天线与电波传播
ANTENNAS AND WAVE PROPAGATION

LECTURE 10

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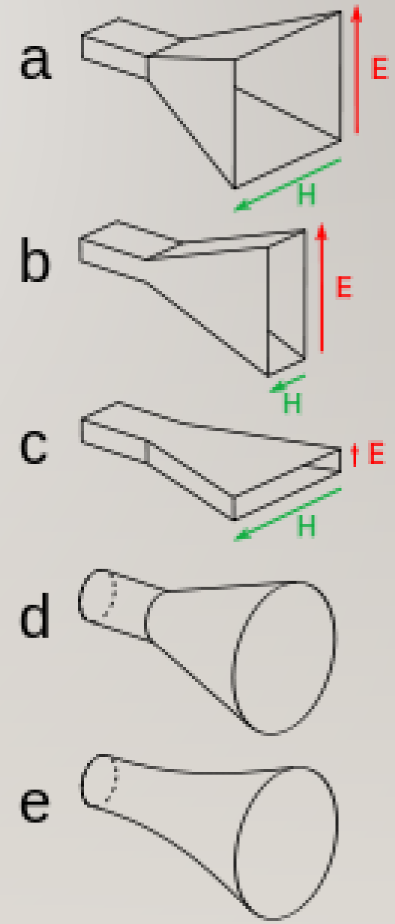
Feed-horns/Horn Antenna

- Horns can be classified:
  - Rectangular horns:
    - Sectoral horns (E-plane and H plane)
    - Pyramidal horns
  - Conical horns:
    - Smooth-walled conical feed-horns
    - Corrugated conical feed-horns
  - Profiled feed-horns
    - Multimode smooth-walled feed-horns
    - Corrugated profiled feed-horns

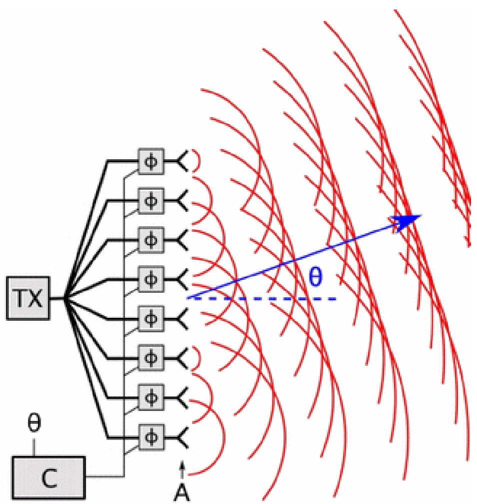


This Week

- Linear arrays: the two-element array
- N-element array with uniform amplitude and spacing
- Broad-side array
- End-fire array
- Phased array
- Planar array
- Circular array

Antenna arrays

- composed of multiple similar radiating elements (e.g., dipoles or horns).
- radiative properties determined by
  - element **spacing**
  - **the relative amplitudes** and **phases** of the element excitation

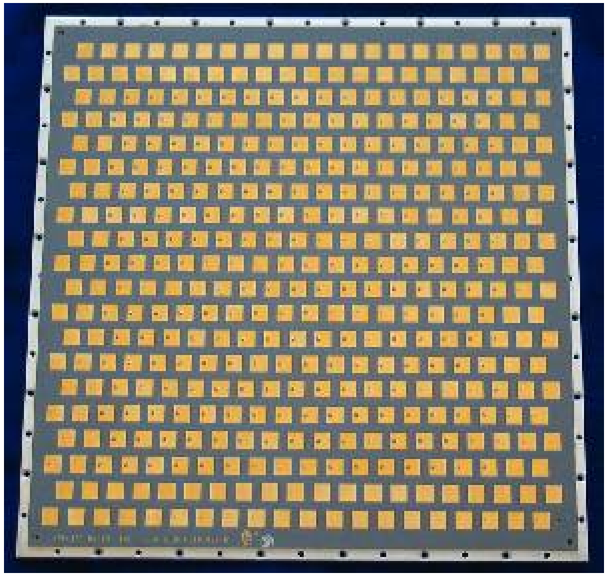


Antenna arrays

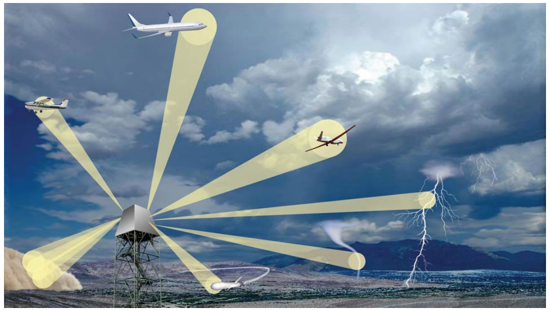
Large planar array antenna of a VHF Russian mobile air defense radar, the Nebo-M. It consists of 175 Yagi folded dipole antennas driven in phase. It radiates a narrow beam of radio waves perpendicular to the antenna.

Antenna arrays

Linear array examples



Two-dimensional array of microstrip patch antennas



Phased Array Radar Video 4’05”

Introduction

- Single-element antennas
  - radiation patterns relatively wide
  - relatively low directivity (gain)
- To increase **directivity** requiring **enlarging** the **radiating aperture**
  - maximum size much larger than λ
  - may lead to the appearance of multiple side lobes
  - usually large and difficult to fabricate.

Introduction

- Another way to **increase** the **electrical size** of an antenna
  - construct an assembly of radiating elements in a proper electrical and geometrical configuration–**antenna array**
- Usually, the array elements are identical
  - not necessary but it is practical and simpler for design and fabrication
  - individual elements may be of any type (wire dipoles, loops, apertures, etc.)

Introduction

- Is this an antenna array?

- NOT AN ARRAY!

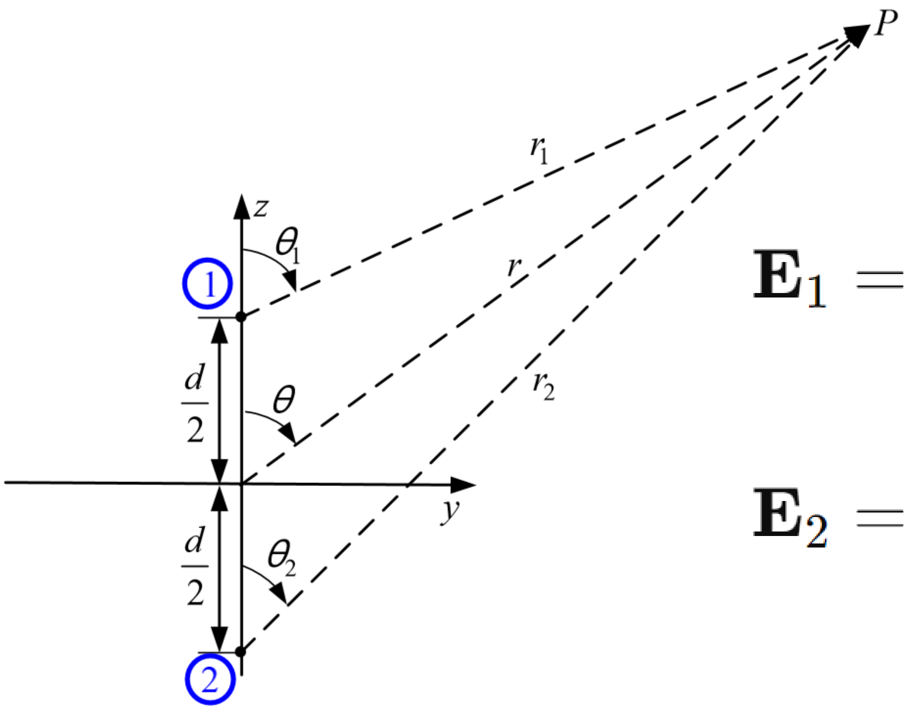
Introduction

Five basic methods to control the overall antenna pattern:

1. the **geometrical configuration** of the **overall array** (linear, circular, spherical, rectangular, etc.),
2. the **relative placement** of the **elements**,
3. the **excitation amplitude** of the **individual elements**,
4. the **excitation phase** of **each element**,
5. the **individual pattern** of **each element**.

*assuming no coupling between the eleme

Two-element Array

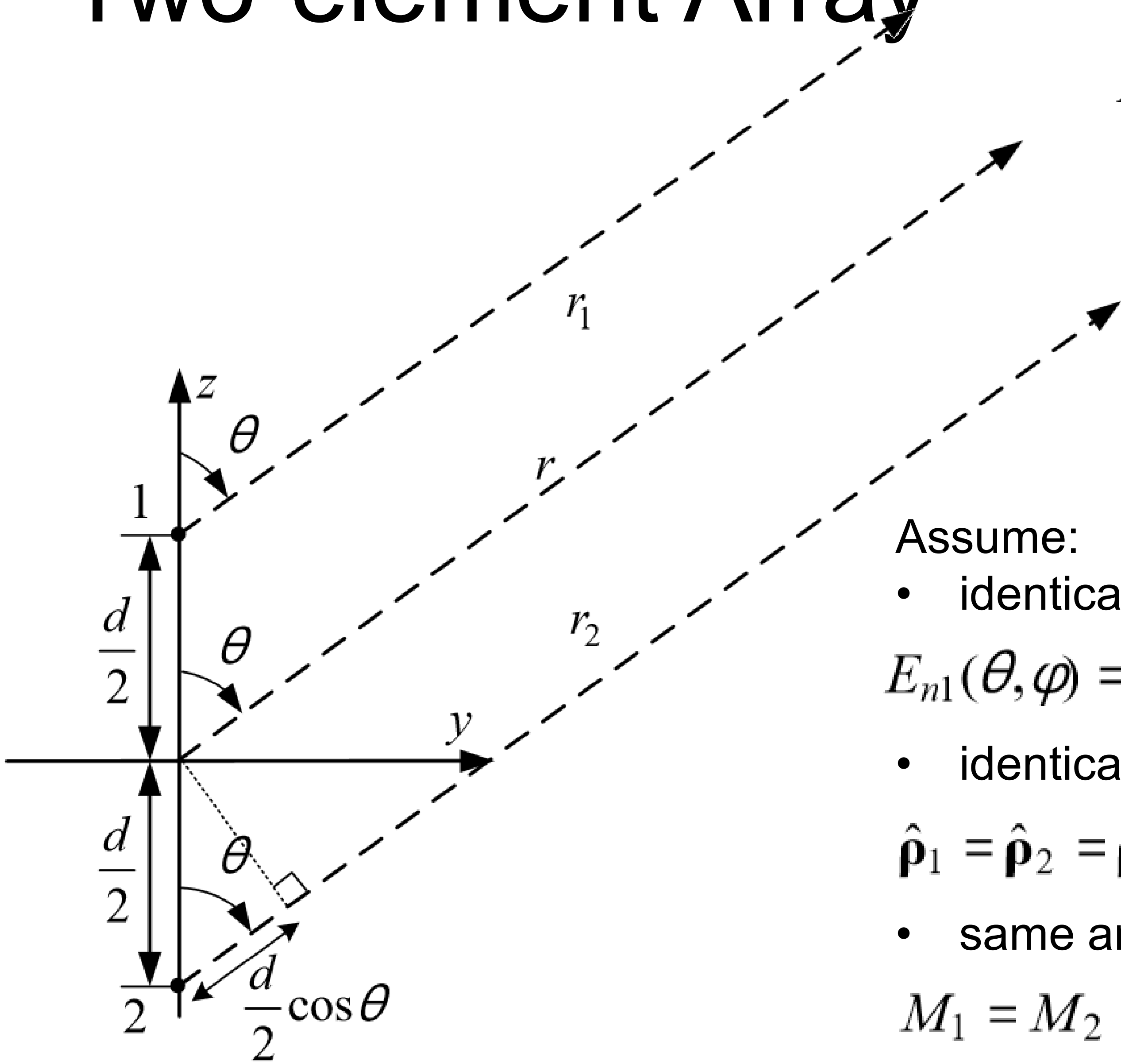


$$\mathbf{E}_1 = M_1 E_{n1}(\theta_1, \varphi_1) \frac{e^{-j\left(k_{r1} - \frac{\beta}{2}\right)}}{r_1} \hat{\boldsymbol{\rho}}_1$$

$$\mathbf{E}_2 = M_2 E_{n2}(\theta_2, \varphi_2) \frac{e^{-j\left(k_{r2} + \frac{\beta}{2}\right)}}{r_2} \hat{\boldsymbol{\rho}}_2$$

M_1, M_1	field magnitudes (do not include the $1/r$ factor);
E_{n1}, E_{n2}	normalized field patterns;
r_1, r_2	distances to the observation point P;
β	phase difference between the feed of the two array elements;
$\hat{\boldsymbol{\rho}}_1, \hat{\boldsymbol{\rho}}_2$	polarization vectors of the far-zone $\mathbf{E}$ fields.

