6-1. Consider a system in which data is transferred at a rate of 100 bits per second over the channel.

(a) Find the symbol duration if we use a sinc pulse for signaling and the channel bandwidth is 10 kHz.

(b) Suppose the received SNR is 10 dB. Find the SNR per symbol and the SNR per bit if 4-QAM is used.

(c) Find the SNR per symbol and the SNR per bit for 16-QAM, and compare with these metrics for 4-QAM.

(a) ∵ for a sinc pulse, $p(t) = \frac{\sin(\pi t/T_{sym})}{\pi t/T_{sym}} \Leftrightarrow P(f) = \begin{cases} T_{sym}, & \frac{-F_{sym}}{2} \le f < \frac{F_{sym}}{2} \\ 0, & \text{otherwise} \end{cases}$ $B = \frac{F_{sym}}{2}$, $F_{sym} = \frac{1}{T_{sym}}$

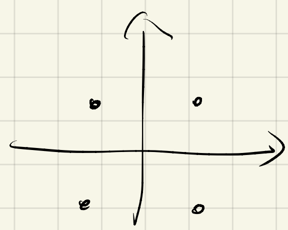
$\therefore B = \frac{1}{2T_{sym}} \Rightarrow T_{sym} = \frac{1}{2B} = 5 \times 10^{-5}$ s.

(b) $SNR_r = 10\,dB = \frac{P_o}{N_o B}$

∵ 4QAM . $E_b = \frac{E_s}{\log_2 4} = \frac{E_s}{2}$

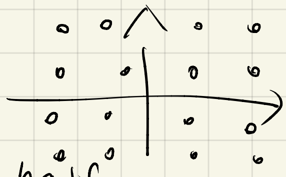
$\therefore$ SNR per symbol $= \frac{E_s}{N_o} = \frac{E_s}{N_o B T_s} = \frac{P_o}{2N_o B} = 5$.

SNR per bit $= \frac{5}{2}$



(C) ∵ 16 QAM , $E_b = \frac{E_s}{\log_2 16} = \frac{E_s}{4}$

$\therefore$ SNR per symbol $= 5$. SNR per bit $= \frac{5}{4}$



compare to 4QAM. SNR per symbol is the same. and SNR per bit is half.

6-2. Consider BPSK modulation where the a priori probability of 0 and 1 is not the same. Specifically, $p(s_n = 0) = 0.3$ and $p(s_n = 1) = 0.7$.

(a) Find the probability of bit error P_b in AWGN assuming we encode a 1 as $s_1(t) = A\cos(2\pi f_c t)$ and a 0 as $s_2(t) = -A\cos(2\pi f_c t)$ for $A > 0$, assuming the receiver structure is as shown in Figure 5.17.

(b) Suppose you can change the threshold value in the receiver of Figure 5.17. Find the threshold value that yields equal error probability regardless of which bit is transmitted – that is, the threshold value that yields $p(\hat{m} = 0 \mid m = 1)p(m = 1) = p(\hat{m} = 1 \mid m = 0)p(m = 0)$.

(c) Now suppose we change the modulation so that $s_1(t) = A\cos(2\pi f_c t)$ and $s_2(t) = -B\cos(2\pi f_c t)$. Find $A > 0$ and $B > 0$ so that the receiver of Figure 5.17 with threshold at zero has $p(\hat{m} = 0 \mid m = 1)p(m = 1) = p(\hat{m} = 1 \mid m = 0)p(m = 0)$.

(d) Compute and compare the expression for P_b in parts (a), (b), and (c) assuming $E_b/N_0 = 10$ dB and $N_0 = .1$. For which system is P_b minimized?

(a)

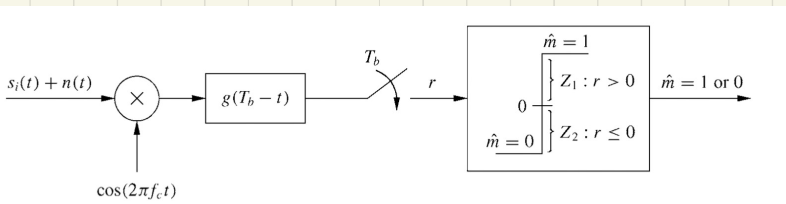


Figure 5.17: Coherent demodulator for BPSK.

