Identification of protoneutron star g-modes in gravitational-wave data [PCD: alt. title]Inference of proto-neutron star properties from gravitational wave data in core-collapse supernovae.

I. INTRODUCTION

The life of massive stars (those born with masses between $\sim 8\ \mathrm{M}_{\odot}$ and $\sim 120\ \mathrm{M}_{\odot}$) ends with the collapse of the iron core under its own gravity, leading to the formation of a neutron star (NS) or a black hole (BH), and followed (typically but not necessarily in the BH case) by a supernova explosion. Nearby core-collapse supernova (CCSN) explosions are expected to be sources of gravitational waves (GW) and are one of the main candidates for the next great discovery by current ground-based observatories. However, these are relative rare events. A neutrino-driven explosion [1] is the most likely outcome in the case of slow rotating cores, which are present in the bulk of CCSN progenitors. This event could be detected with advanced ground-based GW detectors within 5 kpc [2, 3]. Such a galactic event has a rate of about $2-3$ per century [4, 5]. For the case of fast rotating progenitor cores the result is likely a magneto-rotational explosion, with a more powerful signal that could be detected within 50 kpc and for some extreme models up to $5-30$ Mpc [2, 3]. However, only about 1% of the electromagnetically observed events show signatures of fast rotation (broad-lined type Ic SNe [6] or events associated to long GRBs [7]), making this possibility a subdominant channel of detection with an event rate of $\sim 10^{-4}\mathrm{yr}^{-1}$. Therefore, we focus this work only in neutrino-driven CCSNe. Despite the low rates, CCSN are of great scientific interest because they produce a complex GW signals which could provide significant clues about the physical processes that occur in the moments after the collapse.

In the last decade, a significant progress has been made in the development of numerical codes, in particular in the treatment of multidimensioal effects [8]. [PCD: I don't think the next sentence is necessary since we have already discarded magneto-rotational events for this work.]~~The waveforms produced by the magneto-rotational mechanism in particular is well understood. The core-bounce signal can be directly related with the rotational properties of the core~~ [9–11]. ~~However, the low rate of this kind of events and its expected low amplitude in the slow-rotation case, will probably impede its detection.~~ In the case of neutrino-driven explosions, the GW emission is primarly induced by instabilities developed at the newly formed proto-neutron star (PNS) and by the non-spherical accreting flow of hot matter over its surface [12]. This dynamics excite the different modes of oscillation of the PNS, which ultimately leads to the emission of GWs. The frequency and time evolution of these modes carry information about the properties of the GW emitter and could allow to perform PNS asteroseismology.

[PCD: I would remove the discussion on the inference of the progenitor properties (commented now).]

The main feature appearing systematically in the GW spectrum of multidimensional numerical simulations is a strong and relatively narrow feature in the post bounce evolution with raising frequency from about 100 Hz up to a few kHz (at most) and a typical duration of $0.5-1$ s. This feature has been interpreted as a continuously excited gravity mode (g-mode, see [13, 14] for a definition in this context) of the PNS [15–20]. In these models the monotonic raise of the frequency of the mode is related to the contraction of the PNS. The typical frequencies of these modes make them a promising source for ground-based interferometers (aLIGO, aVirgo, KAGRA).

The properties of g-modes in hot PNSs have been studied since the end of last century by means of linear perturbation analysis of background PNS models. The oscillation modes associated to the surface of hot PNSs was first considered by McDermott, van Horn & Scholl [21]. Additionally, the stratified structure of the PNS allows the presence of different types of g-modes related with the fluid core [22]. Many posterior works used simplified neutron star models assuming an equilibrium configuration as a background, to study the effect of rotation [23], general relativity [24], non-linearities [25], phase transition [26] and realistic equation of state [27]. Only recently, there has been an effort to incorporate realistic backgrounds based in numerical simulations in the computation of the mode structure and evolution [28–36].

We base this work in the PNS mode analysis performed by [29, 31], which explored the eigenmode spectrum of the region within the shock (including the PNS and the post-shock region) using results from 2D CCSN numerical simulations as a background. Their results show a good match of the mode frequencies computed and the features observed in the GW spectrum of the same simulation (specially when space-time perturbations are included [31]). This result reveals that it is posible to perform CCSN asteroseismology under realistic conditions and serves as a starting point to carry out inference of astrophysical parameters of PNSs. [32] went one step further showing that it was possible to derive simple relations between the instantaneous frequency of the g-mode and the mass and radius of the PNS at each time of the evolution. These relations are universal in the sense that they do not depend on the equation of state (EOS) used or the mass of the progenitor, and only weakly on the numerical code used (see discussion in section II). Similar relations have been found by [35, 36], which also found that the universal relations do not depend on the dimensionality (1D, 2D or 3D) of the numerical simulation used as a background.

In this work, we present a method to infer from the GW data alone, the time evolution of some properties or the PNS, namely a combination of its mass and radius. For this purpose we have developed an algorithm to extract the time-frequency evolution of the main feature in the spectrograms of the GW emission of 2D simulations of CCSN. This feature corresponds to the $^2g_2$ mode, according to the nomenclature used in [32] (different authors may have slightly different naming convection). Next, we use the universal relations obtained by [32] to infer the time evolution of the ratio $r = M_{\rm PNS}/R^2_{\rm PNS}$, being $M_{\rm PNS}$ and $R_{\rm PNS}$ the mass and radius of the PNS. Using 2D CCSN waveform corresponding to different progenitor masses we estimate the performance of the algorithm for current and future generation of ground-based GW detectors.

