Identification of protoneutron star g-modes in gravitational-wave data

I. INTRODUCTION

CCSN Simulation and GW emission

The life of massive stars ($8\mathrm{M}_\odot - 100\mathrm{M}_\odot$) ends with the collapse of their iron core under their own gravity, leading to the formation of a neutron star or a black hole (BH), followed (typically but not necessarily in the BH case) by the explosion of the star as a supernova. Core-collapse supernova (CCSN) explosions are one of the most important sources of gravitational-waves (GW) that have not yet been detected by current ground-based observatories. This is because even the most common type of CCSN, the neutrino-driven explosion supernova, have a rate of about three per century [1] within our galaxy. The other main type of explosion, those produced by the magneto-rotational mechanism, can produce a more powerful signal and can be detected at distances up to ~ 5 Mpc [1]. However, the rate of events of this kind is much lower than the one for the neutrino driven mechanism $\sim 10^{-4}\mathrm{yr}^{-1}$, which represents less than 1% of all CCSNe. Despite all this, collapsing stars produces a complex GW signal which could provide significant clues about the physical processes that occur in the moments after the explosion.

The computational modelling of the core-collapse phenomena is challenging due the large number of physical processes involved whose role is not completely understood. There are uncertainties in the stellar evolution of massive stars or in the nuclear and weak force interactions necessary for the equation of state (EoS) or the neutrino interactions. These large number of physical ingredients in addition to the necessary accuracy of the modelling of complex multidimensional interactions requires large computational resources. One simulation of one single progenitor in 3D with accurate neutrino transport and realistic equation of state (EoS) can take several months of intense calculations on a scientific supercomputer facility.

PNS oscillation modes

Asteroseismology and universal relationships

Introduce the topic of the paper: method to extract from the GW data the Mass and the radius of the PNS as function of time using the universal relations. Previous work in identifying modes from spectrogram

Paper organization

II. CORE COLLAPSE SUPERNOVA SIMULATIONS

Martin's simulations and code description

1D simlation data to fit the ratio vs frequency model - AA and CoConut outputs

III. METHODS DESCRIPTION

In this section, we outline a strategy for estimating the time evolution of the ratio $r = M_{\mathrm{PNS}}/R^2_{\mathrm{PNS}}$ of the mass of the PNS and its squared radius (in units of solar mass and km) from the observation of the $^2g_2$ oscillation mode in the gravitational wave detector data. An integral part of this strategy is the universal relationships that relate the characteristic frequency of the PNS oscillation f, g and p modes with the mass and the radius of the PNS, the shock radius and the total mass inside the shock as demonstrated in [2].

Using 25 1D simulations obtained with the AENUS-ALCAR code [] and the CoCoNuT [] code, we parametrize the ratio with a cubic polynomial regression with heteroscedastic errors

$$r_i = \beta_1 f_i + \beta_2 f_i^2 + \beta_3 f_i^3 + \epsilon_i \tag{1}$$

where ϵ_i are assumed to be independent zero-mean Gaussian errors with variances σ_i^2 that increase with frequency f_i. The model for frequency-dependent variances is

$$\log \sigma_i = \alpha_0 + \alpha_1 f_i + \alpha_2 f_i^2 + \delta_i \tag{2}$$

with independent and identically zero-mean Gaussian errors δ_i. The R-package LMVAR [3] that implements a maximum likelihood approach was used to fit the model. The best fitting model amongst polynomials of degree 1, 2, and 3 was chosen according to the Aikaike information criterion with coefficients given in Table I,

$$r_i = \beta_1 f_i + \beta_3 f_i^3 + \epsilon_i \tag{3}$$

The best-fitting model achieves a coefficient of determination of $R^2 = 0.9812$. The data and fit of the model including 95% confidence bands are displayed in Figure 1.

