# Inference of proto-neutron star properties from gravitational wave data in core-collapse supernovae.

#### I. INTRODUCTION

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The life of massive stars (those born with masses be- 57 tween  $\sim 8~{\rm M}_{\odot}$  and  $\sim 120~{\rm M}_{\odot})$  ends with the collapse of  $^{58}$ the iron core under its own gravity, leading to the for-59 mation of a neutron star (NS) or a black hole (BH), and 60 followed (typically but not necessarily in the BH case) by 61 a supernova explosion. Nearby core-collapse supernova 62 (CCSN) explosions are expected to be sources of gravi- 63 tational waves (GW) and are one of the main candidates 64 for the next great discovery by current ground-based ob- 65 servatories. However, these are relative rare events. A 66 neutrino-driven explosion [1] is the most likely outcome 67 in the case of slow rotating cores, which are present in 68 the bulk of CCSN progenitors. This event could be de-69 tected with advanced ground-based GW detectors within 70 5 kpc [2, 3]. Such a galactic event has a rate of about 71 2-3 per century [4, 5]. For the case of fast rotating  $_{72}$ progenitor cores the result is likely a magneto-rotational 73 explosion, with a more powerful signal that could be de-74 tected within 50 kpc and for some extreme models up to 75 5-30 Mpc [2, 3]. However, only about 1% of the electromagnetically observed events show signatures of fast 77 rotation (broad-lined type Ic SNe [6] or events associated 78 to long GRBs [7]), making this possibility a subdominant 79 channel of detection with an event rate of  $\sim 10^{-4} {\rm yr}^{-1}$ . Therefore, we focus this work only in neutrino-driven 81 CCSNe. Despite the low rates, CCSN are of great scien-  $_{82}$ tific interest because they produce a complex GW signals 83 which could provide significant clues about the physical 84 processes that occur in the moments after the collapse. 85

In the last decade, a significant progress has been made <sup>86</sup> in the development of numerical codes, in particular in <sup>87</sup> the treatment of multidimensioal effects [8]. In the case of <sup>88</sup> neutrino-driven explosions, the GW emission is primarly <sup>89</sup> induced by instabilities developed at the newly formed <sup>90</sup> proto-neutron star (PNS) and by the non-spherical ac- <sup>91</sup> creting flow of hot matter over its surface [9]. This dy- <sup>92</sup> namics excite the different modes of oscillation of the <sup>93</sup> PNS, which ultimately leads to the emission of GWs. <sup>94</sup> The frequency and time evolution of these modes carry <sup>95</sup> information about the properties of the GW emitter and <sup>96</sup> could allow to perform PNS asteroseismology.

The main feature appearing systematically in the GW  $_{98}$  spectrum of multidimensional numerical simulations is a  $_{99}$  strong and relatively narrow feature in the post bounce  $_{100}$  evolution with raising frequency from about 100 Hz up  $_{101}$  to a few kHz (at most) and a typical duration of  $_{102}$  1 s. This feature has been interpreted as a continuously  $_{103}$  excited gravity mode (g-mode, see [10, 11] for a definition  $_{104}$  in this context) of the PNS [12–17]. In these models the  $_{105}$  monotonic raise of the frequency of the mode is related  $_{106}$  to the contraction of the PNS. The typical frequencies of  $_{107}$ 

these modes make them a promising source for ground-based interferometers (aLIGO, aVirgo, KAGRA).

The properties of g-modes in hot PNSs have been studied since the end of last century by means of linear perturbation analysis of background PNS models. The oscillation modes associated to the surface of hot PNSs was first considered by McDermott, van Horn & Scholl [18]. Additionally, the stratified structure of the PNS allows the presence of different types of g-modes related with the fluid core [19]. Many posterior works used simplified neutron star models assuming an equilibrium configuration as a background, to study the effect of rotation [20], general relativity [21], non-linearities [22], phase transition [23] and realistic equation of state [24]. Only recently, there has been an effort to incorporate realistic backgrounds based in numerical simulations in the computation of the mode structure and evolution [25–33].

We base this work in the PNS mode analysis performed by [26, 28], which explored the eigenmode spectrum of the region within the shock (including the PNS and the post-shock region) using results from 2D CCSN numerical simulations as a background. Their results show a good match of the mode frequencies computed and the features observed in the GW spectrum of the same simulation (specially when space-time perturbations are included [28]). This result reveals that it is possible to perform CCSN asteroseismology under realistic conditions and serves as a starting point to carry out inference of astrophysical parameters of PNSs. [29] went one step further showing that it was possible to derive simple relations between the instantaneous frequency of the g-mode and the mass and radius of the PNS at each time of the evolution. These relations are universal in the sense that they do not depend on the equation of state (EOS) used or the mass of the progenitor, and only weakly on the numerical code used (see discussion in section II). Similar relations have been found by [32, 33], which also found that the universal relations do not depend on the dimensionality (1D, 2D or 3D) of the numerical simulation used as a background.

