Inference of proto-neutron star properties from gravitational wave data in core-collapse supernovae.

I. INTRODUCTION

The life of massive stars (those born with masses between ~ 8 M$_\odot$ and ~ 120 M$_\odot$) ends with the collapse of the iron core under its own gravity, leading to the formation of a neutron star (NS) or a black hole (BH), and followed (typically but not necessarily in the BH case) by a supernova explosion. Nearby core-collapse supernova (CCSN) explosions are expected to be sources of gravitational waves (GW) and are one of the main candidates for the next great discovery by current ground-based observatories. However, these are relative rare events. A neutrino-driven explosion [1] is the most likely outcome in the case of slow rotating cores, which are present in the bulk of CCSN progenitors. This event could be detected with advanced ground-based GW detectors within 5 kpc [2, 3]. Such a galactic event has a rate of about $2-3$ per century [4, 5]. For the case of fast rotating progenitor cores the result is likely a magneto-rotational explosion, with a more powerful signal that could be detected within 50 kpc and for some extreme models up to $5-30$ Mpc [2, 3]. However, only about 1% of the electromagnetically observed events show signatures of fast rotation (broad-lined type Ic SNe [6] or events associated to long GRBs [7]), making this possibility a subdominant channel of detection with an event rate of $\sim 10^{-4}$yr^{-1}. Therefore, we focus this work only in neutrino-driven CCSNe. Despite the low rates, CCSN are of great scientific interest because they produce a complex GW signals which could provide significant clues about the physical processes that occur in the moments after the collapse.

In the last decade, a significant progress has been made in the development of numerical codes, in particular in the treatment of multidimensioal effects [8]. In the case of neutrino-driven explosions, the GW emission is primarly induced by instabilities developed at the newly formed proto-neutron star (PNS) and by the non-spherical accreting flow of hot matter over its surface [9]. This dynamics excite the different modes of oscillation of the PNS, which ultimately leads to the emission of GWs. The frequency and time evolution of these modes carry information about the properties of the GW emitter and could allow to perform PNS asteroseismology.

The main feature appearing systematically in the GW spectrum of multidimensional numerical simulations is a strong and relatively narrow feature in the post bounce evolution with raising frequency from about 100 Hz up to a few kHz (at most) and a typical duration of $0.5-1$ s. This feature has been interpreted as a continuously excited gravity mode (g-mode, see [10, 11] for a definition in this context) of the PNS [12–17]. In these models the monotonic raise of the frequency of the mode is related to the contraction of the PNS. The typical frequencies of these modes make them a promising source for ground-based interferometers (aLIGO, aVirgo, KAGRA).

The properties of g-modes in hot PNSs have been studied since the end of last century by means of linear perturbation analysis of background PNS models. The oscillation modes associated to the surface of hot PNSs was first considered by McDermott, van Horn & Scholl [18]. Additionally, the stratified structure of the PNS allows the presence of different types of g-modes related with the fluid core [19]. Many posterior works used simplified neutron star models assuming an equilibrium configuration as a background, to study the effect of rotation [20], general relativity [21], non-linearities [22], phase transition [23] and realistic equation of state [24]. Only recently, there has been an effort to incorporate realistic backgrounds based in numerical simulations in the computation of the mode structure and evolution [25–33].

We base this work in the PNS mode analysis performed by [26, 28], which explored the eigenmode spectrum of the region within the shock (including the PNS and the post-shock region) using results from 2D CCSN numerical simulations as a background. Their results show a good match of the mode frequencies computed and the features observed in the GW spectrum of the same simulation (specially when space-time perturbations are included [28]). This result reveals that it is posible to perform CCSN asteroseismology under realistic conditions and serves as a starting point to carry out inference of astrophysical parameters of PNSs. [29] went one step further showing that it was possible to derive simple relations between the instantaneous frequency of the g-mode and the mass and radius of the PNS at each time of the evolution. These relations are universal in the sense that they do not depend on the equation of state (EOS) used or the mass of the progenitor, and only weakly on the numerical code used (see discussion in section II). Similar relations have been found by [32, 33], which also found that the universal relations do not depend on the dimensionality (1D, 2D or 3D) of the numerical simulation used as a background.

