Naïve Bayes

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1 Naïve Bayes Algorithm

1.1 Overview

The Naïve Bayes algorithm is a probabilistic machine learning algorithm based on Bayes' Theorem. It assumes that all features are independent of each other given the class label, which is a strong assumption, hence the term "naïve". It is commonly used for classification tasks such as spam detection, text classification, and sentiment analysis.

1.1.1 How Naïve Bayes Works

1. **Training Phase**:
   - The algorithm calculates the prior probabilities of each class from the training data.
   - It computes the likelihood of each feature given each class. For continuous features, it assumes a normal distribution and computes the mean and variance for each feature per class.
2. **Prediction Phase**:
   - For a given test data point, the algorithm calculates the posterior probability for each class using Bayes' Theorem
   - The class with the highest posterior probability is chosen as the predicted class for the test point.

1.1.2 Key Points

- The "naïve" assumption of independence between features simplifies the computation and makes the algorithm fast and efficient.
- Despite its simplicity, Naïve Bayes often performs surprisingly well and is particularly suited for high-dimensional data.
- It works well with small datasets and can handle both continuous and discrete data.

1.1.3 Implementation Objective

In this notebook, we will implement the Naïve Bayes algorithm from scratch using Python. We will test our implementation on a dataset and evaluate its performance using metrics such as confusion matrix, accuracy, recall, precision, and F1-score.

1.1.4 Importing necessary libraries

[59]:
```
import pandas as pd
import numpy as np
from sklearn.metrics import precision_score, recall_score, f1_score,␣
 ↪roc_auc_score, roc_curve, confusion_matrix
import matplotlib.pyplot as plt
import seaborn as sns
from sklearn.model_selection import train_test_split
```

1.1.5 Naive Bayes algorithm implementation

[60]:
```
class NaiveBayes:
    def __init__(self, epsilon=1e-9, distribution='gaussian', smoothing=1.0,␣
 ↪criterion='log_likelihood'):
        """
        Initialize the Naive Bayes model.

        Parameters:
        - epsilon: Small value to avoid division by zero.
        - distribution: Type of distribution ('gaussian', 'bernoulli',␣
 ↪'multinomial').
        - smoothing: Smoothing factor (used for 'bernoulli' and 'multinomial').
        - criterion: Criterion for probability calculation ('log_likelihood',␣
 ↪'gini', 'entropy').
        """
        self.epsilon = epsilon
        self.distribution = distribution
        self.smoothing = smoothing
        self.criterion = criterion

    def fit(self, X, y):
        """
        Fit the model to the training data.

        Parameters:
        - X: Input feature dataset.
        - y: Corresponding output labels.
        """
        n_samples, n_features = X.shape
        self._classes = np.unique(y)
        n_classes = len(self._classes)

        # Initialize parameters for each class
        if self.distribution == 'gaussian':
            self._mean = np.zeros((n_classes, n_features), dtype=np.float64)
            self._var = np.zeros((n_classes, n_features), dtype=np.float64)
```

```python
        elif self.distribution in ['bernoulli', 'multinomial']:
            self._feature_probs = np.zeros((n_classes, n_features), dtype=np.
↪float64)

        self._priors = np.zeros(n_classes, dtype=np.float64)

        for idx, c in enumerate(self._classes):
            X_c = X[y == c]
            self._priors[idx] = X_c.shape[0] / float(n_samples)
            if self.distribution == 'gaussian':
                self._mean[idx, :] = X_c.mean(axis=0)
                self._var[idx, :] = X_c.var(axis=0) + self.epsilon  # Add␣
↪epsilon to avoid division by zero
            elif self.distribution == 'bernoulli':
                # Bernoulli: Calculate the probability of each feature being 1
                self._feature_probs[idx, :] = (np.sum(X_c, axis=0) + self.
↪smoothing) / (X_c.shape[0] + 2 * self.smoothing)
            elif self.distribution == 'multinomial':
                # Multinomial: Calculate the frequency of each feature value
                self._feature_probs[idx, :] = (np.sum(X_c, axis=0) + self.
↪smoothing) / (X_c.sum() + self.smoothing * n_features)

    def predict(self, X):
        """
        Perform prediction on the input dataset X.

        Parameters:
        - X: Input dataset for prediction.

        Returns:
        - y_pred: Predictions for the input data.
        """
        y_pred = [self._predict(x) for x in X]
        return np.array(y_pred)

    def _predict(self, x):
        """
        Predict the label for a single input data point.

        Parameters:
        - x: Input data point.

        Returns:
        - Predicted label for the input point.
        """
        posteriors = []
```

```python
        # Calculate the posterior probability for each class
        for idx, c in enumerate(self._classes):
            prior = np.log(self._priors[idx])
            if self.criterion == 'log_likelihood':
                if self.distribution == 'gaussian':
                    posterior = np.sum(np.log(self._pdf_gaussian(idx, x) + self.epsilon))  # Avoid log(0) by adding epsilon
                elif self.distribution == 'bernoulli':
                    # Clip probabilities to avoid log(0)
                    probs = np.clip(self._feature_probs[idx, :], self.epsilon, 1 - self.epsilon)
                    posterior = np.sum(
                        x * np.log(probs) +
                        (1 - x) * np.log(1 - probs)
                    )
                elif self.distribution == 'multinomial':
                    # Clip probabilities to avoid log(0)
                    probs = np.clip(self._feature_probs[idx, :], self.epsilon, 1 - self.epsilon)
                    posterior = np.sum(x * np.log(probs))
                else:
                    raise ValueError(f"Unsupported distribution for criterion 'log_likelihood': {self.distribution}")
            elif self.criterion == 'gini':
                if self.distribution == 'gaussian':
                    posterior = 1 - np.sum((self._pdf_gaussian(idx, x) ** 2))
                else:
                    raise ValueError(f"Gini criterion not supported for {self.distribution}")
            elif self.criterion == 'entropy':
                if self.distribution == 'gaussian':
                    posterior = -np.sum(self._pdf_gaussian(idx, x) * np.log(self._pdf_gaussian(idx, x) + self.epsilon))
                else:
                    raise ValueError(f"Entropy criterion not supported for {self.distribution}")
            else:
                raise ValueError(f"Unsupported criterion: {self.criterion}")

            posterior = prior + posterior
            posteriors.append(posterior)

        # Return class with highest posterior probability
        return self._classes[np.argmax(posteriors)]

    def _pdf_gaussian(self, class_idx, x):
```

```
        """
        Calculate the probability density function for Gaussian distribution.

        Parameters:
        - class_idx: Index of the class.
        - x: Input data point.

        Returns:
        - Probability density value.
        """
        mean = self._mean[class_idx]
        var = self._var[class_idx]
        numerator = np.exp(- (x - mean) ** 2 / (2 * var))
        denominator = np.sqrt(2 * np.pi * var)
        return numerator / denominator
```

