In [1]:

```
import os
import math
import time
import struct
import json
import pandas as pd
import networkx as nx
import numpy as np
import numpy.linalg as npla
import scipy
import scipy.sparse.linalg as spla
from scipy import sparse
from scipy import linalg
import matplotlib.pyplot as plt
from matplotlib import cm
from mpl_toolkits.mplot3d import axes3d
%matplotlib tk
```

In [2]:

```
"""
This file is trying to implement and optimaze function to solve pagerank problem, insipred by Prof
essor Ziad Matni.
The purpose of these function is to intentify pivoting websites after crawl, that may contain most
valuable infomation.
The function are being test on following files:
PageRankEG1.npy and PageRankEG2.npy, which are the small graphs (as dense adjacency matrices)
PageRankEG3.npy, which is the graph of the 500-node Harvard web crawl.
PageRankEG3.nodelabels, which lists the 500 Harvard site names
PageRankEG*.npz, which are the same graphs as the three above, but are stored as scipy
csr sparse matrices. You can use these to test your sparse code.

And finally webGoogle.npz, which is the graph of a web crawl of about 900,000 pages.
"""
```

Out[2]:

```
'\nThis file is trying to implement and optimaze function to solve pagerank problem, insipred by P
rofessor Ziad Matni.\nThe purpose of these function is to intentify pivoting websites after crawl,
that may contain most valuable infomation.\nThe function are being test on following
files:\nPageRankEG1.npy and PageRankEG2.npy, which are the small graphs (as dense adjacency matric
es)\nPageRankEG3.npy, which is the graph of the 500-node Harvard web
crawl.\nPageRankEG3.nodelabels, which lists the 500 Harvard site names\nPageRankEG*.npz, which are
the same graphs as the three above, but are stored as scipy\ncsr sparse matrices. You can use thes
e to test your sparse code.\n\nAnd finally webGoogle.npz, which is the graph of a web crawl of
about 900,000 pages.\n'
```

In [3]:

```
def pagerank1(E, return_vector = False, max_iters = 1000, tolerance = 1e-6):
    """compute page rank from dense adjacency matrix

    Inputs:
      E: adjacency matrix with links going from cols to rows.
         E is a matrix of 0s and 1s, where E[i,j] = 1 means
         that web page (vertex) j has a link to web page i.
      return_vector = False: If True, return the eigenvector as well as the ranking.
      max_iters = 1000: Maximum number of power iterations to do.
      tolerance = 1e-6: Stop when the eigenvector norm changes by less than this.

    Outputs:
      ranking: Permutation giving the ranking, most important first
      vector (only if return_vector is True): Dominant eigenvector of PageRank matrix

    This computes page rank by the following steps:
    1. Add links from any dangling vertices to all vertices.
    2. Scale the columns to sum to 1.
    3. Add a constant matrix to represent jumping at random 15% of the time.
    4. Find the dominant eigenvector with the power method.
    5. Sort the eigenvector to get the rankings.
```

```
    The homework problem due February 22 asks you to rewrite this code so
    it takes input E as a scipy csr_sparse matrix, and then never creates
    a full matrix or any large matrix other than E.
    """

    if type(E) is not np.ndarray:
        print('Warning, converting input from type', type(E), 'to dense array.')
        E = E.toarray()

    nnz = np.count_nonzero(E) # This call for sparse E may be different
    outdegree = np.sum(E, 0)  # This call for sparse E may be different
    nrows, n = E.shape

    assert nrows == n, 'E must be square'
    assert np.max(E) == 1 and np.sum(E) == nnz, 'E must contain only zeros and ones'

    #  1. Add links from any dangling vertices to all other vertices.
    #     E + F will be the matrix with the added links.

    F = np.zeros((n,n))
    for j in range(n):
        if outdegree[j] == 0:
            F[:,j] = np.ones(n)
            F[j,j] = 0

    #  2. Scale the columns to sum to 1.

    A = (E + F) / np.sum(E + F, 0)

    #  3. Add a constant matrix to represent jumping at random 15% of the time.

    S = np.ones((n,n)) / n
    m = 0.15
    M = (1 - m) * A + m * S

    #  4. Find the dominant eigenvector.
    #  Start with a vector all of whose entries are equal.

    e = np.ones(n)
    v = e / npla.norm(e)

    for iteration in range(max_iters):
        oldv = v

        v = M @ v
        eigval = npla.norm(v)
        v = v / eigval

        if npla.norm(v - oldv) < tolerance:
            break

    if npla.norm(v - oldv) < tolerance:
        print('Dominant eigenvalue is %f after %d iterations.\n' % (eigval, iteration+1))
    else:
        print('Did not converge to tolerance %e after %d iterations.\n' % (tolerance, max_iters))

    # Check that the eigenvector elements are all the same sign, and make them positive
    assert np.all(v > 0) or np.all(v < 0), 'Error: eigenvector is not all > 0 or < 0'
    vector = np.abs(v)

    #  5. Sort the eigenvector and reverse the permutation to get the rankings.
    ranking = np.argsort(vector)[::-1]

    if return_vector:
        return ranking, vector
    else:
        return ranking
```

