

天线与电波传播 ANTENNAS AND WAVE PROPAGATION

LECTURE 6

Qingsha Cheng 程庆沙



Last Week

Polarization (linear, circular and elliptical)

Left and right hand polarization

Clockwise and counter-clockwise polarization

Polarization vector (normalized field vector)

Polarization ratio

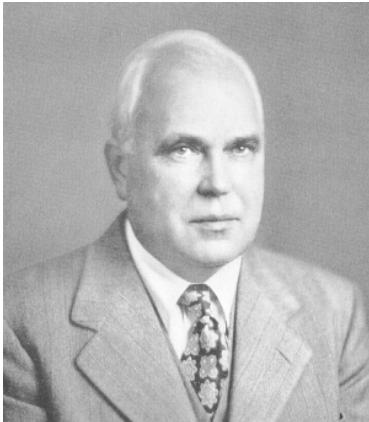
Axial ratio

Polarization loss factor

Friis transmission equation

RCS

Friis Transmission Equation



Harald Trap Friis (22 February 1893 – 15 June 1976), who published as H. T. Friis, was a Danish-American radio engineer whose work at Bell Laboratories included pioneering contributions to radio propagation, radio astronomy, and radar. His two Friis formulas remain widely used.

- Friis formula for noise factor
- Friis transmission equation (formula)

Friis transmission equation characterizes the behavior of a free-space radio circuit

FRIIS FREE-SPACE RADIO CIRCUIT



Video, 6'48"

Friis Transmission Equation

- essential in the analysis and design of wireless communication systems
- relates the power fed to the transmitting antenna and the power received by the receiving antenna
- antennas separated by a sufficiently large distance, i.e., they are in each other's **far zones**.

$$R \gg 2D_{\max}^2 / \lambda$$

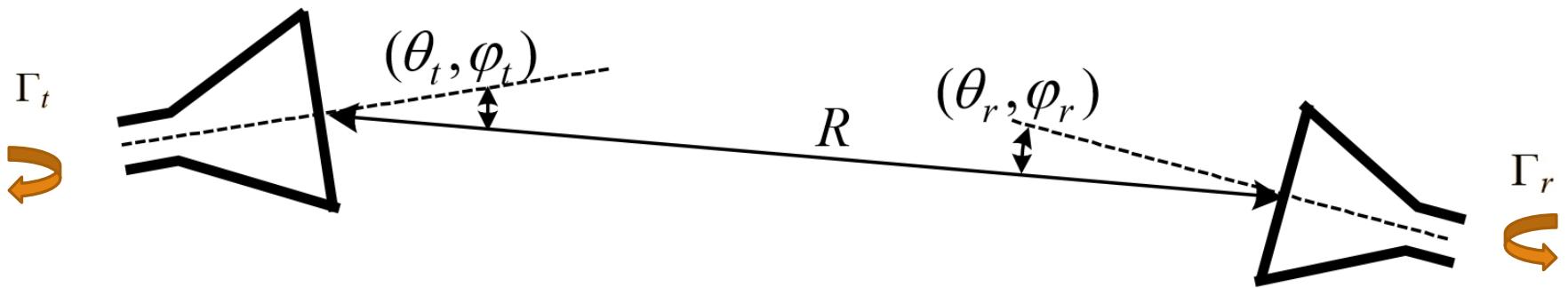
Friis Transmission Equation

ratio of the received to the transmitted power

$$\frac{P_r}{P_t} = \eta_t \eta_r |\boldsymbol{\rho}_t^* \cdot \boldsymbol{\rho}_r|^2 \left(\frac{\lambda}{4\pi R} \right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) (1 - |\Gamma_t|^2) (1 - |\Gamma_r|^2)$$

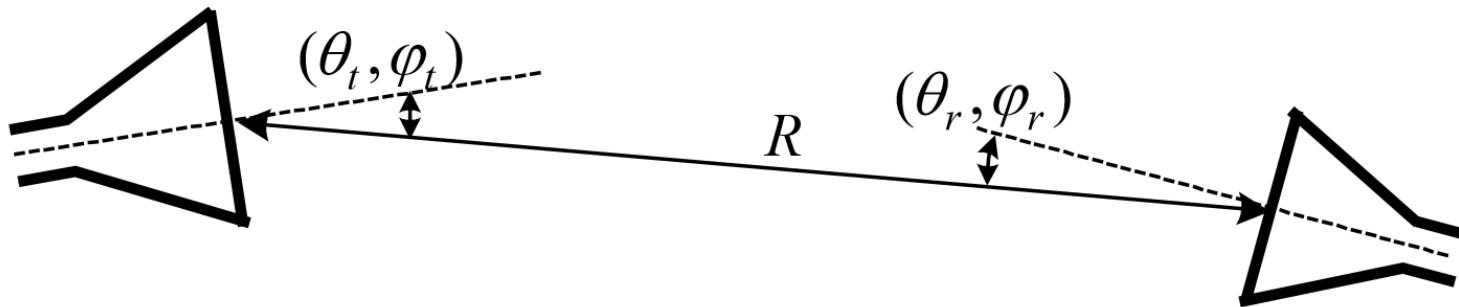
- P_r : Power received at the receiving antenna.
- P_t : Power transmitted from the transmitting antenna.
- η_t : Radiation efficiency of the transmitting antenna, accounting for losses due to non-ideal radiation.
- η_r : Radiation efficiency of the receiving antenna, also accounting for losses due to non-ideal radiation.
- $\boldsymbol{\rho}_t$: Polarization vector of the transmitting antenna, indicating the electric field's orientation and ellipticity.
- $\boldsymbol{\rho}_r$: Polarization vector of the receiving antenna.
- λ : Wavelength of the transmitted signal.
- R : Distance between the antennas.
- $D_t(\theta_t, \phi_t)$: Directivity of the transmitting antenna as a function of spherical coordinates.
- $D_r(\theta_r, \phi_r)$: Directivity of the receiving antenna as a function of spherical coordinates.
- Γ_t : Reflection coefficient at the transmitting antenna due to impedance mismatch.
- Γ_r : Reflection coefficient at the receiving antenna due to impedance mismatch.

Friis Transmission Equation



$$\frac{P_r}{P_t} = \eta_t \eta_r |\boldsymbol{\rho}_t^* \cdot \boldsymbol{\rho}_r|^2 \left(\frac{\lambda}{4\pi R} \right)^2 D_t(\theta_t, \phi_t) D_r(\theta_r, \phi_r) (1 - |\Gamma_t|^2) (1 - |\Gamma_r|^2)$$

Friis Transmission Equation



For **free-space loss factor**, **impedance-matched** and **polarization-matched** transmitting and receiving antennas

$$\frac{P_r}{P_t} = \left(\frac{\lambda}{4\pi R} \right)^2 G_t(\theta_t, \varphi_t) G_r(\theta_r, \varphi_r)$$

free-space loss factor

Friis Transmission Equation

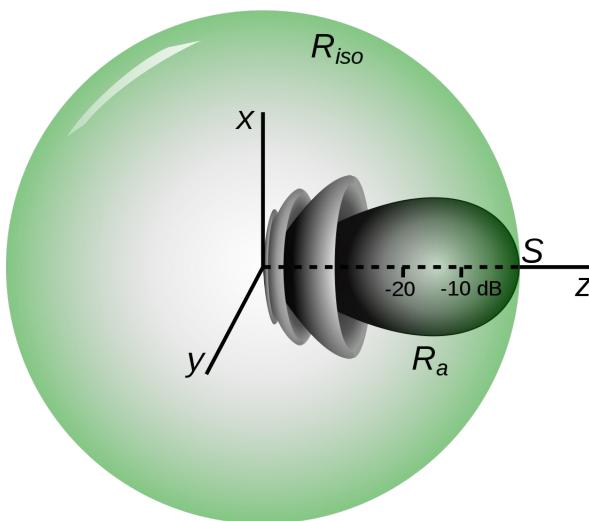
Friis' transmission equation frequently used to estimate the **maximum range** at which a wireless link can operate

$$R_{\max}^2 = \left(\frac{\lambda}{4\pi} \right)^2 \frac{P_t G_t(\theta_t, \phi_t) G_r(\theta_r, \phi_r)}{P_{r_{\min}}}$$

- $P_{r_{\min}}$: Minimum required received power for the receiving antenna to function properly.
- R_{\max} : Maximum distance at which the receiving antenna can detect the signal at the minimum required power level.

Effective Isotropically Radiated Power (EIRP)

a calculation used to estimate the radiated output power of an isotropic antenna (a theoretical half wave dipole antenna that radiates perfectly in all directions). This formula takes into account transmitter output power, cable loss, and antenna gain.



$$EIRP = P_T - L_C + G_a$$

Where,

EIRP (Effective Isotropic Radiated Power) = Output power of a signal when it is concentrated into a smaller area by the antenna

P_T = Output power of the transmitter (dBm)

L_C = Cable Loss (dB)

G_a = Antenna Gain (dBi)

Power Level (dBm)	Power Level (mW)
0 dBm	1 mW
10 dBm	10 mW
20 dBm	100 mW
30 dBm	1 W
40 dBm	10 W

Effective Isotropically Radiated Power (EIRP)

dBm is a unit of power expressed in decibels relative to one milliwatt.

$$P_{\text{dBm}} = 10 \log_{10}(P_{\text{W}} \times 1000)$$

- P_{dBm} is the power in dBm,
- P_{W} is the power in watts,
- The factor of 1000 converts watts to milliwatts since 1 watt = 1000 milliwatts.

Power Level (dBm)	Power Level (mW)
0 dBm	1 mW
10 dBm	10 mW
20 dBm	100 mW
30 dBm	1 W
40 dBm	10 W

Effective Isotropically Radiated Power (EIRP)

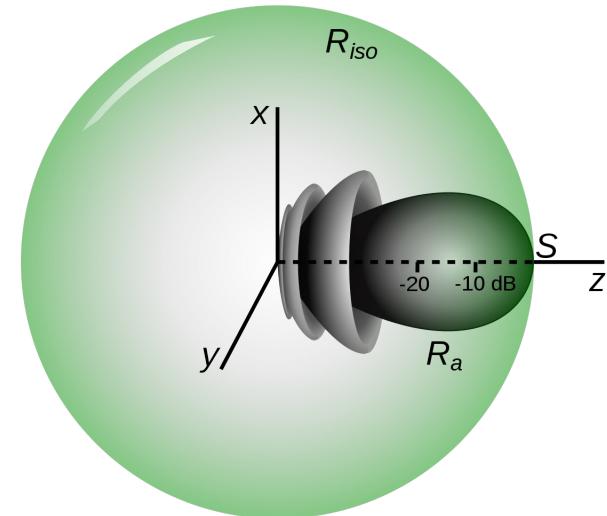
characterize a transmission system

$$EIRP = P_t G_t e_{TL}, \text{ W}$$

or

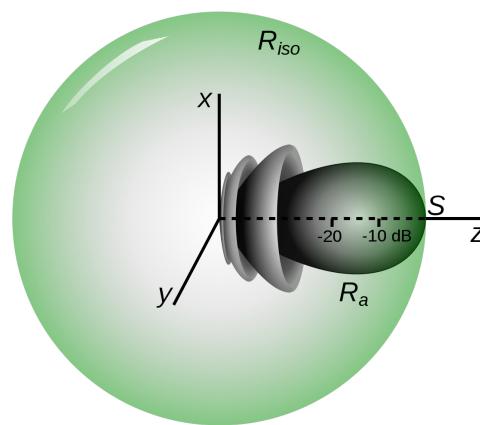
$$EIRP = 4\pi U_{\max,t}, \text{ W}$$

e_{TL} the loss efficiency of the transmission line connecting the transmitter to the antenna



Effective Isotropically Radiated Power (EIRP)

- **a fictitious amount** of power that an isotropic radiator would have to emit in order to produce the peak power density observed in the direction of the maximum radiation
- **much greater than the actual power** an antenna needs in order to achieve a given amount of radiation intensity in its direction of maximum radiation



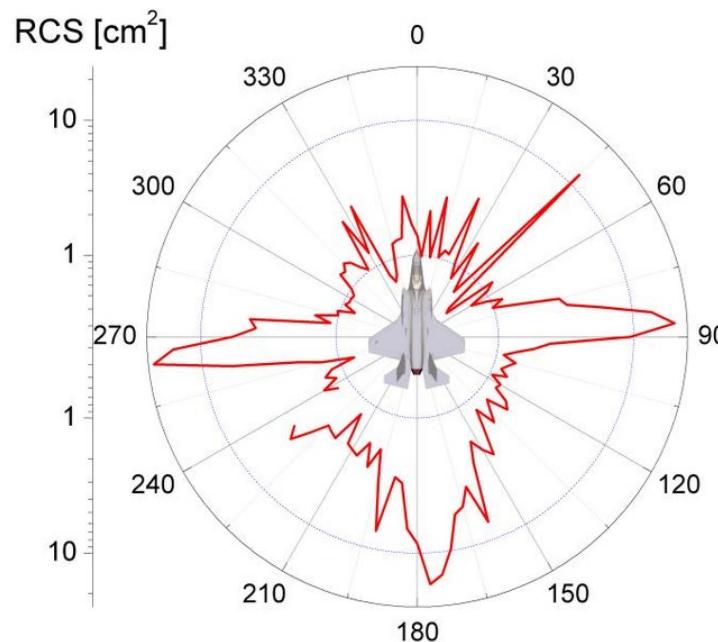
Effective Isotropically Radiated Power (EIRP)

- Max Tx power < 30 dBm (1 watt)
- Max EIRP < 36 dBm (4 watt)
- 2.4 GHz band, EIRP > 36 dBm allowed, but for every 3dBi increase of antenna gain, Tx power reduced by 1 dBm.

Transmit Power (dBm)	Antenna Gain (dBi)	EIRP (dBm)
30	6	36
29	9	38
28	12	40
27	15	42
26	18	44
25	21	46
24	24	48
23	27	50
22	30	52

Radar Cross-section (RCS)

The RCS or Echo Area of a target σ is the equivalent area capturing that amount of power, which, when scattered isotropically, produces at the receiver an amount of power density, which is equal to that scattered by the target itself:



Radar Cross-section (RCS)

$$\sigma = \lim_{R \rightarrow \infty} \left[4\pi R^2 \frac{W_s}{W_i} \right] = \lim_{R \rightarrow \infty} \left[4\pi R^2 \frac{|\mathbf{E}_s|^2}{|\mathbf{E}_i|^2} \right], \text{ m}^2.$$

R is the distance from the target, m;

W_i is the incident power density, W/m²;

W_s is the scattered power density at the receiver, W/m².

$$\lim_{R \rightarrow \infty} \left[\frac{\sigma W_i}{4\pi R^2} \right] = W_s(R).$$

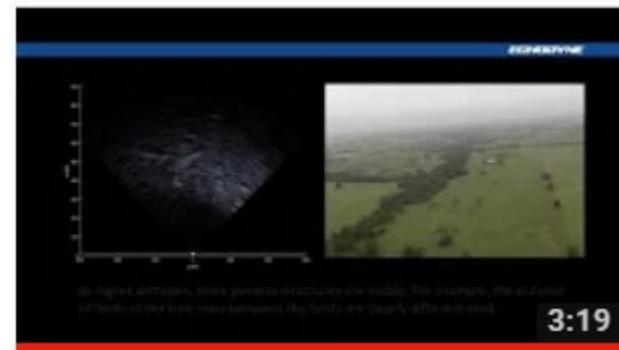
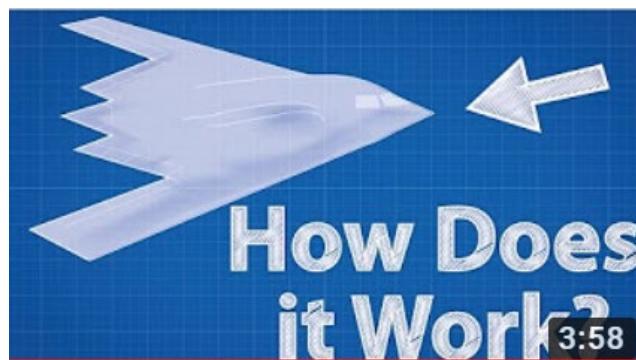
Radar Cross-section (RCS)

- trucks and jumbo jet airliners have large RCS, $\sigma > 100 \text{ m}^2$
- RCS increases also due to sharp metallic or dielectric edges and corners
- stealth military aircraft
 - achieved by careful shaping and coating (with special materials) of the outer surface of the airplane. The materials are mostly designed to absorb EM waves at the radar frequencies (usually S and X bands). The stealth aircraft has RCS smaller than 10^{-4} m^2 , which makes it comparable or smaller than the RCS of a penny.



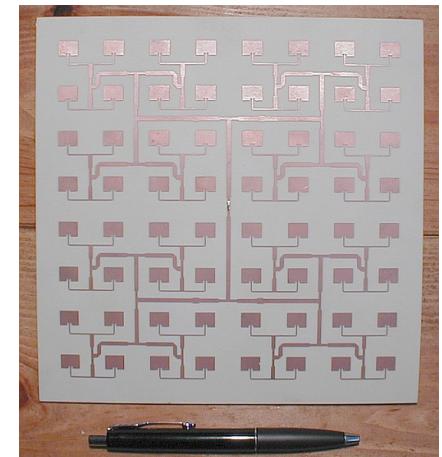
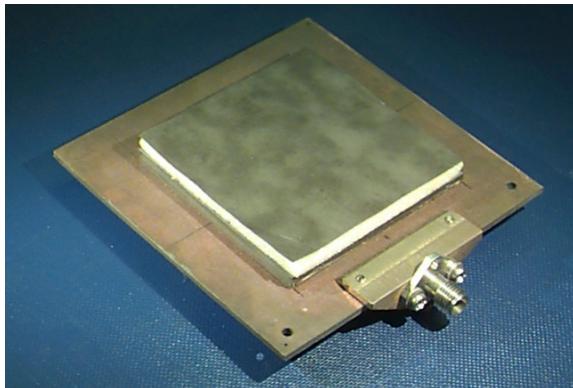
Radar Cross-section (RCS)

- Missile: 0.5 sq m
- Tactical Jet: 5 to 100 sq m;
- Bomber: 10 to 1000 sq m;
- Ships: 3,000 to 1,000,000 sq m.
- RCS can also be expressed in decibels referenced to a square meter (dBsm) which equals $10 \log (\text{RCS in } \text{m}^2)$.



Microstrip Antenna

- introduction
- physical and mathematical basis for understanding how microstrip antennas work.
- physical understanding of the basic physical properties of microstrip antennas.
- overview of some of the recent advances and trends in the area (but not an exhaustive survey – directed towards understanding the fundamental principles).



Topics

- Overview of microstrip antennas
- Basic principles of operation
- Feeding methods
- General characteristics
- CAD Formulas
- Radiation pattern
- Input Impedance
- Polarization

Notation Review

c = speed of light in free space

$$c = 2.99792458 \times 10^8 \text{ [m/s]}$$

λ_0 = wavelength of free space

$$\lambda_0 = c / f$$

k_0 = wavenumber of free space

$$k_0 = \omega \sqrt{\mu_0 \epsilon_0} = 2\pi / \lambda_0$$

k_1 = wavenumber of substrate

$$k_1 = k_0 \sqrt{\epsilon_r}$$

η_0 = intrinsic impedance of free space

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} \approx 376.7303 \text{ [\Omega]}$$

η_1 = intrinsic impedance of substrate

$$\eta_1 = \eta_0 / \sqrt{\epsilon_r}$$

ϵ_r = relative permittivity (dielectric constant) of substrate

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

ϵ_r^{eff} = effective relative permittivity
(accounting for fringing of flux lines at edges)

$$\mu_0 = 4\pi \times 10^7 \text{ [H/m]}$$

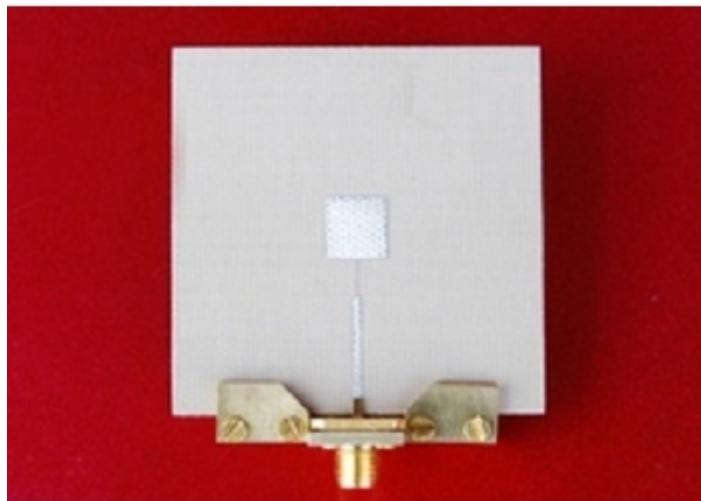
ϵ_{rc}^{eff} = complex effective relative permittivity
(used in the cavity model to account for all losses)

$$\epsilon_0 = \frac{1}{\mu_0 c^2} \approx 8.854188 \times 10^{12} \text{ [F/m]}$$

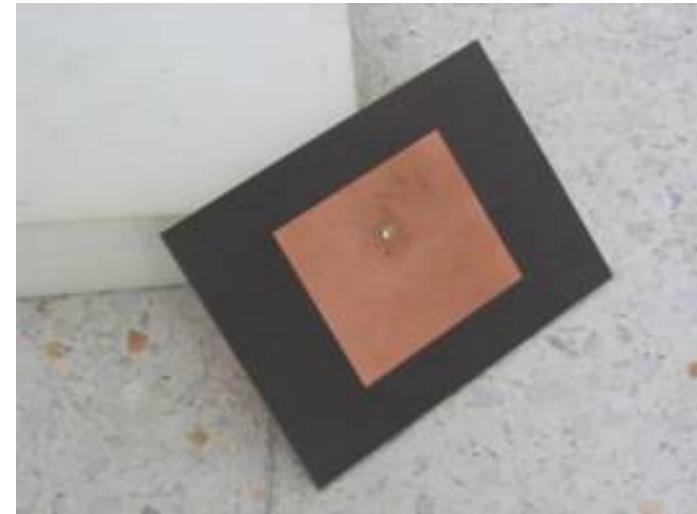
Overview of Microstrip Antennas (MSA)

Also called “**patch antennas**”

- One of the most useful antennas at microwave frequencies ($f > 1$ GHz).
- It usually consists of a metal “patch” on top of a grounded dielectric substrate.
- The patch may be in a variety of shapes, but rectangular and circular are the most common.



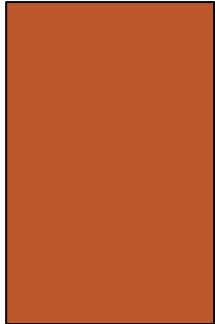
Microstrip line feed



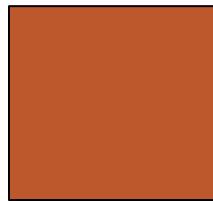
Coax feed

Overview of Microstrip Antennas

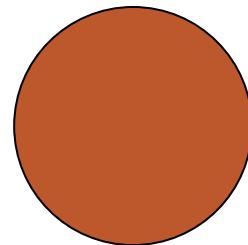
Common Shapes



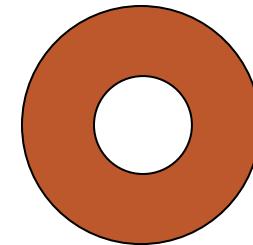
Rectangular



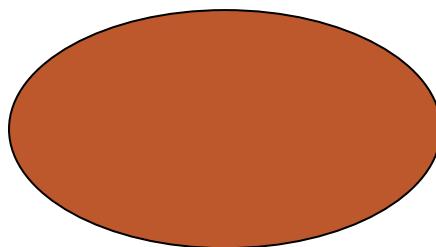
Square



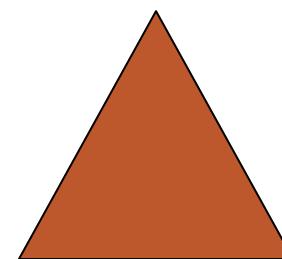
Circular



Annular ring



Elliptical



Triangular

Overview of Microstrip Antennas

History

- Invented by Bob Munson in 1972 (but earlier work by Deschamps goes back to 1953).
- Became popular starting in the 1970s.

G. Deschamps and W. Sichak, "Microstrip Microwave Antennas," Proc. of Third Symp. on USAF Antenna Research and Development Program, October 18–22, 1953.

R. E. Munson, "Microstrip Phased Array Antennas," *Proc. of Twenty-Second Symp. on USAF Antenna Research and Development Program*, October 1972.

R. E. Munson, "Conformal Microstrip Antennas and Microstrip Phased Arrays," *IEEE Trans. Antennas Propagat.*, vol. AP-22, no. 1 (January 1974): 74–78.

Overview of Microstrip Antennas

Advantages of Microstrip Antennas

- Low profile (can even be “conformal,” i.e., 共形, flexible to conform to a surface).
- Easy to fabricate (use etching and photolithography).
- Easy to feed (coaxial cable, microstrip line, etc.).
- Easy to incorporate with other microstrip circuit elements and integrate into systems.
- Patterns are somewhat hemispherical, with a moderate directivity (about 6-8 dB is typical).
- Easy to use in an array to increase the directivity.

Overview of Microstrip Antennas

Disadvantages of Microstrip Antennas

- Low bandwidth (but can be improved by a variety of techniques). Bandwidths of a few percent are typical. Bandwidth is roughly proportional to the substrate thickness and inversely proportional to the substrate permittivity.
- Efficiency may be lower than with other antennas. Efficiency is limited by conductor and dielectric losses*, and by surface-wave loss**.
- Only used at microwave frequencies and above (the substrate becomes too large at lower frequencies).
- Cannot handle extremely large amounts of power (dielectric breakdown).

* Conductor and dielectric losses become more severe for thinner substrates.

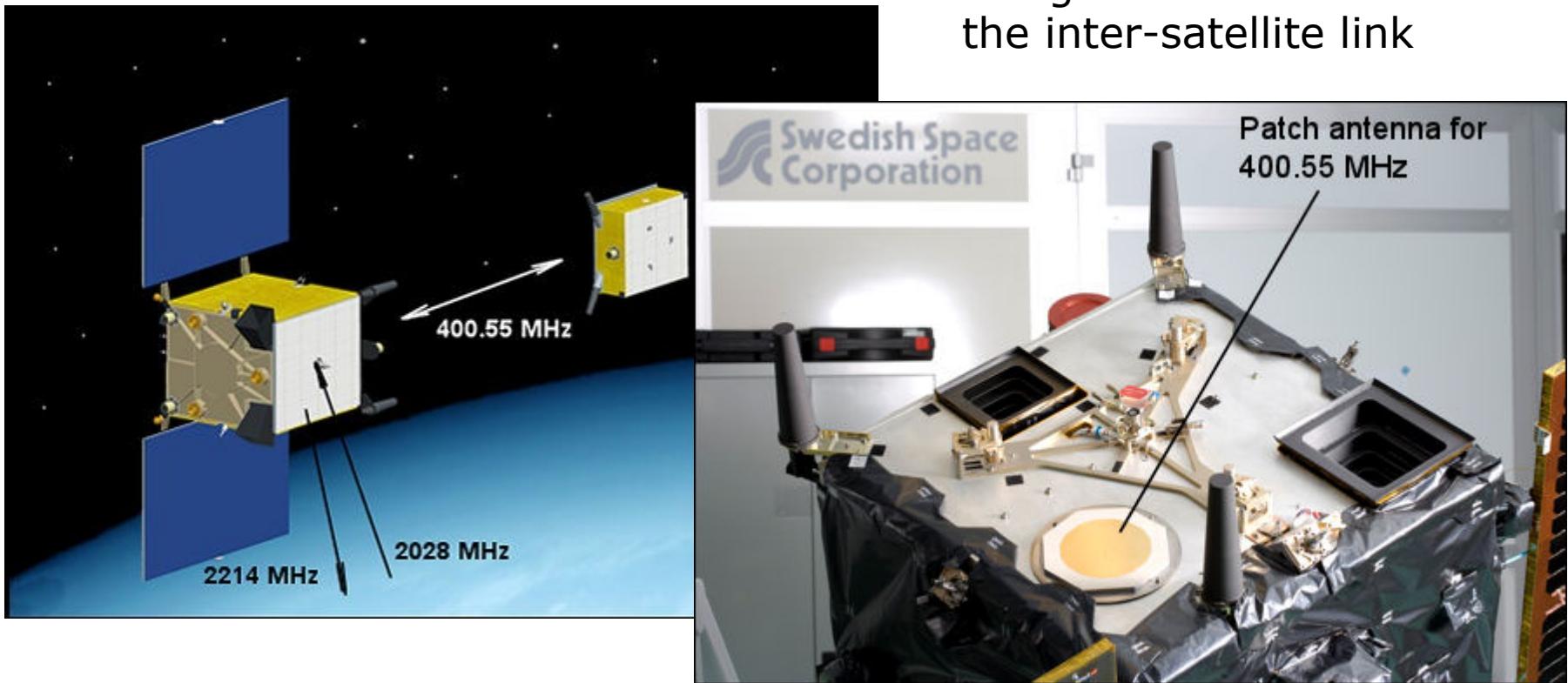
** Surface-wave losses become more severe for thicker substrates (unless air or foam is used).

