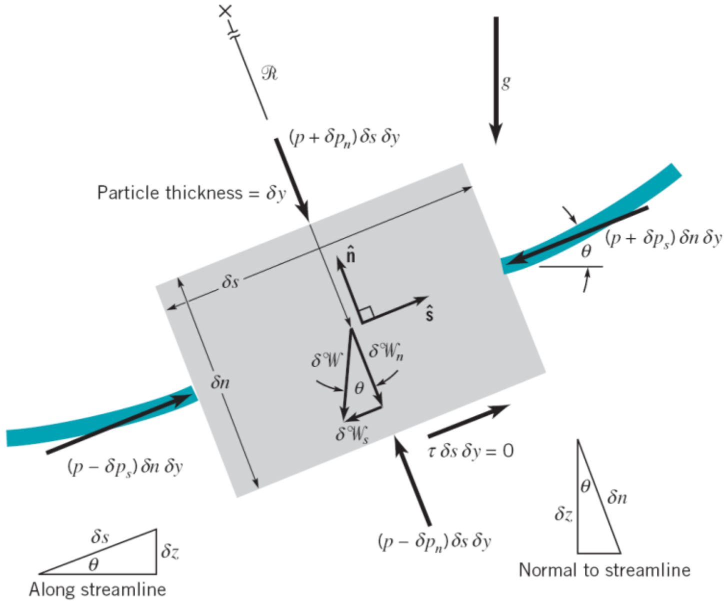
Fluid I, Midterm exam

1. (a) Starting from the Newton's 2nd law of motion, please derive the Bernoulli equation along the stream line. (10%)

$$-\frac{\partial P}{\partial s}-\gamma\sin\theta=\rho v\frac{\partial v}{\partial s}$$



Proof：

Newton's second law $\sum \delta F_s = \delta m a_s = \delta m V\frac{\delta V}{\delta s} = \rho \delta \forall V\frac{\partial V}{\partial s}$ ----(1)

The gravity force on the particle can be written as $\delta w = \gamma\delta\forall$, wherer $\gamma = \rho g$

The component of the weight force in the direction of the streamline is $\delta w_s = -\delta w sin\theta = -\gamma\delta\forall sin\theta$, if the streamline is horizontal at the point of interest, then $\theta = 0$

Use a one-term Taylor series expansion for the pressure field $\delta p_s \approx \frac{\partial p}{\partial s}\frac{\partial s}{2}$

If δF_{ps} is the net pressure force on the particle in the streamline

$\delta F_{ps} = (p-\delta p_s)\delta n\delta y - (p+\delta p_s)\delta n\delta y = -2\delta p_s\delta n\delta y$

$= -\frac{\partial p}{\partial s}\delta s\delta n\delta y = \frac{\partial p}{\partial s}\delta\forall$

The nonzero pressure gradient $\nabla p = \frac{\partial p}{\partial s}\hat{s} + \frac{\partial p}{\partial n}\hat{n}$, is what provides a net pressure force on the particle

Thus, the net force acting in the streamline direction on the particle

$\sum \delta F_s = \delta w_s + \delta F_{ps} = (-\gamma\sin\theta - \frac{\partial p}{\partial s})\delta\forall$ ----(2)

Combine (1) & (2) => $-\gamma\sin\theta - \frac{\partial p}{\partial s} = \rho V \frac{\partial p}{\partial s} = \rho a_s$ ---(3)

(b) If the flow is steady, incompressible and inviscid, please derive the following equation.

$$\frac{P}{\gamma} + \frac{1}{2}\frac{v^2}{g} + z = \text{Constant}$$

Proof：

$\sin\theta = \frac{dz}{ds}$, $V\frac{dV}{ds} = \frac{1}{2}d(V^2)/ds$, finally along the streamline the value of n is

constant (dn=0) so that $dp = \left(\frac{\partial p}{\partial s}\right) ds + \left(\frac{\partial p}{\partial n}\right) dn = \left(\frac{\partial p}{\partial s}\right) ds$

along a given streamline p(s , n)=p(s) and $\frac{\partial p}{\partial s} = \frac{dp}{ds}$

So eq. (3) become $-\gamma\frac{dz}{ds} - \frac{dp}{ds} = \frac{1}{2}\rho\frac{d(V^2)}{ds}$

This simplifies to $dp + \frac{1}{2}\rho d(V^2) + \gamma dz = 0$ (along a streamline)

For constant acceleration of gravity $\int \frac{dp}{\rho} + \frac{1}{2}V^2 + gz = C$ (along a streamline)