Two-element Array



Assume:

- identical array elements

$$E_{n1}(\theta,\varphi) = E_{n2}(\theta,\varphi) = E_n(\theta,\varphi)$$

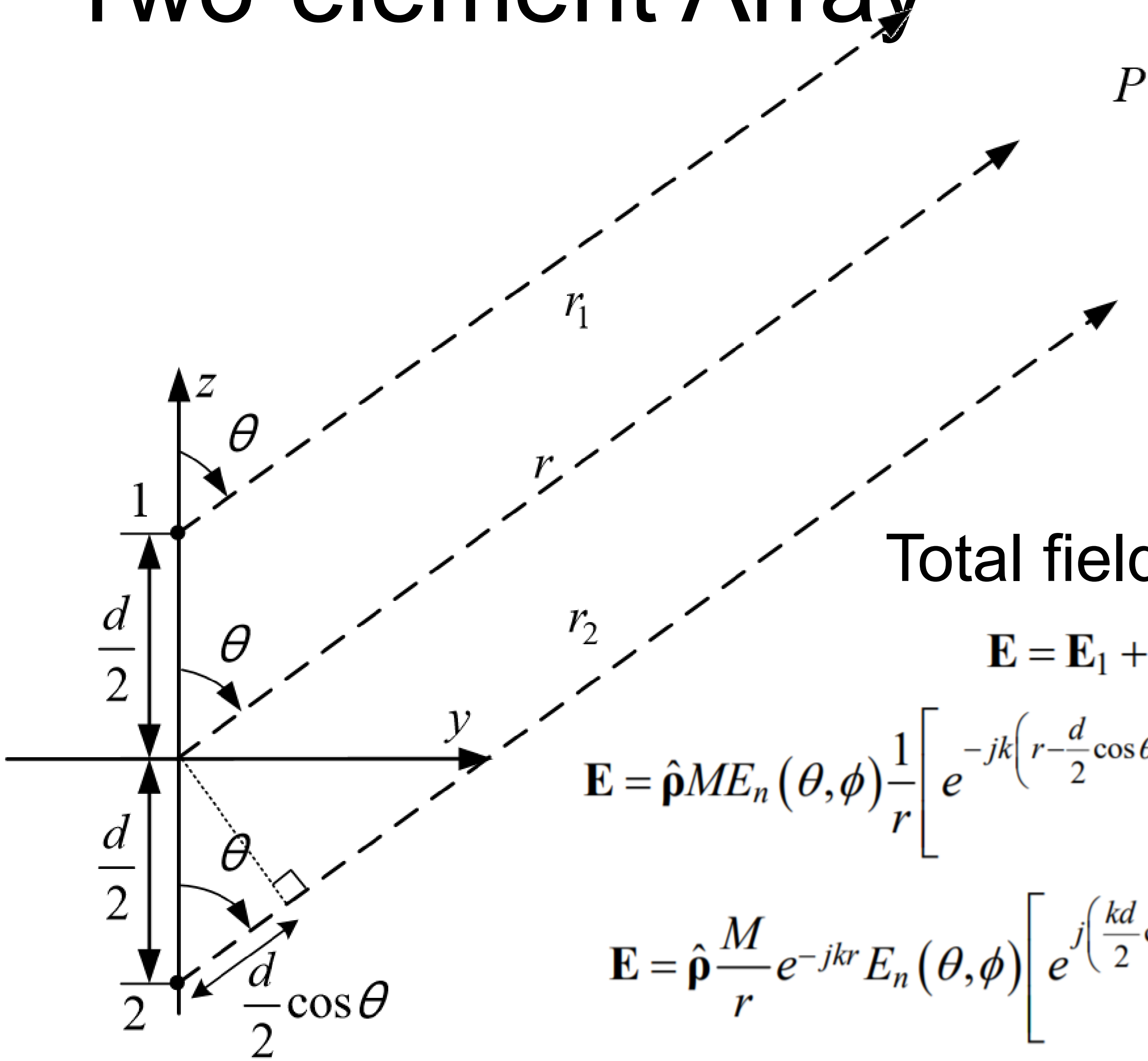
- identical polarizations

$$\hat{\boldsymbol{\rho}}_1 = \hat{\boldsymbol{\rho}}_2 = \hat{\boldsymbol{\rho}}$$

- same amplitude of excitation

$$M_1 = M_2 = M$$

Two-element Array



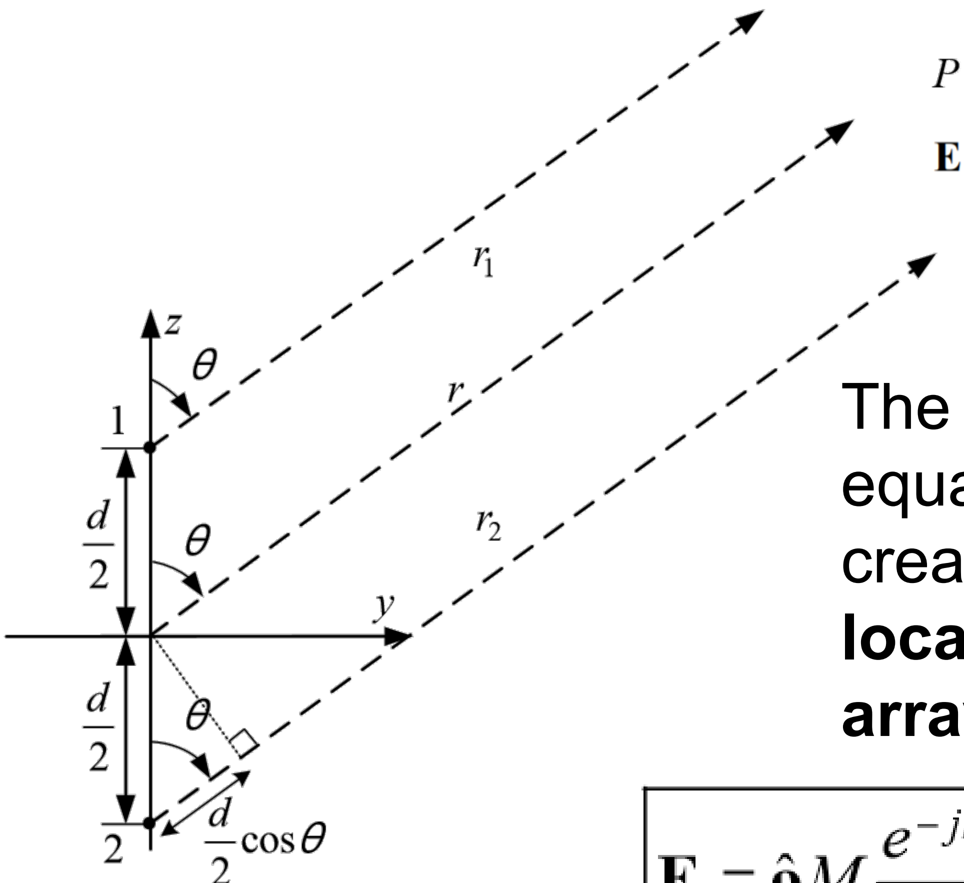
Total field:

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2,$$

$$\mathbf{E} = \hat{\boldsymbol{\rho}} M E_n(\theta,\phi)\frac{1}{r}\left[e^{-jk\left(r-\frac{d}{2}\cos\theta\right)+j\frac{\beta}{2}} + e^{-jk\left(r+\frac{d}{2}\cos\theta\right)-j\frac{\beta}{2}}\right],$$

$$\mathbf{E} = \hat{\boldsymbol{\rho}}\frac{M}{r}e^{-jkr}E_n(\theta,\phi)\left[e^{j\left(\frac{kd}{2}\cos\theta+\frac{\beta}{2}\right)} + e^{-j\left(\frac{kd}{2}\cos\theta+\frac{\beta}{2}\right)}\right],$$

Two-element Array

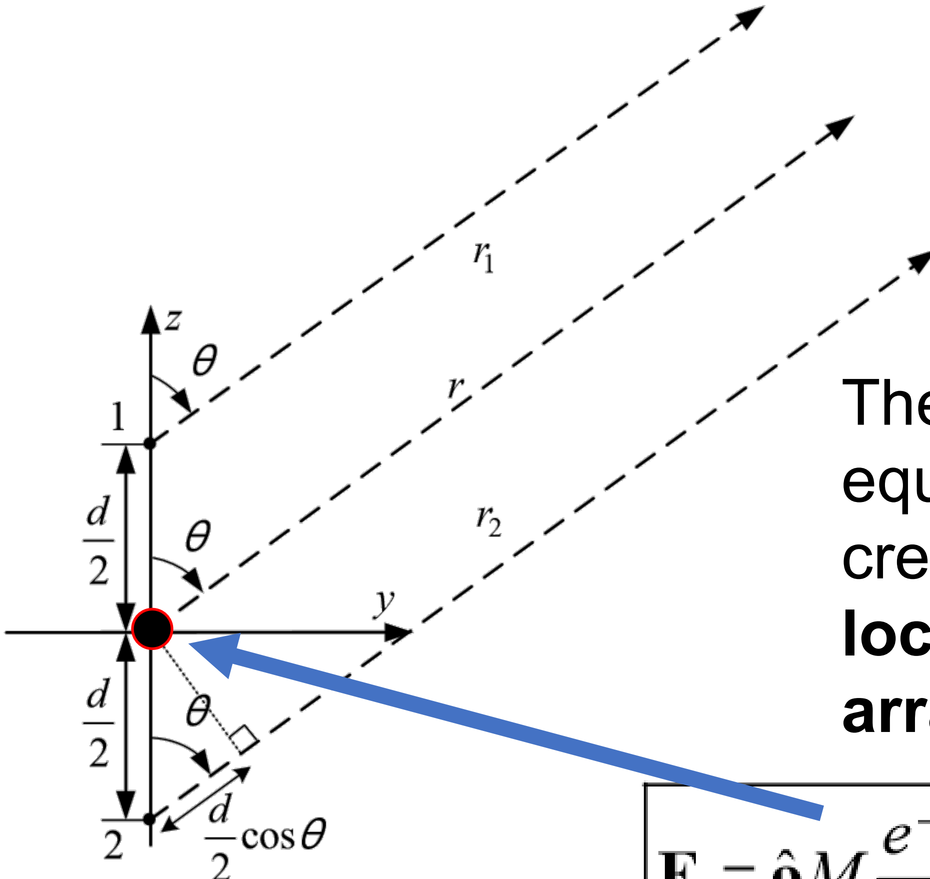


$$\mathbf{E} = \hat{\boldsymbol{\rho}} \frac{M}{r} e^{-jkr} E_n(\theta,\phi) \left[e^{j\left(\frac{kd}{2}\cos\theta + \frac{\beta}{2}\right)} + e^{-j\left(\frac{kd}{2}\cos\theta + \frac{\beta}{2}\right)} \right],$$

The total field of the array is equal to the product of the field created by a **single element located at the origin** and the **array factor (AF)**:

$$\boxed{\mathbf{E} = \underbrace{\hat{\boldsymbol{\rho}} M \frac{e^{-jkr}}{r} E_n(\theta,\varphi)}_{EF} \times \underbrace{2\cos\left(\frac{kd\cos\theta + \beta}{2}\right)}_{AF}}$$

Two-element Array



$$\mathbf{E} = \hat{\boldsymbol{\rho}} \frac{M}{r} e^{-jkr} E_n(\theta,\phi) \left[e^{j\left(\frac{kd}{2}\cos\theta + \frac{\beta}{2}\right)} + e^{-j\left(\frac{kd}{2}\cos\theta + \frac{\beta}{2}\right)} \right],$$

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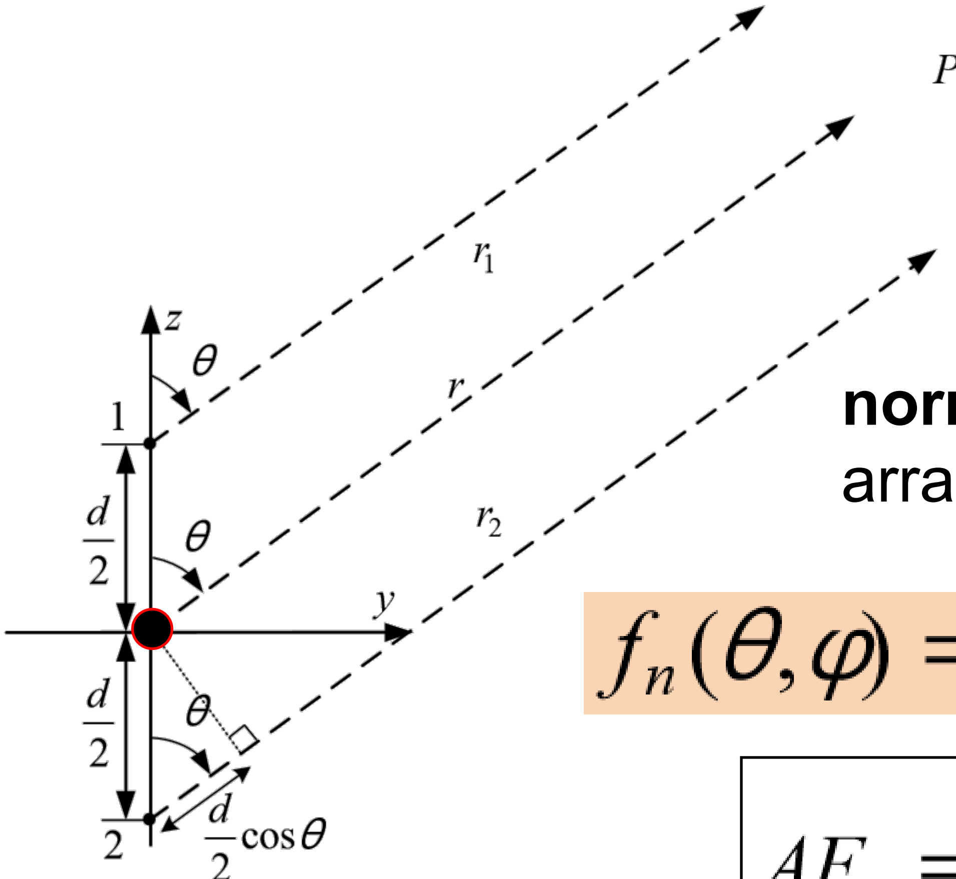
Two-element Array



normalized field pattern of the array is expressed as

$$f_n(\theta,\varphi) = E_n(\theta,\varphi) \times AF_n(\theta,\varphi)$$

$$\boxed{AF_n = \cos\left(\frac{kd\cos\theta + \beta}{2}\right)}$$

Note: in this case, AF does not dependent on φ

Two-element Array

normalized field pattern of the array is expressed as

$$f_n(\theta,\varphi) = E_n(\theta,\varphi) \times AF_n(\theta,\varphi)$$

The principle of **pattern multiplication** states that “the radiation **pattern** of an array is the **product** of the **pattern** of **the individual antenna** with the **array pattern** of **isotropic point sources** each **located at the phase centre of the individual source**.”

