Lecture 29:
PCA (cont.) and Latent Dirichlet Allocation
Big Data and Machine Learning for Applied Economics
Econ 4676

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Announcements

- Problem Set 4 is posted

- Course Perception Survey http://bit.ly/encuesta-cursos-uniandes

- Final Exam Date will be Friday Dec 11 8am to Sunday 13 8am

- Remember to turn in your project proposal by December 7

Recap: Text as Data

- ▶ Text as Data: Tokenization and Bag of Words Representation

- ▶ Tokenization Demo

- ▶ Text Regression

- ▶ Text Regression: Example

- ▶ Topic Models: PCA

Agenda

Topic Models

▶ Text is super high dimensional

▶ Some times unsupervized factor model is a popular and useful strategy with text data

▶ You can first fit a factor model to a giant corpus and use these factors for supervised learning on a subset of labeled documents.

▶ The unsupervised dimension reduction facilitates the supervised learning

Topic Models: Example

▶ We have 6166 reviews, with an average length of 90 words per review, we8there.com.

▶ A useful feature of these reviews is that they contain both text and a multidimensional rating on overall experience, atmosphere, food, service, and value.

▶ For example, one user submitted a glowing review for Waffle House #1258 in Bossier City, Louisiana: *I normally would not revue a Waffle House but this one deserves it. The*

workers, Amanda, Amy, Cherry, James and J.D. were the most pleasant crew I have seen. While it was only lunch, B.L.T. and chili, it was great. The best thing was the 50′ s rock and roll music, not to loud not to soft. This is a rare exception to what you all think a Waffle House is. Keep up the good work.
Overall: 5, Atmosphere: 5, Food: 5, Service: 5, Value: 5.

Topic Models: Example

▶ After cleaning and Porter stemming, we are left with a vocabulary of 2640 bigrams.
▶ For example, the first review in the document-term matrix has nonzero counts on bigrams indicating a pleasant meal at a rib joint:

```r
#load packages
library(textir)
#load data
data(we8there)
x <- we8thereCounts
x[1,x[1,]!=0]
```

```
## even though larg portion  mouth water     red sauc    babi back     back rib chocol mouss
##           1            1            1            1            1            1            1
## veri satisfi
##           1
```

Topic Models: Example

- We can apply PCA to get a factor representation of the review text.
- PC1 looks like it will be big and positive for positive reviews,

```
pca <- prcomp(x, scale=TRUE) # can take a long time

tail(sort(pca$rotation[,1]))
```

```
##      food great       staff veri     excel food high recommend      great food
##    0.007386860     0.007593374      0.007629771    0.007821171     0.008503594
##      food excel
##    0.008736181
```

- while PC4 will be big and negative

```
tail(sort(pca$rotation[,4]))
```

```
##   order got after minut   never came   ask check readi order drink order
## 0.05918712 0.05958572 0.06099509 0.06184512 0.06776281 0.07980788
```

Principal Component Analysis

- Dimensionality via main components

$$X = (x_1, x_2, \ldots, x_K)_{N \times K} \tag{1}$$

- Factor:

$$F = X\delta \ \ \delta \in K \tag{2}$$

- Idea: summarize the K variables in a single (F).
- Vocab: the coefficients of δ are the loadings: how much 'matters' each x s in the factor.
- Dimensionality: summarize the original K variables in a few $q < K$ factors.

Algebra Review

- ▶ Let $A_{m \times m}$. It exists
  - ▶ a scalar λ such that $Ax = \lambda x$ for a vector $x_{m \times 1}$,
  - ▶ if $x \neq 0$, then λ is an eigenvalue of A.
  - ▶ and a vector x is an eigenvector of A corresponding to the eigenvalue λ.

- ▶ $A_{m \times m}$ with eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_m$, then:

$$tr(A) = \sum_{i=1}^{m} \lambda_i \tag{3}$$

$$det(A) = \Pi_{i=1}^{m} \lambda_i \tag{4}$$

- ▶ If $A_{m \times m}$ has m different eigenvalues, then the associated eigenvectors are all linearly independent.

- ▶ Spectral decomposition: $A = P \Lambda P$, where $\Lambda = diag(\lambda_1, \ldots \lambda_n)$ and P is the matrix whose columns are the corresponding eigenvectors.

