1. $Z_0 = 50$ ohm. $L = 7.4$ nH, $f = 3.7$ GHz.

$\therefore \beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{v_p} = \frac{2\pi f}{2.08\,c} = 37.256\ m^{-1}$

$\because$ open circuited

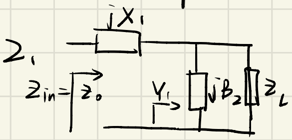
$\therefore \Gamma_0 = 1 \qquad \Gamma_L = \frac{Z_L - Z_c}{Z_L + Z_c} = 1 \quad (Z_L \to \infty)$

$\therefore 1 = e^{-j2\beta l} = [\cos(-2\beta l) + j\sin(-2\beta l)]$

$\therefore \cos(2\beta l) = 1 \Rightarrow 2\beta l = \pi + 2k\pi \quad (k \in Z)$

$\therefore l = \frac{-\pi + 2k\pi}{2\beta} = 84.3k + 76.7$ mm. $(k \in Z)$

2.



$\because \frac{1}{Z_L} = G + jB$

$\therefore Y_1 = (G + jB) + jB_2$

$\because Z_1 = \frac{1}{G + j(B + B_2)} = \frac{G - j(B + B_2)}{G^2 + (B + B_2)^2}$

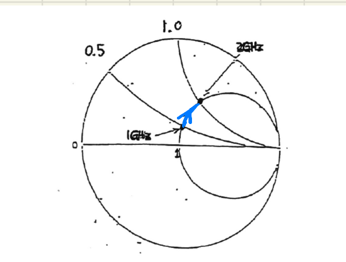
$Z_0 = \frac{1}{Y_1} + jX_1$

$\therefore Z_0 = \frac{G}{G^2 + (B + B_2)^2} \Rightarrow B_2 = \pm\sqrt{\frac{G}{Z_0} - G^2} - B$

$\therefore X_1 = \frac{B + B_2}{G^2 + (B + B_2)^2} = \frac{\pm\sqrt{\frac{G}{Z_0} - G^2}}{G/Z_0} = \pm Z_0\sqrt{\frac{1}{Z_0 G} - 1}$

$S_{11} = \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

3.



$Z_0 = 50$ ohm

$\because$ 阻抗点都在上半平面

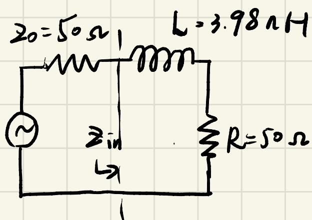
$\therefore$ 为串联RL电路

$Z_{in}(\omega_L) = r + j\omega_L L/Z_0, \ r = 1.$

$\because \omega_L = 2\pi f$

$\therefore jx = j\omega_L L/Z_0, \ j0.5 = j\,2\pi \cdot 1GHz \cdot L/50\Omega$

$\Rightarrow L = 3.98nH$

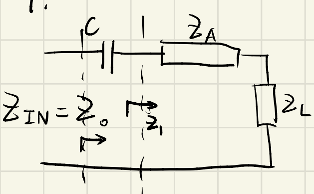


At 2GHz, $Z_L = R + \omega_L L$

$z_L = 1 + j1$

$S_{11} = \Gamma = \frac{z_L - 1}{z_L + 1} = \frac{j}{2 + j} = \frac{1}{5} + \frac{2}{5}j$

4.



$Z_1 = Z_A \frac{Z_L + jZ_A \tan\beta l}{Z_A + jZ_L \tan\beta l} \qquad \tan\beta l = \tan\frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \tan\frac{\pi}{2}$

$\therefore Z_1 = Z_A \cdot \frac{jZ_A}{jZ_L} = \frac{Z_A^2}{Z_L} = \frac{Z_A^2}{R_L + jX_L}$

$\because Z_1 = \cancel{Z_A} + Z_L$

$Z_{in(A)} = \frac{Z_A^2}{Z_L}$

$\therefore (Z_A + Z_L) = \frac{Z_A^2}{Z_L} \Rightarrow \cancel{Z_A^2} - Z_L Z_A - Z_L^2 = 0 \Rightarrow Z_A = (\frac{1}{2}+\frac{\sqrt{5}}{2})Z_L$

$\therefore Z_A = \cancel{\frac{1+\sqrt{5}}{2}R_L + j\frac{1+\sqrt{5}}{2}X_L}$

$\because \underline{Z_{IN}} = Z_0 = \frac{1}{j\omega C} + \underbrace{Z_A + Z_L}_{Z_{in}} = \underbrace{\frac{Z_A^2 R_L}{R_L^2+X_L^2}}_{=Z_0} - j\underbrace{\left(\frac{1}{\omega_C} + \frac{Z_A^2 X_L}{R_L^2+X_L^2}\right)}_{=0}$

