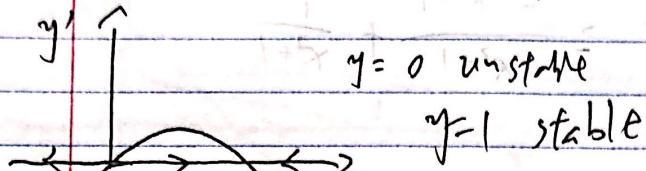


Math 142 HW3

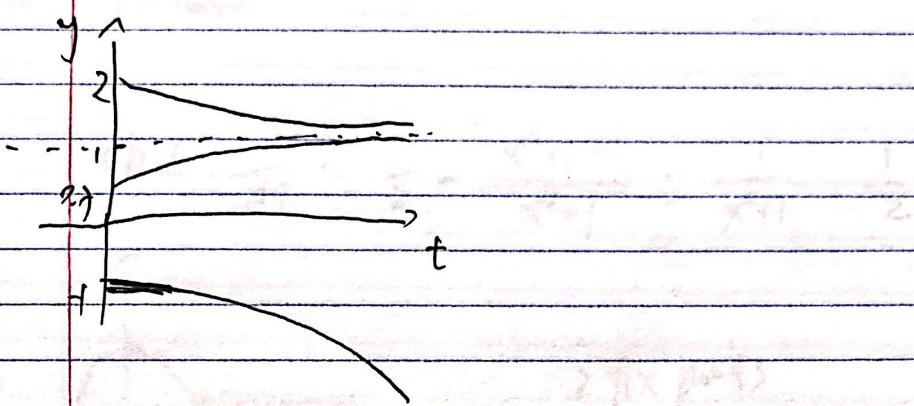
Chen Li 705426669

(1) (a)  $y - y^2 = 0$   $y_1 = 0, y_2 = 1$

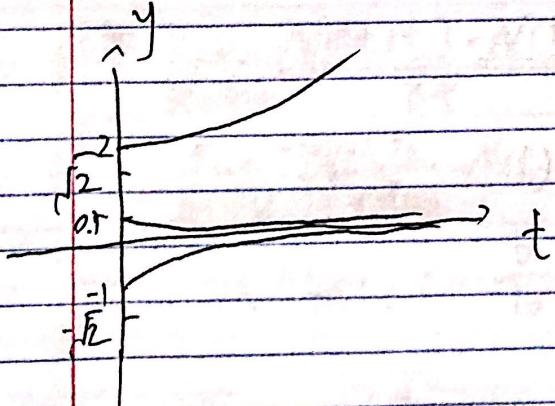
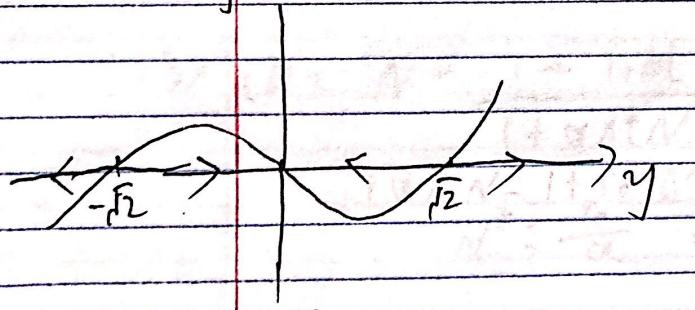


