EE307 assignment4 Homework

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I. Homework1

A. Derive H and E field from Vector Potential on page 19

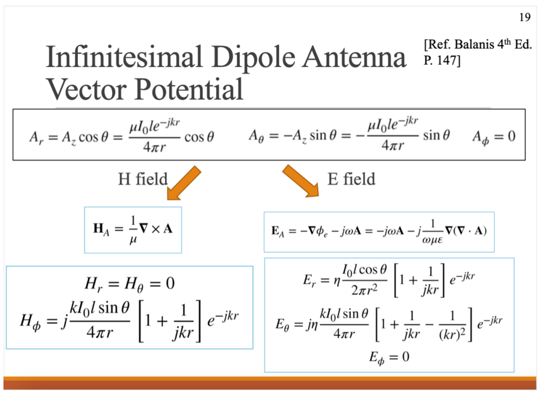

Fig. 1: this is screenshot of the lecture slides

$$
\begin{aligned}
\vec{H}_A &= \frac{1}{\mu}\nabla\times\vec{A}\\
&= \frac{1}{\mu}\left[0\cdot\vec{a}_r + 0\cdot\vec{a}_\theta + \frac{\vec{a}_\phi}{r}\left[\frac{\partial(rA_\theta)}{\partial r} - \frac{\partial A_r}{\partial\theta}\right]\right]\\
&= \frac{\vec{a}_\phi}{\mu r}\left[\frac{jk\mu I_0 l e^{-jkr}}{4\pi}\sin\theta + \frac{\mu I_0 l e^{-jkr}}{4\pi r}\sin\theta\right]\\
&= \frac{jkI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}\vec{a}_\phi\\
\overrightarrow{A} &= \overrightarrow{A_\gamma} + \overrightarrow{A_\theta} + \overrightarrow{A_\phi}\\
&= \frac{\mu I_0/e^{-jk_\gamma}}{4\pi\gamma}\cos\theta\overrightarrow{a_\gamma} + \left(\frac{-\mu I_0/e^{-ik\gamma}}{4\pi\gamma}\sin\theta\right)\overrightarrow{a_\theta}
\end{aligned}
$$

$$
\vec{H}_r = \vec{H}_\theta = \overrightarrow{0}
$$

$$
H_\phi = j\frac{kI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}\right]e^{-jkr}
$$

$$
\vec{E}_A = -\nabla\vec{\phi}_e - j\omega\vec{A} = -j\omega\vec{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\vec{A})
$$

$$
\begin{aligned}
\nabla\cdot\vec{A} &= \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(\frac{\mu I_0\left(r\cdot e^{-jkr}\cos\theta\right)}{4\pi}\right)\\
&+ \frac{1}{r\cdot\sin\theta}\cdot\frac{\partial}{\partial\theta}\left(\frac{-\mu I_0 l e^{-jkr}\sin^2\theta}{4\pi r}\right)\\
&= \frac{1}{r^2}\left(\frac{\mu_0 I l e^{-jkr}\cos\theta}{4\pi} + \frac{-jkr\mu I_0 e^{-jkr}\cos\theta}{4\pi}\right)\\
&+ \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(\frac{\mu I_0 l e^{-jkr}(1-\cos 2\theta)}{8\pi r}\right)\\
&= \frac{1}{r^2}\left(\frac{\mu I_0 l e^{-jkr}\cos\theta}{4\pi} + \frac{-jkr\mu I_0 l e^{-jkr}\cos\theta}{4\pi}\right)\\
&+ \frac{1}{r\sin\theta}\frac{-\mu I_0 l e^{-jkr}\sin 2\theta}{4\pi r}\\
&= \frac{1}{r^2}\left(\frac{\mu I_0 l e^{-jkr}\cos\theta}{4\pi} + \frac{-jkr\mu I_0 e^{-jkr}\cos\theta}{4\pi}\right)\\
&+ \frac{\mu I_0 l e^{-jkr}\cdot\cos\theta}{2\pi r^2}
\end{aligned}
$$

$$
\nabla(\nabla\cdot\vec{A}) = \overrightarrow{a_r}\frac{\partial(\nabla\cdot\vec{A})}{\partial r} + \frac{\rightharpoonup}{a_\theta\cdot\frac{1}{r}}\frac{\partial(\nabla\cdot\vec{A})}{\partial\theta} + \overrightarrow{a_\phi}\cdot\frac{1}{r\sin\theta}\cdot\frac{\partial(\nabla\cdot\vec{A})}{\partial\phi}
$$