The total pattern, therefore, can be controlled via the single–element pattern $E_n(\theta,\varphi)$ or via the AF.

Two-element Array

normalized field pattern of the array is expressed as

$$f_n(\theta,\varphi) = E_n(\theta,\varphi) \times AF_n(\theta,\varphi)$$

The AF, in general, depends on the:
- number of elements,
- mutual placement,
- relative excitation magnitudes and phases.

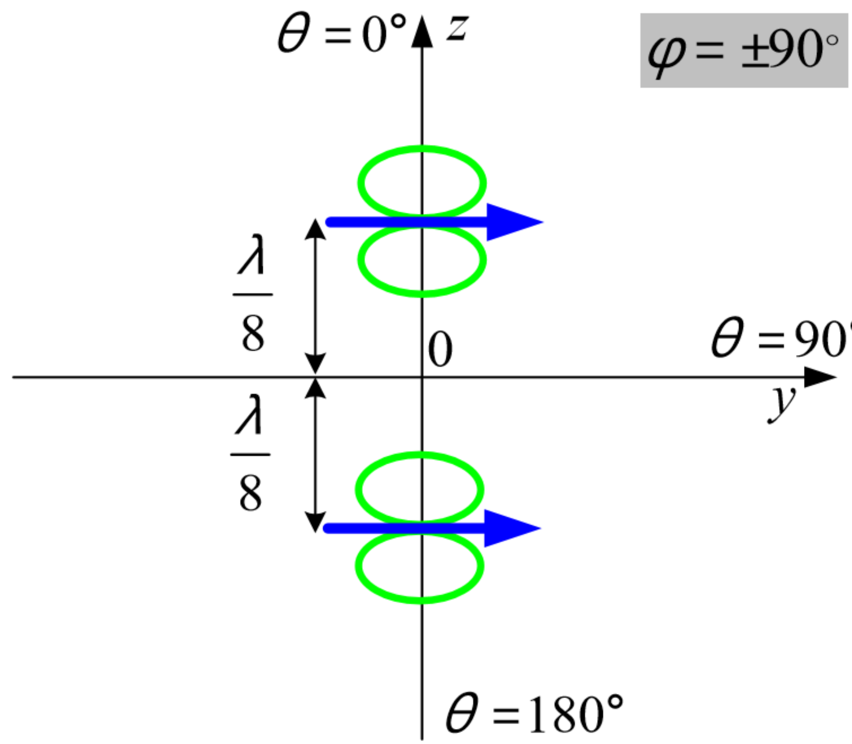
Two-element Array Example

Example 1: An array consists of two horizontal infinitesimal dipoles located at a distance $d = \frac{\lambda}{4}$ from each other. Find the nulls of the total field in the elevation plane $\varphi = \pm 90^\circ$, if the **excitation magnitudes are the same** and **the phase difference** is:

a) $\beta = 0$

b) $\beta = \frac{\pi}{2}$

c) $\beta = -\frac{\pi}{2}$



$$AF_n = \cos\left(\frac{kd\cos\theta + \beta}{2}\right)$$

Two-element Array Example

The element factor $E_n(\theta,\varphi) = \sqrt{1 - \sin^2\theta \sin^2\phi}$ does not depend on β, and it produces in all three cases the same null. For $\varphi = \pm 90^\circ$, $E_n(\theta,\varphi) = |\cos\theta|$ and the null is at

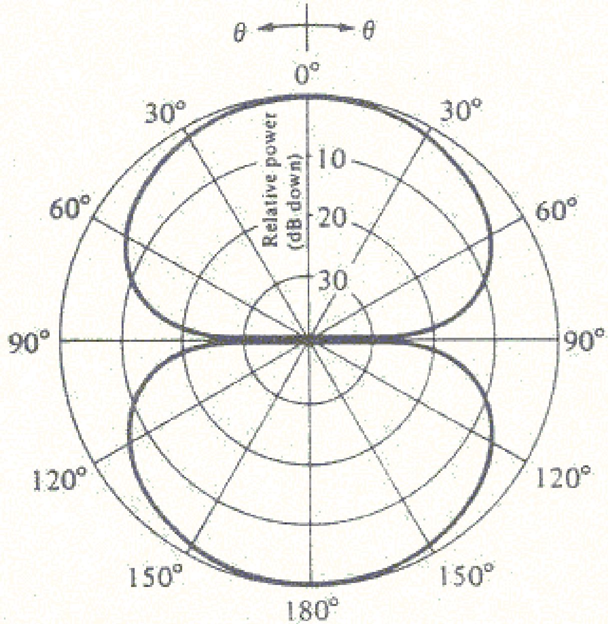
$$\theta_1 = \pi / 2.$$

a) $\beta = 0$, the null is

$$AF_n = \cos\left(\frac{kd\cos\theta_n}{2}\right) = 0 \quad \Rightarrow \cos\left(\frac{\pi}{4}\cos\theta_n\right) = 0,$$

$$\Rightarrow \frac{\pi}{4}\cos\theta_n = (2n+1)\frac{\pi}{2} \quad \Rightarrow \cos\theta_n = (2n+1)\cdot 2, \;\; n = 0, \pm 1, \pm 2, \ldots.$$

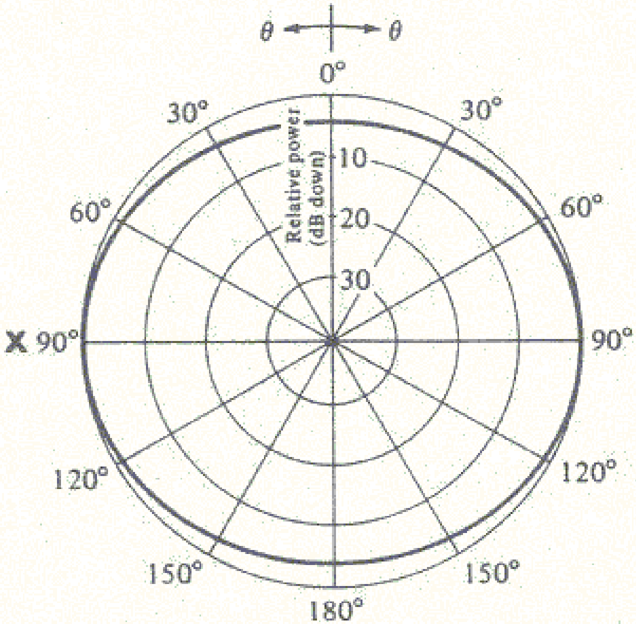
A solution with a real-valued angle does not exist. In this case, the total field pattern has only 1 null at $\theta = 90°$.



Element

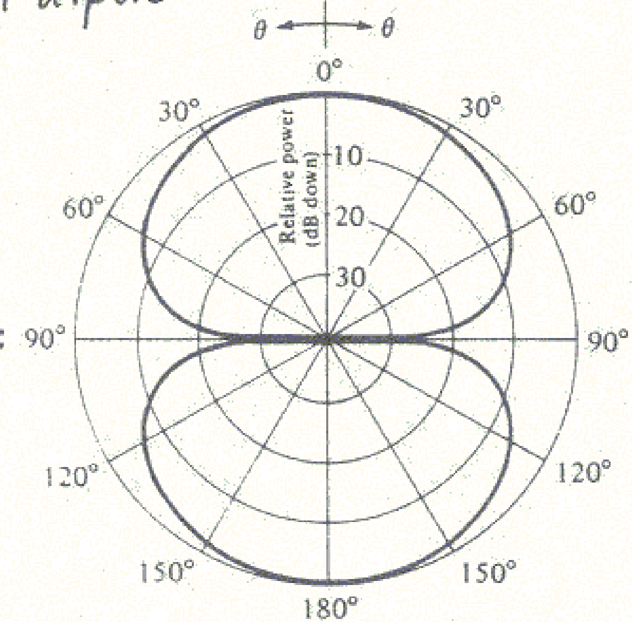
horizontal dipole

×



Array factor

=



Total

$N = 2$

$\beta = 0°$

$d = \frac{\lambda}{4}$

$$AF_n = \cos\left(\frac{kd\cos\theta + \beta}{2}\right)$$

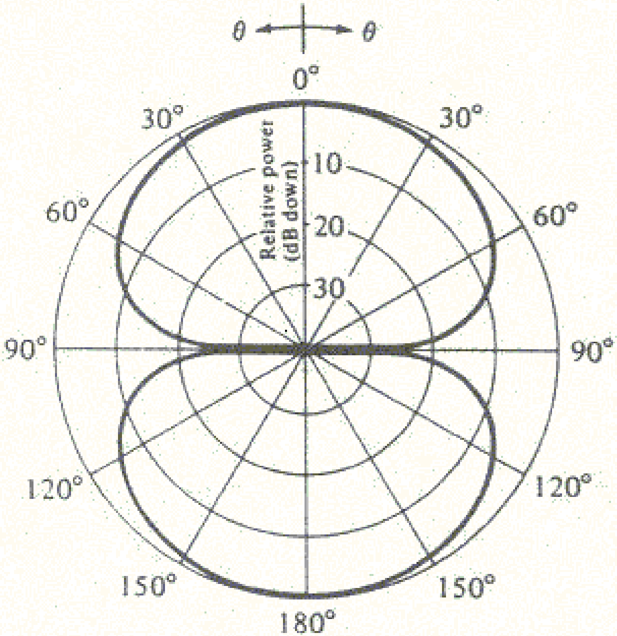
Two-element Array Example

b) $\beta = \pi / 2$, the null is

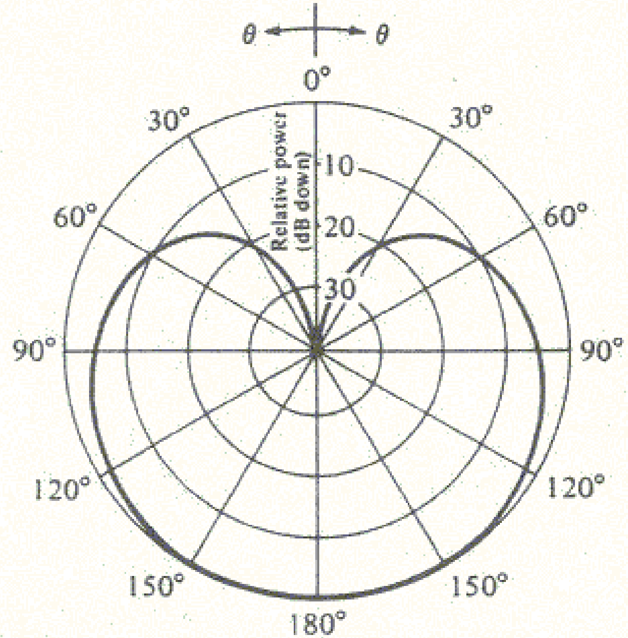
$$AF_n = \cos\left(\frac{\pi}{4}\cos\theta_n + \frac{\pi}{4}\right) = 0 \quad \Rightarrow \frac{\pi}{4}\left(\cos\theta_n + 1\right) = (2n+1)\frac{\pi}{2},$$

$$\Rightarrow \cos\theta_n + 1 = (2n+1)\cdot 2 \quad \Rightarrow \cos\theta_{(n=0)} = 1 \quad \Rightarrow \boxed{\theta_2 = 0}.$$

The solution for $n = 0$ is the only real-valued solution. Thus, the total field pattern has 2 nulls: at $\theta_1 = 90^\circ$ and at $\theta_2 = 0^\circ$:

