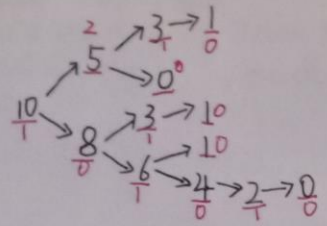
SG函数：

① 设取石子的集合为{1,2,5}，且仅有一堆石子，石子数为10（终点的SG值为0）

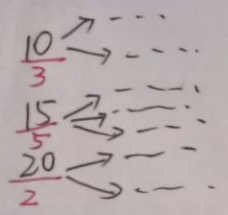


当仅有一堆石子时，如果SG(10)≠0，必胜，SG(10)=0，必败

原因：当SG值不为0时，因为SG值经过3Mex运算，所以其连接的点必有一点为0，当走到SG值为0的这一点时，由于Mex运算，该点所连接的点的值必定不为0，∵终点的SG值为0，所以只要先手的SG值不为0，便可以一直走到SG值为0的点，最终走向终点。反之，同理。

②当有n堆石子的情况时：

任意设每堆石子的SG值：



∵每堆石子的SG值都是与其连接点的SG值中最小的且与连接点的值不同的最小非负整数，所以选择时都可以选择SG值 0-SG(x)之间的任意数。

例：

∵SG(10)=3，∴SG(10)可以选择连接点的SG值的范围是 0-3（0≤SG<3）

同理 SG(15)也可选择[0,5)，SG(20)可选择[0,2)

将每堆石子的SG值取出来，可以发现这些值拼接起来可以成为Nim游戏！

∴当 $SG(X_1)\wedge SG(X_2)\wedge SG(X_3)\underset{=m}{\neq 0}$ 时，可以选择SG值最大的那个，如：$SG(X_2)$，使得

$SG(X_2')=SG(X_2)\wedge m \Rightarrow SG(X_1)\wedge SG(X_2')\wedge SG(X_3)=SG(X_1)\wedge SG(X_2)\wedge SG(X_3)\wedge m=m\wedge m=0$

接下来的步骤可参考Nim游戏。

所以当所有石子堆的异或值不等于0的话，必胜，等于0，必败。

BSGS HASHMAP

```
// B^L == N (mod P)
// Code by KSkun, 2018/4
#include <cstdio>
#include <cmath>
#include <cstring>

#include <algorithm>

typedef long long LL;

inline LL fpow(LL n, LL k, LL p) {
	LL res = 1; n %= p;
	while(k) {
		if(k & 1) res = res * n % p;
		n = n * n % p;
		k >>= 1;
	}
	return res;
}

const int MO = 611977, MAXN = 1000005;

struct HashMap {
	int head[MO + 5], key[MAXN], value[MAXN], nxt[MAXN], tot;
	inline void clear() {
		tot = 0;
		memset(head, -1, sizeof(head));
	}
	HashMap() {
		clear();
	}
	inline void insert(int k, int v) {
		int idx = k % MO;
		for(int i = head[idx]; ~i; i = nxt[i]) {
			if(key[i] == k) {
				value[i] = v;
				return;
			}
		}
		key[tot] = k; value[tot] = v; nxt[tot] = head[idx]; head[idx] = tot++;
	}
```

```
        inline int operator[](const int &k) const {
            int idx = k % MO;
            for(int i = head[idx]; ~i; i = nxt[i]) {
                if(key[i] == k) return value[i];
            }
            return -1;
        }
} x;

inline LL bsgs(LL a, LL b, LL p) {
        a %= p; b %= p;
        if(a == 0) return b == 0 ? 1 : -1;
        if(b == 1) return 0;
        LL m = ceil(sqrt(p - 1)), inv = fpow(a, p - m - 1, p);
        x.clear();
        x.insert(1, 0);
        for(LL i = 1, e = 1; i < m; i++) {
            e = e * a % p;
            if(x[e] == -1) x.insert(e, i);
        }
        for(LL i = 0; i < m; i++) {
            if(x[b] != -1) {
                LL res = x[b];
                return i * m + res;
            }
            b = b * inv % p;
        }
        return -1;
}

LL p, b, n;

int main() {
        while(scanf("%lld%lld%lld", &p, &b, &n) != EOF) {
            LL res = bsgs(b, n, p);
            if(res != -1) printf("%lld\n", res); else puts("no solution");
        }
        return 0;
}
```