In this work, we present a method to infer from the GW data alone, the time evolution of some properties or the PNS, namely a combination of its mass and radius. For this purpose we have developed an algorithm to extract the time-frequency evolution of the main feature in the spectrograms of the GW emission of 2D simulations of CCSN. This feature corresponds to the  $^2$ g<sub>2</sub> mode, according to the nomenclature used in [29] (different authors may have slightly different naming convention). Next, we use the universal relations obtained by [29], based on a set of 1D simulations, to infer the time evolution of the ratio  $M_{\rm PNS}/R_{\rm PNS}^2$ , being  $M_{\rm PNS}$  and  $R_{\rm PNS}$  the mass and radius of the PNS. Using 2D CCSN waveform corre-

sponding to different progenitor masses we estimate the performance of the algorithm for current and future generation of ground-based GW detectors.

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This paper is organised as follows. Section II describes the details of the CCSN simulations used in the paper. Section III focuses on the algorithm that extracts the time evolution of a combination of the mass and radius of the PNS corresponding to a g-mode. Section IV shows the performance of the data analysis method with simulated GW detectors data. Finally, we discuss the results in section V.

#### II. CORE COLLAPSE SUPERNOVA SIMULATIONS

Unless other methods used GW astronomy, the algorithm proposed in this work does not require accu-162 rate waveforms in order to infer the properties of the 163 PNS. Instead, it relies on the evolution of the frequency 164 of oscillation of some particular modes, as seen in the 165 GW spectrum. The frequency of these modes depends, 166 in a universal way, on the surface gravity of the PNS<sub>167</sub>  $(r = M_{\rm PNS}/R_{\rm PNS}^2)$ , in the sense that if at a given time we observe GW emission at a certain frequency f we can de-169 termine univocally the value of the surface gravity, within 170 a certain error, regardless of the details of the numerical 171 simulation. In this work we use two sets of simulations:172 i) The model set, composed by 1D simulations, which is173 used to build the universal relation (model), r(f), link-174 ing the ratio r with the observed frequency f, and ii) the 175 test set, composed by 2D simulations, for which we know 176 both the GW signal and the evolution of the ratio, r(t), 177 and that is used to test performance of the algorithm.

We have used two different numerical codes in our nu-179 merical simulations. CoCoNuT [34, 35] is a code for 180 general relativistic hydrodynamics coupled to the Fast 181 Multigroup Transport scheme [36] providing an approxi-182 mate description of the emission and transport of neutri-183 nos. AENUS-ALCAR [37] combines special relativistic 184 (magneto-)hydrodynamics, a modified Newtonian grav-185 itational potential approximating the effects of general 186 relativity [38], and a spectral two-moment neutrino trans-187 port solver [37]. We included the relevant reactions be-188 tween matter and neutrinos of all flavours, i.e., emission 189 and absorption by nucleons and nuclei, electron-positron 190 pair annihilation, nucleonic bremsstrahlung, and scatter-191 ing off nucleons, nuclei, and electrons.

For the model set, we use the 25 spherically symmet-<sup>193</sup> ric (1D) simulations of [28] including progenitors with<sup>194</sup> zero-age main sequence (ZAMS) masses in the range<sup>195</sup>  $M_{\rm ZAMS} = 11.2 - 75\,M_{\odot}$ . The set contains simulations<sup>196</sup> using the two numerical codes and six different equations<sup>197</sup> of state. Details can be found in [28]. The reason to use<sup>198</sup> one dimensional simulations for the model set is that the<sup>199</sup> computational cost of those is significantly smaller than<sup>200</sup> the cost of multidimensional simulations, so is easier to<sup>201</sup>

Model	$M_{ m ZAMS}$	progenitor	EOS	$t_{ m f}$	$t_{\rm explosion}$	$M_{ m PNS,f}$
name	$[M_{\odot}]$	model		[s]		$[M_{\odot}]$
s11	11.2	[39]	LS220	1.86	×	1.47
s15	15.0	[39]	LS220	1.66	×	2.00
s15S	15.0	[39]	SFHo	1.75	×	2.02
s15G	15.0	[39]	$\operatorname{GShen}$	0.97	×	1.86
s20	20.0	[39]	LS220	1.53	×	1.75
s20S	20.0	[40]	SFHo	0.87	×	2.05
s25	25.0	[39]	LS220	1.60	0.91	2.33
s40	40.0	[39]	LS220	1.70	1.52	2.23

TABLE I. List of axisymmetric simulations used for the *test set*. The last three columns show, the post-bounce time at the end of the simulation, the one at the onset of the explosion (non exploding models marked with  $\times$ ), and the PNS mass at the end of the simulation.

accumulate the statistics necessary to build a good model for r(f). For each time of each simulation we compute the ratio r and the frequency of the  $^2g_2$  mode by means of the linear analysis described in [26, 28, 29].

For the test set, we use 8 axisymmetric (2D) simulations using the AENUS-ALCAR code (see Table I for a list of models). 7 of these simulations use a selection of progenitors with masses in the range  $M_{\rm ZAMS} =$  $11.2-40\,M_{\odot}$  evolved through the hydrostatic phases by [39]. We performed one simulation of each stellar model using the equation of state of [41] with an incompressibility of  $K = 220 \,\mathrm{MeV}$  (LS220) and added comparison simulations with the SFHo EOS [42] and the GShen EOS [43] for the progenitor with  $M_{\rm ZAMS} = 15 \, M_{\odot}$ . To this set of simulations, we add the waveform of a two-dimensional model used in [28], denoted s20S. It corresponds to a star with the same initial mass,  $M_{\rm ZAMS} = 20 M_{\odot}$ , as for one of the other 7 axisymmetric simulations, but was taken from a newer set of stellar-evolution models [40]. It was evolved with the SFHo EOS.