$$
\begin{aligned}
&= \overrightarrow{a_r}\frac{\partial}{\partial r}\left[\frac{\mu I_0 l\cos\theta}{4\pi}\left(\frac{e^{-jkr}}{r^2} + \frac{-jkre^{-jkr}}{r^2} + \frac{-2e^{-jkr}}{r^2}\right]\right.\\
&+ \frac{1}{r}\left[\frac{1}{r^2}\frac{-\mu_0 I_0 l e^{-jkr}\sin\theta}{4\pi} + \frac{1}{r^2}\frac{1+jkr\mu I_0 l e^{-jkr}\sin\theta}{4\pi}\right.\\
&+ \frac{\mu I_0 l e^{-jkr}\sin\theta}{2\pi r^2}\overrightarrow{a_\theta}\\
&= \frac{\mu I_0 l\cos\theta}{4\pi}\left(\frac{-jke^{-jkr}}{r^2} - \frac{2e^{-jkr}}{r^3} + \frac{jke^{-jkr}}{r} - \frac{k^2e^{-jkr}}{r}\right.\\
&+ \left.\frac{jk2e^{-jkr}}{r^2} + \frac{4e^{-jkr}}{r}\right)\overrightarrow{a_r}\\
&+ \frac{1}{r^3}\frac{\mu I_0 l e^{-jkr}\sin\theta}{4\pi}[jkr-1+2]\vec{a}_\theta
\end{aligned}
$$

$$
\eta = \frac{w\mu}{k} = \frac{\sqrt{\mu}}{\sqrt{\varepsilon}}
$$

$$
\begin{aligned}
\vec{E}_A &= -j\omega\vec{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\vec{A}) = -j\omega\left[\frac{\mu I_0 l e^{-jkr}}{4\pi r}\cos\theta\overrightarrow{a_r}\right.\\
&\left.-\frac{\mu I_0 l e^{-jkr}}{4\pi r}\sin\theta\vec{a}_\theta\right] - j\frac{1}{\omega\mu\varepsilon}\left[\frac{\mu I_0 l\cos\theta}{4\pi}\right.\\
&\left(\frac{-jke^{-jkr}}{r^2} - \frac{2e^{-jkr}}{r^2} + \frac{jke^{-jkr}}{r} - \frac{ke^2-jkr}{r} + \frac{jk2e^{-jkr}}{r^2}\right.\\
&\left.\left.+\frac{4e^{-jkr}}{r}\right)\overrightarrow{a_r} + \frac{1}{r^3}\frac{\mu I_0 l e^{-jkr}\sin\theta}{4\pi}[jkr+1]\overrightarrow{a_\theta}\right]\\
&= \frac{\omega\mu I_0 l\cos\theta}{2\pi r^2\cdot k}\left[\frac{1}{jkr}+1\right]e^{-jkr}\vec{a}_r + j\frac{j\mu kI_0 l\sin\theta}{4\pi kr}\vec{a}_\theta\\
&\left[1-\frac{1}{(kr)^2}+\frac{1}{jkr}\right]e^{-jkt}
\end{aligned}
$$

$$
\overrightarrow{E_A} = \frac{\eta I_0 l\cos\theta}{2\pi r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}\,\vec{a_r} + \frac{j\eta kI_0 l\sin\theta}{4\pi r}
$$

$$
\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}\,\vec{a_\theta}
$$

$$
E_r = \eta\frac{I_0 l\cos\theta}{2\pi r^2}\left[1+\frac{1}{jkr}\right]e^{-jkr}
$$

$$
E_\theta = jn\frac{kI_0 l\sin\theta}{4\pi r}\left[1+\frac{1}{jkr}-\frac{1}{(kr)^2}\right]e^{-jkr}
$$

$$
E_\phi = 0
$$

B. *A 1-m–long dipole is excited by a 1-MHz current with an amplitude of 12 A.What is the average power density radiated by the dipole at a distance of 5 km in a direction that is 45○ from the dipole axis?*

$$f = 10^6 \text{ Hz}.$$
$$\lambda = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{10^6 \text{ Hz}} = 300 \text{ m}$$
$$l = 1 \text{ m} = \frac{\lambda}{300} < \frac{\lambda}{50}$$
The antenna is an infinitesimal dipole antenna.
$$W_{ar} = \frac{1}{2} \operatorname{Re}\left(\vec{E} \times \vec{H}^*\right) = \vec{a}_r \cdot \frac{\eta}{2} \left|\frac{kI_0 l}{4\pi}\right|^2 \frac{\sin^2\theta}{r^2}$$
$$I_0 = 12 \text{ A} \cdot r = 5 \times 10^3 \text{ m.} \quad \theta = 45^\circ \cdot \eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$
$$= 377$$
$$K = \frac{2\pi}{\lambda} = \frac{\pi}{150}$$
$$W_{ar} = \frac{377}{2} \times 1 \frac{\frac{\pi}{150} \times 12 \text{ A} \times 1 \text{ m}}{4\pi}\Bigg|^2 \times \frac{\left(\frac{\sqrt{2}}{2}\right)^2}{25 \times 10^6}$$
$$= 1.508 \times 10^{-9} \text{ W/m}^2$$

