

$$1. \quad Z_0 = 50 \text{ ohm}, \quad L = 7.4 \text{ nH}, \quad f = 3.7 \text{ GHz}.$$

$$\therefore P_0 = \frac{2\pi}{\lambda} = \frac{2\pi f}{V_p} = \frac{2\pi f}{2.08 c} = 37.256 \text{ m}^{-1}$$

$\therefore$  open circuited

$$\therefore Z_0 = 1 \quad Z_L = \frac{Z_0 - Z_C}{Z_0 + Z_C} = 1 \quad (Z_L \rightarrow \infty)$$

$$\therefore I = e^{-jZ_0 l} = [\cos(-jZ_0 l) + j \sin(-jZ_0 l)]$$

$$\therefore \cos(-jZ_0 l) = 1 \Rightarrow jZ_0 l = \pi + 2k\pi \quad (k \in \mathbb{Z})$$

$$\therefore l = \frac{\pi + 2k\pi}{jZ_0} = 84.3k + 76.7 \text{ mm.} \quad (k \in \mathbb{Z})$$

$$2. \quad \begin{array}{c} jX_1 \\ \hline Z_{in} = \boxed{Z_0} \end{array} \quad \begin{array}{c} Y_1 \\ \hline \boxed{jB_2} \end{array} \quad \begin{array}{c} Z_L \\ \hline \end{array}$$

$$\therefore \frac{1}{Z_L} = A + jB$$

$$\therefore Y_1 = (A + jB) + jB_2$$

$$\therefore Z_1 = \frac{1}{A + j(B+B_2)} = \frac{A - j(B+B_2)}{A^2 + (B+B_2)^2}$$

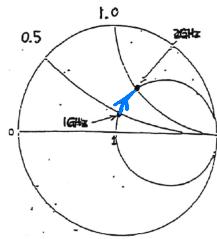
$$Z_0 = \frac{1}{Y_1 + jX_1}$$

$$\therefore Z_0 = \frac{A}{A^2 + (B+B_2)^2} \Rightarrow B_2 = \pm \sqrt{\frac{A}{Z_0} - A^2} - B$$

$$\therefore X_1 = \frac{B+B_2}{A^2 + (B+B_2)^2} = \frac{\pm \sqrt{\frac{A}{Z_0} - A^2}}{A/Z_0} = \pm Z_0 \sqrt{\frac{1}{Z_0 A} - 1}$$

$$S_{11} = T_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

3.



$$Z_0 = 50 \text{ ohm}$$

$\therefore$  阻抗点都在  $\frac{1}{2}$  平面上

$\therefore$  为串联  $R-L$  电路

$$Z_{in}(w_L) = r + j w_L L / Z_0, \quad r = 1.$$

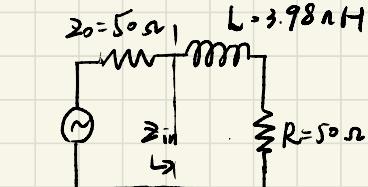
$$w_L = 2\pi f$$

$$\therefore jX = j w_L L / Z_0, \quad j0.5 = j 2\pi \cdot 1 \text{ GHz} \cdot L / 50 \Omega \quad \Rightarrow L = 3.98 \text{ nH}$$

$$\text{At } 2 \text{ GHz, } Z_L = R + w_L L$$

$$jZ_L = 1 + j1$$

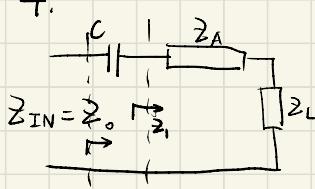
$$S_{11} = P = \frac{jZ_L - 1}{jZ_L + 1} = \frac{1}{2+j} = \frac{1}{5} + \frac{2}{5}j$$



$$4. \quad Z_1 = Z_A \frac{Z_L + jZ_A \tan \beta l}{Z_A + jZ_L \tan \beta l} \quad \tan \beta l = \tan \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \tan \frac{\pi}{2}$$

$$\therefore Z_1 = Z_A \cdot \frac{jZ_A}{Z_L} = \frac{Z_A^2}{Z_L} \quad = \frac{Z_A^2}{R_L + jX_L}$$

$$\therefore Z_1 = Z_A + Z_L$$



$$Z_{in(A)} = \frac{Z_A^2}{Z_L}$$

$$\therefore (Z_A + Z_L) = \frac{Z_A^2}{Z_L} \Rightarrow Z_A - Z_L Z_A - Z_L^2 = 0 \Rightarrow Z_A = \frac{1}{(2 + \frac{\sqrt{5}}{2})^2} Z_L$$

$$\therefore Z_A = \frac{1 + \sqrt{5}}{2} R_L + j \frac{1 + \sqrt{5}}{2} X_L$$

$$\therefore Z_{in} = Z_0 = \frac{1}{j\omega C} + \frac{Z_A + Z_L}{Z_A} = \frac{Z_A K_L}{R_L^2 + X_L^2} - j \left( \frac{1}{\omega C} + \frac{Z_A X_L}{R_L^2 + X_L^2} \right)$$

$$\therefore \frac{1}{j\omega C} = Z_0 - Z_A - Z_L$$

$$C = \frac{1}{j\omega (Z_0 - Z_A - Z_L)} = \frac{1}{j\omega (\frac{1 + \sqrt{5}}{2} X_L + j(Z_0 - \frac{1 + \sqrt{5}}{2} R_L))} \Rightarrow Z_A = \sqrt{N} C$$