For all the simulations, we mapped the pre-collapse state of the stars to a spherical coordinate system with  $n_r=400$  zones in radial direction distributed logarithmically with a minimum grid width of  $(\Delta r)_{\rm min}=400\,{\rm m}$  and an outer radius of  $r_{\rm max}=8.3\times10^9\,{\rm cm}$  and  $n_\theta=128$  equidistant cells in angular direction. For the neutrino energies, we used a logarithmic grid with  $n_e=10$  bins up to 240 MeV. Unlike the model set, the simulations in the test set are not 1D because we need to extract the gravitational wave signal, which is a multi-dimensional effect. For each simulation we extract the gravitational wave signal,  $h_+(t)$ , by means of the quadrupole formula and compute the time evolution of the surface gravity, r(t).

All spherical and most axisymmetric models fail to achieve shock revival during the time of our simulations. Only the two stars with the highest masses,  $\mathfrak{s}25$  and  $\mathfrak{s}40$ , develop relatively late explosions in axisymmetry. Consequently, mass accretion onto the PNSs proceeds at high rates for a long time in all cases and

causes them to oscillate with their characteristic frequencies. The final masses of the PNSs are in the range of  $M_{\rm PNS}=1.47-2.33\,M_{\odot}$ , i.e., insufficient for producing a black hole.

## III. METHODS DESCRIPTION

In this section, we outline a strategy for estimating the time evolution of the ratio  $r = M_{\rm PNS}/R_{\rm PNS}^2$  of the mass of the PNS and its squared radius (in units of solar mass and km) from the observation of the  $^2g_2$  oscillation mode in the gravitational wave detector data. An integral part of this strategy is the universal relations that relate the characteristic frequency of the PNS oscillation f, g and p modes with the mass and the radius of the PNS, the shock radius and the total mass inside the shock as demonstrated in [29].

To build the model of the ratio r as a function of the frequency f we use the spherically symetric (1D) simulations of the  $model\ set$ . Figure 1 shows the data for the 25 numerical simulations. As identified by [29], the only systematic deviation from a single universal relation is the numerical code used in the simulations. To avoid any systematic effect, we only use the 18 simulations performed with the ALCAR-AENUS code, which is the same code that was used in our test set. The consequences of this choice are discussed in the conclusions. Using this data, we parametrize the ratio with a cubic polynomial regression with heteroscedastic errors

$$r_{i} = \beta_{1} f_{i} + \beta_{2} f_{i}^{2} + \beta_{3} f_{i}^{3} + \epsilon_{i}$$
 (1)

where  $\epsilon_i$  are assumed to be independent zero-mean Gaussian errors with variances  $\sigma_i^2$  that increase with frequency  $f_i$ . The model for frequency-dependent variances is

$$\log \sigma_i = \alpha_0 + \alpha_1 f_i + \alpha_2 f_i^2 + \delta_i \tag{2}$$

with independent and identically zero-mean Gaussian errors  $\delta_i$ . The R-package lmvar [44] that implements a maximum likelihood approach was used to fit the model.<sup>250</sup>

The best fitting model amongst polynomials of degree<sub>251</sub> 1, 2, and 3 was chosen according to the Aikaike informa-<sub>252</sub> tion criterion with coefficients given in Table II, which is<sub>253</sub> actually the model defined in (1). The data and fit of the<sub>254</sub> model including 95% confidence bands are displayed in<sub>255</sub> Figure 1.

We use this model to infer the properties of the simula-257 tions in the *test set* described in in Section II. To describe258 the method we focus on the gravitational wave signal of 259  $\pm$  208, originally sampled at 16384 Hz but resampled at 260 4096 Hz. A spectrogram of this signal is shown in Fig-261 ure 2 based on autoregressive estimates [CITATION?]of 262 the local spectra for successive time intervals of length 263 200 with a 90% overlap. The dominant emission mode 264 corresponds to the PNS oscillation  $2g_2$ -mode. We have 265

Coefficient	Estimate	Standard error
$\beta_1$	$1.00 \times 10^{-06}$	$2.12 \times 10^{-08}$
$eta_2$	$-8.22 \times 10^{-10}$	$5.00 \times 10^{-11}$
$eta_3$	$1.01 \times 10^{-12}$	$2.70 \times 10^{-14}$
$lpha_0$	$-1.02 \times 10^{+01}$	$6.80 \times 10^{-02}$
$\alpha_1$	$7.24 \times 10^{-04}$	$1.56 \times 10^{-04}$
$\alpha_2$	$6.23 \times 10^{-07}$	$8.15 \times 10^{-08}$

TABLE II. Estimate and standard error of the coefficients of the best fit model describing the ratio  $r = M_{\rm PNS}/R_{\rm PNS}^2$  as function of the frequency of the  $^2g_2$  mode.