$P_e = p(S=0)Pr(1 \text{ detected}, 0\text{sent}\text{-}0\text{sent})$
$+ p(S=1)\, Pr(0 \text{ detected}, 1\text{sent}\text{-}1\text{sent})$
$= 0.3\,Q\left(\frac{2A}{\sqrt{2N_o}}\right) + 0.7\,Q\left(\frac{2A}{\sqrt{2N_o}}\right)$
$= Q\left(\frac{2A}{\sqrt{2N_o}}\right)$

(b) ∵ $p(\hat{m}=0 \mid m=1)\,p(m=1) = p(\hat{m}=1 \mid m=0)\,p(m=0)$

∴ $0.7\,Q\left(\frac{2(A+a)}{\sqrt{2N_0}}\right) = 0.3\,Q\left(\frac{2(A-a)}{\sqrt{2N_0}}\right)$

(c) $0.7\,Q\left(\frac{2A}{\sqrt{2N_0}}\right) = 0.3\,Q\left(\frac{2B}{\sqrt{2N_0}}\right)$

(d) $\frac{E_b}{N_0} = 10\,dB$, $N_0 = 0.1$ ∵ $A = \sqrt{E_b}$ ∴ $A = (10N_0)^{2} = 1$

对(a), $P_e = Q\left(\frac{2A}{\sqrt{2N_0}}\right) = Q(2\sqrt{5}) = \int_{2\sqrt{5}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}}\,dt = 3.87\times10^{-6}$

对(b). $P_e = 0.7\,Q[\sqrt{5}(1+a)] + 0.3\,Q[\sqrt{5}(1-a)]$, ∵ 可解出 $a = 0.0203$

∴ $P_e = 1.4\,Q[\sqrt{5}(1+a)] = 3.53\times10^{-6}$

对(c). $P_e = 1.4\,Q\left(\frac{2A}{\sqrt{2N_0}}\right) = 1.4 P_e \text{ in (a)} = 5.42\times10^{-6}$

∴ (b) 最好

6-3. Consider a BPSK receiver whose demodulator has a phase offset of ϕ relative to the transmitted signal, so for a transmitted signal $s(t) = \pm g(t)\cos(2\pi f_c t)$ the carrier in the demodulator of Figure 5.17 is $\cos(2\pi f_c t + \phi)$. Determine the threshold level in the threshold device of Figure 5.17 that minimizes probability of bit error, and find this minimum error probability.

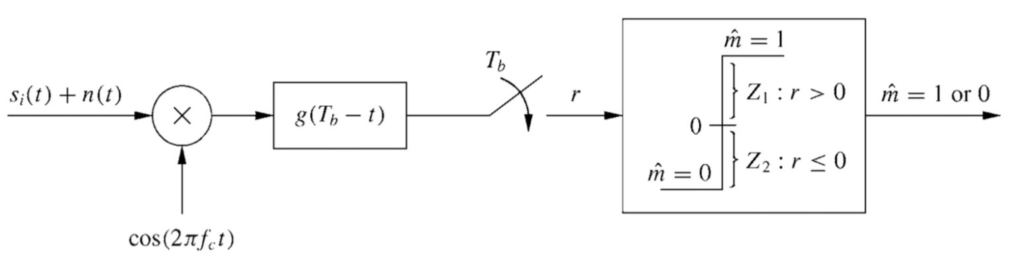


Figure 5.17: Coherent demodulator for BPSK.

$s(t) = \pm g(t)\cos(2\pi f_c t)$ ∴ $A = 1$ (假设 $A > 0$)

$s_1 = 1$, $s_2 = -1$ $-1 \quad 0 \quad 1$ Error: $n < -1$, $n > 1$

∵ 有 phase offset ϕ. ∴ $r = \hat{r}\cos\Delta\phi$.

$P_{e\,min} = Q\left(\frac{d_{min}\cos\Delta\phi}{\sqrt{2N_0}}\right)$