In this work, we present a method to infer from the GW data alone, the time evolution of some properties or the PNS, namely a combination of its mass and radius. For this purpose we have developed an algorithm to extract the time-frequency evolution of the main feature in the spectrograms of the GW emission of 2D simulations of CCSN. This feature corresponds to the $^2g_2$ mode, according to the nomenclature used in [29] (different authors may have slightly different naming convection). Next, we use the universal relations obtained by [29], based on a set of 1D simulations, to infer the time evolution of the ratio $M_{\rm PNS}/R_{\rm PNS}^2$, being $M_{\rm PNS}$ and $R_{\rm PNS}$ the mass and radius of the PNS. Using 2D CCSN waveform corre-

108 sponding to different progenitor masses we estimate the
109 performance of the algorithm for current and future gen-
110 eration of ground-based GW detectors.

111   This paper is organised as follows. Section II describes
112 the details of the CCSN simulations used in the paper.
113 Section III focuses on the algorithm that extracts the
114 time evolution of a combination of the mass and radius
115 of the PNS corresponding to a g-mode. Section IV shows
116 the performance of the data analysis method for different
117 GW detectors. Finally, we discuss the results in section
118 V.

II. CORE COLLAPSE SUPERNOVA SIMULATIONS

121   Unless other methods used GW astronomy, the al-
122 gorithm proposed in this work does not require accu-
123 rate waveforms in order to infer the properties of the
124 PNS. Instead, it relies on the evolution of the frequency
125 of oscillation of some particular modes, as seen in the
126 GW spectrum. The frequency of these modes depends,
127 in a universal way, on the surface gravity of the PNS
128 ($r = M_{\rm PNS}/R_{\rm PNS}^2$), in the sense that if at a given time we
129 observe GW emission at a certain frequency f we can de-
130 termine univocally the value of the surface gravity, within
131 a certain error, regardless of the details of the numerical
132 simulation. In this work we use two sets of simulations:
133 i) The *model set*, composed by 1D simulations, which is
134 used to build the universal relation (model), $r(f)$, link-
135 ing the ratio r with the observed frequency f, and ii) the
136 *test set*, composed by 2D simulations, for which we know
137 both the GW signal and the evolution of the ratio, $r(t)$,
138 and that is used to test performance of the algorithm.

139   We have used two different numerical codes in our nu-
140 merical simulations. CoCoNuT [34, 35] is a code for
141 general relativistic hydrodynamics coupled to the Fast
142 Multigroup Transport scheme [36] providing an approxi-
143 mate description of the emission and transport of neutri-
144 nos. AENUS-ALCAR [37] combines special relativistic
145 (magneto-)hydrodynamics, a modified Newtonian grav-
146 itational potential approximating the effects of general
147 relativity [38], and a spectral two-moment neutrino trans-
148 port solver [37]. We included the relevant reactions be-
149 tween matter and neutrinos of all flavours, i.e., emission
150 and absorption by nucleons and nuclei, electron-positron
151 pair annihilation, nucleonic bremsstrahlung, and scatter-
152 ing off nucleons, nuclei, and electrons.

153   For the *model set*, we use the 25 spherically symmet-
154 ric (1D) simulations of [28] including progenitors with
155 zero-age main sequence (ZAMS) masses in the range
156 $M_{\rm ZAMS} = 11.2 - 75\,M_\odot$. The set contains simulations
157 using the two numerical codes and six different equations
158 of state. Details can be found in [28]. The reason to use
159 one dimensional simulations for the model set is that the
160 computational cost of those is significantly smaller than
161 the cost of multidimensional simulations, so is easier to

Model name	$M_{\rm ZAMS}$ [$M_\odot$]	progenitor model	EOS	$t_{\rm f}$ [s]	$t_{\rm explosion}$	$M_{\rm PNS,f}$ [$M_\odot$]
s11	11.2	[39]	LS220	1.86	×	1.47
s15	15.0	[39]	LS220	1.66	×	2.00
s15S	15.0	[39]	SFHo	1.75	×	2.02
s15G	15.0	[39]	GShen	0.97	×	1.86
s20	20.0	[39]	LS220	1.53	×	1.75
s20S	20.0	[40]	SFHo	0.87	×	2.05
s25	25.0	[39]	LS220	1.60	0.91	2.33
s40	40.0	[39]	LS220	1.70	1.52	2.23