Overview of Microstrip Antennas

Applications Example

- Satellite communications

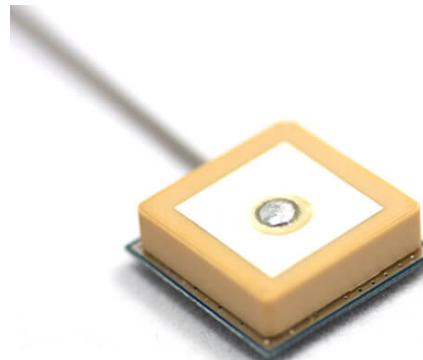
patch antenna on the
Mango satellite used for
the inter-satellite link



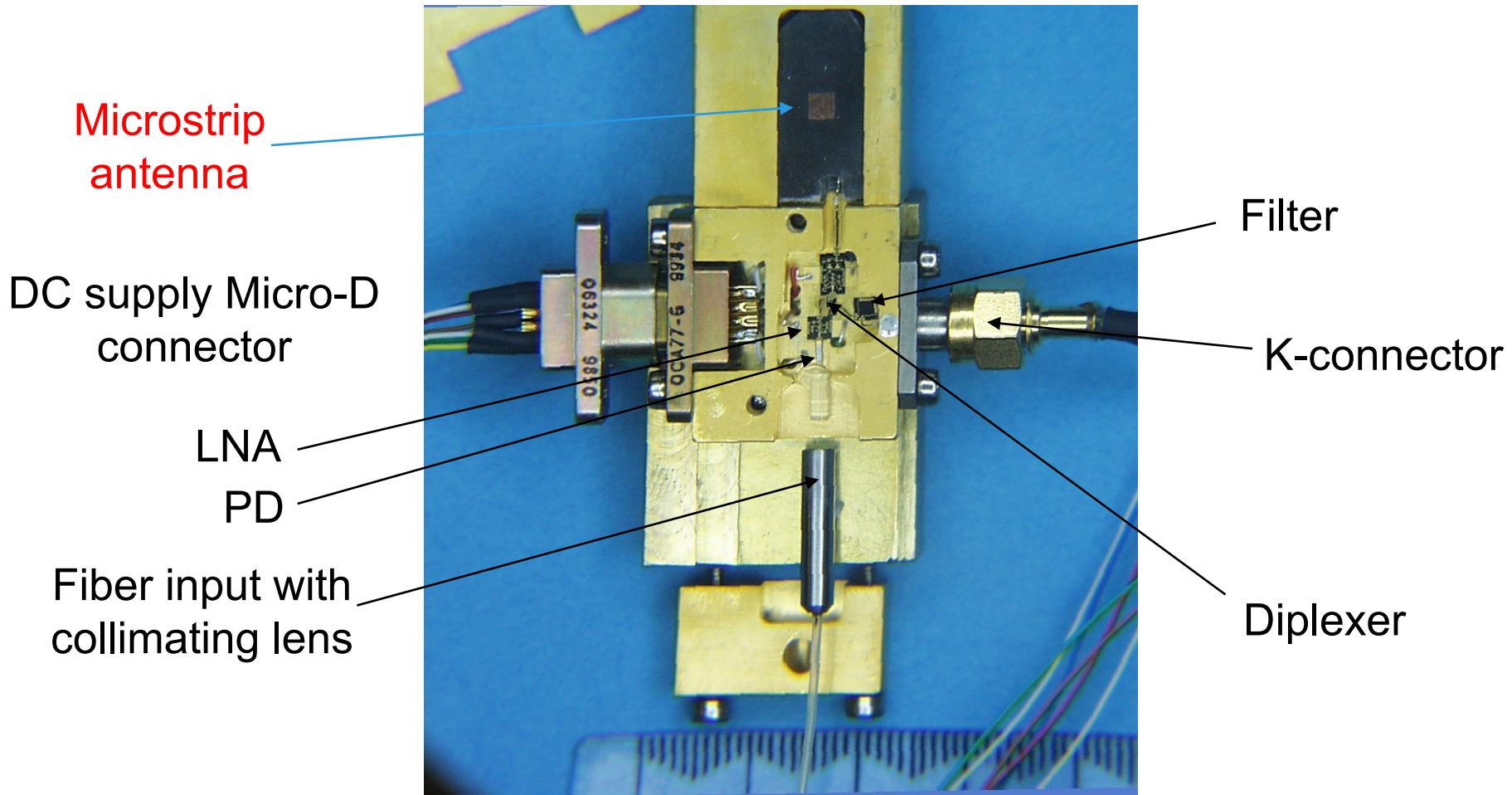
Overview of Microstrip Antennas

Applications Example

- Satellite communications
- Microwave communications
- Cell phone antennas
- GPS antennas

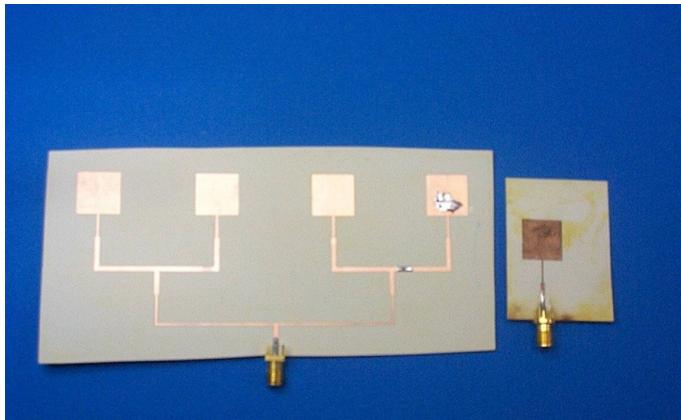


Overview of Microstrip Antennas

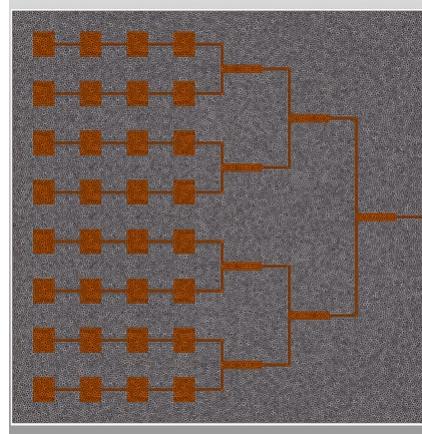


Microstrip Antenna Integrated into a System: HIC Antenna Base-Station for 28-43 GHz
(Photo courtesy of Dr. Rodney B. Waterhouse)

Overview of Microstrip Antennas

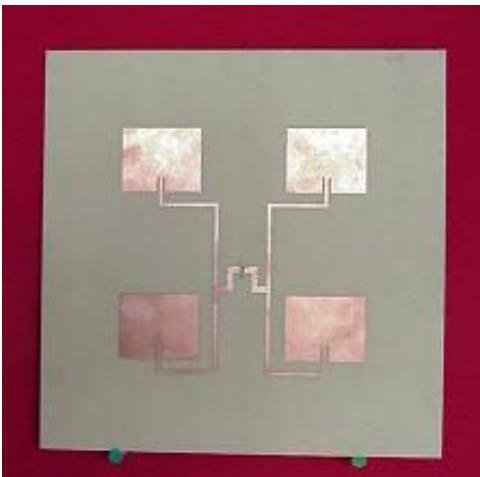


Arrays

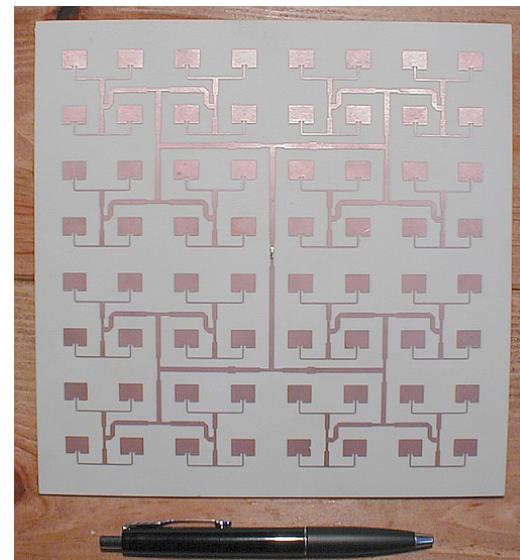


Linear array (1-D corporate feed)

4 × 8 corporate-fed / series-fed array



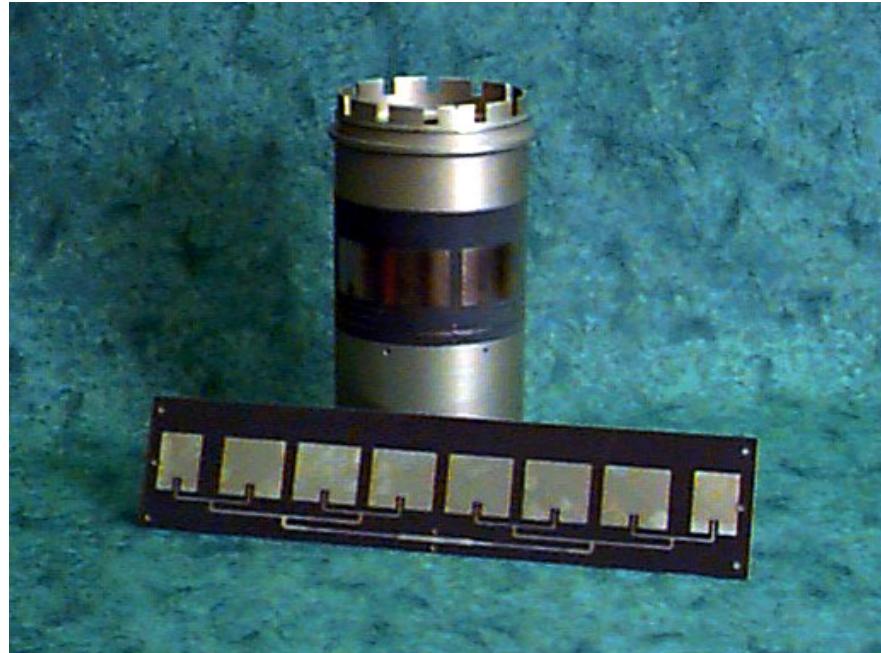
2×2 array



2-D 8X8 corporate-fed array

Overview of Microstrip Antennas

Wraparound Array (conformal)



The substrate is so thin that it can be bent to “conform” to the surface.

(Photo courtesy of Dr. Rodney B. Waterhouse)

Overview of Microstrip Antennas

Textile Antenna for Wearable Applications

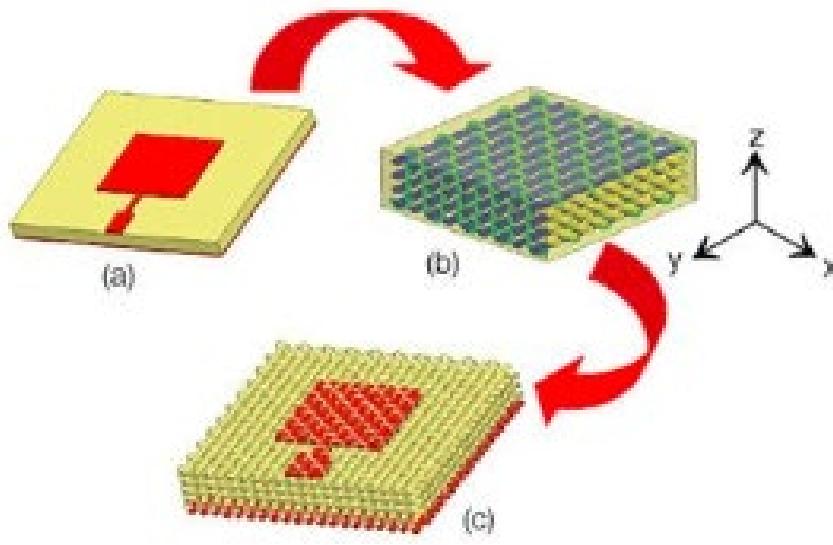


Fig. 6: Geometry of textile antenna

<https://electronicsforu.com/technology-trends/microstrip-antenna-applications/2>

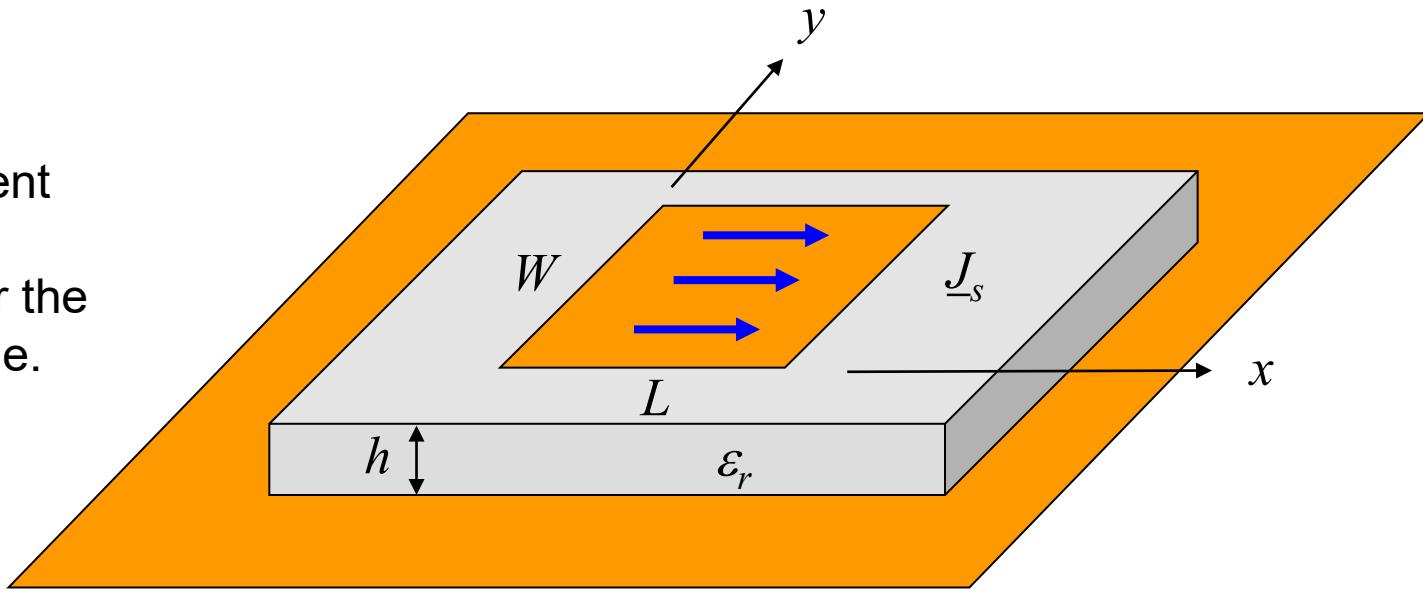


Overview of Microstrip Antennas

Rectangular patch

Note 1:

The fields and current are approximately independent of y for the dominant $(1,0)$ mode.



Note 2:

L is the resonant dimension (direction of current flow).

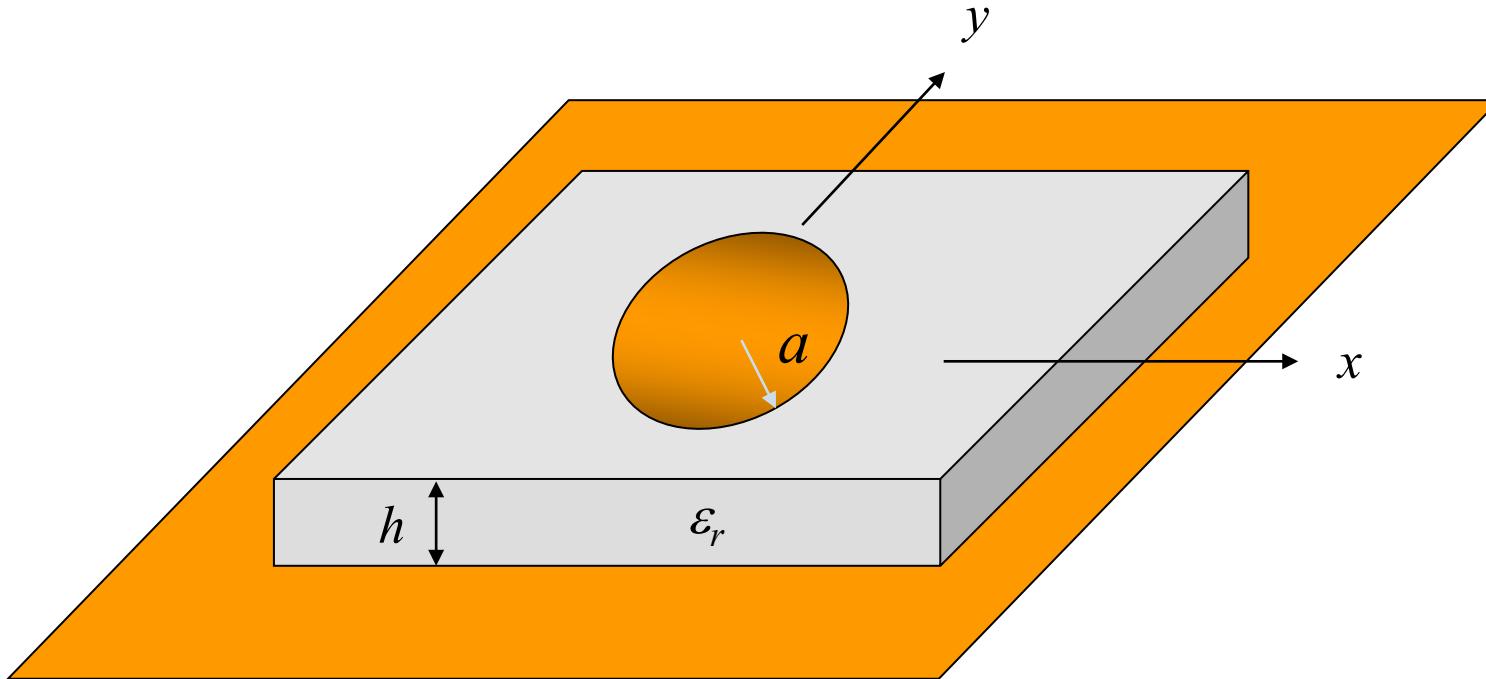
The width W is usually chosen to be larger than L (to get higher bandwidth).

However, usually $W < 2L$ (to avoid problems with the $(0,2)$ mode).

$W = 1.5L$ is typical.

Overview of Microstrip Antennas

Circular Patch



The **location of the feed** determines the direction of current flow and hence the **polarization** of the radiated field.

Basic Principles of Operation

- The basic principles for a rectangular patch illustrated here, but the principles apply similarly for other patch shapes.
- We use the cavity model to explain the operation of the patch antenna.

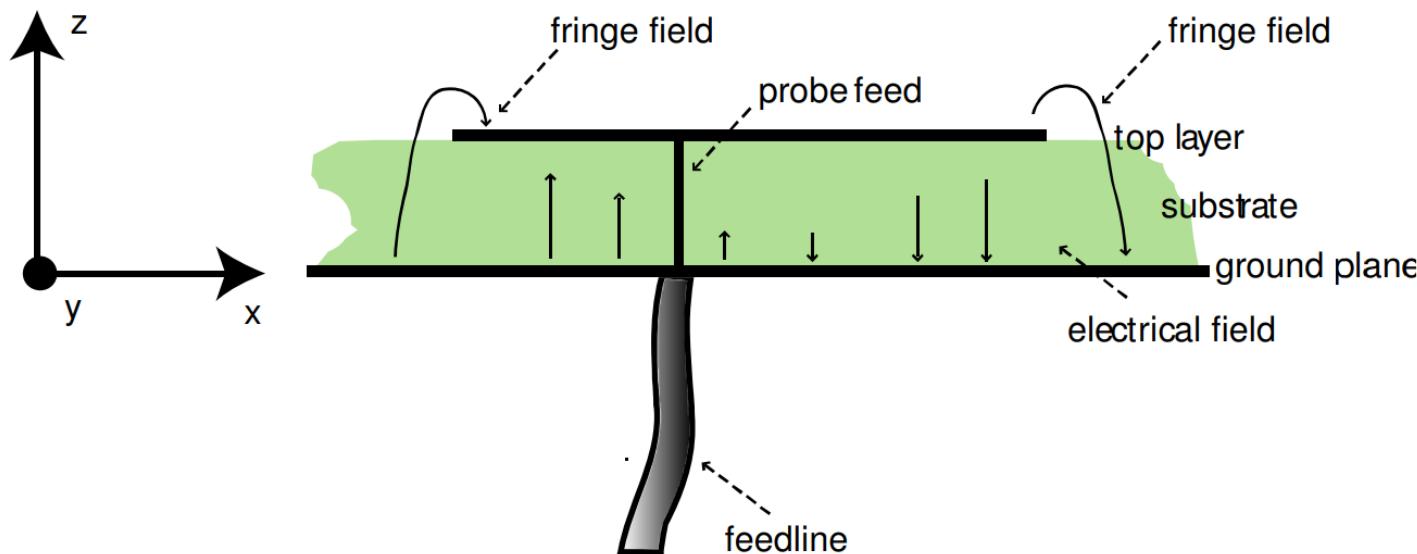
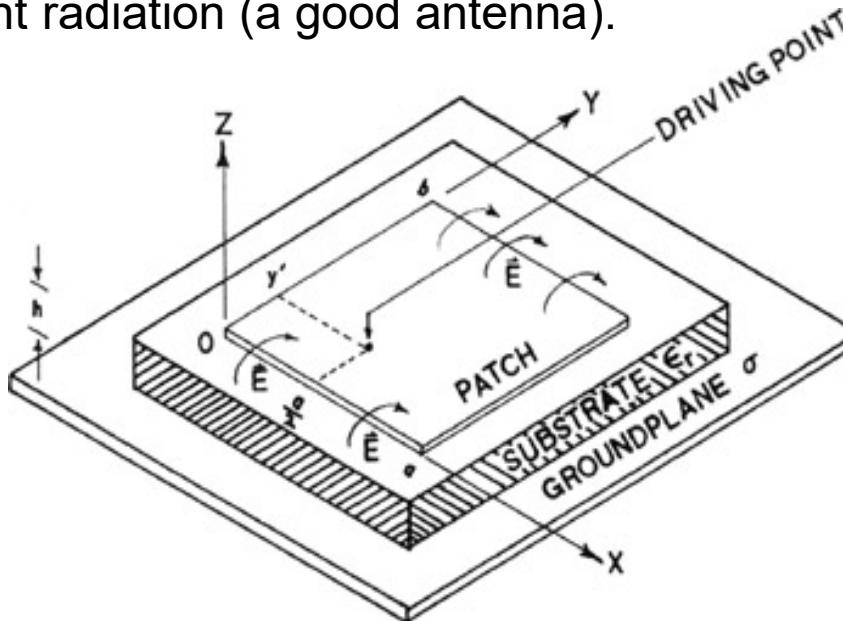


Figure 1: Cross section of a patch antenna in its basic form

Basic Principles of Operation

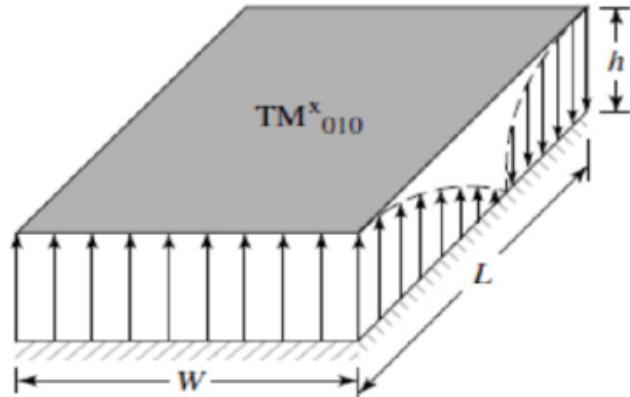
Main Ideas:

- The patch acts approximately as a **resonant cavity** (with short-circuit (PEC) walls on top and bottom, open-circuit (PMC) walls on the edges).
- In a cavity, **only certain modes** are allowed to exist, at different resonance frequencies.
- If the antenna is excited at a **resonance frequency**, a strong field is set up inside the cavity, and a strong current on the (bottom) surface of the patch. This produces significant radiation (a good antenna).

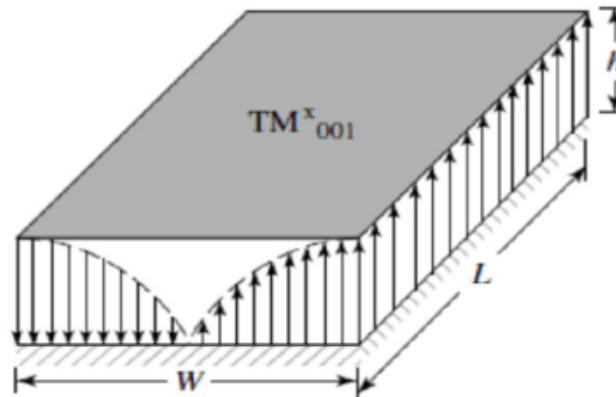


Basic Principles of Operation

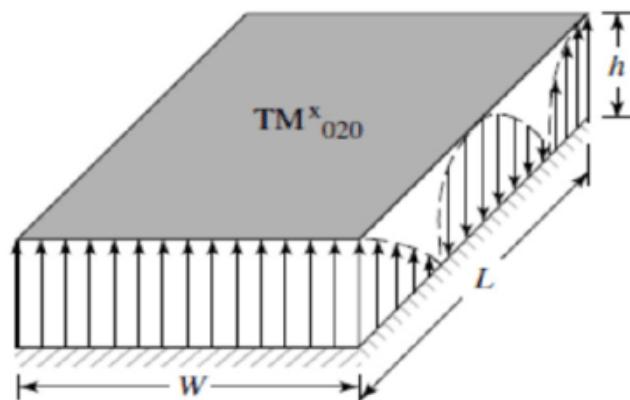
Main Ideas: Resonant Modes



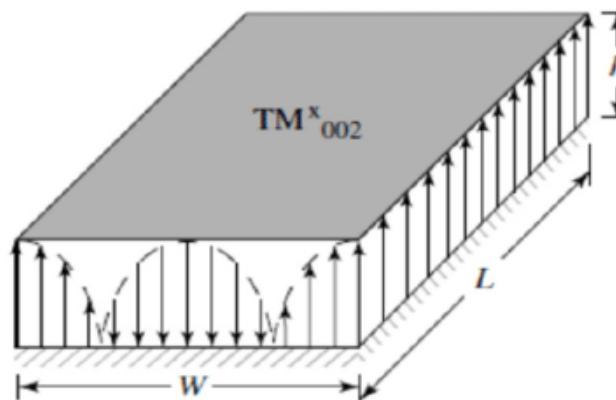
(a) TM^x_{010}



(b) TM^x_{001}



(c) TM^x_{020}



(d) TM^x_{002}

Basic Principles of Operation

Resonance Frequency of Dominant Mode

The **resonance frequency** is mainly controlled by the **patch length L** and the **substrate permittivity**.

Approximately, (assuming PMC walls)

$$k_1^2 = \left(\frac{m\pi}{L} \right)^2 + \left(\frac{n\pi}{W} \right)^2$$

This is equivalent to saying that the length L is one-half of a wavelength in the dielectric.

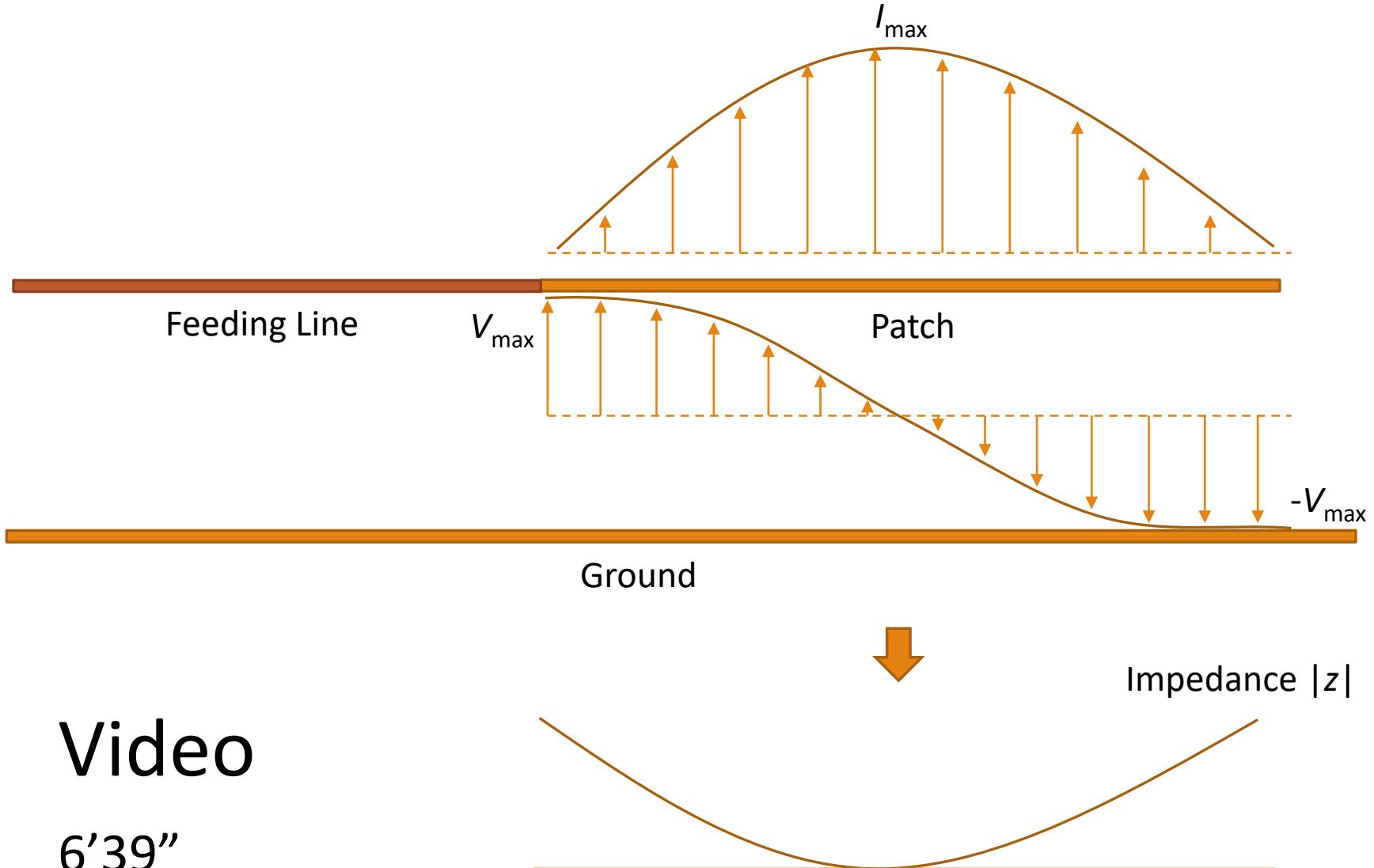
(1,0) mode: $k_1 L = \pi$  $L = \lambda_d / 2 = \frac{\lambda_0 / 2}{\sqrt{\epsilon_r}}$



Comment:

A higher substrate permittivity allows for a smaller antenna (miniaturization), but with a lower bandwidth.

