$\Phi = \{\phi_1(t), \cdots, \phi_N(t)\}.\ t \in (0, \tau).$

$<\phi_i(t), \phi_j(t)> = \int_0^T \phi_i(t)\phi_j(t)\,dt = \begin{cases} 1, & i=j \\ 0, & \text{otherwise.} \end{cases}$

$\{s_1(t), s_2(t), \cdots, s_N(t)\}.$

$s_i(t) = \sum_{j=1}^{N} \underbrace{s_{ij}}_{} \phi_j(t)$

$s_i(t) \longrightarrow (s_{i1}, s_{i2}, \cdots, s_{iN}) \triangleq \vec{s}_i$

$s_{ij} = <s_i(t), \phi_j(t)>.$

$\|\vec{s}_i\| \quad \|\vec{s}_i - \vec{s}_j\|.$

BPSK:

$\phi_i: \{\sqrt{\frac{2}{T}}\cos 2\pi f_c t\},\ \{\vec{s}_1 = a.\ \vec{s}_2 = -a\}.$

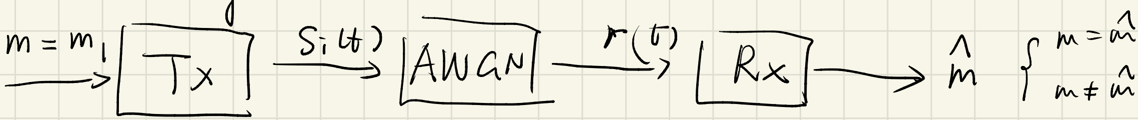
QPSK (QAM).

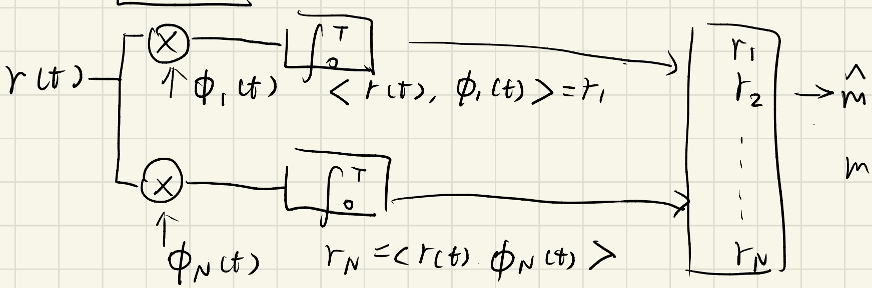
$\phi_i: \{\sqrt{\frac{2}{T}}\cos 2\pi f_c t, \sqrt{\frac{2}{T}}\sin 2\pi f_c t\}. \quad \begin{bmatrix} \vec{s}_1 = (a, a). & \vec{s}_2 = (a. -a) \\ \vec{s}_3 = (-a. a). & \vec{s}_4 = (-a, -a) \end{bmatrix}.$

- Receiver: $r(t) = s(t) + n(t)$

AWGN: $\quad r(t) = h s(t) + n(t)$

Flat Fading Channel:





message is sent

$r(t) = s_i(t) + n(t)$

$r_j = \int_0^T r(t)\phi_j(t)\,dt = \int_0^T s_i(t)\phi_j(t)\,dt + \int_0^T n(t)\phi_j(t)\,dt$

$= s_{ij} + \underbrace{<n(t), \phi_j(t)>}_{n_j(t)}$

$n(t) = \sum_{j=1}^{N} n_j \phi_j(t) + n_r(t)$.

左右与 $\phi_j(t)$ 内积，$\Rightarrow n_j = n_j + \underbrace{\langle n_r(t), \phi_j(t) \rangle}_{=0}$

$n_r(t)$ 与所有正交基正交

信号被分成了 j 维. 噪声 $n_r(t)$ 有更高维度

$E[n(t)] = 0$. $E[n(t)\, n(\tau)] = \begin{cases} 0 & t \neq \tau \\ \frac{N_0}{2} \delta(t-\tau) & t = \tau \end{cases}$

$r_j = s_{ij} + n_j$. $\vec{r} = \vec{s_i} + \vec{n}$.

$n_j = \int_0^T n(t) \cdot \phi_j(t)\, dt$

Gaussian. $E[n_j] = 0$. $N(0, \frac{N_0}{2})$

$E[n_j] = E\left[\int_0^T \int_0^T n(t) \phi_j(t) n(\tau) \phi_j(\tau) dt\, d\tau\right] = \int_0^T \underbrace{\int_0^T \frac{N_0}{2} \delta(t-\tau) \phi_j(\tau) d\tau}\, \phi_j(t) dt$

$= \int_0^T \frac{N_0}{2} \phi_j^2(t) dt = \frac{N_0}{2}$

$E[n_i \cdot n_j] = 0$

$E[r_j | \vec{S_i}] = E[s_{ij} + n_j] = s_{ij}$

$Var[r_j | \vec{S_i}] = \frac{N_0}{2}$

Given: $\vec{S_i}$, $\begin{cases} r_j \sim N(s_{ij}, \frac{N_0}{2}) \\ r_j, r_k \text{ are independent} \end{cases}$

$p(\vec{r} | \vec{s_i}) = \prod_{j=1}^{N} p(r_j | \vec{s_i}) = \prod_{j=1}^{N} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{1}{N_0}(r_j - s_{ij})^2} = \frac{1}{(\pi N_0)^{\frac{N}{2}}} e^{-\frac{1}{N_0} \underbrace{\sum_{i=1}^{N} (r_j - s_{ij})^2}_{\|\vec{r} - \vec{s_i}\|^2}}$

$= \frac{1}{(\pi N_0)^{\frac{N}{2}}} e^{-\frac{1}{N_0} \|\vec{r} - \vec{s_i}\|^2}$