C. *Calculate radiation resistance of a 0.25-meter dipole antenna working at 600MHz*

$$\lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{600 \times 10^6} = 0.5 \text{ m}, l = \frac{\lambda}{2} > \frac{\lambda}{10}$$

It is half wavelength dipole antenna.
$$U(\theta) = \frac{1}{2} r^2 \frac{|E_\theta|^2}{\eta} = \frac{1}{2} \frac{\eta I_m^2}{(2\pi)^2} \frac{\cos^2(\pi/2 \cos\theta)}{\sin^2\theta}$$
The radiated power produced by the dipole is
$$\begin{aligned} W_{rad} &= \int_0^{2\pi} \int_0^{\pi} U(\theta) \sin\theta d\theta d\phi \\ &= \tfrac{1}{2}(2\pi) \tfrac{\eta I_m^2}{(2\pi)^2} \underbrace{\int_0^{\pi} \frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta} d\theta}_{1.2188 \text{ numerically}} \\ &= 30(1.2188) I_m^2 = 36.5640 I_m^2 \\ R_{rad} &= \frac{2W_{rad}}{I_m^2} = 73.1280\ \Omega \end{aligned}$$

D. *Plot radiation pattern for*

1) *half wavelength dipole:*

2) *quarter wavelength monopole (infinite ground)*

: For half wavelength dipole, the figures we get:

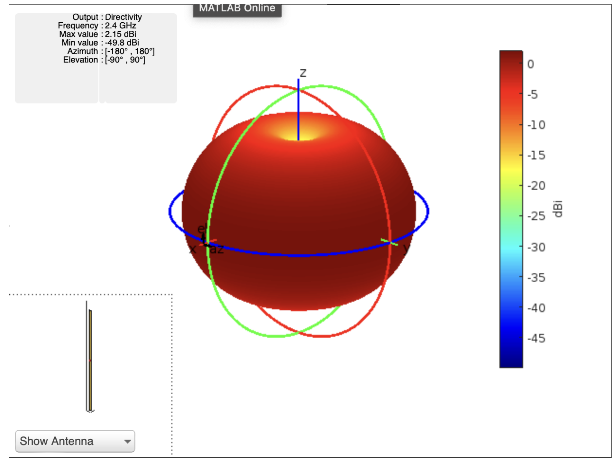

Fig. 2: this is screenshot of 3D radiation pattern in the azimuth for half wavelength dipole

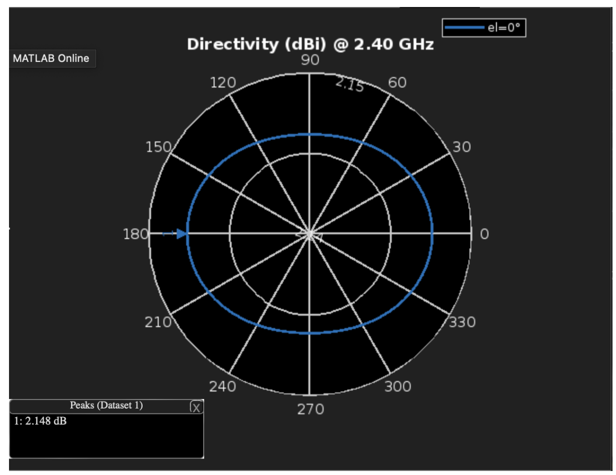

Fig. 3: this is screenshot of radiation pattern half wavelength dipole in the azimuth

