5.4

5-4. Consider the three signal waveforms $\{\phi_1(t), \phi_2(t), \phi_3(t)\}$ shown in Figure 5.32.

(a) Show that these waveforms are orthonormal.

(b) Express the waveform $x(t)$ as a linear combination of $\{\phi_i(t)\}$ and find the coefficients, where $x(t)$ is given as

$$x(t) = \begin{cases} 2 & 0 \le t < 2, \\ 4 & 2 \le t \le 4. \end{cases}$$

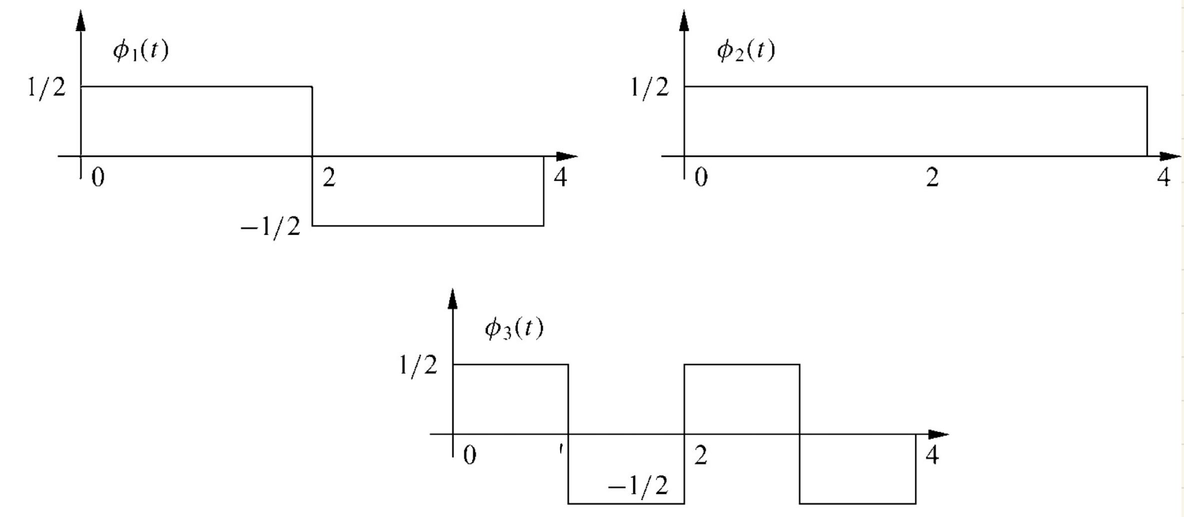


Figure 5.32: Signal waveforms for Problem 5-4.

(a) $\langle f_1(t), f_2(t) \rangle = \int_0^T f_1(t) f_2(t)\, dt = 0$

$\langle f_1(t), f_3(t) \rangle = \int_0^T f_1(t) f_3(t) = 0$

$\langle f_2(t), f_3(t) \rangle = \int_0^T f_2(t) f_3(t) = 0$,

∴ 两两正交，这3个波正交

(b) ∵ linear ∴ 设 $x(t) = a f_1(t) + b f_2(t) + c f_3(t)$.

由图像 $\begin{cases} \frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c = 2 \\ \frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}c = 2 \\ -\frac{1}{2}a + \frac{1}{2}b + \frac{1}{2}c = 4 \\ -\frac{1}{2}a + \frac{1}{2}b - \frac{1}{2}c = 4 \end{cases} \Rightarrow \begin{cases} a = -2 \\ b = 6 \\ c = 0 \end{cases}$

∴ $x(t) = -2 f_1(t) + 6 f_2(t)$

5.6

5-6. Derive a mathematical expression for decision regions Z_i that minimize error probability assuming that messages are not equally likely – that is, assuming $p(m_i) = p_i$ $(i = 1, \ldots, M)$, where p_i is not necessarily equal to $1/M$. Solve for these regions in the case of QPSK modulation with $s_1 = (A_c, 0)$, $s_2 = (0, A_c)$, $s_3 = (-A_c, 0)$, and $s_4 = (0, A_c)$, assuming $p(s_1) = p(s_3) = .2$ and $p(s_1) = p(s_3) = .3$.