$y=0$  unstable

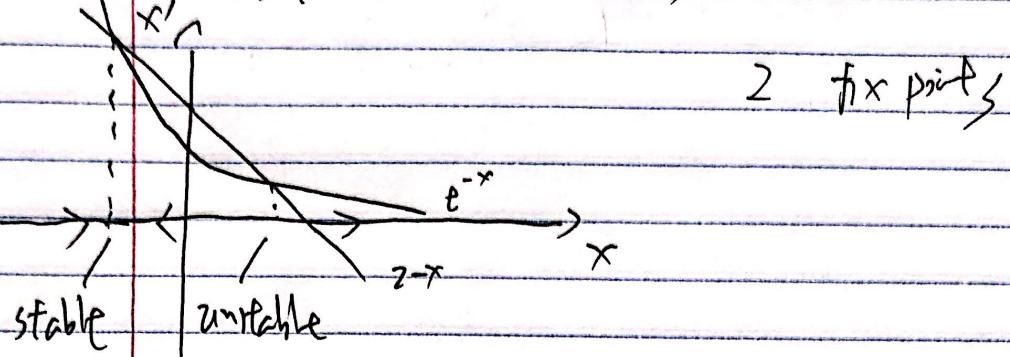
$y=1$  stable



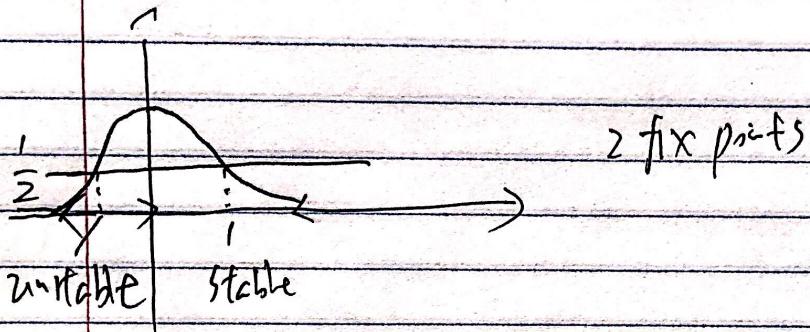
(b)  $y^3 - 2y = 0$   $y_1 = 0, y_2 = \sqrt{2}, y_3 = -\sqrt{2}$   
stable ; unstable ; unstable



$$(2) \text{, a, } \frac{dx}{dt} = e^{-x} - (2-x)$$



$$(b) \frac{dx}{dt} = \frac{1}{2} - \frac{x^2 + 1 - 1}{x^2 + 1} = \frac{1}{x^2 + 1} - \frac{1}{2}$$



$$(3) (a) N = \frac{1 + r_0 \alpha t}{1 + \alpha \Delta t N} N$$

$$1 + \alpha \Delta t N = 1 + r_0 \alpha t$$

$$N^* = \frac{r_0}{\alpha} \quad \text{or} \quad N^* = 0$$

$$(b) \lim_{\Delta t \rightarrow 0} \frac{N(r_0 \Delta t) - N(t)}{\Delta t} = \lim_{\Delta t \rightarrow 0} \frac{1 + r_0 \alpha t - 1 + \alpha \Delta t N(t)}{\Delta t (1 + \alpha \Delta t N(t))} N(t)$$

$$= \lim_{\Delta t \rightarrow 0} \frac{-N \alpha \Delta t + r_0}{1 + \alpha \Delta t N} N(t) = N(t), (r_0 - \alpha N(t))$$

$$= r_0 N(t) \left( 1 - \frac{\alpha}{r_0} N(t) \right) \quad k = \frac{r_0}{\alpha}$$

$$(c) r_0 N \left( 1 - \frac{\alpha}{r_0} N \right) = 0$$

$$\frac{\alpha}{r_0} N = 1 \Rightarrow N = \frac{r_0}{\alpha}$$

$$N^* = \frac{r_0}{\alpha} \quad \text{or} \quad N^* = 0$$

$$4. (i) N(3) = e^{3r} N(0)$$

$$47.2 = e^{3r} \cdot 9.6$$

$$r = 0.530877$$

(ii) as  $t$  goes large,  $N(t) \rightarrow k$   
 $\therefore k \approx 663$

$$(b) \frac{dN}{dt} = 0.530877 \left(1 - \frac{N(t)}{663}\right) N(t) \quad N_0 = 9.6$$

