CS 111 (S19): Homework 2

Due by 6:00pm Monday, April 14

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1. Write the following matrix in the form $A = LU$, where L is a unit lower triangular matrix (that is, a lower triangular matrix with ones on the diagonal) and U is an upper triangular matrix. You can check your answer using Python, but for this exercise, you need to also show the steps you took to get to your answer with some explanation to go with them.

$$A = \begin{pmatrix} 5 & -1 & -1 \\ -1 & 5 & -1 \\ -1 & -1 & 5 \end{pmatrix} \quad \begin{array}{c} R_1 = R_1 + 0.2R_0 \\ R_2 = R_2 + 0.2R_0 + 0.25R_1 \end{array}$$

Therefore, A= $\begin{pmatrix} 1 & 0 & 0 \\ -1/5 & 1 & 0 \\ -1/5 & -1/4 & 1 \end{pmatrix} \begin{pmatrix} 5 & -1 & -1 \\ 0 & 4.8 & -1.2 \\ 0 & 0 & 4.5 \end{pmatrix}$

2. The following three statements are all **false**. For each one, give a counterexample consisting of a 3-by-3 matrix or matrices, and show the computation that proves that the statement fails.

2a. If P is a permutation matrix and A is any matrix, then $PA = AP$.

if we let $P = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, and let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ then, $PA = \begin{pmatrix} 0 & 2 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ and $AP = \begin{pmatrix} 0 & 1 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$, therefore, PA is not equal to AP.

2b. If matrix A is nonsingular, then it has a factorization $A = LU$ where L is lower triangular and U is upper triangular.

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix},$$

Where there is no pivot in second row, so can't be LU factorized.

2c. The product of two symmetric matrices is a symmetric matrix.

let A = $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$, and B = $\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix}$, then AB = $\begin{pmatrix} 4 & 2 \\ 6 & 4 \end{pmatrix}$, which is not a symmetric matrix

3a. Consider the permutation matrix

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Find a 4-element permutation vector `v = np.array(something)` such that, for *every* 4-by-4 matrix A, we have `A[v,:]  == P @ A`. Test your answer by running a few lines of Python, and turn in the result.

If you do P @ A, you switch row of A, This P here switch A's row by sequence of 2, 4, 3, 1, So

$$v = np.array([1, 3, 2, 0])$$

3b. For the same P, find a 4-element permutation vector `w = np.array(something)` such that, for *every* 4-by-4 matrix A, we have `A[:,w] == A @ P`. Test your answer and turn in the result.

If you do P @ A, you switch row of A, This P here switch A's row by sequence of 4, 1, 3, 2, So

$$v = np.array([3, 0, 2, 1])$$