1.1.6 Evaluation function for Naive Bayes

[61]:
```
def evaluate_nb(X_train, X_test, y_train, y_test, distribution='gaussian',␣
 ↪smoothing=1.0, criterion='log_likelihood'):
    # Initialize and train the Naive Bayes model with specified parameters
    nb = NaiveBayes(distribution=distribution, smoothing=smoothing,␣
 ↪criterion=criterion)
    nb.fit(X_train, y_train)

    # Predict on the test set
    predictions = nb.predict(X_test)

    # Calculate accuracy
    accuracy = np.sum(predictions == y_test) / len(y_test)
    print(f'Accuracy: {accuracy:.2f}')

    # Calculate precision, recall, and F1-score
    precision = precision_score(y_test, predictions, average='weighted',␣
 ↪zero_division=0)
    recall = recall_score(y_test, predictions, average='weighted',␣
 ↪zero_division=0)
    f1 = f1_score(y_test, predictions, average='weighted', zero_division=0)

    print(f'Precision: {precision:.2f}')
    print(f'Recall: {recall:.2f}')
    print(f'F1 Score: {f1:.2f}')

    # Confusion matrix
    conf_matrix = confusion_matrix(y_test, predictions)
    classes = np.unique(y_test)
```

```
    # Plot normalized confusion matrix
    plt.figure(figsize=(8, 6))
    sns.heatmap(conf_matrix, annot=True, fmt="d", cmap="Greens",␣
 ↪xticklabels=classes, yticklabels=classes)
    plt.title("Confusion Matrix - Naive Bayes")
    plt.xlabel("Predicted Label")
    plt.ylabel("True Label")
    plt.show()

    # Calculate ROC-AUC score for binary classification
    if len(classes) == 2:
        roc_auc = roc_auc_score(y_test, predictions)
        print(f'ROC AUC Score: {roc_auc:.2f}')

        # Plot ROC Curve
        fpr, tpr, _ = roc_curve(y_test, predictions)
        plt.figure()
        plt.plot(fpr, tpr, label=f'ROC Curve (area = {roc_auc:.2f})')
        plt.plot([0, 1], [0, 1], linestyle='--')
        plt.xlabel('False Positive Rate')
        plt.ylabel('True Positive Rate')
        plt.title('Receiver Operating Characteristic (ROC) Curve')
        plt.legend(loc='lower right')
        plt.show()
    else:
        print("ROC AUC Score and ROC Curve are not applicable for multiclass␣
 ↪classification.")
```

