① Discrete random variable:

What is it? Finite number of values $a < z < b$, not necessarily integer

② Notation

$f(Z = z) = [z]$

Support: all values of $z$ for which $[z] > 0$

③ Probability mass function:

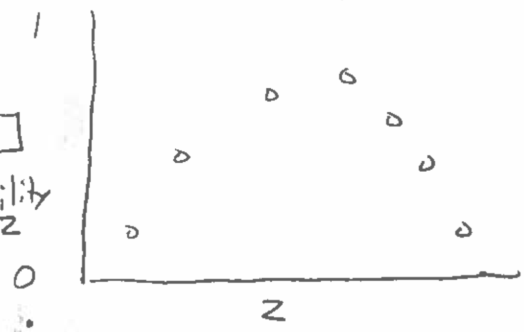
A function for which ; 2 things

$[z] \geq 0$

$\sum_{z \in S} z = 1$

Probability mass function

note scale → $[z]$ Probability of $z$



④ Example using Species Richness on many plots:

$[z|\lambda] = \dfrac{\lambda^z e^{-\lambda}}{\boxed{z!}}$

Moment generating function $j = 1, 2, 3, 4$

$M_j = \sum_{z \in S} (z - c)^j [z]$

Moments

$c = 0$, $j = 1$

$E(z) = \mu = \sum_{z \in S} z [z]$

Expected value or mean

First moment

approximated by many random draws from the distribution $z$ using $\frac{1}{n} \sum_{i=1}^{n} z_i$

$c = \mu$, $j = 2$

$E(z - \mu)^2 = \sigma^2 = \sum_{z \in S} (z - \mu)^2 [z]$ approximated as $\frac{1}{n} \sum_{i=1}^{n} (z_i - \mu)^2$

Second central moment

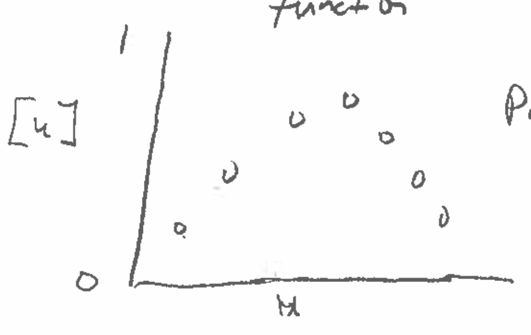
variance

other moments

$j = 3$ skewness

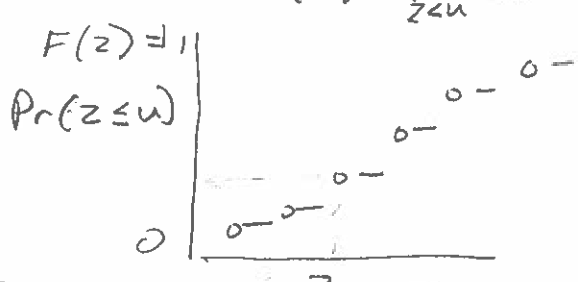
$j = 4$ kurtosis "fatness of tails"

Probability mass function



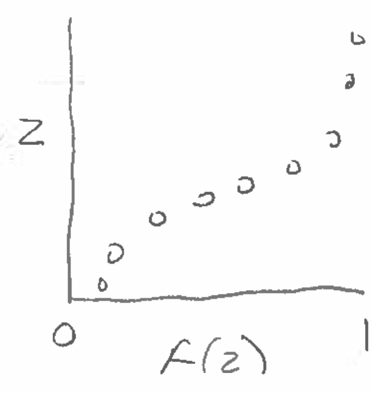
Cumulative distribution function

$F(z) = \sum_{z \leq u} [u]$



Why is it useful?

Quantile function



① Continuous radom varible

$a < z < b$, infinite number of values

Requirements for probability density function

$[z]$ probability density of $z$

① $[z] \geq 0$

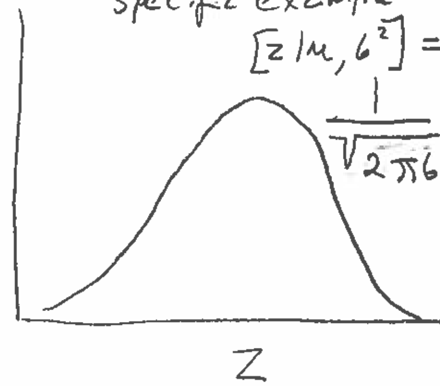
② $\int_a^b [z]\, dz = Pr(a \leq z \leq b)$

③ $\int_{-\infty}^{\infty} [z]\, dz = 1$

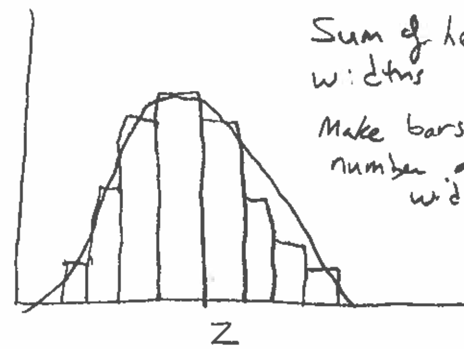
support $[z] \geq 0$

specific example

$[z|\mu, \sigma^2] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$




$[z]$




Sum of heights · widths = 1

Make bars infinite number and 0 width

<u>Moment generating function</u>

$m_j = \int_{-\infty}^{\infty} (z-c)^j [z]dz$

First moment:
Expected value or mean

$c=0$, $j=1$ $\quad m_1 = E(z) = \mu = \int_{-\infty}^{\infty} z[z]\, dz$

approximated as $\frac{1}{n}\sum_{i=1}^{n} z_i$

Second central moment:

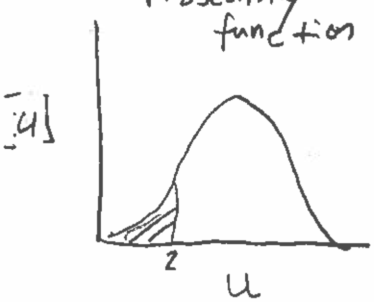
$c=\mu$, $j=2$ $\quad m_2 = E(z-\mu)^2 = \sigma^2 = \int_{-\infty}^{\infty} (z-\mu)^2 [z]dz$

approximated as $\frac{1}{n}\sum_{i=1}^{n} (z_i-\mu)^2$

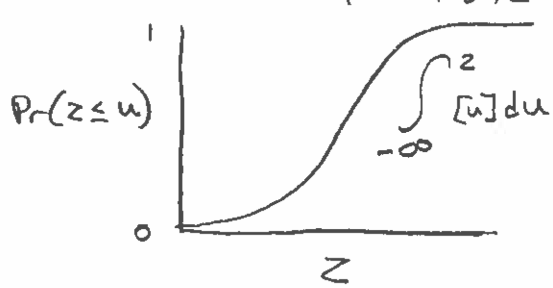
Probability density are values on the continuous curve such that the area under the curve = 1

<u>Scaling of y axis</u>

Probability density function




$F(z)$ = Cummulative distribution function =




$F^{-1}(z)$ = quantile function