5.  $\beta l = \frac{\pi}{6}$

$$Z_{in} = Z_0 \frac{Z_L + j Z_C \tan(\frac{\pi}{6})}{Z_C + j Z_L \tan(\frac{\pi}{6})} = 100 \frac{380 - 270j}{100 + j \frac{\sqrt{3}(380 - 270j)}{3}} = 44.6 - 121.2j$$

$$P_L = \frac{Z_L - Z_C}{Z_L + Z_C} = \frac{280 - 270j}{480 - 270j} = 0.68 - 0.18j$$

$$\tan \theta = \frac{-0.18}{0.68} \Rightarrow \theta = \tan^{-1}(-\frac{0.18}{0.68}) = -14.83^\circ$$

$$Z_L = 0.70 \angle -14.83^\circ$$

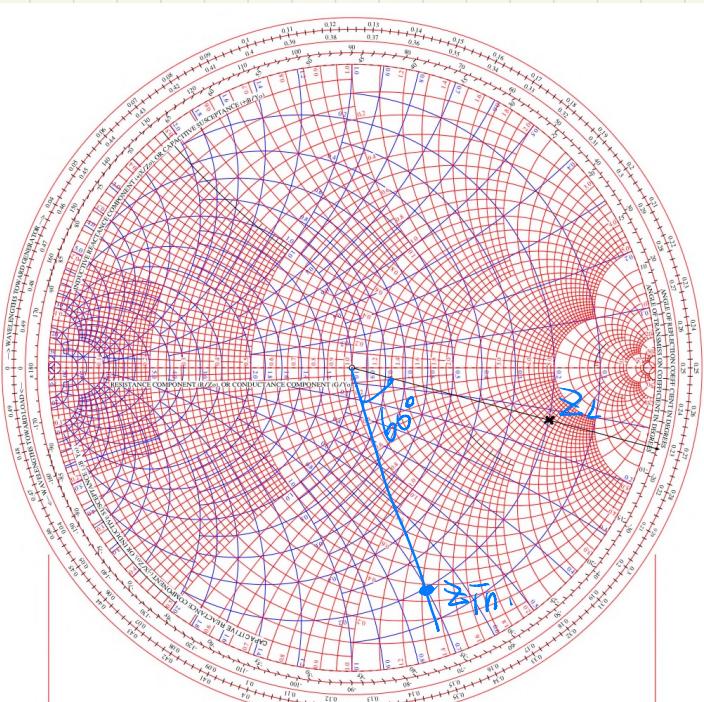
Use smith chart we get:

$$R_1 = 3.8, \quad X_1 = -2.7$$

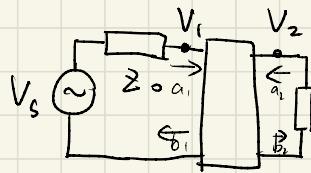
$$E_{in} = E_L e^{-j6^\circ}$$

$$Z_{in} = Z_0 \frac{1 + E_{in}}{1 - E_{in}}$$

-- get  $Z_{in}$ !



$$b. \begin{bmatrix} b_1 \\ \cdot \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} \Rightarrow \begin{aligned} a_1 &= \frac{1}{2\sqrt{Z_0}} (V_1 + Z_0 \bar{v}_1), \quad a_2 = \frac{1}{2\sqrt{Z_0}} (V_2 + Z_0 \bar{v}_2) \\ b_1 &= \frac{1}{2\sqrt{Z_0}} (V_1 - Z_0 \bar{v}_1), \quad b_2 = \frac{1}{2\sqrt{Z_0}} (V_2 - Z_0 \bar{v}_2) \end{aligned}$$



$$\begin{aligned} V_1 &= (a_1 + b_1) \sqrt{Z_0} S_{11} = \frac{b_1}{a_1} \Big|_{a_2=0} = \frac{V_1 - Z_0 \bar{v}_1}{V_1 + Z_0 \bar{v}_1} \\ \bar{v}_1 &= (a_1 - b_1) \frac{1}{\sqrt{Z_0}} S_{21} = \frac{b_2}{a_1} \Big|_{a_2=0} = \frac{\frac{V_2}{Z_0}}{\frac{V_1 + Z_0 \bar{v}_1}{Z_0}} = \frac{2V_2}{V_1 + Z_0 \bar{v}_1} \end{aligned}$$