Factors via main components

▶ $x_1, x_2, \ldots, x_K$, K vectors of N observations each.

▶ Factor: $F = X\delta$

▶ What is the 'best' linear combination of $x_1, x_2, \ldots, x_K$?

▶ Best? Maximum variance. Why? The one that best reproduces variability original of all xs

Factors via main components

- Let
  - $X = (x_1, \ldots, x_K)_{N \times K}$,
  - $\Sigma = V(X)$
  - $\delta \in K$

- $F = X\delta$ is a linear combination of X, with $V(X\delta) = \delta' \Sigma \delta$.

- Let's set up the problem as

$$\max_{\delta} \; \delta' \Sigma \delta \tag{5}$$

  - It is obvious that the solution is to bring δ to infinity.

Factors via main components

▶ Let's "fix" the problem by normalizing δ

$$\max_{\delta} \ \delta'\Sigma\delta \tag{6}$$
$$\text{subject to}$$
$$\delta'\delta = 1$$

▶ Let us call the solution to this problem δ^*.

▶ $F^* = X\delta^*$ is the 'best' linear combination of X.

▶ Intuition: X has K columns and $Y = X\delta$ has only one. The factor built with the first principal component is the best way to represent the K variables of X using a single single variable.

Factors via main components

▶ Result: δ^* is the eigenvector corresponding to the largest eigenvalue of $\Sigma = V(X)$.

▶ $F^* = X\delta^*$ is the first principal component of X.

Factors via main components

- ▶ Solution to the problem of the first principal component
- ▶ Problem:

$$\max_{\delta} \ \delta' \Sigma \delta \tag{7}$$

$$\text{subject to}$$

$$\delta' \delta = 1$$

- ▶ Seting up the Lagrangian

$$\mathcal{L}(\delta, \lambda) = \delta' \Sigma \delta + \lambda(1 - \delta' \delta) \tag{8}$$

- ▶ CPO

$$\Sigma \delta = \lambda \delta \tag{9}$$

- ▶ At the optimum, δ is the eigenvector corresponding to the eigenvalue λ of Σ.
- ▶ Premultiplying by δ and remembering that $\delta' \delta = 1$:

Factors is unsupervised learning

▶ Recall that

  ▶ In regression we had

$$y = X\beta + u \tag{11}$$

  ▶ We minimized the MSE

$$min \sum (y_i - \hat{y})^2 \tag{12}$$

  ▶ Learning is supervised: the difference between y and $\hat{y}$ "guides' the learning.

▶ The factor construction problem is unsupervised: we construct an index (the factor) without ever seeing it.

  ▶ We start with

$$X_{n \times k} \tag{13}$$

  ▶ We end with

$$F^*_{n \times 1} = X\delta^* \tag{14}$$

q main components

► The first main component? There's others?

► Let's consider the following problem:

$$\max_{\delta_2} \delta_2' \Sigma \delta_2 \qquad (15)$$

$$\text{subject to}$$

$$\delta_2' \delta_2 = 1$$

$$\text{and}$$

$$Cov(\delta_2' X, \delta^{*'} X) = 0$$

► $F_2^* = X\delta_2^*$ is the second principal component : the best linear combination which is orthogonal to the best initial linear combination.

► Recursively, using this logic you can form q main components.

► Note that algebraically we could construct $q = K$ factors.

q main components

- Let $\lambda_1, \ldots, \lambda_K$ be the eigenvalues of $\Sigma = V(X)$, ordered from highest to lowest, and $p_1, \ldots, p_K$ the corresponding eigenvectors. Let us call P the matrix of eigenvectors.
- Result: $\delta_j = p_j$, $\forall j$ ('loadings' of the principal components =ordered eigenvectors of Σ).
- Let $F_j = X\delta_j$, $j = 1, \ldots, K$ be the j-th principal component. It's easy to see that

$$V(F_j) = \delta_j'\Sigma\delta_j = p_j P \Lambda P p_j = \lambda_j \tag{16}$$

(the variance of the j-th principal component is the j-th ordered eigenvalue of Σ).