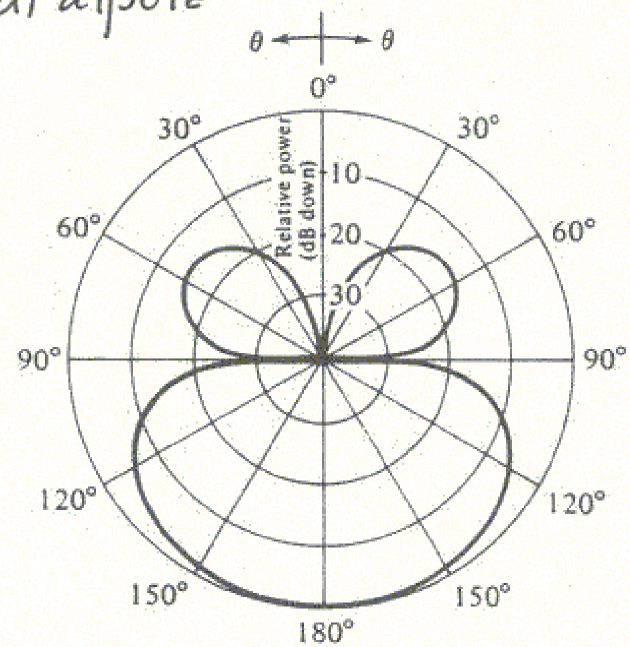


Element



Array factor

horizontal dipole



Total

$N = 2$

$\beta = +90^\circ$

$d = \lambda/4$

$$AF_n = \cos\left(\frac{kd\cos\theta + \beta}{2}\right)$$

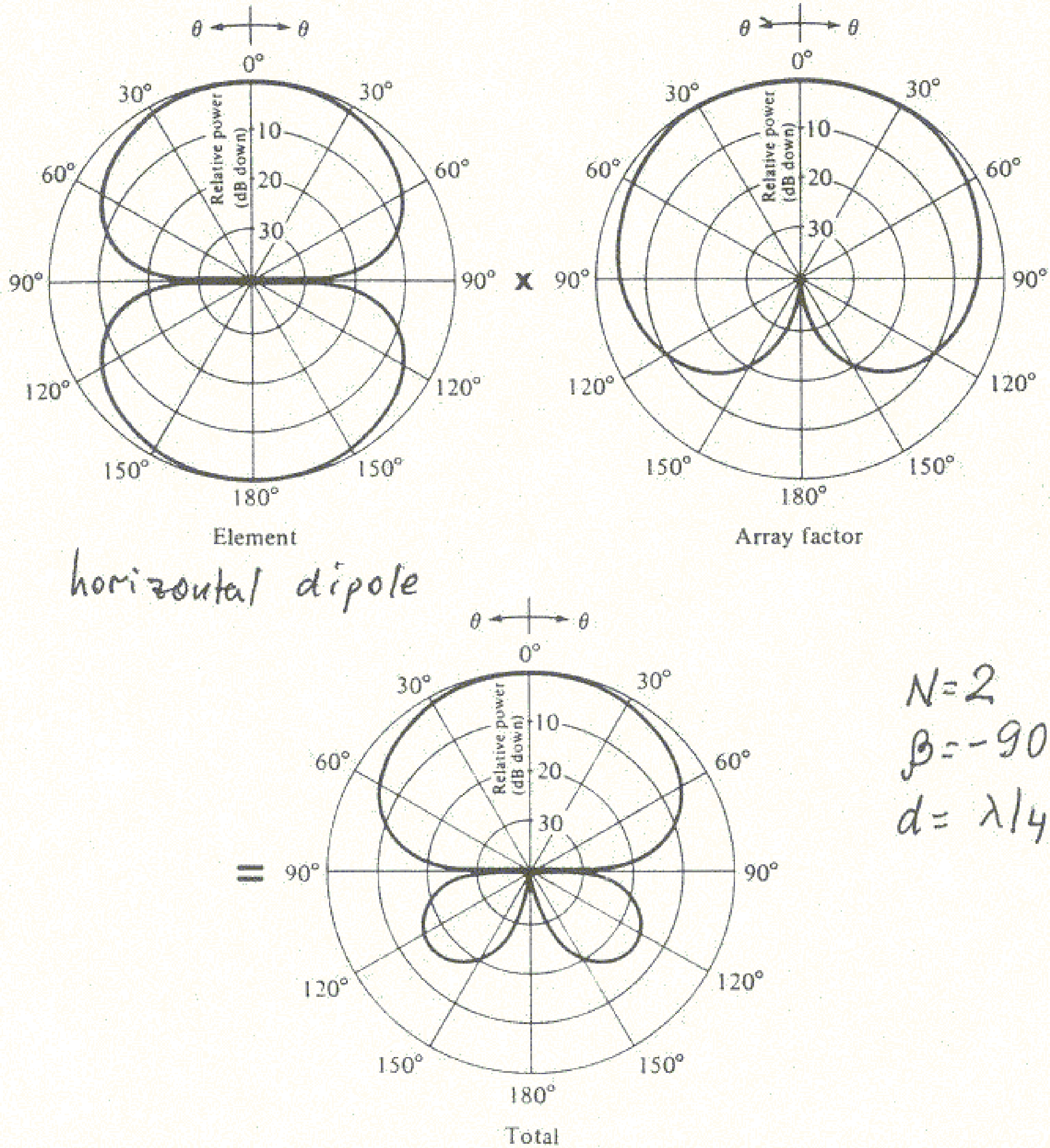
Two-element Array Example

c) $\beta = -\pi / 2$, the null is

$$AF_n = \cos\left(\frac{\pi}{4}\cos\theta_n - \frac{\pi}{4}\right) = 0 \quad \Rightarrow \frac{\pi}{4}(\cos\theta_n - 1) = (2n+1)\frac{\pi}{2},$$

$$\Rightarrow \cos\theta_n - 1 = (2n+1)\cdot 2 \quad \Rightarrow \cos\theta_{(n=-1)} = -1 \quad \Rightarrow \boxed{\theta_2 = \pi}.$$

The total field pattern has 2 nulls: at $\theta_1 = 90^\circ$ and at $\theta_2 = 180^\circ$



horizontal dipole

$N = 2$
$\beta = -90°$
$d = \lambda/4$

Example 2: Consider a 2-element array of identical (infinitesimal) dipoles oriented along the y-axis. Find the angles of observation where the nulls of the pattern occur in the plane $\phi = \pm 90^\circ$ as a function of the distance d between the dipoles and the phase difference β.

normalized total field pattern

$$f_n = \left|\cos\theta\right| \times \cos\left(\frac{kd\cos\theta + \beta}{2}\right)$$

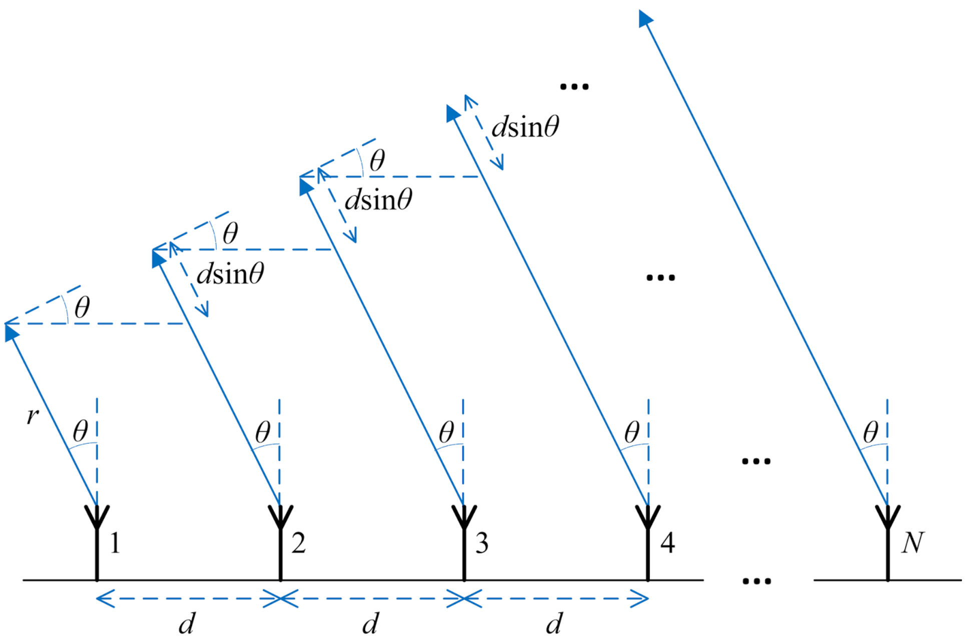
$$\Big\Downarrow \quad \text{Nulls: } f_n = 0$$

$$|\cos\theta| = 0 \longrightarrow \theta_1 = \pi/2$$

$$\cos\left(\frac{kd\cos\theta + \beta}{2}\right) = 0 \longrightarrow \theta_n = \arccos\left\{\frac{\lambda}{2\pi d}\left[-\beta \pm (2n+1)\pi\right]\right\}$$

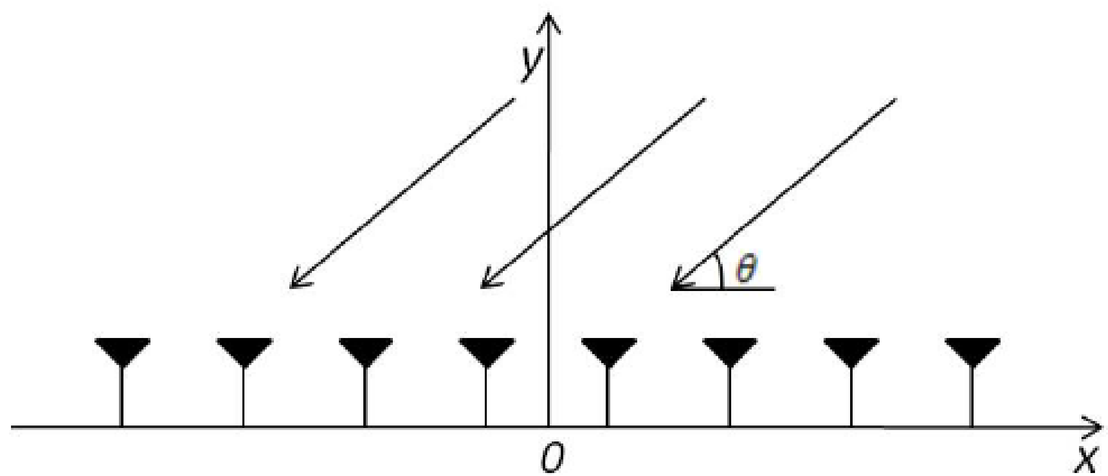
N-element Linear Array with Uniform Amplitude and Spacing

- assume that each succeeding element has a β **progressive phase** lead in the excitation relative to the preceding one.
- An array of ***identical elements*** with ***identical magnitudes*** and with a ***progressive phase*** is called a ***uniform array***.

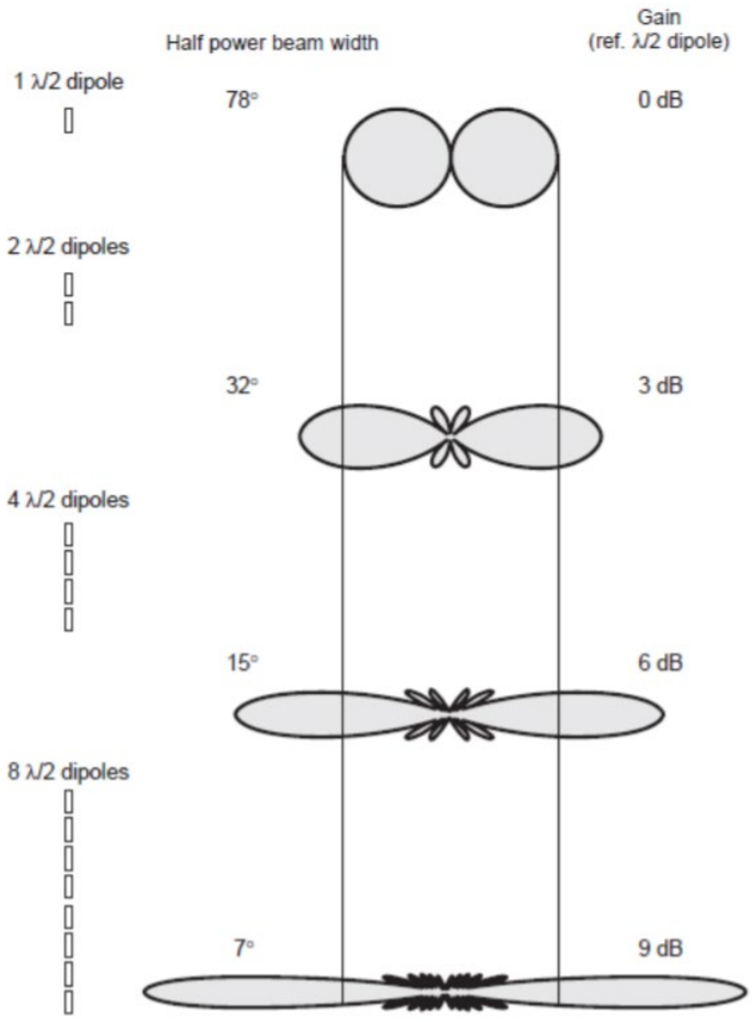


N-element Linear Array with Uniform Amplitude and Spacing

- The *AF* of the uniform array can be obtained by considering **the individual elements** as **point (isotropic) sources**.
- The total field pattern can be obtained by simply **multiplying the *AF* by the normalized field pattern of the individual element** (provided the elements are not coupled).



N-element Linear Array with Uniform Amplitude and Spacing



Broad side array β=0

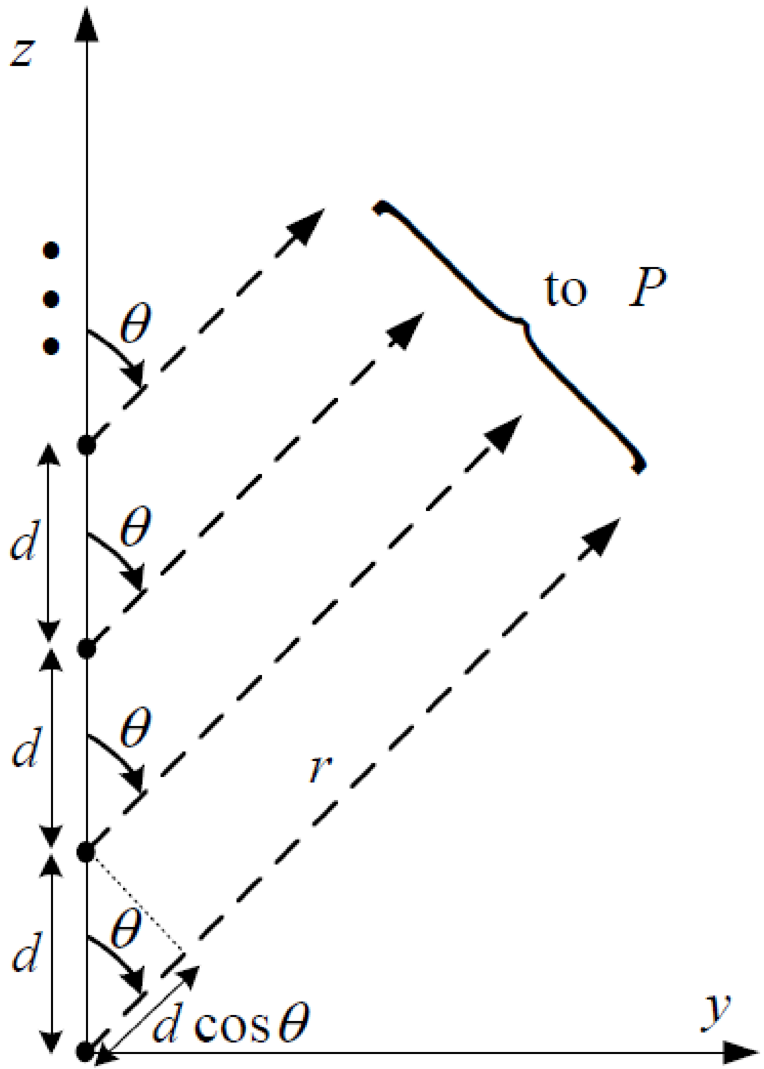
If λ/2 dipole is reference
i.e. its Gain considered to be=0dB
("note that λ/2 dipole has D=2dB")
then

2 element array increase gain by 3dB(double gain 2 time)

4 element array increase gain by 6dB(double gain 4 time)

8 element array could increase gain by 9dB(double gain 8time)

N-element Linear Array with Uniform Amplitude and Spacing



The **total field** of an **array** is a vector superposition of the **fields** radiated by the individual elements.

$$AF = 1 + e^{j(kd\cos\theta+\beta)} + e^{j2(kd\cos\theta+\beta)} + \ldots + e^{j(N-1)(kd\cos\theta+\beta)}$$

N-element Linear Array with Uniform Amplitude and Spacing

$$AF = 1 + e^{j(kd\cos\theta+\beta)} + e^{j2(kd\cos\theta+\beta)} + \ldots + e^{j(N-1)(kd\cos\theta+\beta)}$$

$$AF = \sum_{n=1}^{N} e^{j(n-1)(kd\cos\theta+\beta)},$$

$$\boxed{AF = \sum_{n=1}^{N} e^{j(n-1)\psi} \quad (6\text{-}7)}$$

where $\psi = kd\cos\theta + \beta$

AFs of uniform linear arrays can be controlled by the relative phase β between the elements.