This paper is organised as follows. Section II describes the details of the 2D CCSN used. Section III focuses on the algorithm that extracts the time evolution of a combination of the mass and radius of the PNS corresponding to a g-mode. Section IV shows the performance of the data analysis method for different GW detectors. Finally, we discuss the results in section V.

II. CORE COLLAPSE SUPERNOVA SIMULATIONS

The algorithm proposed in the article does not require accurate waveforms but relies on the evolution of the frequencies of oscillations depending on the PNS mass and radius. To parametrize this dependence, we have considered spherically symmetric [31] and two-dimensional axisymmetric models of stellar core collapse simulated with two codes, CoCoNuT (one-dimensional models) and AENUS-ALCAR [37] (one- and two-dimensional models). CoCoNuT [38] is a code for general relativistic hydrodynamics coupled to the Fast Multigroup Transport scheme [39] providing an approximate description of the emission and transport of neutrinos. AENUS-ALCAR [37] combines special relativistic (magneto-)hydrodynamics, a modified Newtonian gravitational potential approximating the effects of general relativity [40], and a spectral two-moment neutrino transport solver [37]. We included the relevant reactions between matter and neutrinos of all flavours, i.e., emission and absorption by nucleons and nuclei, electron-positron pair annihilation, nucleonic bremsstrahlung, and scattering off nucleons, nuclei, and electrons.

We use two sets of 25 models in the range of initial stellar masses $M_{\rm ZAMS} = 11.2 - 75\,M_\odot$ simulated with the two codes. They were carried out using six equations of state (EOSs). In addition to these simulations data we have considered 8 waveforms. 7 of them are from two-dimensional axisymmetric models consisting of stellar core collapse of five stars with zero-age main-sequence masses of $M_{\rm ZAMS} = 11.2{-}40\,M_\odot$ evolved through the hydrostatic phases by [41]. We performed one simulation of each stellar model using the equation of state of [42] with an incompressibility of $K = 220\,\mathrm{MeV}$ (LS220) and added comparison simulations with the SFHo EOS [43] and the EOS of [44] (GShen) for the one with $M_{\rm ZAMS} = 15\,M_\odot$ (see Table I for a list of models). To this set of simulations, we add the waveform of a two-dimensional model used in [31], denoted `s20S`. It corresponds to a star with the same initial mass, $M_{\rm ZAMS} = 20\,M_\odot$, as for one of the other 7 axisymmetric simulations, but was taken from a newer set of stellar-evolution models [45]. It was evolved with the SFHo EOS.

Simulation	$M_{\rm ZAMS}[M_\odot]$	EOS	$t_{\rm f}$[s]	explosion	$M_{\rm PNS}[M_\odot]$
`s11`	11.2	LS220	1.86	×	1.47
`s15`	15.0	LS220	1.66	×	2.00
`s15S`	15.0	SFHo	1.75	×	2.02
`s15G`	15.0	GShen	0.97	×	1.86
`s20`	20.0	LS220	1.53	×	1.75
`s20S`	20.0	SFHo	0.87	×	2.05
`s25`	25.0	LS220	1.60	0.91	2.33
`s40`	40.0	LS220	1.70	1.52	2.23

TABLE I. List of axisymmetric simulations. We present the name of the models, the initial mass of the progenitors, and the EOS used, and the final post-bounce time of the simulations. For models which explode, we list the time at which the shock starts to expand in column "explosion"; otherwise, a × sign is displayed. The final column indicates the mass of the PNS at the end of the simulation.

We mapped the pre-collapse state of the stars to a spherical coordinate system with $n_r = 400$ zones in radial direction distributed logarithmically with a minimum grid width of $(\Delta r)_{\rm min} = 400\,\mathrm{m}$ and an outer radius of $r_{\rm max} = 8.3 \times 10^9\,\mathrm{cm}$ and $n_\theta = 128$ equidistant cells in angular direction. For the neutrino energies, we used a logarithmic grid with $n_e = 10$ bins up to $240\,\mathrm{MeV}$.

All spherical and most axisymmetric models fail to achieve shock revival during the time of our simulations. Only the two stars with the highest masses, `s25` and `s40`, develop relatively late explosions in axisymmetry. Consequently, mass accretion onto the PNSs proceeds at high rates for a long time in all cases and causes them to oscillate with their characteristic frequencies. The final masses of the PNSs are in the range of $M_{\rm PNS} = 1.47 - 2.33\,M_\odot$, i.e., insufficient for producing a black hole.

III. METHODS DESCRIPTION

In this section, we outline a strategy for estimating the time evolution of the ratio $r = M_{\rm PNS}/R^2_{\rm PNS}$ of the mass of the PNS and its squared radius (in units of solar mass and km) from the observation of the $^2g_2$ oscillation mode in the gravitational wave detector data. An integral part of this strategy is the universal relations that relate the characteristic frequency of the PNS oscillation f, g and p modes with the mass and the radius of the PNS, the shock radius and the total mass inside the shock as demonstrated in [32].

