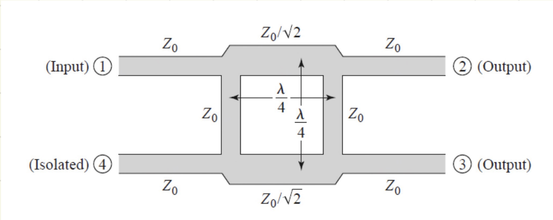
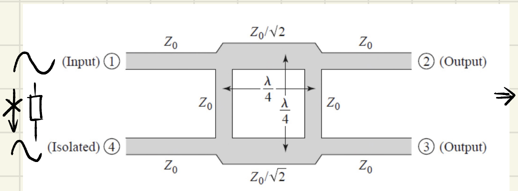
推导



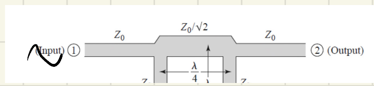
A branch-line coupler

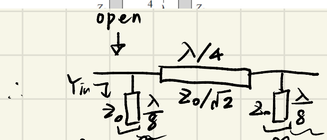
Use even & odd mode analysis

① let mode coming in from the left ports that are in-phase, which is even, we have virtual open.

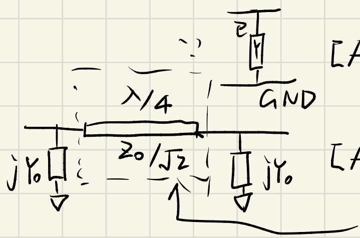


⇒





ABCD Analysis: $Z_{in} = Z_0\left(\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}\right) = \frac{Z_0}{j}$ $\quad Y_{in} = \frac{j}{Z_0} = jY_0$

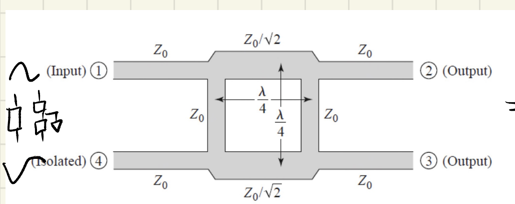


$$[ABCD] = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{jZ_0} & 1 \end{bmatrix}$$

$$[ABCD_{T\text{-}line}] = \begin{bmatrix} \cos(\beta l) & jZ_{Line}\sin(\beta l) \\ (j/Z_{Line})\sin(\beta l) & \cos(\beta l) \end{bmatrix} = \begin{bmatrix} 0 & \frac{jZ_0}{\sqrt{2}} \\ \frac{j\sqrt{2}}{Z_0} & 0 \end{bmatrix}$$

$$\therefore [ABCD] = \begin{bmatrix} 1 & 0 \\ \frac{1}{jZ_0} & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{jZ_0}{\sqrt{2}} \\ \frac{j\sqrt{2}}{Z_0} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{jZ_0} & 1 \end{bmatrix}$$

② let an in-phase signal on one port, and an out-of-phase signal on the other port, we have a virtual ground.



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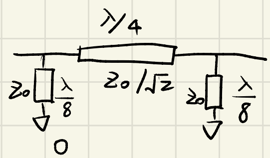
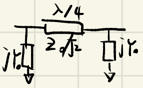


ABCD Analysis: $Z_{in} = Z_0\left(\frac{Z_L + jZ_0\tan(\beta l)}{Z_0 + jZ_L\tan(\beta l)}\right) = jZ_0$, $\quad Y_{in} = \frac{1}{jZ_0} = -jY_0$

$$\therefore [ABCD] = \begin{bmatrix} 1 & 0 \\ \frac{-1}{jZ_0} & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{jZ_0}{\sqrt{2}} \\ \frac{j\sqrt{2}}{Z_0} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -\frac{1}{jZ_0} & 1 \end{bmatrix}$$

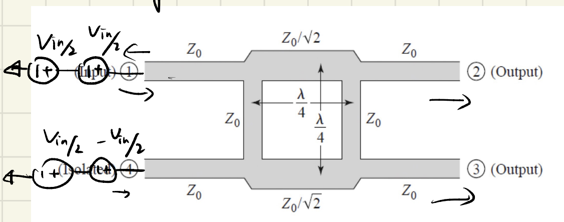
Use Table from Pozar to convert to S-parameter



$$S_E^o = \begin{bmatrix} 0 & \frac{\mp 1 - j}{\sqrt{2}} \\ \frac{\mp 1 - j}{\sqrt{2}} & 0 \end{bmatrix}$$

+ Even
- odd

Put it all together



$$S_{11} = \left.\frac{V_1^-}{V_1^+}\right|_{a_2=a_3=a_4=0} = \frac{V_1^{-e} + V_1^{-o}}{V^+ + V^+}$$

$$= \frac{V_1^{-e} + V_1^{-o}}{2V^+} = \frac{1}{2}\left(\frac{V_1^{-e}}{V^+} + \frac{V_1^{-o}}{V^+}\right)$$

$$= \frac{1}{2}\left(S_{11}^e + S_{11}^o\right) = 0.$$

$$S_{11} = S_{22} = S_{33} = S_{44} = 0.$$

$$S_{31} = \left.\frac{V_3^-}{V_1^+}\right|_{a_2=a_3=a_4} = \frac{V_3^{-e} + V_3^{-o}}{V^+ + V^+} = \frac{1}{2}\left(\frac{V_3^{-e}}{V^+} + \frac{V_3^{-o}}{V^+}\right) = \frac{1}{2}\left(\frac{V_2^{-e}}{V^+} - \frac{V_2^{-o}}{V^+}\right) = \frac{1}{2}\left(S_{21}^e - S_{21}^o\right) = \frac{1}{2}\left[\left(\frac{-1-j}{\sqrt{2}}\right) - \left(\frac{1-j}{\sqrt{2}}\right)\right]$$

$$= -\frac{1}{\sqrt{2}}.$$

$$S_{31} = S_{13} = S_{42} = S_{24} = -\frac{1}{\sqrt{2}}$$

$$S_{41} = \left.\frac{V_4^-}{V_1^+}\right|_{a_2=a_3=a_4=0} = \frac{V_4^{-e} + V_4^{-o}}{V^+ + V^+} = \frac{V_4^{-e} + V_4^{-o}}{2V^+} = \frac{V_1^{-e} - V_1^{-o}}{2V^+} = \frac{1}{2}\left(S_{11}^e - S_{11}^o\right) = 0$$

$$S_{41} = S_{14} = S_{32} = S_{23} = 0$$

$$\therefore [S] = -\frac{1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

$j \leftarrow 1$

(S_{12} has a 90-degree phase shift compared to S_{13}).