1

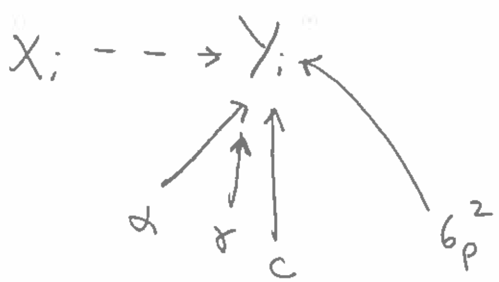
Simple Bayes

Y_1, X_1 $\quad Y_2, X_2 \quad Y_3, X_3 \quad Y_4 X_4$ etc

$$g(\alpha, \gamma, c, x_i) = \frac{\alpha(x_i - c)}{\frac{\alpha}{\gamma} + (x_i - c)}$$

height increment

X's are measured perfectly



Interpretation of σ^2: process variance because Y_i are measured perfectly

$$[\alpha, \gamma, c, \sigma^2 \mid \underline{y}] \propto \prod_{i=1}^{n} [y_i \mid g(\alpha, \gamma, c, x_i), \sigma^2] \underbrace{[\alpha][\gamma][c][\sigma_p^2]}_{\text{put these here}}$$

$$Y_i \sim \text{normal}\left(g(\alpha, \gamma, c, x_i), \sigma^2\right)$$

$$\alpha \sim \text{gamma}\left(\frac{35^2}{4.25^2}, \frac{35}{4.25^2}\right)$$

$$\gamma \sim \text{Uniform}(0, 10)$$

$$\sigma_p \sim \text{uniform}(0, 50)$$ ← mention inverse gamma

talk about alternative notations for likelihood?

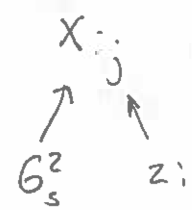
$[y_i \mid \mu_i, \sigma^2]$ $\qquad \mu_i = \alpha$

~~$[y_i \mid \theta]$~~ $\qquad \theta = (\gamma, \alpha, c, \sigma^2)$

$[y \mid \alpha, \gamma, c, \sigma^2]$

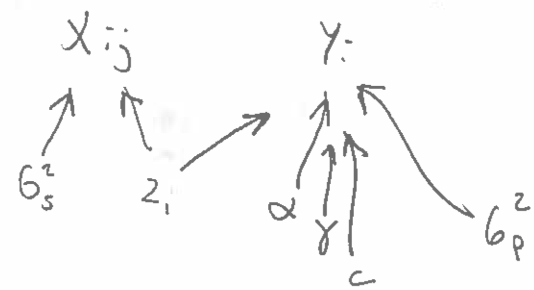
② — Errors in the x's. We have $j = 1 \ldots 8$ replicate measures of light each tree

How would we estimate the mean light seen by the tree?



where z_i is the mean

We now attach this to our model



Talk about σ_s^2 at individual tree level

matrix

note! z_i here

$$[\sigma_s^2, \underline{z}, \alpha, \gamma, c, \sigma_p^2 \mid \underline{X}, \underline{y}] \propto \prod_{i=1}^{n} \prod_{j=1}^{8} [y_i \mid g(\alpha, \gamma, c, x_i), z_i]\,[x_{ij} \mid z_i, \sigma_s^2]$$

vector

$$\times [z_i][\sigma_s^2][\alpha][\gamma][c][\sigma_p^2]$$

Same priors as before except we now add

$z_i \sim \text{uniform}(0, 100)$

$\sigma_s \sim \text{uniform}(0, 100)$

① ask about sep. est. σ_s^2

② show alternative priors

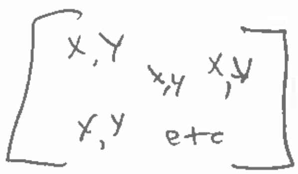
③ show no replication

3

Errors in the Y_i

Calibration equation — model the tree mass of the tree from its ~~current~~ height not height inc.