BSGS STLMAP

```
// B^L == N (mod P)
// Code by KSkun, 2018/4
#include <cstdio>
#include <cmath>

#include <map>

typedef long long LL;

inline LL fpow(LL n, LL k, LL p) {
	LL res = 1; n %= p;
	while(k) {
		if(k & 1) res = res * n % p;
		n = n * n % p;
		k >>= 1;
	}
	return res;
}

std::map<LL, LL> x;

inline LL bsgs(LL a, LL b, LL p) {
	a %= p; b %= p;
	if(a == 0) return b == 0 ? 1 : -1;
	if(b == 1) return 0;
	LL m = ceil(sqrt(p - 1)), inv = fpow(a, p - m - 1, p);
	x.clear();
	x[1] = m; // use m instead of 0
	for(LL i = 1, e = 1; i < m; i++) {
		e = e * a % p;
		if(!x[e]) x[e] = i;
	}
	for(LL i = 0; i < m; i++) {
		if(x[b]) {
			LL res = x[b];
			return i * m + (res == m ? 0 : res);
		}
		b = b * inv % p;
	}
	return -1;
}
```

```
LL p, b, n;

int main() {
	while(scanf("%lld%lld%lld", &p, &b, &n) != EOF) {
		LL res = bsgs(b, n, p);
		if(res != -1) printf("%lld\n", res); else puts("no solution");
	}
	return 0;
}
```

exBSGS HASHMAP

```
// B^L == N (mod P)
// Code by KSkun, 2018/4
#include <cstdio>
#include <cmath>
#include <cstring>

#include <algorithm>

typedef long long LL;

inline char fgc() {
	static char buf[100000], *p1 = buf, *p2 = buf;
	return p1 == p2 && (p2 = (p1 = buf) + fread(buf, 1, 100000, stdin), p1 == p2) ? EOF
		: *p1++;
}

inline LL readint() {
	register LL res = 0, neg = 1;
	char c = fgc();
	while(c < '0' || c > '9') {
		if(c == '-') neg = -1;
		c = fgc();
	}
	while(c >= '0' && c <= '9') {
		res = res * 10 + c - '0';
		c = fgc();
	}
```

```
    return res * neg;
}

inline LL fpow(LL n, LL k, LL p) {
    LL res = 1; n %= p;
    while(k) {
        if(k & 1) res = res * n % p;
        n = n * n % p;
        k >>= 1;
    }
    return res;
}

inline LL exgcd(LL a, LL b, LL &x, LL &y) {
    if(!b) {
        x = 1; y = 0;
        return a;
    }
    LL res = exgcd(b, a % b, x, y);
    LL t = x; x = y; y = t - a / b * y;
    return res;
}

const int MO = 611977, MAXN = 1000005;

struct HashMap {
    LL head[MO + 5], key[MAXN], value[MAXN], nxt[MAXN], tot;
    inline void clear() {
        tot = 0;
        memset(head, -1, sizeof(head));
    }
    HashMap() {
        clear();
    }
    inline void insert(LL k, LL v) {
        int idx = k % MO;
        for(int i = head[idx]; ~i; i = nxt[i]) {
            if(key[i] == k) {
                value[i] = std::min(value[i], v);
                return;
            }
        }
        key[tot] = k; value[tot] = v; nxt[tot] = head[idx]; head[idx] = tot++;
    }
```

```
        inline LL operator[](const LL &k) const {
            int idx = k % MO;
            for(int i = head[idx]; ~i; i = nxt[i]) {
                if(key[i] == k) return value[i];
            }
            return -1;
        }
} x;

inline LL bsgs(LL a, LL b, LL p) {
        a %= p; b %= p;
        if(a == 0) return b == 0 ? 1 : -1;
        if(b == 1) return 0;
        LL m = ceil(sqrt(p - 1)), inv, y;
        exgcd(fpow(a, m, p), p, inv, y); inv = (inv % p + p) % p;
        x.clear();
        x.insert(1, 0);
        for(LL i = 1, e = 1; i < m; i++) {
            e = e * a % p;
            if(x[e] == -1) x.insert(e, i);
        }
        for(LL i = 0; i < m; i++) {
            if(x[b] != -1) {
                LL res = x[b];
                return i * m + res;
            }
            b = b * inv % p;
        }
        return -1;
}

inline LL gcd(LL a, LL b) {
        if(!b) return a;
        return gcd(b, a % b);
}

inline LL exbsgs(LL a, LL b, LL p) {
        if(b == 1) return 0;
        LL tb = b, tmp = 1, k = 0;
        for(int g = gcd(a, p); g != 1; g = gcd(a, p)) {
            if(tb % g) return -1;
            tb /= g; p /= g; tmp = tmp * a / g % p;
            k++;
            if(tmp == tb) return k;
```

```
	}
	return bsgs(a, b, p);
}

LL a, b, p;

int main() {
	for(;;) {
		a = readint(); p = readint(); b = readint();
		if(!a && !b && !p) break;
		LL res = exbsgs(a, b, p);
		if(res != -1) printf("%lld\n", res); else puts("No Solution");
	}
	return 0;
}
```