TABLE I. List of axisymmetric simulations used for the *test set*. The last three columns show, the post-bounce time at the end of the simulation, the one at the onset of the explosion (non exploding models marked with ×), and the PNS mass at the end of the simulation.

accumulate the statistics necessary to build a good model for $r(f)$. For each time of each simulation we compute the ratio r and the frequency of the $^2g_2$ mode by means of the linear analysis described in [26, 28, 29].

For the *test set*, we use 8 axisymmetric (2D) simula-tions using the AENUS-ALCAR code (see Table I for a list of models). 7 of these simulations use a selec-tion of progenitors with masses in the range $M_{\rm ZAMS} = 11.2 - 40\,M_\odot$ evolved through the hydrostatic phases by [39]. We performed one simulation of each stellar model using the equation of state of [41] with an incompress-ibility of $K = 220\,{\rm MeV}$ (LS220) and added comparison simulations with the SFHo EOS [42] and the GShen EOS [43] for the progenitor with $M_{\rm ZAMS} = 15\,M_\odot$. To this set of simulations, we add the waveform of a two-dimensional model used in [28], denoted s20S. It corresponds to a star with the same initial mass, $M_{\rm ZAMS} = 20\,M_\odot$, as for one of the other 7 axisymmetric simulations, but was taken from a newer set of stellar-evolution models [40]. It was evolved with the SFHo EOS.

For all the simulations, we mapped the pre-collapse state of the stars to a spherical coordinate system with $n_r = 400$ zones in radial direction distributed logarith-mically with a minimum grid width of $(\Delta r)_{\rm min} = 400\,{\rm m}$ and an outer radius of $r_{\rm max} = 8.3 \times 10^9\,{\rm cm}$ and $n_\theta = 128$ equidistant cells in angular direction. For the neutrino energies, we used a logarithmic grid with $n_e = 10$ bins up to 240 MeV. Unlike the model set, the simulations in the test set are not 1D because we need to extract the gravitational wave signal, which is a multi-dimensional effect. For each simulation we extract the gravitational wave signal, $h_+(t)$, by means of the quadrupole formula and compute the time evolution of the surface gravity, $r(t)$.

All spherical and most axisymmetric models fail to achieve shock revival during the time of our simula-tions. Only the two stars with the highest masses, s25 and s40, develop relatively late explosions in axisym-metry. Consequently, mass accretion onto the PNSs proceeds at high rates for a long time in all cases and

causes them to oscillate with their characteristic frequencies. The final masses of the PNSs are in the range of $M_{\mathrm{PNS}} = 1.47 - 2.33\,M_\odot$, i.e., insufficient for producing a black hole.

III. METHODS DESCRIPTION

In this section, we outline a strategy for estimating the time evolution of the ratio $r = M_{\mathrm{PNS}}/R_{\mathrm{PNS}}^2$ of the mass of the PNS and its squared radius (in units of solar mass and km) from the observation of the $^2g_2$ oscillation mode in the gravitational wave detector data. An integral part of this strategy is the universal relations that relate the characteristic frequency of the PNS oscillation f, g and p modes with the mass and the radius of the PNS, the shock radius and the total mass inside the shock as demonstrated in [29].

To build the model of the ratio r as a function of the frequency f we use the spherically symetric (1D) simulations of the *model set*. Figure 1 shows the data for the 25 numerical simulations. As identified by [29], the only systematic deviation from a single universal relation is the numerical code used in the simulations. To avoid any systematic effect, we only use the 18 simulations performed with the ALCAR-AENUS code, which is the same code that was used in our test set. The consequences of this choice are discussed in the conclusions. Using this data, we parametrize the ratio with a cubic polynomial regression with heteroscedastic errors

$$r_i = \beta_1 f_i + \beta_2 f_i^2 + \beta_3 f_i^3 + \epsilon_i \tag{1}$$

where ϵ_i are assumed to be independent zero-mean Gaussian errors with variances σ_i^2 that increase with frequency f_i. The model for frequency-dependent variances is

$$\log \sigma_i = \alpha_0 + \alpha_1 f_i + \alpha_2 f_i^2 + \delta_i \tag{2}$$

with independent and identically zero-mean Gaussian errors δ_i. The R-package `lmvar` [44] that implements a maximum likelihood approach was used to fit the model.

