2.4 Ray Tracing

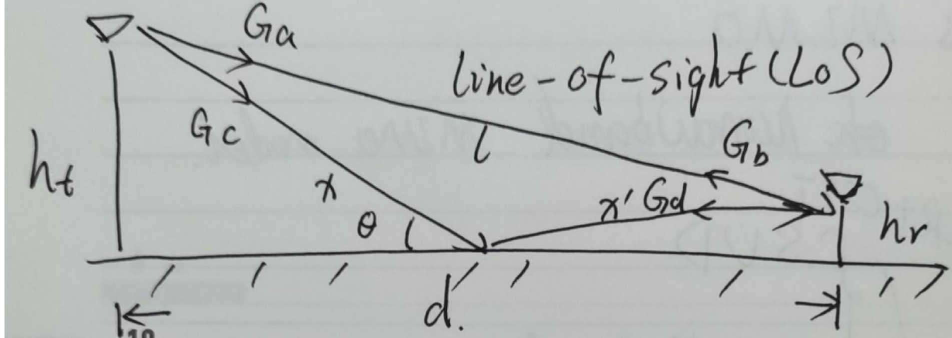
Use finite # of rays to approximate the signal propagation from Tx to Rx

$\Rightarrow$ The received signal is the summation of the signals along the rays

$\Rightarrow$ Scenario-dependent.

2.4.1 Two-Ray Model.

Open space : LoS + Single reflection off the ground.



10 廿九
Monday / 星期一

$$V(t) = u_1(t) + u_2(t)$$

$u_1(t)$: Free-space model.

$$= \frac{\sqrt{G_l}\cdot\lambda}{4\pi l} e^{-j2\pi l/\lambda} \cdot u(t)$$

$$\sqrt{G_l} = \sqrt{G_a G_b}$$

11 三十
Tuesday / 星期二

$u_2(t)$: ~~Ignore the reflection loss~~

$$= \frac{\lambda R \sqrt{G_r}\, e^{-j2\pi(x+x')/\lambda}}{4\pi(x+x')} u(t)$$

$$\sqrt{G_r} = \sqrt{G_c G_d}$$

12 四月
Wednesday / 星期三

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$$V(t) = \frac{\lambda}{4\pi}\left[\frac{\sqrt{G_l}}{l} e^{-j2\pi l/\lambda} + \frac{R\sqrt{G_r}}{x+x'} e^{-j2\pi(x+x')/\lambda}\right] \cdot u(t)$$

Delay spread: $x+x'-l/c$

5 MAY

~~Linear~~ Path ~~Loss~~ Gain

$$P_G = \frac{P_r}{P_t} = \cancel{\left|\frac{\sqrt{G_l}}{l} + \frac{R\sqrt{G_r}}{x+x'} e^{-j\Delta\phi}\right|^2 e^{-j2\pi l/\lambda}}$$

$$= \frac{\frac{1}{2}\frac{1}{T}\int_0^T |v(t)|^2 dt}{\frac{1}{2}\frac{1}{T}\int_0^T |u(t)|^2 dt}$$

$$= \left(\frac{\lambda}{4\pi}\right)^2 \left|e^{-j2\pi l/\lambda}\right|^2 \left|\frac{\sqrt{G_l}}{l} + \frac{R\sqrt{G_r}}{x+x'} e^{-j\Delta\phi}\right|^2$$

where $\Delta\phi = 2\pi(x+x'-l)/\lambda$.

If $\theta \to 0$, $x+x' \approx l \approx d$, $R \approx -1$, $G_r \approx G_l$.

$$\approx \left(\frac{\lambda}{4\pi}\right)^2 \frac{G_l}{d^2} \left|1-e^{-j\Delta\phi}\right|^2$$

$$\approx \left(\frac{\lambda}{\quad}\right)^2 G \quad |1-(1-j\Delta\phi)|^2 = \left(\frac{\lambda}{4\pi d}\right)^2 G_l \cdot \Delta\phi^2$$

$$V(t) = \frac{\lambda}{4\pi}\left[\frac{\sqrt{G_l}}{l} e^{-j2\pi l/\lambda} + \frac{R\sqrt{G_r}}{x+x'} e^{-j2\pi(x+x')/\lambda}\right]$$

$$\cdot u(t)$$

Delay spread: $x+x'-l/c$

~~Linear~~ Path ~~Loss~~ Gain

$$P_G = \frac{P_r}{P_t} \cancel{= \left|\frac{\sqrt{G_l}}{l} + \frac{R\sqrt{G_r}}{x+x'} e^{-j\Delta\phi}\right|^2 e^{-j2\pi l/\lambda}}$$

$$= \frac{\frac{1}{2}\frac{1}{T}\int_0^T |v(t)|^2 dt}{\frac{1}{2}\frac{1}{T}\int_0^T |u(t)|^2 dt}$$

$$= \left(\frac{\lambda}{4\pi}\right)^2 \left|e^{-j2\pi l/\lambda}\right|^2 \left|\frac{\sqrt{G_l}}{l} + \frac{R\sqrt{G_r}}{x+x'} e^{-j\Delta\phi}\right|^2$$

where $\Delta\phi = 2\pi(x+x'-l)/\lambda$.