高斯消元解异或线性方程组

```
#include <iostream>
#include <algorithm>

using namespace std;

const int N = 110;

int n;
int a[N][N];
int gauss()
{
	int c,r;
	for(c=0,r=0;c<n;c++)
	{
		// 找主元
		int t=-1;
		for(int i=r;i<n;i++)
			if(a[i][c])
			{
				t=i;
				break;
			}
```

```
            if(t==-1) continue;
            // 交换主元行
            for(int j=c;j<=n;j++) swap(a[r][j],a[t][j]);
            // 左下角消
            for(int i=r+1;i<n;i++)
                if(a[i][c])//漏了
                    for(int j=n;j>=c;j--)//漏了
                        a[i][j] ^= a[r][j];
            r++;
        }
        // 判断
        if(r<n)
        {
            for(int i=r;i<n;i++)//i=r
                if(a[i][n])
                    return 2;
            return 1;
        }
        // 消右上角
        for(int i=n-1;i>=0;i--)
            for(int j=i+1;j<n;j++)
            //如果是 0 就不用下面的 a[j][j] 来^a[i][j]了
            //如果不是 0 才需要用第 j 行第 j 列 a[j][j]来^第 i 行第 j 列 a[i][j]
            //进而进行整行 row[i]^row[j] 间接导致 a[i][n]^a[j][n]
                if(a[i][j])
                    a[i][n]^=a[j][n];
        return 0;
    }

    int main()
    {
        cin >> n;
        for(int i=0;i<n;i++)
            for(int j=0;j<=n;j++)
                cin >> a[i][j];
        int t = gauss();
        if(t==0)
        {
            for(int i=0;i<n;i++) cout << a[i][n] << endl;
        }
        else if(t==1) puts("Multiple sets of solutions");
        else puts("No solution");
        return 0;
    }
```

模意义下的高斯消元

```
#include<cstdio>
#define maxn 110
#define r register
using namespace std;
typedef long long ll;
int n,p,maxi;
ll tmp,ans[maxn],a[maxn][maxn];
int read()
{
	r char ch=getchar();r int in=0;
	while(ch>'9'||ch<'0') ch=getchar();
	while(ch>='0'&&ch<='9') in=(in<<3)+(in<<1)+ch-'0',ch=getchar();
	return in;
}
ll ksm(r ll x,r int y)
{
	if(!y) return 1;
	r ll ret=ksm(x,y>>1);
	if(y&1) return ret*ret%p*x%p;
	return ret*ret%p;
}
int main()
{
	n=read(),p=read();
	for(r int i=1;i<=n;i++)
		for(r int j=1;j<=n+1;j++)
			a[i][j]=read();
	for(r int i=1;i<=n;i++)
	{
		if(!a[i][i])//主元不能为 0
		{
			maxi=0;
			for(r int j=i+1;j<=n&&!maxi;j++)
				if(a[j][i]) maxi=j;
			if(!maxi) continue;//如果一整列都为 0，不需要消元
			for(r int j=i;j<=n+1;j++)
				tmp=a[maxi][j],a[maxi][j]=a[i][j],a[i][j]=tmp;
		}
		for(r int j=i+1;j<=n;j++)
		{
			tmp=a[j][i];
```

```
                if(!tmp) continue;//已经为 0，不需要消元
                for(r int k=i;k<=n+1;k++)
                    a[j][k]=((a[j][k]*a[i][i]-a[i][k]*tmp)%p+p)%p;
            }
        }
        for(r int i=n;i;i--)
        {
            for(r int j=i+1;j<=n;j++)
                a[i][n+1]=((a[i][n+1]-ans[j]*a[i][j])%p+p)%p;
            ans[i]=a[i][n+1]*ksm(a[i][i],p-2)%p;
        }
        for(r int i=1;i<=n;i++) printf("%lld ",ans[i]);
        return 0;
}
```