1.1.7 Load All Datasets

Iris Dataset

[62]:
```
iris = pd.read_csv('https://archive.ics.uci.edu/ml/machine-learning-databases/
 ↪iris/iris.data', header=None)
iris.columns = ['sepal_length', 'sepal_width', 'petal_length', 'petal_width',␣
 ↪'class']
iris['class'] = iris['class'].astype('category').cat.codes
X_iris = iris.iloc[:, :-1].values
y_iris = iris.iloc[:, -1].values
```

Penguins Dataset

[63]:
```
penguins = sns.load_dataset('penguins').dropna()
penguins['species'] = penguins['species'].astype('category').cat.codes
penguins['island'] = penguins['island'].astype('category').cat.codes
penguins['sex'] = penguins['sex'].astype('category').cat.codes
X_penguins = penguins.drop('species', axis=1).values
y_penguins = penguins['species'].values
```

Titanic Dataset

[64]:
```
# Carregar conjunto de dados Titanic
titanic = sns.load_dataset('titanic')

# Preencher valores faltantes sem usar inplace=True
titanic['age'] = titanic['age'].fillna(titanic['age'].mean())
titanic['embarked'] = titanic['embarked'].fillna(titanic['embarked'].mode()[0])
titanic = titanic.dropna(subset=['embark_town', 'sex', 'fare', 'class'])

# Transformar características categóricas em numéricas
titanic['sex'] = titanic['sex'].astype('category').cat.codes
titanic['embarked'] = titanic['embarked'].astype('category').cat.codes
titanic['class'] = titanic['class'].astype('category').cat.codes

# Separar características e rótulos
X_titanic = titanic[['pclass', 'sex', 'age', 'sibsp', 'parch', 'fare',␣
↪'embarked']].values
y_titanic = titanic['survived'].values
```