Using 25 spherically symetric (1D) simulations obtained with the AENUS-ALCAR code [37] and the CoCoNuT [38] code, we parametrize the ratio with a cubic polynomial regression with heteroscedastic errors

$$r_i = \beta_1 f_i + \beta_2 f_i^2 + \beta_3 f_i^3 + \epsilon_i \qquad (1)$$

where ϵ_i are assumed to be independent zero-mean Gaussian errors with variances σ_i^2 that increase with frequency f_i. The model for frequency-dependent variances is

$$\log \sigma_i = \alpha_0 + \alpha_1 f_i + \alpha_2 f_i^2 + \delta_i \qquad (2)$$

with independent and identically zero-mean Gaussian errors δ_i. The R-package `lmvar` [46] that implements a maximum likelihood approach was used to fit the model.

The best fitting model amongst polynomials of degree 1, 2, and 3 was chosen according to the Aikaike information criterion with coefficients given in Table II, which is actually the model defined in (1). The data and fit of the model including 95% confidence bands are displayed in Figure 1.

Coefficient	Estimate	Standard error
β_1	1.00×10^{-06}	2.12×10^{-08}
β_2	-8.22×10^{-10}	5.00×10^{-11}
β_3	1.01×10^{-12}	2.70×10^{-14}
α_0	$-1.02 \times 10^{+01}$	6.80×10^{-02}
α_1	7.24×10^{-04}	1.56×10^{-04}
α_2	6.23×10^{-07}	8.15×10^{-08}

TABLE II. Estimate and standard error of the coefficients of the best fit model describing the ratio $r = M_{\rm PNS}/R_{\rm PNS}^2$ as function of the frequency of the $^2g_2$ mode.

To develop the method we considered the gravitational wave signal `s20S` described in Section II, originally sampled at 16384 Hz but resampled at 4096 Hz. A spectrogram of this signal is shown in Figure 2 based on autoregressive estimates of the local spectra for successive time intervals of length 200 with a 90% overlap. The dominant emission mode corresponds to the PNS oscillation $^2g_2$-mode. We have developed a time-frequency method to track the ridge $m(t)$ in the spectrogram, taking into account that it is monotonically increasing as time goes, a property of the $^2g_2$-mode. Starting from either the left- or right-most column of the time-frequency matrix we identify and trace the sequence of amplitude peaks within a certain frequency band given the monotonicity constraint. Appendix A is providing more details on the reconstruction of the g mode ridge.

We collect the instantaneous frequency $f(t_i)$ corresponding to the ridge $m(t_i)$ for the midpoint t_i of each

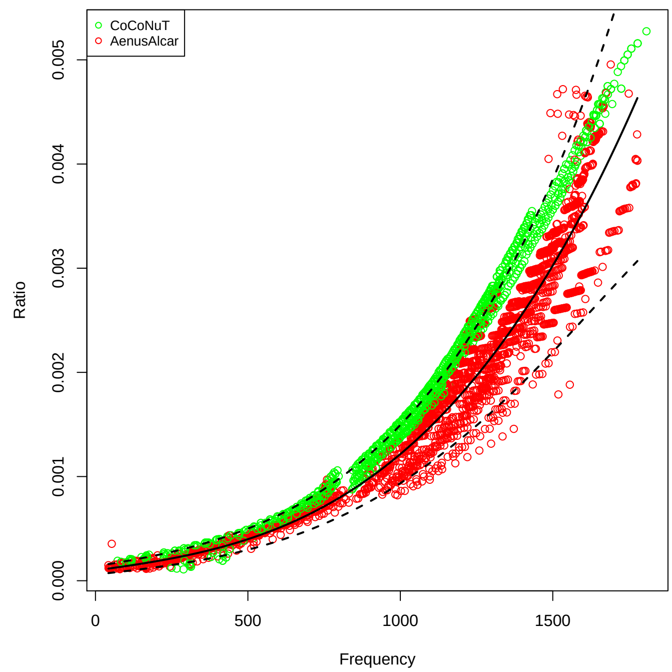

FIG. 1. Ratio $M_{\rm PNS}/R_{\rm PNS}^2$ from 25 1D simulations AENUS-ALCAR (red) and CoCoNuT (green) code. The solid line is the maximum likelihood estimate of heteroscedastic cubic model with 95% confidence bands (dashed lines) considering the AENUS-ALCAR data points.

local time interval of the spectrogram and interpolating $f(t)$ for values in between the t_i. We then use equation (1) to obtain estimates of the time evolution of the ratio together with 95% confidence intervals. An exemple is given in Figure 3 where the red points are the point estimates and the grey bands represent 95% confidence bands. Ratio values computed using the mass and radius values obtained from the simulation code are shown in black.

In this case, for a GW signal without any noise, the coverage of our 95% confidence band is 94%. In the next section we investigate the performance of reconstruction of $r(t)$ when the gravitational wave signal is embedded in noise.