Basic Principles of Operation



Basic Principles of Operation

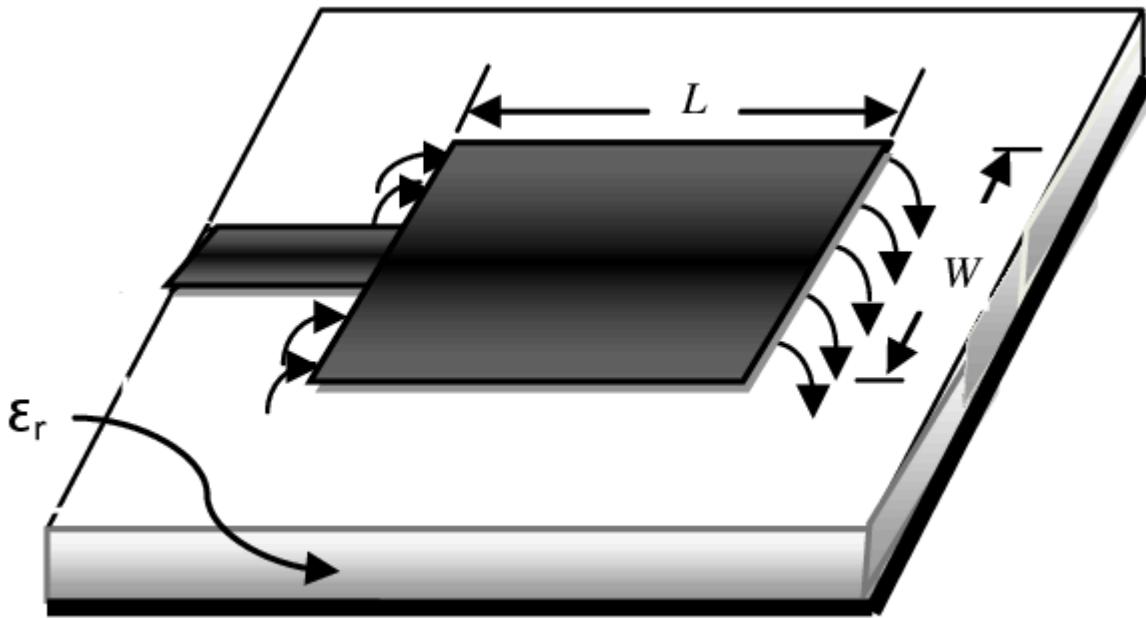


Figure 1. Rectangular microstrip antenna

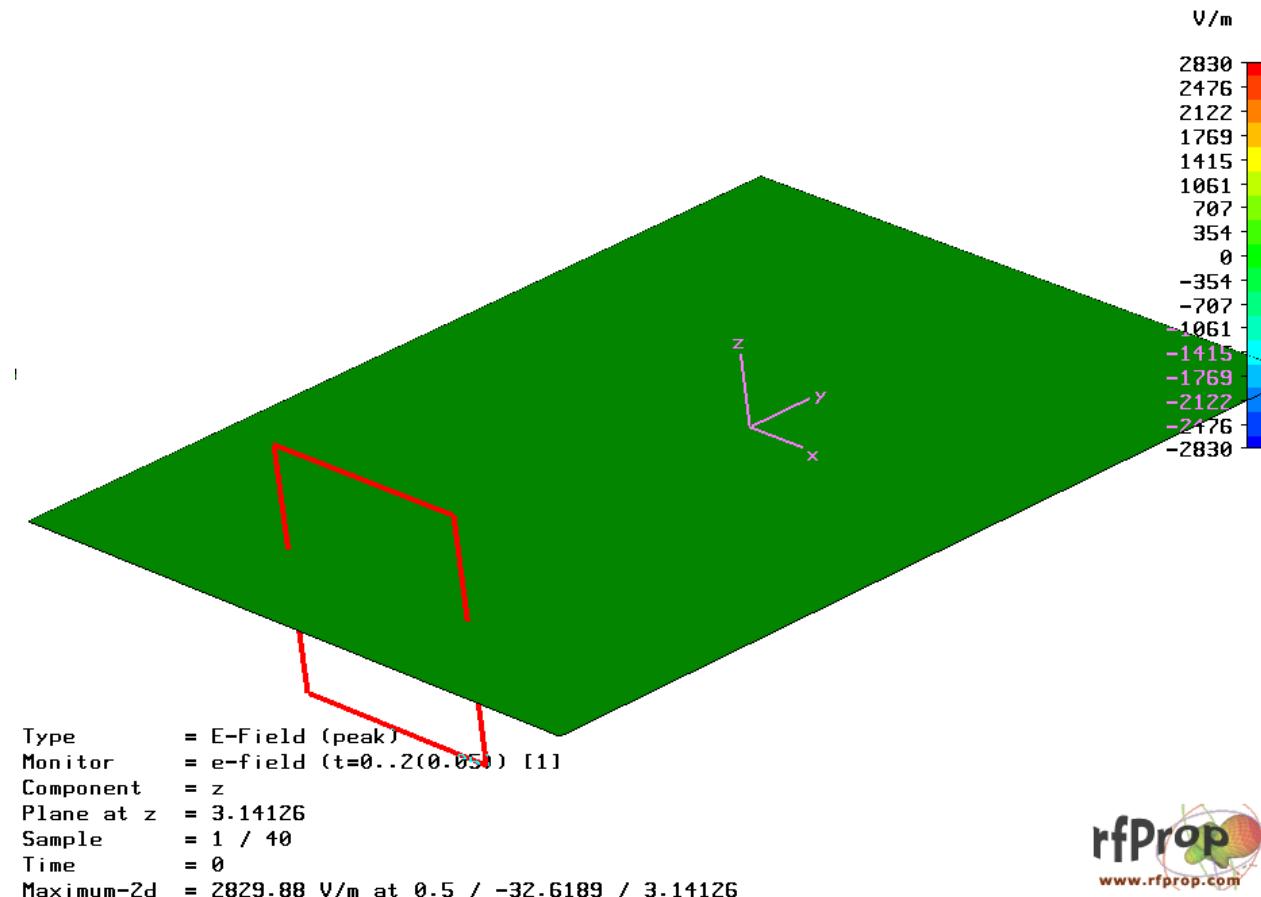
Fringing Field:

- Electromagnetic fields that extend beyond the physical boundaries of the patch, particularly at its edges.
- Induced by surface currents on the patch, creating time-varying electromagnetic fields.
- Critical components for transition of electromagnetic energy from the antenna structure to free space.

Basic Principles of Operation

Animation

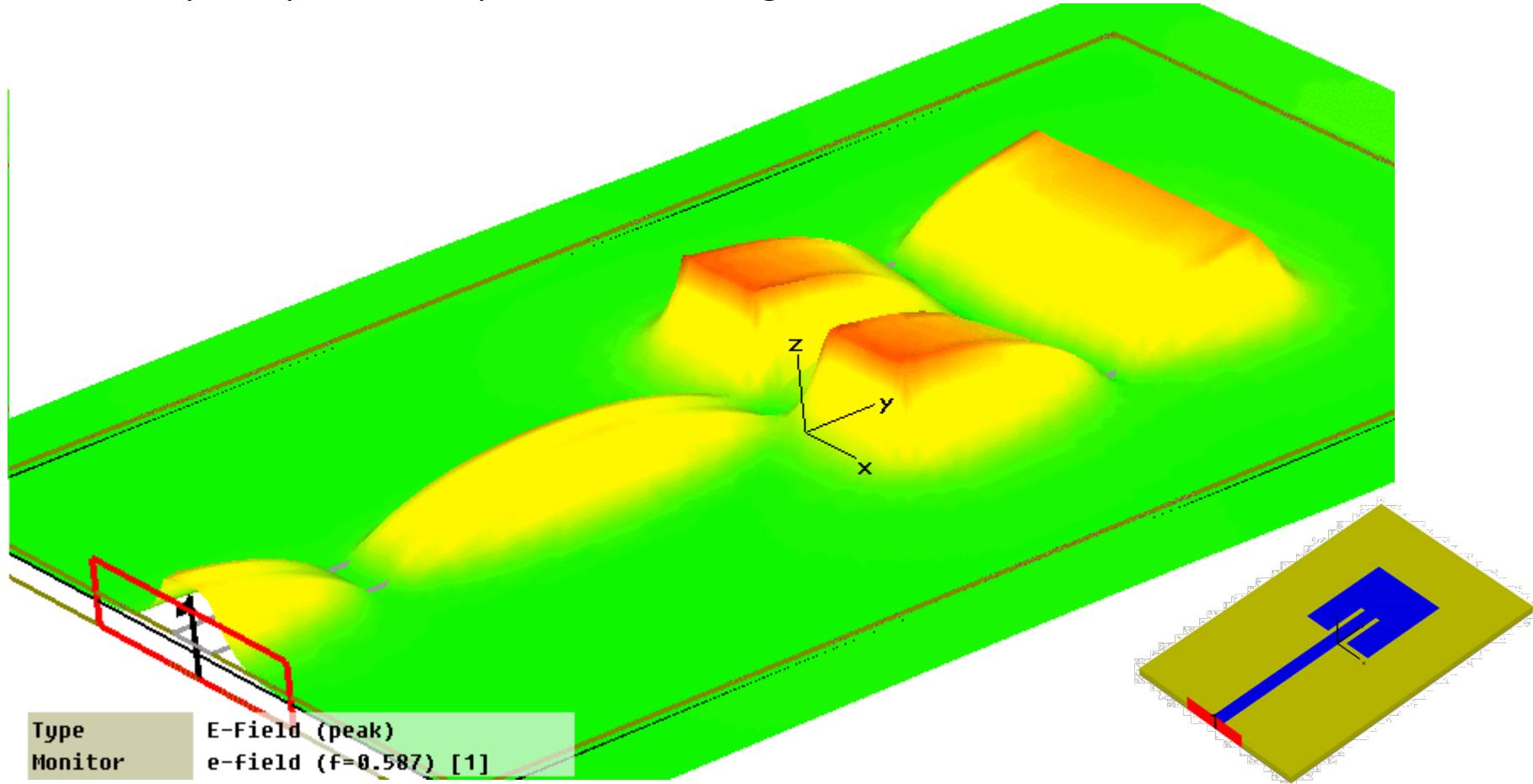
Rectangular patch antenna fed by microstrip line - animation of the vertical component of the electric field (dominant mode). The antenna is not perfectly matched, note standing waves at the feeding line



Basic Principles of Operation

Animation

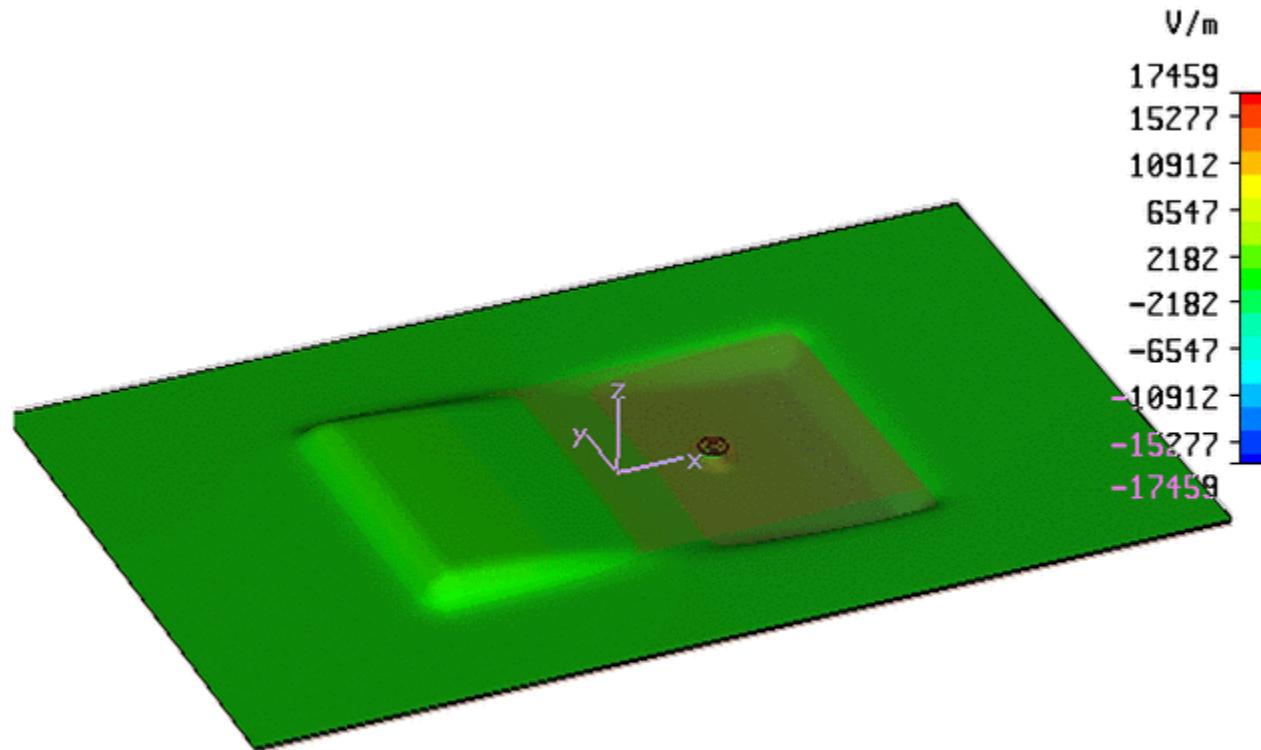
the 3-D absolute value of the electric field for a rectangular microstrip patch antenna fed with a microstrip line. The antenna operates at 587 MHz. The inset termination of the microstrip line provides impedance matching with the antenna close to 50 Ohms



Basic Principles of Operation

Animation

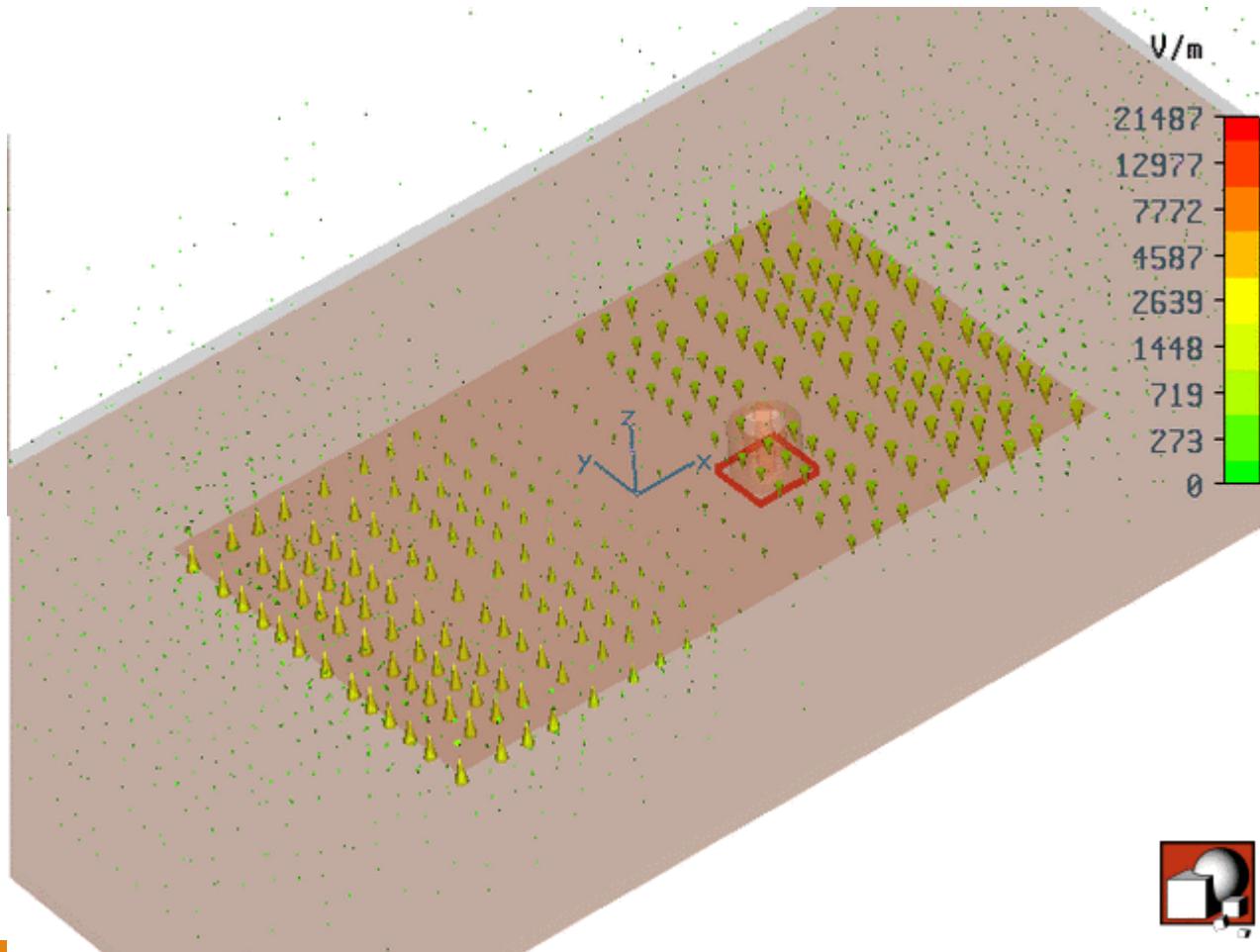
Rectangular patch antenna fed by coaxial line - animation of the vertical component of the electric field (dominant mode).



Basic Principles of Operation

Animation

now the fully vector E-field is shown. Note the fringing field at the shorter (radiating) edges.

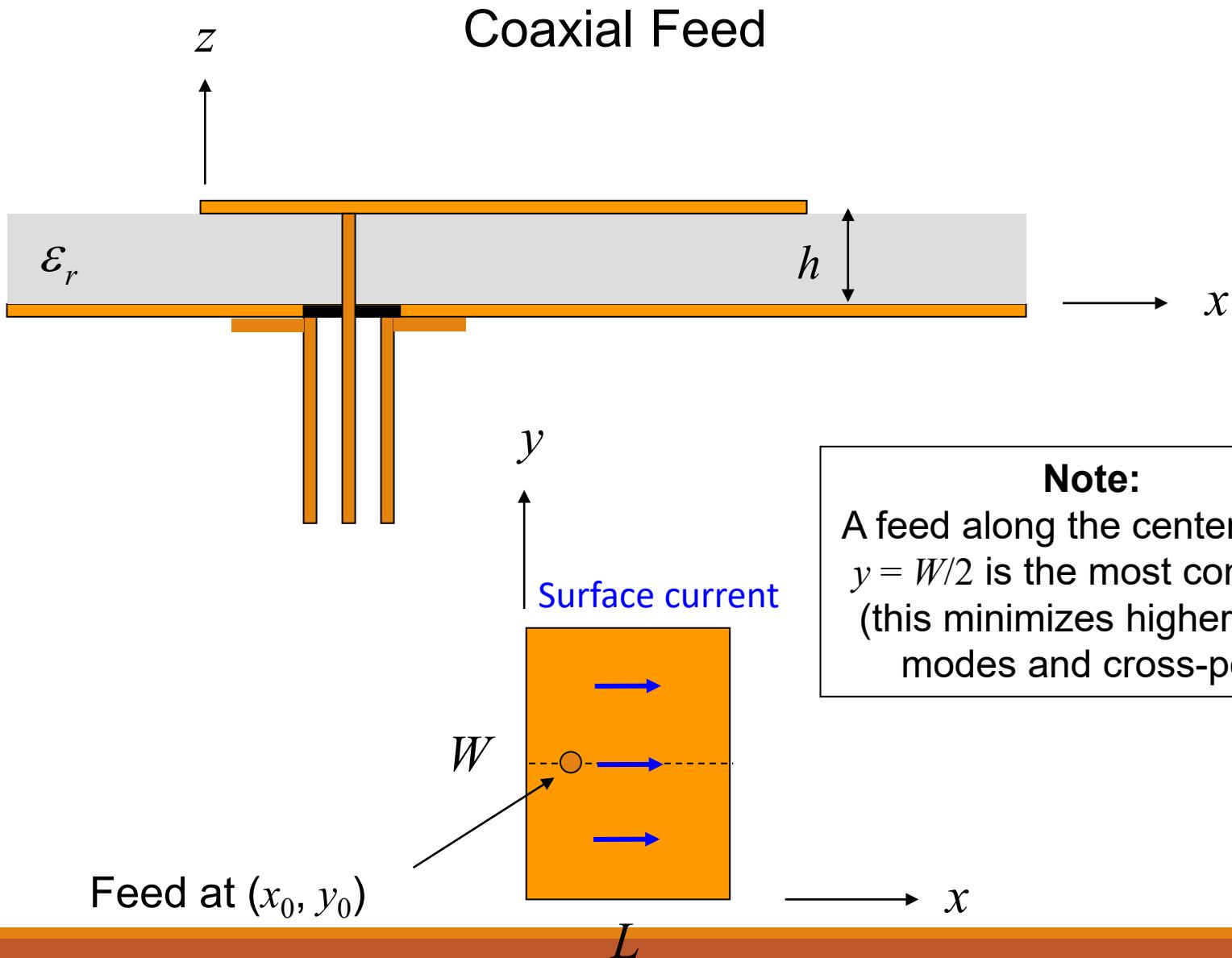


Feeding Methods

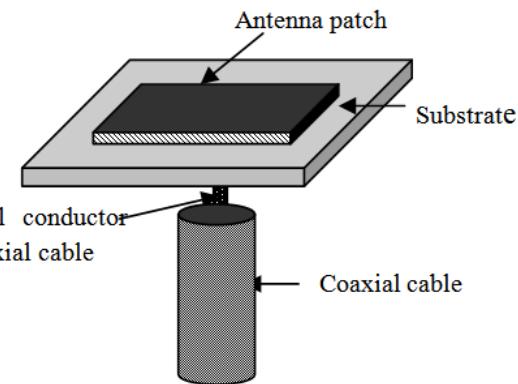
Some of the more common methods for feeding microstrip antennas are shown.

The feeding methods are illustrated for a rectangular patch, but the principles apply for circular and other shapes as well.

Feeding Methods



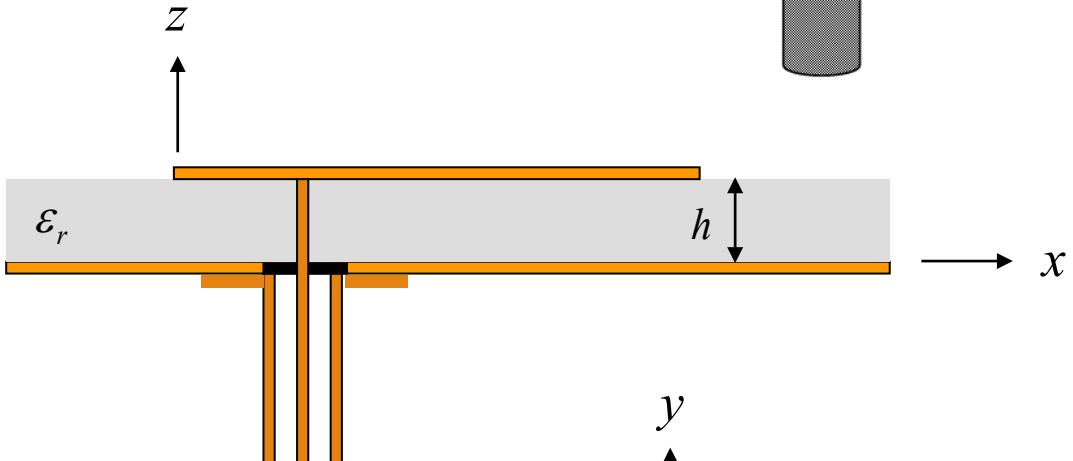
Feeding Methods



Coaxial Feed

$$R = R_{edge} \cos^2 \left(\frac{\pi x_0}{L} \right)$$

(The resistance varies as the square of the modal field shape.)

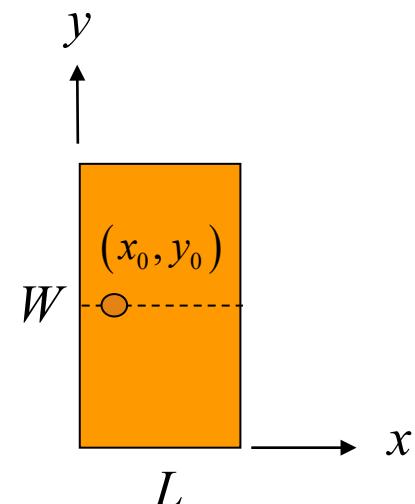


Advantages:

- Simple
- Directly compatible with coaxial cables
- Easy to obtain input match by adjusting feed position

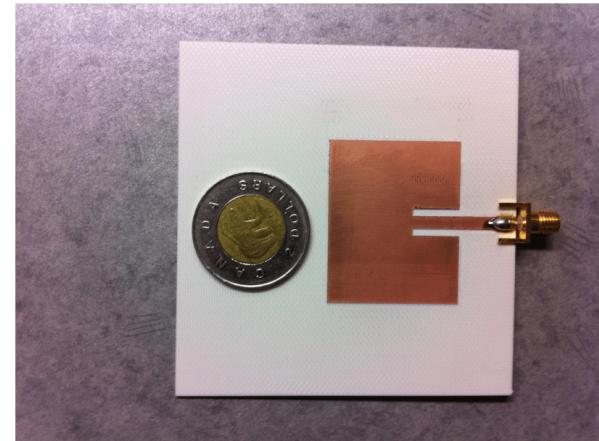
Disadvantages:

- Significant probe (feed) radiation for thicker substrates
- Significant probe inductance for thicker substrates (limits bandwidth)
- Not easily compatible with arrays



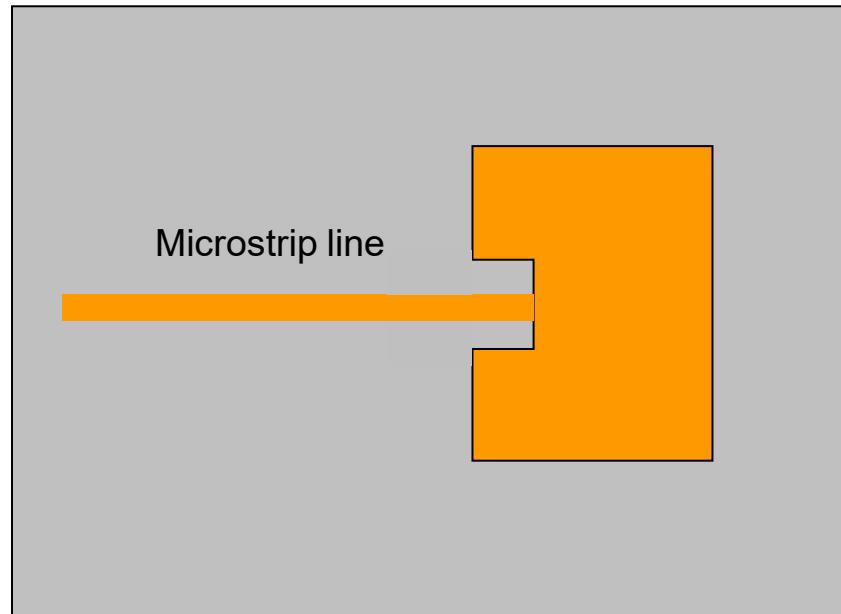
Feeding Methods

Inset Feed



Advantages:

- Simple
- Allows for planar feeding
- Easy to use with arrays
- Easy to obtain input match



Disadvantages:

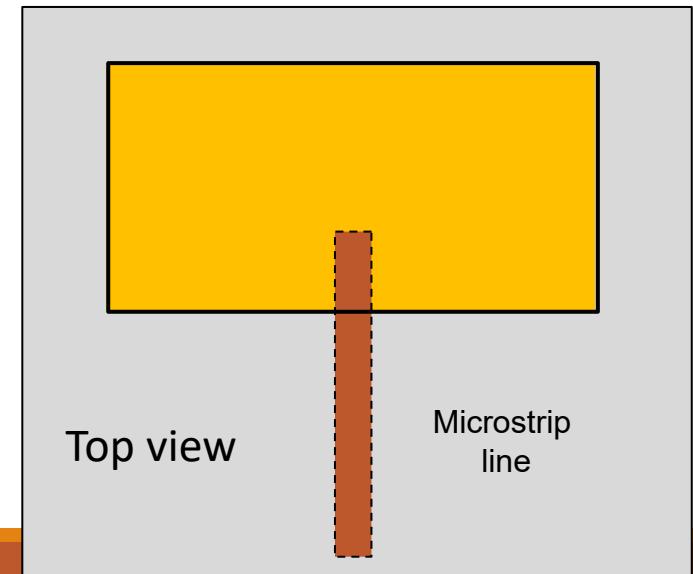
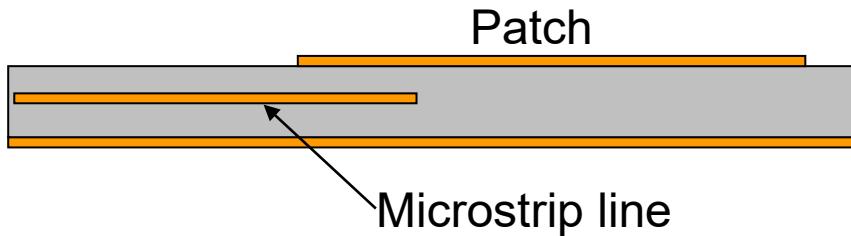
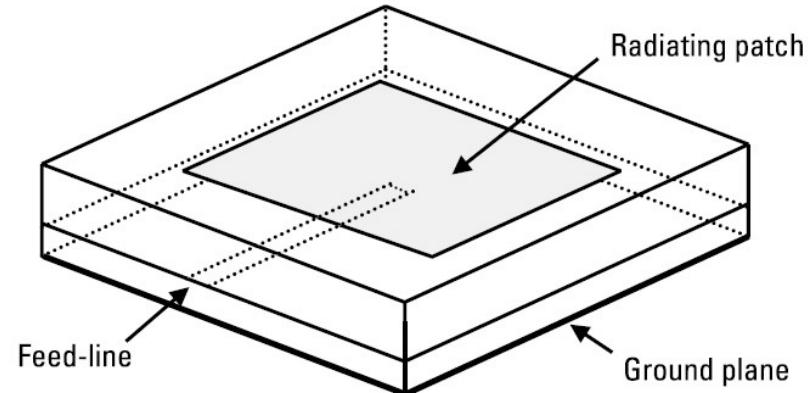
- Significant line radiation for thicker substrates
- For deep notches, patch current and radiation pattern may show distortion

Feeding Methods

Proximity-coupled Feed (Electromagnetically-coupled Feed)

Advantages:

- Allows for planar feeding
- Less line radiation compared to microstrip feed
- Can allow for higher bandwidth (no probe inductance, so substrate can be thicker)



Disadvantages:

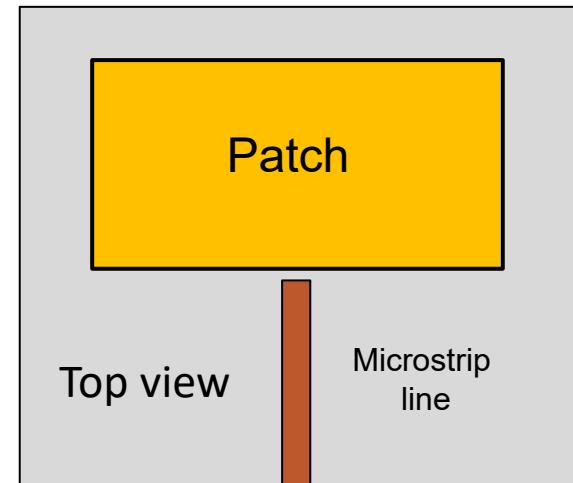
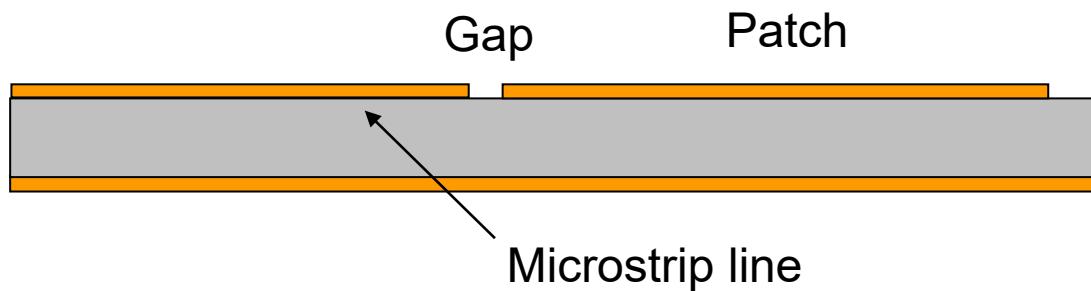
- Requires multilayer fabrication
- Alignment is important for input match