Census Income Dataset

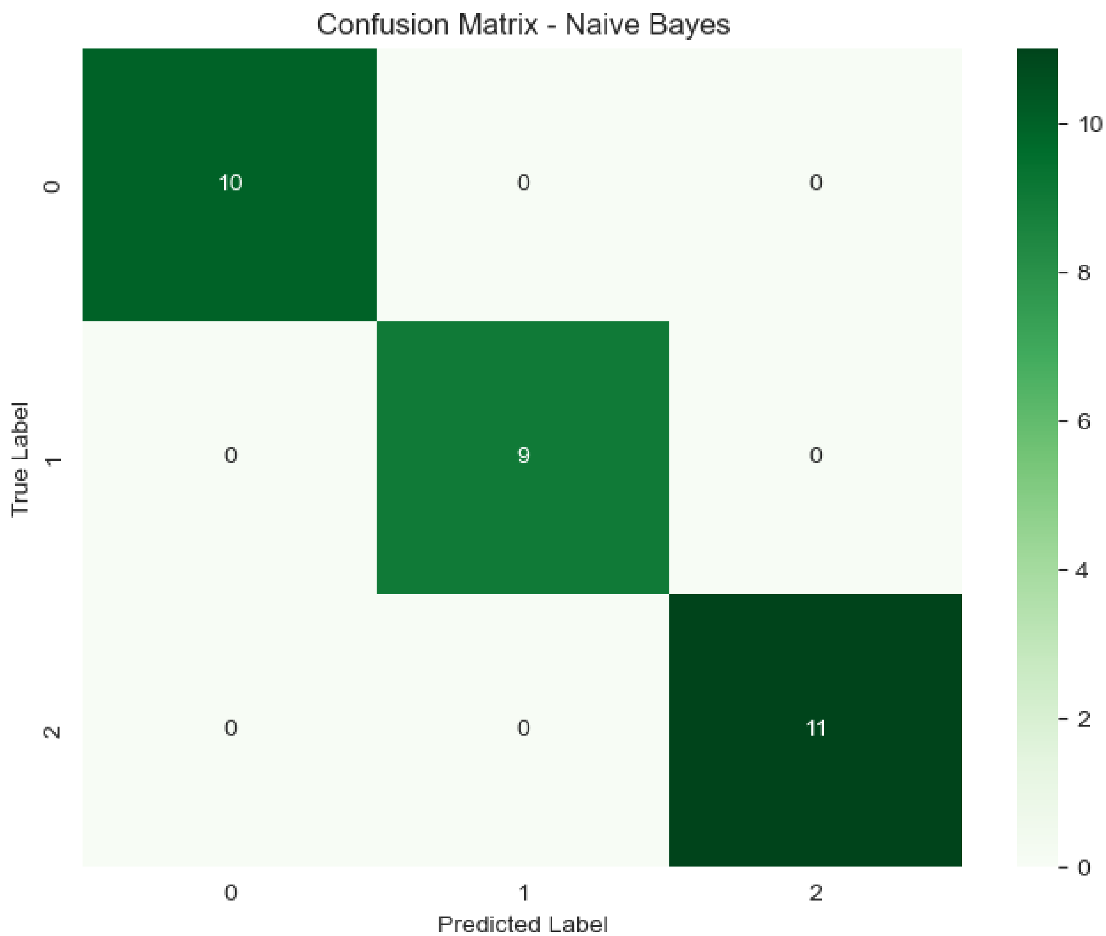
[65]:
```
census = pd.read_csv('https://archive.ics.uci.edu/ml/machine-learning-databases/
↪adult/adult.data', header=None)
census.columns = ['age', 'workclass', 'fnlwgt', 'education', 'education-num',␣
↪'marital-status',
                  'occupation', 'relationship', 'race', 'sex', 'capital-gain',␣
↪'capital-loss',
                  'hours-per-week', 'native-country', 'income']
for column in ['workclass', 'education', 'marital-status', 'occupation',␣
↪'relationship', 'race', 'sex', 'native-country', 'income']:
    census[column] = census[column].astype('category').cat.codes
X_census = census.drop('income', axis=1).values
y_census = census['income'].values
```

