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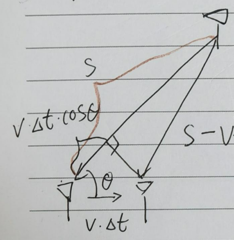
## △ Doppler Shift.




$\cos 2\pi f_c t. \rightarrow \cos 2\pi f_c(t+\Delta t)$

$\cos 2\pi f_c(t-\frac{S}{c}) \rightarrow \cos 2\pi f_c(t+\Delta t-\frac{S-V\Delta t\cos\theta}{c})$

$=\cos(2\pi f_c t - 2\pi f_c\frac{S}{c}) \quad = \cos\{2\pi f_c t - 2\pi f_c\frac{S}{c}$

$\quad + 2\pi f_c\Delta t + 2\pi f_c\frac{V\cdot\Delta t\cdot\cos\theta}{c})$

Phase difference $= 2\pi f_c\Delta t + 2\pi f_c\frac{V\cos\theta\cdot\Delta t}{c}$

frequency of received ~~封~~ signal

$= f_c + f_c\frac{V\cos\theta}{c}$

$= f_c + \underbrace{\frac{V}{\lambda}\cos\theta}_{f_d}$ $\quad\lambda$: wavelength

$f_d$: Doppler Frequency Shift.

SUSTech

③ $E[r_I(t)\cdot r_Q(t)]$

$= E\left[\sum_n \alpha_n \cos\phi_n(t) \cdot \sum_m \alpha_m \sin\phi_m(t)\right]$

$= \sum_m \sum_n E[\alpha_m \alpha_n]$
$\qquad E[\cos\phi_n(t) \cdot \sin\phi_m(t)]$

$= \sum_n E[\alpha_n^2] \cdot E[\cos\phi_n(t) \cdot \sin\phi_n(t)]$

$= \sum_n E[\alpha_n^2] \cdot E\left[\frac{1}{2}\sin 2\phi_n(t)\right]$

$= 0$

$\Rightarrow$ $r_I(t)$ and $r_Q(t)$ are uncorrelated

If $r_I(t)$ and $r_Q(t)$ are Gaussian,
they are independent.

④ $A_{r_I}(t;t+\tau) = E[r_I(t)\, r_I(t+\tau)]$

$= \sum_n E[\alpha_n^2] \cdot E[\cos\phi_n(t) \cdot \cos\phi_n(t+\tau)]$

$\phi_n(t) = 2\pi f_c \tau_n - 2\pi f_{D_n} t - \phi_0$

$\phi_n(t+\tau) = 2\pi f_c \tau_n - 2\pi f_{D_n}(t+\tau) - \phi_0$

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$$= \sum_n E[\alpha_n^2]\, E[0.5 \cos 2\pi f_{Dn} \tau]$$

$$+ \sum_n E[\alpha_n^2] \cdot E[0.5 \cos(4\pi f_c \tau_n - 4\pi f_{Dn} t - 2\pi f_{Dn} \tau - 2\phi_n)]$$

$$= 0.5 \sum_n E[\alpha_n^2] \cdot \cancel{E} \cos 2\pi f_{Dn} \tau \cancel{]}$$

$$f_{Dn} = \frac{v}{\lambda} \cdot \cos\theta_n$$

$$= 0.5 \sum_n E[\alpha_n^2] \cdot \cancel{E} \cos(2\pi \cancel{f_{Dn}} \frac{v \cdot \tau}{\lambda} \cos\theta_n) \cancel{]}$$



$A_{r_I}(t, t+\tau)$ depends only on $\tau$.

$\Rightarrow r_I(t)$ is WSS

$A_{r_I}(t, t+\tau) = A_{r_I}(\tau)$

⑤ Similarly, $A_{r_Q}(t, t+\tau)$ is also WSS

$$A_{r_Q}(t, t+\tau) = A_{r_Q}(\tau) = A_{r_I}(\tau)$$

⑥ Cross-correlation of $r_I(t)$, $r_Q(t)$

$A_{r_I, r_Q}(t, t+\tau) = E[r_I(t) \cdot r_Q(t+\tau)]$

$= \cdots\cdots$

Uniform Scattering Environment.

$$\theta_n = n \cdot \Delta\theta, \quad \Delta\theta = 2\pi/N. \quad E[\alpha_n^2] = 2P_r/N$$

$$\Rightarrow A_{r_I}(\tau) = \frac{P_r}{N} \sum_{n=1}^{N} \cos\left(2\pi \frac{v\tau}{\lambda} \cdot \cos(n \cdot \Delta\theta)\right)$$

$$N = 2\pi/\Delta\theta \Rightarrow \frac{1}{N} = \frac{\Delta\theta}{2\pi}$$

$$\Rightarrow A_{r_I}(\tau) = \frac{P_r}{2\pi} \sum_{n=1}^{N} \cos\left(\frac{2\pi v\tau}{\lambda} \cdot \cos(n \cdot \Delta\theta)\right) \cdot \Delta\theta$$

$$N \to \infty \qquad f_D = \frac{v}{\lambda}$$

$$= \frac{P_r}{2\pi} \int_0^{2\pi} \cos(2\pi f_D \tau \cos\theta)\, d\theta$$

$$= P_r J_0(2\pi f_D \tau)$$

$\uparrow$

Bessel function of zeroth order

Autocorrelation is 0 for $f_D \tau = 0.4$

$v\tau = 0.4\lambda$.

3.2.2. Envelope and Power Distributions

$u(t) = e^{j\phi_0} \Leftrightarrow s(t) = \cos\phi_0 \cos 2\pi f_c t - \sin\phi_0 \sin 2\pi f_c t$

$r(t) = r_I(t) \cdot \cos 2\pi f_c t + r_Q(t) \cdot \sin 2\pi f_c t.$

$r_I(t) = \sum_{n=1}^{N(t)} \alpha_n(t) \cdot \cos\phi_n(t)$

$r_Q(t) = \sum_{n=1}^{N(t)} \alpha_n(t) \cdot \sin\phi_n(t)$

$\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{D_n}(t) - \phi_0$

Baseband: $r_I(t) - j\, r_Q(t)$

$z(t) \triangleq \sqrt{r_I^2(t) + r_Q^2(t)}$

signal envelope

Average received power $\bar{P_r}$

$= E\, r_I^2(t) + E\, r_Q^2(t)$

$= E\left[\sum_{n=1}^{N(t)} \alpha_n^2(t)\right] = \sum_{n=1}^{N(t)} E\, \alpha_n^2(t)$

$\Delta$ No LoS. - Rayleigh fading

$r_I(t) \sim N(0, \sigma^2) \Rightarrow \bar{P_r} = 2\sigma^2.$

$r_Q(t) \sim N(0, \sigma^2)$

$z(t) \sim$ ~~Raylen~~ Rayleigh.

PDF: $p_z(z) = \frac{z}{\sigma^2} e^{-z^2/2\sigma^2}$ , $z \geq 0$.

PDF of $z^2(t)$ received power

$$p_{z^2}(x) = \frac{1}{2\sigma^2} e^{-x/2\sigma^2}$$

$$= \frac{1}{\bar{P}_r} e^{-x/\bar{P}_r}$$

exponential distribution with expectation

$\bar{P}_r = 2\sigma^2$

$\Delta$ Fixed LoS + NLoS : Rician fading.