Coefficient	Estimate	standard error
β_1	6.09×10^{-7}	1.75×10^{-8}
β_3	6.24×10^{-13}	8.79×10^{-15}

TABLE I. Estimate and standard error of the coefficients of the best fit model describing the ratio $r = M_{\mathrm{PNS}}/R^2_{\mathrm{PNS}}$ as function of the frequency of the $^2g_2$ mode.

To develop the method we considered the gravitational wave signal `s20-gw-10kpc` described in Section II, originally sampled at 16384 Hz but resampled at 4096 Hz. A

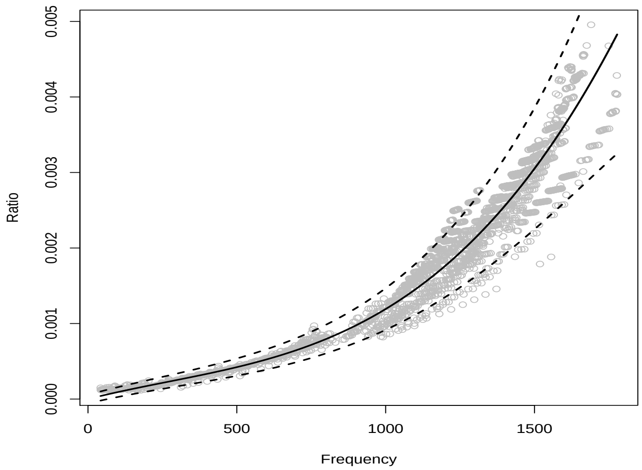

FIG. 1. Data from 25 1D simulations AENUS-ALCAR and CoCoNuT code, solid line is the maximum likelihood estimate of heteroscedastic cubic model with 95% confidence bands.

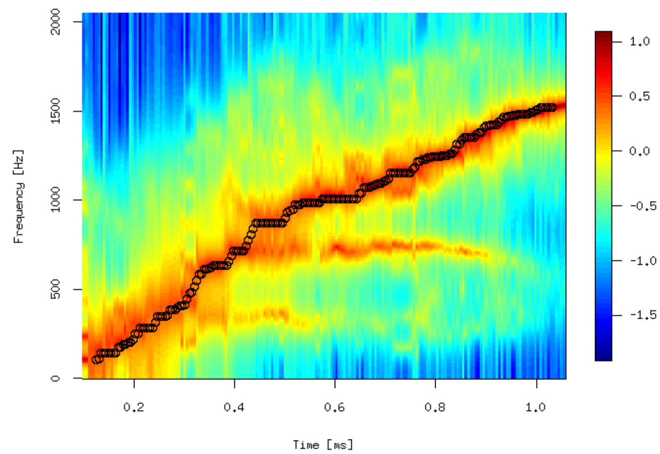

FIG. 2. Spectrogram of the gravitational wave signal `s20-gw-10kpc` sampled at 4096 Hz. The spectrogram is obtained using data streach of 200 samples overlapping at 90% with each other.

spectrogram of this signal is shown in Figure 2 based on autoregressive estimates of the local spectra for successive time intervals of length 200 with a 90% overlap. The dominant emission mode corresponds to the ${}^2g_2$-mode and we have developed a time-frequency method to track the ridge $m(t)$ in the spectrogram, taking into account that it is monotonically increasing as time goes. Starting from either the left- or right-most column of the time-frequency matrix we identify and trace the sequence of amplitude peaks within a certain frequency band given the monotonicity constraint. Appendix **??** is providing more details on the reconstruction of the g mode ridge.

We identify the instantaneous frequency $f(t_i)$ corresponding to the ridge $m(t_i)$ for the midpoint t_i of each local time interval of the spectrogram and interpolating $f(t)$ for values in between the t_i. We then use the equation (3) to obtain estimates of the time evolution of the ratio together with 95% confidence intervals. An exemple is given in Figure 3 where the red points are the point estimates and the grey bands represent 95% confidence bands. The black points are the true ratio values computed using the mass and radius values obtained from the simulation code.