$\because \hat{m} = m_i$ ∴有 $\hat{s_i} = \arg\max_{s_i} p(y|s_i)p(s_i)$

$\therefore \max_{s_i} l(s_i) = \log L(s_i) = \log p(y|s_i) + \log p(s_i)$

$= \max_{s_i} -\frac{N}{2}\log(\pi N_0) - \frac{1}{N_0}\sum_{j=1}^{N}(y_i - s_{ij})^2 + \log p(s_i)$

$= \max_{s_i} -\frac{1}{N_0}\|y_i - s_{ij}\|^2 + \log p(s_i) = \min_{s_i} \frac{1}{N_0}\|y_j - s_{ij}\|^2 + \log\frac{1}{p(s_i)}$

$\therefore Z_i = \{x \in R \mid \frac{1}{N_0}\|x - s_i\|^2 - \log p(s_i) < \frac{1}{N_0}\|x - s_j\|^2 - \log p(s_j),\ \forall j \neq i\}$

$= \{x \in R \mid \frac{1}{N_0}(\|x - s_i\|^2 - \|x - s_j\|^2) < \log\frac{p(s_i)}{p(s_j)},\ \forall i \neq j\}$

$\because S_1 = (A_c, 0)$. $S_2 = (0, A_c)$, $s_3 = (-A_c, 0)$, $S_4(0, -A_c)$ $p(S_1) = p(S_3) = 0.2$

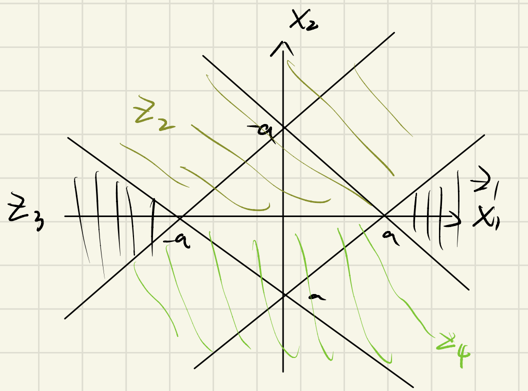
$p(S_2) = p(S_4) = 0.4$

$\therefore i = 1$ 时, $Z_1 \begin{cases} j = 2 \Rightarrow x_2 - x_1 < -\frac{N_0}{2A_c}\log\frac{3}{2} \\ j = 3 \Rightarrow x_1 > 1 \\ j = 4 \Rightarrow x_2 + x_1 > \frac{N_0}{2A_c}\log\frac{3}{2} \end{cases}$

$i = 2$ 时. $Z_2: \begin{cases} j = 1 \Rightarrow x_2 - x_1 > -\frac{N_0}{2A_c}\log\frac{3}{2} \\ j = 3 \Rightarrow x_2 + x_1 > \frac{N_0}{2A_c}\log\frac{3}{2} \\ j = 4 \Rightarrow x_2 > 0 \end{cases}$

$i = 3$ 时. $Z_3 \begin{cases} j = 1 \Rightarrow x_1 < 0 \\ j = 2 \Rightarrow x_2 + x_1 > \frac{N_0}{2A_c}\log\frac{3}{2} \\ j = 4 \Rightarrow x_2 - x_1 > -\frac{N_0}{2A_c}\log\frac{3}{2} \end{cases}$

$i = 4$ 时 $Z_4 \begin{cases} j = 1 & x_1 + x_2 < \frac{N_0}{2A_c}\log\frac{3}{2} \\ j = 2 & x_2 < 0 \\ j = 3 & x_2 - x_1 < \frac{N_0}{2A_c}\log\frac{3}{2} \end{cases}$



令 $\frac{N_0}{2A_c}\log\frac{3}{2} = a$, 则:

5.10

5-10. Show that the ML receiver of Figure 5.4 is equivalent to the matched filter receiver of Figure 5.7.

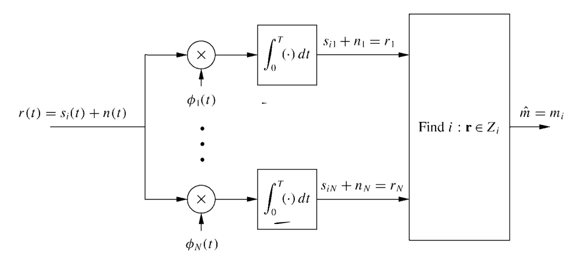


Figure 5.4: Receiver structure for signal detection in AWGN.

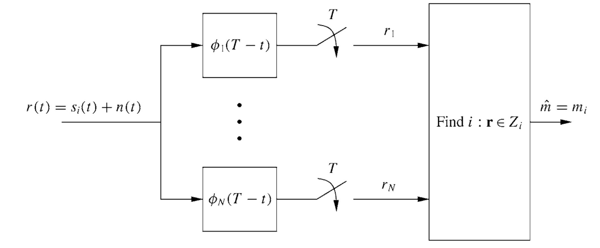


Figure 5.7: Matched filter receiver structure.

$\because r_K = \int_0^T r(\tau)\phi_K(\tau)\,d\tau$

$\psi(t) = \phi(T-t)$

$r_K = \int_0^T \phi_K(T-(T-\tau))\,d\tau$

$= \int_0^T r(\tau)\,d\tau$

输出的抽样完全一样，两种接收机等价