10 多天线通信

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AWGN $y(t)=x(t)+n(t)$

$\vec{y}=\vec{x}+\vec{n}$ baseband

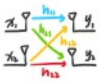
$\int_0^T x^2(t)\,dt=\langle x(t),x(t)\rangle=x_1^2+x_2^2=|\vec{x}|^2$

Flat Fading $y=hx+n$

$x\to\boxed{}\to y=hx+n$

SIMO
MIMO
SIMO
MISO

S: 单 M: 多



△ Narrowband MIMO

$y_1=h_{11}x_1+h_{21}x_2+n_1$
$=(h_{11},h_{21})\binom{x_1}{x_2}+n_1$
$y_2=(h_{12},h_{22})\binom{x_1}{x_2}+n_2$

$\Rightarrow \begin{bmatrix}y_1\\y_2\end{bmatrix}=\begin{bmatrix}h_{11}&h_{21}\\h_{12}&h_{22}\end{bmatrix}\begin{pmatrix}x_1\\x_2\end{pmatrix}+\begin{pmatrix}n_1\\n_2\end{pmatrix}$

⇓ 推广

M_t: transmit antennas
M_r: receive antennas

$$\begin{bmatrix}y_1\\y_2\\\vdots\\y_{M_r}\end{bmatrix}=H_{M_r\times M_t}\begin{bmatrix}x_1\\x_2\\\vdots\\x_{M_t}\end{bmatrix}+\begin{bmatrix}n_1\\n_2\\\vdots\\n_{M_r}\end{bmatrix}$$

$$\vec{y}=H\vec{x}+\vec{n}$$

$$R_n=E[\vec{n}\,\vec{n}^H]=E\left[\begin{pmatrix}n_1\\\vdots\\n_{M_r}\end{pmatrix}(n_1^*\cdots n_{M_r}^*)\right]=\begin{bmatrix}n_1n_1^* & n_1n_2^* & \cdots & n_1n_{M_r}^*\\ \vdots & \ddots & & \vdots\\ n_{M_r}n_1^* & \cdots & \cdots & n_{M_r}n_{M_r}^*\end{bmatrix}=\sigma^2 I$$

① 非对角线上的值平均都为0

② 对角线上为 $n_i\cdot n_i^*=\sigma^2=E|n_i|^2$

$\sum_{i=1}^{M_t}E|x_i|^2=P$ $R_x=E[\vec{x}\,\vec{x}^H]$

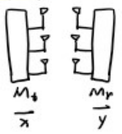
$tr(R_x)=\sum_{i=1}^{M_t}E|x_i|^2=P$

$M_r\geq M_t$

▲ Parallel Decomposition

$H\sim M_r\times M_t$ $Rank(H)=R_H$

Narrowband MIMO



M_t $\vec{x}$ M_r $\vec{y}$

$\vec{y}=H\vec{x}+\vec{n}$

$n_i\sim CN(0,\sigma^2)$

$R_n=E[\vec{n}\,\vec{n}^H]=\sigma^2 I$

↳ 噪声的统计特性

$E|x_i|^2=Ex_ix_i^*$

$P_总=\sum_{i=1}^{M_t}E|x_i|^2=tr\,\underbrace{E[\vec{x}\,\vec{x}^H]}_{R_x}$

↳ 信号的协方差矩阵

SVD (matlab中有) $H=U\Lambda V^H$

let $[\Lambda]_{i,i}=\sigma_i$ $\sigma_1\geq\sigma_2\geq\sigma_3\geq\cdots\geq\sigma_{R_H}>0$

$U\sim M_r\times M_r$ $U\cdot U^H=U^HU=I$
$V\sim M_t\times M_t$ $VV^H=V^HV=I$
$\Lambda\sim M_r\times M_t$
$[\Lambda]_{i,j}=0,\ i\neq j$ $(i>R_H)$ ← Rank of H
$\sigma_i=0$

$\vec{y}=H\vec{x}+\vec{n}$
$=U\Lambda V^H\vec{x}+\vec{n}$ $\Rightarrow U^H\vec{y}=\Lambda V^H\vec{x}+U^H\vec{n}$