The best fitting model amongst polynomials of degree 1, 2, and 3 was chosen according to the Aikaike information criterion with coefficients given in Table II, which is actually the model defined in (1). The data and fit of the model including 95% confidence bands are displayed in Figure 1.

We use this model to infer the properties of the simulations in the *test set* described in in Section II. To describe the method we focus on the gravitational wave signal of `s20S`, originally sampled at 16384 Hz but resampled at 4096 Hz. A spectrogram of this signal is shown in Figure 2 based on autoregressive estimates [CITATION?] of the local spectra for successive time intervals of length 200 with a 90% overlap. The dominant emission mode corresponds to the PNS oscillation $^2g_2$-mode. We have

Coefficient	Estimate	Standard error
β_1	1.00×10^{-06}	2.12×10^{-08}
β_2	-8.22×10^{-10}	5.00×10^{-11}
β_3	1.01×10^{-12}	2.70×10^{-14}
α_0	$-1.02 \times 10^{+01}$	6.80×10^{-02}
α_1	7.24×10^{-04}	1.56×10^{-04}
α_2	6.23×10^{-07}	8.15×10^{-08}

TABLE II. Estimate and standard error of the coefficients of the best fit model describing the ratio $r = M_{\mathrm{PNS}}/R_{\mathrm{PNS}}^2$ as function of the frequency of the $^2g_2$ mode.
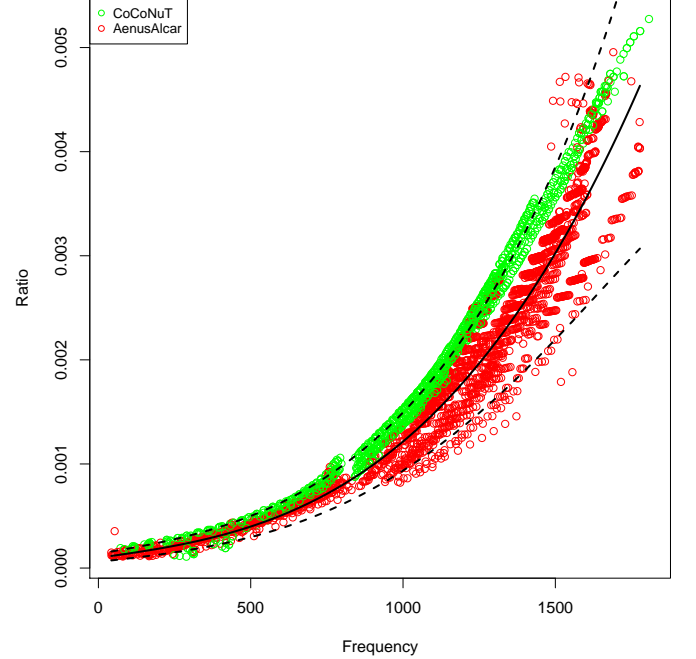

FIG. 1. Ratio $M_{\mathrm{PNS}}/R_{\mathrm{PNS}}^2$ from 25 1D simulations AENUS-ALCAR (red) and CoCoNuT (green) code. The solid line is the maximum likelihood estimate of heteroscedastic cubic model with 95% confidence bands (dashed lines) considering only the AENUS-ALCAR data points (18 simulations).

developed a time-frequency method to track the ridge $m(t)$ in the spectrogram, taking into account that it is monotonically increasing as time goes. This is a property of the $^2g_2$-mode, the frequency of which increases as the object becomes more massive and compact. Starting from either the left- or right-most column of the time-frequency matrix we identify and trace the sequence of amplitude peaks within a certain frequency band given the monotonicity constraint. Appendix A is providing more details on the reconstruction of the g mode ridge.