Relative importance of factors

▶ The total variance of X is the sum of the variances of x_j, $j = 1, ..., K$, that is $trace(\Sigma)$

▶ It is easy to show that:

$$trace(\Sigma) = trace(P\Lambda P') = trace(PP'\Lambda) = \sum_{j=1}^{K} \lambda_j = \sum_{j=1}^{K} V(F_j) \tag{17}$$

▶ Then

$$\frac{\lambda_k}{\sum_{j=1}^{K} \lambda_j} \tag{18}$$

▶ measures the relative importance of the jth principal component.

Selection of factors

- Look at the importance of the first major components. If the first one explains a lot, there is really only one dimension (one dimension explains almost everything).

- The coefficients of the eigenvectors are weights. See how each of the variables 'participate' in each leading coefficient.

- Beware of differences in scale. Always standardize

Selection of factors

- ▶ Let the columns of X be standardized, so that each variable has unit variance.
- ▶ In this case:

$$trace(\Sigma) = \sum_{j=1}^{K} V(F_j) = K \tag{19}$$

- ▶ and recall $\sum_{j=1}^{K} \lambda_j = \sum_{j=1}^{K} V(F_j)$ then

$$\sum_{j=1}^{K} \lambda_j = K \tag{20}$$

- ▶ On average, each factor contributes one unit. When $\lambda_j > 1$, that factor it explains the total variance more than the average. $\rightarrow$ Retain the factors with $\lambda_j > 1$

Useful Tips: Factor Computation

▶ As a practical aside, note that `prcomp` converts x here from sparse to dense matrix storage.

▶ For really big text DTMs, which will be very sparse, this will cause you to run out of memory.

▶ A big data strategy for PCA is to first calculate the covariance matrix for x and then obtain PC rotations as the eigenvalues of this covariance matrix.

▶ The first step can be done using sparse matrix algebra.

▶ The rotations are then available as

```
## eigen( xvar, symmetric = TRUE)$vec.
```

▶ There are also approximate PCA algorithms available for fast factorization on big data. See, for example, the `irlba` package for R.

Factor Interpretation

- $F_s = X\delta_s$: 'loadings' often suggest that a factor works as a 'index' of a group of variables.

- Idea: look at the 'loadings'

- Caution: factors via principal components are orthogonal recursively.

Factor Interpretation: Example

- **Congress and Roll Call Voting**

  - Votes in which names and positions are recorded are called 'roll calls'.
  - The site `voteview.com` archives vote records and the R package `pscl` has tools for this data.
  - 445 members in the last US House (the 111th)
  - 1647 votes: nea = -1, yea=+1, missing = 0.
  - This leads to a large matrix of observations that can probably be reduced to simple factors (party).
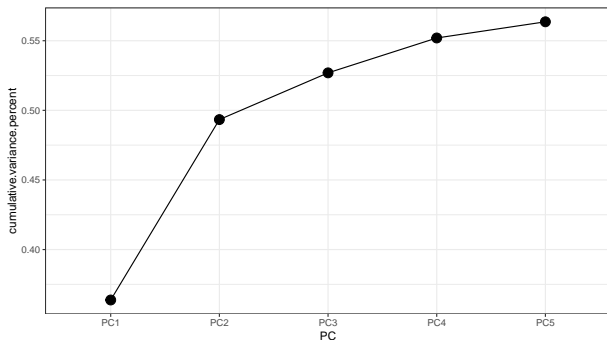
Factor Interpretation
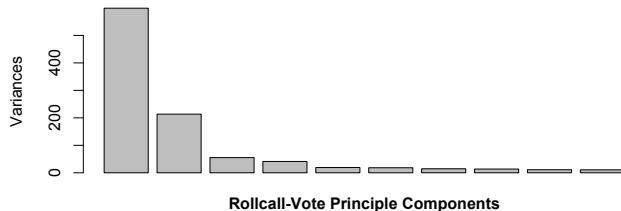
- ▶ Vote components in the **111th** house
- ▶ Each PC is $F_s = X\delta_s$

Rollcall-Vote Principle Components

- ▶ Huge drop in variance from 1st to 2nd and 2nd to 3rd PC.
- ▶ Poli-Sci holds that PC1 is usually enough to explain congress.
  2nd component has been important twice: 1860's and 1960's.