IV. DETECTION SENSITIVITY WITH ADVANCED GRAVITATIONAL WAVE DETECTORS

To estimate how accurately we can infer the time evolution of $r = M_{\rm PNS}/R_{\rm PNS}^2$ in the gravitational wave detector data, we have added `s20S` GW signal to 100 Gaussian noise realisations whose power spectral density follows advanced LIGO (aLIGO) spectrum [47] shown on Figure 5. We have varied the distance to the source, covering a large range of distances for which a detection in

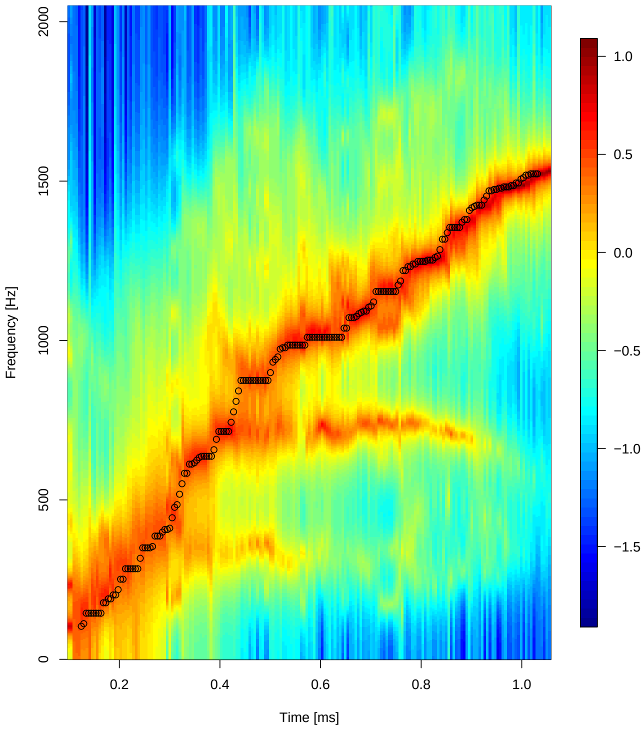

FIG. 2. Spectrogram of the gravitational wave signal `s20S` sampled at 4096 Hz. The spectrogram is obtained using data streach of 200 samples overlapping at 90% with each other.

second generation of gravitational wave detectors is feasible. The source is optimally oriented with respect to the gravitational wave detector. We are assuming a GW signal from a core collapse phenomena has been identified in the data and that the beginning of the GW signal is known within $O(10\ ms)$. The data (signal embedded in noise) are whitened using the function *prewhiten* of the R-package TSA. An auto-regressive model with maximal 100 coefficients has been used.

For each of the noise realisations, we reconstruct the ratio time series r_i of length N starting from the left side of the spectrogram and constraining the beginning of the track to be smaller than 200 Hz. The reconstructed ratio is then compare to the "true" ratio r_i^0 derived from the PNS mass and radius generated by the simulation code that produced `s20S`.

Figure 4 is showing the fraction of the ratio r_i^0 values that fall within the 95% confidence interval of r_i. This quantity, *coverage*, is taking maximal values when the source is located within few kpc and then decreases with the distance.

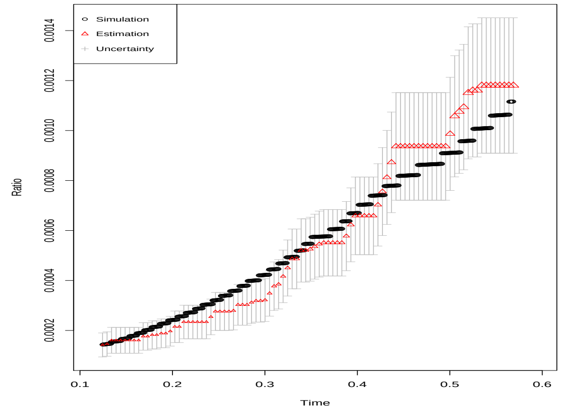

FIG. 3. Ratio $M_{\rm PNS}/R^2_{\rm PNS}$ as function of time extracted from the ${}^2g_2$-mode of the `s20S` signal (red points and the 95% confidence belt in grey) compared to the ratio value derived from the PNS mass and radius given by the simulation code (black points).

To better quantify how well we reconstruct the ratio, we have also considered Δ the mean over the track of the relative error of r_i.

$$\Delta = \frac{1}{N}\sum_1^N \frac{|r_i - r_i^0|}{r_i^0} \tag{3}$$

Δ values of each of the 100 noise realisation are shown as well as function of the distance on Figure 4. For a source located up to ∼10 kpc the relative error remains smaller than 20%. At small distance Δ is small but not null. This reflects the approximation of the model used for r. It is nevertheless remarkable that one can reconstruct the ratio time series with a good precision at distance up to ∼ 10 kpc for this particular waveform, with *coverage* value larger than 80%. We have tested that the method does not depend on features of `s20S` using 7 other waveforms described in section II covering a large range of progenitor masses. Figure 6 shows that apart `s11.2-LS220`, the ratio is well reconstructed for all waveforms up to ∼ 10kpc. On this figure we also show the *coverage* value in case of absence of signal. The median value is significantly different from 0 because the g-mode reconstruction algorithm is looking for a continuously frequency increasing track in the spectrogram, starting between 0 and 200 Hz. In Table **??** we are reporting the distance at which *coverage* median is lower than 80%.

