Identification of protoneutron star g-modes in gravitational-wave data

I. INTRODUCTION

II. METHODS DESCRIPTION

In this section, we outline a strategy for estimating the time evolution of the ratio $r = M_{\mathrm{PNS}}/R^2_{\mathrm{PNS}}$ of the mass of the proto-neutron star (PNS) and its squared radius (units, solar Mass and km?) based on the CCSN gravitational wave observations. An integral part of this strategy is the universal relationship between the characteristic frequency and the ratio of mass and radius as demonstrated by [1] on 25 1D simulations with the AENUS-ALCAR code [] and the CoCoNuT [] code. Here we are using the data from only AENUS-ALCAR or both?, maybe colour-code the two groups of data points to fit a cubic polynomial regression with heteroscedastic errors

$$r_i = \beta_1 f_i + \beta_2 f_i^2 + \beta_3 f_i^3 + \epsilon_i \tag{1}$$

where ϵ_i are assumed to be independent zero-mean Gaussian errors with variances σ_i^2 that increase with f_i. The model for frequency-dependent variances is

$$\log \sigma_i = \alpha_0 + \alpha_1 f_i + \alpha_2 f_i^2 + \delta_i \tag{2}$$

with independent and identically zero-mean Gaussian errors δ_i. The R-package LMVAR add citation that implements a maximum likelihood approach was used to fit the model. The best fitting model amongst polynomials of degree 1, 2, and 3 was chosen according to the AIC, i.e. insert correct estimates

$$r_i = f_i + f_i^3 + \epsilon_i \tag{3}$$

The best-fitting model achieves a coefficient of determination of R^2 = insert Rsquared. Parameter estimates and their standard errors are given in add Table xxx. The data and fit of the model including 95% confidence bands are displayed in Figure 1.

Here we analyse the gravitational wave signal `s20-gw-10kpc` insert more detailed description, originally sampled at 10 kHz but resampled to the LIGO sampling rate of 16384 Hz. A spectrogram of this signal is shown in Figure insert spectrogram of signal based on autoregressive estimates of the local spectra for successive time intervals of length 200 with a 90% overlap.The dominant emission mode correspondes to the ${}^2g_1$-mode [] and we have developed a time-frequency method to track the ridge $m(t)$ in the spectrogram, taking into account that it is monotonically increasing. Starting from either the left- or rightmost column of the time-frequency matrix we identify and trace the sequence of amplitude peaks within a certain frequency band given the monotonicity constraint. Are more details regarding the ridge tracking required here?

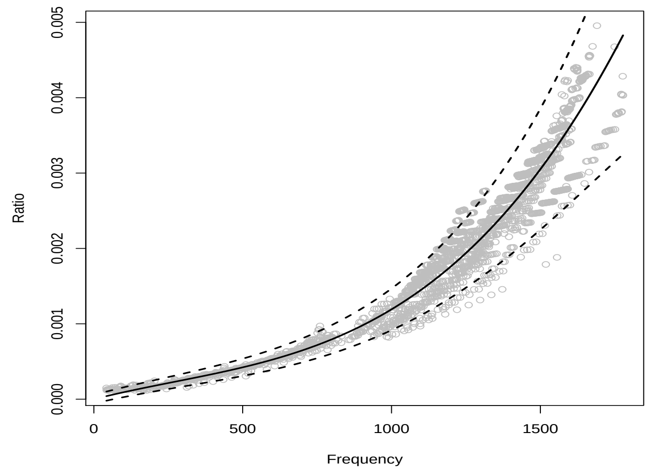

FIG. 1. Data from 25 1D simulations AENUS-ALCAR and CoCoNuT code, solid line is the maximum likelihood estimate of heteroscedastic cubic model with 95% confidence bands.

We identify the instantaneous frequency $f(t_i)$ corresponding the ridge $m(t_i)$ for the midpoint t_i of each local time interval of the spectrogram and interpolating $f(t)$ for values in between the t_i. Now we can use the universal relationship in (3) to obtain estimates of the time evolution of the ratios together with 95% confidence intervals. These are given in Figure ?? insert figure where the black points are the true ratio values, the red points the estimates and the grey bands represent 95% confidence bands. In this case without any noise, the coverage of our 95% confidence band is xx%.

In the following simulation study we explore how accurately we can estimate the parameters when the gravitational wave signal is embedded in noise. For that purpose, we inject the gravitational wave signal into simulated Advanced LIGO noise using the noise power spectral density insert formula for varying SNRs, respectively distances to the source. We estimate the coverage probability of the 95% confidence band by calculating the proportion of times that the true ratio lies outside one of the pointwise 95% confidence intervals. These coverage probabilities together for varying SNRs are given in Table insert Table and displayed in the form of boxplots in Figure insert Figure

III. DISCUSSION

Acknowledgments —

[1] Alejandro Torres-Forné, Pablo Cerdá-Durán, Martin Obergaulinger, Bernhard Müller, and José A. Font, "Universal relations for gravitational-wave asteroseismology of proto-neutron stars," Physical Review Letters **123**, 051102 (2019).