1.1.8 Test Naive Bayes on all datasets

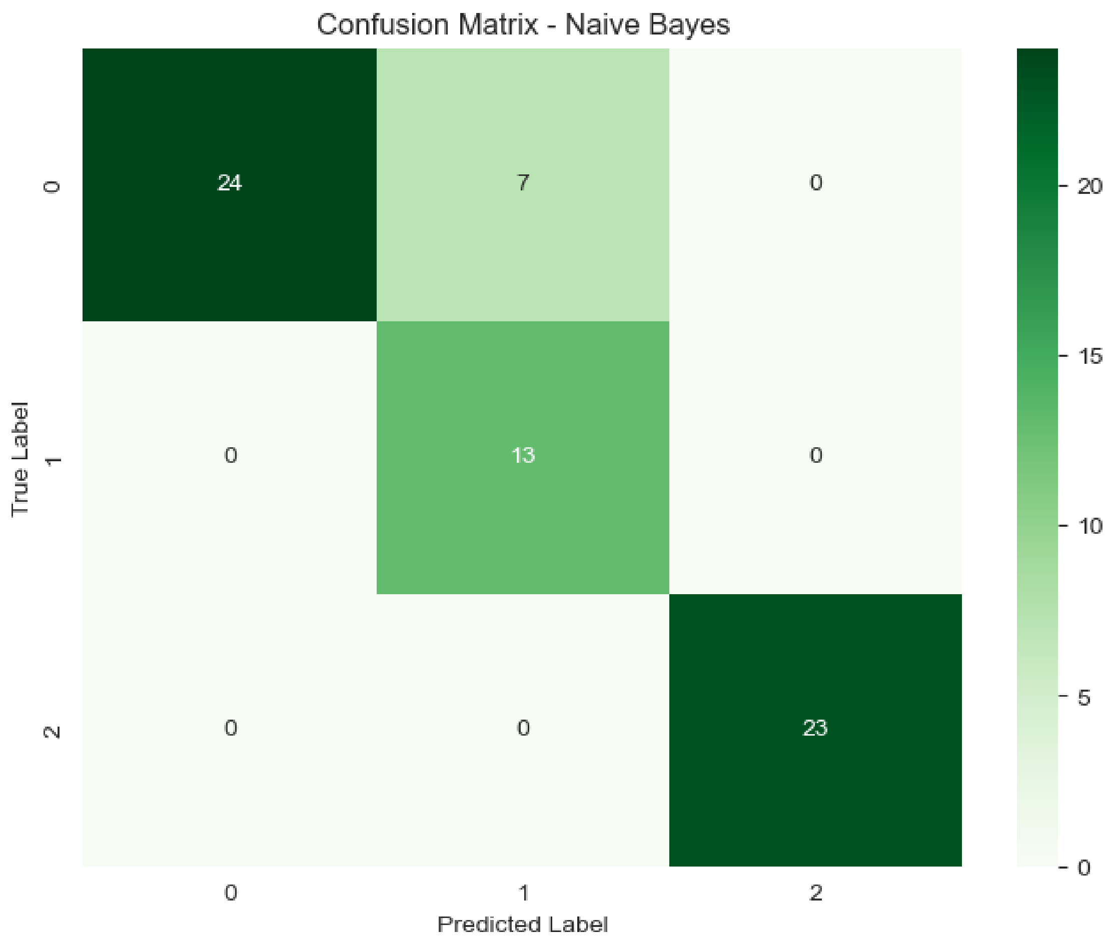
[66]:
```
datasets = {
    'Iris': (X_iris, y_iris, 'gaussian', 1.0, 'log_likelihood'),
    # 'gaussian' distribution is chosen for the Iris dataset because all the␣
↪features
    # (sepal length, sepal width, petal length, petal width) are continuous␣
↪numeric variables.
    # Smoothing is set to 1.0 as a default precaution, but it is not typically␣
↪used for Gaussian.
    # The 'log_likelihood' criterion is standard for continuous data with␣
↪Gaussian distribution.
```

```
    'Penguins': (X_penguins, y_penguins, 'gaussian', 1.0, 'log_likelihood'),
    # 'gaussian' distribution is selected because the features of the Penguins␣
↪dataset
    # (e.g., bill length, flipper length, body mass) are continuous numeric␣
↪variables.
    # Smoothing of 1.0 helps avoid any numerical instabilities.
    # The 'log_likelihood' criterion is used for calculating the likelihood of␣
↪the Gaussian distribution.

    'Titanic': (X_titanic, y_titanic, 'bernoulli', 1.0, 'log_likelihood'),
    # 'bernoulli' distribution is appropriate for the Titanic dataset because␣
↪many features
    # (e.g., sex, survived) are binary (0 or 1). The Bernoulli distribution␣
↪handles binary features well.
    # Smoothing is set to 1.0 to handle cases where a particular feature-class␣
↪combination does not appear in the training data.
    # The 'log_likelihood' criterion is used to compute the log-probability for␣
↪binary features.

    'Census': (X_census, y_census, 'multinomial', 1.0, 'log_likelihood')
    # 'multinomial' distribution is suitable for the Census dataset because it␣
↪contains categorical features
    # (e.g., workclass, education, marital-status) which have been encoded as␣
↪integers. The multinomial distribution models the counts of categorical␣
↪variables.
    # Smoothing is set to 1.0 to ensure non-zero probabilities for all␣
↪categories.
    # The 'log_likelihood' criterion is standard for calculating probabilities␣
↪in a multinomial Naive Bayes model.
}


for name, (X, y, distribution, smoothing, criterion) in datasets.items():
    print(f"Testing Naive Bayes on {name} dataset with␣
↪distribution={distribution}, smoothing={smoothing}, criterion={criterion}")
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=0.2,␣
↪random_state=42)
    evaluate_nb(X_train, X_test, y_train, y_test, distribution=distribution,␣
↪smoothing=smoothing, criterion=criterion)
```