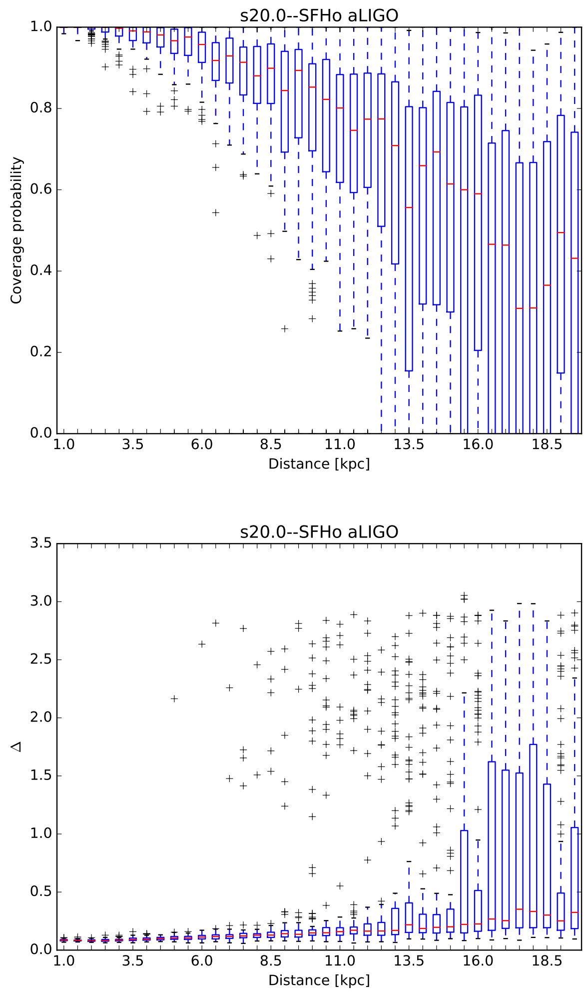

FIG. 4. Boxplots of *coverage* (upper panel) and Δ (lower panel) for s20S signal embedded in aLIGO noise at different distances from the Earth. 100 noise realisations is considered for each distance.

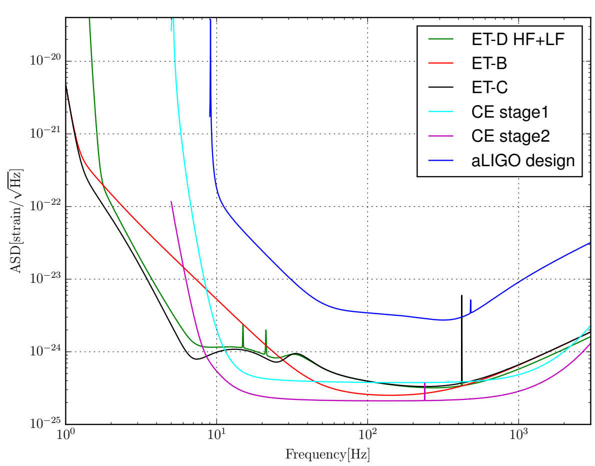

FIG. 5.

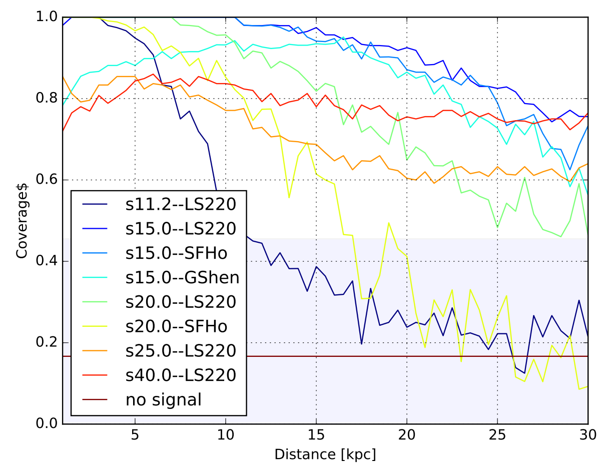

FIG. 6. Median of *coverage* for 8 CCSN waveforms embedded in aLIGO noise and located at different distance from the Earth. The "no signal" line and band show the median and first and third quartile of *coverage* in absence of any signal.

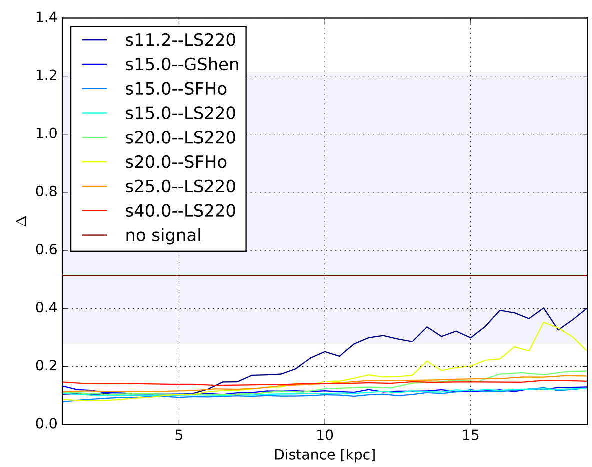

FIG. 7. Median of *Delta* for 8 CCSN waveforms embedded in aLIGO noise and located at different distance from the Earth. The "no signal" line and band show the median and first and third quartile of *Delta* in absence of any signal.

