Implicit handling of multilayered material substrates in full-wave SCUFF-EM calculations

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$$\epsilon_1, \mu_1$$
 $z=z_1$
 $z=z_2$
 ϵ_2, μ_2
 ϵ_3, μ_3
 $z=z_{N-1}$
 $z=z_{N-1}$
 $z=z_{N-1}$
 $z=z_{N-1}$
 $z=z_{N-1}$
 $z=z_{N-1}$

Figure 1: Geometry of the layered substrate. The *n*th layer has relative permittivity and permeability ϵ_n, μ_n , and its lower surface lies at $z = z_n$. The ground plane, if present, lies at $z = z_{\text{GP}}$.

1 Overview

In a previous memo¹ I considered SCUFF-STATIC electrostatics calculations in the presence of a multilayered dielectric substrate. In this memo I extend that discussion to the case of *full-wave* (i.e. nonzero frequencies beyond the quasistatic regime) scattering calculations in the SCUFF-EM core library.

Substrate geometry

As shown in Figure 1, I consider a multilayered substrate consisting of N material layers possibly terminated by a perfectly-conducting ground plane. The uppermost layer (layer 1) is the infinite half-space above the substrate. The nth layer has relative permittivity and permeability ϵ_n, μ_n , and its lower surface lies at $z=z_n$. The ground plane, if present, lies at $z\equiv z_{\rm N}\equiv z_{\rm GP}$. If the ground plane is absent, layer N is an infinite half-space² ($z_{\rm N}=-\infty$).

Mechanics of implementation in Scuff-em

The full-wave substrate implementation in SCUFF-EM involves multiple working parts.

¹ "Implicit handling of multilayered dielectric substrates in SCUFF-STATIC"

 $^{^2}$ As in the electrostatic case, this means that a finite-thickness substrate consisting of N material layers is described as a stack of N+1 layers in which the bottommost layer is an infinite vacuum half-space.

- 2 Computation of Fourier-space DGF
- 3 Reduction of 2D integrals over q to 1D integrals over q

4 Evaluation of 1D integrals

5 Substrate contributions to panel and panelpanel integrals

$$\begin{split} \boldsymbol{\mathcal{G}}^{\scriptscriptstyle{\mathrm{PQ}}}(\rho,\theta,z_{\scriptscriptstyle{\mathrm{D}}},z_{\scriptscriptstyle{\mathrm{S}}}) &= \sum_{\nu p} g^{\scriptscriptstyle{\mathrm{PQ}}\nu p}(\rho,z_{\scriptscriptstyle{\mathrm{D}}},z_{\scriptscriptstyle{\mathrm{S}}}) \boldsymbol{\Lambda}^{\scriptscriptstyle{\mathrm{PQ}}\nu p}(\theta) \\ g^{\scriptscriptstyle{\mathrm{PQ}}\nu p}(\rho) &= \int \frac{q dq}{(2\pi)} \widetilde{g}^{\scriptscriptstyle{\mathrm{PQ}}\nu p}(q) J_{\nu}(q\rho) e^{i\alpha(q,z_{\scriptscriptstyle{\mathrm{D}}},z_{\scriptscriptstyle{\mathrm{S}}})} \boldsymbol{\Lambda}^{\scriptscriptstyle{\mathrm{PQ}}\nu p}(\theta) \\ & \left\langle \mathbf{b}_{\alpha} \middle| \boldsymbol{\mathcal{G}}^{PQ} \middle| \mathbf{b}_{\beta} \right\rangle \end{split}$$