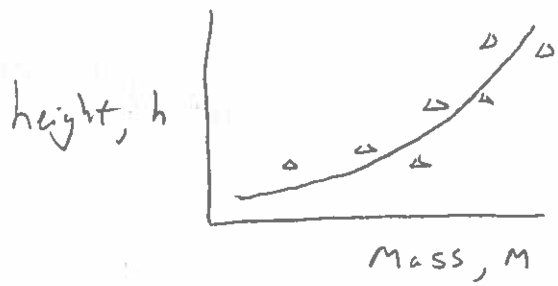
Single plot



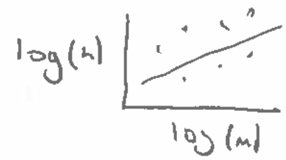
Mass = m
Height = h

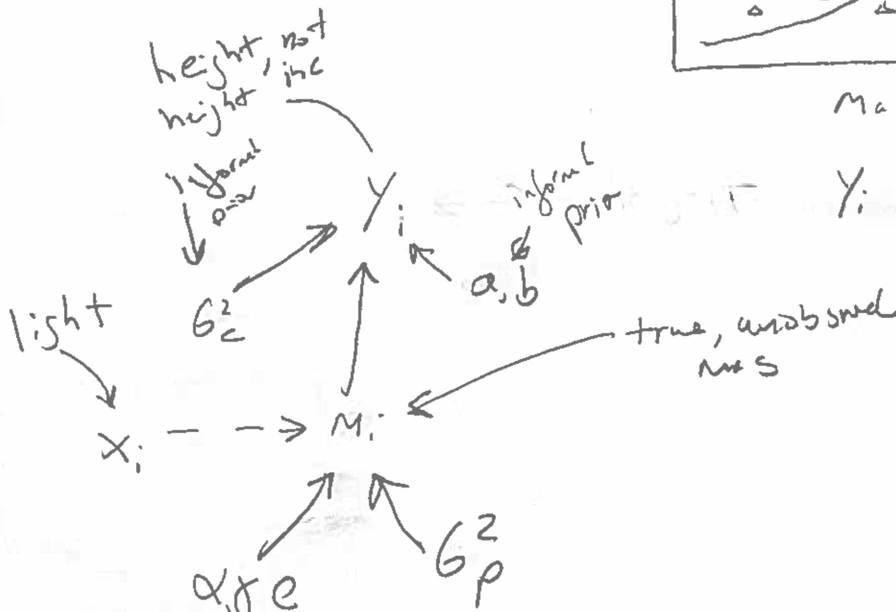
$m = a h^b$

$h = \left(\frac{m}{a}\right)^{\frac{1}{b}}$



σ_c^2 is scatter





Y_i = height of ith tree

$$[\underline{M}, a, b, \sigma_c^2, \alpha, \gamma, c, \sigma_p^2 \mid \underline{Y}] = \prod_{i=1}^{n} \left[Y_i \mid (M_i/a)^{\frac{1}{b}}, \sigma_c^2\right]\left[M_i \mid g(\alpha, \gamma, c, x_i), \sigma_p^2\right]$$

$$\times [a][b][\sigma_c^2][\alpha][\gamma][c][\sigma_p^2]$$

Choices of distributions

$\log(Y_i) \sim \text{normal}\left(\log\left((M_i/a)^{\frac{1}{b}}\right), \sigma_c^2\right)$

$Y_i \sim \text{lognormal}\left(\log\left((M_i/a)^{1/b}\right), \sigma_c^2\right)$

$Y_i \sim \text{gamma}\left(\frac{(M_i/a)^{\frac{2}{b}}}{\sigma_c^2}, \frac{(M_i/a)^{\frac{1}{b}}}{\sigma_c^2}\right)$ not

Be careful here!

on log scale will not work! unless regression is done on log scale

What could you use to model mass?