N-element Linear Array with Uniform Amplitude and Spacing

$$\boxed{AF = \sum_{n=1}^{N} e^{j(n-1)\psi} \quad \text{(6-7)}}$$

where $\psi = kd\cos\theta + \beta$

(6-7)x$e^{j\psi}$ on both sides

$$\boxed{AF \cdot e^{j\psi} = \sum_{n=1}^{N} e^{jn\psi} \quad \text{(6-8)}}$$

(6-8) – (6-7)

$$\boxed{AF \cdot e^{j\psi} - AF = e^{jN\psi} - 1}$$

closed form

$$\boxed{AF = \frac{e^{jN\psi} - 1}{e^{j\psi} - 1}}$$

N-element Linear Array with Uniform Amplitude and Spacing

AF in a **closed form** can be further derived applying trigonometric identities.

$$AF = \frac{e^{jN\psi} - 1}{e^{j\psi} - 1} = \frac{e^{j\frac{N}{2}\psi}\left(e^{j\frac{N}{2}\psi} - e^{-j\frac{N}{2}\psi}\right)}{e^{j\frac{\psi}{2}}\left(e^{j\frac{\psi}{2}} - e^{-j\frac{\psi}{2}}\right)},$$

$$\boxed{AF = e^{j\left(\frac{N-1}{2}\right)\psi} \cdot \frac{\sin(N\psi/2)}{\sin(\psi/2)}.}$$

N-element Linear Array with Uniform Amplitude and Spacing

the **phase factor** represents shift of the array's phase center relative to the origin, if the reference point is the physical center of the array, this term can be neglected

$$AF = \boxed{e^{j\left(\frac{N-1}{2}\right)\psi}} \cdot \frac{\sin\left(N\psi/2\right)}{\sin\left(\psi/2\right)}.$$

$$\boxed{AF = \frac{\sin\left(N\psi/2\right)}{\sin\left(\psi/2\right)}}$$

For small values of ψ

$$AF = \frac{\sin\left(N\psi/2\right)}{\psi/2} \Rightarrow AF_{max}=N$$

N-element Linear Array with Uniform Amplitude and Spacing

Normalized *AF* (It can be shown $AF_{max}=N$)

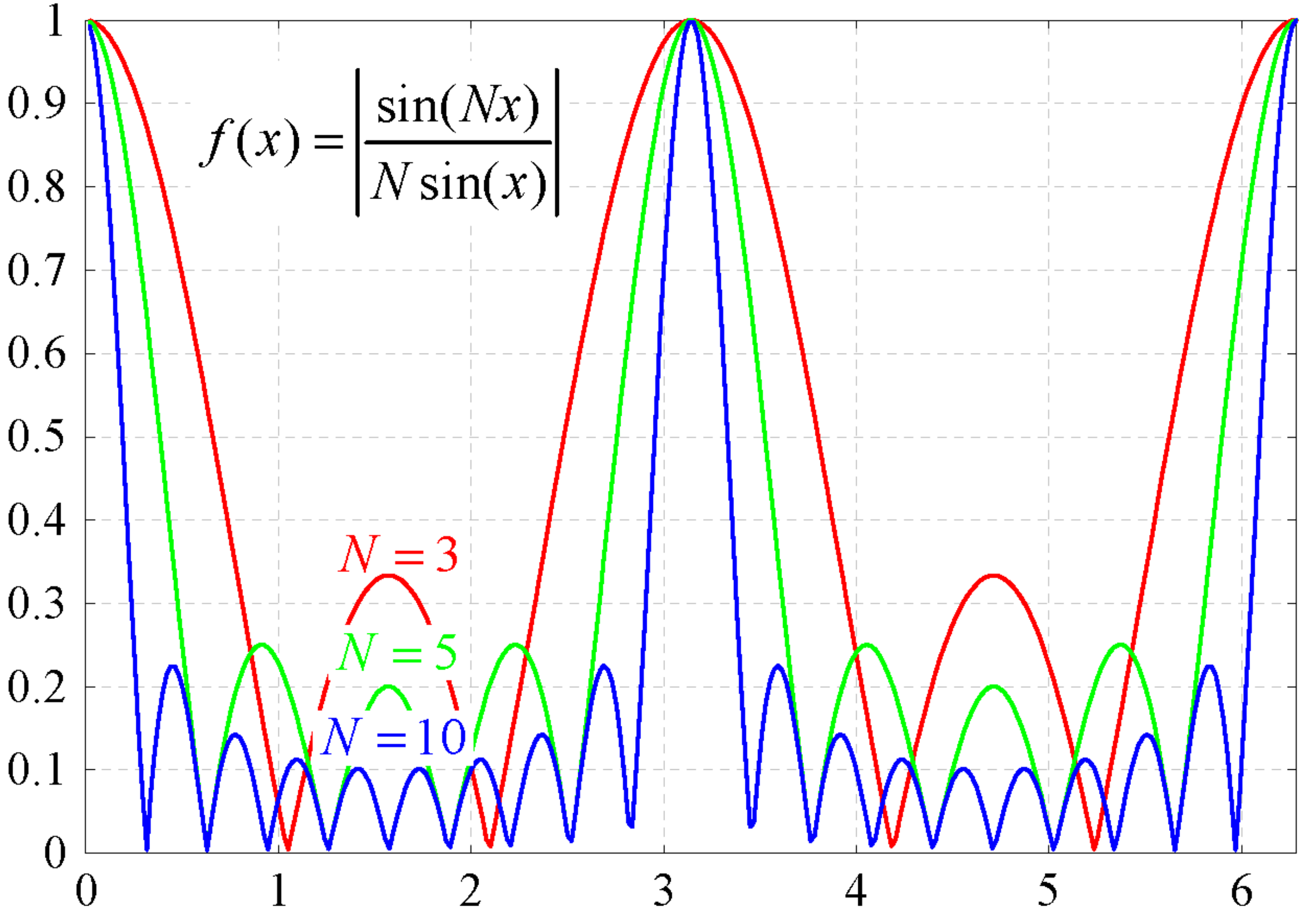
$$\boxed{AF_n = \frac{\sin\left(N\psi/2\right)}{N\sin\left(\psi/2\right)}}$$

Normalized *AF* for small values of ψ

$$\boxed{AF_n = \frac{1}{N}\left[\frac{\sin\left(N\psi/2\right)}{\psi/2}\right]}$$

N-element Linear Array with Uniform Amplitude and Spacing

Normalized AF



$$AF_n = \frac{\sin(N\psi/2)}{N\sin(\psi/2)}$$

N-element Linear Array with Uniform Amplitude and Spacing

Null of the AF

$$\sin\left(\frac{N}{2}\psi\right) = 0 \quad \Rightarrow \frac{N}{2}\psi = \pm n\pi \quad \Rightarrow \frac{N}{2}\left(kd\cos\theta_n + \beta\right) = \pm n\pi$$

$$\theta_n = \arccos\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2n}{N}\pi\right)\right],\ n = 1,2,3\ldots(n \neq \underline{0, N, 2N, 3N}\ldots)$$

When n = 0, N ,2N ,3N …, the *AF* attains its **maximum** values not nulls.

The values of n determine the order of the nulls. For a null to exist, the argument of the arccosine must be between –1 and +1.

N-element Linear Array with Uniform Amplitude and Spacing

Major maxima of the AF

$$\boxed{AF_n = \frac{\sin\left(N\psi/2\right)}{N\sin\left(\psi/2\right)}}$$

$$\frac{\psi}{2} = \frac{1}{2}\left(kd\cos\theta_m + \beta\right) = \pm m\pi,$$

$$\theta_m = \arccos\left[\frac{\lambda}{2\pi d}\left(-\beta \pm 2m\pi\right)\right], \quad m = 0,1,2\dots.$$

Many major maxima exist, m is the maximum's order.

N-element Linear Array with Uniform Amplitude and Spacing

For small d/λ, single maximum is obtained

$$\theta_m = \theta_0 = \arccos\left(-\frac{\beta\lambda}{2\pi d}\right)$$

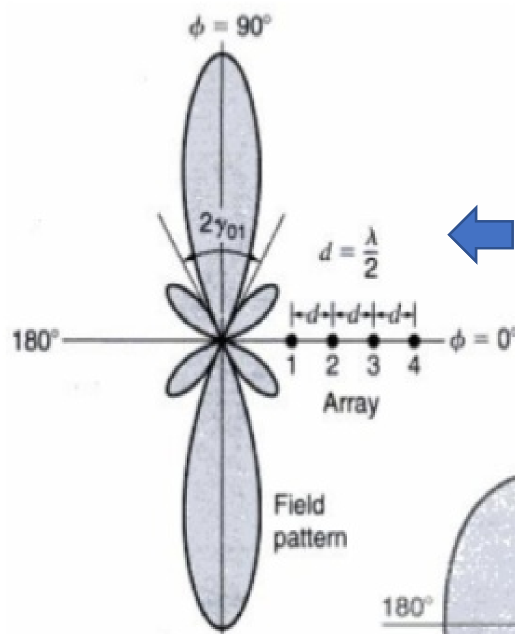
HPBW of a major lobe

$$HPBW = 2\,|\,\theta_m - \theta_h\,|$$

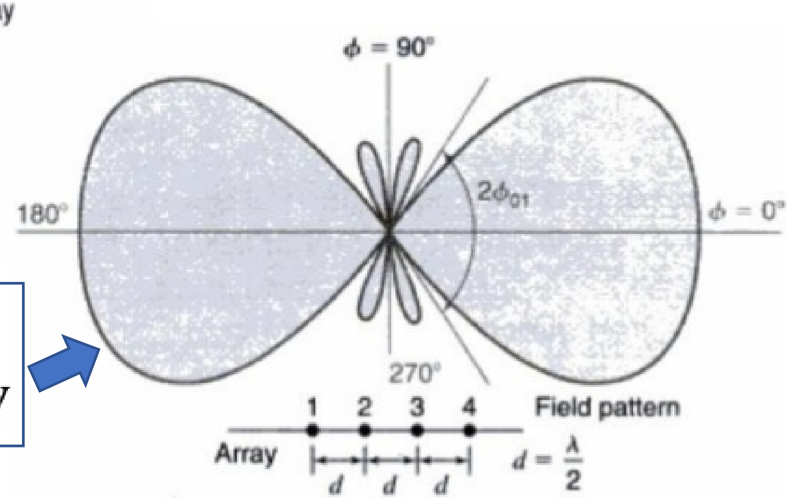
$$\theta_h = \arccos\left[\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right]\Bigg|_{AF_n = 1/\sqrt{2}}$$

N-element Linear Array with Uniform Amplitude and Spacing

Broadside array (maximum radiation at θ = 90° or (normal to the axis of the array)



Broad side type of array



- Endfire

Type of array

N-element Linear Array with Uniform Amplitude and Spacing

Broadside array (maximum radiation at $\theta_m = 90°$ or (normal to the axis of the array)

$$\psi = kd\cos\theta_m + \beta = 0$$

$$\Downarrow$$

$$\boxed{\beta = 0}$$

The uniform linear array has its maximum radiation at $\theta = 90°$, if all array elements have their **excitation with the same phase**.

To ensure that there are **no maxima in other directions**, the separation has to satisfy $d < \lambda$.

