

① Discrete random variable:

What is it? Finite number of values $a \leq z \leq b$

② Notation

$$f(Z=z) = [z]$$

Support all values of z for which $[z] > 0$

Probability mass function

③ Probability mass function: not necessarily integer

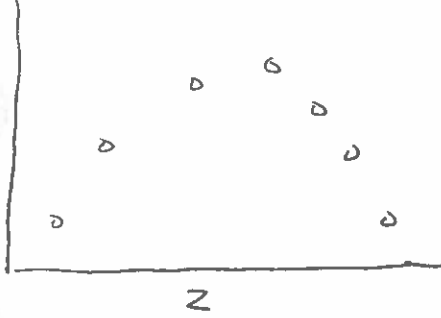
A function for which: 2 things

$$[z] \geq 0$$

$$\sum_{z \in S} [z] = 1$$

note scale \rightarrow

$[z]$
Probability of z



④ Example using Species Richness on many plots:

$$[z|\lambda] = \frac{\lambda^z e^{-\lambda}}{z!}$$

Moment generating function

$j = 1, 2, 3, 4$

$$M_j = \sum_{z \in S} (z - c)^j [z]$$

Moments

$c = 0$
 $j = 1$

$$E(z) = \mu = \sum_{z \in S} z [z]$$

Expected value or mean

First moment

approximated by many random draws from the distribution of z using

$$\frac{1}{n} \sum_{i=1}^n z_i$$

$c = \mu$
 $j = 2$

$$E(z - \mu)^2 = \sigma^2 = \sum_{z \in S} (z - \mu)^2 [z]$$

Second central moment
variance

approximated as

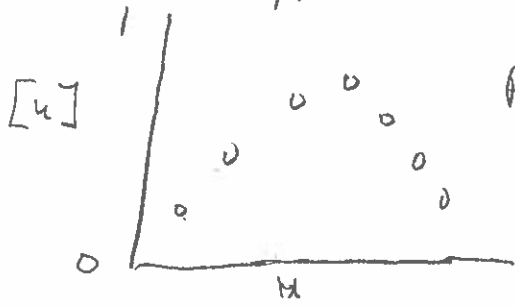
$$\frac{1}{n} \sum_{i=1}^n (z_i - \mu)^2$$

Other moments

$j = 3$ skewness

$j = 4$ kurtosis "fatness of tails"

Probability mass function



Cumulative distribution function

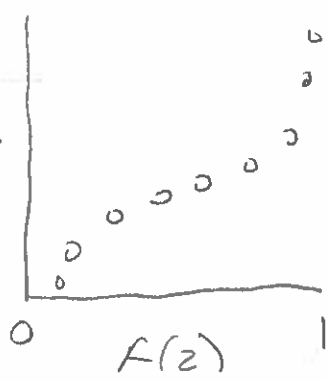
$$F(z) = \sum_{z \leq u} [u]$$

$$F(z) = 1$$

$$\Pr(z \leq u)$$

when is it useful?

Quantile function



① Continuous random variable
 $a < z < b$, infinite number of values

Requirements for probability density function

① $[z] \geq 0$

② $\int_a^b [z] dz = \Pr(a \leq z \leq b)$

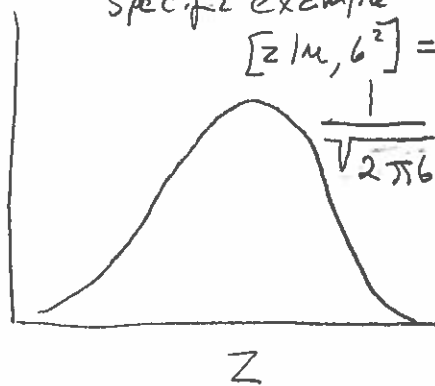
③ $\int_{-\infty}^{\infty} [z] dz = 1$

$[z]$
 probability density of z

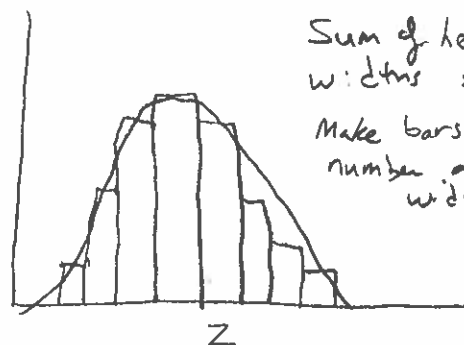
support $[z] \geq 0$

specific example

$[z|m, \sigma^2] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-m)^2}{2\sigma^2}}$



$[z]$



Sum of heights \cdot widths = 1

Make bars infinite number and 0 width

Moment generating function

$m_j = \int_{-\infty}^{\infty} (z-c)^j [z] dz$

First moment:

Expected value or mean

$c=0$ $m_1 = E(z) = \mu = \int_{-\infty}^{\infty} z [z] dz$
 approximated as $\frac{1}{n} \sum_{i=1}^n z_i$

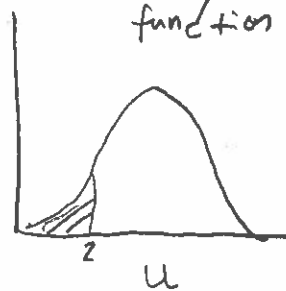
Probability density are values on the continuous curve such that the area under the curve = 1

Scaling of y axis

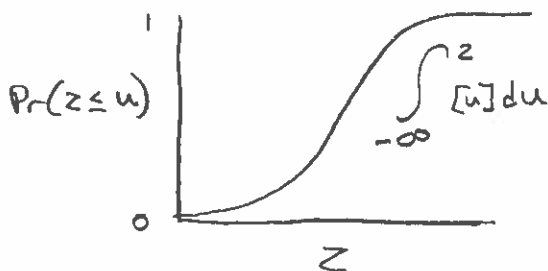
Second central moment:

$c=\mu$ $m_2 = E(z-\mu)^2 = \sigma^2 = \int_{-\infty}^{\infty} (z-\mu)^2 [z] dz$
 $j=2$ approximated as $\frac{1}{n} \sum_{i=1}^n (z_i - \mu)^2$

Probability density function



$F(z)$ = Cumulative distribution function =



$F'(z)$ = quantile function