Feeding Methods

Gap-coupled Feed

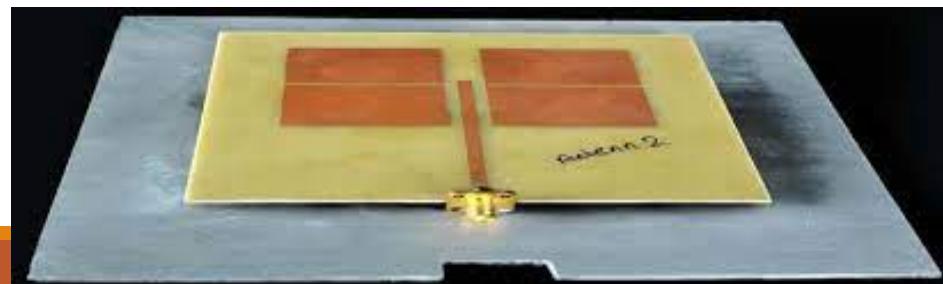
Advantages:

- Allows for planar feeding
- Can allow for a match even with high edge impedances, where a notch might be too large (e.g., when using high permittivity)



Disadvantages:

- Requires accurate gap fabrication
- Requires full-wave design

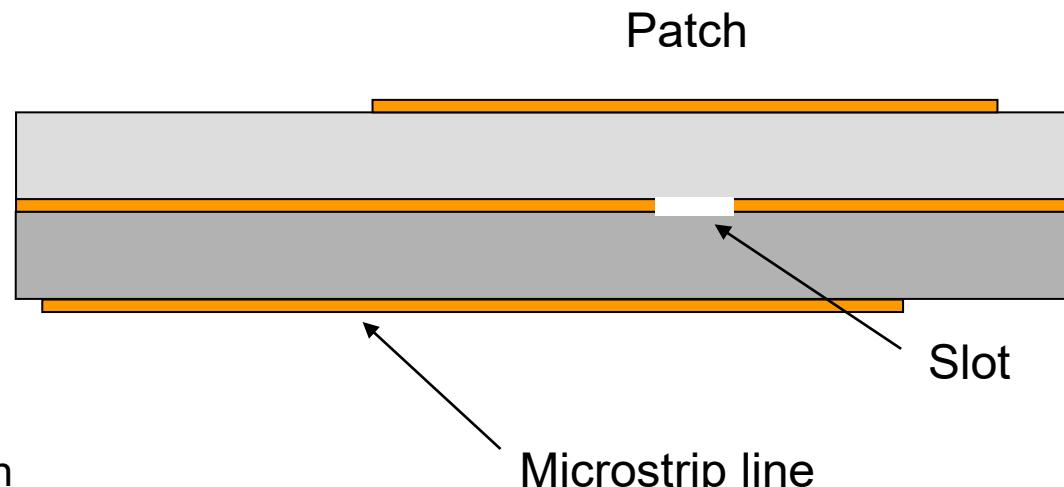
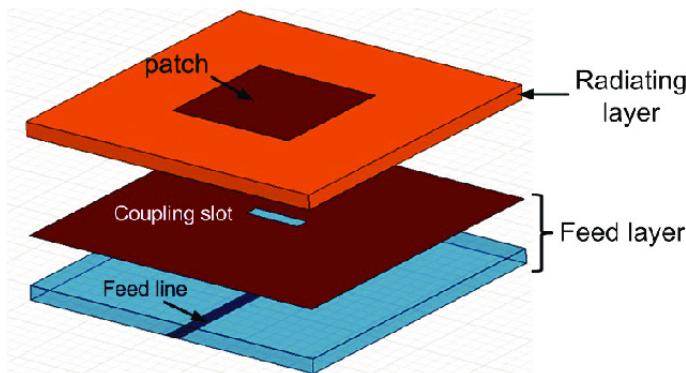
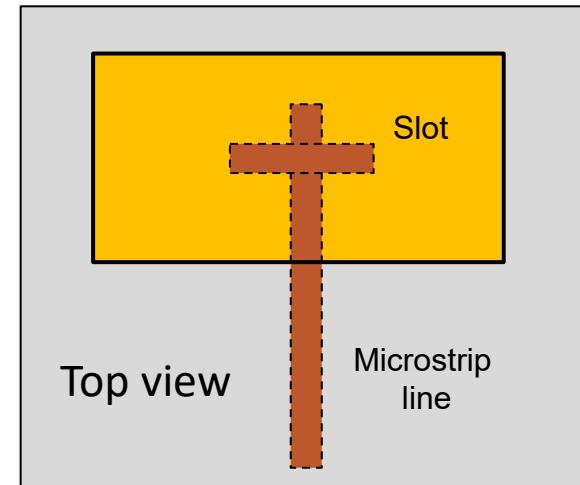


Feeding Methods

Aperture-coupled Patch (ACP)

Advantages:

- Allows for planar feeding
- Feed-line radiation is isolated from patch radiation
- Higher bandwidth is possible since probe inductance is eliminated (allowing for a thick substrate), and also a double-resonance can be created
- Allows for use of different substrates to optimize antenna and feed-circuit performance



Disadvantages:

- Requires multilayer fabrication
- Alignment is important for input match

General Characteristics

Bandwidth

- The bandwidth is directly proportional to substrate thickness h .
- However, if $h > 0.05\lambda_0$, the probe inductance (for a coaxial feed) becomes large enough so that matching is difficult – the bandwidth will decrease.
- The bandwidth is inversely proportional to ε_r (a foam substrate gives a high bandwidth).
- The bandwidth of a rectangular patch is proportional to the patch width W (but we need to keep $W < 2L$; see the next slide).

General Characteristics

Width Restriction for a Rectangular Patch

$$W < 2L$$

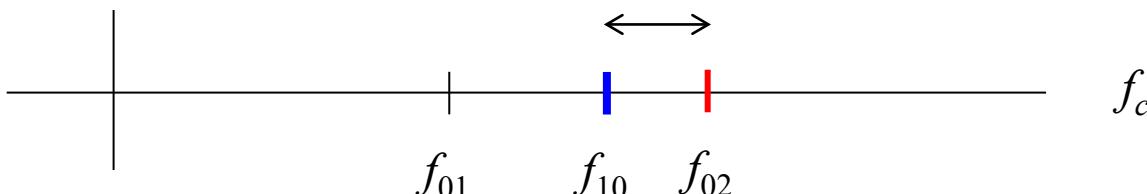
$$f_{01} = \frac{c}{2\sqrt{\epsilon_r}} \left(\frac{1}{W} \right)$$

$$f_{mn} = \frac{c}{2\pi\sqrt{\epsilon_r}} \sqrt{\left(\frac{m\pi}{L} \right)^2 + \left(\frac{n\pi}{W} \right)^2}$$

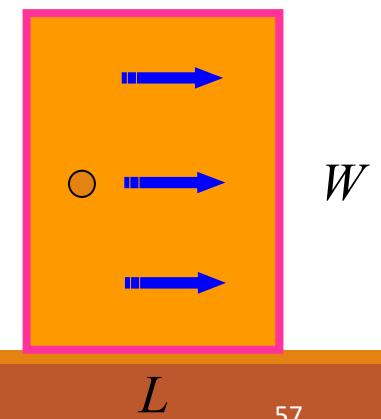
$$f_{10} = \frac{c}{2\sqrt{\epsilon_r}} \left(\frac{1}{L} \right)$$

$$f_{02} - f_{01} = \frac{c}{\sqrt{\epsilon_r}} \left(\frac{1}{W} - \frac{1}{2L} \right)$$

$$f_{02} = \frac{c}{2\sqrt{\epsilon_r}} \left(\frac{2}{W} \right)$$



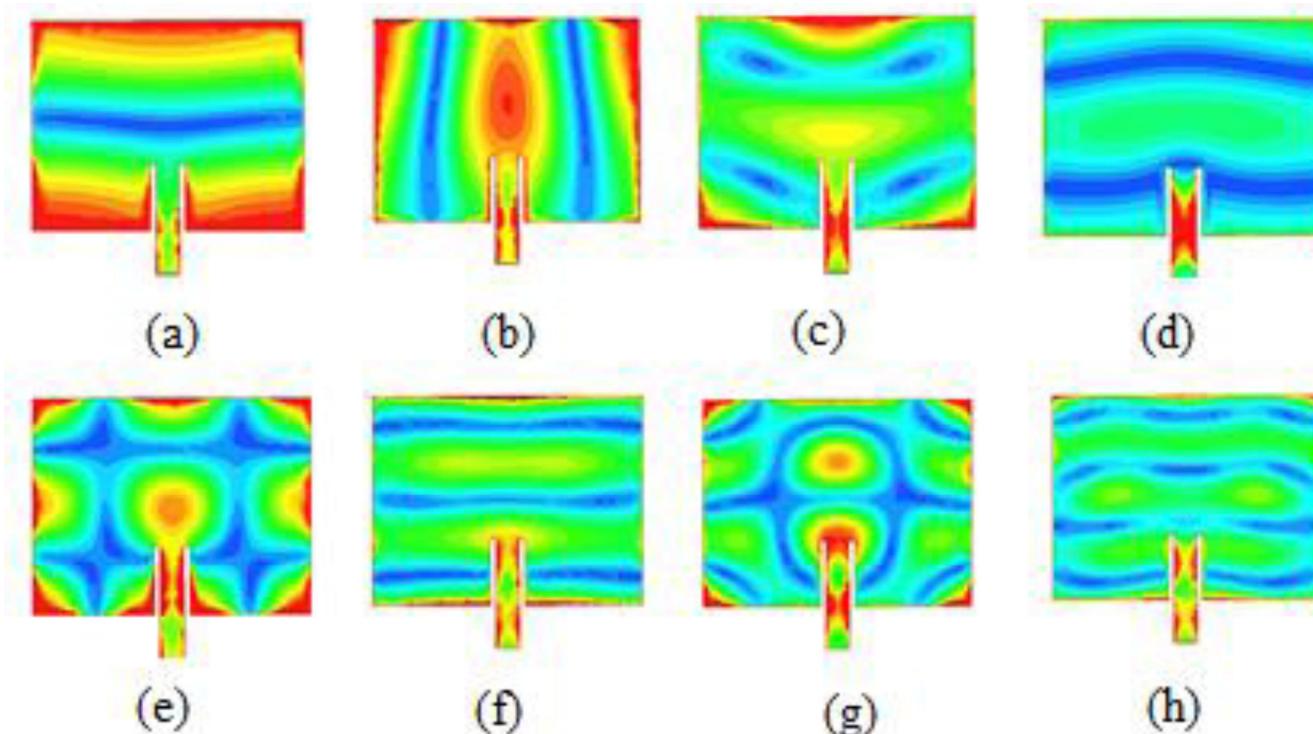
to avoid higher mode: $W < 2L$ $W = 1.5 L$ is typical.



General Characteristics

Width Restriction for a Rectangular Patch

$$W < 2L$$



Simulated electric field in the patch: (a) TM₁₀ (fundamental), (b) TM₀₂, (c) spurious, (d) TM₂₀, (e) TM₂₂, (f) TM₃₀, (g) spurious, and (h) TM₄₀.

General Characteristics

Width Restriction for a Rectangular Patch

$$W < 2L$$

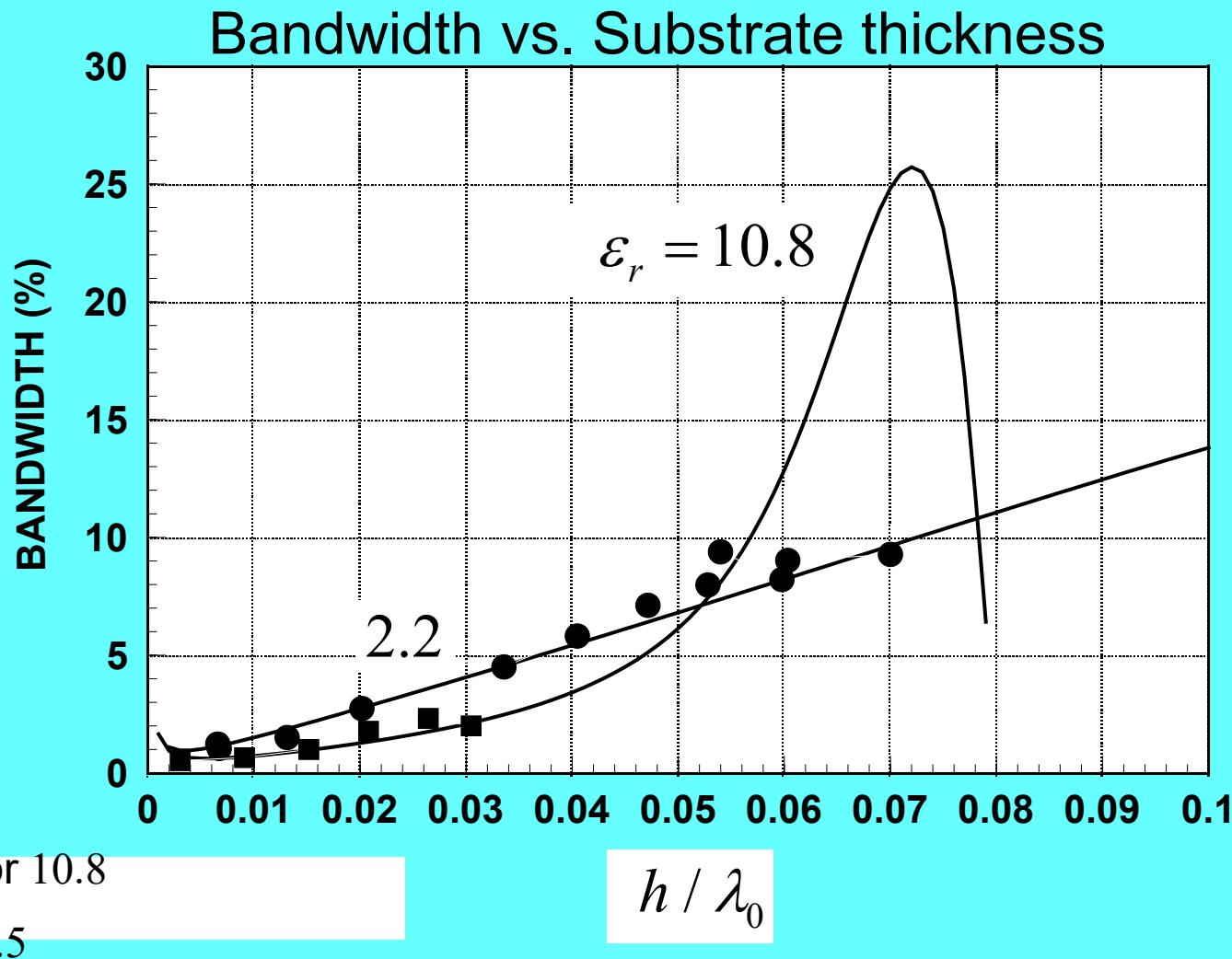
- Fundamental Mode (desired): TM10
- Radiation Pattern: Main lobe perpendicular to the patch plane, nulls at patch edges
- Higher Order Modes: Complex patterns with additional lobes and nulls

General Characteristics

Some Bandwidth Observations

- For a typical substrate thickness ($h / \lambda_0 = 0.02$), and a typical substrate permittivity ($\epsilon_r = 2.2$) the bandwidth is about 3%.
- By using a thick foam substrate, bandwidth of about 10% can be achieved.
- By using special feeding techniques (aperture coupling) and stacked patches, bandwidths of 100% have been achieved.

General Characteristics

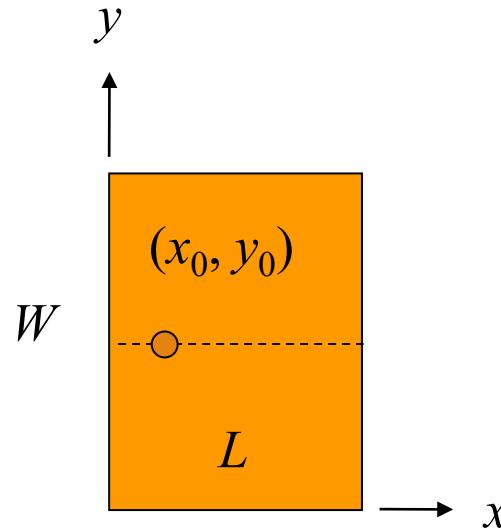


The discrete data points are measured values.
The solid curves are from a CAD formula (given later).

General Characteristics

Resonant Input Resistance

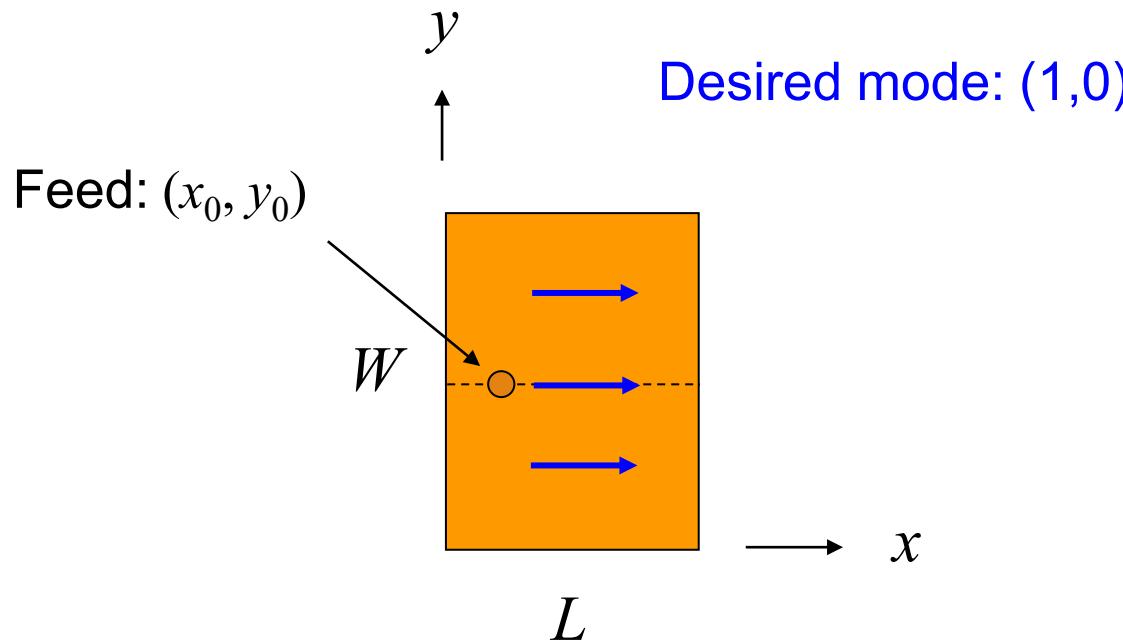
- The resonant input resistance is fairly independent of the substrate thickness h unless h gets small (the variation is then mainly due to dielectric and conductor loss).
- The resonant input resistance is proportional to **dielectric constant ϵ_r** .
- The resonant input resistance is directly controlled by **the location** of the feed point (maximum at edges $x = 0$ or $x = L$, zero at center of patch).



General Characteristics

Resonant Input Resistance (cont.)

The patch is usually fed along the centerline ($y_0 = W / 2$) to maintain symmetry and thus minimize excitation of undesirable modes (which cause cross-pol).

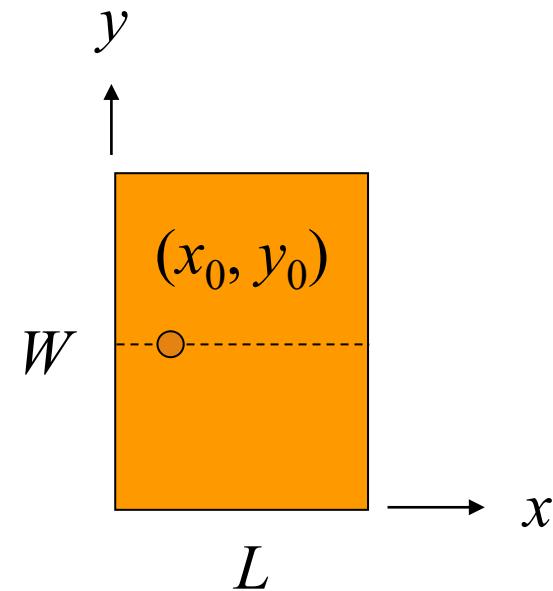


General Characteristics

Resonant Input Resistance (cont.)

Hence, for (1,0) mode:

$$R_{in} = R_{edge} \cos^2 \left(\frac{\pi x_0}{L} \right)$$



The value of R_{edge} depends strongly on the substrate permittivity (it is proportional to the permittivity).

For a typical patch, it is often in the range of 100-200 Ohms.

General Characteristics

Radiation Efficiency

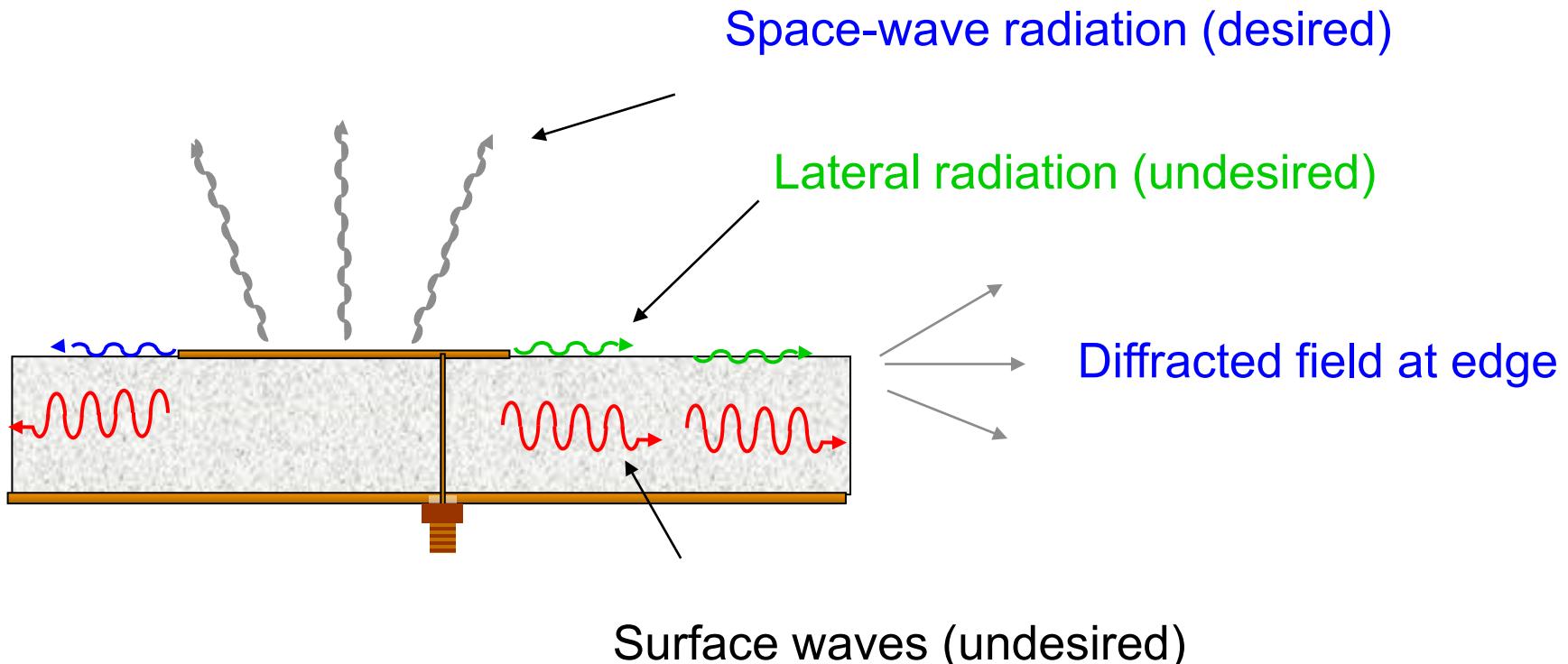
- Radiation efficiency is the ratio of power radiated into space, to the total input power.

$$e_r = \frac{P_r}{P_{tot}}$$

- The radiation efficiency is less than 100% due to
 - Conductor loss
 - Dielectric loss
 - Surface-wave excitation

General Characteristics

Radiation Efficiency (cont.)



Ref: Reduced Surface Wave Microstrip Antennas

General Characteristics

Radiation Efficiency (cont.)

Hence,

$$e_r = \frac{P_r}{P_{tot}} = \frac{P_r}{P_r + (P_c + P_d + P_{sw})}$$

P_r = radiated power

P_{tot} = total input power

P_c = power dissipated by conductors

P_d = power dissipated by dielectric

P_{sw} = power launched into surface wave

General Characteristics

Radiation Efficiency (cont.)

Some observations:

- **Conductor and dielectric loss is more important for thinner substrates** (the Q of the cavity is higher, and thus the resonance is more seriously affected by loss).
- **Conductor loss increases with frequency** (proportional to $f^{1/2}$) due to the skin effect. It can be very serious at millimeter-wave frequencies.
- **Conductor loss** is usually **more important than dielectric loss** for typical substrate thicknesses and loss tangents.

$$R_s = \frac{1}{\sigma \delta} \quad \delta = \sqrt{\frac{2}{\omega \mu \sigma}}$$

R_s is the surface resistance of the metal.
The skin depth of the metal is δ .

➡ $R_s = \sqrt{\frac{\omega \mu_0}{2\sigma}} \propto \sqrt{f}$

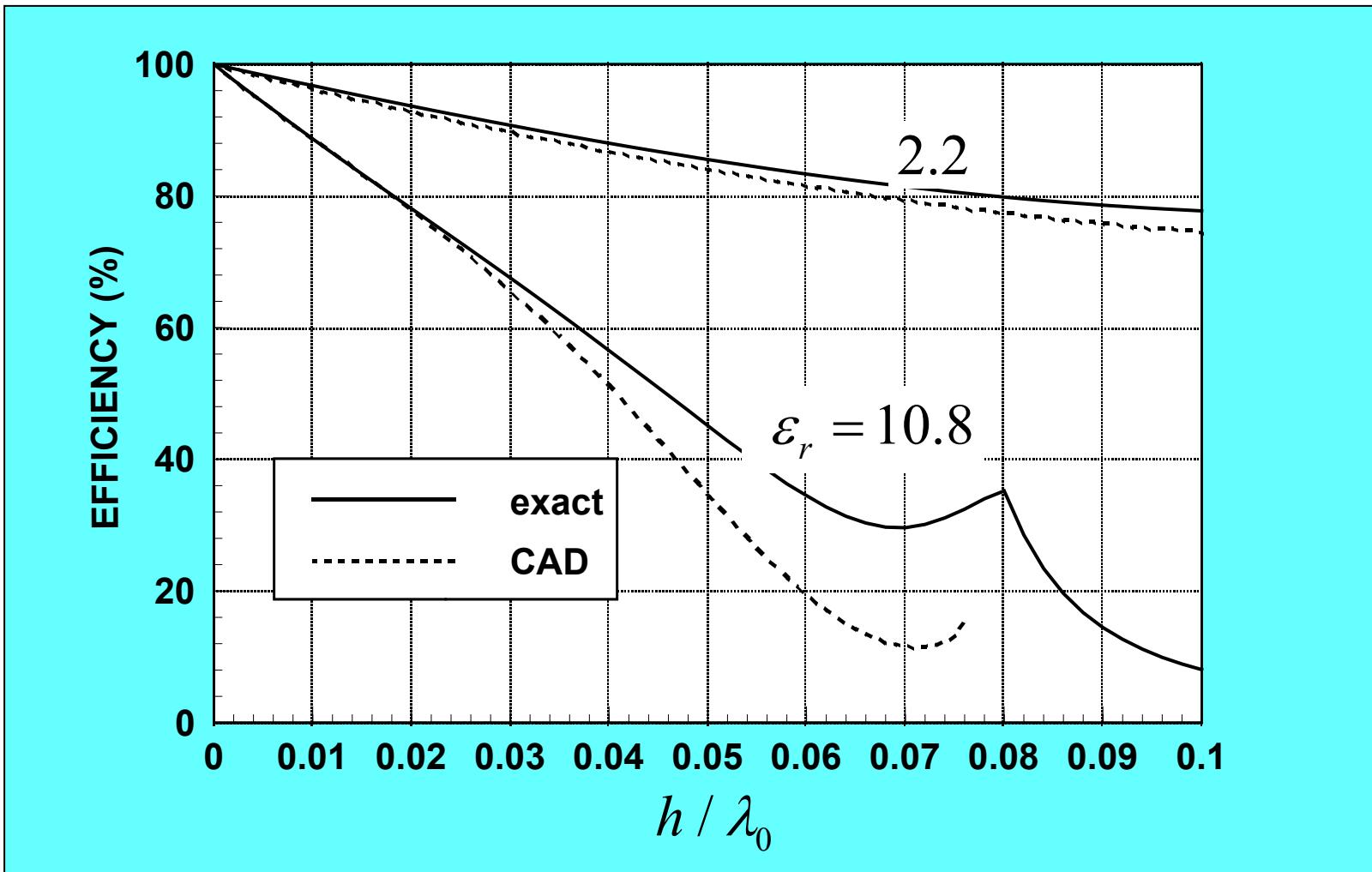
General Characteristics

Radiation Efficiency (cont.)

- **Surface-wave power** is more important for **thicker substrates** or for higher-substrate permittivities. (The surface-wave power can be minimized by using a thin substrate or a foam substrate.)
- For a **foam substrate**, a high radiation efficiency is obtained by making the substrate thicker (minimizing the conductor and dielectric losses). There is no surface-wave power to worry about.
- For a **typical substrate** such as $\varepsilon_r = 2.2$, the radiation efficiency is maximum for $h / \lambda_0 \approx 0.02$.

General Characteristics

Results: Efficiency (Conductor and dielectric losses are neglected.)