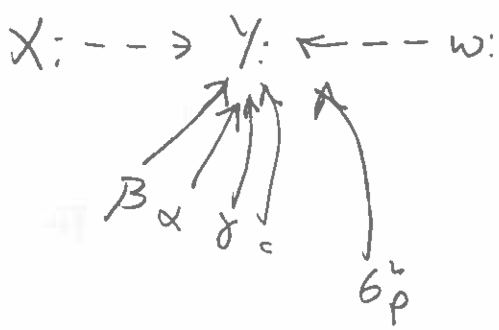
(4) Differences due to treatment

$$g(\alpha,\gamma,c,\beta) = \frac{\alpha(x_i - c)}{\frac{\alpha}{\gamma} + (x_i - c)} + \beta w_i$$

treatment for ith tree

$w_i = 0$ if control

$w_i = 1$ if treated

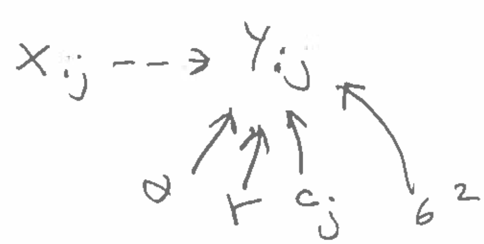


Differences in shade tolerance between species

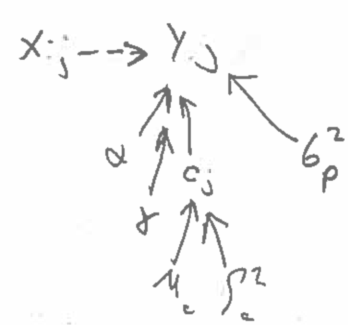
i = individual

j = species

< 4 species

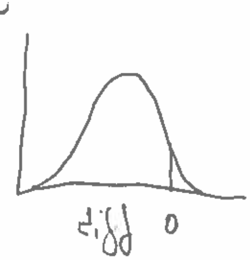


> 4 species



posterior distribution of difference

$$diff = c_1 - c_2 \quad [diff]$$



5

Multi-level model including effect of locations

$j = 1, \ldots, J$ sites

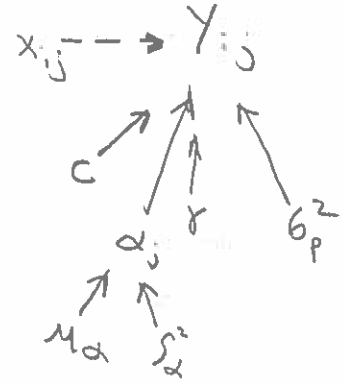
Acknowledge differences ...

Put on board →

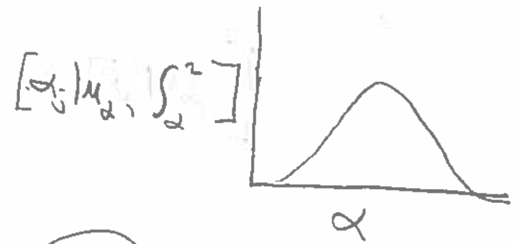
What are the options for modeling α_j?

(1) pooled

(2) individual



ith plant at location ~~plot~~ j



Talk about borrowing strength

model

note vector →

$$[\underline{\alpha}, \gamma, c, \mu_\alpha, \varsigma_\alpha^2, \sigma^2 \mid \underline{y}] \propto \prod_{i=1}^{n_j} \prod_{j=1}^{J} [y_{ij} \mid g(\alpha_j, \gamma, c, x_{ij}), \sigma^2][\alpha_j \mid \mu_\alpha, \varsigma_\alpha^2]$$

$$\times [\gamma][c][\sigma_p^2][\varsigma_\alpha^2][\mu_\alpha]$$

Note no prior on α_j

$y_{ij} \sim \text{normal}(g(\alpha_j, \gamma, c, x_{ij}), \sigma^2)$

$\gamma \sim \text{uniform}(0, 10)$

$\sigma_p \sim \text{uniform}(0, 50)$

$\varsigma \sim \text{uniform}(0, 50)$

$\mu_\alpha \sim \text{gamma}(.001, .001)$ or $\mu_\alpha \sim \text{gamma}\left(\frac{35^2}{4.25^2}, \frac{35}{4.25^2}\right)$