In [4]:

```
def pagerank2(E, return_vector = False, max_iters = 1000, tolerance = 1e-6):
    """compute page rank from dense adjacency matrix

    Inputs:
      E: adjacency matrix with links going from cols to rows.
```

```
        E is a matrix of 0s and 1s, where E[i,j] = 1 means
        that web page (vertex) j has a link to web page i.
      return_vector = False: If True, return the eigenvector as well as the ranking.
      max_iters = 1000: Maximum number of power iterations to do.
      tolerance = 1e-6: Stop when the eigenvector norm changes by less than this.

    Outputs:
      ranking: Permutation giving the ranking, most important first
      vector (only if return_vector is True): Dominant eigenvector of PageRank matrix

    This computes page rank by the following steps:
    1. Add links from any dangling vertices to all vertices.
    2. Scale the columns to sum to 1.
    3. Add a constant matrix to represent jumping at random 15% of the time.
    4. Find the dominant eigenvector with the power method.
    5. Sort the eigenvector to get the rankings.

    The homework problem due February 22 asks you to rewrite this code so
    it takes input E as a scipy csr_sparse matrix, and then never creates
    a full matrix or any large matrix other than E.
    """
    """
    if type(E) is not np.ndarray:
        print('Warning, converting input from type', type(E), 'to dense array.')
        E = E.toarray()
    """
    np.seterr(divide='ignore', invalid = 'ignore')
    nnz = E.nnz # This call for sparse E may be different
    outdegree = np.sum(E, 0).A1   # This call for sparse E may be different
    #D = np.diag(1/outdegree)
    nrows, n = E.shape

    assert nrows == n, 'E must be square'
    assert np.max(E) == 1 and np.sum(E) == nnz, 'E must contain only zeros and ones'

    #preparation to save memory
    #E = E@D now is link matrix
    #print(E.toarray())
    ZeroCol = np.where(outdegree == 0)
    e = np.ones(n)
    v = e / npla.norm(e)
    for iteration in range(max_iters):
        oldv = v
        #E*V   E = E/np.sum(E,0)
        v = E.dot(0.85*oldv/outdegree)
        #mSv
        v += 0.15/n*sum(oldv)
        #F*v
        v += sum(oldv[ZeroCol]*(0.85/(n-1)))
        v[ZeroCol] -= oldv[ZeroCol]*(0.85/(n-1))
        v = v/npla.norm(v)
        if npla.norm(v - oldv) < tolerance:
            eigval = np.average(oldv/v)
            break

    if npla.norm(v - oldv) < tolerance:
        print('Dominant eigenvalue is %f after %d iterations.\n' % (eigval, iteration+1))
    else:
        print('Did not converge to tolerance %e after %d iterations.\n' % (tolerance, max_iters))
    # Check that the eigenvector elements are all the same sign, and make them positive
    assert np.all(v > 0) or np.all(v < 0), 'Error: eigenvector is not all > 0 or < 0'
    vector = np.abs(v)

    #  5. Sort the eigenvector and reverse the permutation to get the rankings.
    ranking = np.argsort(vector)[::-1]
    if return_vector:
        return ranking, vector
    else:
        return ranking
```

In [5]:

```
#Comparing the output of PG1 with PG2
E = np.load('PageRankEG1.npy')
r, v = pagerank1(E, return_vector = True)
print('r =', r)
```

```
print('v =', v)
print("maxvalue:",v[r[0]])
print("minvalue:",v[r[-1]])
```

```
Dominant eigenvalue is 1.000000 after 19 iterations.

r = [0 2 3 1]
v = [0.69648305 0.26828106 0.54477799 0.38230039]
maxvalue: 0.6964830451765137
minvalue: 0.2682810590636119
```

In [6]:

```
E = sparse.load_npz('PageRankEG1.npz')
r, v = pagerank2(E, return_vector = True)
print('r =', r)
print('v =', v)
```

```
Dominant eigenvalue is 1.000000 after 19 iterations.

r = [0 2 3 1]
v = [0.69648305 0.26828106 0.54477799 0.38230039]
```

