① Simple Bayes

$$g(\alpha, \gamma, c, x_i) = \frac{\alpha(x_i - c)}{\frac{\alpha}{\gamma} + (x_i - c)}$$

↳ height increment

Y_1, x_1	$Y_4 X_4$
	Y_2, x_2 etc
Y_3, x_3	

x's
are measured
perfectly

$$x_i \dashrightarrow Y_i$$

$\alpha \quad \gamma \quad c \quad 6_p^2$
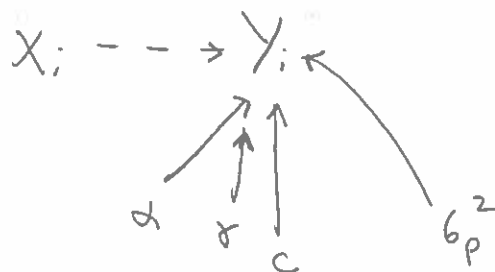
Interpretation of
6^2: process varian
because Y_i are
measured perfectly

$$[\alpha, \gamma, c, 6^2 \mid \underline{Y}] \propto \prod_{i=1}^{n} [Y_i \mid g(\alpha, \gamma, c, x_i, 6^2] [\alpha][\gamma][c][6_p^2]$$

← put these here

$$Y_i \sim normal\left(g(\alpha, \gamma, c, x_i), 6^2\right)$$

$$\alpha \sim gamma\left(\frac{35^2}{4.25^2}, \frac{35}{4.25^2}\right)$$

$$\gamma \sim uniform(0, 10)$$

$$6_p \sim uniform(0, 50) \quad \leftarrow mention\ inverse\ gamma$$
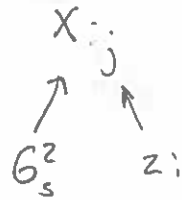
talk about alternative notations for likelihood?

$$[Y_i \mid \mu_i, 6^2] \qquad \mu_i = \alpha$$

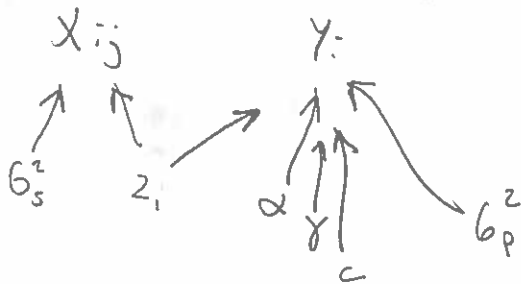$$\overline{[Y_i \mid \theta]} \qquad \theta = (\gamma, \alpha, c, 6^2)$$

$$[Y \mid \alpha, \gamma, c, 6^2]$$

(2) ~~Hierarchical~~ — Errors in te x's. We have

$j = 1-8$ replicate measures of light each tree

How would we estimate te mea light seen by the tree?

$$X_{\cdot j} \xleftarrow{} \quad \xleftarrow{}$$
$$\sigma_s^2 \qquad z_i$$

where z_i is the mean

We now attach this to our model

$$X_{\cdot j} \qquad Y_\cdot$$
$$\sigma_s^2 \quad z_i \quad \alpha \quad \gamma \quad c \quad \sigma_p^2$$

Talk about σ_s^2 at individual tree level

note!
z_i here

$$[\sigma_s^2, \underline{z}, \alpha, \gamma, c, \sigma_p^2 \mid \underline{x}, \underline{y}] \propto \prod_{i=1}^{n} \prod_{j=1}^{8} [y_i \mid g(\alpha, \gamma, c, x_i), z_i] [x_{\cdot j} \mid z_i, \sigma_s^2]$$

matrix
vector

$$\times [z_i][\sigma_s^2][\alpha][\gamma][c][\sigma_p^2]$$

Same priors as before

except we now add

$$z_i \sim \text{uniform}(0, 100)$$
$$\sigma_s \sim \text{uniform}(0, 100)$$

① ask about spect σ_s^2

② show alternative priors

③ show no replication

(3)

Errors in the Y_i:

Calibration equation — model the true mass y of the tree from its **current height** not height inc.