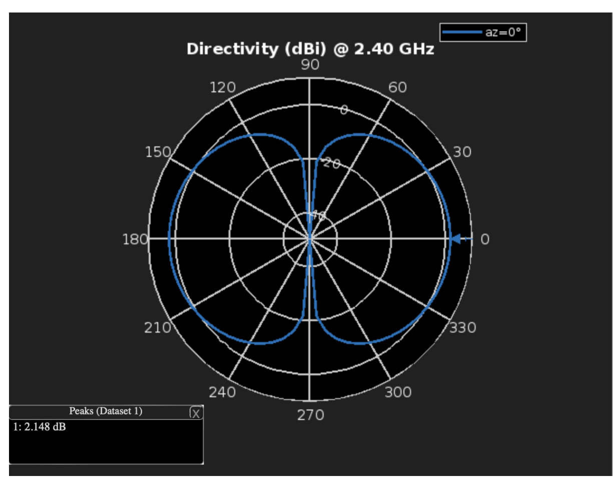

Fig. 4: this is screenshot of radiation pattern half wavelength dipole in the elevation

Listing 1: A Sample

```
%% Antenna Properties
antennaObject = design(dipole, 2400*1e6);
antennaObject.Length = 0.062;
%% Antenna Analysis
% Define plot frequency
plotFrequency = 2400*1e6;
% Define frequency range
freqRange = (2160:24:2640)*1e6;
% pattern
figure;
pattern(antennaObject, plotFrequency)
% azimuth
figure;
patternAzimuth(antennaObject, plotFrequency,
    0, 'Azimuth', 0:5:360)
% elevation
figure;
patternElevation(antennaObject, plotFrequency
    ,0,'Elevation',0:5:360)
```

For quarter wavelength dipole, the figures we get:

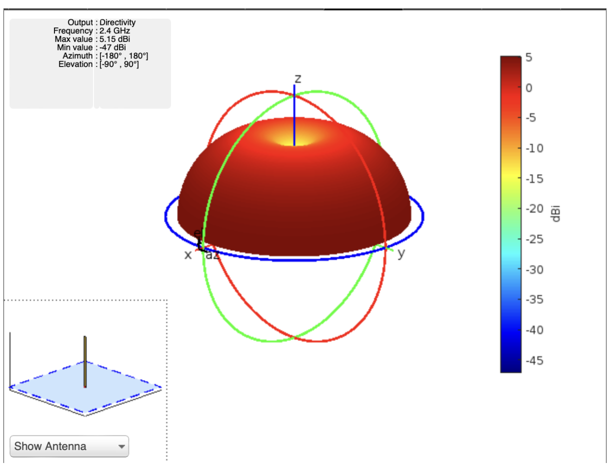

Fig. 5: this is screenshot of 3D radiation pattern in the azimuth for quarter wavelength dipole

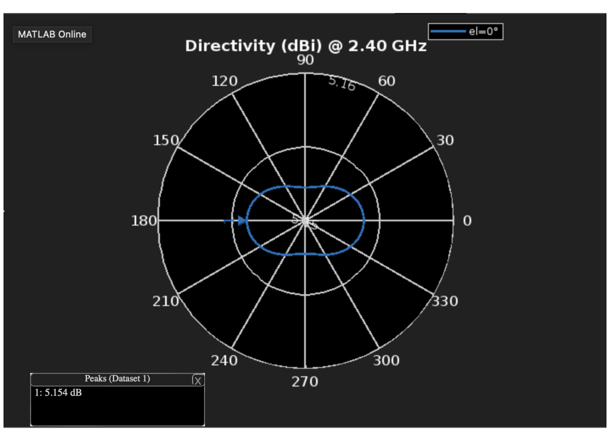

Fig. 6: this is screenshot of radiation pattern quarter wavelength dipole in the azimuth

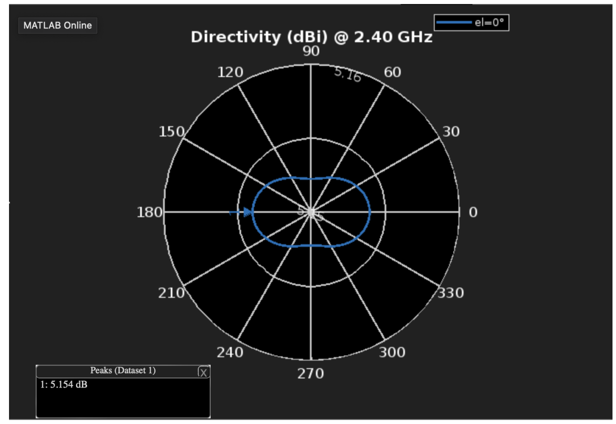

Fig. 7: this is screenshot of radiation pattern quarter wavelength dipole in the elevation