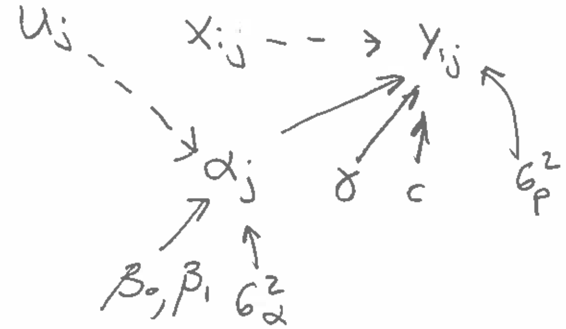
Could make these broader

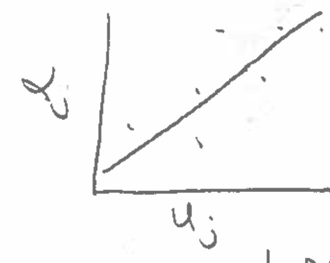
Ask about σ^2, how to model at plot scale

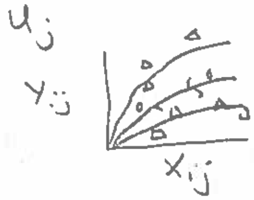
(6)

Multi-level with location data, covariates for each plot, say soil ~~nitrogen~~ water

— add u_j to diagram to represent site data, assume normal perfectly





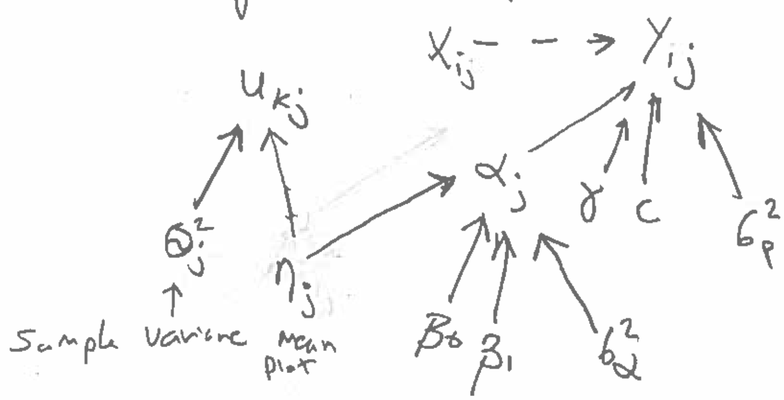


$$[\underline{\alpha},\gamma,c,\beta_0,\beta_1,\sigma^2_\alpha,\sigma^2_p \mid \underline{Y}] \propto \prod_{i=1}^{n_j}\prod_{j=1}^{J}[Y_{ij}\mid g(\alpha_j,\gamma,c)\,\sigma^2]\,[\alpha_j\mid\beta_0+\beta_1 u_j,\sigma^2_\alpha]$$

$$\times[\gamma][c][\sigma^2_p][\beta_0][\beta_1][\sigma^2_\alpha]$$

normal priors

What if we have replicate (10) measures of soil water? (sampling error in covariate)



Talk about spatial interpolation

note j on α_j

$$[\underline{\eta},\underline{\Theta}^2,\beta_0,\beta_1,\underline{\alpha},\gamma,c,\sigma^2_p\mid \underline{u},\underline{Y}]\propto\prod_{i=1}^{n_j}\prod_{j=1}^{J}\prod_{k=1}^{10}[Y_{ij}\mid g(\alpha_j,\gamma,c,x_{ij})\,\sigma^2_p]\,[\alpha_j\mid\beta_0+\beta_1\eta_j,\Theta^2_j]$$

$$[\Theta^2_j][\eta_j][\beta_0][\beta_1][\sigma^2_\alpha][\gamma][c][\sigma^2_p]$$