3-6. Assume a Rayleigh fading channel with average signal power $2\sigma^2 = -80$ dBm. What is the power outage probability of this channel relative to the threshold $P_0 = -95$ dBm? How about $P_0 = -90$ dBm?

$\overline{P_r} = 2\sigma^2 = -80\,\text{dBm} = 10^{-11}\,W$

$P(z^2 < P_o) = \int_0^{P_o} \frac{1}{2\sigma^2} e^{-\frac{x}{2\sigma^2}} dz^2 = -e^{-\frac{x}{2\sigma^2}} \Big|_0^{P_o} = 1 - e^{-\frac{P_o}{2\sigma^2}}$

when $P_o = -95\,\text{dBm} = 10^{-12.5}\,W$. $\frac{P_o}{2\sigma^2} = 10^{-1.5} \Rightarrow P = 1 - e^{-10^{-1.5}} = 0.0311$

when $P_o = -90\,\text{dBm} = 10^{-12}\,W$. $\frac{P_o}{2\sigma^2} = 10^{-1} \Rightarrow P = 1 - e^{-0.1} = 0.0952$

3-7. Suppose we have an application that requires a power outage probability of .01 for the threshold $P_0 = -80$ dBm. For Rayleigh fading, what value of the average signal power is required?

$P(z^2 < P_o) = \int_0^{P_o} \frac{1}{2\sigma^2} e^{-\frac{x}{2\sigma^2}} dz^2 = -e^{-\frac{x}{2\sigma^2}} \Big|_0^{P_o} = 1 - e^{-\frac{P_o}{2\sigma^2}}$

$\because P_o = -80\,\text{dBm} = 10^{-11}\,W$

$1 - e^{-\frac{P_o}{2\sigma^2}} = 0.01$

$\therefore -\ln 0.99 = \frac{10^{-11}}{2\sigma^2} \Rightarrow 2\sigma^2 = -\frac{10^{-11}}{\ln 0.99} = 9.95 \times 10^{-10}\,W$

$\therefore \overline{P_r} = 2\sigma^2 = 9.95 \times 10^{-10}\,W = -60.02\,\text{dBm}.$