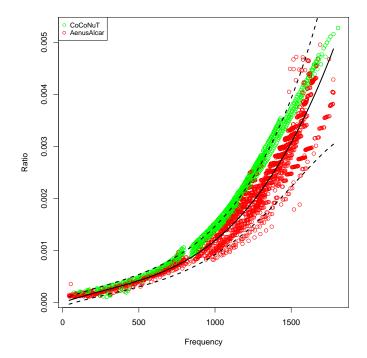
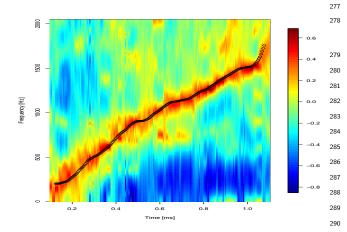


FIG. 1. Ratio  $M_{\rm PNS}/R_{\rm PNS}^2$  from 25 1D simulations AENUS-ALCAR (red) and CoCoNuT (green) code. The solid line is the maximum likelihood estimate of heteroscedastic cubic model with 95% confidence bands (dashed lines) considering only the AENUS-ALCAR data points (18 simulations).

developed a time-frequency method to track the ridge m(t) in the spectrogram, taking into account that it is monotonically increasing as time goes. This is a property of the  $^2g_2$ -mode, the frequency of which increases as the object becomes more massive and compact. Starting from either the left- or right-most column of the time-frequency matrix we identify and trace the sequence of amplitude peaks within a certain frequency band given the monotonicity constraint. Appendix  $\bf A$  is providing more details on the reconstruction of the g mode ridge.

We collect the instantaneous frequency  $f(t_i)$  corresponding to the ridge  $m(t_i)$  for the midpoint  $t_i$  of each local time interval of the spectrogram and interpolating f(t) for values in between the  $t_i$ . We then use our model given by Eq. (1) to obtain estimates of the time evolution of the ratio together with 95% confidence intervals.



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FIG. 2. Spectrogram of the gravitational wave signal  $s20S^{293}$  sampled at 4096 Hz. The spectrogram is obtained using data<sup>294</sup> streach of 200 samples overlapping at 90% with each other. <sup>295</sup>

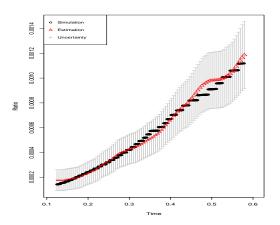


FIG. 3. Ratio  $M_{\rm PNS}/R_{\rm PNS}^2$  as function of time extracted from the  $^2g_2$ -mode of the s20S signal (red points and the 95% con-311 fidence belt in grey) compared to the ratio value derived from the PNS mass and radius given by the simulation code (black points) To be redone in python.

An exemple is given in Figure 3 where the red points are<sub>317</sub> the point estimates and the grey bands represent 95%<sub>318</sub> confidence bands. Ratio values computed using the mass<sub>319</sub> and radius values obtained from the simulation code (true<sub>320</sub> values) are shown in black.

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In this case, for a GW signal without any noise, the  $^{322}$  coverage of our 95% confidence band is 94% of the true  $^{323}$  values. In the next section we investigate the perfor- $^{324}$  mance of the reconstruction of r(t) when the gravita- $^{325}$  tional wave signal is embedded in noise.

## IV. DETECTION SENSITIVITY WITH ADVANCED GRAVITATIONAL WAVE DETECTORS

To estimate how accurately we can infer the time evolution of  $r = M_{PNS}/R_{PNS}^2$  in the gravitational wave detector data, we have added the GW signal from \$20S to 100 Gaussian noise realisations whose power spectral density follows the advanced LIGO (aLIGO) spectrum [45] shown on Figure 5. We have varied the distance to the source, covering a large range of distances for which a detection in second generation of gravitational wave detectors is feasible. The source is optimally oriented with respect to the gravitational wave detector. We are assuming a GW signal from a core collapse phenomena has been identified in the data and that the beginning of the GW signal is known within O(10 ms). The data (signal embedded in noise) are whitened using the function prewhiten of the R-package TSA. An auto-regressive model with maximal 100 coefficients has been used.

For each of the noise realisations, we reconstruct the ratio time series  $r_i$  of length N starting from the left side of the spectrogram and constraining the beginning of the track to be smaller than 200 Hz. The reconstructed ratio is then compare to the "true" ratio  $r_i^0$  derived from the PNS mass and radius computed from the \$20S simulation.

Figure 4 is showing the distribution of the fraction of the ratio  $r_i^0$  values that fall within the 95% confidence interval of  $r_i$ . This quantity, coverage, is taking maximal values when the source is located within few kpc and then decreases with the distance.