Simulation	s11	s15	s15S	s15G	s20	s20S	s25	s40
aLIGO char. distance (kpc)	7	26	25	21	16	11	9	12
SNR in aLIGO	19.5	55.4	59.0	60.0	34.3	35.8	116.5	98.5
CE2 char. distance (kpc)								
SNR in CE2								

TABLE III. Matched filter signal-to-noise ratio (SNR) of the simulated waveforms for the different GW detectors considered in this study. The source is located at 10 kpc and is optimally oriented with respect to the detector.

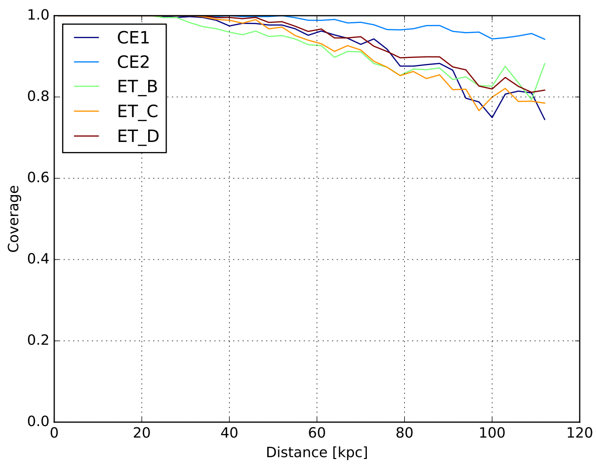

FIG. 8. Median of *coverage* for s20.0–SFHo CCSN waveform embedded in 3G detectors noise and located at different distance from the Earth.

V. DISCUSSION

Acknowledgments —

Appendix A G-MODE RECONSTRUCTION

Given the spectrogram and an specified time interval for the g-mode reconstruction, our proposal method works as follows. The starting point must be specified. It can be either at the beginning or at the end of the signal. Then, in one of these extremes, the maximum energy value is identified, registering its frequency. This is done independently for a number of consecutive time intervals. Then we calculate the median of these frequency values, providing a robust starting value for the g-mode reconstruction.

The starting frequency value is the first g-mode estimate for the first or the last time interval, depending on the specified starting location. If the reconstruction is set to start at the beginning of the signal, the reconstruction will be done progressively over the time intervals, where each maximum frequency value will be calculated within a frequency range specified by the previous g-mode estimate. Given the non-decreasing behaviour of the true g-mode values, the g-mode estimates will be forced to be greater or equal than the one estimated for its previous time interval, and lower than a specified upper limit. As a result, the g-modes estimates will be a non-decreasing sequence of frequency values.

If the reconstruction is set to start at the end of the signal, the g-modes will be estimated backward in time. Each maximum frequency is calculated within a range determined by its successor (in time) g-mode estimate. These estimates are forced to be lower or equal than its successor (in time) estimate, but greater than a specified lower limit. Thus, a non-decreasing sequence of g-mode estimates is guaranteed.

This g-mode reconstruction method works if and only if the signal is strong enough to provide information about the g-mode, which is reflected in the spectrogram.

Given the sequence of g-mode estimates, the confidence band will be calculated by using the model defined in (**??**). The g-mode estimates are frequency values which we use as predictors in the model in order to generate confidence intervals for the ratios. Since the g-mode estimates are indexed by time, the confidence intervals for the ratios are too. Thus, we generate the confidence band by interpolating the lower and upper limits of the collection of consecutive confidence intervals, which will be valid for the time range of the g-mode estimates. This confidence band is used to estimate the coverage probabilities in our simulation studies presented below.

---

[1] H. A. Bethe, "Supernova mechanisms," Rev. Mod. Phys. **62**, 801–866 (1990).

[2] S.E. Gossan, P. Sutton, A. Stuver, M. Zanolin, K. Gill, and C. Ott, "Observing gravitational waves from core-collapse supernovae in the advanced detector era," Physical Review D **93** (2016), 10.1103/physrevd.93.042002.

[3] B. P. Abbott and et al, "Optically targeted search for gravitational waves emitted by core-collapse supernovae during the first and second observing runs of advanced LIGO and advanced Virgo," Phys. Rev. D **101**, 084002 (2020), arXiv:1908.03584 [astro-ph.HE].

[4] Scott M. Adams, C. S. Kochanek, John F. Beacom, Mark R. Vagins, and K. Z. Stanek, "OBSERVING THE NEXT GALACTIC SUPERNOVA," The Astrophysical Journal **778**, 164 (2013).

[5] Karolina Rozwadowska, Francesco Vissani, and Enrico Cappellaro, "On the rate of core collapse supernovae in the milky way," New A **83**, 101498 (2021), arXiv:2009.03438 [astro-ph.HE].