Factor Interpretation

- Vote components in the **111th** house
- Each PC is $F_s = X\delta_s$

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- Poli-Sci holds that PC1 is usually enough to explain congress.
  2nd component has been important twice: 1860's and 1960's.
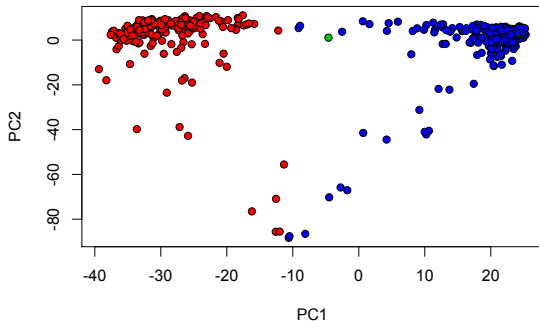
Factor Interpretation

▶ Top two PC directions in the **111th** house

▶ Republicans in red and Democrats in blue:
  ▶ Clear separation on the first principal component.
  ▶ The second component looks orthogonal to party.

Factor Interpretation

```
## Far right (very conservative)
> sort(votepc[,1])
     BROUN (R GA-10)       FLAKE (R AZ-6)    HENSARLIN (R TX-5)
        -39.3739409          -38.2506713          -37.5870597

## Far left (very liberal)
> sort(votepc[,1], decreasing=TRUE)
   EDWARDS (D MD-4)     PRICE (D NC-4)     MATSUI (D CA-5)
        25.2915083         25.1591151         25.1248117

## social issues?  immigration?  no clear pattern
> sort(votepc[,2])
    SOLIS (D CA-32) GILLIBRAND (D NY-20)       PELOSI (D CA-8)
        -88.31350926         -87.58871687          -86.53585568
   STUTZMAN (R IN-3)        REED (R NY-29)       GRAVES (R GA-9)
        -85.59217310         -85.53636319          -76.49658108
```
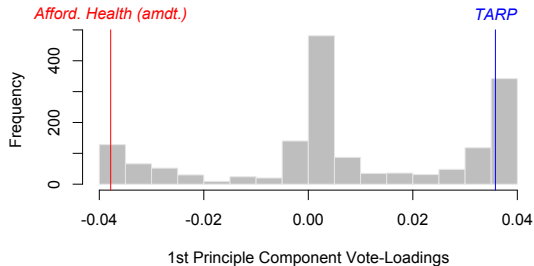
▶ PC1 is easy to read, PC2 is ambiguous (is it even meaningful?)

Factor Interpretation

► **High PC1-loading votes are ideological battles.**

► These tend to have informative voting across party lines.

► A vote for Repulican amendments to 'Affordable Health Care for America' strongly indicates a negative PC1 (more conservative), while
a vote for Troubled Asset Relief Program (TARP) indicates a positive PC1 (more progressive).

Factor Interpretation

▶ Look at the largest loadings in δ_2 to discern an interpretation.
```
> loadings[order(abs(loadings[,2]), decreasing=TRUE)[1:5],2]
  Vote.1146    Vote.658   Vote.1090   Vote.1104   Vote.1149
 0.05605862  0.05461947  0.05300806  0.05168382  0.05155729
```

▶ These votes all correspond to near-unanimous symbolic action.

▶ For example, 429 legislators voted for resolution 1146:
'Supporting the goals and ideals of a Cold War Veterans Day'
If you didn't vote for this, you weren't in the house.

▶ Mystery Solved: the second PC is just attendance!
```
> sort(rowSums(votes==0), decreasing=TRUE)
    SOLIS (D CA-32) GILLIBRAND (D NY-20)        REED (R NY-29)
               1628                 1619                  1562
  STUTZMAN (R IN-3)       PELOSI (D CA-8)       GRAVES (R GA-9)
               1557                 1541                  1340
```

Factor Interpretation

PCR: **Principal Component Regression**

The concept is very simple: instead of regressing onto **x**, use a lower dimension set of principal components **z** as covariates.

This works well for a few reasons:

- ▶ PCA reduces dimension, which is always good.
- ▶ Higher variance covariates are good in regression, and we choose the top PCs to have highest variance.
- ▶ The PCs are independent: no multicollinearity.