Assume the flow is steady incompressible inviscid the equation become

$\frac{p}{\gamma} + \frac{1}{2}\frac{V^2}{g} + z = C$ $(p = \rho RT)$

(c) If the flow is steady, compressible, ideal, isothermal and inviscid, please derive the following equation.

$$RT \ln P + \frac{1}{2}v^2 + gz = const$$

Proof：

From problem1-(b) equation $\int \frac{dp}{\rho} + \frac{1}{2}V^2 + gz = c$ $(p = \rho RT)$

=> $\int \frac{RT}{p} dp + \frac{1}{2}V^2 + gz = c$

=> $RT \ln p + \frac{1}{2}V^2 + gz = c$

(d) For an airplane flying at an elevation of 3000 m, the pressure difference between the stagnation and static pressures of the Pitot-tube installed on the airplane is 3000 N/m^2. Please determine the speed of the airplane if the air is (1) incompressible (5%) or (2) ideal and isothermal.

$$(1)\ P_2 = P_1 + \frac{\rho {V_1}^2}{2}\ \ (V_2 = 0)$$

$$P_2 - P_1 = \frac{\rho {V_1}^2}{2} = 3000$$

$$\frac{0.9093}{2} \times V^2 = 3000\ \rightarrow\ \mathrm{v} = 81.23\,\frac{m}{s}$$ (3%)

$$(2)\mathrm{RT}\ln p_1 + \frac{1}{2}{V_1}^2 = \mathrm{RT}\ln p_2 + \frac{1}{2}{V_2}^2\ \ (V_2 = 0)$$

$$\frac{1}{2}{V_1}^2 = \mathrm{RT}\ln\frac{p_2}{p_1} \rightarrow\ V_1 = \sqrt{2RT\ln(\frac{p_2}{p_1})}$$ (2%)

2. A drop of water in a zero-g environment (as in the International Space Station) will assume a spherical shape as shown in Fig.P2a. A raindrop in the cartoons is typically drawn as in Fig.P2b. The shape of an actual raindrop is more nearly like that shown in Fig.2c. Discuss why these shapes are as indicated.

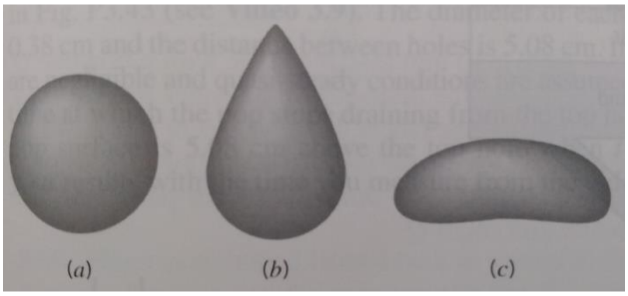


Fig.P2

圖 A 是假設在無重力狀況之下，所以水只受到表面張力的影響，故成球形；圖 B 是加入了重力影響的狀況下，所以水滴質量集中在下方；圖 C 是是理想狀況，再多加了空氣阻力 (平均向上的施力)，所以水滴呈現平坦狀。

3.

A 50 mm diameter plastic tube is used to siphon water from the large tank shown in Fig.3. If the pressure on the outside of the tube is more than40 kPa greater than the pressure within the tube, the tube will collapse and siphon will stop. If viscous effects are negligible, determine the minimum value of h allowed without the siphon stopping.(20%)

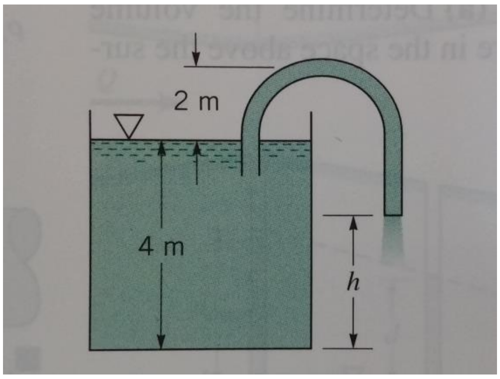


Fig.3

At any location within the tube $V = V_3$so that with $V_1 = 0$, $p_1 =$ and $z_1 = 0$

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p}{\gamma} + \frac{V^2}{2g} + z \quad \text{gives}$$

$$\frac{p}{\gamma} = -z - \frac{V_3^2}{2g}$$

Thus, the lowest pressure occurs at the point of maximum, z.