Z_0 is real

$\therefore \frac{1}{j\omega C} = Z_0 - Z_A - Z_L$

$C = \frac{1}{j\omega(Z_0 - Z_A - Z_L)} = \frac{1}{\omega[\frac{\sqrt{5}+3}{2}X_L + j(Z_0 - \frac{\sqrt{5}+3}{2}R_L)]}$

$\Rightarrow Z_A =$

$C =$

5. $\beta l = \frac{\pi}{6}$,

$Z_{in} = Z_c \frac{Z_L + jZ_c\tan(\frac{\pi}{6})}{Z_c + jZ_L\tan(\frac{\pi}{6})} = 100\frac{380-270j+\frac{100\sqrt{3}}{3}j}{100+j\frac{\sqrt{3}(380-270j)}{3}} = 44.6 - 121.2j$

$z_{in} = 0.44 - j1.21$

$\Gamma_L = \frac{Z_L - Z_c}{Z_L+Z_c} = \frac{280-270j}{480-270j} = 0.68 - 0.18j$

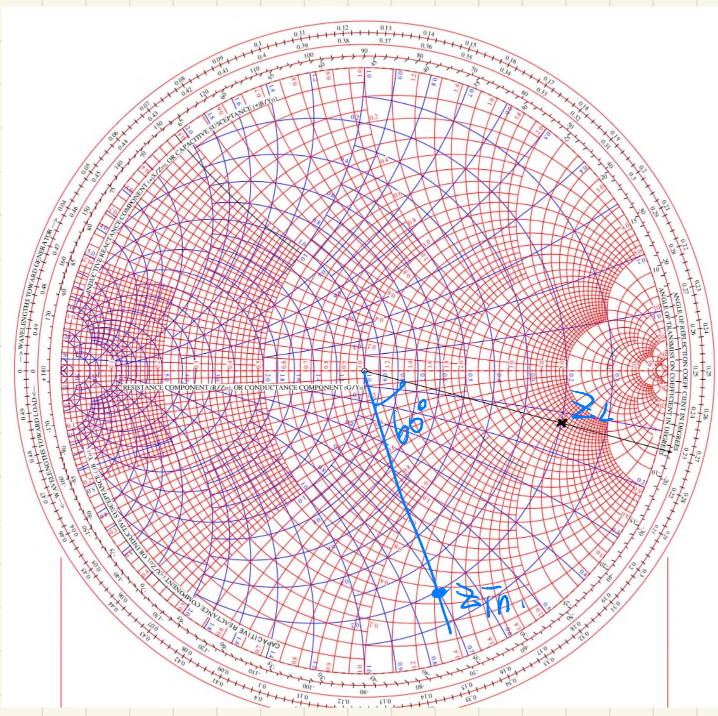
$\therefore \tan\theta = \frac{-0.18}{0.68} \Rightarrow \theta = \tan^{-1}(-\frac{0.18}{0.68}) = -14.83^\circ$

$\underline{\Gamma_L = 0.70\angle -14.83^\circ}$

$\bar{R}_1 = 3.8$, $X_1 = -2.7$

Use smith chart we get:

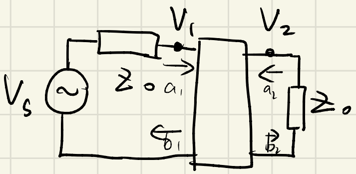
$\Gamma_{in} = \Gamma_L e^{-j60^\circ}$



$Z_{in} = Z_c\frac{1+\Gamma_{in}}{1-\Gamma_{in}}$

... get Z_{in}!

6. $$\begin{bmatrix} b_1 \\ \vdots \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \Rightarrow \begin{aligned} a_1 &= \frac{1}{2\sqrt{Z_0}}(V_1 + Z_0 i_1), \quad a_2 = \frac{1}{2\sqrt{Z_0}}(V_2 + Z_0 i_2) \\ b_1 &= \frac{1}{2\sqrt{Z_0}}(V_1 - Z_0 i_1) \quad b_2 = \frac{1}{2\sqrt{Z_0}}(V_2 - Z_0 i_2) \end{aligned}$$



$V_1 = (a_1 + b_1)\sqrt{Z_0}$

$i_1 = (a_1 - b_1)\frac{1}{\sqrt{Z_0}}$

$$S_{11} = \frac{b_1}{a_1}\Big|_{a_2=0} = \frac{V_1 - Z_0 i_1}{V_1 + Z_0 i_1}$$

$$S_{21} = \frac{b_2}{a_1}\Big|_{a_2=0} = \frac{\frac{V_2}{\sqrt{Z_0}}}{\frac{V_1 + Z_0 i_1}{2\sqrt{Z_0}}} = \frac{2V_2}{V_1 + Z_0 i_1}$$

$$S_{22} = \frac{b_2}{a_2}\Big|_{a_1=0} = \frac{V_2 - Z_0 i_2}{V_2 + Z_0 i_2}$$

$$S_{12} = \frac{b_1}{a_2}\Big|_{a_1=0} = \frac{\frac{V_1}{\sqrt{Z_0}}}{\frac{V_2 + Z_0 i_2}{2\sqrt{Z_0}}} = \frac{2V_1}{V_2 + Z_0 i_2}$$

