3-10. In order to improve the performance of cellular systems, multiple base stations can receive the signal transmitted from a given mobile unit and combine these multiple signals either by selecting the strongest one or summing the signals together, perhaps with some optimized weights. This typically increases SNR and reduces the effects of shadowing. Combining of signals received from multiple base stations is called *macrodiversity,* and here we explore the benefits of this technique. Diversity will be covered in more detail in Chapter 7.

Consider a mobile at the midpoint between two base stations in a cellular network. The received signals (in dBW) from the base stations are given by

$$P_{r,1} = W + Z_1,$$
$$P_{r,2} = W + Z_2,$$

where $Z_{1,2}$ are $N(0, \sigma^2)$ random variables. We define outage with macrodiversity to be the event that both $P_{r,1}$ and $P_{r,2}$ fall below a threshold T.

(a) Interpret the terms W, Z_1, Z_2 in $P_{r,1}$ and $P_{r,2}$.

(b) If Z_1 and Z_2 are independent, show that the outage probability is given by

$$P_{\text{out}} = [Q(\Delta/\sigma)]^2,$$

where $\Delta = W - T$ is the fade margin at the mobile's location.

(c) Now suppose that Z_1 and Z_2 are correlated in the following way:

$$Z_1 = aY_1 + bY,$$
$$Z_2 = aY_2 + bY,$$

where Y, Y_1, Y_2 are independent $N(0, \sigma^2)$ random variables and where a, b are such that $a^2 + b^2 = 1$. Show that

$$P_{\text{out}} = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}} \left[Q\left(\frac{\Delta + by\sigma}{|a|\sigma} \right) \right]^2 e^{-y^2/2}\, dy.$$

(d) Compare the outage probabilities of (b) and (c) for the special case of $a = b = 1/\sqrt{2}$, $\sigma = 8$, and $\Delta = 5$ (this will require a numerical integration).

(a) W: average received power

Z_1: shadowing of base station 1.

Z_2: shadowing of base station 2.

$P_{r,1}$: received power of base station 1. in dBW

$P_{r,2}$: received power of base station 2 in dBW

(b) $\because Z \sim N(0, \sigma^2)$.

$\therefore P_{r,1}, P_{r,2} \sim N(W, \sigma^2)$

$\therefore P(P_{r,1} < T) = Q\left(\frac{W-T}{\sigma}\right),\ P(P_{r,2} < T) = Q\left(\frac{W-T}{\sigma}\right)$

$\because P_{r,1} \cdot P_{r,2}$ independent

$\therefore P_{out} = P(P_{r,1} < T \cap P_{r,2} < T) = P[P_{r,1} < T] \cdot P[P_{r,2} < T] = [Q(\frac{W-T}{\sigma})]^2 = [Q(\frac{\Delta}{\sigma})]^2$

(c) $P_{out} = P(P_{r,1} < T \cap P_{r,2} < T]$

$= \int_{-\infty}^{\infty} P[P_{r,1} \leq T,\ P_{r,2} < T \mid Y = y] \cdot f_Y(y)\, dy$

$\because P_{r,1}\big|_{Y=y} \sim N(W + by, a^2\sigma^2)$, $[P_{r,1} \mid Y = y] \perp [P_{r,2} \mid Y = y]$

$\therefore P_{out} = \int_{-\infty}^{\infty} [Q(\frac{W+by-T}{a\sigma})]^2 \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}} dy$

let $\frac{y}{\sigma} = u$.

$\Rightarrow P_{out} = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} [Q(\frac{W-T+bu\sigma}{a\sigma})]^2 e^{-\frac{u^2}{2}} du$

$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} [Q(\frac{\Delta + by\sigma}{a\sigma})]^2 e^{-\frac{y^2}{2}} dy$

(d) $\because a = b = \frac{1}{\sqrt{2}}$, $\sigma = 8$. $\Delta = 5$.

in (b), $P_{out} = [Q(\frac{\Delta}{\sigma})]^2 = [Q(\frac{5}{8})] = 0.07$

in (c), $P_{out} = [Q(\frac{\Delta + by\sigma}{a\sigma})]^2 e^{-\frac{y^2}{2}} dy = \int_{-\infty}^{\infty} [Q(\frac{8 + \frac{8}{\sqrt{2}}y}{\frac{1}{\sqrt{2}} \times 8})]^2 e^{-\frac{y^2}{2}} dy$

$= \int_{-\infty}^{\infty} [Q(\sqrt{2} + y)]^2 e^{-\frac{y^2}{2}} dy = 0.13$

$\therefore$ Independent shadowing is prefarable for diversity.

3-15. Consider the following channel scattering function obtained by sending a 900-MHz sinusoidal input into the channel:

$$S(\tau, \rho) = \begin{cases} \alpha_1 \delta(\tau) & \rho = 70 \text{ Hz}, \\ \alpha_2 \delta(\tau - .022\ \mu\text{s}) & \rho = 49.5 \text{ Hz}, \\ 0 & \text{else}, \end{cases}$$

where α_1 and α_2 are determined by path loss, shadowing, and multipath fading. Clearly this scattering function corresponds to a two-ray model. Assume the transmitter and receiver used to send and receive the sinusoid are located 8 m above the ground.

(a) Find the distance and velocity between the transmitter and receiver.

(b) For the distance computed in part (a), is the path loss as a function of distance proportional to d^{-2} or d^{-4}? *Hint:* Use the fact that the channel is based on a two-ray model.

(c) Does a 30-kHz voice signal transmitted over this channel experience flat or rather frequency-selective fading?

(a) $\tau = 0.022 \mu s.$ $\quad f_c = 900 MHz$

$\because \Delta x = 2\sqrt{(\frac{d}{2})^2 + 8^2} - d = \tau \cdot c = 0.022 \mu s \cdot 3 \times 10^8 m/s$

$\Rightarrow d = 16.1 m$

$\because f_D = \frac{v}{\lambda} \cos\theta, \Rightarrow v = \frac{f_D \lambda}{\cos\theta} = \frac{f_D c}{f_c \cdot \cos\theta}$

$\because$ LOS. $\therefore \theta = 0.$ $\cos\theta = 1$, $f_D = 70 Hz$

$v = \frac{70 Hz \cdot 3 \times 10^8 m/s}{900 \times 10^6 Hz} = 23.3 m/s$

(b) $\because d_c = \frac{4 h_t h_r}{\lambda} = 768 m >> d = 8 m$

$\therefore$ to d^{-2}

(c) $T_m = 0.022 \mu S$ $\quad \because B^{-1} = \frac{1}{f} = \frac{1}{30 \times 10^3 Hz} = 3.3 \times 10^{-5} s >> T_m$

$\therefore$ flat fading.