The 2-stage algorithm is straightforward. For example,

```
mypca = prcomp(X, scale=TRUE)
z = predict(mypca)[,1:K]
reg = glm(y~., data=as.data.frame(z))
```

Latent Dirichlet Allocation

▶ The approach of using PCA to factorize text was common before the 2000s.

▶ Versions of this algorithm were referred to under the label latent semantic analysis.

▶ However, this changed with the introduction of topic modeling, also known as Latent Dirichlet Allocation (LDA), by Blei et al. in 2003.

▶ These authors pointed out that the squared error loss (i.e., Gaussian model) implied by PCA is inappropriate for analysis of sparse word-count data.

▶ Instead, they proposed you take the bag-of-words representation seriously and model token counts as realizations from a multinomial distribution.

Latent Dirichlet Allocation

▶ That is, they proposed topic models as a multinomial factor model.

▶ Topic models are built on a simple document generation process:

  ▶ For each word, pick a "topic" k. This topic is defined through a probability vector over words, say, θ_k with probability θ_{kj} for each word j.

  ▶ Then draw the word according to the probabilities encoded in θ_k .

▶ After doing this over and over for each word in the document, you have proportion ω_{i1} from topic 1, ω_{i2} from topic 2, and so on.

Latent Dirichlet Allocation

▶ This basic generation process implies that the full vector of word counts, x_i, has a multinomial distribution:

$$x_i \sim MN(\omega_{i1}\theta_1 + \cdots + \omega_{iK}\theta_K, m_i) \tag{21}$$

▶ where $m_i = \sum_j x_{ij}$ is the total document length and, for example,
▶ the probability of word j in document i will be $\sum_k \omega_{ik}\theta_{kj}$

Latent Dirichlet Allocation vs PCA

▶ Recall our PC model:

$$E(x_i) = \delta_{i1}F_1 + \cdots + \delta_{iK}F_K \tag{22}$$

▶ The analogous topic model representation, implied by the above equation, is

$$E(x_i) = \omega_{i1}\theta_1 + \cdots + \omega_{iK}\theta_K \tag{23}$$

▶ such that topic score ω_{ik} is like PC score δ_{ik} and
▶ θ_k topic probabilities are like rotations F_k.
▶ The distinction is that the multinomial in implies a different loss function (from a multinomial) rather than the sums of squared errors that PCA minimizes.
▶ Note that we condition on document length here so that topics are driven by relative rather than absolute term usage.

LDA: Example

```
library(textir)

library(maptpx) # for the topics function

data(we8there)

# you need to convert from a Matrix to a `slam' simple_triplet_matrix
x <- as.simple_triplet_matrix(we8thereCounts)

# to fit, just give it the counts, number of `topics' K, and any other args
tpc <- topics(x,K=10)
```

```
##
## Estimating on a 6166 document collection.
## Fitting the 10 topic model.
## log posterior increase: 4441.8, 461.4, 101.5, 57.4, 51, 19.2, 26.2, 15.3, 15.4, 11.7, 6.7, 12.2, 8, 10.1,
4.8, 5.3, 3.2, 6.6, 2.8, 7, 3.6, 3.9, 6.7, 5.5, 8.6, 5, 11, 10.3, 12, 7.9, 12.1, 9, 8.8, 13.9, 8.6, 7.3, 6.1,
4.9, 4.3, 12, 11.1, 8.7, 3.2, 2.8, 5.1, 1.9, 2.6, 2.4, 4.9, 2.9, 1.5, 2.5, 4.7, 1.7, 0.9, 1.4, 0.7, 2.5, 2.2,
1.7, 1, 1.3, 1.5, 2, 0.8, 1.7, 0.5, 0.2, 0.5, 0.6, 0.9, 3.9, 0.5, 0.6, 0.4, 0.2, 0.8, 0.2, 1.4, 0.3, 0.5, 0.6, done.
```