[6] Weidong Li, Jesse Leaman, Ryan Chornock, Alexei V. Filippenko, Dovi Poznanski, Mohan Ganeshalingam, Xiaofeng Wang, Maryam Modjaz, Saurabh Jha, Ryan J. Foley, and Nathan Smith, "Nearby supernova rates from the Lick Observatory Supernova Search - II. The observed luminosity functions and fractions of supernovae in a complete sample," MNRAS **412**, 1441–1472 (2011), arXiv:1006.4612 [astro-ph.SR].

[7] R. Chapman, N. R. Tanvir, R. S. Priddey, and A. J. Levan, "How common are long Gamma-Ray Bursts in the Local Universe?" Mon. Not. R. Astron. Soc. **L21**, 382 (2007).

[8] Bernhard Müller, "Hydrodynamics of core-collapse supernovae and their progenitors," Living Reviews in Computational Astrophysics **6**, 3 (2020), arXiv:2006.05083 [astro-ph.SR].

[9] C. D. Ott, H. Dimmelmeier, A. Marek, H.-T. Janka, B. Zink, I. Hawke, and E. Schnetter, "Rotating collapse of stellar iron cores in general relativity," Classical and Quantum Gravity **24**, 139–+ (2007), arXiv:astro-ph/0612638.

[10] E. Abdikamalov, S. Gossan, A. M. DeMaio, and C. D. Ott, "Measuring the angular momentum distribution in core-collapse supernova progenitors with gravitational waves," Phys. Rev. D **90**, 044001 (2014), arXiv:1311.3678 [astro-ph.SR].

[11] Sherwood Richers, Christian D. Ott, Ernazar Abdikamalov, Evan O'Connor, and Chris Sullivan, "Equation of state effects on gravitational waves from rotating core collapse," Phys. Rev. D **95**, 063019 (2017), arXiv:1701.02752 [astro-ph.HE].

[12] Kei Kotake and Takami Kuroda, "Gravitational Waves from Core-Collapse Supernovae," in Handbook of Supernovae, edited by Athem W. Alsabti and Paul Murdin (2017) p. 1671.

[13] K.D. Kokkotas and B.G. Schmidt, "Quasi-normal modes of stars and black holes," Living Rev. Rel. **2**, 2 (1999).

[14] John L. Friedman and Nikolaos Stergioulas, Rotating Relativistic Stars (2013).

[15] J. W. Murphy, C. D. Ott, and A. Burrows, "A Model for Gravitational Wave Emission from Neutrino-Driven Core-Collapse Supernovae," ApJ **707**, 1173 (2009).

[16] B. Müller, H.-T. Janka, and A. Marek, "A New Multi-dimensional General Relativistic Neutrino Hydrodynamics Code of Core-collapse Supernovae. III. Gravitational Wave Signals from Supernova Explosion Models," ApJ **766**, 43 (2013), arXiv:1210.6984 [astro-ph.SR].

[17] Pablo Cerdá-Durán, Nicolas DeBrye, Miguel A. Aloy, José A. Font, and Martin Obergaulinger, "Gravitational Wave Signatures in Black Hole Forming Core Collapse," Astrophys. J. Lett. **779**, L18 (2013), arXiv:1310.8290 [astro-ph.SR].

[18] Konstantin N. Yakunin, Anthony Mezzacappa, Pedro Marronetti, Shin'ichirou Yoshida, Stephen W. Bruenn, W. Raphael Hix, Eric J. Lentz, O. E. Bronson Messer, J. Austin Harris, Eirik Endeve, John M. Blondin, and Eric J. Lingerfelt, Phys. Rev. D **92**, 084040 (2015), arXiv:1505.05824 [astro-ph.HE].

[19] Takami Kuroda, Kei Kotake, and Tomoya Takiwaki, "A New Gravitational-wave Signature from Standing Accretion Shock Instability in Supernovae," Astrophys. J. Lett. **829**, L14 (2016), arXiv:1605.09215 [astro-ph.HE].

[20] H. Andresen, B. Müller, E. Müller, and H. Th. Janka, "Gravitational wave signals from 3D neutrino hydrodynamics simulations of core-collapse supernovae," MNRAS **468**, 2032–2051 (2017), arXiv:1607.05199 [astro-ph.HE].

[21] P. N. McDermott, H. M. van Horn, and J. F. Scholl, "Nonradial g-mode oscillations of warm neutron stars," ApJ **268**, 837–848 (1983).

[22] A. Reisenegger and P. Goldreich, "A new class of g-modes in neutron stars," ApJ **395**, 240–249 (1992).

[23] V. Ferrari, L. Gualtieri, J. A. Pons, and A. Stavridis, "Gravitational waves from rotating proto-neutron stars," Classical and Quantum Gravity **21**, S515–S519 (2004), astro-ph/0409578.

[24] A. Passamonti, M. Bruni, L. Gualtieri, and C. F. Sopuerta, "Coupling of radial and nonradial oscillations of relativistic stars: Gauge-invariant formalism," Phys. Rev. D **71**, 024022 (2005), gr-qc/0407108.

[25] H. Dimmelmeier, N. Stergioulas, and J. A. Font, "Nonlinear axisymmetric pulsations of rotating relativistic stars in the conformal flatness approximation," MNRAS **368**, 1609–1630 (2006), astro-ph/0511394.

