Special case: $c(\tau, t_1) = c(\tau, t_2) = c(\tau)$

Time-invariant channel.

$$c(\tau) = \sum_{n=0}^{N} \alpha_n e^{-j\phi_n} u(t-\tau_n)$$

Delay spread: $T_m = \max_n [\tau_n - \tau_0]$

3.2 Narrowband

$$\sum_{n=0}^{N(t)} \alpha_n(t) \cdot e^{-j\phi_n(t)} \cdot u(t-\tau_n(t))$$

Narrowband $B^{-1} \gg T_m$

$$\approx \underbrace{\left(\sum_{n=0}^{N(t)} \alpha_n(t) \cdot e^{-j\phi_n(t)}\right)} \cdot u(t)$$

Complex Gaussian.

① Large $N(t) \Rightarrow$ CLT

② Small $N(t)$ + Rayleigh Amplitude + uniform $\phi_n(t)$

Gaussian process + j Gaussian process.

Bandpass: $s(t) = \cos(2\pi f_c t + \phi_0) = Re\{ e^{j\phi_0} \cdot e^{j2\pi f_c t} \}$

Baseband: $u(t) = e^{j\phi_0}$

Received signal in bandpass:

$$r(t) = Re\{ \underbrace{\sum_{n=1}^{N(t)} \alpha_n(t) \cdot e^{-j\phi_n(t)} \cdot e^{j\phi_0}}_{\text{Received signal in baseband.}} \cdot e^{j2\pi f_c t} \}$$

$$= Re\{ \sum_{n=1}^{N(t)} \alpha_n(t)\, e^{j(2\pi f_c t - \phi_n(t) + \phi_0)} \}$$

$$= \sum_{n=1}^{N(t)} \alpha_n(t) \cdot \cos(2\pi f_c t - \phi_n(t) + \phi_0)$$

$$= \sum_{n=1}^{N(t)} \alpha_n(t) \cdot \cos[2\pi f_c t - (\underbrace{2\pi f_c \tau_n(t) - \phi_{D_n} - \phi_0}_{\text{Defined as new } \phi_n(t)})]$$

$$= \sum_{n=1}^{N(t)} \alpha_n(t) \cdot \cos(2\pi f_c t - \phi_n(t))$$

$$= \sum_{n=1}^{N(t)} \alpha_n(t) \cdot \cos\phi_n(t) \cdot \cos 2\pi f_c t$$

$$+ \sum_{n=1}^{N(t)} \alpha_n(t) \cdot \sin\phi_n(t) \cdot \sin 2\pi f_c t$$

$$r_I(t) = \sum_{n=1}^{N(t)} \alpha_n(t)\cdot \cos\phi_n(t)$$

$$r_Q(t) = \sum_{n=1}^{N(t)} \alpha_n(t)\cdot \sin\phi_n(t)$$

where $\phi_n(t) = 2\pi f_c \tau_n(t) - \phi_{D_n} - \phi_0$

3.2.1. Autocorrelation, Cross-Correlation, PSD

Assumption: $\alpha_n(t) \approx \alpha_n$, $\tau_n(t) \approx \tau_n$

f_{D_n} is quasi-static.

No LoS, $\phi_n(t)$ is uniformly dist.

$$\phi_n(t) = 2\pi f_c \tau_n - 2\pi f_{D_n} t - \phi_0$$

$$\begin{cases} r_I(t) = \sum_n \alpha_n \cdot \cos\phi_n(t) \\ r_Q(t) = \sum_n \alpha_n \cdot \sin\phi_n(t) \end{cases}$$

① $E[r_I(t)] = \sum_n E[\alpha_n] \cdot \underbrace{E[\cos\phi_n(t)]}_{=0}$

as $\phi_n(t)$ is uniformly distributed on $[-\pi, \pi]$ (No LoS)

$= 0.$

② Similarly, $E[r_Q(t)] = 0$

③ $E[r_I(t) \cdot r_Q(t)]$

$= E\left[\sum_n \alpha_n \cos\phi_n(t) \cdot \sum_m \alpha_m \sin\phi_m(t)\right]$

$= \sum_m \sum_n E[\alpha_m \alpha_n] \, E[\cos\phi_n(t) \cdot \sin\phi_m(t)]$

$= \sum_n E[\alpha_n^2] \cdot E[\cos\phi_n(t) \cdot \sin\phi_n(t)]$

$= \sum_n E[\alpha_n^2] \cdot E\left[\frac{1}{2}\sin 2\phi_n(t)\right]$

$= 0$

$\Rightarrow$ $r_I(t)$ and $r_Q(t)$ are uncorrelated

If $r_I(t)$ and $r_Q(t)$ are Gaussian, they are independent.

④ $A_{r_I}(t, t+\tau) = E[r_I(t) \, r_I(t+\tau)]$

$= \sum_n E[\alpha_n^2] \cdot E\{\cos\phi_n(t) \cdot \cos\phi_n(t+\tau)\}$

$\phi_n(t) = 2\pi f_c \tau_n - 2\pi f_{D_n} t - \phi_0$

$\phi_n(t+\tau) = 2\pi f_c \tau_n - 2\pi f_{D_n}(t+\tau) - \phi_0$