N-element Linear Array with Uniform Amplitude and Spacing

End-fire array (maximum radiation at $\theta_m = 0°$ or $\theta_m = 180°$ (normal to the axis of the array)

$$\psi = kd\cos\theta_m + \beta = 0$$

$$\Rightarrow \boxed{\beta = -kd, \quad \text{for} \quad \theta_{\max} = 0°}$$

$$\Rightarrow \boxed{\beta = kd, \quad \text{for} \quad \theta_{\max} = 180°}$$

To ensure that there are **no maxima in other directions**, the separation has to satisfy $d < \lambda$

N-element Linear Array with Uniform Amplitude and Spacing

- **Phased (Scanning) Arrays** The direction of the main beam can be **controlled by the phase shift β**.
- When the scanning is required to be continuous, the feeding system must be capable of **continuously varying the progressive phase β** between the elements. This is accomplished by ferrite or diode shifters (varactors).
- This is the **basic principle of electronic scanning** for phased arrays.
- maximum (m=0) of 0^{th} order AF_n occurs when

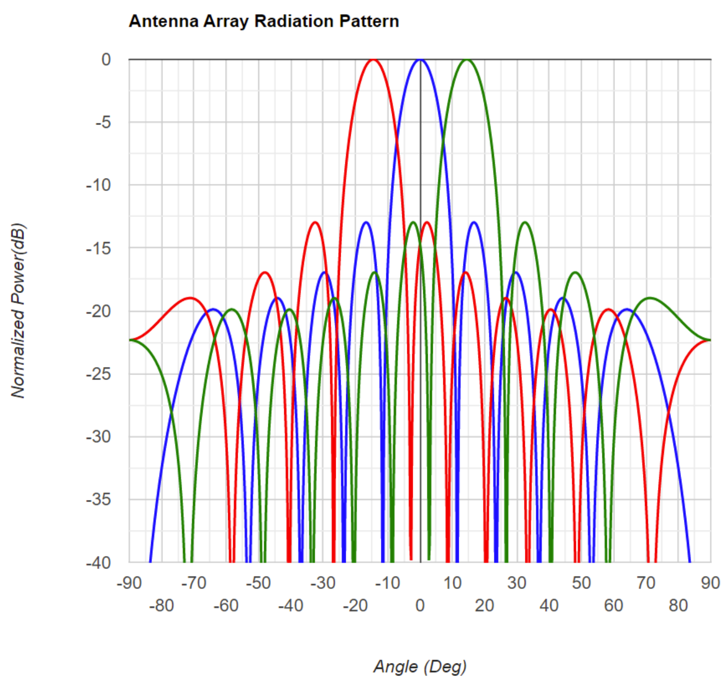
$$\boxed{\psi = kd\cos\theta_0 + \beta = 0}$$

N-element Linear Array with Uniform Amplitude and Spacing

<u>Example:</u> Derive the values of the progressive phase shift β as dependent on the direction of the main beam θ_0 for a uniform linear array with $d = \lambda / 4$.

$$\beta = -kd\cos\theta_0 = -\frac{2\pi}{\lambda}\frac{\lambda}{4}\cos\theta_0 = -\frac{\pi}{2}\cos\theta_0$$

θ_0	β
0°	-90°
60°	-45°
120°	$+45^\circ$
180°	$+90^\circ$

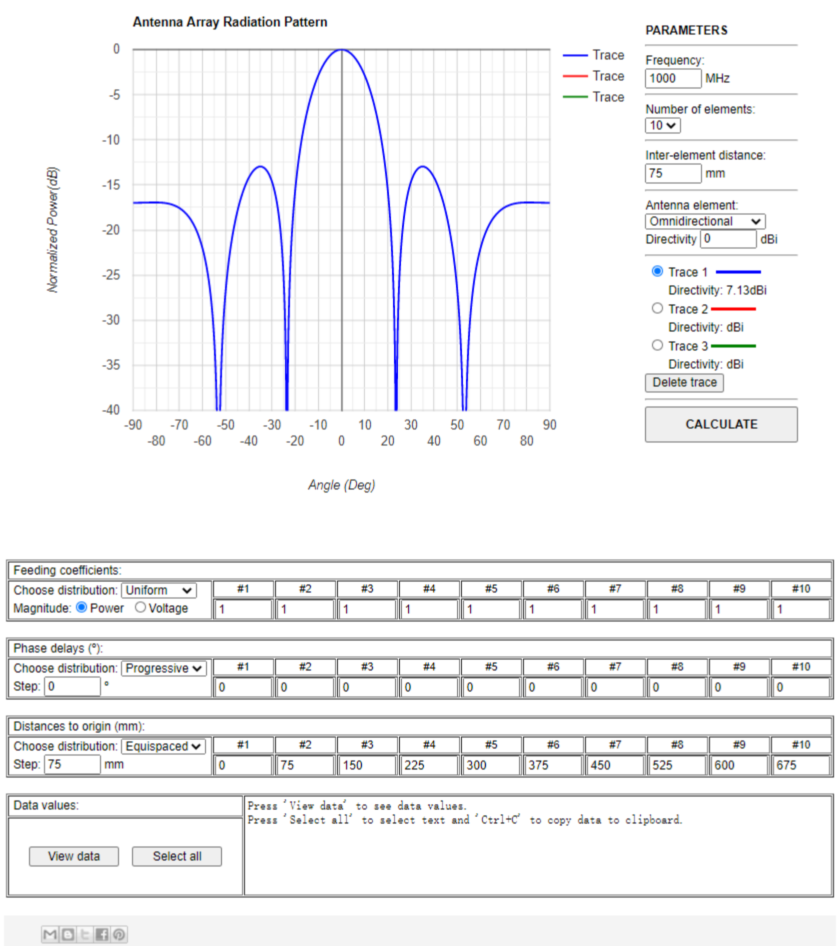


0^o
45^o
-45^o

f=1GHz
N=10
d=150mm

N-element Linear Array with Uniform Amplitude and Spacing

Antenna Array Factor Calculator



https://antennaarraycalculator.blogspot.com/p/calculator.html

N-element Linear Array with Uniform Amplitude and Spacing

- ***Phased (Scanning) Arrays***

broadside array

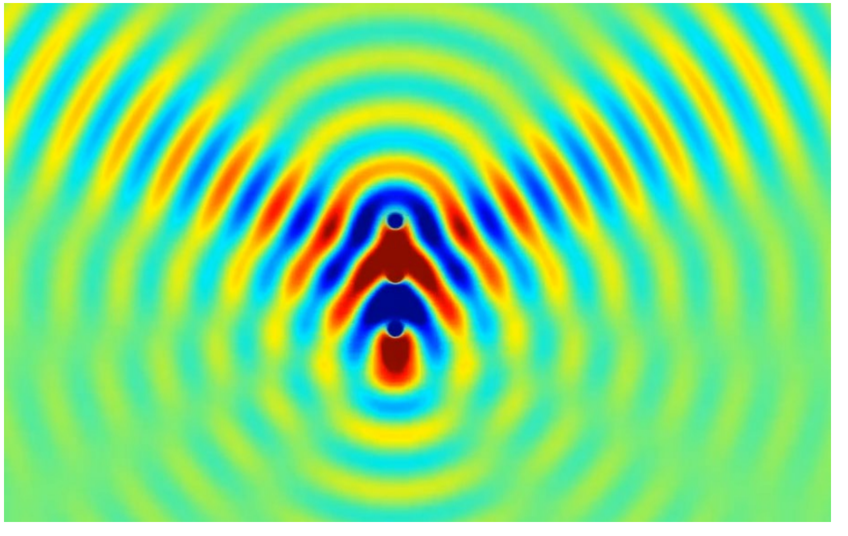
$$\boxed{HPBW = \arccos\left[\cos\theta_0 - \frac{2.782}{Nkd}\right] - \arccos\left[\cos\theta_0 + \frac{2.782}{Nkd}\right]}$$

end-fire arrays

$$HPBW = 2\arccos\left(1 - \frac{2.782}{Nkd}\right)$$

N-element Linear Array with Uniform Amplitude and Spacing

- Radiation by a 6-element array of Hertzian dipole antennas.
- Perspective taken along the planar at z = 0 (looking down).
- Frequency of excitation is 300 MHz with a progressive phase shift along each element.
- A steered beam.



N-element Linear Array with Uniform Amplitude and Spacing

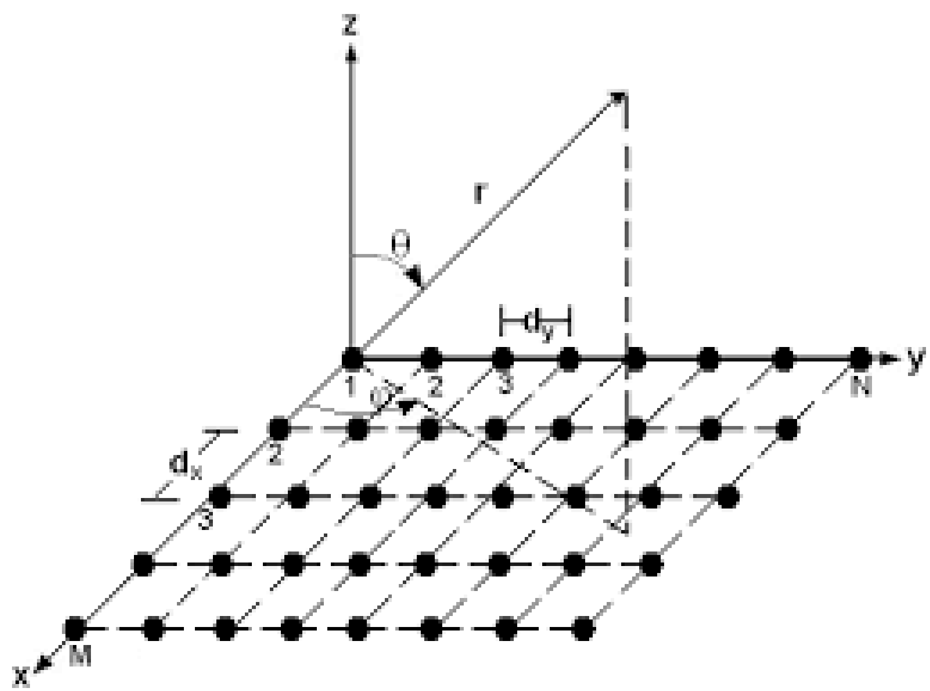
N-element Linear Array with Uniform Amplitude and Spacing

All calculations performed analytically on the Hertzian dipole model.

N-element Linear Array with Uniform Amplitude and Spacing

Planar Arrays

- individual radiators positioned **along a rectangular grid** to form a rectangular or planar array.
- provide **additional variables** used **to control and shape the pattern** of the array.



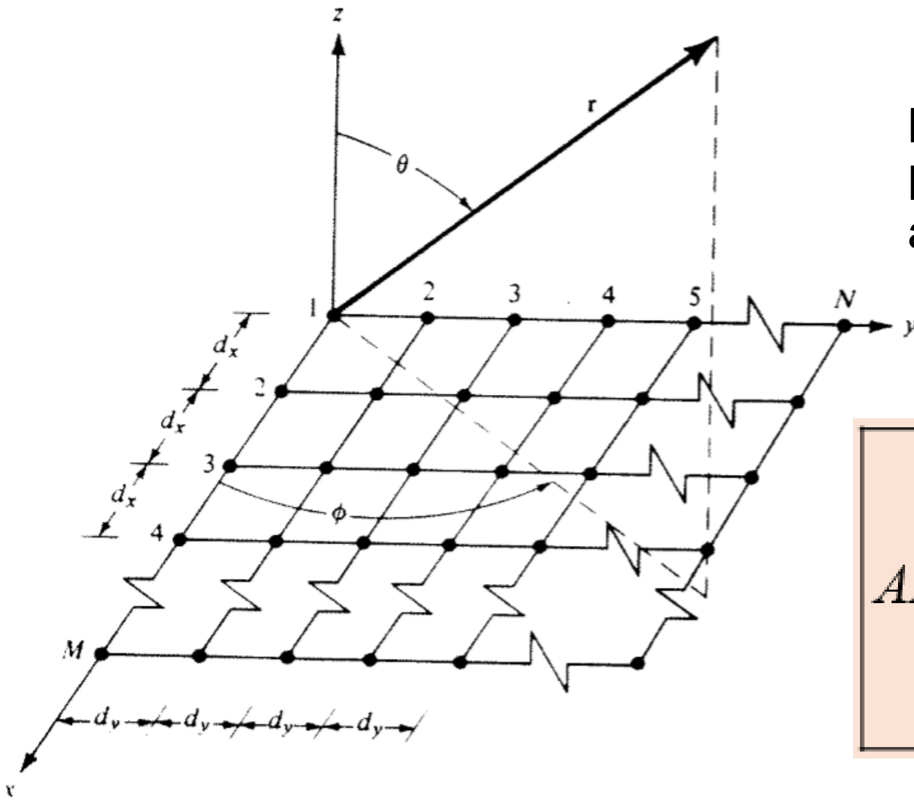
Planar Arrays

- **more versatile**
- **more symmetrical patterns** with **lower side lobes**.
- scan the main beam of the antenna **toward any point in space**.
- Applications include tracking radar, search radar, remote sensing, communications, and many others.

Planar Arrays

A planar array of slots, used in the Airborne Warning and Control System (AWACS), is shown. It utilizes rectangular waveguide sticks placed vertically, with slots on the narrow wall of the waveguides. The system has 360° view of the area, and at operating altitudes can detect targets hundreds of kilometers away. It is usually mounted at a height above the fuselage of an aircraft.

