Review:

$$c(\tau, t) = \sum_n \alpha_n(t)\,\delta(\tau - \tau_n(t))\, e^{-j\phi_n(t)}$$

$$WB: \ T_m \underset{\gg}{\approx} B^{-1}$$

22 初六
Thursday / 星期四

$$A_c(\tau_1, \tau_2; t, t+\Delta t)$$

$$\Downarrow \ WSS$$

$$A_c(\tau_1, \tau_2; \Delta t)$$

$$\Downarrow \ US$$

23 霜降
Friday / 星期五

$$A_c(\tau_1 \tau_2; \Delta t) = A_c(\tau_1; \Delta t)\, \delta(\tau_1 - \tau_2)$$

24 初八
Saturday / 星期六

$$A_c(\tau_1 \tau_1; \Delta t) = A_c(\tau_1; \Delta t) \cdot \delta(0)$$

$$A_c(\tau; \Delta t) = \sum_n E\left[\alpha_n^2(t) \cdot \delta(\tau - \tau_n(t))\right] e^{j2\pi f_{D_n}\Delta t}$$

25 重阳节
Sunday / 星期日

$$S_c(\tau, \rho) = F\left\{A_c(\tau, \Delta t)\right\}$$

$S_c(\tau,\rho)$ shows the Dopple frequencies in the paths with delay τ.

3.3.1
Power Delay Profile.

Let $\Delta t=0$ in $A_c(\tau;\Delta t)$, denote as $A_c(\tau)$

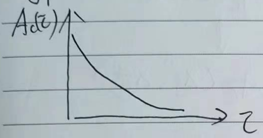
Thus, $A_c(\tau) = A_c(\tau;0)$ Power Delay Profile

$$A_c(\tau) = E[C^*(\tau,t)\,C(\tau,t)]/\delta(0)$$
$$= \sum_{n=1}^{N(t)} E[\alpha_n^2(t)\,\delta(\tau-\tau_n(t))]$$
$$\underbrace{= \sum_{n=1}^{N(t)} \underbrace{E[\alpha_n^2(t)]}_{\text{Average power (gain) of nth path}} \cdot \underbrace{E[\delta(\tau-\tau_n(t))]}_{\text{Chance of delay } =\tau}}_{\text{Average channel power intensity at delay } \tau}$$

Average power (gain) of nth path ; Chance of delay $=\tau$.

Average channel power ~~gain~~ intensity at delay τ.

Typical $A_c(\tau)$ curve.



~~α~~ Average delay spread.

$$\mu_{T_m} = \frac{\int_0^\infty \tau A_c(\tau)\,d\tau}{\int_0^\infty A_c(\tau)\,d\tau} \quad \text{(like mean)}$$

RMS delay spread

$$\sigma_{T_m} = \sqrt{\frac{\int_0^\infty (\tau-\mu_{T_m})^2 A_c(\tau)\,d\tau}{\int_0^\infty A_c(\tau)\,d\tau}} \quad (\sigma_{T_m}^2 \text{ like variance})$$

3.3.2

△ Coherent Bandwidth:

How flat of the frequency selective fading

$$C(f,t) = \int_{-\infty}^{+\infty} c(\tau, t) e^{-j2\pi f\tau} d\tau.$$

Frequency domain autocorrelation function.

$A_{\bar{C}}(f_1, f_2; t, t+\Delta t)$

WSS

$= A_{\bar{C}}(f_1, f_2; \Delta t)$

$= E[C^*(f_1, t)\, C(f_2, t+\Delta t)]$

$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E[c^*(\tau_1, t)\, c(\tau_2, t+\Delta t)] e^{j2\pi f_1\tau_1}$

$e^{-j2\pi f_2\tau_2} d\tau_1 d\tau_2$

uncorrelated scattering

$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A_c(\tau_1, \Delta t)\cdot\delta[\tau_1-\tau_2] e^{-j2\pi(f_2\tau_2 - f_1\tau_1)}$

$d\tau_1 d\tau_2$

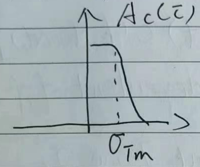
$= \int_{-\infty}^{+\infty} A_c(\tau_1, \Delta t) e^{-j2\pi\underbrace{(f_2-f_1)}_{\Delta f}\tau_1} d\tau_1 \triangleq A_{\bar{C}}(\Delta f, \Delta t)$

