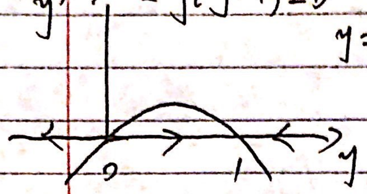
Math 142 HW3 Chen Li 705426669

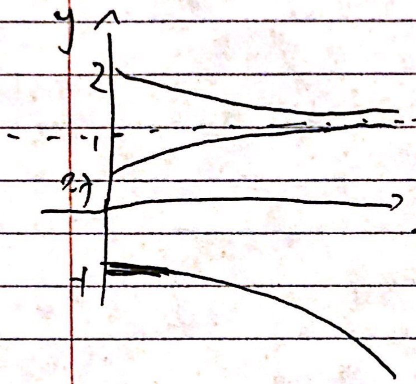
(1) (a) $y-y^2=0$ $y_1=0, y_2=1$ are fix points

$-y(y-1)=0$

$y=0$ unstable

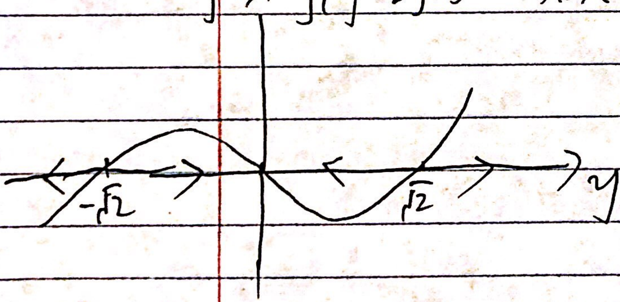
$y=1$ stable

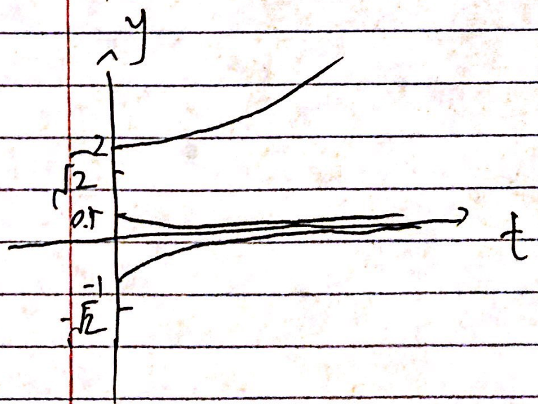




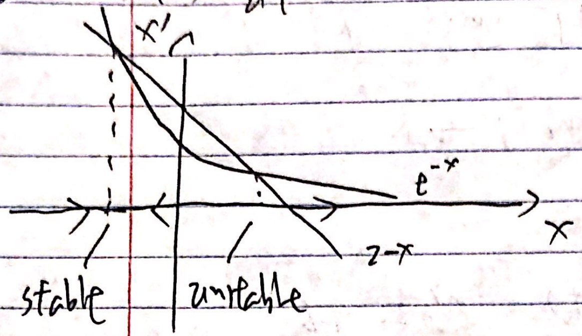
(b) $y^3-2y=0$ $y_1=0$, $y_1=\sqrt{2}$, $y_2=-\sqrt{2}$ are fix points

$y(y^2-2)=0$ stable ; unstable ; unstable



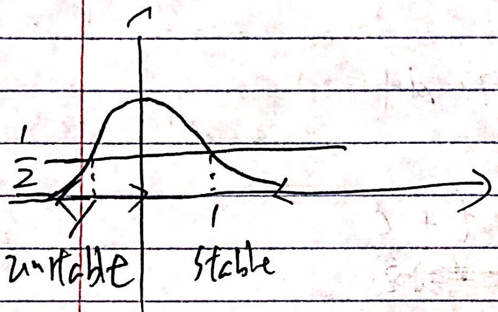


(2) (a) $\frac{dx}{dt} = e^{-x} - (2-x)$



2 fix points

(b) $\frac{dx}{dt} = \frac{1}{2} - \frac{x^2+1-1}{x^2+1} = \frac{1}{x^2+1} - \frac{1}{2}$



2 fix points

(3) (a) $N = \frac{1+r_0\Delta t}{1+\alpha\Delta t N} N$

$1+\alpha\Delta t N = 1 + r_0 \Delta t$

$N^* = \frac{r_0}{\alpha}$ or $N^* = 0$

(b) $\lim_{\Delta t \to 0} \frac{N(t+\Delta t) - N(t)}{\Delta t} = \lim_{\Delta t \to 0} \frac{1 + r_0\Delta t - 1 - \alpha\Delta t N(t)}{\Delta t(1+\alpha\Delta t N(t))} N(t)$

$= \lim_{\Delta t \to 0} \frac{-N\alpha + r_0}{1+\alpha\Delta t N_t} N(t) = N(t)(r_0 - \alpha N(t))$

$= r_0 N(t)\left(1 - \frac{\alpha}{r_0} N(t)\right)$ let $K = \frac{r_0}{\alpha}$

$= r_0 N(t)\left(1 - \frac{N(t)}{K}\right)$

(c) $r_0 N\left(1 - \frac{\alpha}{r_0} N\right) = 0$

$1 = \frac{\alpha}{r_0} N$

$N^* = \frac{r_0}{\alpha}$ or $N^* = 0$

4. (a) (i) $N(3) = e^{3r} N(0)$

$47.2 = e^{3r} \cdot 9.6$ $\quad \ln(e^{3r}) = \ln(47.2/9.6)$

$r = 0.530877$

(ii) as t goes large, $N(t) \to k$

so $k \simeq 663$

(b) $\frac{dN}{dt}(t) = 0.530877\left(1 - \frac{N(t)}{663}\right) N(t)$ $\quad N_0 = 9.6$