We collect the instantaneous frequency $f(t_i)$ corresponding to the ridge $m(t_i)$ for the midpoint t_i of each local time interval of the spectrogram and interpolating $f(t)$ for values in between the t_i. We then use our model given by Eq. (1) to obtain estimates of the time evolution of the ratio together with 95% confidence intervals.
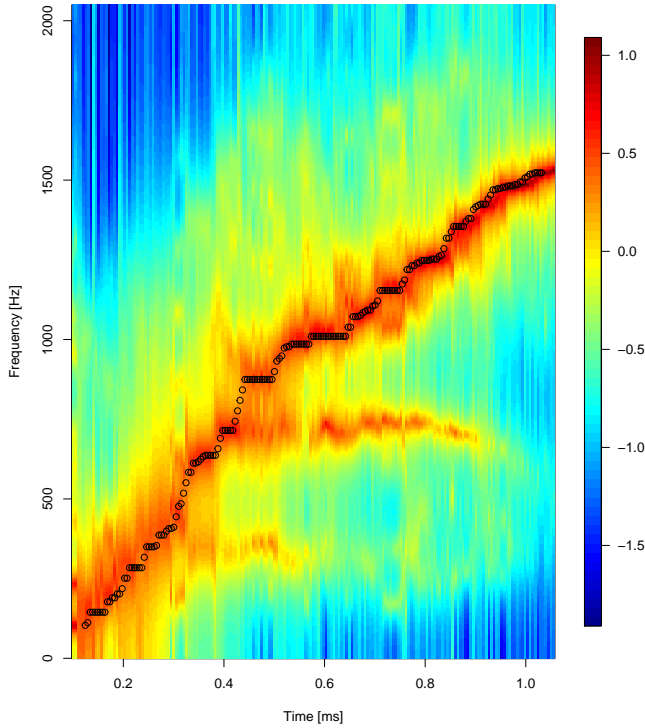
FIG. 2. Spectrogram of the gravitational wave signal $\mathbf{s}20\mathrm{S}$ sampled at 4096 Hz. The spectrogram is obtained using data streach of 200 samples overlapping at 90% with each other.

An exemple is given in Figure 3 where the red points are the point estimates and the grey bands represent 95% confidence bands. Ratio values computed using the mass and radius values obtained from the simulation code (true values) are shown in black.

In this case, for a GW signal without any noise, the coverage of our 95% confidence band is 94% of the true values. In the next section we investigate the performance of the reconstruction of $r(t)$ when the gravitational wave signal is embedded in noise.

IV.   DETECTION SENSITIVITY WITH ADVANCED GRAVITATIONAL WAVE DETECTORS

To estimate how accurately we can infer the time evolution of $r = M_{\mathrm{PNS}}/R_{\mathrm{PNS}}^2$ in the gravitational wave detector data, we have added the GW signal from $\mathbf{s}20\mathrm{S}$ to 100 Gaussian noise realisations whose power spectral density
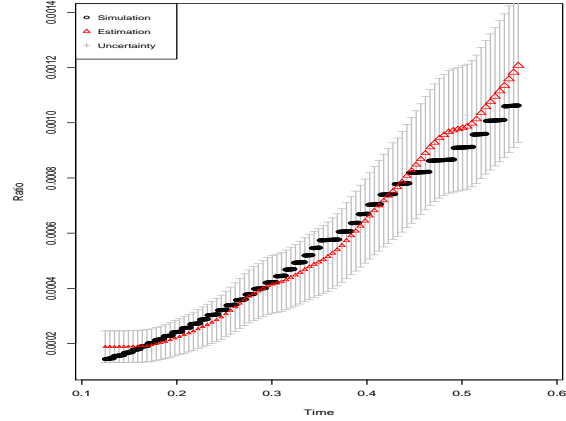

FIG. 3. Ratio $M_{\mathrm{PNS}}/R_{\mathrm{PNS}}^2$ as function of time extracted from the $^2g_2$-mode of the $\mathbf{s}20\mathrm{S}$ signal (red points and the 95% confidence belt in grey) compared to the ratio value derived from the PNS mass and radius given by the simulation code (black points) To be redone in python.

follows the advanced LIGO (aLIGO) spectrum [45] shown on Figure 5. We have varied the distance to the source, covering a large range of distances for which a detection in second generation of gravitational wave detectors is feasible. The source is optimally oriented with respect to the gravitational wave detector. We are assuming a GW signal from a core collapse phenomena has been identified in the data and that the beginning of the GW signal is known within $O(10 \ ms)$. The data (signal embedded in noise) are whitened using the function *prewhiten* of the R-package TSA. An auto-regressive model with maximal 100 coefficients has been used.

