4-1. Capacity in AWGN is given by $C = B\log_2(1 + P/N_0B)$, where P is the received signal power, B is the signal bandwidth, and $N_0/2$ is the noise PSD. Find capacity in the limit of infinite bandwidth $B \to \infty$ as a function of P.

$$C = B\log_2\left(1+\frac{P}{N_0B}\right) = \frac{\log_2\left(1+\frac{P}{N_0B}\right)}{\frac{1}{B}}$$

$B \to \infty$. $C = \frac{P}{N_0} \cdot \ln 2$.

4-2. Consider an AWGN channel with bandwidth 50 MHz, received signal power 10 mW, and noise PSD $N_0/2$ where $N_0 = 2\cdot 10^{-9}$ W/Hz. How much does capacity increase by doubling the received power? How much does capacity increase by doubling the channel bandwidth?

由题、$B = 50\,\text{MHz}$, $P_r = 10\,\text{mW}$, $N_0 = 2\times10^{-9}\,\text{W/Hz}$,

$$\therefore C = B\log_2\left(1+\frac{P}{N_0B}\right) = 50\times10^{6}\log_2\left(1+\frac{10\times10^{-3}}{2\times10^{-9}\times50\times10^{6}}\right) = 6.875\,\text{Mbps}$$

① $P_r' = 20\,\text{mW}$

$$C = B\log_2\left(1+\frac{P}{N_0B}\right) = 50\times10^{6}\log_2\left(1+\frac{20\times10^{-3}}{2\times10^{-9}\times50\times10^{6}}\right) = 13.15\,\text{Mbps}$$

② $B' = 100\,\text{MHz}$

$$C = B\log_2\left(1+\frac{P}{N_0B}\right) = 100\times10^{6}\log_2\left(1+\frac{10\times10^{-3}}{2\times10^{-9}\times100\times10^{6}}\right) = 7.039\,\text{Mbps}$$

4-3. Consider two users simultaneously transmitting to a single receiver in an AWGN channel. This is a typical scenario in a cellular system with multiple users sending signals to a base station. Assume the users have equal received power of 10 mW and total noise at the receiver in the bandwidth of interest of 0.1 mW. The channel bandwidth for each user is 20 MHz.

(a) Suppose that the receiver decodes user 1's signal first. In this decoding, user 2's signal acts as noise (assume it has the same statistics as AWGN). What is the capacity of user 1's channel with this additional interference noise?

(b) Suppose that, after decoding user 1's signal, the decoder re-encodes it and subtracts it out of the received signal. Now, in the decoding of user 2's signal, there is no interference from user 1's signal. What then is the Shannon capacity of user 2's channel?

$B = 20\,\text{MHz}$. $P = 10\,\text{mW}$. $P_{noise} = 0.1\,\text{mW}$.

(a) $C = B\log_2\left(1+\frac{P}{N_0B}\right)$. $\quad N_0 = \frac{P + P_{noise}}{B}$

$$\therefore C_1 = B\log_2\left(\frac{2P + P_{noise}}{P + P_{noise}}\right) = 20\times10^6 \log_2\left(\frac{20.1}{10.1}\right) = 19.86\,\text{Mbps}.$$

(b) $$C_{总} = B\log_2\left(1 + \frac{2P}{P_{noise}}\right) = 20\times10^6 \log_2(1+200) = 152.02\,\text{Mbps}$$

$$\therefore C_2 = C_{总} - C_1 = 132.16\,\text{Mbps}$$