Listing 2: A Sample

```
%% Antenna Properties
antennaObject = design(monopole, 2400*1e6);
antennaObject.Height = 0.0307;
antennaObject.GroundPlaneLength = inf;
antennaObject.GroundPlaneWidth = inf;
%% Antenna Analysis
% Define plot frequency
plotFrequency = 2400*1e6;
% Define frequency range
freqRange = (2160:24:2640)*1e6;
% pattern
figure;
pattern(antennaObject, plotFrequency)
% azimuth
figure;
patternAzimuth(antennaObject, plotFrequency,
    0, 'Azimuth', 0:5:360)
% elevation
figure;
patternElevation(antennaObject, plotFrequency
    ,0,'Elevation',0:5:360)
```

E. Calculate directivity and HPBW for each antenna

First, let's evaluate the directivity of the half-wave dipole at the frequency where the dipole is exactly half a wavelength long. We begin by calculating the radiation intensity produced by the dipole: The radiated power produced by the dipole is

$$\begin{aligned} W_{rad} &= \int_0^{2\pi}\int_0^{\pi} U(\theta)\sin\theta\, d\theta d\phi \\ &= \frac{1}{2}(2\pi)\frac{\eta I_m^2}{(2\pi)^2}\underbrace{\int_0^{\pi}\frac{\cos^2(\frac{\pi}{2}\cos\theta)}{\sin\theta}d\theta}_{\text{1.2188 numerically}} \\ &= 30(1.2188)I_m^2 = 36.5640 I_m^2. \end{aligned}$$

The directivity relative to an isotropic radiator is then calculated as

$$D_m = \frac{4\pi U_m}{W_{rad}} = \frac{4\pi}{8\pi^2}\eta I_m^2 \cdot \frac{1}{36.5640 I_m^2} = 1.64$$

Therefore,

$$D_{dipole} = 1.64 = 2.15\text{dBi} = 0\text{dBd}.$$

Because,

$$D_0 = 1.64 = \frac{2F(\theta)}{Q}$$

$$F(\theta) = \left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right)^2$$

So, when

$$F(\theta) = \frac{1}{2}$$

$$\left(\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\right)^2 = \frac{1}{2}$$

We get $\theta = c$ or $\theta = 129^\circ$
So $HPBW = (129^\circ - 51^\circ) = 78^\circ$
For quarter wavelength dipole antenna, it is obviously that

$$D_0 = 2 \cdot D_{\lambda/2dipole} = 2 \cdot 1.64 = 3.28 = 5.15\text{dBi}$$

$$HPBW = 1/2 \cdot (129^\circ - 51^\circ) = 39^\circ$$

II. Homework2

A. Please discuss if the following ChatGPT answer has any problem

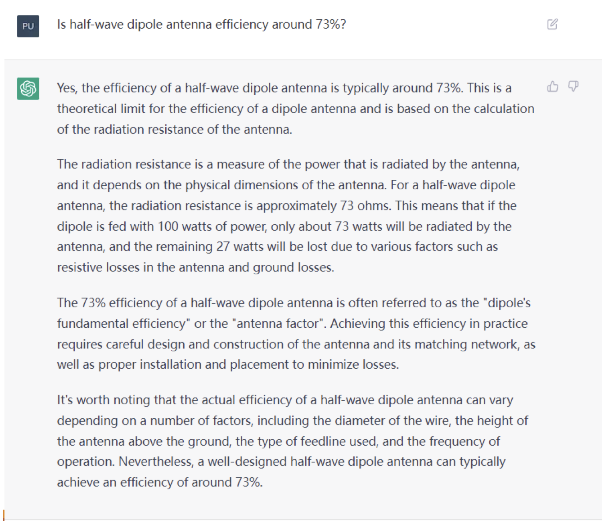

Fig. 8: the Answer of ChatGPT

B. Answer

A half-wave antenna has a radiation resistance often

$$R_{rad} = \frac{1.219}{2\pi} Z_0 \approx 73\Omega$$

However, the radiation efficiency is depended not only radiation resistance but also the resistance itself.
The ohmic loss in a half-wave dipole is$R_{ohmic} = \frac{R_s}{2\pi a}\frac{\lambda}{4}$.
For example, suppose there is a steel rod of length $L = 1.5$ meters, radius $a = 1$ mm is used as an antenna for radiation at $f = 100$ MHz (FM radio). This frequency corresponds to $\lambda = 3$ meters, so this is a half-wave dipole. The resistance of the metal wire is given by: $R_{Ohmic} = \sqrt{\frac{\pi f \mu}{\sigma}}\frac{L}{2\pi\alpha} = 3.4\Omega$ The radiation efficiency is therefore: $\eta = \frac{R_{rad}}{R_{rad}+R_{Ohmic}} = 0.955$ (often expressed in decibels: $\eta = -0.2$ dB)