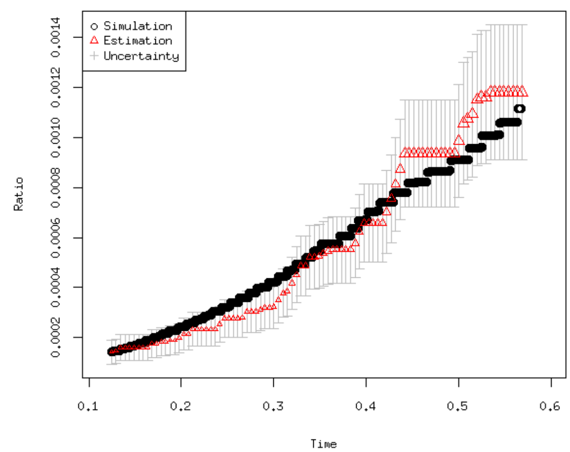

FIG. 3.

In the following simulation study we explore how accurately we can estimate the parameters when the gravitational wave signal is embedded in noise. For that purpose, we inject the gravitational wave signal into simulated Advanced LIGO noise using the noise power spectral density insert formula for varying SNRs, respectively distances to the source. We estimate the coverage probability of the 95% confidence band by calculating the proportion of times that the true ratio lies outside one of the pointwise 95% confidence intervals. These coverage probabilities together for varying SNRs are given in Table insert Table and displayed in the form of boxplots in Figure insert Figure

IV. SIMULATION STUDY

V. DISCUSSION

Acknowledgments —

Appendix A G-MODE RECONSTRUCTION

Given the spectrogram and an specified time interval for the g-mode reconstruction, our proposal method works as follows. The starting point must be specified. It can be either at the beginning or at the end of the signal. Then, in one of these extremes, the maximum energy value is identified, registering its frequency. This is done independently for a number of consecutive time intervals. Then we calculate the median of these frequency values, providing a robust starting value for the g-mode reconstruction.

The starting frequency value is the first g-mode estimate for the first or the last time interval, depending on the specified starting location. If the reconstruction is set to start at the beginning of the signal, the reconstruction will be done progressively over the time intervals, where each maximum frequency value will be calculated within a frequency range specified by the previous g-mode estimate. Given the non-decreasing behaviour of the true g-mode values, the g-mode estimates will be forced to be greater or equal than the one estimated for its previous time interval, and lower than a specified upper limit. As a result, the g-modes estimates will be a non-decreasing sequence of frequency values.

If the reconstruction is set to start at the end of the signal, the g-modes will be estimated backward in time. Each maximum frequency is calculated within a range determined by its successor (in time) g-mode estimate. These estimates are forced to be lower or equal than its successor (in time) estimate, but greater than a specified lower limit. Thus, a non-decreasing sequence of g-mode estimates is guaranteed.

This g-mode reconstruction method works if and only if the signal is strong enough to provide information about the g-mode, which is reflected in the spectrogram.

Given the sequence of g-mode estimates, the confidence band will be calculated by using the model defined in (3). The g-mode estimates are frequency values which we use as predictors in the model in order to generate confidence intervals for the ratios. Since the g-mode estimates are indexed by time, the confidence intervals for the ratios are too. Thus, we generate the confidence band by interpolating the lower and upper limits of the collection of consecutive confidence intervals, which will be valid for the time range of the g-mode estimates. This confidence band is used to estimate the coverage probabilities in our simulation studies presented below.

[1] S.E. Gossan, P. Sutton, A. Stuver, M. Zanolin, K. Gill, and C. Ott, "Observing gravitational waves from core-collapse supernovae in the advanced detector era," Physical Review D **93** (2016), 10.1103/physrevd.93.042002.

[2] A. Torres-Forné, P. Cerdá-Durán, M. Obergaulinger, B. Müller, and J. Font, "Universal relations for gravitational-wave asteroseismology of proto-neutron stars," Physical Review Letters **123**, 051102 (2019).

[3] .