$Mass = M$
$Height = h$

$M = ah^b$

$h = \left(\frac{m}{a}\right)^{\frac{1}{b}}$

Single plot

$$\begin{bmatrix} x,y & & \\ & x,y & x,y \\ x,y & etc & \end{bmatrix}$$

height, h

Mass, M

σ_c^2 is scatter

$\log(h)$

$\log(m)$

$Y_i = height$ of ith tree

height, not inc. height inc.

informal prior

informal prior of a,b

light

σ_c^2

$x_i \dashrightarrow M_i$

true, unobserved mass

α, γ, c

σ_ρ^2
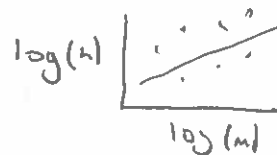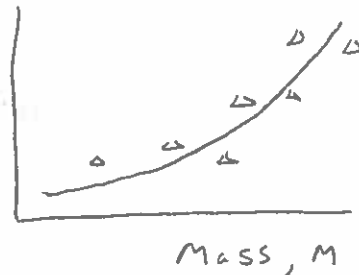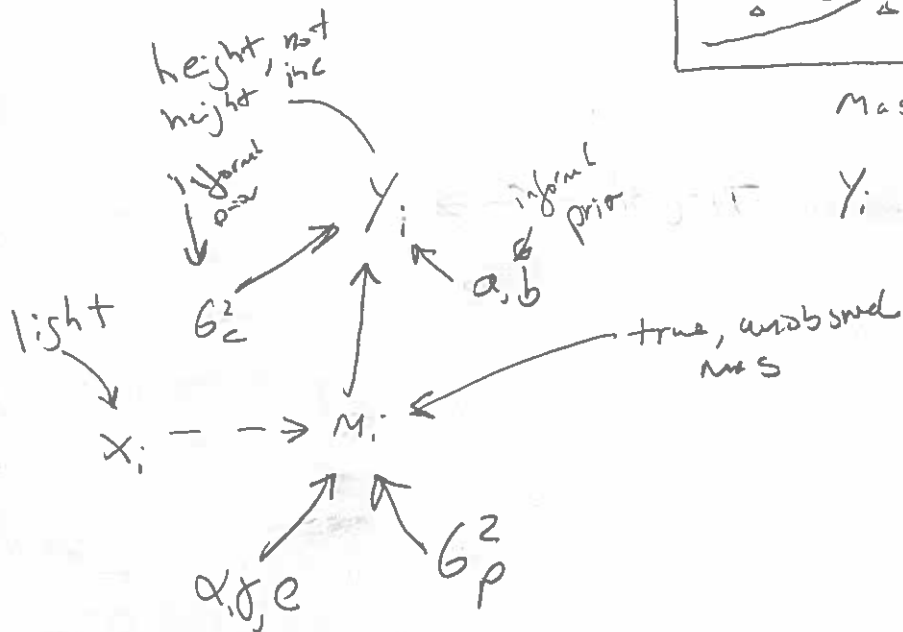
$$\left[\underline{m}, a, b, \sigma_c^2, \alpha, \gamma, c, \sigma_\rho^2 \mid \underline{y} \right] = \prod_{i=1}^{n} \left[Y_i \mid (M_i/a)^{\frac{1}{b}}, \sigma_c^2 \right] \left[M_i \mid g(\alpha, \gamma, c, x_i), \sigma_\rho^2 \right]$$

$$\times [a][b][\sigma_c^2] [\alpha][\gamma][c][\sigma_\rho^2]$$

choices of distributions

$$\log(Y_i) \sim normal\left(\log\left((M_i/a)^{\frac{1}{b}}\right), \sigma_c^2 \right)$$

$$Y_i \sim lognormal\left(\log\left((M_i/a)^{1/b}\right), \sigma_c^2 \right)$$

$$Y_i \sim gamma\left(\frac{(M_i/a)^{\frac{2}{b}}}{\sigma_c^2}, \frac{(M_i/a)^{\frac{1}{b}}}{\sigma_c^2} \right)$$

Be careful here!

on log scale will **not** work! unless regression is done on log scale

what could you use to model mass?

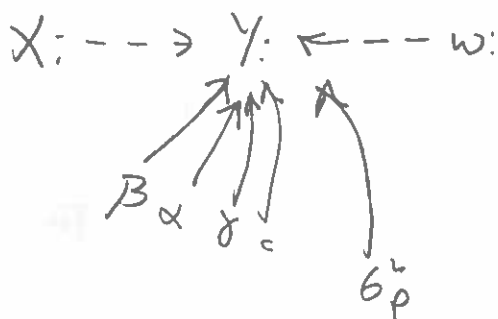(4)   Differences due to treatment

$$g(\alpha, \gamma, c, \beta) = \frac{\alpha(X_i - c)}{\frac{\alpha}{\gamma} + (X_i - c)} + \beta w_i$$

