3.3 WB Fading Model.

△ Autocorrelation function.

$$A_c(\tau_1,\tau_2;t,t+\Delta t)=E[C^*(\tau_1,t)C(\tau_2,t+\Delta t)]$$

⇓ Wide sense stationary: WSS

$A_c(\tau_1,\tau_2;t,t+\Delta t)$ is independent of t

$$A_c(\tau_1,\tau_2;\Delta t)=E[C^*(\tau_1,t)C(\tau_2,t+\Delta t)]$$

⇓ uncorrelated scattering: US

$$E[\alpha_m^*(t_1)\alpha_n(t_2)]=E[\alpha_m^*(t_1)]\cdot E[\alpha_n(t_2)] \quad m\neq n.$$

$$=0$$

$$A_c(\tau_1,\tau_2;\Delta t)$$

$$= E\left[C^*(\tau_1,t)\; C(\tau_2,t+\Delta t)\right]$$

$$= E\Big[\sum_{n=1}^{N(t)} \alpha_n(t)\, e^{j\phi_n(t)}\, \delta(\tau_1-\tau_n(t)) \sum_{m=1}^{N(t)} \alpha_m(t+\Delta t)\, e^{-j\phi_m(t+\Delta t)}\, \delta(\tau_2-\tau_m(t+\Delta t))\Big]$$

$$= E\Big[\sum_{m,n=1}^{N(t)} \alpha_n(t)\,\alpha_m(t+\Delta t)\, e^{j[\phi_n(t)-\phi_m(t+\Delta t)]}\, \delta(\tau_1-\tau_n(t))\cdot\delta(\tau_2-\tau_m(t+\Delta t))\Big]$$

$$= E\Big[\sum_{n=1}^{N(t)} \alpha_n(t)\,\alpha_n(t+\Delta t)\, e^{j[\phi_n(t)-\phi_n(t+\Delta t)]}\, \delta(\tau_1-\tau_n(t))\, \delta(\tau_2-\tau_n(t+\Delta t))\Big]$$

$$\approx E\Big[\sum_{n=1}^{N(t)} \alpha_n^2(t)\, e^{j[\phi_n(t)-\phi_n(t+\Delta t)]}\, \delta(\tau_1-\tau_n(t))\, \delta(\tau_2-\tau_n(t))\Big]$$

$$= E\Big[\sum_{n=1}^{N(t)} \alpha_n^2(t)\, e^{j[\phi_n(t)-\phi_n(t+\Delta t)]}\, \delta(\tau_1-\tau_n(t))\Big]\, \delta[\tau_1-\tau_2]$$

$\Rightarrow A_c(\tau_1,\tau_2;\Delta t)$ is non-zero only when $\tau_1=\tau_2$

$$A_c(\tau_1;\Delta t) = A_c(\tau_1,\tau_1;\Delta t)/\delta(0)$$

or $$A_c(\tau;\Delta t) = A_c(\tau,\tau;\Delta t)/\delta(0)$$

$$A_c(\tau_1,\tau_2;\Delta t) = A_c(\tau_1;\Delta t)\cdot\delta(\tau_1-\tau_2)$$

Note that $\phi_n(t) = 2\pi f_c \tau_n(t) - 2\pi f_{D,n}(t)\cdot t$

$$\phi_n(t) - \phi_n(t+\Delta t) = 2\pi f_c \tau_n(t) - 2\pi f_{D,n}(t)\cdot t - 2\pi f_c \tau_n(t+\Delta t) + 2\pi f_{D,n}(t+\Delta t)(t+\Delta t)$$

$$\approx 2\pi f_{D,n} \cancel{(t)} \cdot \Delta t$$

$\boxed{\text{Suppose constant Doppler}}$

$$A_c(\tau;\Delta t) = E\left[\sum_{n=1}^{N(t)} \alpha_n^2(t) e^{j(\phi_n(t)-\phi_n(t+\Delta t))} \delta(\tau-\tau_n(t))\right]$$

$$= E\left[\sum_{n=1}^{N(t)} \alpha_n^2(t) e^{j2\pi f_{D,n}\cancel{(t)}\cdot\Delta t} \cdot \delta(\tau-\tau_n(t))\right]$$

$$\cancel{= \sum_{n=1}^{N(t)} E[\alpha_n^2(t)]\, e^{j2\pi f_{D,n}\cdot\Delta t} \cdot \delta(\tau-\tau_n(t))}$$

$$= \sum_{n=1}^{N(t)} E[\alpha_n^2(t)\cdot\delta(\tau-\tau_n(t))]\, e^{j2\pi f_{D,n}\cdot\Delta t}$$

Observation:

Given τ, treat Δt as independent variable, the frequencies of $A_c(\tau;\Delta t)$'s frequency components are the Doppler frequencies.

$\Rightarrow$ Taking Fourier Transform w.r.t. Δt, we get Doppler frequencies.

Scattering function

$$S_c(\tau,\rho) = \int_{-\infty}^{+\infty} A_c(\tau,\Delta t) e^{-j2\pi\rho\cdot\Delta t}\, d\Delta t.$$

$S_c(\tau,\rho)$ shows the Dopple frequencies in the paths with delay τ.

3.3.1

△ Power Delay Profile.

Let $\Delta t=0$ in $A_c(\tau;\Delta t)$, denote as $A_c(\tau)$

Thus, $A_c(\tau) = A_c(\tau;0)$ Power Delay Profile

$$A_c(\tau) = E[\,c^*(\tau,t)\,c(\tau,t)]/\delta(0)$$
$$= \sum_{n=1}^{N(t)} E[\alpha_n^2(t)\,\delta(\tau-\tau_n(t))]$$
$$= \sum_{n=1}^{N(t)} E[\alpha_n^2(t)]\cdot E[\,\delta(\tau-\tau_n(t))]$$

Average power $\overset{\text{gain}}{\text{of}}$ nth path ; Chance of delay $=\tau$.

Average channel power ~~gain~~ intensity at delay τ.