$$\begin{aligned} S_{22} &= \frac{b_2}{a_2} \Big|_{a_1=0} = \frac{V_2 - Z_0 \bar{v}_2}{V_2 + Z_0 \bar{v}_2} \\ S_{12} &= \frac{b_1}{a_2} \Big|_{a_1=0} = \frac{\frac{V_1}{Z_0}}{\frac{V_2 + Z_0 \bar{v}_2}{Z_0}} = \frac{2V_1}{V_2 + Z_0 \bar{v}_2} \end{aligned}$$

$$\begin{aligned} \Rightarrow \begin{cases} Z_0 \bar{v}_1 = V_1 \left( 1 - \frac{2S_{11}}{1+S_{11}} \right) \\ Z_0 \bar{v}_2 = V_2 \left( 1 - \frac{2S_{22}}{1+S_{22}} \right) \end{cases} \\ \therefore \frac{V_1}{V_2} = \frac{S_{12} (V_2 + Z_0 \bar{v}_2)}{S_{21} (V_1 + Z_0 \bar{v}_1)} = \frac{S_{12}}{S_{21}} \cdot \frac{V_2}{V_1} \cdot \left( \frac{1 - \frac{S_{22}}{1+S_{22}}}{1 - \frac{S_{11}}{1+S_{11}}} \right) \end{aligned}$$

$$\Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{2S_{12}(1+S_{11})}{2S_{21}(1+S_{22})}}, \quad \frac{V_2}{V_1} = \sqrt{\frac{2S_{21}(1+S_{22})}{2S_{12}(1+S_{11})}}$$

$$V_m = \frac{V_s}{2} = \frac{V_1 + Z_0 \bar{v}_1}{2} = \frac{V_1}{2} \left( 2 - \frac{2S_{11}}{1+S_{11}} \right) = V_1 \left( 1 - \frac{S_{11}}{1+S_{11}} \right) = V_1 \cdot \frac{1}{1+S_{11}}$$

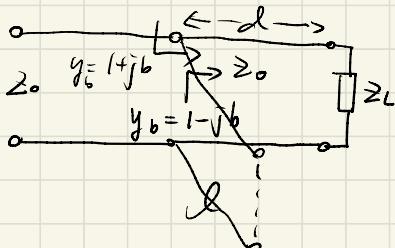
$$\therefore \frac{V_2}{V_m} = \frac{V_2}{V_1} \left( 1 + S_{11} \right) = \sqrt{\frac{2S_{21}(1+S_{22})}{2S_{12}(1+S_{11})}}$$

$$\theta = \tan^{-1}\left(\frac{1.8}{0.8}\right) = 66.04^\circ$$

$$7. f_L = 1 \text{ GHz}, \quad Z_0 = 50 \text{ ohm}, \quad Z_L = 40 + 90j \text{ ohm}, \quad \Rightarrow j_L = 0.8 + 1.8j = 1.97e^{j0^\circ}$$

(a) open stub:

$$Z_{oc} = -j \cot(\beta l) \Rightarrow y_{oc} = j \tan(\beta l), \quad \beta = \frac{2\pi}{\lambda}$$



~~$$Z_{oc} = \frac{1}{2\pi} \tan^{-1}(-Y_1) \lambda = -0.176\lambda + 0.5\lambda$$~~

~~$$= 0.324\lambda$$~~

~~$$Z_{oc2} = \frac{1}{2\pi} \tan^{-1}(-Y_2) \lambda = 0.01/\lambda$$~~

~~$$(b) Z_{sc} = j \tan(\beta l) \Rightarrow y_{sc} = -j \cot(\beta l), \quad l_{sc1} = \frac{1}{2\pi} \tan^{-1}\left(\frac{1}{Y_1}\right) \lambda = 0.426\lambda$$~~

$$l_{sc2} = \frac{1}{2\pi} \tan^{-1} \left( \frac{1}{r_2} \right) \lambda = 0.07\lambda$$

Use Smith chart. We get  $Z_{b1} = 0.2 - j0.4$ .  $Z_{b2} = 0.2 + j0.4$

$\therefore$  short stub.  $\begin{cases} \cot(\beta l_1) = 0.4 \\ \cot(\beta l_2) = -0.4 \end{cases} \Rightarrow \begin{cases} \beta l_1 = 68.2^\circ \\ \beta l_2 = 291.8^\circ \end{cases} \Rightarrow \begin{cases} l_1 = 0.19\lambda \\ l_2 = 0.81\lambda \end{cases}$

open stub  $\begin{cases} \tan(\beta l_1) = 0.4 \\ \tan(\beta l_2) = -0.4 \end{cases} \Rightarrow \begin{cases} \beta l_1 = 21.8^\circ \\ \beta l_2 = -338.2^\circ \end{cases} \Rightarrow \begin{cases} l_1 = 0.06\lambda \\ l_2 = 0.94\lambda \end{cases}$

$\therefore \begin{cases} 2\beta d_1 = 219.8^\circ \Rightarrow d_1 = 0.61\lambda \\ 2\beta d_2 = 272.6^\circ \Rightarrow d_2 = 0.76\lambda \end{cases}$