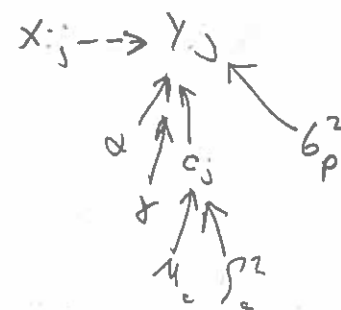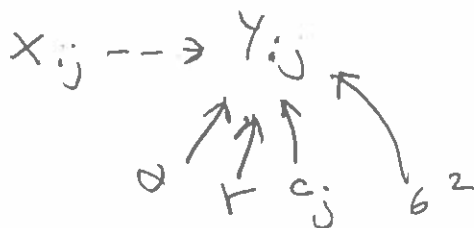treatment for ith tree
$w_i = 0$ if control
$w_i = 1$ if treated

$$X_i \dashrightarrow Y_i \dashleftarrow w_i$$

$\beta \quad \alpha \quad \gamma \quad c$

σ_p^2

in shade tolerance

Differences between species

$i = $ individual
$j = $ species

< 4 species

$$X_{ij} \dashrightarrow Y_{ij}$$

$\alpha \quad \gamma \quad c_j \quad \sigma^2$

> 4 species

$$X_{ij} \dashrightarrow Y_{ij}$$

$\alpha \quad c_j \quad \sigma_p^2$

γ

$\mu_c \quad S_c^2$
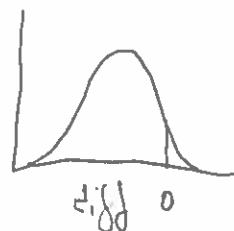
posterior distribution of difference
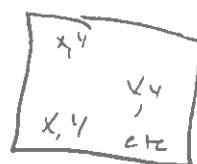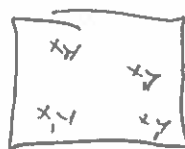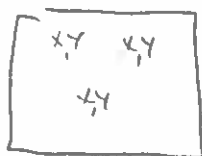
$$diff = C_1 - C_2 \quad [diff]$$

diff   0

(5) Multi-level model including effect g
locations

$j = 1, ..., S$ sites
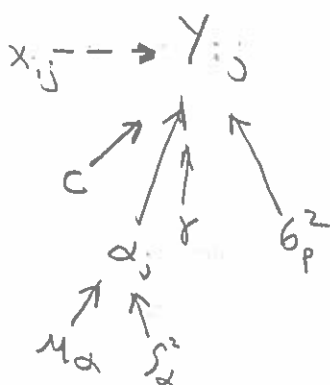
4 knowledge differences :-

Put on → board

What are the options for modeling α_j?

① pooled
② individual

$x_{ij} \dashrightarrow Y_{ij}$

ith plant in at location plot j

$[\alpha_j | \mu_\alpha, S_\alpha^2]$

α
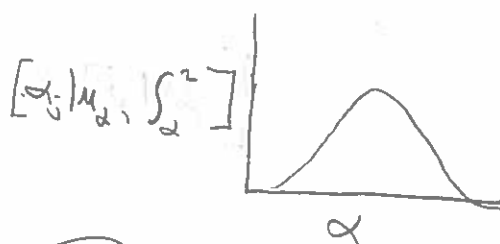
Talk about borrowing strength

model

note vector →

$$[\underline{\alpha}, \gamma, c, \mu_\alpha, S_\alpha^2, \sigma^2 | \underline{Y}] \propto \prod_{i=1}^{n_j} \prod_{j=1}^{J} [Y_{ij} | g(\alpha_j, \gamma, c, x_{ij}), \sigma^2] [\alpha_j | \mu_\alpha, S_\alpha^2]$$

$$\times [\gamma] [c] [\sigma_p^2] [S_\alpha^2] [\mu_\alpha]$$

<u>Note no prior on α_j</u>

$Y_{ij} \sim normal(g(\alpha_j, \gamma, c, x_{ij}), \sigma^2)$
$\gamma \sim uniform(0, 10)$
$\sigma_p \sim uniform(0, 50)$
$S_\alpha \sim uniform(0, 50)$
$\mu_\alpha \sim$ gamma(.001, .001) or $\mu_\alpha \sim gamma\left(\frac{35^2}{4.25^2}, \frac{35}{4.25^2}\right)$

→ Cont make tose broader

7th Ask about σ^2, how to model at plot scale

Multi-level with location data, covariates
for each plot, say soil ~~Nutrient~~ water