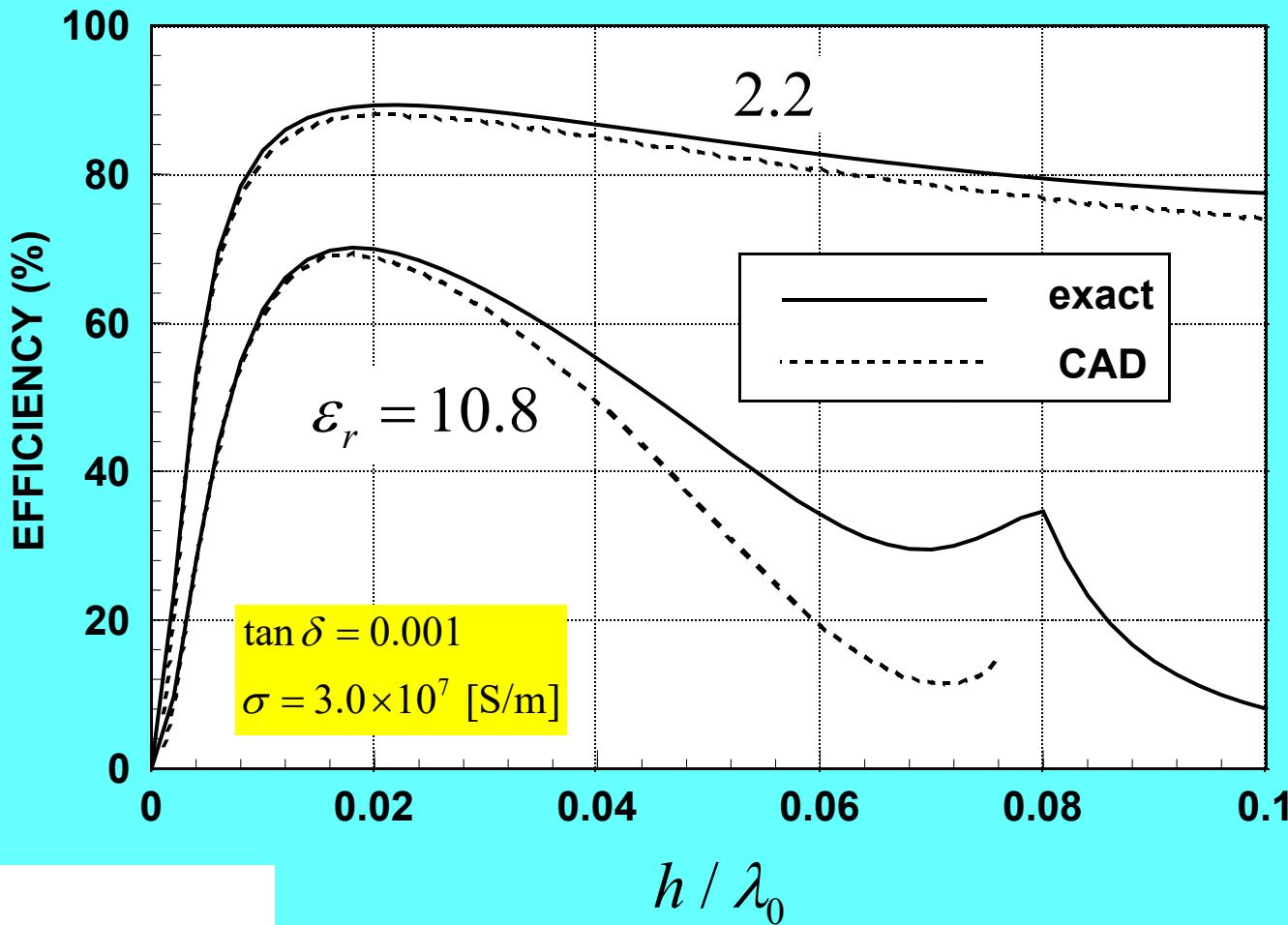
$$\epsilon_r = 2.2 \text{ or } 10.8$$

$$W/L = 1.5$$

Note: CAD plot uses the Pozar formula (given later).

General Characteristics

Results: Efficiency (All losses are accounted for.)



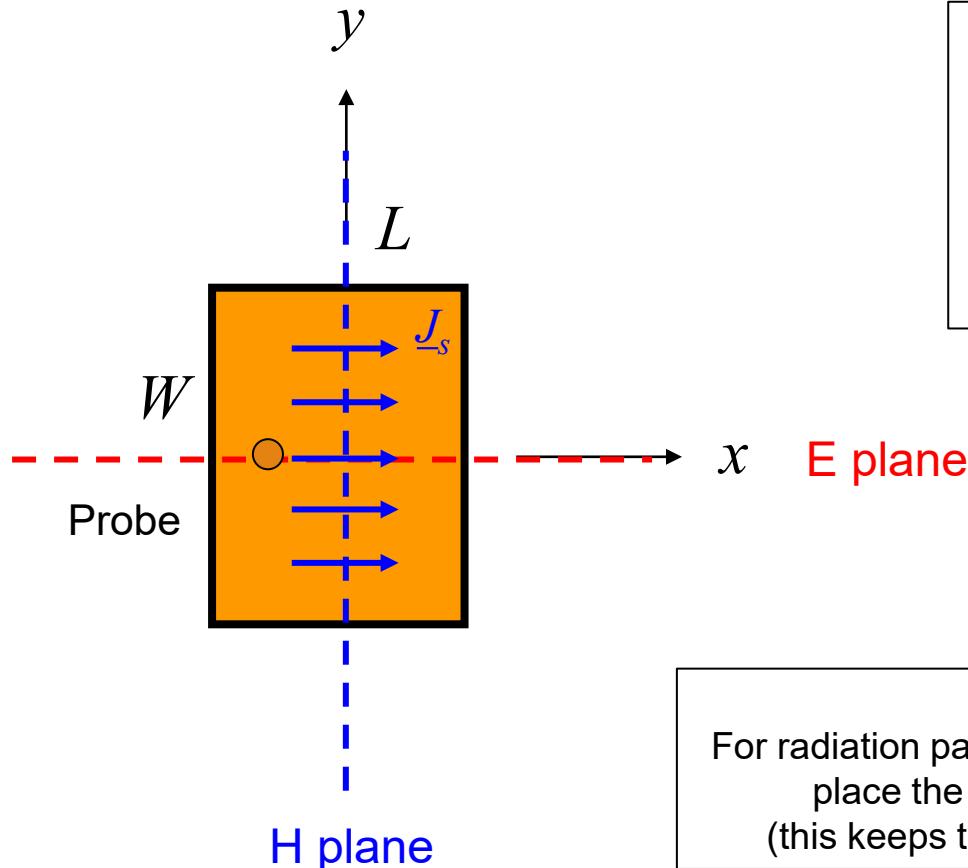
$\epsilon_r = 2.2$ or 10.8

$W/L = 1.5$

Note: CAD plot uses the Pozar formula (given later).

General Characteristics

Radiation Pattern



E-plane: co-pol is E_θ

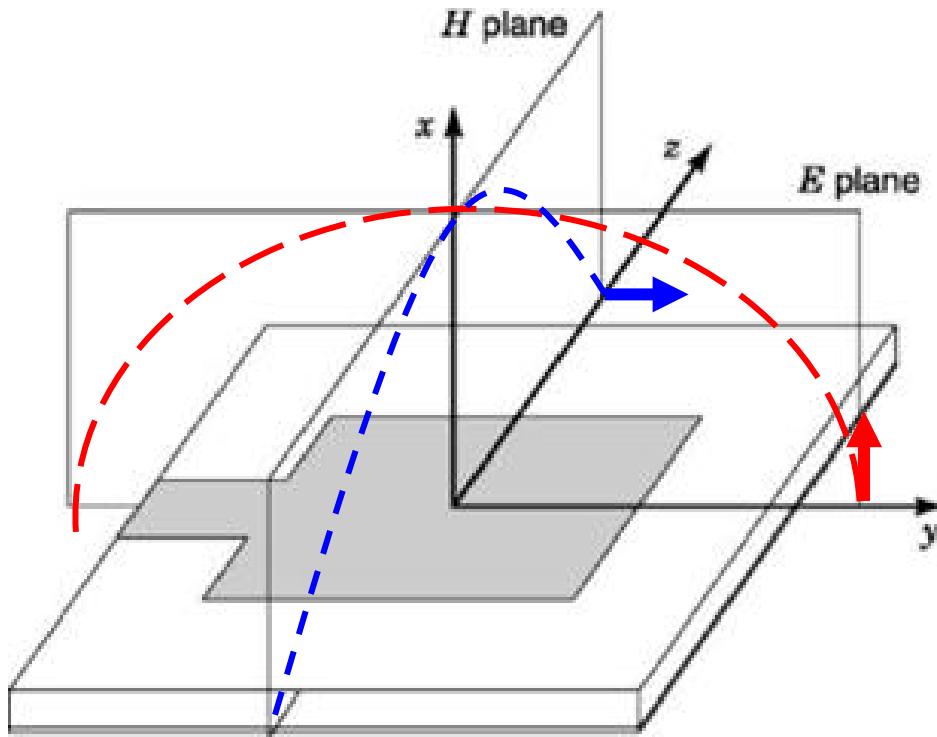
H-plane: co-pol is E_ϕ

Note:

For radiation patterns, it is usually more convenient to place the origin at the middle of the patch (this keeps the formulas as simple as possible).

General Characteristics

Radiation Pattern



E-plane: co-pol is E_θ

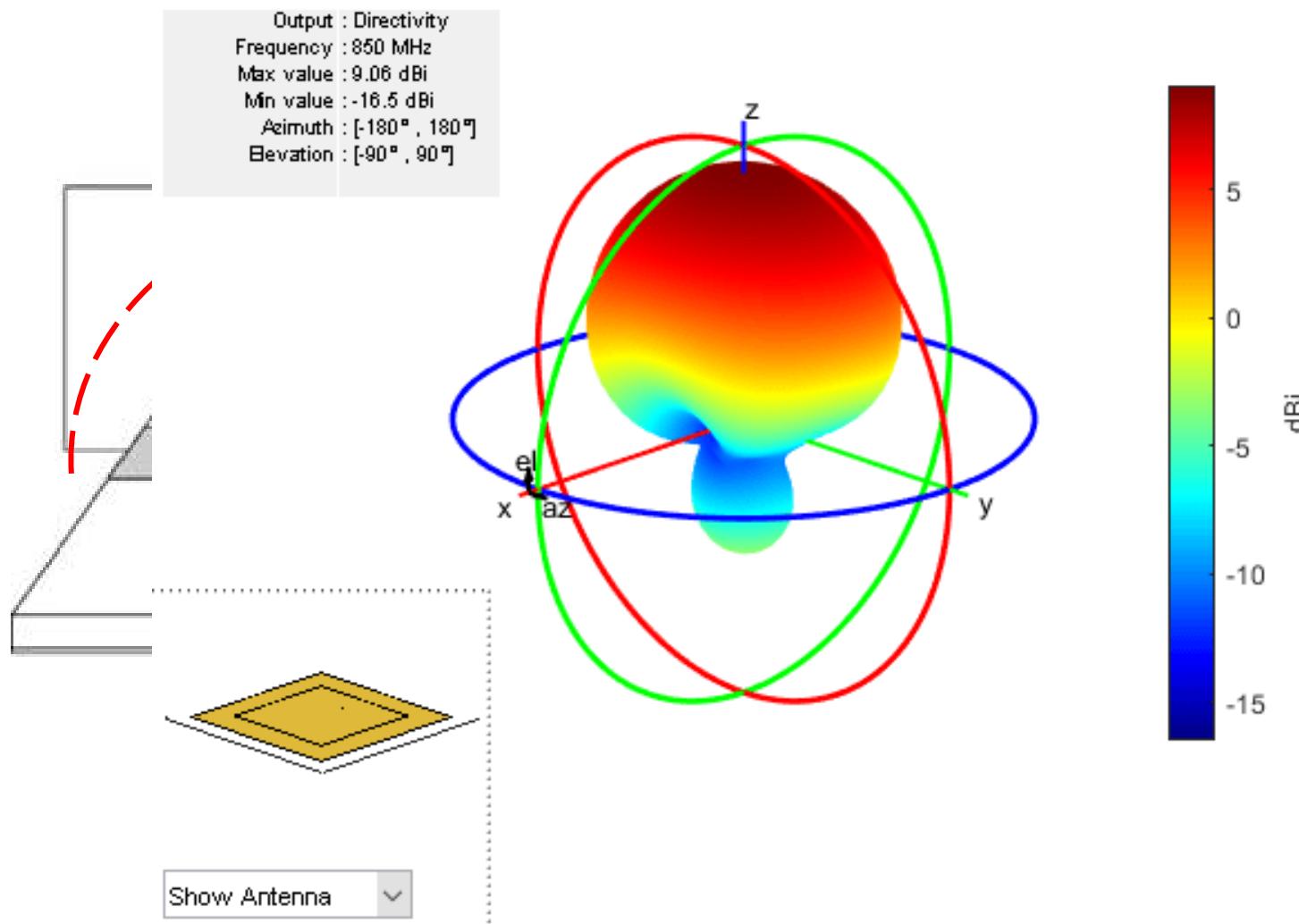
H-plane: co-pol is E_ϕ

Note:

For radiation patterns, it is usually more convenient to place the origin at the middle of the patch (this keeps the formulas as simple as possible).

General Characteristics

Radiation Pattern



General Characteristics

Radiation Patterns (cont.)

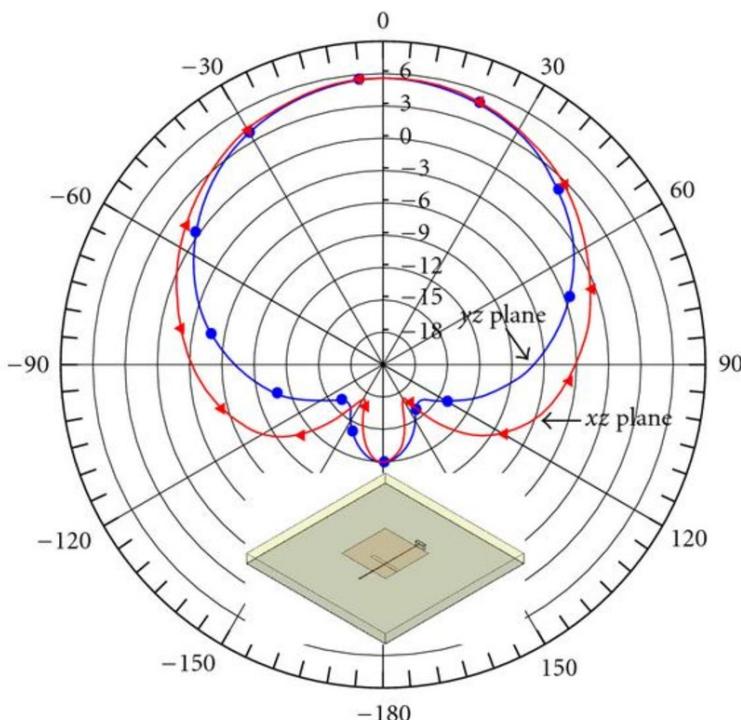
Comments on radiation patterns:

- The E-plane pattern is typically broader than the H-plane pattern.

- The truncation of the pattern which tends to do

- Rippling in the main lobe
 - Back-radiation

- Pattern distortion due to the angle dependence of the ground plane.
(It varies as $\cos^2(\theta)$)



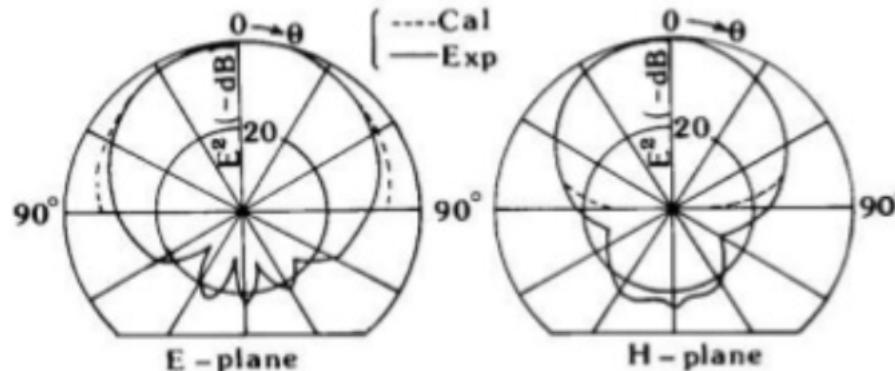
due to diffraction, :
the angle of the ground plane.

General Characteristics

Radiation Patterns (cont.)

Comments on:

- The E-plane pattern.

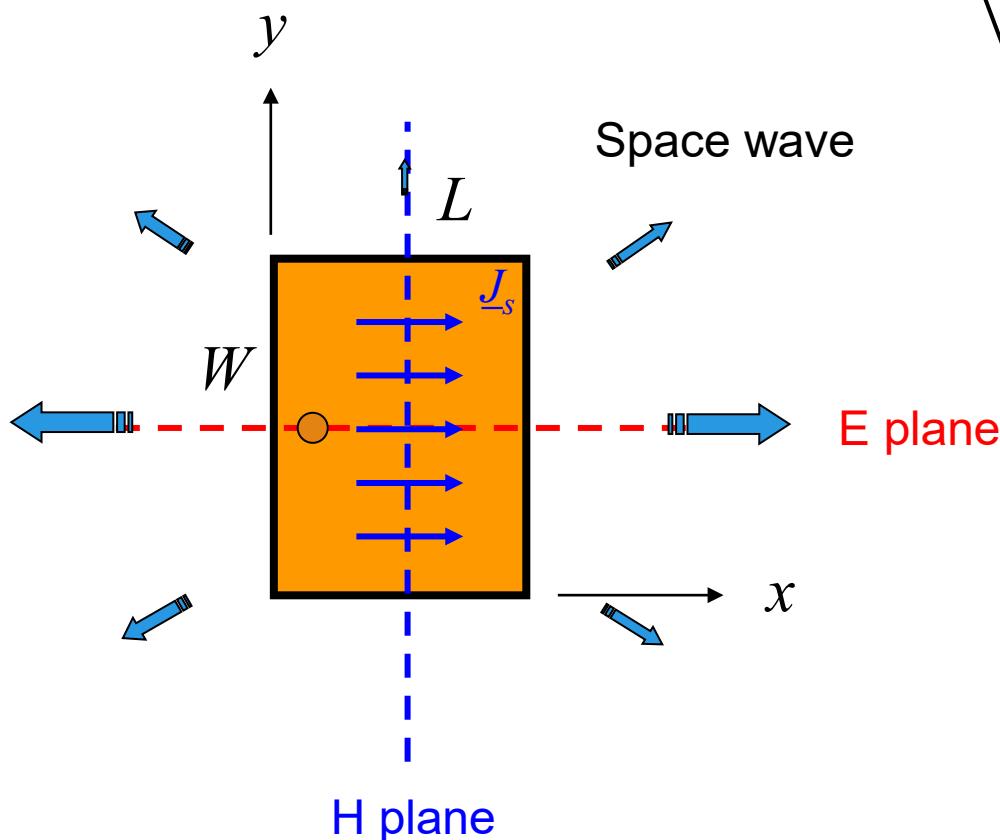


- The truncation of the ground plane will cause edge diffraction, which tends to degrade the pattern by introducing:
 - Rippling in the forward direction
 - Back-radiation
- Pattern distortion is more severe in the E-plane, due to the angle dependence of the vertical polarization E_θ on the ground plane.
(It varies as $\cos(\phi)$).

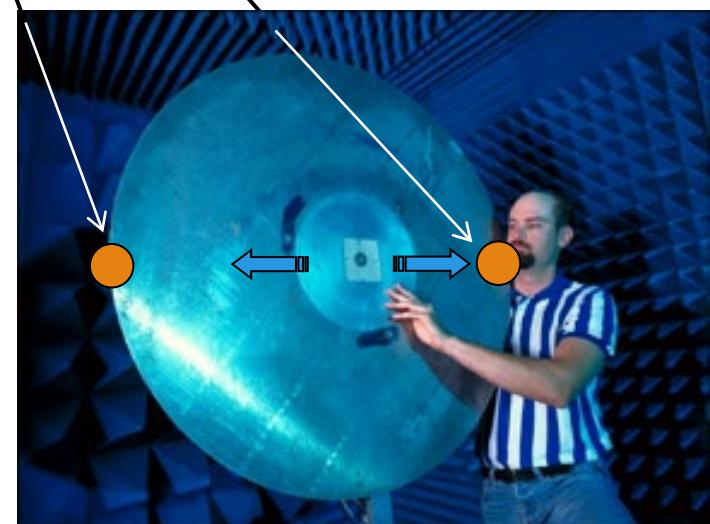
General Characteristics

Radiation Patterns

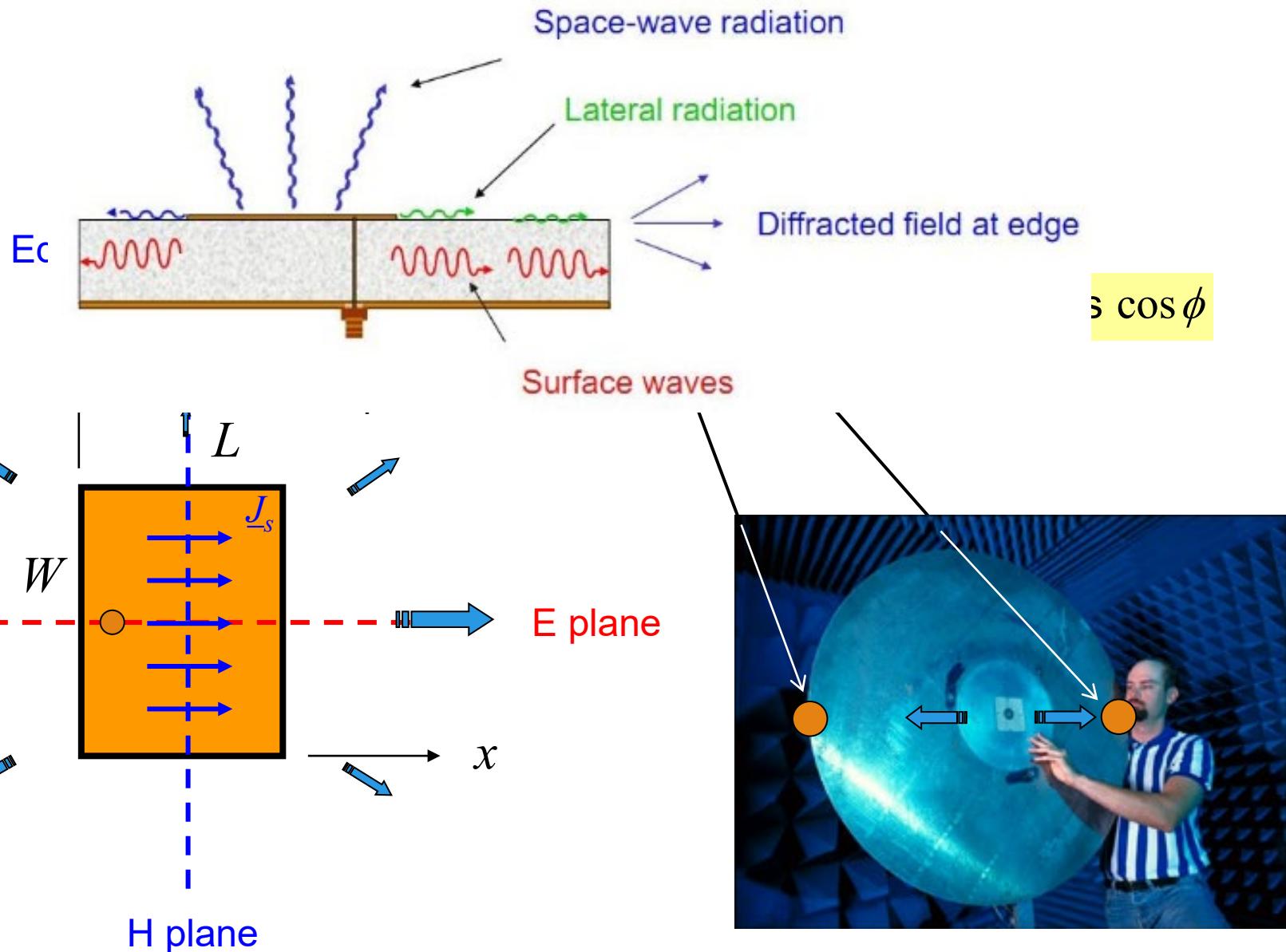
Edge diffraction is the most serious in the E plane.



E_θ varies as $\cos \phi$



General Characteristics



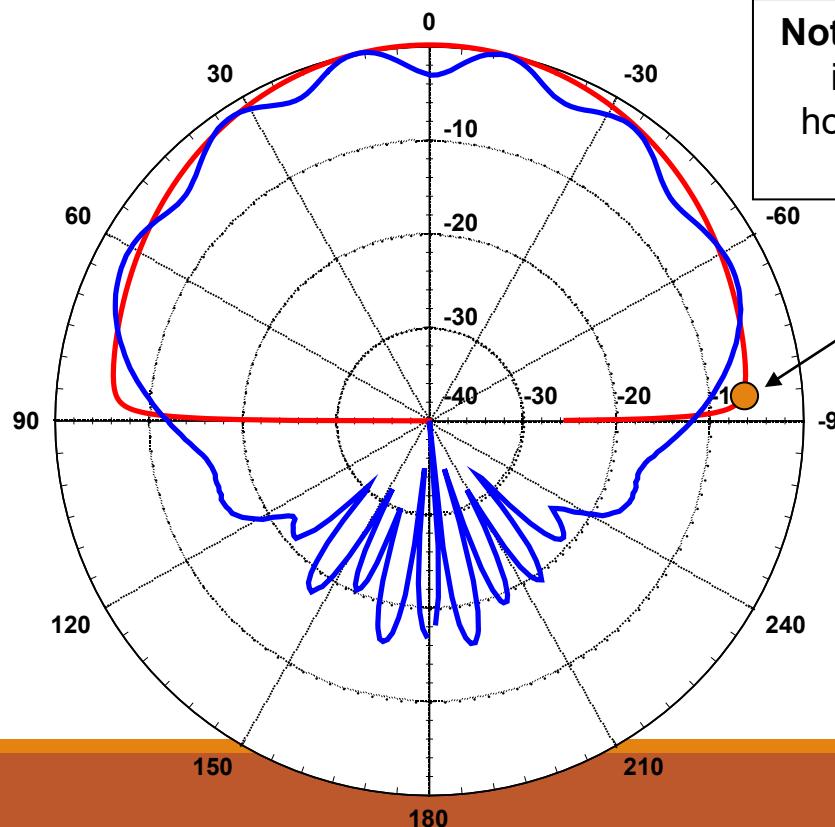
General Characteristics

Radiation Patterns

E-plane pattern

Red: infinite substrate and ground plane

Blue: 1 meter ground plane



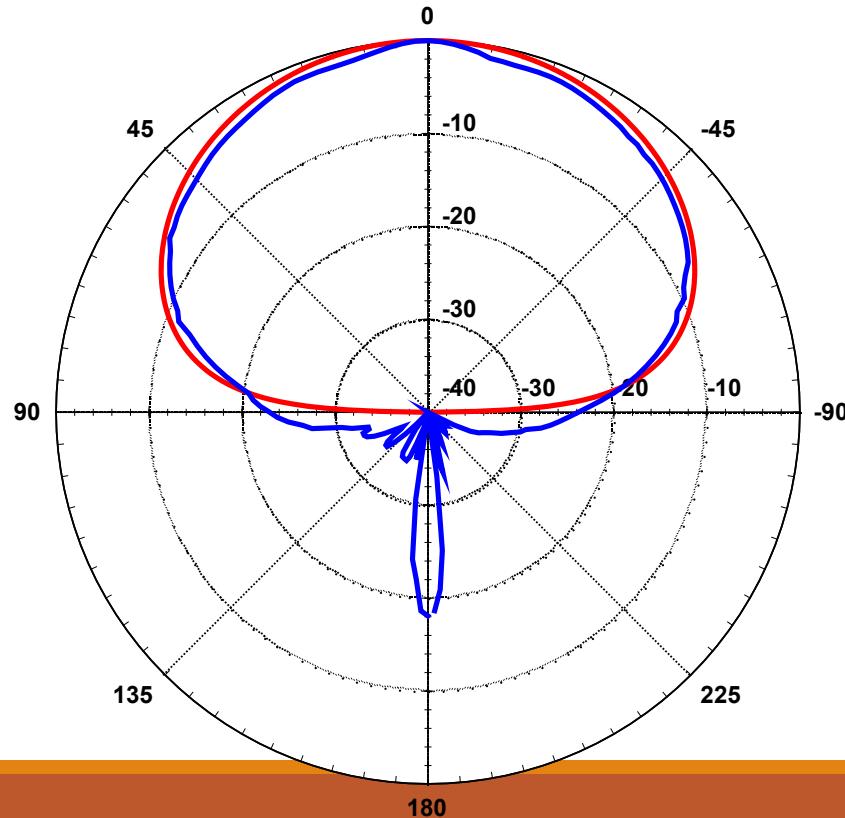
General Characteristics

Radiation Patterns

H-plane pattern

Red: infinite substrate and ground plane

Blue: 1 meter ground plane

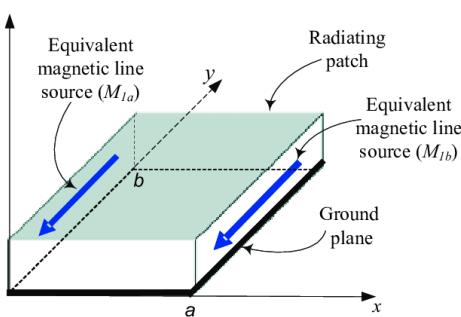


General Characteristics

Directivity

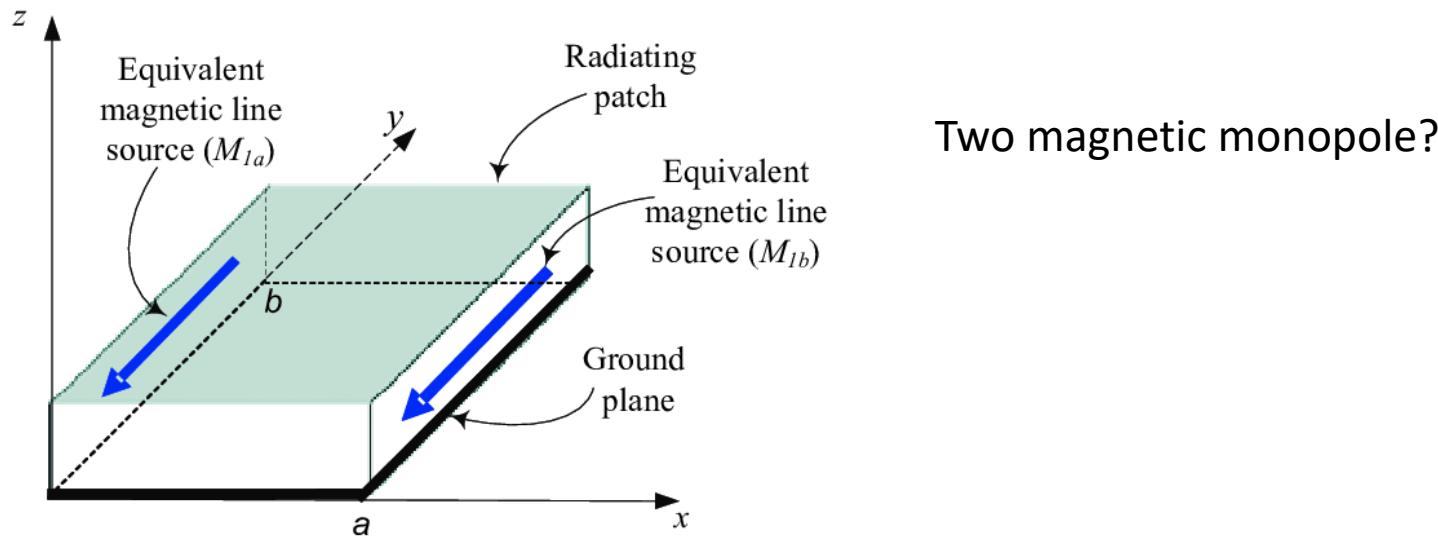
- The directivity is fairly insensitive to the substrate thickness.
- The directivity is higher for lower permittivity, because the radiation resistance is larger.