Planar Arrays



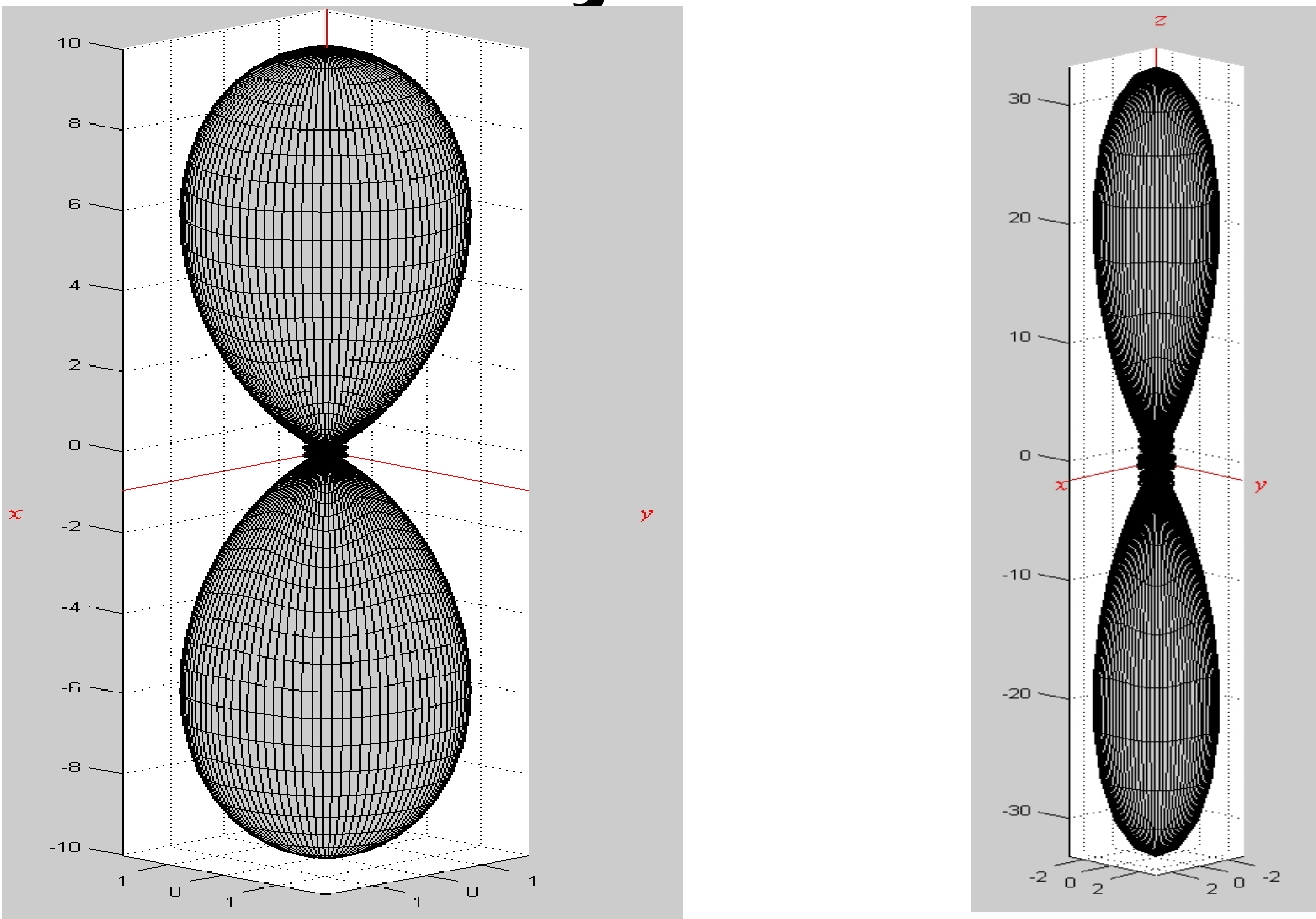
pattern of a rectangular array is the **product** of the **array factors** of the linear arrays in the *x* and *y* directions.

$$AF_n(\theta,\phi) = \left[\frac{\sin\left(M\dfrac{\psi_x}{2}\right)}{M\sin\left(\dfrac{\psi_x}{2}\right)}\right]\cdot\left[\frac{\sin\left(N\dfrac{\psi_y}{2}\right)}{N\sin\left(\dfrac{\psi_y}{2}\right)}\right]$$

$$\psi_x = kd_x \sin\theta\cos\phi + \beta_x$$

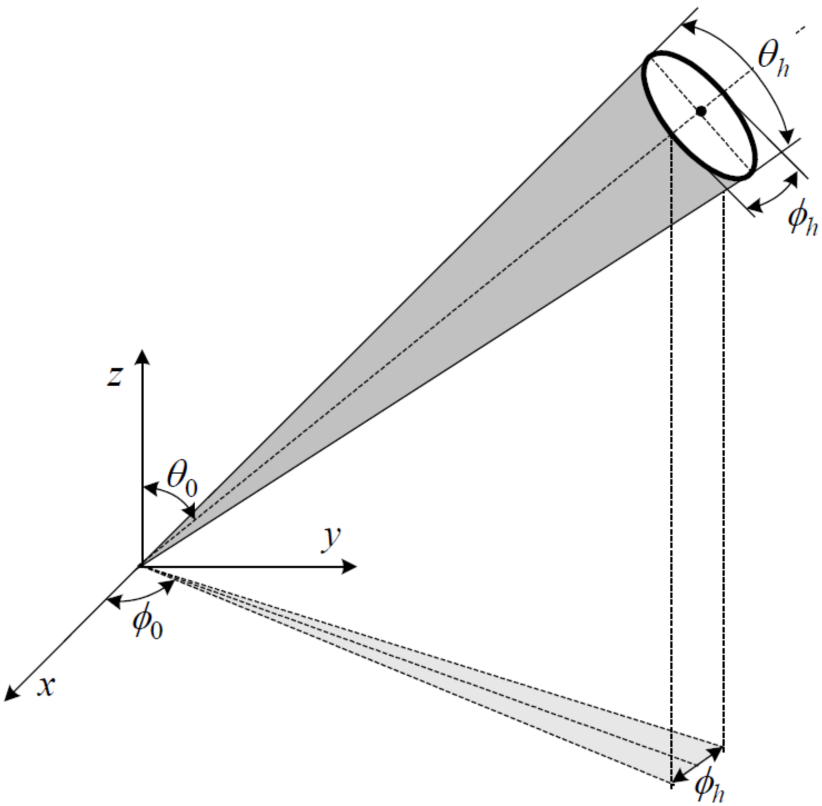
$$\psi_y = kd_y \sin\theta\sin\phi + \beta_y$$

Planar Arrays



directivity patterns of a 5-element square planar uniform array $\beta x = \beta y = 0$
(a) $d=\lambda/4$, $D_0=10.0287$ (10.0125 dB) (b) $d=\lambda/2$, $D_0= 33.2458$ (15.2174 dB)

Planar Arrays-beamwidth



For a large array, whose maximum is near the broad side, the elevation plane HPBW is approximately:

$$\theta_h = \sqrt{\frac{1}{\cos^2\theta_0\left[\Delta\theta_x^{-2}\cos^2\phi_0 + \Delta\theta_y^{-2}\sin^2\phi_0\right]}}$$

$$\phi_h = \sqrt{\frac{1}{\Delta\theta_x^{-2}\sin^2\phi_0 + \Delta\theta_y^{-2}\cos^2\phi_0}}$$

(θ_0, ϕ_0) specify the main-beam direction;

$\Delta\theta_x$ is the HPBW of a linear broadside array whose number of elements M and amplitude distribution is the same as that of the x-axis linear arrays building the planar array;

$\Delta\theta_y$ is the HPBW of a linear BSA whose number of elements N and amplitude distribution is the same as those of the y-axis linear arrays building the planar array.

Planar Arrays-beamwidth

beam solid angle of the planar array

$$\Omega_A = \theta_h \cdot \phi_h ,$$

or

$$\boxed{\Omega_A = \frac{\Delta\theta_x \Delta\theta_y}{\cos\theta_0 \sqrt{\left[\sin^2\phi_0 + \frac{\Delta\theta_y^2}{\Delta\theta_x^2}\cos^2\phi_0\right]\left[\sin^2\phi_0 + \frac{\Delta\theta_x^2}{\Delta\theta_y^2}\cos^2\phi_0\right]}}}.$$

(θ_0, ϕ_0) specifies the main-beam direction;

$\Delta\theta_x$ is the HPBW of a linear BSA of M elements and an amplitude distribution which is the same as that of the x-axis linear arrays building the planar array;

$\Delta\theta_y$ is the HPBW of a linear BSA of N elements and amplitude distribution is the same as those of the y-axis linear arrays building the planar array.

Planar Arrays-Directivity

$$D_0 = \pi D_x D_y \cos\theta_0$$

D_x is the directivity of the respective linear BSA, x-axis;

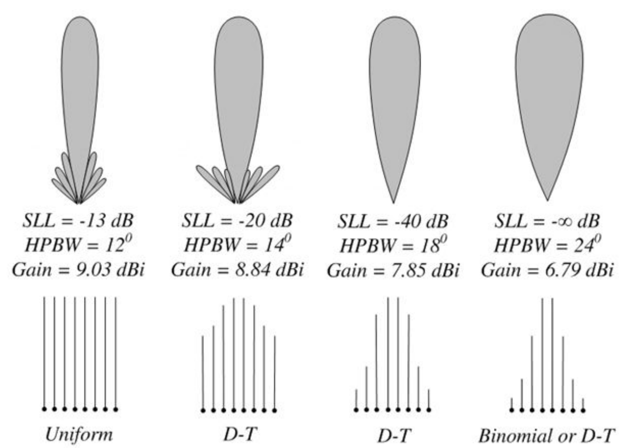
D_y is the directivity of the respective linear BSA, y-axis.

We can also use the array solid beam angle Ω_A to calculate the approximate directivity of a nearly broadside planar array

$$D_0 \approx \frac{\pi^2}{\Omega_{A[\mathrm{Sr}]}} \approx \frac{32400}{\Omega_{A[\mathrm{deg}^2]}}$$

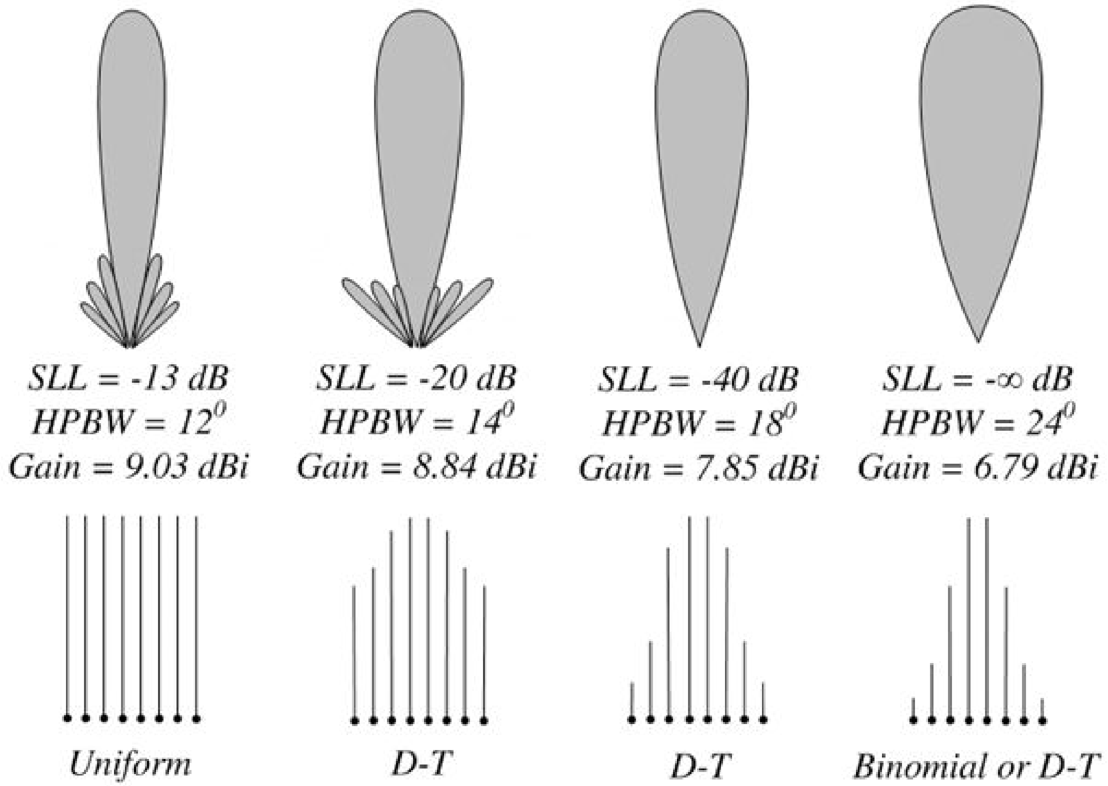
Planar Arrays-Directivity

1. The main beam direction is controlled through the phase shifts, βx and βy
2. The beamwidth and side-lobe levels are controlled through the amplitude distribution.