For each of the noise realisations, we reconstruct the ratio time series r_i of length N starting from the left side of the spectrogram and constraining the beginning of the track to be smaller than 200 Hz. The reconstructed ratio is then compare to the "true" ratio r_i^0 derived from the PNS mass and radius computed from the ~~generated by the simulation code that produced~~ $\mathbf{s}20\mathrm{S}$ simulation.

Figure 4 is showing the fraction of the ratio r_i^0 values that fall within the 95% confidence interval of r_i. This quantity, *coverage*, is taking maximal values when the source is located within few kpc and then decreases with the distance.

To better quantify how well we reconstruct the ratio, we have also considered Δ the mean over the track of the relative error of r_i.

$$\Delta = \frac{1}{N}\sum_1^N \frac{|r_i - r_i^0|}{r_i^0} \qquad (3)$$

Δ values of each of the 100 noise realisation are shown as well as function of the distance on Figure 4. For a

		s11	s15	s15S	s15G	s20	s20S	s25	s40
aLIGO	d_{max}	7	26	24	22	16	10	13	28
	SNR	14	46	33	35	27	21	96	80
CE1	d_{max}	75	285	279	230	179	111	164	349
	SNR	149	490	352	366	282	218	1006	822
CE2	d_{max}	147	–	–	–	–	207	–	–
	SNR	256	843	608	635	487	382	1751	1430
ET_B	d_{max}	75	278	274	244	159	107	79	62
	SNR	138	473	356	379	381	260	1046	865
ET_C	d_{max}	71	271	239	222	154	119	104	165
	SNR	126	432	320	338	252	213	945	783
ET_D	d_{max}	79	306	264	252	179	131	134	349
	SNR	140	477	352	371	277	227	1034	859

TABLE III. Maximal distance d_{max} at which the ratio $r = M_{\mathrm{PNS}}/R_{\mathrm{PNS}}^2$ is reconstructed with good accuracy for a source optimally oreinted with respect to the GW detectors considered in this study. Matched filter signal-to-noise ratio (SNR) of the simulated waveforms for the different GW detectors. The source is located at 10 kpc and is optimally oriented with respect to the detector.

source located up to ∼10 kpc the relative error remains smaller than 20% [PCD: should we comment on the outliers?]. At small distance Δ is small but not null. This reflects the approximation of the model used for r. It is nevertheless remarkable that one can reconstruct the ratio time series with a good precision at distance up to ∼ 10 kpc for this particular waveform, with *coverage* value larger than 80%. We have tested that the method does not depend on features of s20S using the 7 other waveforms of the *test set* described in section II covering a large range of progenitor masses. Figure 6 shows that apart s11.2-LS220, the ratio is well reconstructed for all waveforms up to ∼ 10kpc. On this figure we also show the *coverage* value in case of absence of signal. The median value is significantly different from 0 because the g-mode reconstruction algorithm is looking for a continuously frequency increasing track in the spectrogram, starting between 0 and 200 Hz. In Table III we are reporting the distance at which *coverage* median is lower than 80%.

FIG. 4. Boxplots of *coverage* (upper panel) and Δ (lower panel) for s20S signal embedded in aLIGO noise at different distances from the Earth. 100 noise realisations is considered for each distance. [PCD: I guess you use "coverage probability" because we are showing the box plots. Shouln't we say Δ probability as well? Or just "coverage"? Also, how are outliers computed? Why are there so many outliers in $|delta|$? If they are a significant fraction of the data points why are they considered outliers?]