— add u_j to diagram to represent site
data, assume normal
Poperly

$U_j \dashrightarrow X_{ij} \dashrightarrow Y_{ij}$

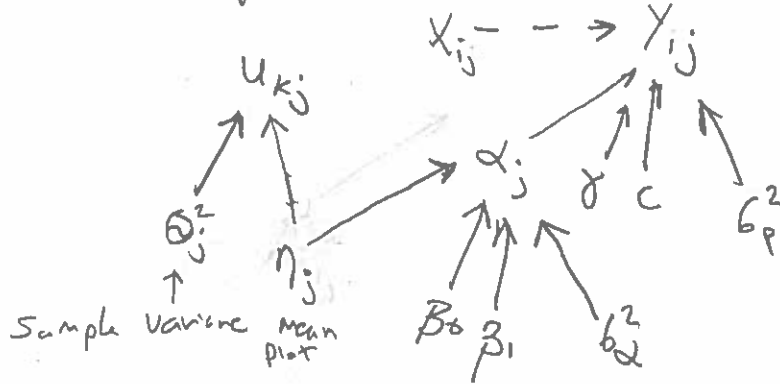$\alpha_j \quad \gamma \quad c \quad \sigma_p^2$

$\beta_0, \beta_1 \; \sigma_\alpha^2$
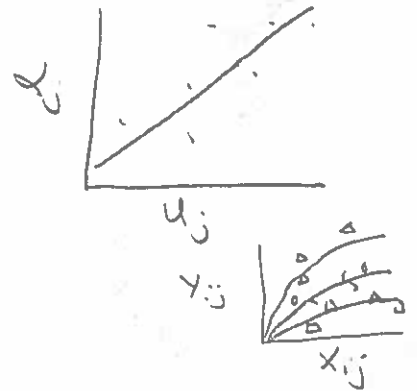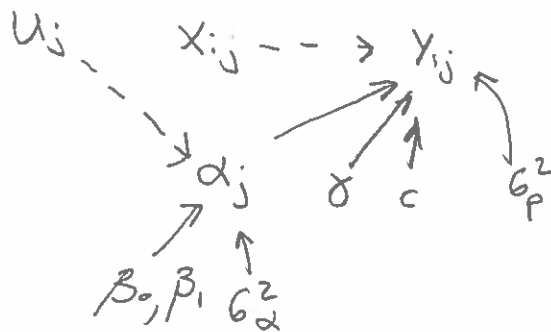
$$\left[\underline{\alpha}, \gamma, c, \beta_0, \beta_1, \sigma_\alpha^2, \sigma_p^2 \mid \underline{Y} \right] \propto \prod_{i=1}^{n_j} \prod_{j=1}^{J} \left[Y_{ij} \mid g(\alpha_j, \gamma, c) \, \sigma_p^2 \right] \left[\alpha_j \mid \beta_0 + \beta_1 u_j, \sigma_\alpha^2 \right]$$

$$\times \left[\gamma \right] \left[c \right] \left[\sigma_p^2 \right] \left[\beta_0 \right] \left[\beta_1 \right] \left[\sigma_\alpha^2 \right]$$

normal
priors

what if we have replicate^{10} measures of soil water?
(samples)
(error in covariate)

$X_{ij} \dashrightarrow Y_{ij}$

u_{kj}

$\alpha_j \quad \gamma \quad c \quad \sigma_p^2$

Talk about
spatial interpolation

$\sigma_u^2 \quad \eta_j$

$\beta_0 \; \beta_1 \quad \sigma_\alpha^2$

Sample variance Mean plot

notij on
v_j

$$\left[\underline{\eta}, \underline{\sigma^2}, \beta_0, \beta_1, \underline{\alpha}, \gamma, c, \sigma_p^2 \mid \underline{u}, \underline{Y} \right] \propto \prod_{i=1}^{n_j} \prod_{j=1}^{J} \prod_{k=1}^{10} \left[Y_{ij} \mid g(\alpha_j, \gamma, c, x_{ij}) \, \sigma_p^2 \right] \left[\alpha_j \mid \beta_0 + \beta_1 \eta_j, \sigma_j^2 \right]$$

$$\left[\sigma_j^2 \right] \left[\eta_j \right] \left[\beta_0 \right] \left[\beta_1 \right] \left[\sigma_\alpha^2 \right] \left[\gamma \right] \left[c \right] \left[\sigma_p^2 \right]$$