To better quantify how well we reconstruct the ratio, we have also considered  $\Delta$  the mean over the track of the relative error of  $r_i$ .

$$\Delta = \frac{1}{N} \sum_{i=1}^{N} \frac{|r_i - r_i^0|}{r_i^0} \tag{3}$$

 $\Delta$  values of each of the 100 noise realisation are shown as well as function of the distance on Figure 4. For a source located up to  $\sim 10 \,\mathrm{kpc}$  the relative error remains smaller than 20% [PCD: should we comment on the outliers?]. At small distance  $\Delta$  is small but not null. This reflects the approximation of the model used for r. It is nevertheless remarkable that one can reconstruct the ratio time series with a good precision at distance up to  $\sim 9$  kpc for this particular waveform, with coverage value larger than 80%. We have tested that the method does not depend on features of \$20S using the 7 other waveforms of the test set described in section II covering a large range of progenitor masses. Figure 6 shows that apart s11.2-LS220, the ratio is well reconstructed for all waveforms up to  $\sim$  9kpc. To measure the level of contamination of *coverage* by reconstructed ratios not associated to a signal, we have run 100 simulations without injecting a signal. The median of coverage as well

as the 95 quartile are shown on Figure 6. The noise only median value is significantly different from zero because the g-mode reconstruction algorithm is looking for a continuously frequency increasing track in the spectrogram, starting between 0 and 200 Hz. Outliers can also reach values as high as 80%. Figure 7 shows  $\Delta$  as function of the distance for the same signals as well as the result when only noise is considered. In Table III we are reporting the distance at which coverage median is lower than 95% of the noise only values. These numbers are an estimate of the order of magnitude of the source maximal distance at which a reconstruction of the ratio could be possible with current and future generations of gravitational wave detectors. We have checked that coverage and  $\Delta$  provide similar values.

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The same analysis has been performed using expected sensitivity curves for the third generation of gravitational wave detectors. The US based project Cosmic Explorer [] is proposing to reach out its design sensitivity circa 2040 through two phases labeled CE1 and CE2 also shown in Figure 5. In Europe, the Einstein Telescope is composed of three nested interferometers in a configuration of an equilateral triange [].

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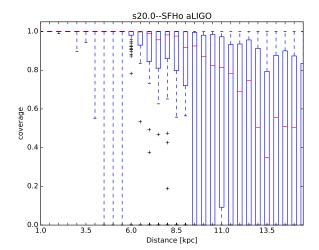
## V. DISCUSSION

Acknowledgments —

# Appendix A G-MODE RECONSTRUCTION

Given the spectrogram and an specified time interval for the g-mode reconstruction, our proposal method works as follows. The starting point must be specified. It can be either at the beginning or at the end of the signal. Then, in one of these extremes, the maximum energy value is identified, registering its frequency. This is done independently for a number of consecutive time intervals. Then we calculate the median of these frequency values, providing a robust starting value for the g-mode reconstruction.

The starting frequency value is the first g-mode esti-<sup>379</sup> mate for the first or the last time interval, depending on <sup>380</sup> the specified starting location. If the reconstruction is set <sup>381</sup> to start at the beginning of the signal, the reconstruction <sup>382</sup> will be done progressively over the time intervals, where <sup>383</sup> each maximum frequency value will be calculated within <sup>384</sup> a frequency range specified by the previous g-mode es-<sup>385</sup> timate. Given the non-decreasing behaviour of the true <sup>386</sup> g-mode values, the g-mode estimates will be forced to be <sup>387</sup> greater or equal than the one estimated for its previous <sup>388</sup> time interval, and lower than a specified upper limit. As <sup>389</sup>



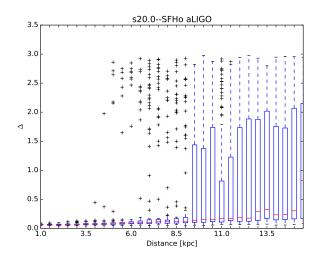


FIG. 4. Boxplots of coverage (upper panel) and  $\Delta$  (lower panel) for s20S signal embedded in aLIGO noise at different distances from the Earth. 100 noise realisations is considered for each distance. [PCD: I guess you use "coverage probability" because we are showing the box plots. Shouln't we say  $\Delta$  probability as well? Or just "coverage"? Also, how are outliers computed? Why are there so many outliers in |delta? If they are a significant fraction of the data points why are they considered outliers?]

a result, the g-modes estimates will be a non-decreasing sequence of frequency values.

If the reconstruction is set to start at the end of the signal, the g-modes will be estimated backward in time. Each maximum frequency is calculated within a range determined by its successor (in time) g-mode estimate. These estimates are forced to be lower or equal than its successor (in time) estimate, but greater than a specified lower limit. Thus, a non-decreasing sequence of g-mode estimates is guaranteed.

This g-mode reconstruction method works if and only if the signal is strong enough to provide information about the g-mode, which is reflected in the spectrogram.

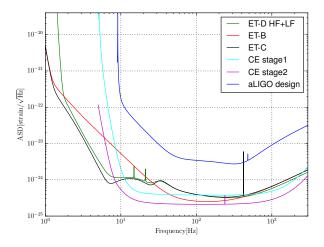


FIG. 5.

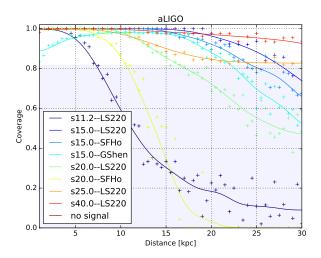


FIG. 6. Median of *coverage* for 8 CCSN waveforms embedded in aLIGO noise and located at different distance from the Earth. The "no signal" line and band show the median and first and third quartile of *coverage* in absence of any signal.