LDA: Example

▶ Choosing the number of topics

```
# If you supply a vector of topic sizes, it uses a Bayes factor to choose
# (BF is like exp(-BIC), so you choose the bigggest BF)
# the algo stops if BF drops twice in a row
tpcs <- topics(x,K=5*(1:5), verb=1) # it chooses 10 topics
```

```
##
## Estimating on a 6166 document collection.
## Fit and Bayes Factor Estimation for K = 5 ... 25
## log posterior increase: 2853.9, 327.1, 85.3, 36.7, 25.9, 19.9, 13.8, 11.6, 9.6, 11.4, 20.3, 7.1, ..., done.
## log BF( 5 ) = 79521.94
## log posterior increase: 4626.7, 197.4, 53, 24.9, 19, 9.3, 7.4, 4.6, 5.2, 3.4, 2.3, 1.7, 0.8, ..., done.
## log BF( 10 ) = 87157.28
## log posterior increase: 3445, 170.2, 49.8, 23.6, 14.1, 31.4, 16.2, 4.8, 6.6, 5.5, 1.9, 5.9, ..., done.
## log BF( 15 ) = 3334.33
## log posterior increase: 2327.1, 139.8, 39.5, 16.7, 20.1, 5.3, 4.5, 3, 3.4, 2.9, 4.4, 1.8, ..., done.
## log BF( 20 ) = -66254.44
```

Topic Models: Example

▶ Interpretation

```
# summary prints the top `n' words for each topic,
# under ordering by `topic over aggregate' lift:
#     the topic word prob over marginal word prob.
summary(tpcs, n=10)

##
## Top 10 phrases by topic-over-null term lift (and usage %):
##
## [1] 'food great', 'great food', 'great servic', 'veri good', 'food veri', ... (14.6)
## [2] 'high recommend', 'italian food', 'best italian', 'mexican food', ... (11.6)
## [3] 'over minut', 'never go', 'go back', 'flag down', 'anoth minut', ... (10.4)
## [4] 'enough share', 'open daili', 'highlight menu', 'until pm', ... (10.4)
## [5] 'never return', 'one worst', 'don wast', 'wast time', ... (9.4)
## [6] 'good work', 'best kept', 'out world', 'great experi', ... (9.1)
## [7] 'thai food', 'veri pleasant', 'ice cream', 'breakfast lunch', ... (9)
## [8] 'take out', 'best bbq', 'can get', 'pork sandwich', 'home cook', ... (9)
## [9] 'food good', 'food place', 'chees steak', 'good select', 'food pretti',... (8.7)
## [10] 'wasn whole', 'came chip', 'got littl', 'over drink', 'took seat',... (7.8)
##
## Log Bayes factor and estimated dispersion, by number of topics:
##
##              5        10       15        20
## logBF 79521.94 87157.28 3334.33 -66254.44
## Disp      7.09     4.96     3.95      3.33
##
## Selected the K = 10 topic model
```

Topic Models: Example

```r
# alternatively, you can look at words ordered by simple in-topic prob
## the topic-term probability matrix is called 'theta',
## and each column is a topic
## we can use these to rank terms by probability within topics
rownames(tpcs$theta)[order(tpcs$theta[,1], decreasing=TRUE)[1:10]]
```

```
## [1] "veri good"    "great food"   "food great"   "great place"  "veri friend"
## [6] "veri nice"    "good food"    "great servic" "food excel"   "food servic"
```

```r
rownames(tpcs$theta)[order(tpcs$theta[,2], decreasing=TRUE)[1:10]]
```

```
## [1] "dine experi"   "high recommend" "wait staff"      "wine list"
## [5] "mexican food"  "italian food"   "italian restaur" "fine dine"
## [9] "staff friend"  "make feel"
```

Review & Next Steps

- ▶ Topic Models
- ▶ PCA Theory
- ▶ Factor Computation Factor Interpretation
- ▶ Latent Dirichlet Allocation LDA: Example

- ▶ Next class: More on text as data

- ▶ Questions? Questions about software?

Further Readings

▶ Blei, D. M., Ng, A. Y., & Jordan, M. I. (2003). Latent dirichlet allocation. Journal of machine Learning research, 3(Jan), 993-1022.

▶ James, G., Witten, D., Hastie, T., & Tibshirani, R. (2013). An introduction to statistical learning (Vol. 112, p. 18). New York: springer.

▶ Taddy, M. (2019). Business data science: Combining machine learning and economics to optimize, automate, and accelerate business decisions. McGraw Hill Professional.