In [7]:

```
E = sparse.load_npz('PageRankEG2.npz')
r, v = pagerank2(E, return_vector = True)
print('r =', r)
print('v =', v)
```

```
Dominant eigenvalue is 1.000000 after 2 iterations.

r = [3 2 1 0 4]
v = [0.40542854 0.40542854 0.57773567 0.57773567 0.06081428]
```

In [8]:

```
E = np.load('PageRankEG3.npy')
sitename = open('PageRankEG3.nodelabels').read().splitlines()
%time r = pagerank1(E)
print('r[:10] =', r[:10])
print()
for i in range(10):
    print('rank %d is page %3d: %s' % (i, r[i], sitename[r[i]]))
```

```
Dominant eigenvalue is 1.000000 after 56 iterations.

CPU times: user 22.8 ms, sys: 10.7 ms, total: 33.5 ms
Wall time: 30.1 ms
r[:10] = [  0   9  41 129  17  14   8  16  45  12]

rank 0 is page   0: http://www.harvard.edu
rank 1 is page   9: http://www.hbs.edu
rank 2 is page  41: http://search.harvard.edu:8765/custom/query.html
rank 3 is page 129: http://www.med.harvard.edu
rank 4 is page  17: http://www.gse.harvard.edu
rank 5 is page  14: http://www.hms.harvard.edu
rank 6 is page   8: http://www.ksg.harvard.edu
rank 7 is page  16: http://www.hsph.harvard.edu
rank 8 is page  45: http://www.gocrimson.com
rank 9 is page  12: http://www.hsdm.med.harvard.edu
```

In [9]:

```
E = sparse.load_npz('PageRankEG3.npz')
sitename = open('PageRankEG3.nodelabels').read().splitlines()
%time r= pagerank2(E)
print('r[:10] =', r[:10])
print()
for i in range(10):
```

```
    print('rank %d is page %3d: %s' % (i, r[i], sitename[r[i]]))
```

```
Dominant eigenvalue is 1.000000 after 56 iterations.

CPU times: user 51.8 ms, sys: 410 µs, total: 52.2 ms
Wall time: 13 ms
r[:10] = [  0   9  41 129  17  14   8  16  45  12]

rank 0 is page   0: http://www.harvard.edu
rank 1 is page   9: http://www.hbs.edu
rank 2 is page  41: http://search.harvard.edu:8765/custom/query.html
rank 3 is page 129: http://www.med.harvard.edu
rank 4 is page  17: http://www.gse.harvard.edu
rank 5 is page  14: http://www.hms.harvard.edu
rank 6 is page   8: http://www.ksg.harvard.edu
rank 7 is page  16: http://www.hsph.harvard.edu
rank 8 is page  45: http://www.gocrimson.com
rank 9 is page  12: http://www.hsdm.med.harvard.edu
```

In [10]:

```
#webGoogle
E = sparse.load_npz('webGoogle.npz')
```

In [11]:

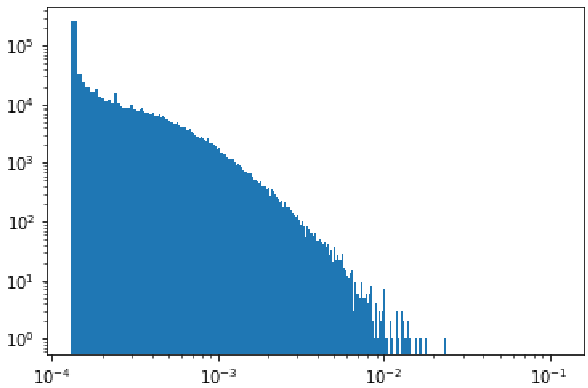
```
%time r, v = pagerank2(E, return_vector = True)
```

```
Dominant eigenvalue is 1.000000 after 71 iterations.

CPU times: user 25.5 s, sys: 143 ms, total: 25.7 s
Wall time: 10.5 s
```

In [12]:

```
%matplotlib inline
plt.hist(v,bins="auto")
plt.gca().set_yscale("log")
plt.gca().set_xscale("log")
```



In [13]:

```
print("maxvalue:",v[r[0]])
print("minvalue:",v[r[-1]])
```

```
maxvalue: 0.11427415903139658
minvalue: 0.00013008094286810808
```

In [14]:

```
%time eigVal, eigVec = spla.eigs(E)
```

```
CPU times: user 21.3 s, sys: 241 ms, total: 21.5 s
```

```
Wall time: 5.81 s
```