That is, $p_2 = -40 \text{ kpa}$ and $z_2 = 2\text{m}$ so that

$$-\frac{40 \times 10^3 \frac{N}{m^2}}{9.8 \times 10^3 \frac{N}{m^3}} = -2\text{m} - \frac{V_3^2}{2(9.81 \frac{m}{s^2})}$$

or

$$V_3 = 6.39 \frac{m}{s}$$

But

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_3}{\gamma} + \frac{V_3^2}{2g} + z_3 \quad \text{where } z_3 = -(4 - h) \text{ and } p_3 = 0$$

Thus,

$$0 = \frac{\left(6.39 \frac{m}{s}\right)^2}{2\left(9.81 \frac{m}{s^2}\right)} - (4 - h)$$

or

$$h = 1.92 \text{ m}$$

4. A 2 m wide, 8m high rectangular gate is located at the end of a rectangular passage that is connected to a large open tank filled with water as shown in Fig. 4 . The gate is hinged at its bottom and held closed by a horizontal force, F_H, located at the center of the gate. The maximum value for F_H is 3000 kN. **(a)** Determine the maximum water depth, h, above the center of the gate that can exist without the gate opening. **(b)** Is the answer the same if the gate is hinged at the top? **Explain your answer**.(20%)

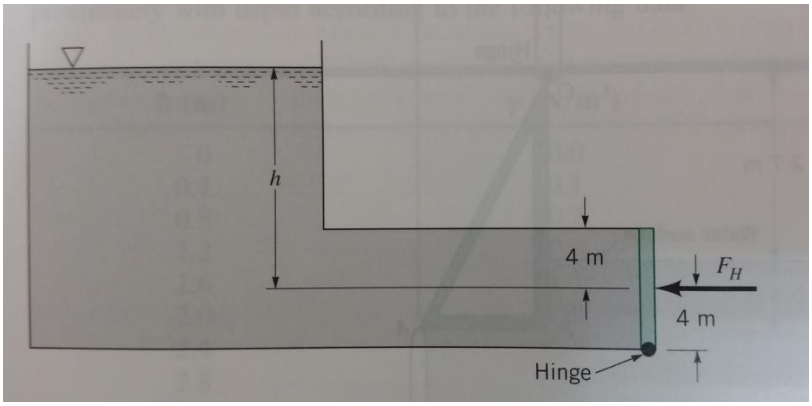


Fig.4

For gate hinged at bottom

$$\sum M_H = 0$$

So that

$(4\mathrm{m})F_H = lF_R$ (see figure) (1)

and

$$F_R = \gamma h_c A = \left(9.80\frac{\mathrm{kN}}{\mathrm{m}^3}\right)(\mathrm{h})(2\mathrm{m} \times 8\mathrm{m}) = (9.80 \times 16\,\mathrm{h})\mathrm{kN}$$

$$y_R = \frac{I \times c}{y_c A} + y_c = \frac{\frac{1}{12}(2\mathrm{m})(8\mathrm{m})^3}{\mathrm{h}(2\mathrm{m} \times 8\mathrm{m})} + \mathrm{h} = \frac{5.33}{\mathrm{h}} + \mathrm{h}$$

Thus,

$$l(\mathrm{m}) = \mathrm{h} + 4 - \left(\frac{5.33}{\mathrm{h}} + \mathrm{h}\right) = 4 - \frac{5.33}{\mathrm{h}}$$

and from Eq.(1)

$$(4\mathrm{m})(3000\mathrm{kN}) = \left(4 - \frac{5.33}{\mathrm{h}}\right)(9.80 \times 16)(\mathrm{h})\ \mathrm{kN}$$

So that

$$\mathrm{h} = 20.5\ \mathrm{m}$$

For gate hinged at Top

$$\sum M_H = 0$$

So that

$(4\text{m})F_H = l_1 F_R$(see figure)

$$(4\text{m})(3000\text{kN}) = \left(\frac{5.33}{h} + 4\right)(9.8 \times 16)h \ \text{kN}$$

$$h = 17.8 \text{ m}$$

Maximum depth for gate hinged at top is less than maximum depth for gate hinged at bottom

5. A 3 m long curved gate is located in the side of a reservoir containing water as shown in Fig. 5 . Determine the magnitude of the horizontal and vertical components of the force of the water on the gate. Will this force pass through point A? **Explain**. (20%)