令 $\begin{cases}\tilde{y}=U^H\vec{y}\\ \tilde{x}=V^H\vec{x}\\ \tilde{n}=U^H\vec{n}\end{cases}\Rightarrow \tilde{y}=\Lambda\tilde{x}+\tilde{n}$

若 $M_r=M_t=M$

$$\begin{bmatrix}\tilde{y}_1\\\vdots\\\tilde{y}_m\end{bmatrix}=\begin{bmatrix}\sigma_1&&0\\&\ddots&\\0&&\sigma_m\end{bmatrix}\begin{bmatrix}\tilde{x}_1\\\vdots\\\tilde{x}_m\end{bmatrix}+\begin{bmatrix}\tilde{n}_1\\\vdots\\\tilde{n}_m\end{bmatrix}$$

$\tilde{y}_i=\sigma_i\tilde{x}_i+\tilde{n}_i$ $i=1,2,\ldots R_H$ (同时可传信号数 $max=R_H$)

$\vec{x}\Rightarrow\boxed{H}\Rightarrow\vec{y}=H\vec{x}+\vec{n}$

把互相干扰的M信道
拆为若干相不干扰的S

$\tilde{x}\Rightarrow\boxed{V}\Rightarrow\vec{x}\Rightarrow\boxed{H}\Rightarrow\vec{y}=H\vec{x}+\vec{n}\Rightarrow\boxed{U^H}\Rightarrow\tilde{y}=U^H\vec{y}$

precoding (预编码) shaping

想发的信号 真实发的 真实收的 比从$\vec{y}\to\vec{x}$
$\tilde{y}$更易解出$\tilde{x}$

$=U^HU\vec{x}+U^H\vec{n}$
$=U^HHV\tilde{x}+U^H\vec{n}$
$=\Lambda\tilde{x}+U^H\vec{n}$

$\Rightarrow\tilde{y}_i=\sigma_i\tilde{x}_i+\tilde{n}_i$ (i路信号互不干扰)
↳ 上限看 R_H

$H\sim \overset{m收}{m}\times\overset{n发}{n}$

即使 $R_H<m\&n$
只有 R_H 路在接收有用

$$\begin{bmatrix}y_1\\\vdots\\y_m\end{bmatrix}=\begin{bmatrix}\sigma_1&&&0\\&\ddots&&\\&&\sigma_{R_H}&\\0&&&\ddots\ 0\end{bmatrix}\begin{bmatrix}x_1\\\vdots\\x_n\end{bmatrix}$$

△MIMO 带 CSIR/CSIT 的信道容量

$\tilde{y}_i=\sigma_i\tilde{x}_i+\tilde{n}_i$ $i=1,\ldots R_H$

$\tilde{n}=U^H\vec{n}$ $E[\tilde{n}]=E[U^H\vec{n}]=U^HE[\vec{n}]=0$

$R_{\tilde{n}}=E[\tilde{n}\tilde{n}^H]=[U^H\vec{n}\,\vec{n}^HU]=U^HR_nU$
$=\sigma^2U^HIU=\sigma^2I$

协方差矩阵

$\tilde{n}=U^H\vec{n}$ $E\tilde{n}=E(U^H\vec{n})=0$

$E[\tilde{n}\tilde{n}^H]=E[U^H\vec{n}\vec{n}^HU]=U^H(\sigma^2I)U=\sigma^2I$

$\Rightarrow E[\tilde{n}_i\tilde{n}_j^*]=\begin{cases}0, & i\neq j\\ \sigma^2, & i=j\end{cases}\Rightarrow\tilde{n}\sim CN(0,\sigma^2)$

$P=tr\,E[\vec{x}\cdot\vec{x}^H]$
$=tr\,E[V\tilde{x}\tilde{x}^HV^H]$
$=tr[V\,E(\tilde{x}\tilde{x}^H)V^H]$
$=tr[V^HV\,E(\tilde{x}\tilde{x}^H)]$
$=tr\,E(\tilde{x}\tilde{x}^H)$
$=\sum_{i=1}^{M_t}E|\tilde{x}_i|^2$ ⇒ M_t 个信号只有 R_H 个能被接收
$=\sum_{i=1}^{R_H}E|\tilde{x}_i|^2$ ⇒ 把不能传的功率置0

