$$[\boldsymbol{\lambda}, \alpha, \beta|\mathbf{y}] \propto \prod_{i=1}^{n} \text{Poisson}\left(y_i|\lambda_i\right) \boxed{\text{gamma}\left(\lambda_i|\alpha, \beta\right)} \text{gamma}\left(\beta|.001, .001\right) \boxed{\text{gamma}\left(\alpha|.001, .001\right)}$$

Writing the full-conditional distribution for α:

$$[\alpha \mid \cdot] \propto \prod_{i=1}^{n} \text{gamma}(\lambda_i \mid \alpha, \beta)\text{gamma}(\alpha \mid .001, .001)$$