If $\theta \to 0$, $x+x' \approx l \approx d$. $R \approx -1$, $G_r \approx G_l$.

$$\approx \left(\frac{\lambda}{4\pi}\right)^2 \frac{G_l}{d^2} \left|1 - e^{-j\Delta\phi}\right|^2$$

$$\approx \left(\frac{\lambda}{4\pi d}\right)^2 G_l \left|1-(1-j\Delta\phi)\right|^2 = \left(\frac{\lambda}{4\pi d}\right)^2 G_l \cdot \Delta\phi^2$$

$$x + x' - l = \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$

$$\approx d\left(1 + \frac{1}{2}\left(\frac{h_t + h_r}{d}\right)^2\right) - d\left(1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2\right)$$

$$= \frac{2h_t h_r}{d}$$

$$\Delta\phi \approx \frac{4\pi h_t h_r}{d\lambda}.$$

$$\Rightarrow P_G \approx \left(\frac{\lambda}{4\pi d}\right)^2 \cdot G_l \cdot \left(\frac{4\pi h_t h_r}{d\lambda}\right)^2.$$

$$\Rightarrow \frac{P_r}{P_t} \approx \left(\frac{h_t h_r \sqrt{G_l}}{d^2}\right)^2$$

$$P_r\,\mathrm{dBm} = P_t\,\mathrm{dBm} + 10\log_{10} G_l + 20\log_{10}(h_t h_r)$$
$$- 40\log_{10} d.$$

Measure and average $\frac{P_r}{P_t}$ versus distance d in a number of typical ~~scen~~ scenarios. e.g., NYC, SF, and etc.

$\begin{cases} \text{Hata Model.} \\ \text{COST 231 Extension} \end{cases}$

urban, suburban, indoor.
Macrocell, Microcell.

2.6. Simplified Path-Loss Model.

Let K be the path gain at a distance d_0,

$$P_r = P_t \, K \cdot \left[\frac{d_0}{d}\right]^{\gamma}.$$

γ: path-loss exponent.

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$$P_r\,\mathrm{dBm} = P_t\,\mathrm{dBm} + K\,\mathrm{dB} - 10\gamma \log_{10}\left[\frac{d}{d_0}\right]$$

where

$$K\,\mathrm{dB} = 10 \cdot \log_{10} K.$$

Free-space : $\gamma = 2$

Two-Ray : $\gamma = 4$

2.7 Shadow Fading

Without Shadowing : $P_L = \dfrac{P_t}{P_r}$

With Shadowing : $\cancel{P_L} \cdot \psi = \dfrac{P_t}{P_r}$

constant PL X

random SH

ψ follows a log-normal distribution:

Let $\psi_{dB} = 10 \cdot \log_{10} \psi = 10 \log_{10} P_t / P_r$.

$$p(\psi_{dB}) = \frac{1}{\sqrt{2\pi}\,\sigma_{\psi_{dB}}} \exp\left[- \frac{(\psi_{dB} - \mu_{\psi_{dB}})^2}{2\sigma^2_{\psi_{dB}}} \right]$$

where $\mu_{\psi_{dB}} = E[\psi_{dB}] = E[10 \log_{10} \psi]$

$$\sigma^2_{\psi_{dB}} = E(\psi_{dB} - \mu_{\psi_{dB}})^2.$$

Let $\psi = P_L \cdot P_{SH}$

P_L: path-loss; P_{SH}: shadowing

$$\mu_{\psi_{dB}} = E[10 \cdot \log_{10} \psi]$$

$$= 10 \cdot \log_{10} \cdot P_L + E[10 \cdot \log_{10} \cdot P_{SH}]$$

$$= P_L \, dB + \text{Average shadowing loss(dB)}$$

Remark:

$$\mu_{\psi} \triangleq E[\psi] = P_L \cdot E[P_{SH}]$$

$$10 \cdot \log_{10} \mu_{\psi} = 10 \cdot \log_{10} P_L + 10 \cdot \log_{10} E[P_{SH}]$$

$$\neq \mu_{\psi_{dB}}$$

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2.8. Combined PL and SH

$$\frac{P_r}{P_t}\,dB = -\frac{P_t}{P_r}\,dB = -\psi_{dB}$$

$$= \underbrace{-\mu_{\psi_{dB}}}_{\#\text{ Simplified PL model.}} - \underbrace{(\psi_{dB} - \mu_{\psi_{dB}})}$$

$$= K\,dB - 10\gamma' \log \frac{d}{d_0} - \boxed{\psi'_{dB}}$$

$$= 10 \cdot \log_{10} K - 10\gamma' \log_{10} \frac{d}{d_0} - \psi'_{dB}$$

γ' : ~~path-loss~~; path-loss (and average shadowing) exponent.

$\psi'_{dB} \sim N(0, \sigma^2_{\psi_{dB}})$