CS 111: Review Quiz: Due by 6:00pm Monday, April 8

Note: You will get full credit for this quiz just by handing it in. The purpose of the quiz is to review some of the math background for the class, and to help you decide if there is anything you want to brush up on. If any of these questions baffles you, it will be easier to pick up the concept right now than struggle with it later. The TAs will answer questions about the quiz in discussion section on April 11.

1. Let

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & -1 \end{pmatrix}.$$

What is A^T? What is A^2? What is $A^T A$? (Do these computations both by hand and with `numpy`.)

$$A^T = \begin{pmatrix} 3 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & 2 & -1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 3 & -1 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 3 & -1 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 11 & -4 & 2 \\ 2 & 1 & 0 \\ 2 & -1 & 3 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 3 & -1 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} 3 & -1 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & -1 \end{pmatrix} = \begin{pmatrix} 10 & -3 & 5 \\ -3 & 2 & 0 \\ 5 & 0 & 9 \end{pmatrix}$$

2. The notation $||x||_2$ means the Euclidean norm (also called the 2-norm, or just the length) of the vector x. What is $||(3,1,4,1,5)^T||_2$?

$$||(3,1,4,1,5)^T||_2 = \sqrt{3^2 + 1^2 + 4^2 + 1^2 + 5^2} = \sqrt{52}$$

3. Consider the following system of three equations in three unknowns.

$$\begin{aligned} 2x_1 - 3x_2 + x_3 &= 1 && (1) \\ 2x_2 + 3x_3 &= 7 && (2) \\ x_1 + x_3 &= 4 && (3) \end{aligned}$$

First write this system in the form $Ax = b$, where A is a matrix and x and b are vectors. Second, write two lines of `numpy` code that use `np.array()` to create A and b as `numpy` arrays.

$$\begin{bmatrix} 2 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 4 \end{bmatrix}$$

$A = np.array([2,-3,1],[0,2,3],[1,0,1]])$
$b = np.array([1,7,4])$

4. What vector x solves the system above?

$$x = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

5. Write down a 2-by-2 matrix A and a 2-vector b such that $Ax = b$ has no solution. Explain in a sentence why there are no solutions.

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

6. Write down a 2-by-2 matrix A and a 2-vector b such that $Ax = b$ has more than one solution. Give two different solutions.

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

7. Is there a 2-by-2 matrix A and a 2-vector b such that $Ax = b$ has exactly two solutions? Why or why not?
   No, it has either one solution, no solution, or infinite solution

8. Recall that a number λ is an *eigenvalue* of a matrix A if there is some vector x (an *eigenvector*) for which $Ax = \lambda x$. Give an eigenvalue of the following matrix, and a corresponding eigenvector.

$$A = \begin{pmatrix} 4 & -1 \\ -1 & 4 \end{pmatrix}.$$

$$\lambda_1 = 5, v_1 = [0.70710678, 0.70710678]$$

$$\lambda_2 = 3, v_2 = [-0.70710678, 0.70710678]$$

9. If $f(x) = 7x^3 - 2x^2 + 4x - 5$, what is $f'(x)$, the derivative of $f(x)$?

$$\frac{\partial f}{\partial x} = 21x^2 - 4x + 4$$

10. If $z = xe^{y/2}$, what is $\partial z/\partial x$? What is $\partial z/\partial y$?

$$\frac{\partial z}{\partial x} = e^{y/2}, \quad \frac{\partial z}{\partial y} = \frac{xy}{2}e^{y/2}$$

11. If $f'(x) = x^2 + \sin x$, what is $f(x)$?

$$f_{(x)} = x^3/3 - cosx$$

12. The height in feet of a bullet fired straight up is given by $h = 1280t - 16t^2$, where t is in seconds. What is the maximum height the bullet will reach? When will it hit the ground?

$$f_{max} = 50560$$

13. Suppose y is a function of x that satisfies $dy/dx = xy$, and also suppose $y = 1$ when $x = 0$. Write y as a function of x.

$$dy/y = xdx$$

$$ln(y) = x^2 + C$$

$$y = e^{x^2+C} = C'e^{x^2}$$

$$y = e^{x^2}$$