

3. Consider a time-invariant indoor wireless channel with LOS component at delay 23 ns, multipath component at delay 48 ns, and another multipath component at delay 67 ns. Find the delay spread assuming that the demodulator synchronizes to the LOS component. Repeat assuming that the demodulator synchronizes to the first multipath component.

① To the LOS component: $T_m = \max \tau_n - \tau_0 = 44 \text{ ns}$

② To the first multipath component: $T_m = \max \tau_n - \tau_1 = 19 \text{ ns}$

3-4. Show that the minimum value of $f_c \tau_n$ for a system at $f_c = 1 \text{ GHz}$ with a fixed transmitter and a receiver separated by more than 10 m from the transmitter is much greater than 1.

$$\because \tau_{n \min} = \frac{10 \text{ m}}{3 \times 10^8 \text{ m/s}} = \frac{10}{3} \times 10^{-8} \text{ s}$$

$$\therefore \min \text{ of } f_c \tau_n = 1 \times 10^9 \times \frac{10}{3} \times 10^{-8} = 3.3 \gg 1$$

3-5. Prove, for X and Y independent zero-mean Gaussian random variables with variance σ^2 , that the distribution of $Z = \sqrt{X^2 + Y^2}$ is Rayleigh distributed and that the distribution of Z^2 is exponentially distributed.

$$F_Z(z) = P[X^2 + Y^2 \leq z^2] = \iint_{x^2 + y^2 \leq z^2} \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}} dx dy = \int_0^{2\pi} \int_0^z \frac{1}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}} dr d\theta$$

$$= 1 - e^{-\frac{z^2}{2\sigma^2}} \quad (z \geq 0)$$

$$\therefore \frac{dF_Z(z)}{dz} = \frac{z}{\sigma^2} e^{-\frac{z^2}{2\sigma^2}}$$

$\therefore Z$ is Rayleigh distributed.

$$F_{Z^2}(z) = P[Z \leq \sqrt{z}] = 1 - e^{-\frac{z}{2\sigma^2}}$$

$$\frac{dF_{Z^2}(z)}{dz} = \frac{1}{2\sigma^2} e^{-\frac{z}{2\sigma^2}}$$

$\therefore Z^2$ is exponentially distributed.