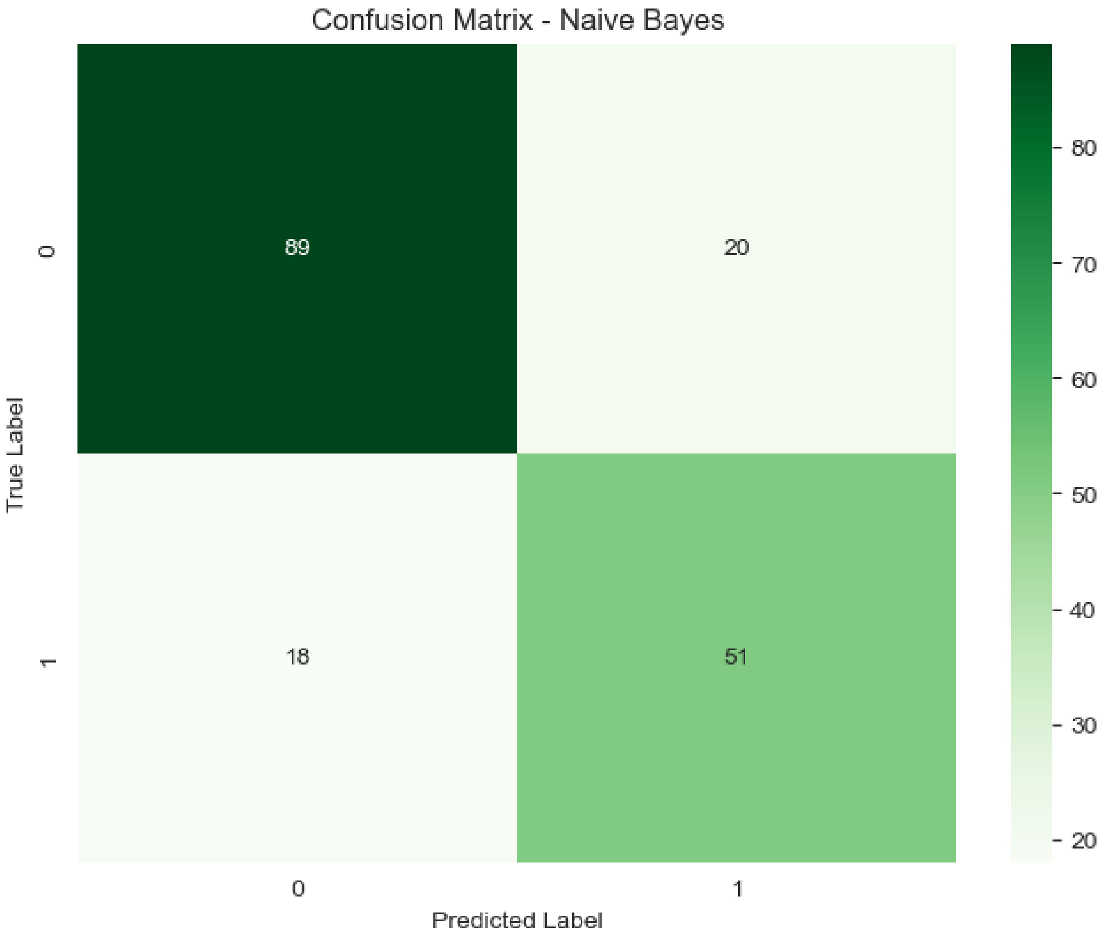
```
Testing Naive Bayes on Iris dataset with distribution=gaussian, smoothing=1.0,
criterion=log_likelihood
Accuracy: 1.00
Precision: 1.00
Recall: 1.00
F1 Score: 1.00
```



