3-16. Consider a wideband channel characterized by the autocorrelation function

$$A_c(\tau, \Delta t) = \begin{cases} \text{sinc}(W\Delta t) & 0 \le \tau \le 10\ \mu\text{s}, \\ 0 & \text{else}, \end{cases}$$

where $W = 100$ Hz and $\text{sinc}(x) = \sin(\pi x)/(\pi x)$.

(a) Does this channel correspond to an indoor channel or an outdoor channel, and why?

(b) Sketch the scattering function of this channel.

(c) Compute the channel's average delay spread, rms delay spread, and Doppler spread.

~~(d)~~ Over approximately what range of data rates will a signal transmitted via this channel exhibit frequency-selective fading?

(e) Would you expect this channel to exhibit Rayleigh or rather Rician fading statistics? Why?

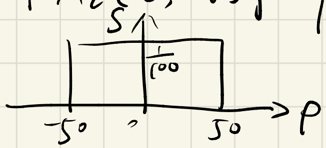
(a) $T_m = \max_n [\tau_n - \tau_0] = 10\mu s - 0 = 10\mu s \Rightarrow 50 ns$.

$\because d = ct = 3\times 10^8 m/s \times 10\mu s = 3km$

$\therefore$ correspond to an outdoor channel.

(b) $S(\tau,\rho) = \mathcal{F}\{A_c(\tau,\Delta t)\} = \begin{cases} \frac{1}{100}\,\text{rect}(\frac{\rho}{100}). & 0<\tau<10\mu s \\ 0 & \text{else.} \end{cases}$

$\therefore$ plot:



(c) $A_c(\tau) = A_c(\tau, \Delta t = 0) = \begin{cases} 1, & 0 \le \tau \le 10\mu s \\ 0 & \text{else.} \end{cases}$

$$\therefore \mu_{T_m} = \frac{\int_0^\infty \tau A_c(\tau)\,d\tau}{\int_0^\infty A_c(\tau)\,d\tau} = \frac{\int_0^{10} \tau\,d\tau}{\int_0^{10} 1\,d\tau} = \frac{50}{10} = 5\mu s$$

$$\sigma_{T_m} = \sqrt{\frac{\int_0^\infty (\tau-\mu_{T_m})^2 A_c(\tau)\,d\tau}{\int_0^\infty A_c(\tau)\,d\tau}} = \sqrt{\frac{\int_0^{10}(\tau-5)^2\,d\tau}{\int_0^{10} 1\,d\tau}} = \sqrt{\frac{250}{3\times 10}} \approx 2.89\ \mu s$$

$B_D = \max[\rho \mid S_c(0,\rho) \ne 0] = 50Hz$

(e) Rayleigh.

Because A_c and τ are independent when $0 \le \tau \le 10\mu s$. so No LOS.

3-17. Let a scattering function $S_c(\tau, \rho)$ be nonzero over $0 \le \tau \le .1$ ms and $-.1 \le \rho \le .1$ Hz. Assume that the power of the scattering function is approximately uniform over the range where it is nonzero.

(a) What are the multipath spread and the Doppler spread of the channel?

(b) Suppose you input to this channel two identical sinusoids separated in time by Δt. What is the minimum value of Δf for which the channel response to the first sinusoid is approximately independent of the channel response to the second sinusoid?

(c) For two sinusoidal inputs to the channel $u_1(t) = \sin 2\pi f t$ and $u_2(t) = \sin 2\pi f(t + \Delta t)$, find the minimum value of Δt for which the channel response to $u_1(t)$ is approximately independent of the channel response to $u_2(t)$.

(d) Will this channel exhibit flat fading or frequency-selective fading for a typical voice channel with a 3-kHz bandwidth? For a cellular channel with a 30-kHz bandwidth?

(a) $T_m = \max_n [\tau_n - \tau_0] = 0.1\,ms$

$B_D = \max[\rho \mid S_c(0,\rho) \neq 0] = 0.1\,Hz$

(b) $B_c \approx \frac{1}{T_m} = 10^4\,Hz$, $\Delta f \geq B_c$.

$\therefore$ minimum value of Δf is $10\,kHz$.

(c) $B_D = 0.1\,Hz$. $\therefore T_c \approx \frac{1}{B_D} = 10\,s$.

$\because \Delta t \gg T_c = 10\,s$

$\therefore$ minimum value of Δt is $10\,s$.

(d) $\because 3\,kHz \ll B_c = 10\,kHz$

$\therefore$ flat fading

$\because 30\,kHz \gg B_c = 10\,kHz$

$\therefore$ frequency-selective fading