➤ 7-9 cm usually



General Characteristics

Directivity



➤ 7-9 dBi directivity usually

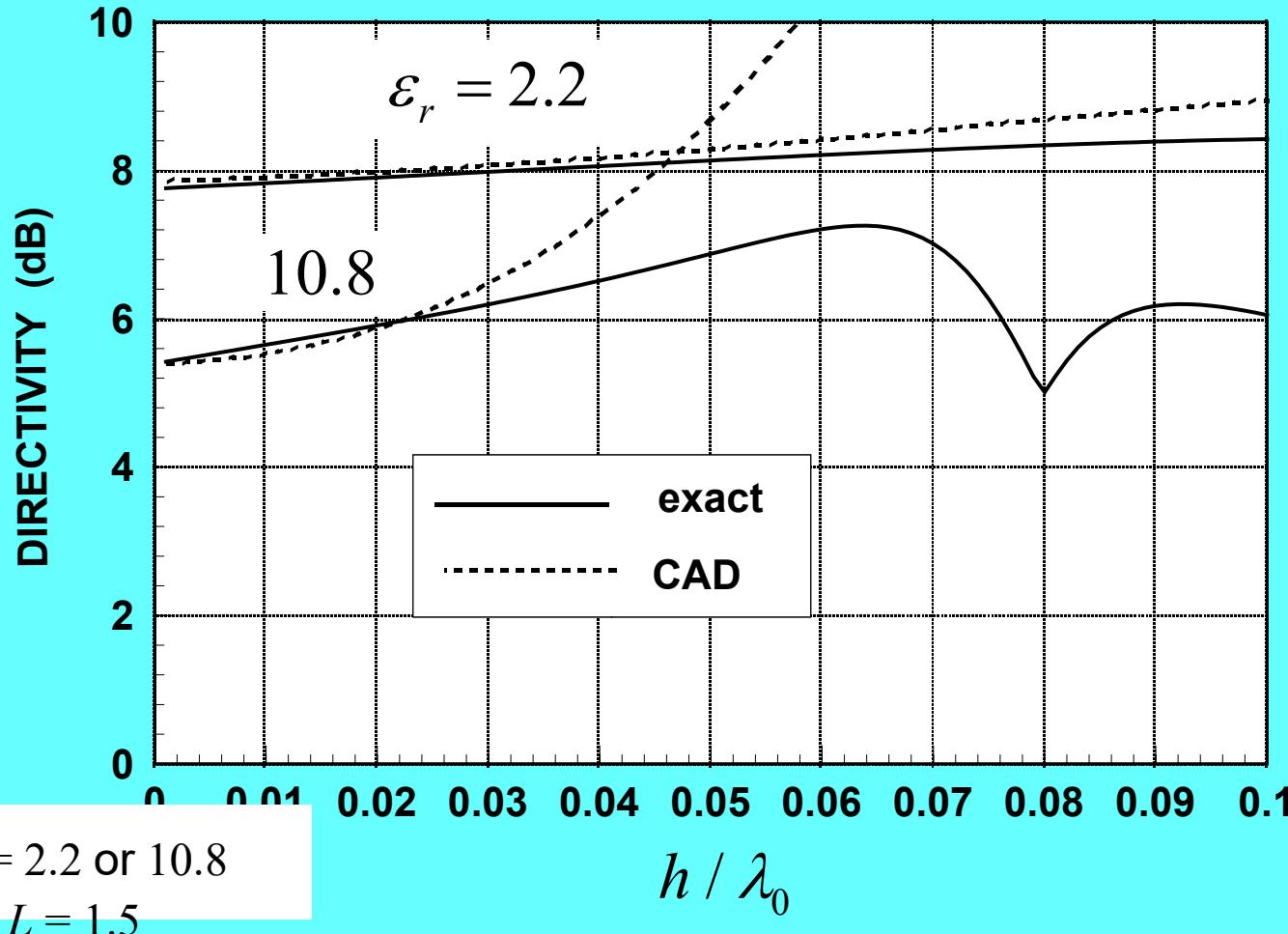
General Characteristics

Directivity

- The directivity is fairly insensitive to the substrate thickness.
- The directivity is higher for lower permittivity, because the patch is larger.

General Characteristics

Results: Directivity (relative to isotropic)



CAD Formulas

CAD formulas for the important properties of the *rectangular microstrip antenna* will be shown.

- Radiation efficiency
- Bandwidth (Q)
- Resonant input resistance
- Directivity

- D.R. Jackson, “Microstrip Antennas,” Chapter 7 of *Antenna Engineering Handbook*, J. L. Volakis, Editor, McGraw Hill, 2007.
- D.R. Jackson, S.A. Long, J.T. Williams, and V.B. Davis, “Computer-Aided Design of Rectangular Microstrip Antennas,” Ch. 5 of *Advances in Microstrip and Printed Antennas*, K. F. Lee and W. Chen, Eds., John Wiley, 1997.
- D.R. Jackson and N.G. Alexopoulos, “Simple Approximate Formulas for Input Resistance, Bandwidth, and Efficiency of a Resonant Rectangular Patch,” IEEE Trans. Antennas and Propagation, Vol. 39, pp. 407-410, March 1991.

CAD Formulas

Radiation Efficiency

$$e_r = \frac{e_r^{hed}}{1 + e_r^{hed} \left[\ell_d + \left(\frac{R_s^{ave}}{\pi \eta_0} \right) \left(\frac{1}{h/\lambda_0} \right) \right] \left[\left(\frac{3}{16} \right) \left(\frac{\epsilon_r}{pc_1} \right) \left(\frac{L}{W} \right) \left(\frac{1}{h/\lambda_0} \right) \right]}$$

Comment:

The efficiency becomes small as the substrate gets thin,
if there is dielectric or conductor loss.

where

$$\ell_d = \tan \delta = \text{loss tangent of substrate}$$

$$R_s = \text{surface resistance of metal} = \frac{1}{\sigma \delta} = \sqrt{\frac{\omega \mu}{2\sigma}} \quad R_s^{ave} = \left(R_s^{patch} + R_s^{ground} \right) / 2$$

Note: “hed” refers to a unit-amplitude horizontal electric dipole.

CAD Formulas

Radiation Efficiency (cont.)

$$e_r^{hed} = \frac{P_{sp}^{hed}}{P_{sp}^{hed} + P_{sw}^{hed}} = \frac{1}{1 + \frac{P_{sw}^{hed}}{P_{sp}^{hed}}}$$

where

$$P_{sp}^{hed} = \frac{1}{\lambda_0^2} (k_0 h)^2 (80\pi^2 c_1)$$

$$P_{sw}^{hed} = \frac{1}{\lambda_0^2} (k_0 h)^3 \left[60\pi^3 \left(1 - \frac{1}{\epsilon_r} \right)^3 \right]$$

Note: “hed” refers to a unit-amplitude horizontal electric dipole.

Note: When we say “unit amplitude” here, we assume peak (not RMS) values.

CAD Formulas

Radiation Efficiency (cont.)

Hence, we have

$$e_r^{hed} = \frac{1}{1 + \frac{3}{4}\pi(k_0 h)\left(\frac{1}{c_1}\right)\left(1 - \frac{1}{\epsilon_r}\right)^3}$$

Physically, this term is the radiation efficiency of a horizontal electric dipole (hed) on top of the substrate.

CAD Formulas

Radiation Efficiency (cont.)

The constants c_1 and p are defined as follows:

$$c_1 = 1 - \frac{1}{\varepsilon_r} + \frac{2/5}{\varepsilon_r^2}$$

$$p = 1 + \frac{a_2}{10} (k_0 W)^2 + (a_2^2 + 2a_4) \left(\frac{3}{560} \right) (k_0 W)^4 + c_2 \left(\frac{1}{5} \right) (k_0 L)^2 + a_2 c_2 \left(\frac{1}{70} \right) (k_0 W)^2 (k_0 L)^2$$

$$c_2 = -0.0914153$$

$$a_2 = -0.16605$$

$$a_4 = 0.00761$$

CAD Formulas

Improved formula for HED surface-wave power (due to Pozar)

$$P_{sw}^{hed} = \frac{\eta_0 k_0^2}{8} \frac{\varepsilon_r (x_0^2 - 1)^{3/2}}{\varepsilon_r (1 + x_1) + (k_0 h) \sqrt{x_0^2 - 1} (1 + \varepsilon_r^2 x_1)}$$

Note: x_0 in this formula is not the feed location!

$$x_1 = \frac{x_0^2 - 1}{\varepsilon_r - x_0^2}$$

$$x_0 = 1 + \frac{-\varepsilon_r^2 + \alpha_0 \alpha_1 + \varepsilon_r \sqrt{\varepsilon_r^2 - 2\alpha_0 \alpha_1 + \alpha_0^2}}{\varepsilon_r^2 - \alpha_1^2}$$

$$\alpha_0 = s \tan[(k_0 h)s] \quad \alpha_1 = -\frac{1}{s} \left[\tan[(k_0 h)s] + \frac{(k_0 h)s}{\cos^2[(k_0 h)s]} \right]$$

$$s = \sqrt{\varepsilon_r - 1}$$

D. M. Pozar, "Rigorous Closed-Form Expressions for the Surface-Wave Loss of Printed Antennas," *Electronics Letters*, vol. 26, pp. 954-956, June 1990.

Note: The above formula for the surface-wave power is different from that given in Pozar's paper by a factor of 2, since Pozar used RMS instead of peak values.

CAD Formulas

Bandwidth

$$BW = \frac{1}{\sqrt{2}} \left[\ell_d + \left(\frac{R_s^{ave}}{\pi \eta_0} \right) \left(\frac{1}{h/\lambda_0} \right) + \left(\frac{16}{3} \right) \left(\frac{p c_1}{\epsilon_r} \right) \left(\frac{h}{\lambda_0} \right) \left(\frac{W}{L} \right) \left(\frac{1}{e_r^{hed}} \right) \right]$$

$$Q = \frac{1}{\sqrt{2} BW}$$

Comments:

For a lossless patch, the bandwidth is approximately proportional to the patch width and to the substrate thickness. It is inversely proportional to the substrate permittivity.

For very thin substrates the bandwidth will increase for a lossy patch, but at the expense of efficiency.

BW is defined from the frequency limits f_1 and f_2 at which $SWR = 2.0$.

$$BW = \frac{f_2 - f_1}{f_0}$$

(multiply by 100 if you want to get %)

CAD Formulas

Quality Factor Q

$$Q \equiv \omega_0 \frac{U_s}{P}$$

U_s = energy stored in patch cavity

P = power that is radiated and dissipated by patch

$$\frac{1}{Q} = \frac{P}{\omega_0 U_s}$$

$$P = P_d + P_c + P_{sp} + P_{sw}$$

$$\frac{1}{Q} = \frac{1}{Q_d} + \frac{1}{Q_c} + \frac{1}{Q_{sp}} + \frac{1}{Q_{sw}}$$

CAD Formulas

Q Components

$$Q_d = 1 / \tan \delta$$

$$Q_c = \left(\frac{\eta_0}{2} \right) \left[\frac{(k_0 h)}{R_s^{ave}} \right]$$

$$Q_{sp} \approx \frac{3}{16} \left(\frac{\varepsilon_r}{pc_1} \right) \left(\frac{L}{W} \right) \left(\frac{1}{h/\lambda_0} \right)$$

$$Q_{sw} = Q_{sp} \left(\frac{e_r^{hed}}{1 - e_r^{hed}} \right)$$

$$R_s^{ave} = \left(R_s^{patch} + R_s^{ground} \right) / 2$$

The constants p and c_1 were defined previously.

$$e_r^{hed} = \frac{1}{1 + \frac{3}{4} \pi (k_0 h) \left(\frac{1}{c_1} \right) \left(1 - \frac{1}{\varepsilon_r} \right)^3}$$

CAD Formulas

Resonant Input Resistance

Probe-feed Patch

$$R = R_{in}^{max} = R_{edge} \cos^2\left(\frac{\pi x_0}{L}\right)$$

$$R_{edge} = \frac{\left(\frac{4\eta_0}{\pi}\right)\left(\frac{L}{W}\right)\left(\frac{h}{\lambda_0}\right)}{\ell_d + \left(\frac{R_s}{\pi\eta_0}\right)\left(\frac{1}{h/\lambda_0}\right) + \left(\frac{16}{3}\right)\left(\frac{pc_1}{\epsilon_r}\right)\left(\frac{W}{L}\right)\left(\frac{h}{\lambda_0}\right)\left(\frac{1}{e_r^{hed}}\right)}$$

Comments:

For a lossless patch, the resonant resistance is approximately independent of the substrate thickness. For a lossy patch it tends to zero as the substrate gets very thin. For a lossless patch it is inversely proportional to the square of the patch width and it is proportional to the substrate permittivity.

CAD Formulas

Approximate CAD formula for probe (feed) reactance (in Ohms)

a = probe radius

h = probe height

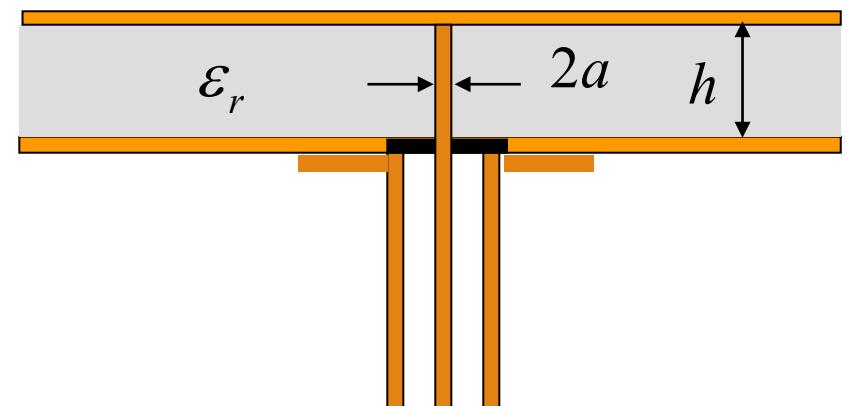
$$X_p = \frac{\eta_0}{2\pi} (k_0 h) \left[-\gamma + \ln \left(\frac{2}{\sqrt{\epsilon_r} (k_0 a)} \right) \right]$$

This is based on an infinite parallel-plate model.

$$X_p = \omega L_p$$

$$\gamma \doteq 0.577216 \text{ (Euler's constant)}$$

$$\eta_0 = \sqrt{\mu_0 / \epsilon_0} = 376.7303 \Omega$$



CAD Formulas

Observations:

- Feed (probe) reactance increases proportionally with substrate thickness h .
- Feed reactance increases for smaller probe radius a .

$$X_p = \frac{\eta_0}{2\pi} (k_0 h) \left[-\gamma + \ln \left(\frac{2}{\sqrt{\epsilon_r} (k_0 a)} \right) \right]$$

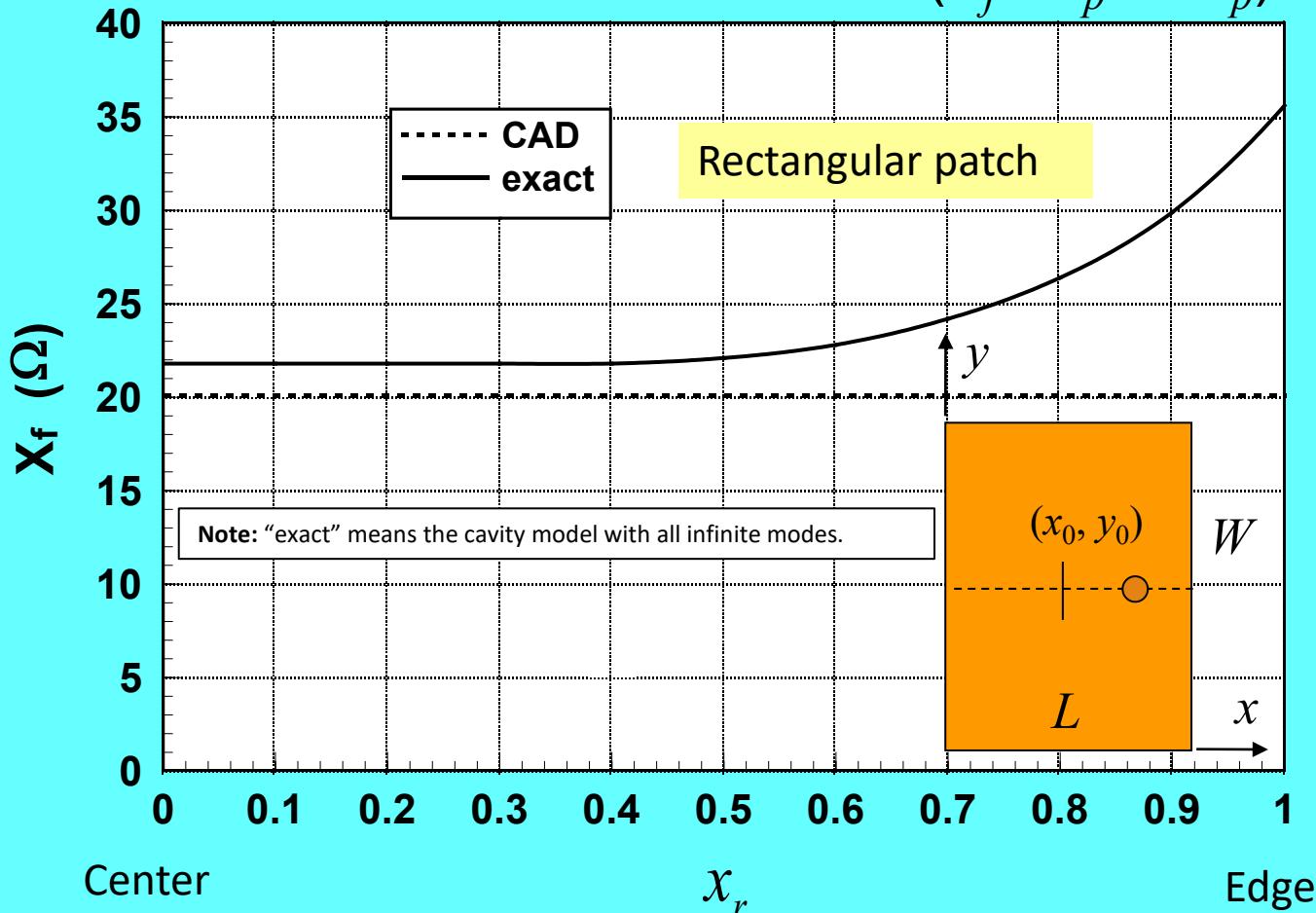
Important point:

If the substrate gets too thick, the probe reactance will make it difficult to get an input match, and the bandwidth will suffer.

(Compensating techniques will be discussed later.)

CAD Formulas

Results: Probe Reactance ($X_f = X_p = \omega L_p$)



$$\epsilon_r = 2.2$$

$$W/L = 1.5$$

$$h = 0.0254 \lambda_0$$

$$a = 0.5 \text{ mm}$$

$$x_r = 2(x_0/L) - 1$$

The normalized feed location ratio x_r is zero at the center of the patch ($x = L/2$), and is 1.0 at the patch edge ($x = L$).

CAD Formulas

Directivity

$$D = \left(\frac{3}{pc_1} \right) \left[\frac{\epsilon_r}{\epsilon_r + \tan^2(k_1 h)} \right] (\text{tanc}^2(k_1 h))$$

$$k_1 = k_0 \sqrt{\epsilon_r}$$

where

$$\text{tanc}(x) \equiv \tan(x)/x$$

The constants p and c_1 were defined previously.

CAD Formulas

Directivity (cont.)

For thin substrates:

$$D \approx \frac{3}{p c_1}$$

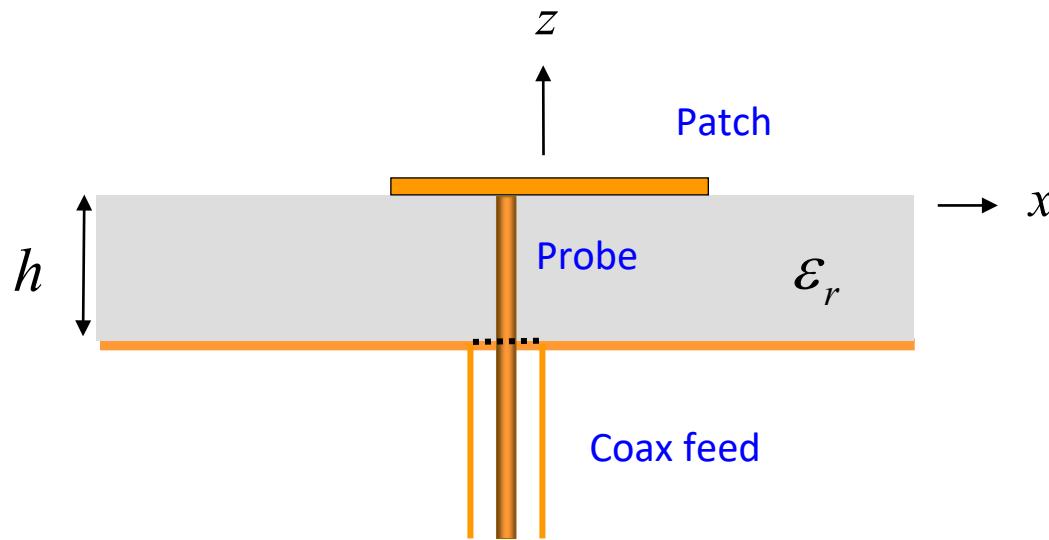
(The directivity is essentially independent of the substrate thickness.)

Radiation Pattern

There are two models often used for calculating the radiation pattern:

- Electric current model
- Magnetic current model

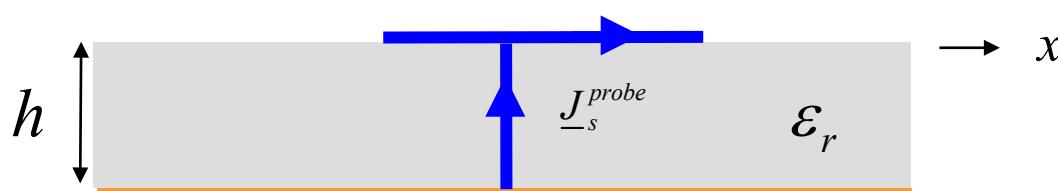
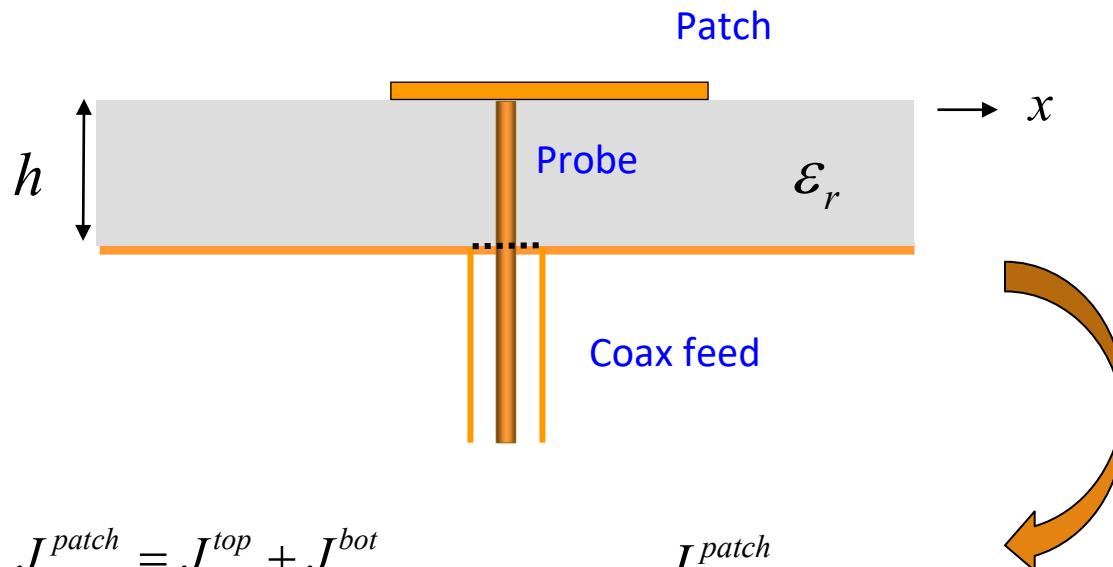
Note: The origin is placed at the center of the patch, at the top of the substrate, for the pattern calculations.



Radiation Pattern

Electric current model:

We keep the physical currents flowing on the patch (and feed).



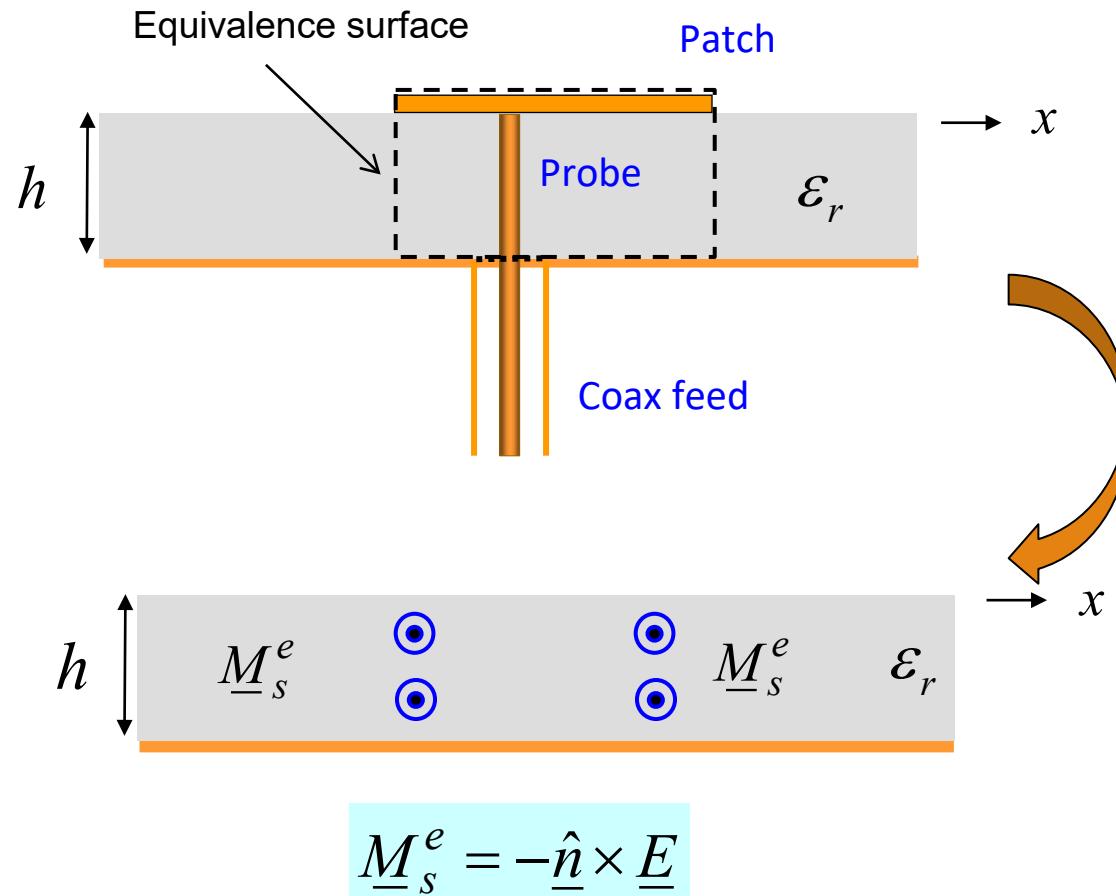
Radiation Pattern

Magnetic current model:

We apply the *equivalence principle* and invoke the (approximate) PMC condition at the edges.

$$\underline{J}_s^e = \hat{\underline{n}} \times \underline{H}$$
$$\underline{M}_s^e = -\hat{\underline{n}} \times \underline{E}$$

The equivalent surface current is approximately zero on the top surface (weak fields) and the sides (PMC). We can ignore it on the ground plane (it does not radiate).



Radiation Pattern

Theorem

The electric and magnetic models yield identical patterns at the **resonance frequency** of the cavity mode.

Assumption:

The electric and magnetic current models are based on the fields of a single cavity mode, corresponding to an ideal cavity with PMC walls.

D. R. Jackson and J. T. Williams, "A Comparison of CAD Models for Radiation from Rectangular Microstrip Patches," *Intl. Journal of Microwave and Millimeter-Wave Computer Aided Design*, vol. 1, no. 2, pp. 236-248, April 1991.