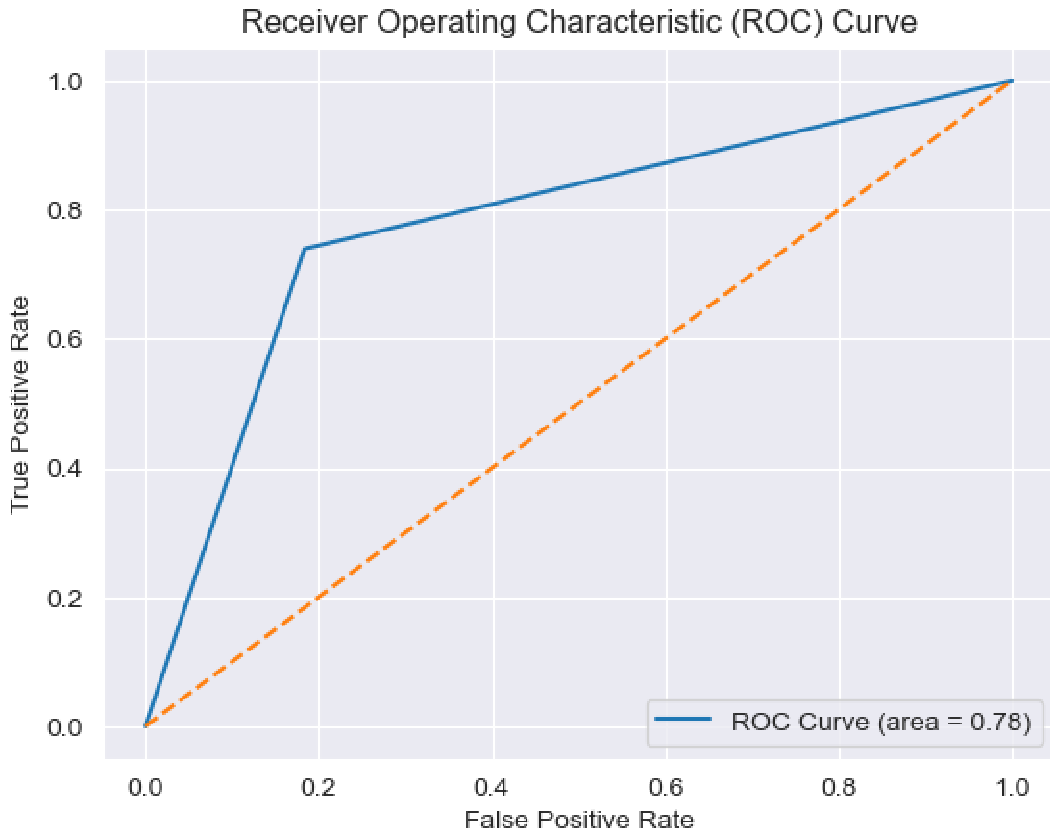

```
ROC AUC Score and ROC Curve are not applicable for multiclass classification.
Testing Naive Bayes on Penguins dataset with distribution=gaussian,
smoothing=1.0, criterion=log_likelihood
Accuracy: 0.90
Precision: 0.93
Recall: 0.90
F1 Score: 0.90
```



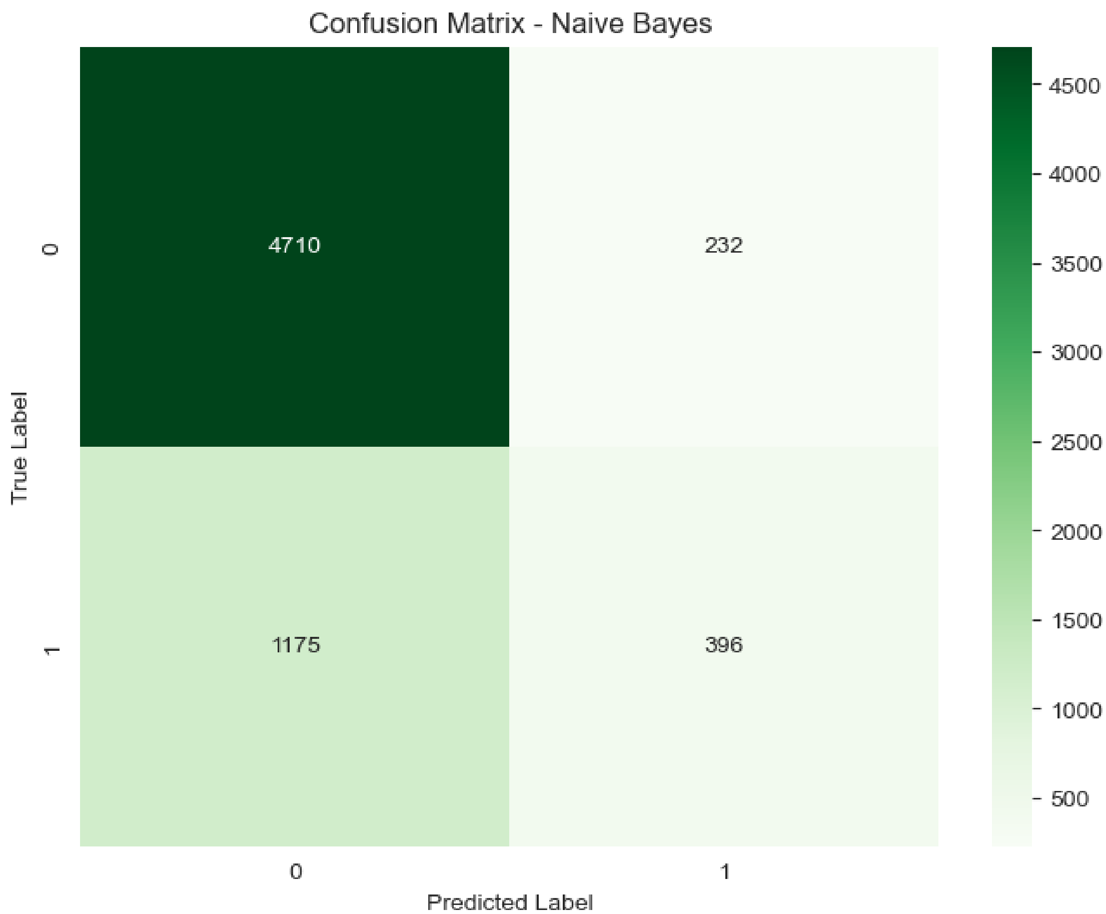

```
ROC AUC Score and ROC Curve are not applicable for multiclass classification.
Testing Naive Bayes on Titanic dataset with distribution=bernoulli,
smoothing=1.0, criterion=log_likelihood
Accuracy: 0.79
Precision: 0.79
Recall: 0.79
F1 Score: 0.79
```