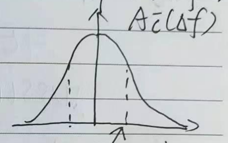
$A_{\bar{C}}(\Delta f, \Delta t)$: Fourier Transform of $A_c(\tau, \Delta t)$ w.r.t. τ

Let $A_{\tilde{c}}(\Delta f) = A_{\tilde{c}}(\Delta f; 0)$

$A_c(\tau) = A_c(\tau; 0)$.

$A_{\tilde{c}}(\Delta f)$ is the Fourier transform of $A_c(\tau)$





B_c : coherent bandwidth.

$$B_c = k/\sigma_{Tm}$$

$A_c(\tau) \approx 0$, for $\tau \geq \sigma_{Tm}$ $\quad$ $A_{\tilde{c}}(\Delta f) \approx 0$. for $\Delta f > B_c$

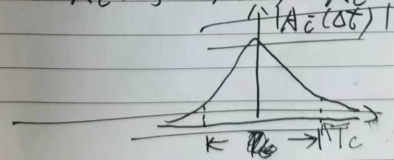
Narrowband : $B \ll B_c$

$\Rightarrow 1/T \ll 1/\sigma_{Tm}$

$\Rightarrow T \gg \sigma_{Tm}$.

~~3.3.3 Doppler Power Spectrum~~

~~Channel coherent time can be observed from $A_{\tilde{c}}(\Delta f = 0, \Delta t) \triangleq A_{\tilde{c}}(\Delta t)$~~



3.3.3. Doppler Power Spectrum.

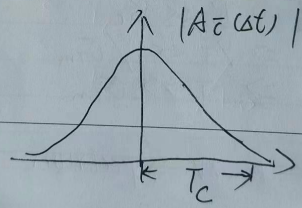
$$A_{\bar{c}}(\Delta f=0, \Delta t) \triangleq A_{\bar{c}}(\Delta t)$$

$$= E[C^*(f,t)\, C(f, t+\Delta t)]$$

WSSUS $\Rightarrow$ independent of f, t.

Define Channel Coherent Time T_c as.

$$\begin{cases} \Delta t \le T_c \Rightarrow A_{\bar{c}}(\Delta t) > 0 \\ \Delta t > T_c \Rightarrow A_{\bar{c}}(\Delta t) \approx 0. \end{cases}$$



Fourier transform of $A_{\bar{\tau}}(\Delta f, \Delta t)$. w.r.t. Δt

$$\int_{-\infty}^{+\infty} A_{\bar{\tau}}(\Delta f, \Delta t)\, e^{-j2\pi\rho\Delta t} \cdot d\Delta t$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A_c(\tau, \Delta t)\, e^{-j2\pi\Delta f\cdot\tau}\, e^{-j2\pi\rho\Delta t}\, d\tau \cdot d\Delta t$$

$$= \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A_c(\tau, \Delta t)\, e^{-j2\pi\rho\Delta t}\, d\Delta t\, e^{-j2\pi\Delta f\tau}\, d\tau.$$

$$= \int_{-\infty}^{+\infty} \underbrace{S_c(\tau, \rho)}_{\text{scattering function.}} \cdot e^{-j2\pi\Delta f\tau}\, d\tau$$

$$\triangleq S_{\bar{\tau}}(\Delta f, \rho)$$

$$\begin{tikzcd}
S_{\bar{\tau}}(\Delta f, \rho) \arrow[d, leftrightarrow, "F"] & & \\
A_{\bar{\tau}}(\Delta f, \Delta t) & & S_c(\tau, \rho) \arrow[ull, bend right, leftrightarrow, "F"'] \\
& A_c(\tau, \Delta t) \arrow[ul, leftrightarrow, "F"] \arrow[ur, leftrightarrow, "F"'] & \\
& A_c(\tau_1, \tau_2; t_1, t_2) \arrow[u, Rightarrow, "\text{WSS. uncorrelated scattering}"'] &
\end{tikzcd}$$