Radiation Pattern

Comments on the Substrate Effects

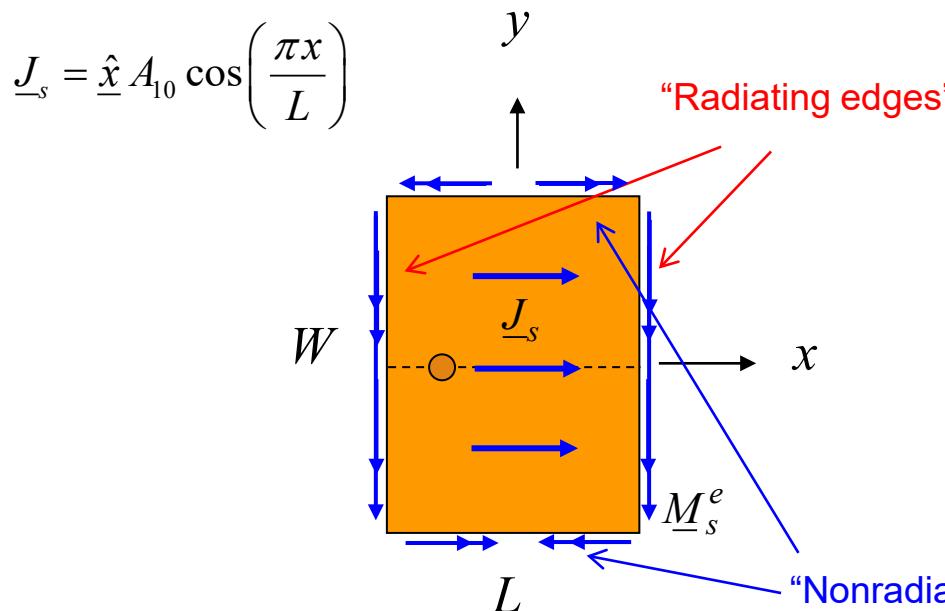
- The substrate can be neglected to simplify the far-field calculation.
- When considering the substrate, it is most convenient to assume an infinite substrate (in order to obtain a closed-form solution).
- Reciprocity can be used to calculate the far-field pattern of electric or magnetic current sources inside of an infinite layered structure.
- When an infinite substrate is assumed, the far-field pattern always goes to zero at the horizon.

D. R. Jackson and J. T. Williams, "A Comparison of CAD Models for Radiation from Rectangular Microstrip Patches," *Intl. Journal of Microwave and Millimeter-Wave Computer Aided Design*, vol. 1, no. 2, pp. 236-248, April 1991.

Radiation Pattern

Comments on the Two Models

- For the rectangular patch, the **electric current model** is the simplest since there is only one electric surface current (as opposed to four edges).
- For the rectangular patch, the **magnetic current model** allows us to classify the “**radiating**” and “**nonradiating**” edges.



$$\underline{M}_s^e = -\hat{n} \times \underline{E}$$

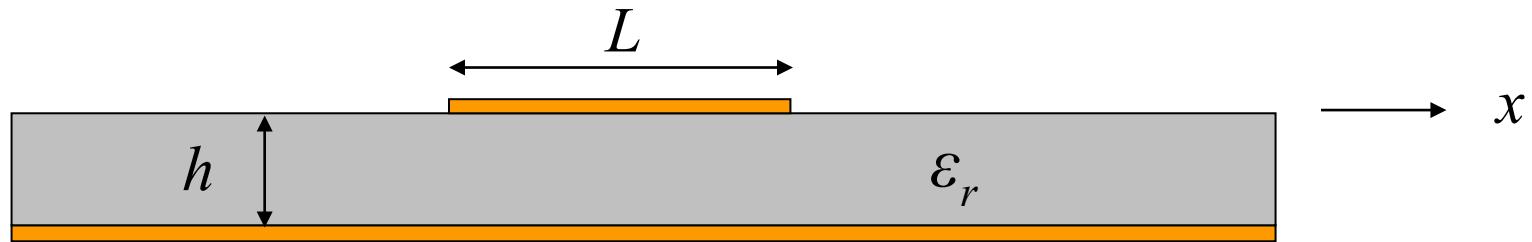
$$E_z = -\sin\left(\frac{\pi x}{L}\right)$$

Note:
On the nonradiating edges, the magnetic currents are in opposite directions across the centerline ($x = 0$).

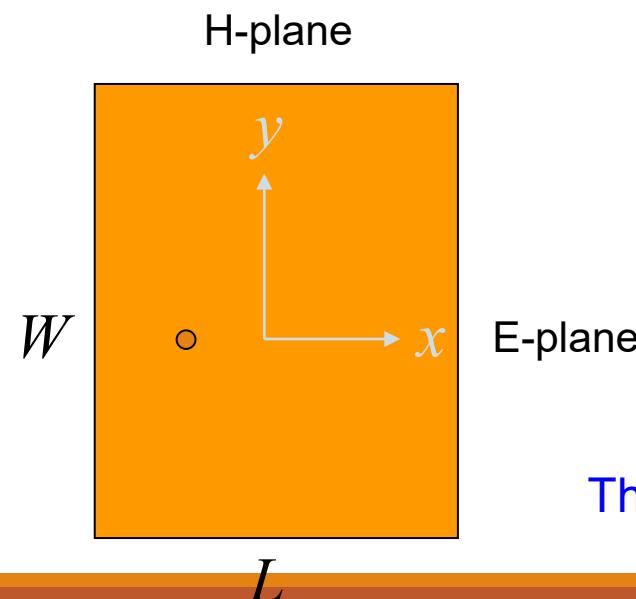
Radiation Pattern

Rectangular Patch Pattern Formula

(The formula is based on the electric current model.)



The origin is at the center of the patch.



(1,0) mode

$$\underline{J}_s = \hat{x} \cos\left(\frac{\pi x}{L}\right)$$

The probe is on the x axis.

Radiation Pattern

The far-field pattern can be determined by reciprocity.

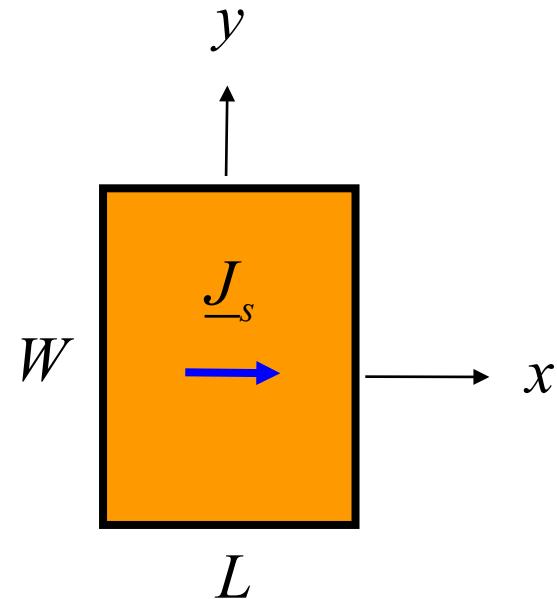
$$E_i(r, \theta, \phi) = E_i^{\text{hex}}(r, \theta, \phi) \left(\frac{\pi WL}{2} \right) \left[\frac{\sin\left(\frac{k_y W}{2}\right)}{\frac{k_y W}{2}} \right] \left[\frac{\cos\left(\frac{k_x L}{2}\right)}{\left(\frac{\pi}{2}\right)^2 - \left(\frac{k_x L}{2}\right)^2} \right]$$

$i = \theta$ or ϕ

$$k_x = k_0 \sin \theta \cos \phi$$

$$k_y = k_0 \sin \theta \sin \phi$$

The “hex” pattern is for a horizontal electric dipole in the x direction, sitting on top of the substrate.



D. R. Jackson and J. T. Williams, “A Comparison of CAD Models for Radiation from Rectangular Microstrip Patches,” *Intl. Journal of Microwave and Millimeter-Wave Computer Aided Design*, vol. 1, no. 2, pp. 236-248, April 1991.

Radiation Pattern

$$E_{\phi}^{hex}(r, \theta, \phi) = -E_0 \sin \phi \ F(\theta)$$

$$E_{\theta}^{hex}(r, \theta, \phi) = E_0 \cos \phi \ G(\theta)$$

where

$$E_0 = \left(\frac{-j\omega \mu_0}{4\pi r} \right) e^{-jk_0 r}$$

$$F(\theta) = 1 + \Gamma^{TE}(\theta) = \frac{2 \tan(k_0 h N(\theta))}{\tan(k_0 h N(\theta)) - j N(\theta) \sec \theta}$$

$$G(\theta) = \cos \theta (1 + \Gamma^{TM}(\theta)) = \frac{2 \tan(k_0 h N(\theta)) \cos \theta}{\tan(k_0 h N(\theta)) - j \frac{\epsilon_r}{N(\theta)} \cos \theta}$$

$$N(\theta) = \sqrt{\epsilon_r - \sin^2(\theta)}$$

Note: To account for lossy substrate, use

$$\epsilon_r \rightarrow \epsilon_{rc} = \epsilon_r (1 - j \tan \delta)$$

Input Impedance

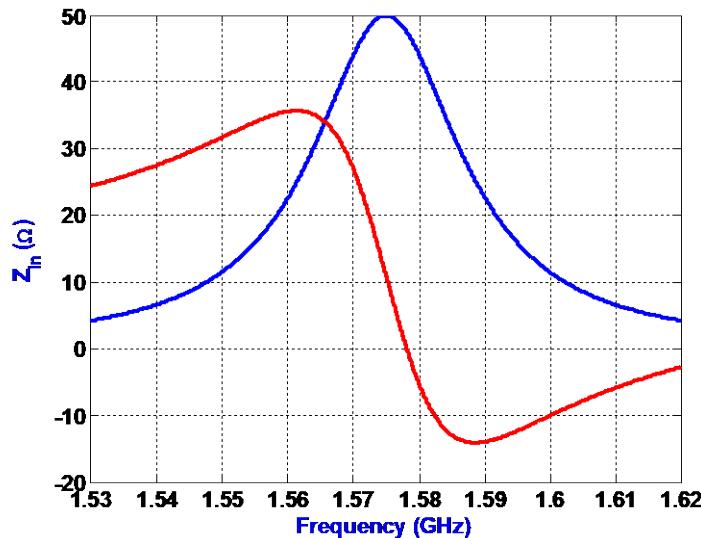
Various models have been proposed over the years for calculating the input impedance of a microstrip patch antenna.

- **Transmission line model**
 - The first model introduced
 - Very simple
- **Cavity model (eigenfunction expansion)**
 - Simple yet accurate for thin substrates
 - Gives physical insight into operation
- **CAD circuit model**
 - Extremely simple and almost as accurate as the cavity model
- **Spectral-domain method**
 - More challenging to implement
 - Accounts rigorously for both radiation and surface-wave excitation
- **Commercial software**
 - Very accurate
 - Can be time consuming

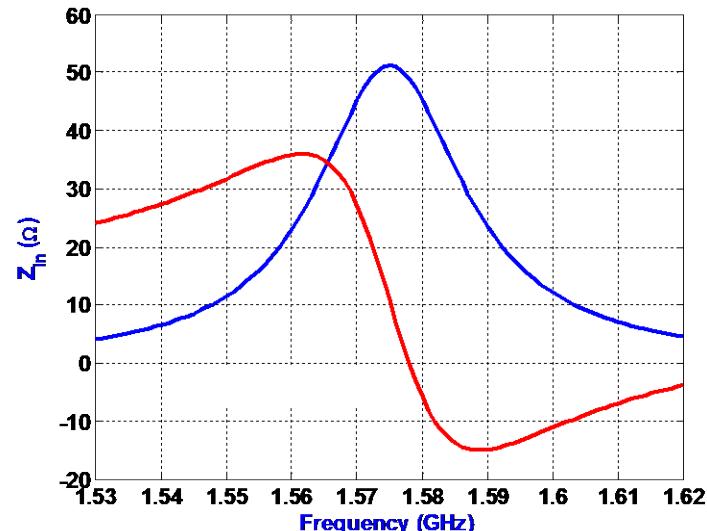
Input Impedance

Comparison of the Three Simplest Models

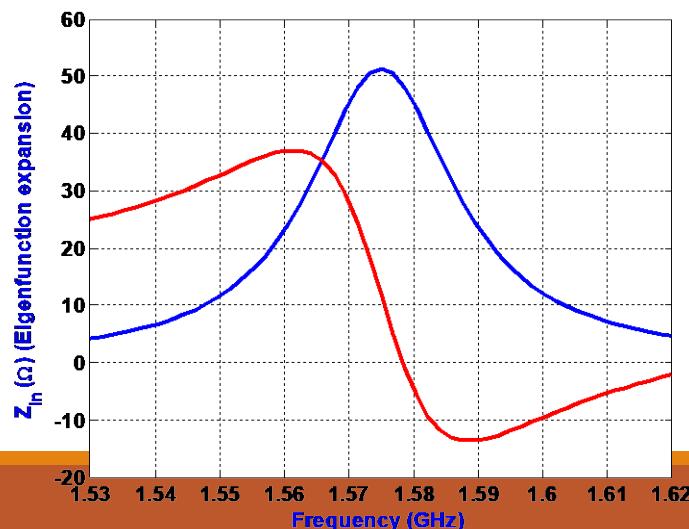
Circuit model of patch



Transmission line model of patch



Cavity model (eigenfunction expansion) of patch



$$\epsilon_r = 2.2$$

$$\tan \delta = 0.001$$

$$h = 1.524 \text{ mm}$$

$$L = 6.255 \text{ cm}$$

$$W / L = 1.5$$

$$\sigma = 3.0 \times 10^7 \text{ S/m}$$

$$x_0 = 6.255 \text{ cm}$$

$$y_0 = 0$$

$$a = 0.635 \text{ mm}$$

Results for a typical patch show that the first three methods agree very well, provided the correct Q is used and the probe inductance is accounted for.

Input Impedance

CAD Circuit Model for Input Impedance

The circuit model discussed assumes a probe feed.
Other circuit models exist for other types of feeds.

Note:

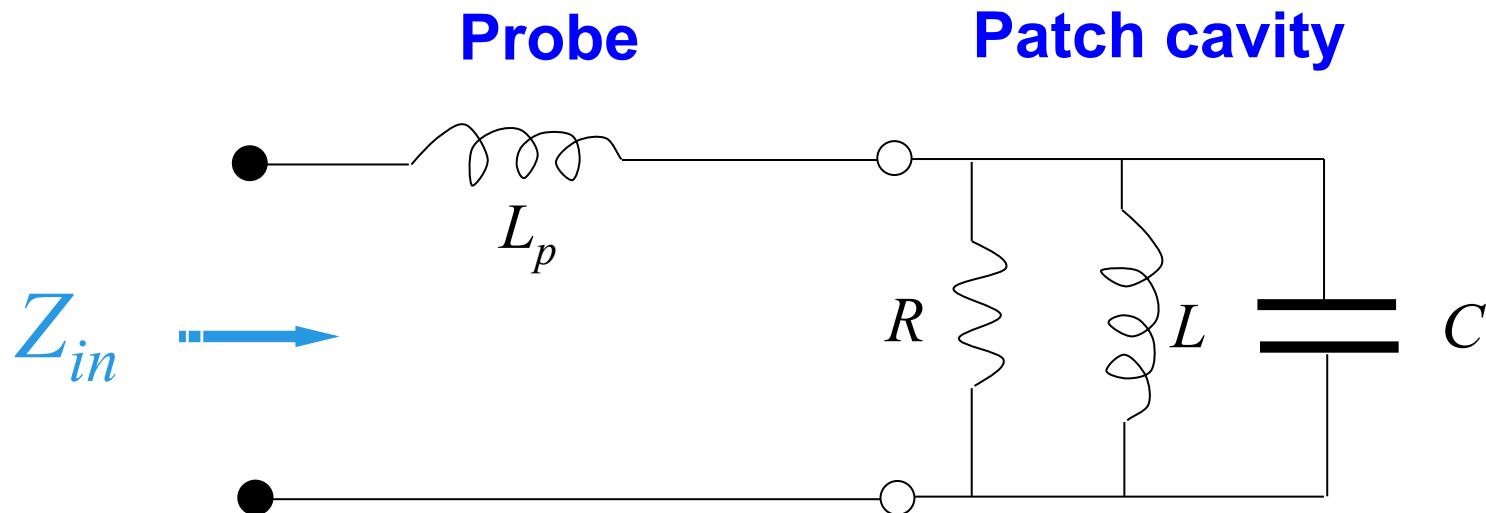
The mathematical justification of the CAD circuit model comes from a cavity-model eigenfunction analysis.

Y. T. Lo, D. Solomon, and W. F. Richards, "Theory and Experiment on Microstrip Antennas," *IEEE Trans. Antennas Propagat.*, vol. AP-27, no. 3 (March 1979): 137–145.

Input Impedance

Probe-fed Patch

- Near the resonance frequency, the patch cavity can be approximately modeled as a resonant RLC circuit.
- The resistance R accounts for radiation and losses.
- A probe inductance L_p is added in series, to account for the “probe inductance” of a probe feed.



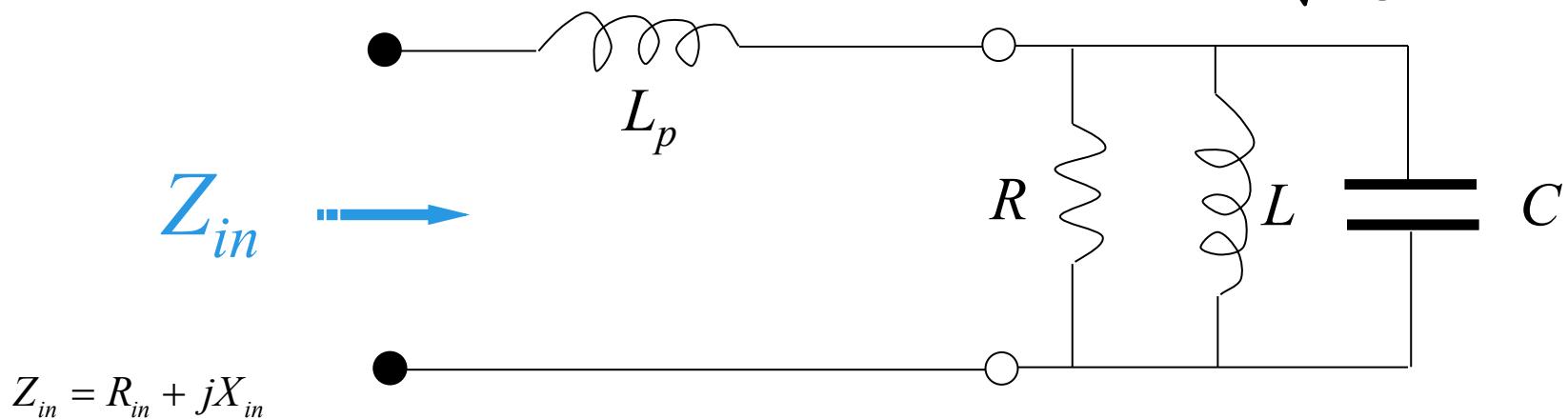
Input Impedance

$$Z_{in} \approx j\omega L_p + \frac{R}{1 + jQ \left(\frac{f}{f_0} - \frac{f_0}{f} \right)}$$

$$Q = \frac{R}{\omega_0 L} \quad BW = \frac{1}{\sqrt{2} Q}$$

BW is defined here by $SWR < 2.0$ when the RLC circuit is fed by a matched line ($Z_0 = R$).

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$$



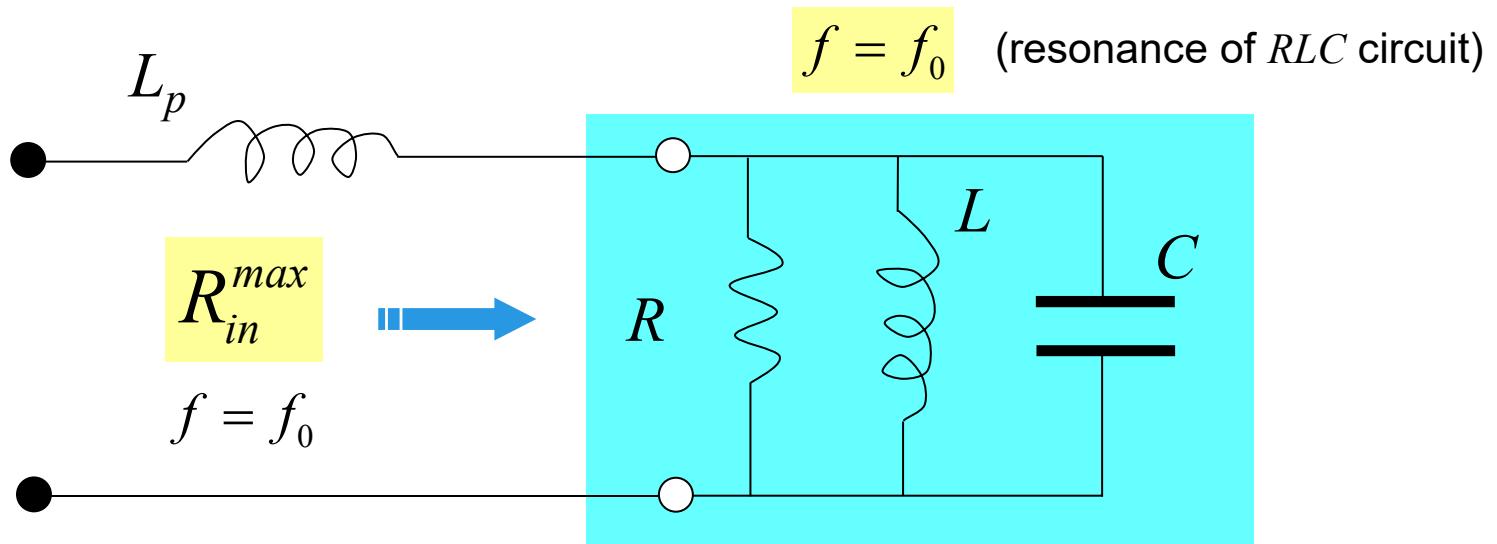
Input Impedance

$$R_{in} = \frac{R}{1 + \left[Q \left(\frac{f}{f_0} - \frac{f_0}{f} \right) \right]^2}$$



$$R_{in}^{max} = R_{in} \Big|_{f=f_0} = R$$

R is the input resistance at the resonance of the patch cavity
(the frequency that maximizes R_{in}).



Input Impedance

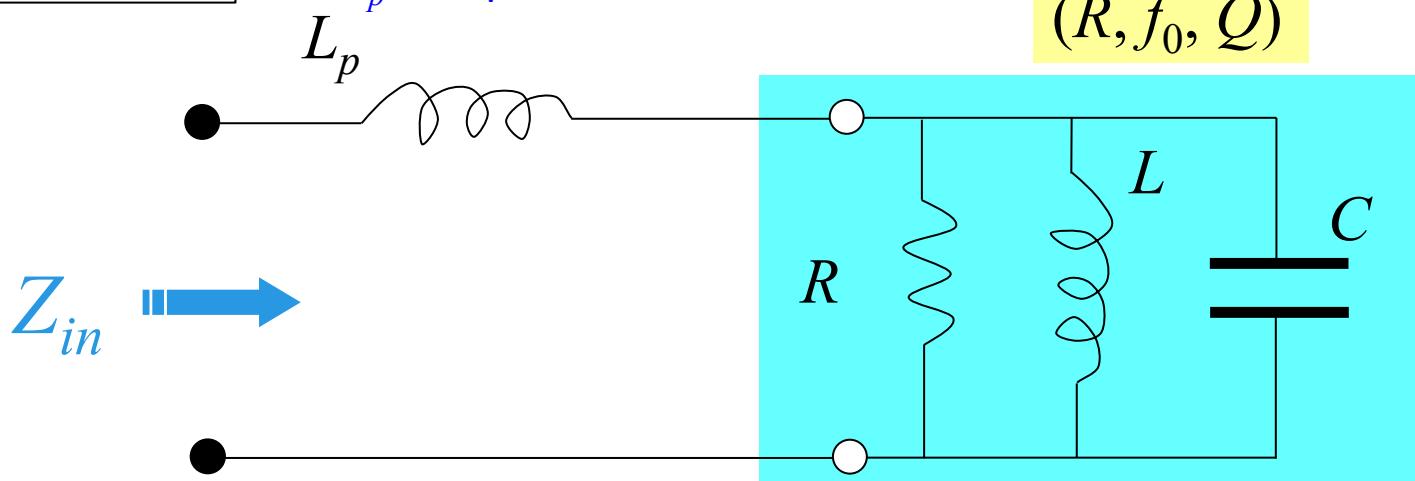
$$Z_{in} \approx j\omega L_p + \frac{R}{1 + jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}$$

The input resistance is determined once we know four parameters:

CAD formulas
for all of these
four parameters
have been given
earlier.

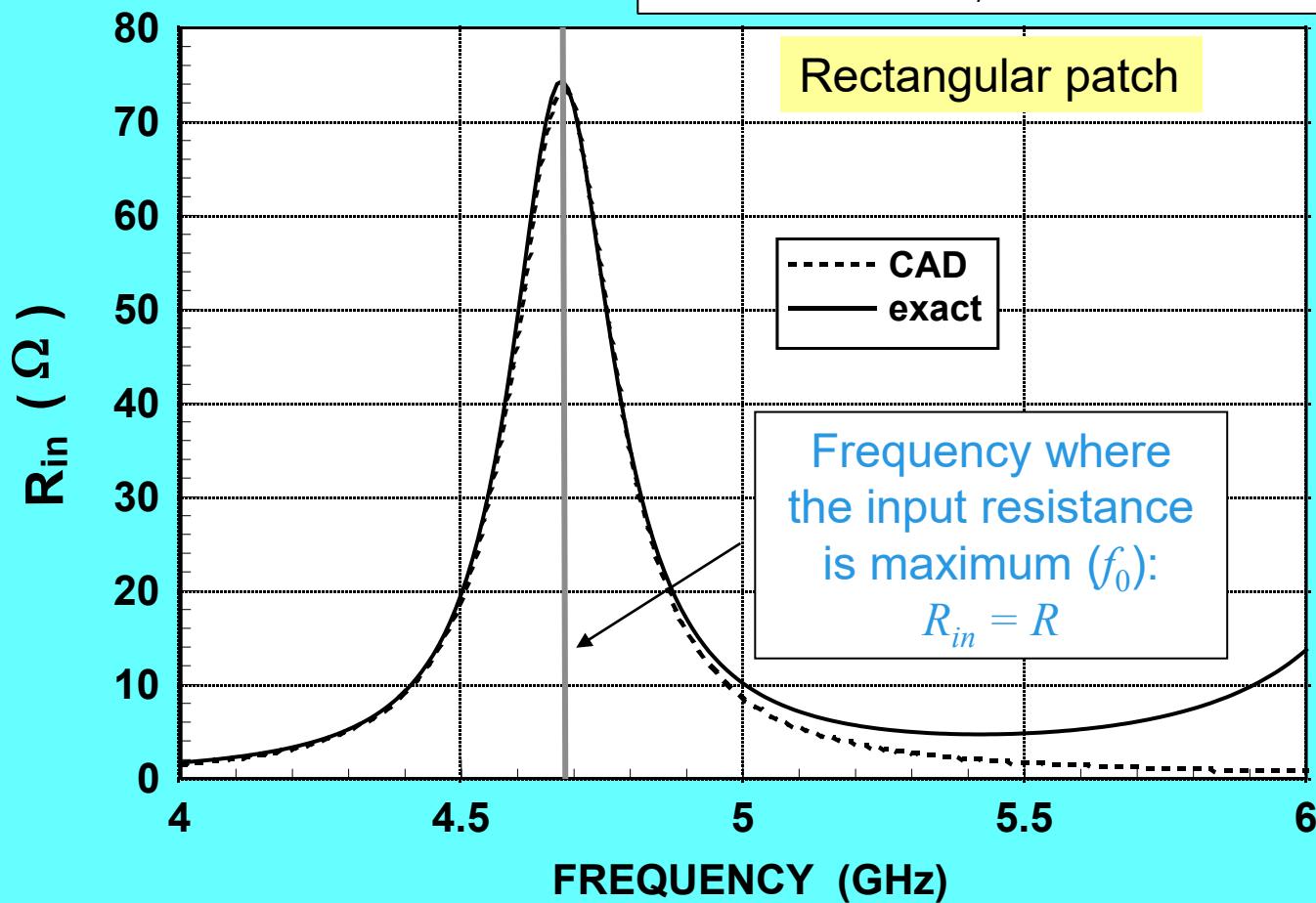
- f_0 : the resonance frequency of the patch cavity
- R : the input resistance at the cavity resonance frequency f_0
- Q : the quality factor of the patch cavity
- L_p : the probe inductance

(R, f_0, Q)



Input Impedance

Results: Input Resistance vs. Frequency



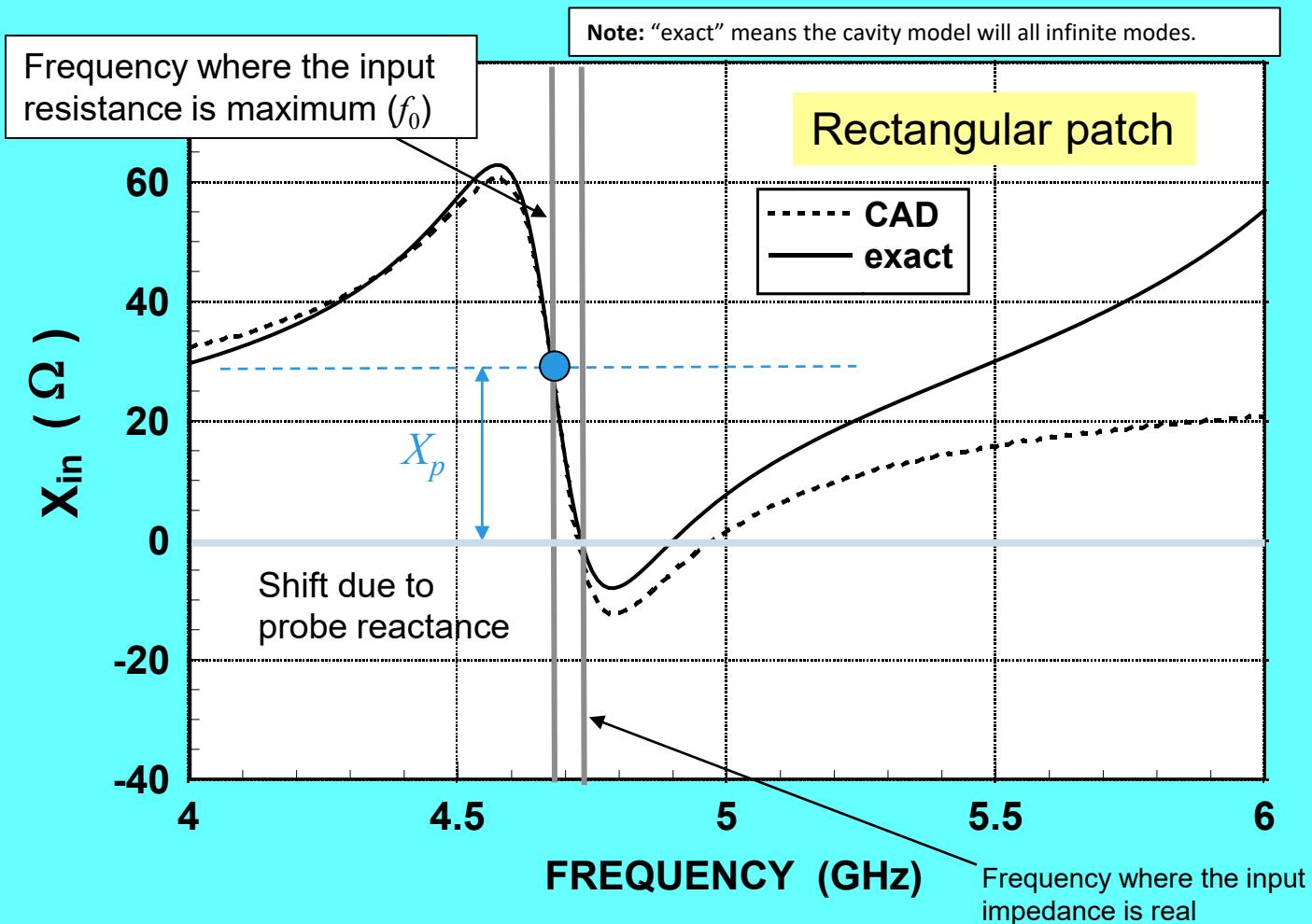
$$\epsilon_r = 2.2$$

$$W/L = 1.5$$

$$L = 3.0 \text{ cm}$$

Input Impedance

Results: Input Reactance vs. Frequency



$$\epsilon_r = 2.2$$

$$W/L = 1.5$$

$$L = 3.0 \text{ cm}$$

Design Example

Design a probe-fed rectangular patch antenna on a substrate having a relative permittivity of 2.33 and a thickness of 62 mils (0.1575 cm). (This is Rogers RT Duroid 5870.) Choose an aspect ratio of $W / L = 1.5$. The patch should resonate at the operating frequency of 1.575 GHz (the GPS L1 frequency). Ignore the probe inductance in your design, but account for fringing at the patch edges when you determine the dimensions. At the operating frequency the input impedance should be 50Ω (ignoring the probe inductance). Assume an SMA connector is used to feed the patch along the centerline (at $y = W / 2$), and that the inner conductor of the SMA connector has a radius of 0.635 mm. The copper patch and ground plane have a conductivity of $\sigma = 3.0 \times 10^7 \text{ S/m}$ and the dielectric substrate has a loss tangent of $\tan\delta = 0.001$.