$$\Rightarrow \begin{cases} Z_0 i_1 = V_1\left(1 - \frac{2S_{11}}{1+S_{11}}\right) \\ Z_0 i_2 = V_2\left(1 - \frac{2S_{22}}{1+S_{22}}\right) \end{cases}$$

$$\therefore \frac{V_1}{V_2} = \frac{S_{12}(V_2 + Z_0 i_2)}{S_{21}(V_1 + Z_0 i_1)} = 2\frac{S_{12}}{S_{21}} \cdot \frac{V_2}{V_1} \cdot \left(\frac{1 - \frac{S_{22}}{1+S_{22}}}{1 - \frac{S_{11}}{1+S_{11}}}\right)$$

$$\Rightarrow \frac{V_1}{V_2} = \sqrt{2\frac{S_{12}(1+S_{11})}{S_{21}(1+S_{22})}}, \quad \frac{V_2}{V_1} = \sqrt{\frac{S_{21}(1+S_{22})}{2S_{12}(1+S_{11})}}$$

$$V_m = \frac{V_s}{2} = \frac{V_1 + Z_0 i_1}{2} = \frac{V_1}{2}\left(2 - \frac{2S_{11}}{1+S_{11}}\right) = V_1\left(1 - \frac{S_{11}}{1+S_{11}}\right) = V_1 \cdot \frac{1}{1+S_{11}}$$

$$\therefore \frac{V_2}{V_m} = \frac{V_2}{V_1}(1+S_{11}) = \sqrt{\frac{S_{21}(1+S_{22})(1+S_{11})}{2S_{12}}}$$

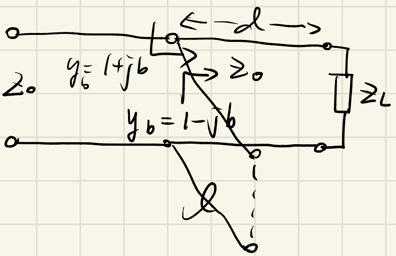
$$\theta = \tan^{-1}\left(\frac{1.8}{0.8}\right) = 66.04^\circ$$

7. $f_\ell = 1\,\text{GHz}$, $Z_0 = 50$ ohm, $Z_L = 40 + 90j$ ohm, $\Rightarrow z_L = 0.8 + 1.8j = 1.97e^{j\theta}$

$z_0 = \frac{4}{9} + \frac{5}{9}j \qquad y_L = 0.21 - 0.4j$

(a) open stub:

$Z_{oc} = -j\cot(\beta \ell) \Rightarrow y_{oc} = j\tan(\beta \ell)$, $\quad \beta = \frac{2\pi}{\lambda}$



$\therefore \ell_{oc_1} = \frac{1}{2\pi}\tan^{-1}(-Y_1)\lambda = -0.176\lambda + 0.5\lambda$

$= 0.324\lambda$

$\ell_{oc_2} = \frac{1}{2\pi}\tan^{-1}(-Y_2)\lambda = 0.017\lambda$

(b) $Z_{sc} = j\tan(\beta \ell) \Rightarrow y_{sc} = -j\cot(\beta \ell)$, $\ell_{sc_1} = \frac{1}{2\pi}\tan^{-1}\left(\frac{1}{Y_1}\right)\lambda$

$= 0.426\lambda$

$$\ell_{sc_2} = \frac{1}{2\pi}\tan^{-1}\left(\frac{1}{Y_2}\right)\lambda = 0.07\lambda$$

Use smith chart. We get $z_{b1} = 0.2 - j0.4$. $z_{b2} = 0.2 + j0.4$

$\therefore$ short stub: $\begin{cases} \cot(\beta \ell_1) = 0.4 \\ \cot(\beta \ell_2) = -0.4 \end{cases} \Rightarrow \begin{cases} \beta\ell_1 = 68.2^\circ \\ \beta\ell_2 = 291.8^\circ \end{cases} \Rightarrow \begin{cases} \ell_1 = 0.19\lambda \\ \ell_2 = 0.81\lambda \end{cases}$

open stub $\begin{cases} \tan(\beta \ell_1) = 0.4 \\ \tan(\beta \ell_2) = -0.4 \end{cases} \Rightarrow \begin{cases} \beta\ell_1 = 21.8^\circ \\ \beta\ell_2 = -338.2^\circ \end{cases} \Rightarrow \begin{cases} \ell_1 \; 0.06\lambda \\ \ell_2 = 0.94\lambda \end{cases}$

$\because \begin{cases} 2\beta d_1 = 219.8^\circ & \Rightarrow d_1 = 0.61\lambda \\ 2\beta d_2 = 272.6^\circ & \Rightarrow d_2 = 0.76\lambda \end{cases}$