(c) $\frac{dN}{N(t)(663 - N(t))} = \frac{0.530877}{663} dt$

$\int \frac{1}{N(t)} + \frac{1}{(663 - N(t))} dN = \int 0.530877 \, dt$

$\ln N - \ln|663 - N| = 0.530877 t + c$

$c = \ln \frac{9.6}{663 - 9.6}$

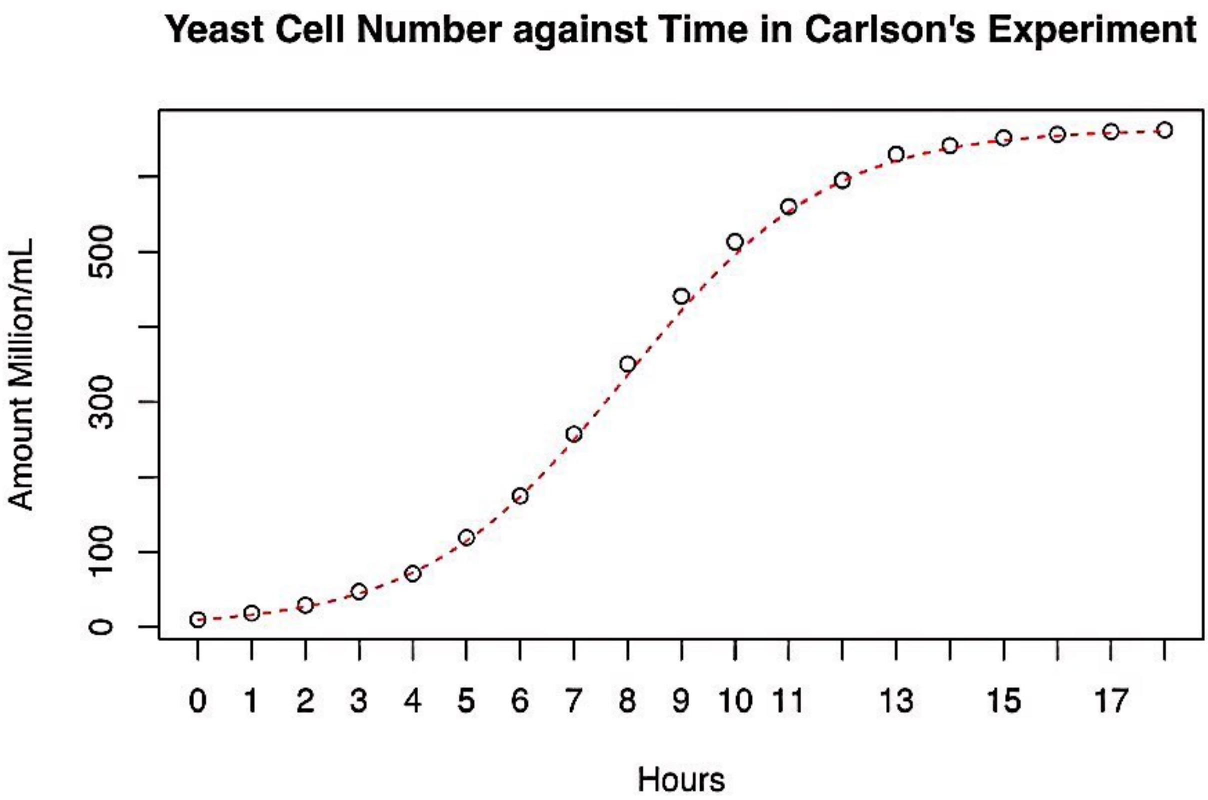
$9.6 < k \Rightarrow 663 - N > 0$

$\ln\left(\frac{N}{663 - N}\right) = 0.530877t + \ln\frac{9.6}{663-9.6}$

$\frac{N}{663 - N} = e^{0.530877t} \cdot \frac{9.6}{663-9.6}$

$N = e^{0.530877t} \cdot \frac{9.6}{663-9.6}(663 - N)$

$$N = \frac{\frac{9.6 \cdot 663}{663 - 9.6} e^{0.530877t}}{1 + \frac{9.6}{663-9.6} e^{0.530877t}} = \frac{9.741 e^{0.530877t}}{1 + 0.01469 e^{0.530877t}}$$



Logistic growth is similar to exponential growth model when N is small. Exponential growth model is valid under ideal condition where there is no enviroment adverse force. However, there is enviroment limit apply upon every system. So exponential growth would only work when population is small. On the other hand, logictic growth model consider enviroment force, and fits bitter than exponential.

(5.)

(a) $$\frac{de}{dt} = -esk_f + ck_b + ck_p$$

$$\frac{ds}{dt} = -esk_f + ck_b$$

$$\frac{dc}{dt} = esk_f - ck_b - ck_p$$

$$\frac{dp}{dt} = ck_p$$

(b) ① E and C are converting in 1:1 ration in both reactn, so $E + C = A_0$ is constant

② S, C, P are chained in a series of reacth in 1:1:1 ration, so $S + C + P = B_0$ is constant

$e = A_0 - C$, $s = B_0 - C - P$

$$\begin{cases} \frac{dc}{dt} = (A_0 - c)(B_0 - c - p)k_f - ck_b - ck_p \\ \frac{dp}{dt} = ck_p \end{cases}$$

the equath has been reducted to only c and p.