V.   DISCUSSION

Acknowledgments —

Appendix A   G-MODE RECONSTRUCTION

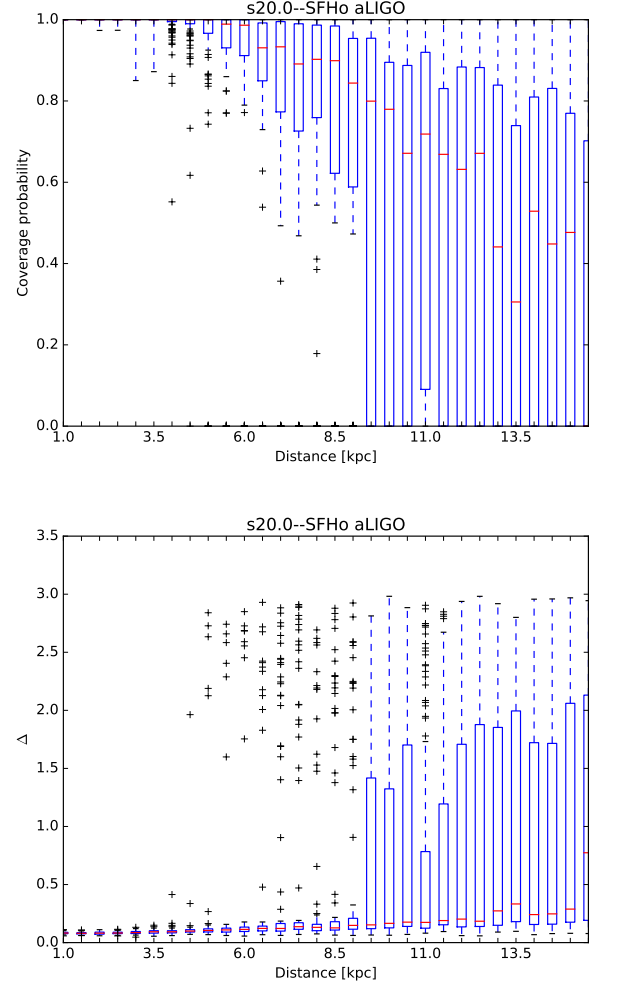Given the spectrogram and an specified time interval for the g-mode reconstruction, our proposal method works as follows. The starting point must be specified. It can be either at the beginning or at the end of the signal. Then, in one of these extremes, the maximum energy value is identified, registering its frequency. This is done independently for a number of consecutive time intervals. Then we calculate the median of these frequency values, providing a robust starting value for the g-mode reconstruction.

The starting frequency value is the first g-mode estimate for the first or the last time interval, depending on the specified starting location. If the reconstruction is set to start at the beginning of the signal, the reconstruction will be done progressively over the time intervals, where each maximum frequency value will be calculated within
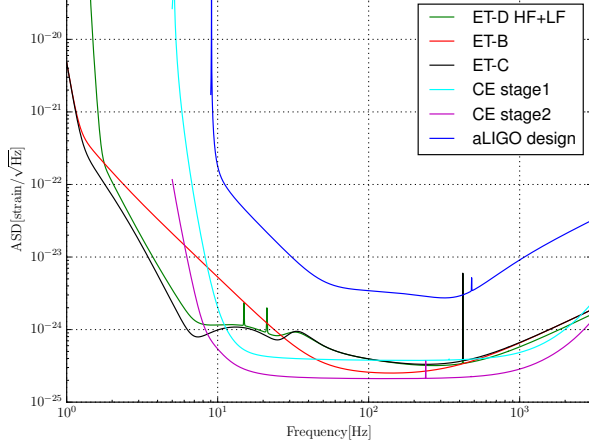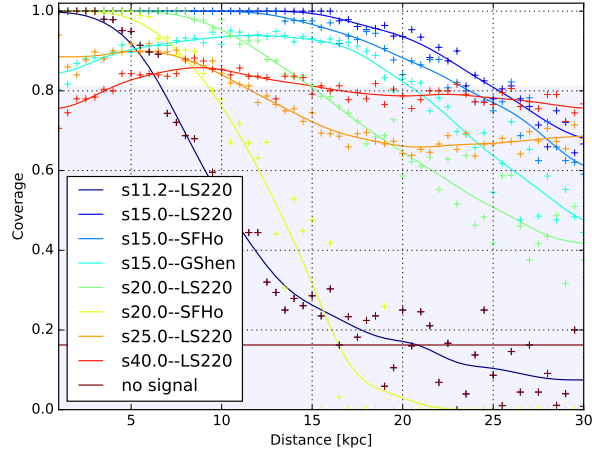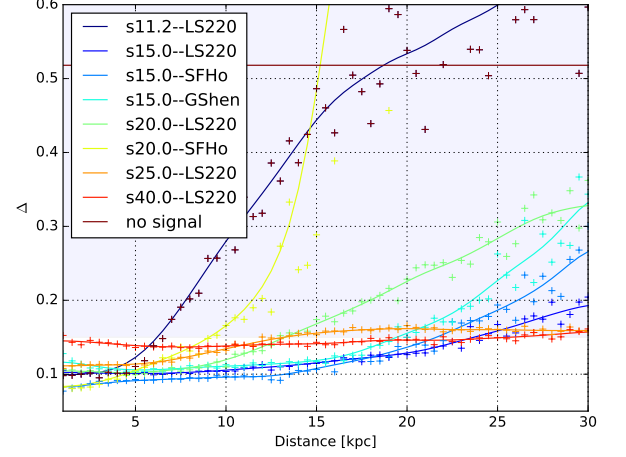
FIG. 5.