[1] H. A. Bethe, "Supernova mechanisms," Rev. Mod. Phys. 416
 62, 801–866 (1990).

- [2] S.E. Gossan, P. Sutton, A. Stuver, M. Zanolin, K. Gill, 418 and C. Ott, "Observing gravitational waves from core-419 collapse supernovae in the advanced detector era," Phys-420 ical Review D 93 (2016), 10.1103/physrevd.93.042002. 421
- [3] B. P. Abbott and et al, "Optically targeted search for 422 gravitational waves emitted by core-collapse supernovae 423 during the first and second observing runs of advanced 424 LIGO and advanced Virgo," Phys. Rev. D 101, 084002425 (2020), arXiv:1908.03584 [astro-ph.HE].
- [4] Scott M. Adams, C. S. Kochanek, John F. Bea-427 com, Mark R. Vagins, and K. Z. Stanek, "Observing428 the Next Galactic Supernova," ApJ 778, 164 (2013),

Given the sequence of g-mode estimates, the confidence band will be calculated by using the model defined in (1). The g-mode estimates are frequency values which we use as predictors in the model in order to generate confidence intervals for the ratios. Since the g-mode estimates are indexed by time, the confidence intervals for the ratios are too. Thus, we generate the confidence band by interpolating the lower and upper limits of the collection of consecutive confidence intervals, which will be valid for the time range of the g-mode estimates. This confidence band is used to estimate the coverage probabilities in our simulation studies presented below above.

arXiv:1306.0559 [astro-ph.HE].

- [5] Karolina Rozwadowska, Francesco Vissani, and Enrico Cappellaro, "On the rate of core collapse supernovae in the milky way," New A 83, 101498 (2021), arXiv:2009.03438 [astro-ph.HE].
- [6] Weidong Li, Jesse Leaman, Ryan Chornock, Alexei V. Filippenko, Dovi Poznanski, Mohan Ganeshalingam, Xiaofeng Wang, Maryam Modjaz, Saurabh Jha, Ryan J. Foley, and Nathan Smith, "Nearby supernova rates from the Lick Observatory Supernova Search II. The observed luminosity functions and fractions of supernovae in a complete sample," MNRAS 412, 1441–1472 (2011), arXiv:1006.4612 [astro-ph.SR].

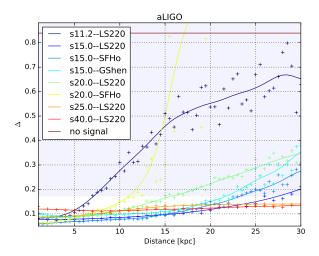


FIG. 7. Median of Delta for 8 CCSN waveforms embedded in aLIGO noise and located at different distance from the Earth. The "no signal" line and band show the median and first and third quartile of Delta in absence of any signal.

		s11	s15	s15S	s15G	s20	s20S	s25	s40
aLIGO	$d_r$	7	27	24	22	16	10	37	45
	$d_{det}$	11	36	26	27	21	16	74	61
	SNR	14	46	33	35	27	21	96	80
ADV	$d_r$	6	25	20	19	14	10	43	42
	$d_{det}$ SNR								
CE1	$d_r$	79	303	257	229	186	115	523	490
	$d_{det}$	115	377	270	282	217	168	774	633
	SNR	149	490	352	366	282	218	1006	822
CE2	$d_r$	135	518	464	405	305	183	391	700
	$d_{det}$	197	649	468	489	375	294	1347	1100
	SNR	256	843	608	635	487	382	1751	1430
ET_B	$d_r$	71	293	248	245	158	123	113	392
	$d_{det}$	106	364	274	391	216	200	805	665
	SNR	138	473	356	379	381	260	1046	865
ET_C	$d_r$	75	302	239	237	172	131	239	446
	$d_{det}$	97	332	246	260	194	164	727	603
	SNR	126	432	320	338	252	213	945	783
ET_D	$d_r$	83	329	257	261	186	139	369	523
	$d_{det}$	107	368	271	285	213	174	796	661
	SNR	140	477	352	371	277	227	1034	859

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TABLE III. Maximal distance  $d_{max}$  at which the ratio  $r=^{438}$   $M_{\rm PNS}/R_{\rm PNS}^2$  is reconstructed with good accuracy for a source<sup>439</sup> optimally oreinted with respect to the GW detectors consid-<sup>440</sup> ered in this study. Matched filter signal-to-noise ratio (SNR)<sup>441</sup> of the simulated waveforms for the different GW detectors.<sup>442</sup> The source is located at 10 kpc and is optimally oriented with<sup>443</sup> respect to the detector.

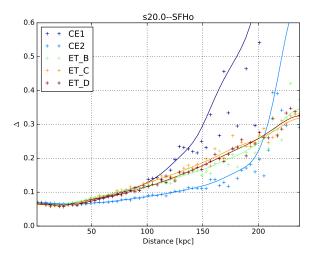


FIG. 8. Median of coverage for s20.0–SFHo CCSN waveform embedded in 3G detectors noise and located at different distance from the Earth.