1) Calculate the following:

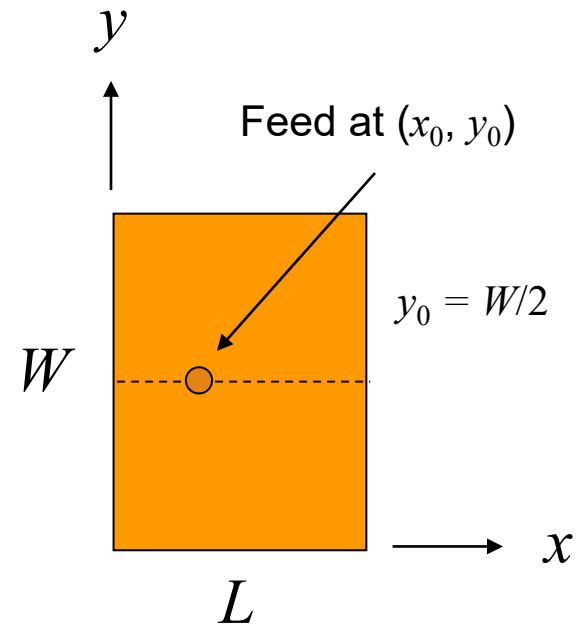
1. The final patch dimensions L and W (in cm)
2. The feed location x_0 (distance of the feed from the closest patch edge, in cm)
3. The bandwidth of the antenna (SWR < 2 definition, expressed in percent)
4. The radiation efficiency of the antenna (accounting for conductor, dielectric, and surface-wave loss, and expressed in percent)
5. The probe reactance X_p at the operating frequency (in Ω)
6. The expected complex input impedance (in Ω) at the operating frequency, accounting for the probe inductance
7. Directivity
8. Gain

2) Plot the input impedance vs. frequency.

Design Example

Results from the CAD formulas

1. $L = 6.07 \text{ cm}$, $W = 9.11 \text{ cm}$
2. $x_0 = 1.82 \text{ cm}$
3. $\text{BW} = 1.24\%$
4. $e_r = 81.9\%$
5. $X_p = 11.1 \Omega$
6. $Z_{in} = 50.0 + j(11.1) \Omega$
7. $D = 5.85$ (7.67 dB)
8. $G = (D)(e_r) = 4.80$ (6.81 dB)



Design Example

Results from the CAD formulas:

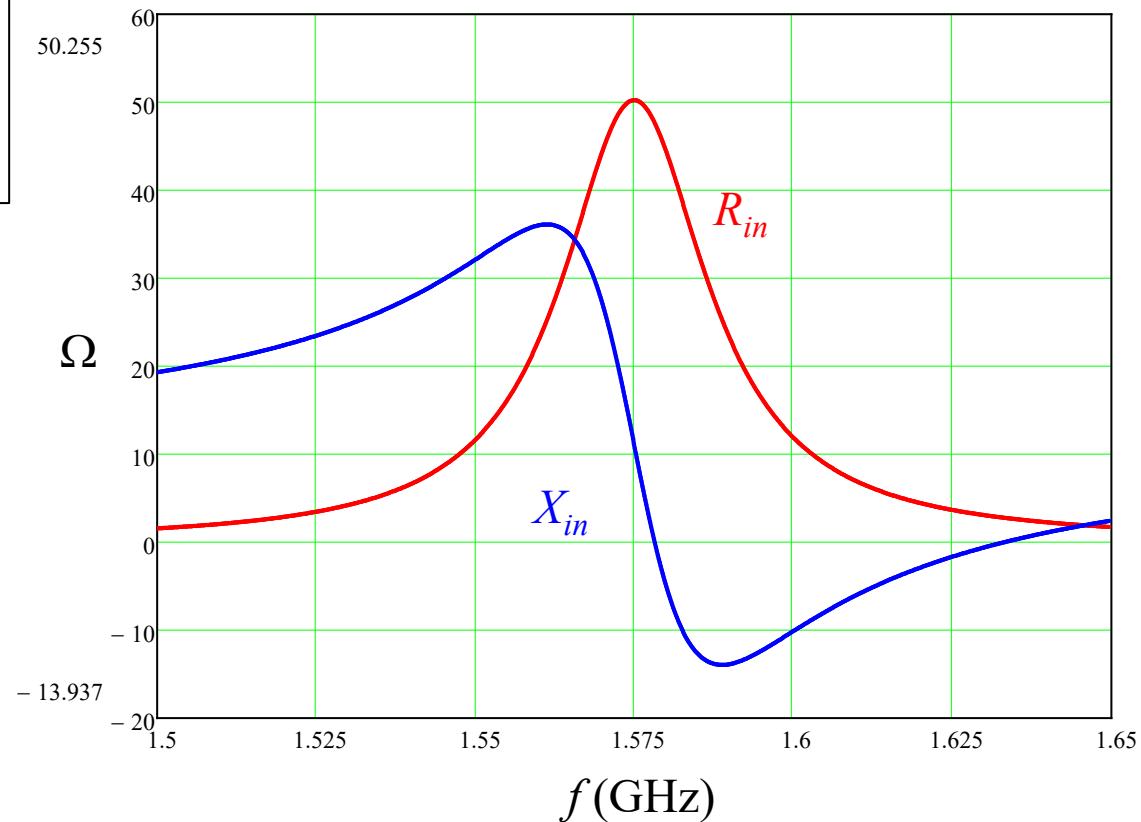
$$f_0 = 1.575 \times 10^9 \text{ Hz}$$

$$R = 50 \Omega$$

$$Q = 56.8$$

$$X_p = 11.1 \Omega$$

$$Z_{in} \approx jX_p + \frac{R}{1 + jQ\left(\frac{f}{f_0} - \frac{f_0}{f}\right)}$$



Circular Polarization

Three main techniques:

- 1) Single feed with “nearly degenerate” eigenmodes (compact but small CP bandwidth).
- 2) Dual feed with delay line or 90° hybrid phase shifter (broader CP bandwidth but uses more space).
- 3) Synchronous subarray technique (produces high-quality CP due to cancellation effect, but requires even more space).

The techniques will be illustrated with a rectangular patch.

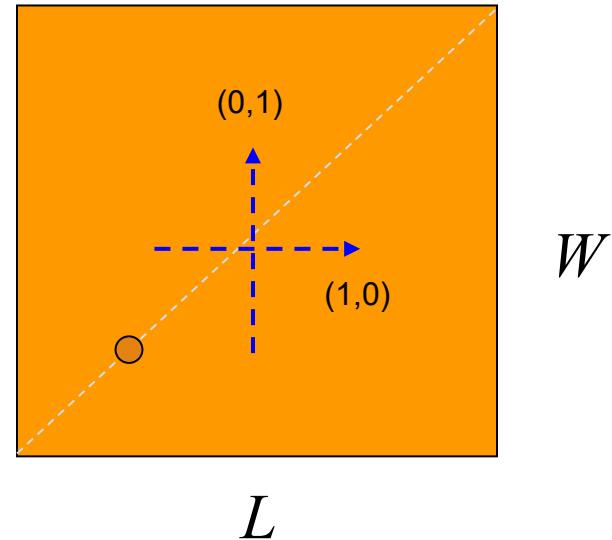
Circular Polarization

Single Feed Method

The feed is on the diagonal.

The patch is nearly
(but not exactly) square.

$$L \approx W$$



Basic principle: The two dominant modes $(1,0)$ and $(0,1)$ are excited with equal amplitude, but with a $\pm 45^\circ$ phase.

They are called degenerate modes.

Degenerate Mode: The modes having same cut off frequency but different field configuration are called **Degenerate Mode**

Circular Polarization

Design equations:

$$f_{CP} = \frac{f_x + f_y}{2}$$

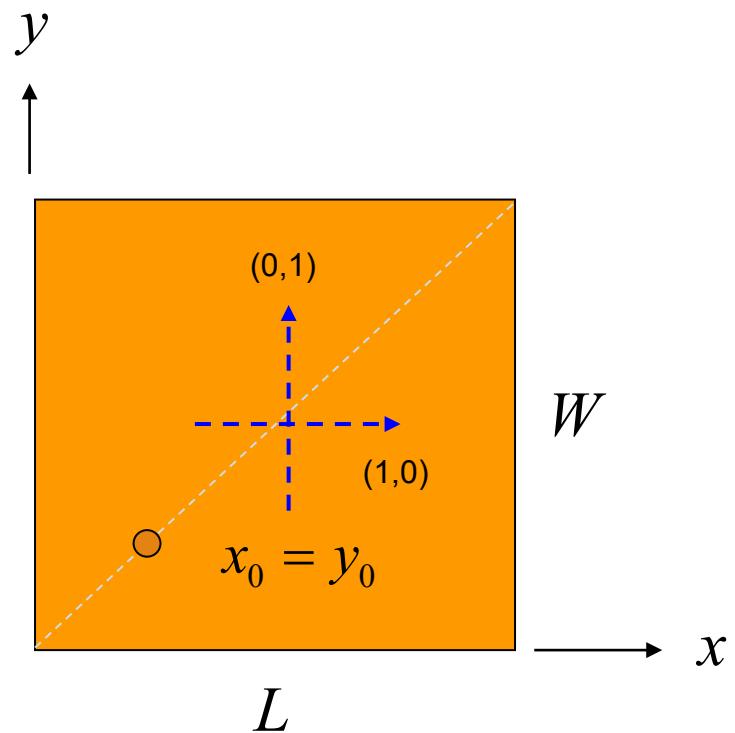
The optimum CP frequency is the average of the x and y resonance frequencies.

$$\left. \begin{array}{l} f_x = f_{CP} \left(1 \mp \frac{1}{2Q} \right) \\ f_y = f_{CP} \left(1 \pm \frac{1}{2Q} \right) \end{array} \right\}$$

Top sign for LHCP,
bottom sign for RHCP.

$$BW = \frac{1}{\sqrt{2}Q}$$

(SWR < 2)



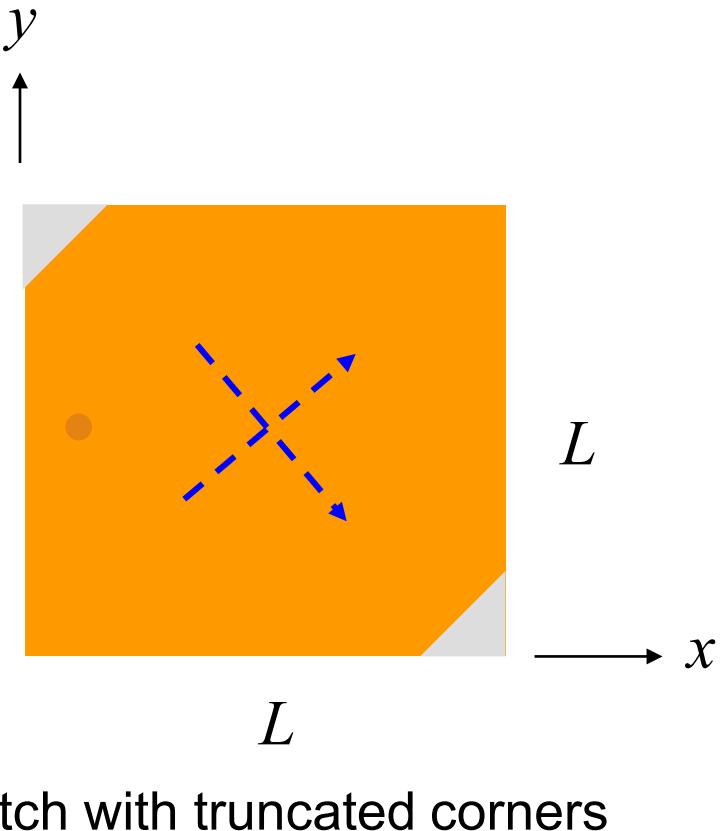
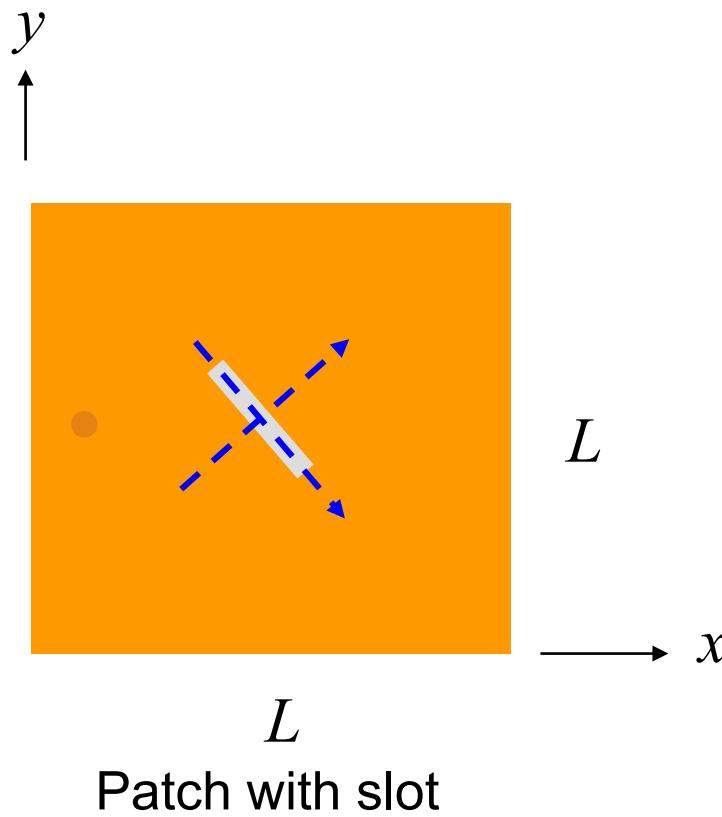
The frequency f_{CP} is also the resonance frequency: $Z_{in} = R_{in} = R_x = R_y$

The resonant input resistance of the CP patch at f_{CP} is the same as what a *linearly-polarized patch* fed at the same position would be.

Circular Polarization

Other Variations

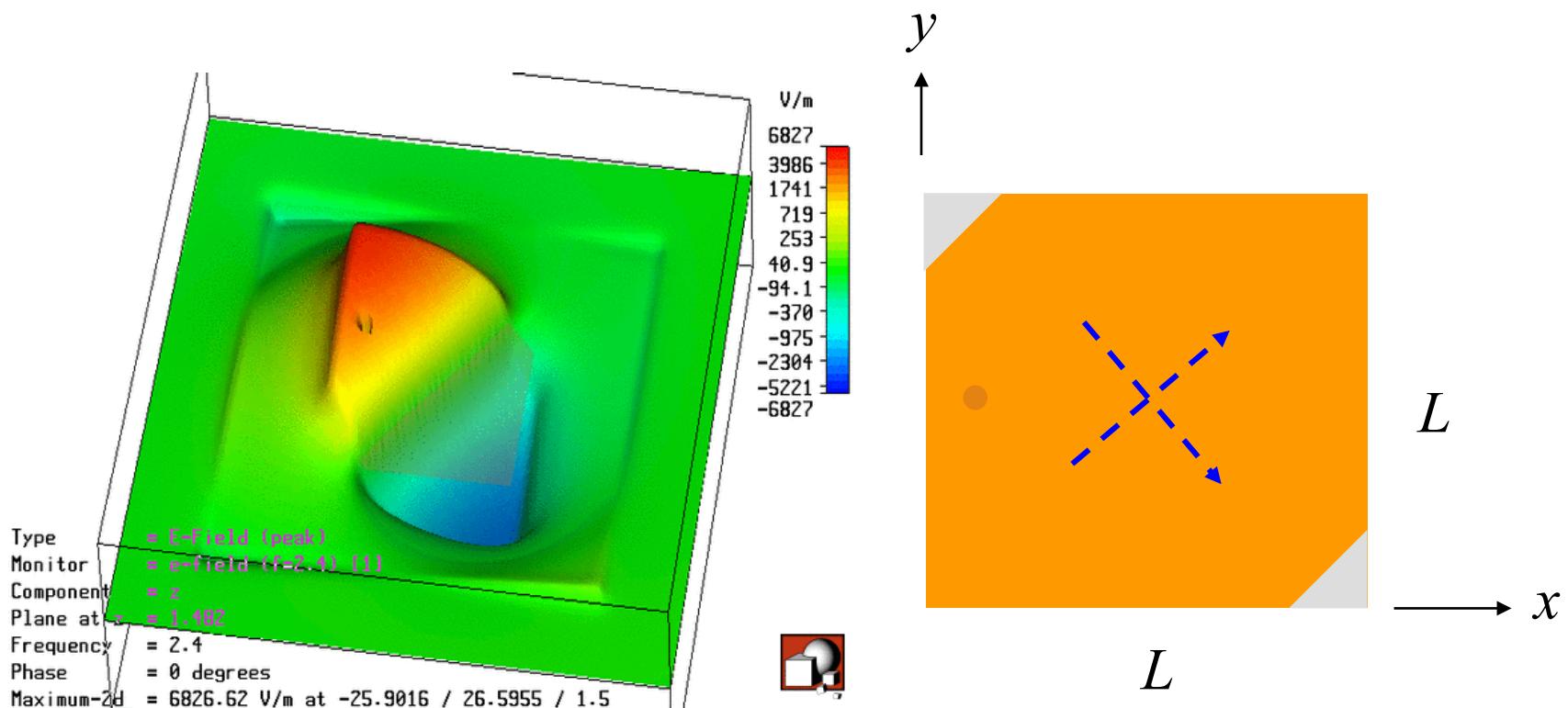
Note: Diagonal modes are used as degenerate modes



Circular Polarization

Other Variations

Note: Diagonal modes are used as degenerate modes



Patch with truncated corners

Circular Polarization

Here we compare bandwidths (impedance and axial-ratio):

Linearly-polarized (LP) patch:

$$BW_{SWR}^{LP} = \frac{1}{\sqrt{2}Q} \quad (\text{SWR} < 2)$$

Circularly-polarized (CP) single-feed patch:

$$BW_{SWR}^{CP} = \frac{\sqrt{2}}{Q} \quad (\text{SWR} < 2)$$

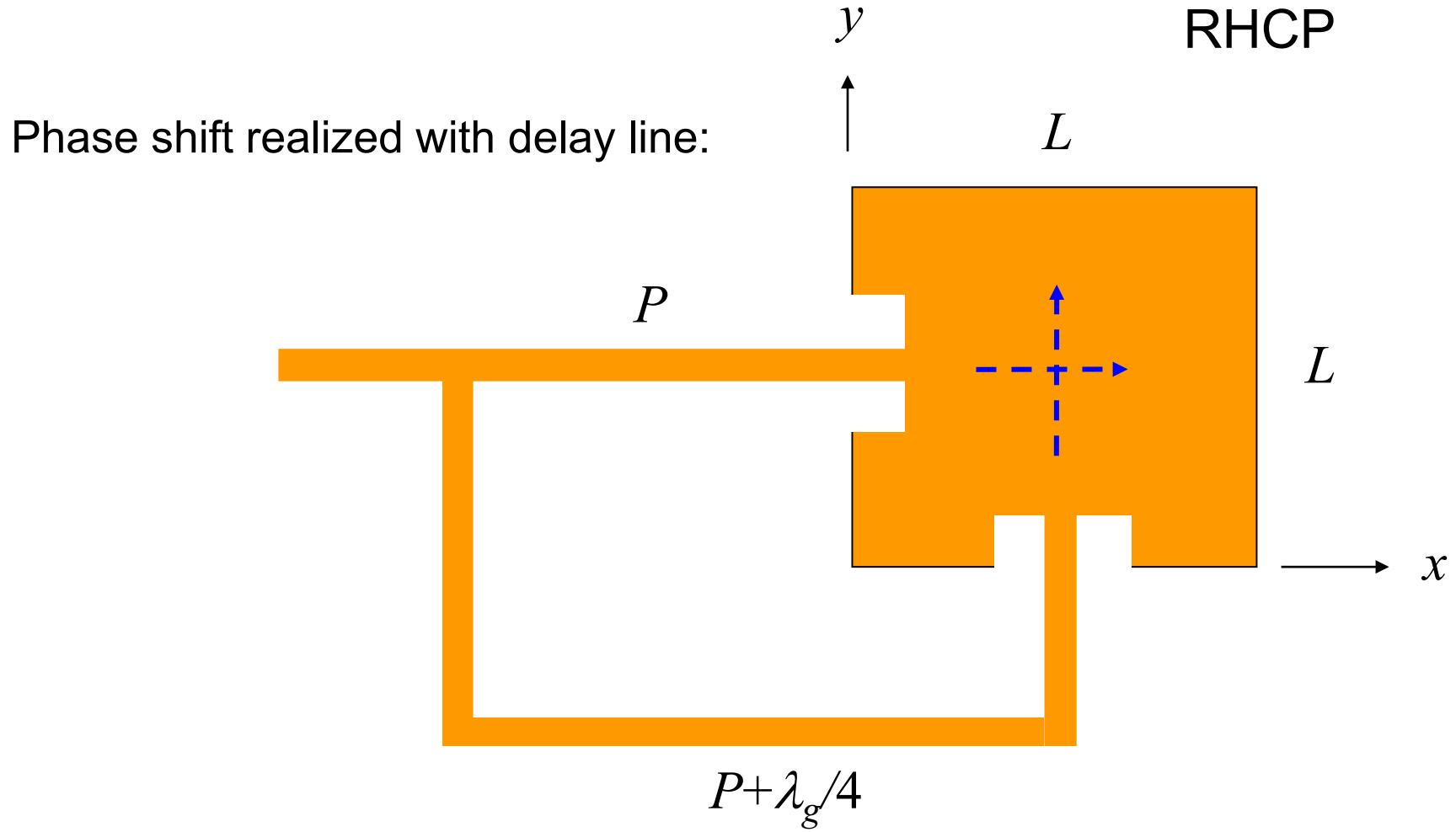
$$BW_{AR}^{CP} = \frac{0.348}{Q} \quad (\text{AR} < \sqrt{2} \text{ (3 dB)})$$

The axial-ratio bandwidth is small when using the single-feed method.

W. L. Langston and D. R. Jackson, "Impedance, Axial-Ratio, and Receive-Power Bandwidths of Microstrip Antennas," *IEEE Trans. Antennas and Propagation*, vol. 52, pp. 2769-2773, Oct. 2004.

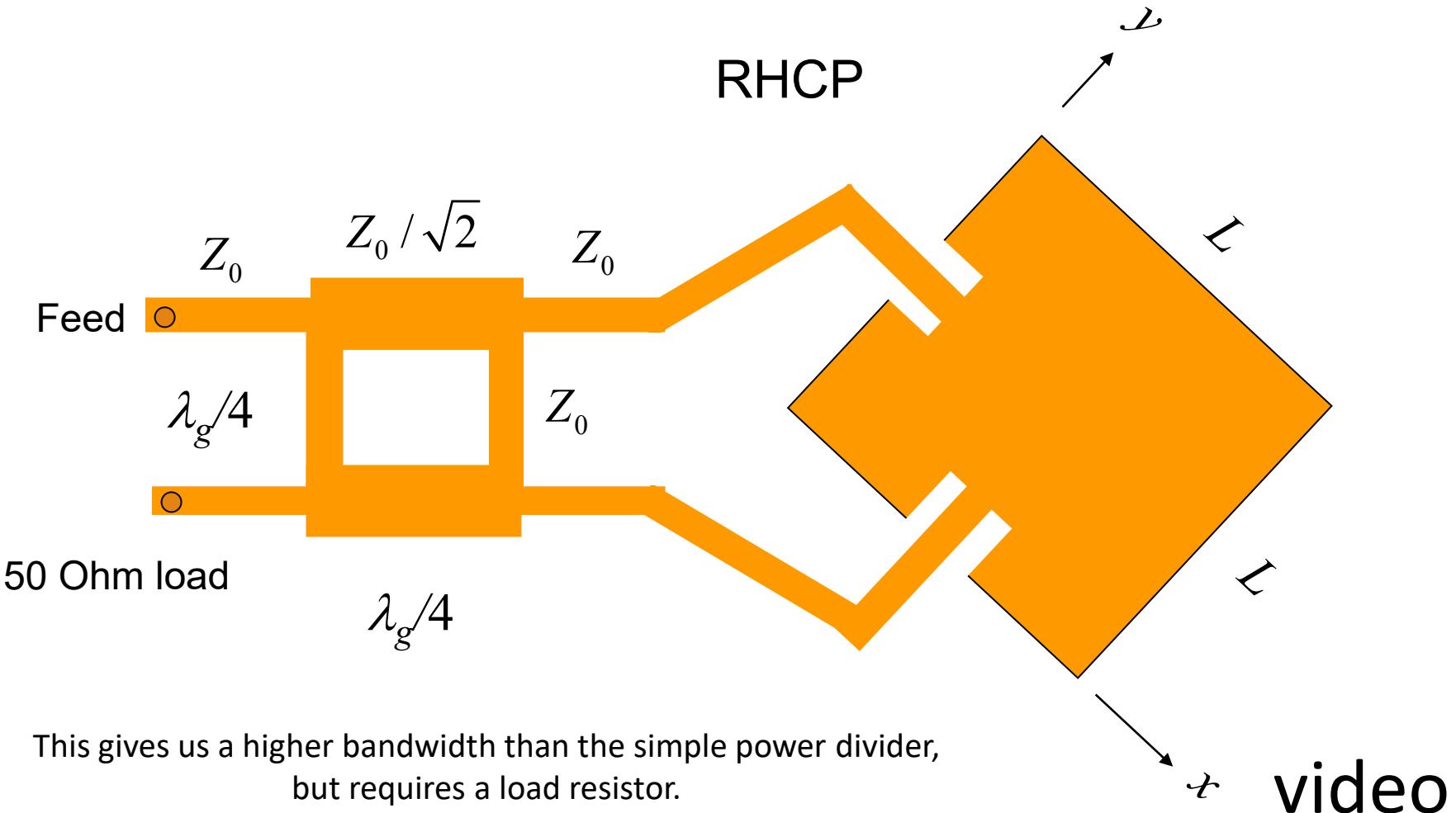
Circular Polarization

Dual-Feed Method



Circular Polarization

Phase shift realized with 90° quadrature hybrid (branchline coupler)

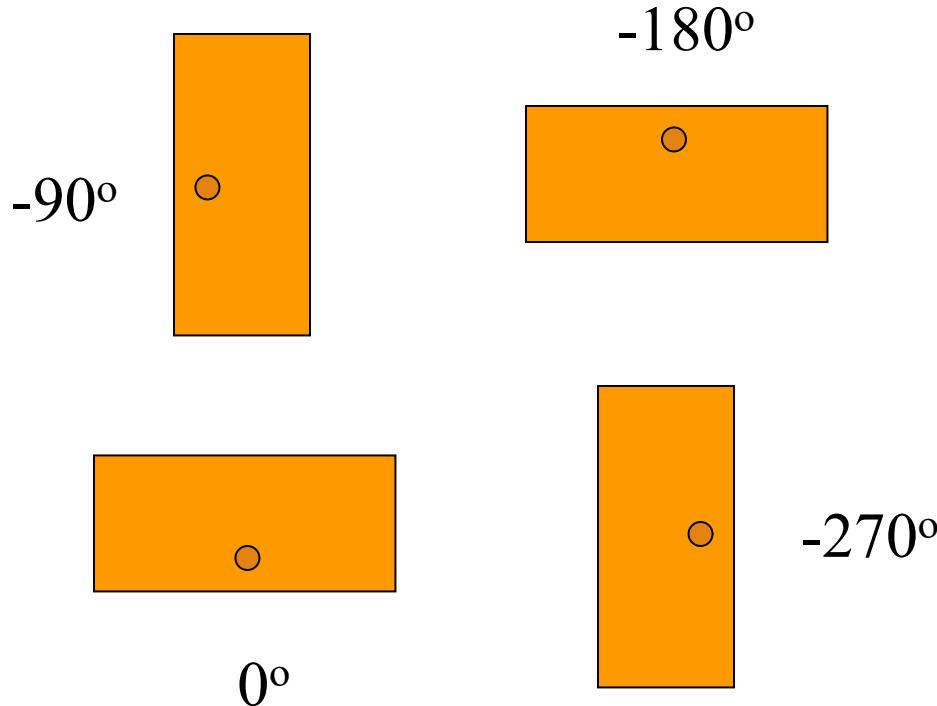


This gives us a higher bandwidth than the simple power divider,
but requires a load resistor.

Circular Polarization

Synchronous Rotation

Multiple elements are rotated in space and fed with phase shifts.



Because of symmetry, radiation from higher-order modes (or probes) tends to be reduced, resulting in good cross-pol.

Homework

[UG & PG] PP. [116-118](#), use Matlab or a programming language to verify the results.

[PG] Use your program to design a patch antenna of same specification using a FR4 substrate ($\epsilon = 4.3$, $h = 1.55$ mm)