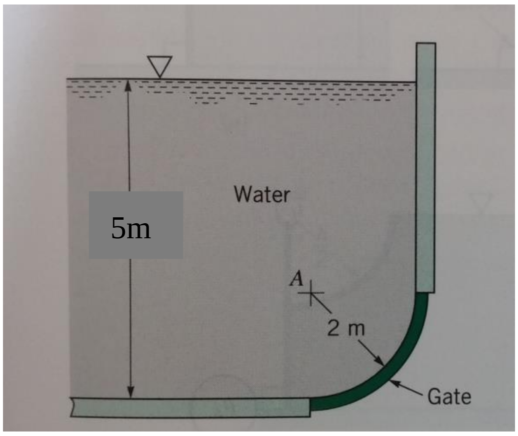


Fig.5

For equilibrium ,

$$\sum F_x = 0$$

or

$$F_H = F_2 = \gamma h_{c2} A_2 = \gamma(3\text{m} + 1\text{m})(2\text{m} \times 3\text{m})$$

So that

$$F_H = \left(9.80 \frac{\text{kN}}{\text{m}^3}\right)(4\text{m})(6\text{m}^2) = 235.2 \text{ kN}$$

Similarly,

$$\sum F_y = 0$$

$F_V = F_1 + W$ where:

$$F_1 = [\gamma(3\text{m})](2\text{m} \times 3\text{m}) = (9.80 \frac{\text{kN}}{\text{m}^3})(3\text{m})(6\text{m}^2)$$

$$W = \gamma \forall = (9.80\frac{\text{kN}}{\text{m}^2})(3\pi\ \text{m}^3)$$

Thus,

$$F_V = \left(9.80\frac{\text{kN}}{\text{m}^3}\right)[18\text{m}^3 + 3\pi\ \text{m}^3] = 269\ \text{kN}$$

(Note : Force of water on gate will be opposite in direction to that shown on figure.)

The direction of all differential forces acting on the curved surface is perpendicular to surface , and therefore , the resultant must pass through the intersection of all these forces which is at point A. <u>Yes</u>

6. The pressure difference, Δp, across a partial blockage in an artery (called a stenosis) is approximated by the equation

$$\Delta p = K_\upsilon \frac{\mu V}{D} + K_u(\frac{A_0}{A_1} - 1)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity $(FL^{-2}T)$, ρ the blood density (ML^{-3}), D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the stenosis. Deter mine the dimensions of the constants K_υ and K_u. Would this equation be valid in any system of units? (10%)

$$\Delta p = K_v \frac{\mu V}{D} + K_v(\frac{A_0}{A_1} - 1)^2 \rho V^2$$

$$[FL^{-2}] = [K_v]\left[\left(\frac{FT}{L^2}\right)\left(\frac{L}{T}\right)\left(\frac{1}{L}\right)\right] + [K_u][\frac{(L^2)}{(L^2)} - 1]^2 \left[\frac{FT^2}{L^4}\right][\frac{L}{T}]^2$$

$$[FL^{-2}] = [K_v][FL^{-2}] + [K_u][FL^{-2}]$$

Since each term must have the same dimensions , $K_v$and K_u are <u>dimensionless</u>. Thus , the equation is a general homogeneous equation that would be valid in any consistent system of units . <u>Yes</u>

7. Vehicle shock absorbers damp out oscillations caused by road roughness. Describe how a temperature change may affect the operation of a shock absorber. (5%)

溫度會影響阻尼里潤滑油的黏滯係數，溫度越低潤滑越稠，摩擦力較大，減震效果也較好，反之溫度高減震效果較差。

8. Air is enclosed by a rigid cylinder containing a piston. A pressure gage attached to the cylinder indicates an initial reading of 180 kPa. Determine the reading on the gage when the piston has compressed the air to one-third its original volume. Assume the compression process to be isothermal and the local atmospheric pressure to be 101.3 kPa. (5%)

for isothermal compression , $\frac{p}{\rho} = \text{constant}$

so that

$\frac{p_i}{\rho_i} = \frac{p_f}{\rho_f}$ where is initial state and f ~ final state

Thus,

$p_f = \frac{\rho_f}{\rho_i} \cdot p_i$

since

$\rho = \frac{\text{mass}}{\text{volume}}, \ \frac{\rho_f}{\rho_i} = \frac{\text{initial volume}}{\text{final volume}} = 3 \text{ (for constant mass)}$

and therefore

$$p_f = (3)[(180 \text{ Kpa} + 101.3 \text{ Kpa})] = 844 \text{ Kpa (abs)}$$

or

$p_f(\text{gage}) = (844 \text{ Kpa} - 101.3\text{Kpa}) = 743 \text{ Kpa (abs)}$