FIG. 7. Median of *Delta* for 8 CCSN waveforms embedded in aLIGO noise and located at different distance from the Earth. The "no signal" line and band show the median and first and third quartile of *Delta* in absence of any signal.

FIG. 6. Median of *coverage* for 8 CCSN waveforms embedded in aLIGO noise and located at different distance from the Earth. The "no signal" line and band show the median and first and third quartile of *coverage* in absence of any signal.
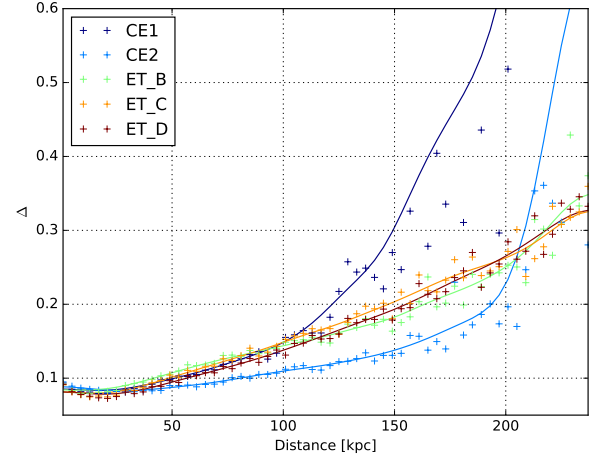

FIG. 8. Median of *coverage* for s20.0–SFHo CCSN waveform embedded in 3G detectors noise and located at different distance from the Earth.

a frequency range specified by the previous g-mode estimate. Given the non-decreasing behaviour of the true g-mode values, the g-mode estimates will be forced to be greater or equal than the one estimated for its previous time interval, and lower than a specified upper limit. As a result, the g-modes estimates will be a non-decreasing sequence of frequency values.

If the reconstruction is set to start at the end of the signal, the g-modes will be estimated backward in time. Each maximum frequency is calculated within a range determined by its successor (in time) g-mode estimate. These estimates are forced to be lower or equal than its successor (in time) estimate, but greater than a specified lower limit. Thus, a non-decreasing sequence of g-mode estimates is guaranteed.

This g-mode reconstruction method works if and only if the signal is strong enough to provide information about the g-mode, which is reflected in the spectrogram.

Given the sequence of g-mode estimates, the confidence band will be calculated by using the model defined in (1). The g-mode estimates are frequency values which we use as predictors in the model in order to generate confidence intervals for the ratios. Since the g-mode estimates are indexed by time, the confidence intervals for the ratios are too. Thus, we generate the confidence band by interpolating the lower and upper limits of the collection of consecutive confidence intervals, which will be valid for the time range of the g-mode estimates. This confidence

band is used to estimate the coverage probabilities in our₃₇₉ simulation studies presented ~~below~~ above.

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