Scattering of Cylindrical Waves from a Dielectric Cylinder

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Contents

1 The Setup

1.1 Cylindrical wave functions

$$\begin{aligned}
\mathbf{M}_\nu(k_0,k_z;\mathbf{x}) &= \Big[i\nu\frac{R_\nu(\zeta)}{\zeta}\hat{\boldsymbol{\rho}} - R'_\nu(\zeta)\hat{\boldsymbol{\varphi}}\Big]e^{i\nu\varphi}e^{ik_z z}\\
\mathbf{N}_\nu(k_0,k_z;\mathbf{x}) &= \frac{1}{ik_0}\Big[-ik_z R'_\nu(\zeta)\hat{\boldsymbol{\rho}} + \nu k_z\frac{R_\nu(\zeta)}{\zeta}\hat{\boldsymbol{\varphi}} - R_\nu(\zeta)\hat{\mathbf{z}}\Big]e^{i\nu\varphi}e^{ik_z z}
\end{aligned}$$

$$\zeta = k_\rho\rho, \qquad k_\rho \equiv \sqrt{k_0^2-k_z^2}, \qquad R(\zeta) = \begin{cases} H_\nu^{(1)}(\zeta), & \text{outgoing}\\ H_\nu^{(2)}(\zeta), & \text{incoming}\\ J_\nu(\zeta), & \text{regular}\end{cases}$$

We have

$$\nabla\times\mathbf{M} = -ik_0\mathbf{N}, \qquad \nabla\times\mathbf{N} = ik_0\mathbf{M}.$$

Incident fields:

$$\begin{aligned}
\mathbf{E}^{\text{inc}}(\mathbf{r}) &= P_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x}) + Q_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x})\\
\mathbf{H}^{\text{inc}}(\mathbf{r}) &= -\frac{1}{Z_0}\Big\{P_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x}) - Q_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x})\Big\}
\end{aligned}$$

Interior fields:

$$\begin{aligned}
\mathbf{E}^{\text{int}}(\mathbf{r}) &= A_\nu(k_z)\mathbf{M}_\nu^{\text{regular}}(nk_0,k_z;\mathbf{x}) + B_\nu(k_z)\mathbf{N}_\nu^{\text{regular}}(nk_0,k_z;\mathbf{x})\\
\mathbf{H}^{\text{int}}(\mathbf{r}) &= -\frac{1}{Z'Z_0}\Big\{A_\nu(k_z)\mathbf{N}_\nu^{\text{regular}}(nk_0,k_z;\mathbf{x}) - B_\nu(k_z)\mathbf{M}_\nu^{\text{regular}}(nk_0,k_z;\mathbf{x})\Big\}
\end{aligned}$$

Scattered fields:

$$\begin{aligned}
\mathbf{E}^{\text{scat}}(\mathbf{r}) &= C_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x}) + D_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x})\\
\mathbf{H}^{\text{scat}}(\mathbf{r}) &= -\frac{1}{Z_0}\Big\{C_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x}) - D_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0,k_z;\mathbf{x})\Big\}
\end{aligned}$$