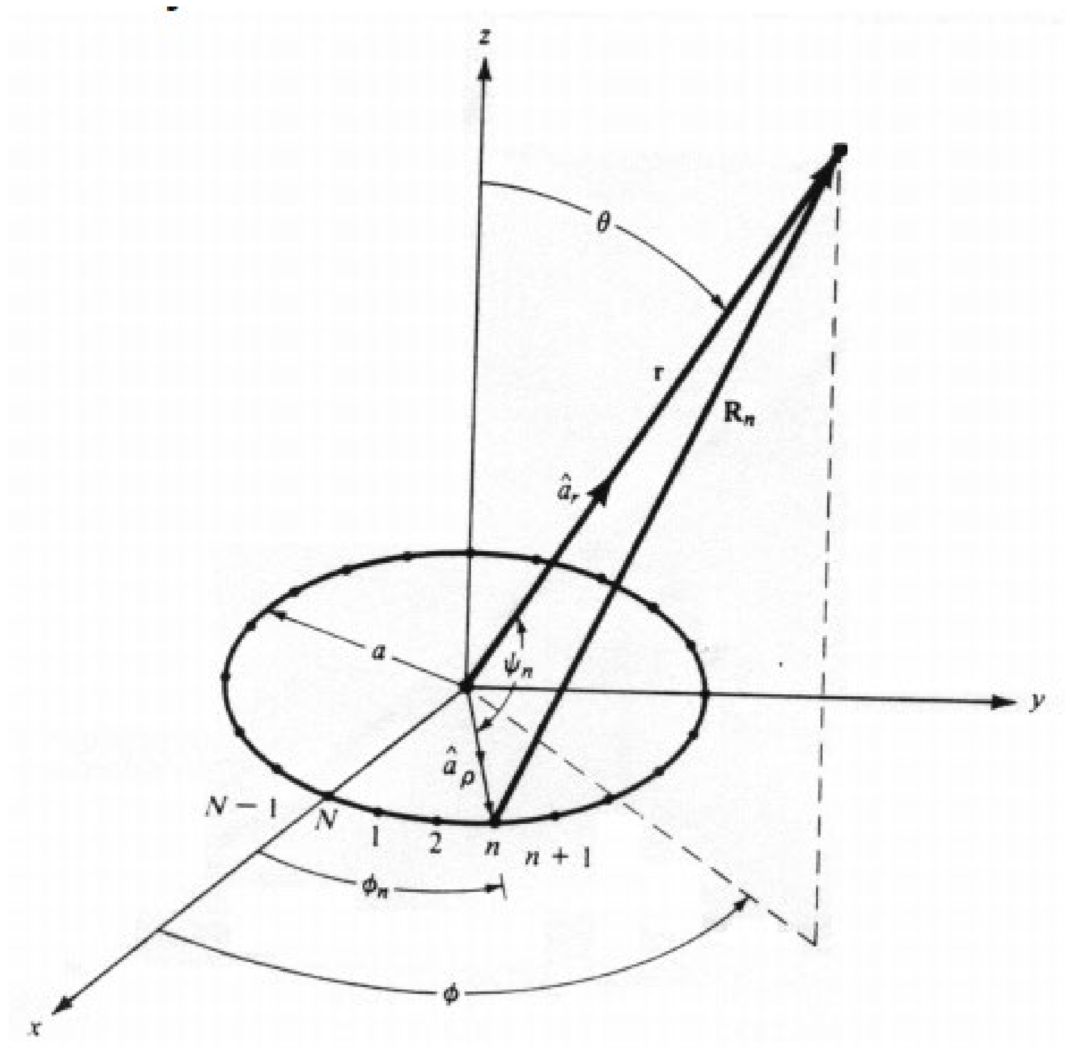
The radiation pattern, SLL, HPBW, and gain for four different source distributions of eight in-phase isotropic sources spaced by $\lambda/2$; **there are trade-offs!**

Antennas: *from Theory to Practice*



The radiation pattern, SLL, HPBW, and gain for four different source distributions of eight in-phase isotropic sources spaced by $\lambda/2$; **there are trade-offs!**

Circular Array



Circular Array

$$AF(\theta,\phi)=\sum_{n=1}^{N} I_n e^{jka[\sin\theta\cos(\phi-\phi_n)-\sin\theta_0\cos(\phi_0-\phi_n)]}$$

$$AF(\theta,\phi)=\sum_{n=1}^{N} I_n e^{jka(\cos\psi_n-\cos\psi_{0n})}$$

Here:

$\psi_n=\cos^{-1}[\sin\theta\cos(\phi-\phi_n)]$ is the angle between $\hat{r}$ and $\hat{a}_{\rho_n}$;

$\psi_{0_n}=\cos^{-1}[\sin\theta_0\cos(\phi_0-\phi_n)]$ is the angle between $\hat{a}_{\rho_n}$ and $\hat{r}_{\max}$ pointing in the direction of maximum radiation.

$I_n e^{jka_n}$ excitation coefficient of the nth element

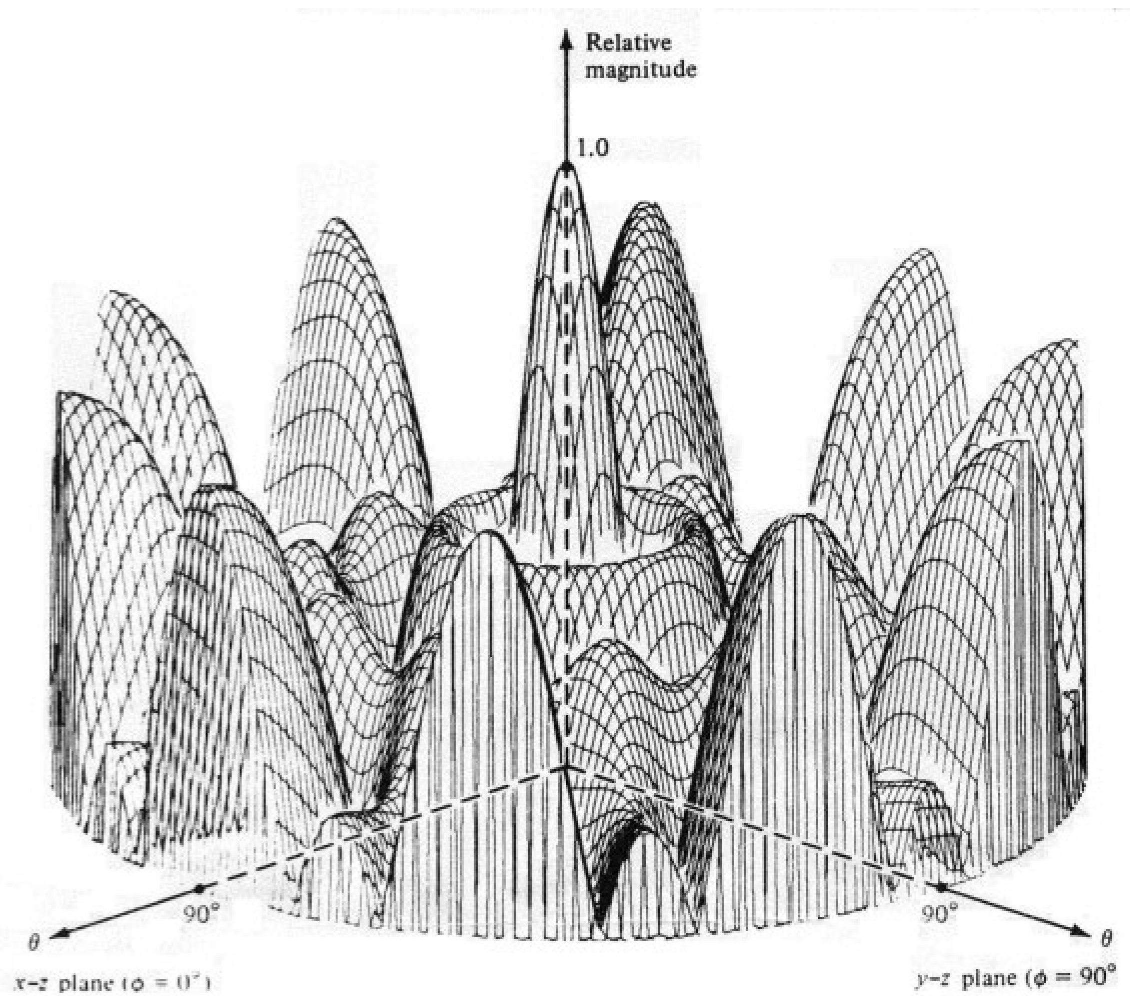
I_n amplitude excitation of nth element

a_n phase excitation (relative to the arracenter) of the nth element

$\phi_n = 2\pi\left(\frac{n}{N}\right)$ angular position of the nth element on x-y plane

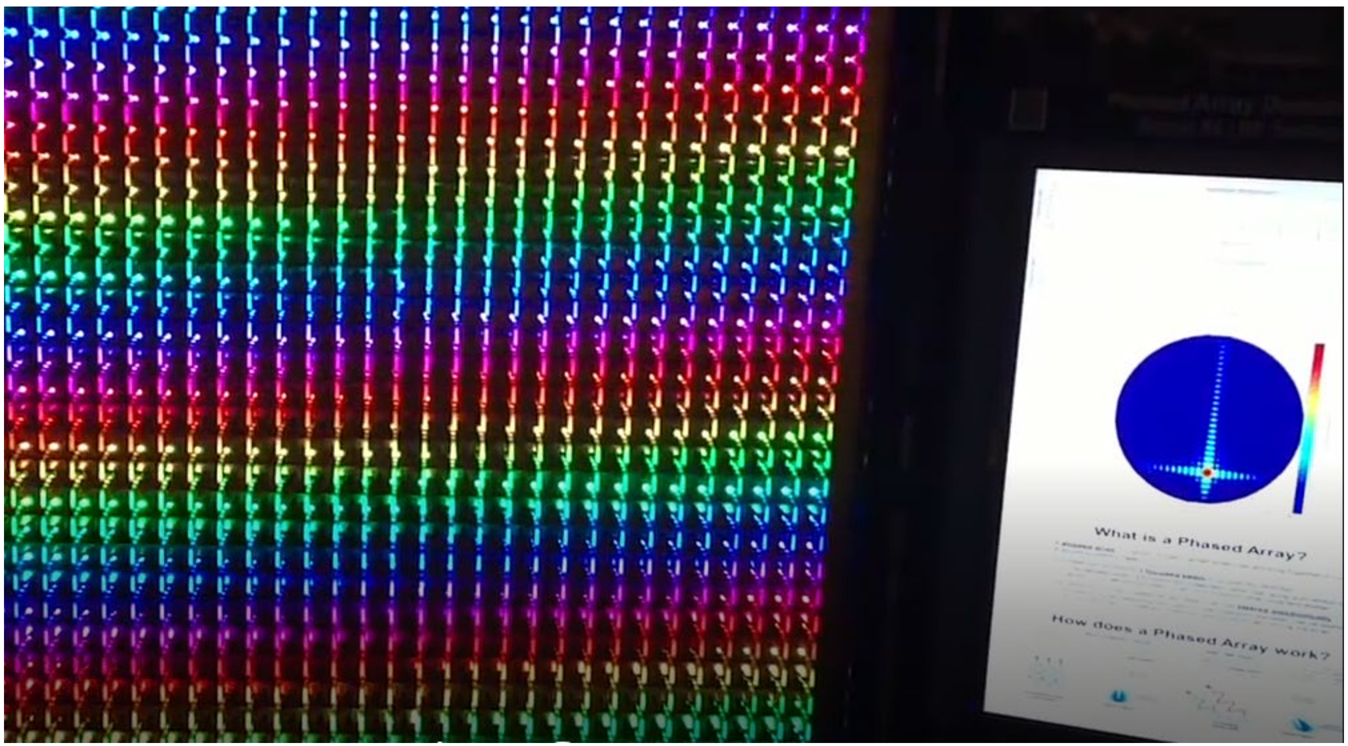
Circular Array

Uniform circular array 3-D pattern (N=10, $ka = \frac{2\pi}{\lambda} a = 10$)



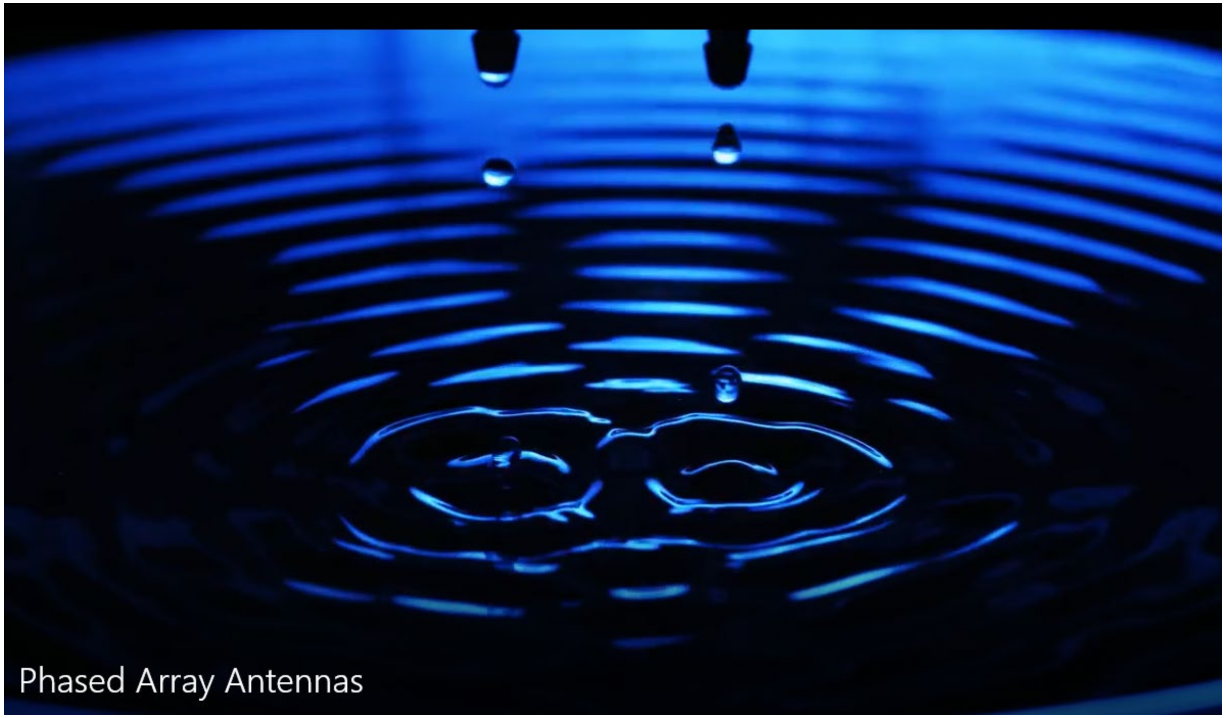
Phased Array demo

Video demo 4’23”



Phased Array Explained

Video 5’



Homework

UG:

- Write a (Matlab) program to plot 2D pattern of a linear array
- Test program using a 10 element array spaced at $\lambda/2$. Show phase shift changes the main beam direction

- PG:
- Write a (Matlab) program to plot 3D pattern of planar array
- Test program using a 5x5 array spaced at $\lambda/2$. Show phase shift changes the main beam direction