Scattering of Cylindrical Waves from a Dielectric Cylinder

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Contents

1 The Setup

1.1 Cylindrical wave functions

$$\mathbf{M}_\nu(k_0, k_z; \mathbf{x}) = \Big[i\nu \frac{R_\nu(\rho)}{\rho}\hat{\boldsymbol{\rho}} - R'_\nu(\rho)\hat{\boldsymbol{\varphi}}\Big] e^{i\nu\varphi} e^{ik_z z}$$

$$\mathbf{N}_\nu(k_0, k_z; \mathbf{x}) = \frac{1}{ik_0}\Big[- ik_z R'_\nu(\rho)\hat{\boldsymbol{\rho}} + \nu k_z \frac{R_\nu(\rho)}{\rho}\hat{\boldsymbol{\varphi}} - R_\nu(\rho)\hat{\mathbf{z}}\Big] e^{i\nu\varphi} e^{ik_z z}$$

$$k_\rho \equiv \sqrt{k_0^2 - k_z^2}, \qquad R(\rho) = \begin{cases} H_\nu^{(1)}(k_\rho \rho), & \text{outgoing} \\ H_\nu^{(2)}(k_\rho \rho), & \text{incoming} \\ J_\nu(k_\rho \rho), & \text{regular} \end{cases}$$

We have

$$\nabla \times \mathbf{M} = -ik_0 \mathbf{N}, \qquad \nabla \times \mathbf{N} = ik_0 \mathbf{M}.$$

Incident fields:

$$\mathbf{E}^{\text{inc}}(\mathbf{r}) = P_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0, k_z; \mathbf{x}) + Q_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0, k_z; \mathbf{x})$$

$$\mathbf{H}^{\text{inc}}(\mathbf{r}) = -\frac{1}{Z_0}\Big\{P_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0, k_z; \mathbf{x}) - Q_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0, k_z; \mathbf{x})\Big\}$$

Interior fields:

$$\mathbf{E}^{\text{int}}(\mathbf{r}) = A_\nu(k_z)\mathbf{M}_\nu^{\text{regular}}(nk_0, k_z; \mathbf{x}) + B_\nu(k_z)\mathbf{N}_\nu^{\text{regular}}(nk_0, k_z; \mathbf{x})$$

$$\mathbf{H}^{\text{int}}(\mathbf{r}) = -\frac{1}{Z' Z_0}\Big\{A_\nu(k_z)\mathbf{N}_\nu^{\text{regular}}(nk_0, k_z; \mathbf{x}) - B_\nu(k_z)\mathbf{M}_\nu^{\text{regular}}(nk_0, k_z; \mathbf{x})\Big\}$$

Scattered fields:

$$\mathbf{E}^{\text{scat}}(\mathbf{r}) = C_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0, k_z; \mathbf{x}) + D_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0, k_z; \mathbf{x})$$

$$\mathbf{H}^{\text{scat}}(\mathbf{r}) = -\frac{1}{Z_0}\Big\{C_\nu(k_z)\mathbf{N}_\nu^{\text{incoming}}(k_0, k_z; \mathbf{x}) - D_\nu(k_z)\mathbf{M}_\nu^{\text{incoming}}(k_0, k_z; \mathbf{x})\Big\}$$