$(tr(ABC)=tr(CAB)=tr(BCA))$

$$C = \sum_{i=1}^{R_H} B \cdot \log_2\left(1 + \frac{\sigma_i^2 E|\hat{x}_i|^2}{E|\hat{n}_i|^2}\right) \qquad \left(P_i = E|\hat{x}_i|^2,\ \sum_{i=1}^{R_H} P_i = P\right)$$

$$= \sum_{i=1}^{R_H} B \cdot \log_2\left(1 + \frac{\sigma_i^2 P_i}{\sigma^2}\right) \qquad \rho_i \triangleq \frac{P_i}{\sigma^2}$$

$$\max_{\rho_1\rho_2\cdots\rho_{R_H}} \sum_{i=1}^{R_H} B \log_2(1+\sigma_i^2\rho_i) \quad \text{s.t.} \sum_{i=1}^{R_H} \rho_i = \frac{P}{\sigma^2}$$

$$\max_{P_1P_2\cdots P_{R_H}} \sum_{i=1}^{R_H} B \log_2\left(1+\sigma_i^2\frac{P_i}{\sigma^2}\right) \quad \text{s.t.} \sum_{i=1}^{R_H} P_i = P$$

Lagrangian:

$$J(\lambda) = \sum_{i=1}^{R_H} B \log_2\left(1+\sigma_i^2\frac{P_i}{\sigma^2}\right) - \lambda\left(\sum_{i=1}^{R_H} P_i - P\right)$$

$$C = \max_{\lambda} \max_{\{P_i\}} J(\lambda) \qquad (A^+) = \max\{0, A\}$$

fix λ

$$\frac{\partial J(\lambda)}{\partial P_i} = 0 \Rightarrow P_i = \left(\frac{B}{\lambda \cdot \ln 2} - \frac{\sigma^2}{\sigma_i^2}\right)^+$$

$$\frac{P_i}{P} = \left(\frac{B}{\lambda P \ln 2} - \frac{\sigma^2}{P\sigma_i^2}\right)^+ = \left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right)^+$$

$$C = \sum_{i=1}^{R_H} B \log_2\left(1 + \frac{\sigma_i^2 P}{\sigma}\cdot\frac{P_i}{P}\right)$$

$$\Rightarrow \quad = \sum_{i=1}^{R_H} B \log_2\left[1 + \gamma_i\left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right)^+\right]$$

$$= \sum_{i:\gamma_i \geq \gamma_0} B \log_2 \frac{\gamma_i}{\gamma_0}$$

$$\sum_{i=1}^{R_H} \frac{P_i}{P} = 1 \qquad \sum_{i=1}^{R_H}\left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right)^+ = 1 \qquad \gamma_i = \frac{P\cdot\sigma_i^2}{\sigma^2} \qquad \begin{cases} \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_{R_H} \\ \gamma_1 \geq \gamma_2 \geq \cdots \geq \gamma_{R_H} \\ \frac{1}{\gamma_1} \leq \frac{1}{\gamma_2} \leq \cdots \leq \frac{1}{\gamma_{R_H}} \end{cases}$$

$\Downarrow$

① Suppose $\gamma_{R_H} \geq \gamma_0$

$$\sum_{i=1}^{R_H}\left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) = 1 \Rightarrow \gamma_0 = \frac{R_H}{1 + \sum_{i=1}^{R_H}\frac{1}{\gamma_i}}$$

if $\gamma_0 \leq \gamma_{R_H}$

$\Downarrow$ No

② suppose $\gamma_{R_H-1} \geq \gamma_0 \geq \gamma_{R_H}$

$$\sum_{i=1}^{R_H-1}\left(\frac{1}{\gamma_0} - \frac{1}{\gamma_i}\right) = 1 \Rightarrow \gamma_0 = \frac{R_H - 1}{1 + \sum_{i=1}^{R_H-1}\frac{1}{\gamma_i}}$$

$\Downarrow$

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