4-4. Consider a flat fading channel of bandwidth 20 MHz and where, for a fixed transmit power $\bar{P}$, the received SNR is one of six values: $\gamma_1 = 20$ dB, $\gamma_2 = 15$ dB, $\gamma_3 = 10$ dB, $\gamma_4 = 5$ dB, $\gamma_5 = 0$ dB, and $\gamma_6 = -5$ dB. The probabilities associated with each state are $p_1 = p_6 = .1$, $p_2 = p_4 = .15$, and $p_3 = p_5 = .25$. Assume that only the receiver has CSI.

(a) Find the Shannon capacity of this channel.

(b) Plot the capacity versus outage for $0 \le P_{\text{out}} < 1$ and find the maximum average rate that can be correctly received (maximum C_{out}).

(a) $C = B\left[\sum_{i=1}^{6} \log_2(1+\gamma_i)\,p(\gamma_i)\right] = B\left[0.1\log_2(1+10^2) + 0.15\log_2(1+10^{1.5}) + 0.25\log_2(1+10^1) + 0.15\log_2(1+10^{0.5}) + 0.25\log_2(1+10^0) + 0.1\log_2(1+10^{-0.5})\right]$

$= 0.2883 \times 20 \times 10^6 = 57.66$ Mbps

(b) $P_{out} = \Pr(\gamma < \gamma_{min})$. $C_o = (1-P_{out})B\log_2(1+\gamma_{min})$.

$\therefore \gamma_{min} > 20$ dB. $P_{out} = 1$. $C_o = 0$.

$15\text{ dB} < \gamma_{min} < 20\text{ dB}$. $P_{out} = 0.9$. $C_o = 0.1B\log_2(1+\gamma_{min})$ $C_{o\,max} = 13.32$ Mbps.

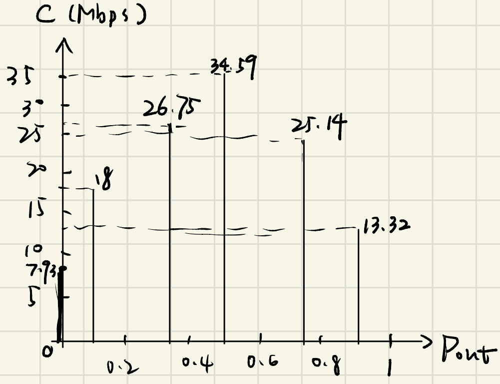
$10\text{ dB} < \gamma_{min} < 15\text{ dB}$. $P_{out} = 0.75$. $C_o = 0.25B\log_2(1+\gamma_{min})$ $C_o = 25.14$ Mbps.

$5\text{ dB} < \gamma_{min} < 10\text{ dB}$. $P_{out} = 0.5$. $C_o = 0.5B\log_2(1+\gamma_{min})$ $C_o = 34.59$ Mbps

$0\text{ dB} < \gamma_{min} < 5\text{ dB}$. $P_{out} = 0.25$. $C_o = 0.75B\log_2(1+\gamma_{min})$ $C_o = 26.75$ Mbps

$-5\text{ dB} < \gamma_{min} < 0\text{ dB}$. $P_{out} = 0.1$. $C_o = 0.9B\log_2(1+\gamma_{min})$. $C_o = 7.93$ Mbps

$\gamma_{min} < -5\text{ dB}$. $P_{out} = 0$. $C_o = B\log_2(1+\gamma_{min})$



$\therefore$ when $\gamma_{min} = 10$ dB. $P_{out} = 0.5$.

C_o has the maximum $= 34.59$ Mbps.

4-6. Consider a cellular system where the power falloff with distance follows the formula $P_r(d) = P_t(d_0/d)^\alpha$, where $d_0 = 100$ m and α is a random variable. The distribution for α is $p(\alpha = 2) = .4$, $p(\alpha = 2.5) = .3$, $p(\alpha = 3) = .2$, and $p(\alpha = 4) = .1$. Assume a receiver at a distance $d = 1000$ m from the transmitter, with an average transmit power constraint of $P_t = 100$ mW and a receiver noise power of .1 mW. Assume that both transmitter and receiver have CSI.

(a) Compute the distribution of the received SNR.
(b) Derive the optimal power adaptation policy for this channel and its corresponding Shannon capacity per unit hertz (C/B).
(c) Determine the zero-outage capacity per unit bandwidth of this channel.
(d) Determine the maximum outage capacity per unit bandwidth of this channel.

(a) $SNR_r = \dfrac{P_r(d)}{P_{noise}} = \dfrac{P_t(d_0/d)^\alpha}{P_{noise}} = \dfrac{1000\,mW\cdot(0.1)^\alpha}{0.1\,mW} = \begin{cases} 10\,dB & \alpha=2,\ p=0.4 \\ 5\,dB & \alpha=2.5\ p=0.3 \\ 0\,dB & \alpha=3\ p=0.2 \\ -10\,dB & \alpha=4\ p=0.1 \end{cases}$

$\because \dfrac{1}{\gamma_0} = 1+\sum_{i=1}^{4}\dfrac{1}{\gamma_i}P_i = 1+\left(\dfrac{1}{10^1}\times 0.4+\dfrac{1}{10^{0.5}}\times 0.3+\dfrac{1}{10^0}\times 0.2+\dfrac{1}{10^{-1}}\times 0.1\right) = 2.3349$

$\therefore \gamma_0 = 0.4283 > 0.1.$

$\therefore$ assume best 3 channel:

$\dfrac{0.9}{\gamma_0} = 1+\sum_{i=1}^{3}\dfrac{1}{\gamma_i}P_i = 1+\left(\dfrac{1}{10^1}\times 0.4+\dfrac{1}{10^{0.5}}\times 0.3+\dfrac{1}{10^0}\times 0.2\right) = 0.3349$

$\gamma_0 = 0.6742 < 1.$

$\therefore \dfrac{S(\gamma)}{\bar{S}} = \begin{cases} 1.3832 & \gamma = 10\,dB \\ 1.1670 & \gamma = 5\,dB \\ 0.4832 & \gamma = 0\,dB \\ 0 & \gamma = -10\,dB \end{cases}$

(b) $\dfrac{C}{B} = 2.3389\ bps/Hz$

(c) $\sigma = \dfrac{1}{E(\frac{1}{\gamma})} = 0.7491$

$\dfrac{C}{B} = \log_2(1+\sigma) = 0.8066\,bps/Hz.$

(d) $\left(\dfrac{C}{B}\right)_{max} = \log_2\left(1+\dfrac{1}{E(\frac{1}{\gamma})_{min}}\right)$

when use 2 channel ($P_{out} = 0.1+0.2 = 0.3$), $\dfrac{C}{B} = 2.1510\ bps/Hz$