[26] C. J. Krüger, W. C. G. Ho, and N. Andersson, "Seismology of adolescent neutron stars: Accounting for thermal effects and crust elasticity," Phys. Rev. D **92**, 063009 (2015), arXiv:1402.5656 [gr-qc].

[27] G. Camelio, A. Lovato, L. Gualtieri, O. Benhar, J. A. Pons, and V. Ferrari, "Evolution of a proto-neutron star with a nuclear many-body equation of state: neutrino luminosity and gravitational wave frequencies," ArXiv e-prints (2017), arXiv:1704.01923 [astro-ph.HE].

[28] H. Sotani and T. Takiwaki, "Gravitational wave asteroseismology with protoneutron stars," Phys. Rev. D **94**, 044043 (2016), arXiv:1608.01048 [astro-ph.HE].

[29] A. Torres-Forné, P. Cerdá-Durán, A. Passamonti, and J. A. Font, "Towards asteroseismology of core-collapse supernovae with gravitational-wave observations - I. Cowling approximation," MNRAS **474**, 5272–5286 (2018), arXiv:1708.01920 [astro-ph.SR].

[30] Viktoriya Morozova, David Radice, Adam Burrows, and David Vartanyan, "The Gravitational Wave Signal from Core-collapse Supernovae," ApJ **861**, 10 (2018), arXiv:1801.01914 [astro-ph.HE].

[31] A. Torres-Forné, P. Cerdá-Durán, A. Passamonti, M. Obergaulinger, and J. A. Font, "Towards asteroseismology of core-collapse supernovae with gravitational wave observations - II. Inclusion of space-time perturbations," MNRAS **482**, 3967–3988 (2019), arXiv:1806.11366 [astro-ph.HE].

[32] A. Torres-Forné, P. Cerdá-Durán, M. Obergaulinger, B. Müller, and J. Font, "Universal relations for gravitational-wave asteroseismology of proto-neutron stars," Physical Review Letters **123**, 051102 (2019).

[33] Hajime Sotani, Takami Kuroda, Tomoya Takiwaki, and Kei Kotake, "Dependence of the outer boundary condition on protoneutron star asteroseismology with gravitational-wave signatures," Phys. Rev. D **99**, 123024 (2019), arXiv:1906.04354 [astro-ph.HE].

[34] John Ryan Westernacher-Schneider, Evan O'Connor, Erin O'Sullivan, Irene Tamborra, Meng-Ru Wu, Sean M. Couch, and Felix Malmenbeck, "Multimessenger asteroseismology of core-collapse supernovae," Phys. Rev. D **100**, 123009 (2019), arXiv:1907.01138 [astro-ph.HE].

[35] Hajime Sotani and Tomoya Takiwaki, "Dimension dependence of numerical simulations on gravitational waves from protoneutron stars," Phys. Rev. D **102**, 023028 (2020), arXiv:2004.09871 [astro-ph.HE].

[36] Hajime Sotani and Tomoya Takiwaki, "Avoided crossing in gravitational wave spectra from protoneutron star," MNRAS (2020), 10.1093/mnras/staa2597, arXiv:2008.00419 [astro-ph.HE].

[37] O. Just, M. Obergaulinger, and H.-T. Janka, "A new multidimensional, energy-dependent two-moment transport code for neutrino-hydrodynamics," MNRAS **453**, 3386–3413 (2015), arXiv:1501.02999.

[38] P. Cerdá-Durán, J. A. Font, L. Antón, and E. Müller, "A new general relativistic magnetohydrodynamics code for dynamical spacetimes," A&A **492**, 937–953 (2008),

arXiv:0804.4572.

[39] B. Müller and H. Th. Janka, "Non-radial instabilities and progenitor asphericities in core-collapse supernovae," MNRAS **448**, 2141–2174 (2015), arXiv:1409.4783 [astro-ph.SR].

[40] A. Marek, H. Dimmelmeier, H.-T. Janka, E. Müller, and R. Buras, "Exploring the relativistic regime with Newtonian hydrodynamics: an improved effective gravitational potential for supernova simulations," A&A **445**, 273–289 (2006).

[41] S. E. Woosley, A. Heger, and T. A. Weaver, "The evolution and explosion of massive stars," Reviews of Modern Physics **74**, 1015–1071 (2002).

[42] J. M. Lattimer and F. Douglas Swesty, "A generalized equation of state for hot, dense matter," Nuclear Physics A **535**, 331–376 (1991).

[43] A. W. Steiner, M. Hempel, and T. Fischer, "Core-collapse Supernova Equations of State Based on Neutron Star Observations," ApJ **774**, 17 (2013), arXiv:1207.2184 [astro-ph.SR].

[44] G. Shen, C. J. Horowitz, and S. Teige, "New equation of state for astrophysical simulations," Phys. Rev. C **83**, 035802 (2011), arXiv:1101.3715 [astro-ph.SR].

[45] S. E. Woosley and A. Heger, "Nucleosynthesis and remnants in massive stars of solar metallicity," Phys. Rep. **442**, 269–283 (2007), astro-ph/0702176.

[46] .

[47] Lisa Barsotti, Peter Fritschel, Matthew Evans, and Slawomir Gras, "Updated advanced ligo sensitivity design curve," (2018).