.3, 15.4, 11.7, 6.7, 12.2, 8, 10.1,
4.8, 5.3, 3.2, 6.6, 2.8, 7, 3.6, 3.9, 6.7, 5.5, 8.6, 5, 11, 10.3, 12, 7.9, 12.1, 9, 8.8, 13.9, 8.6, 7.3, 6.1,
4.9, 4.3, 12, 11.1, 8.7, 3.2, 2.8, 5.1, 1.9, 2.6, 2.4, 4.9, 2.9, 1.5, 2.5, 4.7, 1.7, 0.9, 1.4, 0.7, 2.5, 2.2,
1.7, 1, 1.3, 1.5, 2, 0.8, 1.7, 0.5, 0.2, 0.5, 0.6, 0.9, 3.9, 0.5, 0.6, 0.4, 0.2, 0.8, 0.2, 1.4, 0.3, 0.5, 0.6, done.
```

LDA: Example

- Choosing the number of topics

```
# If you supply a vector of topic sizes, it uses a Bayes factor to choose
# (BF is like exp(-BIC), so you choose the biggest BF)
# the algo stops if BF drops twice in a row
tpcs <- topics(x,K=5*(1:5), verb=1) # it chooses 10 topics
```

```
##
## Estimating on a 6166 document collection.
## Fit and Bayes Factor Estimation for K = 5 ... 25
## log posterior increase: 2853.9, 327.1, 85.3, 36.7, 25.9, 19.9, 13.8, 11.6, 9.6, 11.4, 20.3, 7.1, ..., done.
## log BF( 5 ) = 79521.94
## log posterior increase: 4626.7, 197.4, 53, 24.9, 19, 9.3, 7.4, 4.6, 5.2, 3.4, 2.3, 1.7, 0.8, ..., done.
## log BF( 10 ) = 87157.28
## log posterior increase: 3445, 170.2, 49.8, 23.6, 14.1, 31.4, 16.2, 4.8, 6.6, 5.5, 1.9, 5.9, ..., done.
## log BF( 15 ) = 3334.33
## log posterior increase: 2327.1, 139.8, 39.5, 16.7, 20.1, 5.3, 4.5, 3, 3.4, 2.9, 4.4, 1.8, ..., done.
## log BF( 20 ) = -66254.44
```

Topic Models: Example

- Interpretation

```r
# summary prints the top `n' words for each topic,
# under ordering by `topic over aggregate' lift:
#    the topic word prob over marginal word prob.
summary(tpcs, n=10)
```

```
##
## Top 10 phrases by topic-over-null term lift (and usage %):
##
## [1] 'food great', 'great food', 'great servic', 'veri good', 'food veri', ... (14.6)
## [2] 'high recommend', 'italian food', 'best italian', 'mexican food', ...  (11.6)
## [3] 'over minut', 'never go', 'go back', 'flag down', 'anoth minut', ...  (10.4)
## [4] 'enough share', 'open daili', 'highlight menu', 'until pm', ...  (10.4)
## [5] 'never return', 'one worst', 'don wast', 'wast time', ...  (9.4)
## [6] 'good work', 'best kept', 'out world', 'great experi', ... (9.1)
## [7] 'thai food', 'veri pleasant', 'ice cream', 'breakfast lunch', ...  (9)
## [8] 'take out', 'best bbq', 'can get', 'pork sandwich', 'home cook', ...  (9)
## [9] 'food good', 'food place', 'chees steak', 'good select', 'food pretti',...  (8.7)
## [10] 'wasn whole', 'came chip', 'got littl', 'over drink', 'took seat',...  (7.8)
##
## Log Bayes factor and estimated dispersion, by number of topics:
##
##              5        10      15        20
## logBF 79521.94 87157.28 3334.33 -66254.44
## Disp      7.09     4.96    3.95      3.33
##
## Selected the K = 10 topic model
```

Topic Models: Example

```r
# alternatively, you can look at words ordered by simple in-topic prob
## the topic-term probability matrix is called 'theta',
## and each column is a topic
## we can use these to rank terms by probability within topics
rownames(tpcs$theta)[order(tpcs$theta[,1], decreasing=TRUE)[1:10]]
```

```
##  [1] "veri good"    "great food"   "food great"   "great place"  "veri friend"
##  [6] "veri nice"    "good food"    "great servic" "food excel"   "food servic"
```

```r
rownames(tpcs$theta)[order(tpcs$theta[,2], decreasing=TRUE)[1:10]]
```

```
##  [1] "dine experi"     "high recommend"  "wait staff"      "wine list"
##  [5] "mexican food"    "italian food"    "italian restaur" "fine dine"
##  [9] "staff friend"    "make feel"
```

Review & Next Steps

- Text as Data: Tokenization
- Tokenization Demo
- Text Regression
- Text Regression: Example
- Topic Models PCA

- Next class: More on text as data

- Questions? Questions about software?

Further Readings

- Blei, D. M., Ng, A. Y., & Jordan, M. I. (2003). Latent dirichlet allocation. Journal of machine Learning research, 3(Jan), 993-1022.
- James, G., Witten, D., Hastie, T., & Tibshirani, R. (2013). An introduction to statistical learning (Vol. 112, p. 18). New York: springer.
- Taddy, M. (2019). Business data science: Combining machine learning and economics to optimize, automate, and accelerate business decisions. McGraw Hill Professional.