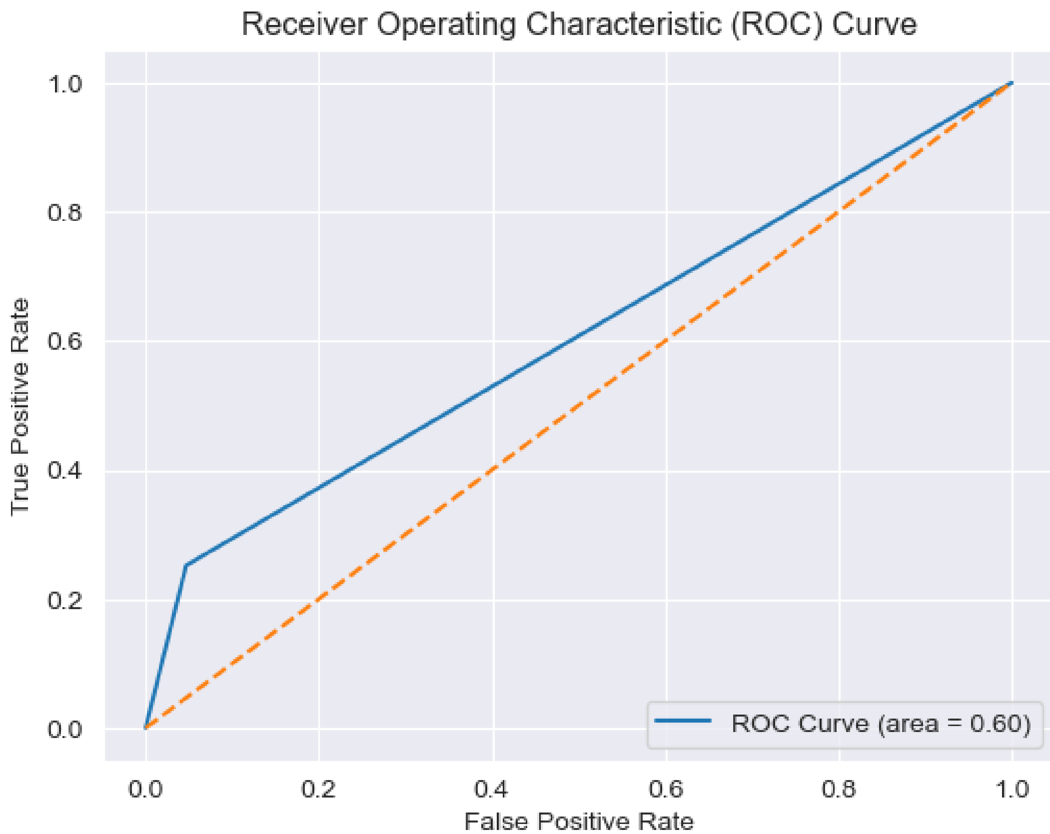

```
ROC AUC Score: 0.78
```




```
Testing Naive Bayes on Census dataset with distribution=multinomial,
smoothing=1.0, criterion=log_likelihood
Accuracy: 0.78
Precision: 0.76
Recall: 0.78
F1 Score: 0.75
```



ROC AUC Score: 0.60


Receiver Operating Characteristic (ROC) Curve
True Positive Rate
False Positive Rate
0.0
0.2
0.4
0.6
0.8
1.0
ROC Curve (area = 0.60)