1) Discrete random variable:

what is it? Finite number of values $a < z < b$, not necessarily integer

2) Notation

$$f(Z = z) = [z]$$

3) Probability mass function:

A function for which ; 2 things

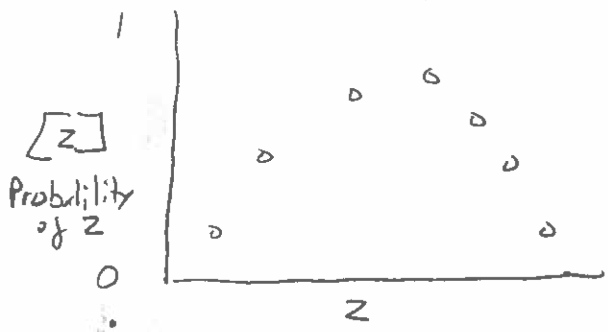
$$[z] \geq 0$$

$$\sum_{z \in S} z = 1$$

Support: all values of z for which $[z] > 0$ and defined

Probability mass function

note scale →



4) Example using Species Richness on many plots:

$$[z|\lambda] = \frac{\lambda^z e^{-\lambda}}{z!}$$

Moment generating function $j = 1, 2, 3, 4$

$$M_j = \sum_{z \in S} (z - c)^j [z]$$

Moments

$c = 0$
$j = 1$

$$E(z) = \mu = \sum_{z \in S} z[z]$$

Expected value or mean

First moment

approximated by many random draws from the distribution z using $\frac{1}{n} \sum_{i=1}^{n} z_i$

$c = \mu$
$j = 2$

$$E(z - \mu)^2 = \sigma^2 = \sum_{z \in S} (z - \mu)^2 [z]$$

Second central moment

variance

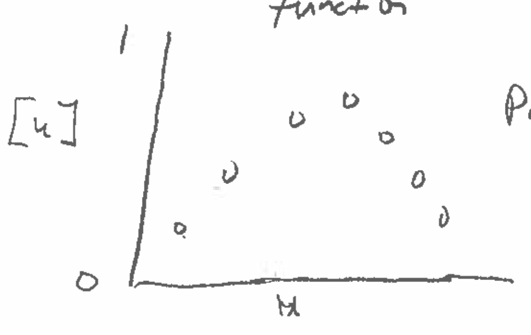
approximated as $\frac{1}{n} \sum_{i=1}^{n} (z_i - \mu)^2$

other moments

$j = 3$ skewness

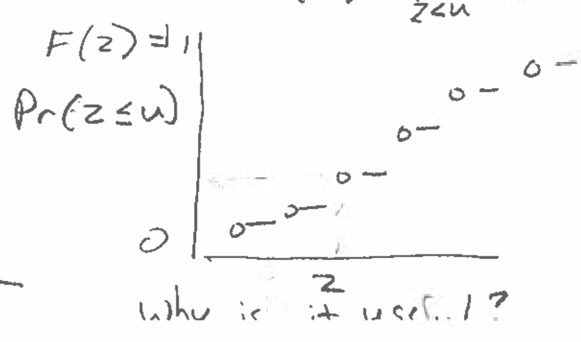
$j = 4$ kurtosis "fatness of tails"

Probability mass function

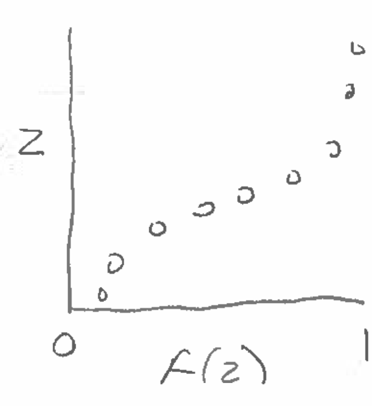


Cummulative distribution function

$$F(z) = \sum_{z \leq u} [u]$$



Quantile function



why is it useful?

① Continuous random variable

$a < z < b$, infinite number of values

Requirements for probability density function

① $[z] \geq 0$

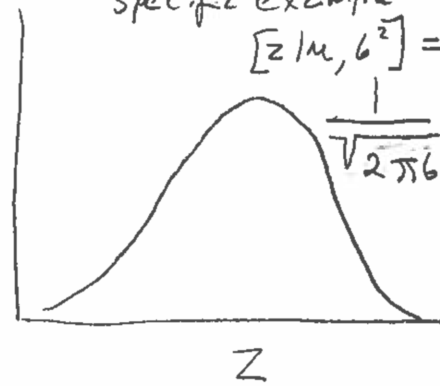
② $\int_a^b [z]\,dz = \Pr(a \leq z \leq b)$

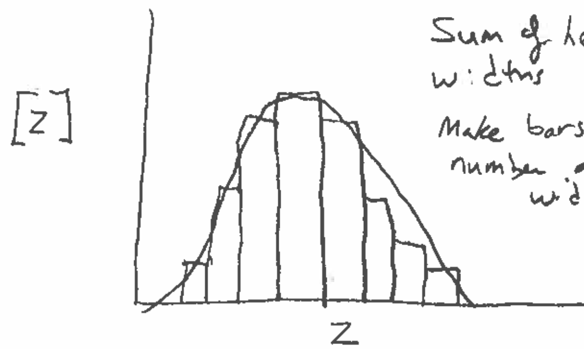
③ $\int_{-\infty}^{\infty} [z]\,dz = 1$

$[z]$ probability density of z

Support $[z] \geq 0$ and defined

specific example $[z \mid \mu, \sigma^2] = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(z-\mu)^2}{2\sigma^2}}$





Sum of heights · widths = 1

Make bars infinite number and 0 width

Probability density are values on the continuous curve such that the area under the curve = 1

Scaling of y axis

Moment generating function

$$m_j = \int_{-\infty}^{\infty} (z-c)^j [z]\,dz$$

First moment:
Expected value or mean

$c=0$, $j=1$: $M_1 = E(z) = \mu = \int_{-\infty}^{\infty} z[z]\,dz$

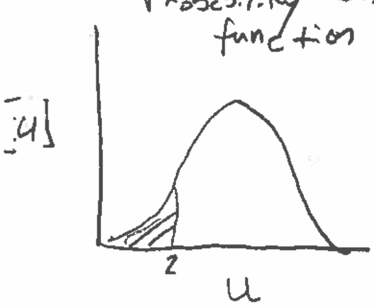
approximated as $\frac{1}{n}\sum_{i=1}^{n} z_i$

Second central moment:

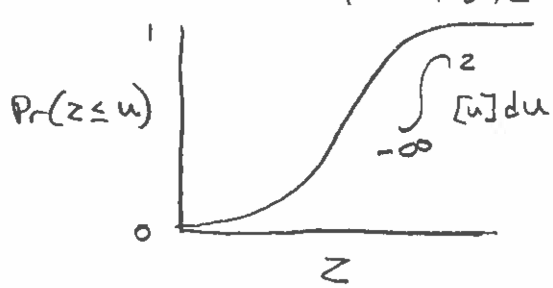
$c=\mu$, $j=2$: $M_2 = E(z-\mu)^2 = \sigma^2 = \int_{-\infty}^{\infty} (z-\mu)^2 [z]\,dz$

approximated as $\frac{1}{n}\sum_{i=1}^{n} (z_i-\mu)^2$

Probability density function



$F(z)$ = Cummulative distribution function = $\int_{-\infty}^{z} [u]\,du$



$F'(z)$ = quantile function