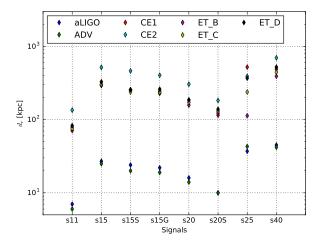


FIG. 9.

- [7] Robert Chapman, Nial R. Tanvir, Robert S. Priddey, and Andrew J. Levan, "How common are long gammaray bursts in the local Universe?" MNRAS 382, L21–L25 (2007), arXiv:0708.2106 [astro-ph].
- [8] Bernhard Müller, "Hydrodynamics of core-collapse supernovae and their progenitors," Living Reviews in Computational Astrophysics 6, 3 (2020), arXiv:2006.05083 [astro-ph.SR].
- [9] Kei Kotake and Takami Kuroda, "Gravitational Waves from Core-Collapse Supernovae," in Handbook of Supernovae, edited by Athem W. Alsabti and Paul Murdin (2017) p. 1671.
- [10] K.D. Kokkotas and B.G. Schmidt, "Quasi-normal modes of stars and black holes," Living Rev. Rel. 2, 2 (1999).
- [11] John L. Friedman and Nikolaos Stergioulas, Rotating Relativistic Stars (2013).
- [12] J. W. Murphy, C. D. Ott, and A. Burrows, "A Model for Gravitational Wave Emission from Neutrino-Driven

Core-Collapse Supernovae," ApJ **707**, 1173 (2009).

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- [13] B. Müller, H.-T. Janka, and A. Marek, "A New Multi-511 dimensional General Relativistic Neutrino Hydrodynam-512 ics Code of Core-collapse Supernovae. III. Gravitational513 Wave Signals from Supernova Explosion Models," ApJ514 766, 43 (2013), arXiv:1210.6984 [astro-ph.SR].
- [14] Pablo Cerdá-Durán, Nicolas DeBrye, Miguel A. Aloy,516 José A. Font, and Martin Obergaulinger, "Gravitational517 Wave Signatures in Black Hole Forming Core Collapse,"518 Astrophys. J. Lett. 779, L18 (2013), arXiv:1310.8290519 [astro-ph.SR].
- Konstantin N. Yakunin, Anthony Mezzacappa, Pedros21
   Marronetti, Shin'ichirou Yoshida, Stephen W. Bruenn,522
   W. Raphael Hix, Eric J. Lentz, O. E. Bronson Messer,523
   J. Austin Harris, Eirik Endeve, John M. Blondin, and524
   Eric J. Lingerfelt, Phys. Rev. D 92, 084040 (2015),525
   arXiv:1505.05824 [astro-ph.HE].
- [16] Takami Kuroda, Kei Kotake, and Tomoya Takiwaki, "A<sub>527</sub> New Gravitational-wave Signature from Standing Accre-<sub>528</sub> tion Shock Instability in Supernovae," Astrophys. J. Lett.<sub>529</sub> 829, L14 (2016), arXiv:1605.09215 [astro-ph.HE]. <sub>530</sub>
- [17] H. Andresen, B. Müller, E. Müller, and H. Th. Janka, 531 "Gravitational wave signals from 3D neutrino hydrody-532 namics simulations of core-collapse supernovae," MNRAS 533 468, 2032–2051 (2017), arXiv:1607.05199 [astro-ph.HE]. 534
- [18] P. N. McDermott, H. M. van Horn, and J. F. Scholl, 535
   "Nonradial g-mode oscillations of warm neutron stars," 536
   ApJ 268, 837–848 (1983).
- [19] A. Reisenegger and P. Goldreich, "A new class of g-modes538 in neutron stars," ApJ 395, 240-249 (1992).
- [20] V. Ferrari, L. Gualtieri, J. A. Pons, and A. Stavridis,<sup>540</sup>
   "Gravitational waves from rotating proto-neutron stars,"<sup>541</sup>
   Classical and Quantum Gravity 21, S515-S519 (2004),<sup>542</sup>
   astro-ph/0409578.
- [21] A. Passamonti, M. Bruni, L. Gualtieri, and C. F.544 Sopuerta, "Coupling of radial and nonradial oscilla-545 tions of relativistic stars: Gauge-invariant formalism," 546 Phys. Rev. D 71, 024022 (2005), gr-qc/0407108.
- [22] H. Dimmelmeier, N. Stergioulas, and J. A. Font, "Non-548 linear axisymmetric pulsations of rotating relativistics49 stars in the conformal flatness approximation," MNRAS550 368, 1609–1630 (2006), astro-ph/0511394.
- [23] C. J. Krüger, W. C. G. Ho, and N. Andersson, "Seis-552 mology of adolescent neutron stars: Accounting for ther-553 mal effects and crust elasticity," Phys. Rev. D 92, 063009554 (2015), arXiv:1402.5656 [gr-qc].
- [24] G. Camelio, A. Lovato, L. Gualtieri, O. Benhar, J. A.556 Pons, and V. Ferrari, "Evolution of a proto-neutron star557 with a nuclear many-body equation of state: neutrino558 luminosity and gravitational wave frequencies," ArXiv e-559 prints (2017), arXiv:1704.01923 [astro-ph.HE]. 560
- [25] H. Sotani and T. Takiwaki, "Gravitational wave astero-561 seismology with protoneutron stars," Phys. Rev. D 94,562 044043 (2016), arXiv:1608.01048 [astro-ph.HE].
- [26] A. Torres-Forné, P. Cerdá-Durán, A. Passamonti, and J. A. Font, "Towards asteroseismology of core-collapse su-565 pernovae with gravitational-wave observations I. Cowl-566 ing approximation," MNRAS 474, 5272-5286 (2018),567 arXiv:1708.01920 [astro-ph.SR].
- [27] Viktoriya Morozova, David Radice, Adam Burrows, 569 and David Vartanyan, "The Gravitational Wave Signals 70 from Core-collapse Supernovae," ApJ 861, 10 (2018), 571 arXiv:1801.01914 [astro-ph.HE].

