Scattering of Cylindrical Waves from a Dielectric Cylinder

Homer Zeid

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Contents

1 The Setup

1.1 Cylindrical wave functions

$$\begin{aligned}
\mathbf{M}_\nu(k_0,k_z;\mathbf{x}) &= \Big[i\nu\frac{Z_\nu(\zeta)}{\zeta}\hat{\boldsymbol{\rho}} - Z'_\nu(\zeta)\hat{\boldsymbol{\varphi}}\Big]e^{i\nu\varphi}e^{ik_z z}\\
\mathbf{N}_\nu(k_0,k_z;\mathbf{x}) &= \frac{1}{ik_0}\Big[-ik_z Z'_\nu(\zeta)\hat{\boldsymbol{\rho}} + \nu k_z\frac{Z_\nu(\zeta)}{\zeta}\hat{\boldsymbol{\varphi}} - k_\rho Z_\nu(\zeta)\hat{\mathbf{z}}\Big]e^{i\nu\varphi}e^{ik_z z}
\end{aligned}$$

$$\zeta \equiv k_\rho\rho, \qquad k_\rho \equiv \sqrt{k_0^2 - k_z^2}, \qquad Z(\rho) = \begin{cases} H_\nu^{(1)}(k_\rho\rho), & \text{outgoing}\\ H_\nu^{(2)}(k_\rho\rho), & \text{incoming}\\ J_\nu(k_\rho\rho), & \text{regular}\end{cases}$$

We have

$$\nabla\times\mathbf{M} = -ik_0\mathbf{N}, \qquad \nabla\times\mathbf{N} = ik_0\mathbf{M}.$$

1.2 Scattering Coefficients

Incident fields:

$$\begin{aligned}
\mathbf{E}^{\text{inc}}(\mathbf{r}) &= P_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x}) + Q_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x})\\
\mathbf{H}^{\text{inc}}(\mathbf{r}) &= -\frac{1}{Z_0}\Big\{P_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x}) - Q_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x})\Big\}
\end{aligned}$$

Interior fields:

$$\begin{aligned}
\mathbf{E}^{\text{int}}(\mathbf{r}) &= A_\nu(k_z)\mathbf{M}_\nu^{\text{regular}}(nk_0,k_z;\mathbf{x}) + B_\nu(k_z)\mathbf{N}_\nu^{\text{regular}}(nk_0,k_z;\mathbf{x})\\
\mathbf{H}^{\text{int}}(\mathbf{r}) &= -\frac{1}{Z'Z_0}\Big\{A_\nu(k_z)\mathbf{N}_\nu^{\text{regular}}(nk_0,k_z;\mathbf{x}) - B_\nu(k_z)\mathbf{M}_\nu^{\text{regular}}(nk_0,k_z;\mathbf{x})\Big\}
\end{aligned}$$

Scattered fields:

$$\begin{aligned}
\mathbf{E}^{\text{scat}}(\mathbf{r}) &= C_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x}) + D_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x})\\
\mathbf{H}^{\text{scat}}(\mathbf{r}) &= -\frac{1}{Z_0}\Big\{C_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x}) - D_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x})\Big\}
\end{aligned}$$

Match tangential fields at $\rho = R$ ($\eta = 1/Z'$:)

$$\begin{pmatrix}
M_\varphi^{\text{reg}}(R) & N_\varphi^{\text{reg}}(R) & -M_\varphi^{\text{out}}(R) & -N_\varphi^{\text{out}}(R)\\
\eta N_\varphi^{\text{reg}}(R) & -\eta M_\varphi^{\text{reg}}(R) & -N_\varphi^{\text{out}}(R) & +M_\varphi^{\text{out}}(R)\\
M_z^{\text{reg}}(R) & N_z^{\text{reg}}(R) & -M_z^{\text{out}}(R) & -N_z^{\text{out}}(R)\\
\eta N_z^{\text{reg}}(R) & -\eta M_z^{\text{reg}}(R) & -N_z^{\text{out}}(R) & +M_z^{\text{out}}(R)
\end{pmatrix}
\begin{pmatrix} A\\ B\\ C\\ D\end{pmatrix}
=
\begin{pmatrix}
PM_\varphi^{\text{inc}}(R) + QN_\varphi^{\text{inc}}(R)\\
PN_\varphi^{\text{inc}}(R) - QM_\varphi^{\text{inc}}(R)\\
PM_z^{\text{inc}}(R) + QN_z^{\text{inc}}(R)\\
PN_z^{\text{inc}}(R) - QM_z^{\text{inc}}(R)
\end{pmatrix}$$