$$\int \frac{dN}{N(t)(663-N(t))} = \frac{0.530877}{663} dt$$

$$\int \frac{663^1}{N(t)} + \frac{663^1}{663-N(t)} dN = \int 0.530877 dt$$

$$\ln N - \ln |663-N| = 0.530877 t + C$$

$$C = \ln \frac{9.6}{663-9.6}$$

$$9.6 < k \Rightarrow 663 - N > 0$$

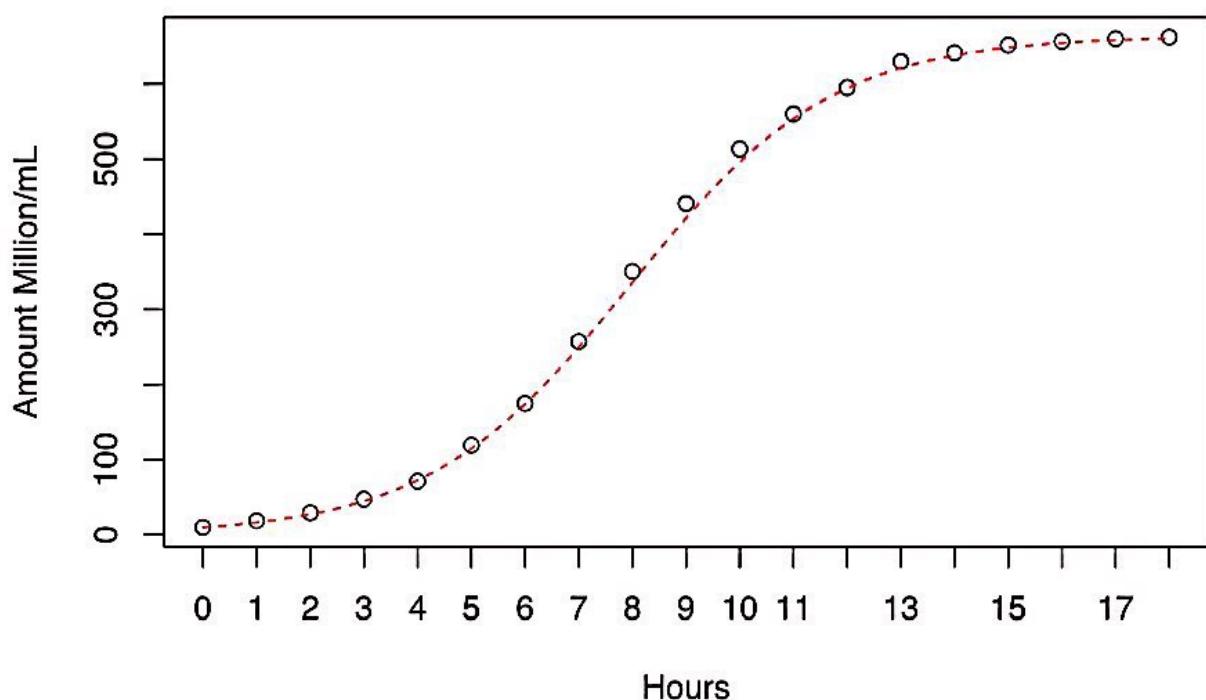
$$\ln \left( \frac{N}{663-N} \right) = 0.530877 t + \text{[crossed out]} \ln \frac{9.6}{663-9.6}$$

$$\frac{N}{663-N} = e^{0.530877 t} \cdot \frac{9.6}{663-9.6}$$

$$N = e^{0.530877 t} \cdot \frac{9.6}{663-9.6} (663-N)$$

$$N = \frac{9.6 \cdot 663}{663-9.6} e^{0.530877 t} \cdot \frac{9.741 e^{0.530877 t}}{1 + \frac{9.6}{663-9.6} e^{0.530877 t}} \cdot \frac{1 + 0.01469 e^{0.530877 t}}{1 + 0.01469 e^{0.530877 t}}$$

### Yeast Cell Number against Time in Carlson's Experiment



Logistic growth is similar to exponential growth model when  $N$  is small. Exponential growth model is valid under ideal condition where there is no environment adverse force. However, there is environment limit apply upon every system. So exponential growth would only work when population is small. On the other hand, logistic growth model consider environment force, and fits better than exponential.

(5.)

$$(a) \frac{de}{dt} = -es k_f + ck_b + ck_p$$

$$\frac{ds}{dt} = -es k_f + ck_b$$

$$\frac{dc}{dt} = es k_f - ck_b - ck_p$$

$$\frac{dp}{dt} = ck_p$$

b) ① E and C are converting in 1:1 ratio in both reaction, so  $B + C = A_0$  is constant

② S, C, P are changing in a series of reaction in 1:1:1 ratio, so  $S + C + P = B_0$  is constant

$$E = A_0 - C, \quad S = B_0 - C - P$$

$$\begin{cases} \frac{dc}{dt} = (A_0 - C)(B_0 - C - P) k_f - ck_b - ck_p \\ \frac{dp}{dt} = ck_p \end{cases}$$

The equation has been reduced to only C and P.