573

- [28] A. Torres-Forné, P. Cerdá-Durán, A. Passamonti, M. Obergaulinger, and J. A. Font, "Towards asteroseismology of core-collapse supernovae with gravitational wave observations - II. Inclusion of spacetime perturbations," MNRAS 482, 3967–3988 (2019), arXiv:1806.11366 [astro-ph.HE].
- [29] A. Torres-Forné, P. Cerdá-Durán, M. Obergaulinger, B. Müller, and J. Font, "Universal relations for gravitational-wave asteroseismology of proto-neutron stars," Physical Review Letters 123, 051102 (2019).
- [30] Hajime Sotani, Takami Kuroda, Tomoya Takiwaki, and Kei Kotake, "Dependence of the outer boundary condition on protoneutron star asteroseismology with gravitational-wave signatures," Phys. Rev. D 99, 123024 (2019), arXiv:1906.04354 [astro-ph.HE].
- [31] John Ryan Westernacher-Schneider, Evan O'Connor, Erin O'Sullivan, Irene Tamborra, Meng-Ru Wu, Sean M. Couch, and Felix Malmenbeck, "Multimessenger asteroseismology of core-collapse supernovae," Phys. Rev. D 100, 123009 (2019), arXiv:1907.01138 [astro-ph.HE].
- [32] Hajime Sotani and Tomoya Takiwaki, "Dimension dependence of numerical simulations on gravitational waves from protoneutron stars," Phys. Rev. D 102, 023028 (2020), arXiv:2004.09871 [astro-ph.HE].
- [33] Hajime Sotani and Tomoya Takiwaki, "Avoided crossing in gravitational wave spectra from protoneutron star," MNRAS (2020), 10.1093/mnras/staa2597, arXiv:2008.00419 [astro-ph.HE].
- [34] H. Dimmelmeier, J. A. Font, and E. Müller, "Relativistic simulations of rotational core collapse I. Methods, initial models, and code tests," A&A 388, 917–935 (2002), arXiv:astro-ph/0204288 [astro-ph].
- [35] Harald Dimmelmeier, Jérôme Novak, José A. Font, José M. Ibáñez, and Ewald Müller, "Combining spectral and shock-capturing methods: A new numerical approach for 3D relativistic core collapse simulations," Phys. Rev. D 71, 064023 (2005), arXiv:astro-ph/0407174 [astro-ph].
- [36] B. Müller and H. Th. Janka, "Non-radial instabilities and progenitor asphericities in core-collapse supernovae," MNRAS 448, 2141–2174 (2015), arXiv:1409.4783 [astro-ph.SR].
- [37] O. Just, M. Obergaulinger, and H.-T. Janka, "A new multidimensional, energy-dependent two-moment transport code for neutrino-hydrodynamics," MNRAS 453, 3386–3413 (2015), arXiv:1501.02999.
- [38] A. Marek, H. Dimmelmeier, H.-T. Janka, E. Müller, and R. Buras, "Exploring the relativistic regime with Newtonian hydrodynamics: an improved effective gravitational potential for supernova simulations," A&A 445, 273–289 (2006).
- [39] S. E. Woosley, A. Heger, and T. A. Weaver, "The evolution and explosion of massive stars," Reviews of Modern Physics 74, 1015–1071 (2002).
- [40] S. E. Woosley and A. Heger, "Nucleosynthesis and remnants in massive stars of solar metallicity," Phys. Rep. 442, 269–283 (2007), astro-ph/0702176.
- [41] J. M. Lattimer and F. Douglas Swesty, "A generalized equation of state for hot, dense matter," Nuclear Physics A 535, 331–376 (1991).
- [42] A. W. Steiner, M. Hempel, and T. Fischer, "Corecollapse Supernova Equations of State Based on Neutron Star Observations," ApJ 774, 17 (2013), arXiv:1207.2184 [astro-ph.SR].

- 574 [43] G. Shen, C. J. Horowitz, and S. Teige, "New equations78 575 of state for astrophysical simulations," Phys. Rev. C **83**,579 576 035802 (2011), arXiv:1101.3715 [astro-ph.SR]. 580 577 [44] .
- [45] Lisa Barsotti, Peter Fritschel, Matthew Evans, and Slawomir Gras, "Updated advanced ligo sensitivity design curve," (2018).