Standard complex stuff

$e^{jz} = \cos z + j \sin z$	$\sin z = \frac{1}{2j}\left(e^{jz} - e^{-jz}\right);\ \cos z = \frac{1}{2}\left(e^{jz} + e^{-jz}\right)$

Laplace transform stuff

$\int_{\sigma_1 - j\infty}^{\sigma_1 + j\infty} F(s)e^{st}\, ds = f(t) \overset{\mathcal{L}}{\leftrightarrow} F(s) = \int_{0^-}^{\infty} f(t)e^{-st}\, dt$		**ROC:** $\left\lvert \int_{0^-}^{\infty} f(t)e^{-\sigma_1 t}\, dt \right\rvert < \infty$ for real σ_1
$a_1 f_1(t) + a_2 f_2(t) \overset{\mathcal{L}}{\leftrightarrow} a_1 F_1(s) + a_2 F_2(s)$		$e^{s_0 t} f\big(a(t + t_0)\big) \overset{\mathcal{L}}{\leftrightarrow} \frac{1}{a} e^{st_0} F\left(\frac{1}{a}(s - s_0)\right)$
$tf(t) \overset{\mathcal{L}}{\leftrightarrow} -\frac{d}{ds}F(s)$		$\frac{1}{t} f(t) \overset{\mathcal{L}}{\leftrightarrow} \int_{s}^{\infty} F(s)\, ds$
$\delta(t) \overset{\mathcal{L}}{\leftrightarrow} 1$	$u(t) \overset{\mathcal{L}}{\leftrightarrow} \frac{1}{s}$	$t^n e^{at} \overset{\mathcal{L}}{\leftrightarrow} \frac{n!}{(s-a)^{n+1}}$
$\sin(\omega t + \theta) \overset{\mathcal{L}}{\leftrightarrow} \frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2}$		$\cos(\omega t + \theta) \overset{\mathcal{L}}{\leftrightarrow} \frac{s \cos\theta - \omega \sin\theta}{s^2 + \omega^2}$
$e^{at} \sin \omega t \overset{\mathcal{L}}{\leftrightarrow} \frac{\omega}{(s-a)^2 + \omega^2}$		$e^{at} \cos \omega t \overset{\mathcal{L}}{\leftrightarrow} \frac{s-a}{(s-a)^2 + \omega^2}$

Circuits stuff

Resistor: $R \cdot i(t) = v(t) \overset{\mathcal{L}}{\leftrightarrow} V(s) = R \cdot I(s)$
Inductor: $L \frac{di(t)}{dt} = v(t) \overset{\mathcal{L}}{\leftrightarrow} V(s) = j\omega L \cdot I(s) = sL \cdot I(s) - L \cdot i(0^-)$
Capacitor: $C \frac{dv(t)}{dt} = i(t) \overset{\mathcal{L}}{\leftrightarrow} I(s) = j\omega C \cdot V(s) = sC \cdot V(s) - C \cdot v(0^-)$

Convolution stuff

Convolution: $x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau \sim x[n] \otimes h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

Fourier Series stuff

$x(t) = \frac{C_0}{2} + \sum_{m=1}^{\infty} C_m \cos(2\pi m f_0 t + \theta_m)$	$x(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} (a_m \cos m\omega_0 t + b_m \sin m\omega_0 t)$
$x(t) = \sum_{m=-\infty}^{\infty} X_m e^{jm\omega_0 t}$	$X_m = \frac{1}{T} \int_0^T x(t) e^{-jm\omega_0 t}\, dt$
$ax(t) + by(t) \overset{\mathcal{FS}}{\leftrightarrow} aX_m + bY_m$	$e^{2\pi j M f_0 t} x(t + t_0) \overset{\mathcal{FS}}{\leftrightarrow} e^{2\pi j m f_0 t_0} X_{m-M}$
$\frac{d}{ds} x(t) \overset{\mathcal{FS}}{\leftrightarrow} 2\pi j m f_0 X_m$	$\int_{-\infty}^{t} x(\tau)\, d\tau \overset{\mathcal{FS}}{\leftrightarrow} \frac{1}{2\pi j m f_0} X_m$

Continuous-time Fourier transform stuff

$\frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega = x(t) \overset{\mathcal{FT}}{\leftrightarrow} X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$	$ax(t) + by(t) \overset{\mathcal{FT}}{\leftrightarrow} aX(\omega) + bY(\omega)$	
$e^{j\omega_0 t} x\big(a(t + t_0)\big) \overset{\mathcal{FT}}{\leftrightarrow} \frac{1}{\lvert a \rvert} e^{j\omega t_0} X\left(\frac{1}{a}(\omega - \omega_0)\right)$	$\delta(t) \overset{\mathcal{FT}}{\leftrightarrow} 1$	$e^{j\omega_0 t} \overset{\mathcal{FT}}{\leftrightarrow} 2\pi\delta(\omega - \omega_0)$
$\sum_{n=-\infty}^{\infty} \delta(t - nT) \overset{\mathcal{FT}}{\leftrightarrow} \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta\left(\omega - \frac{2k\pi}{T}\right)$	$u(t) \overset{\mathcal{FT}}{\leftrightarrow} \pi\delta(\omega) + \frac{1}{j\omega}$	$sign(t) \overset{\mathcal{FT}}{\leftrightarrow} \frac{2}{j\omega}$

$\sin(\omega_0 t) \overset{\mathcal{FT}}{\leftrightarrow} j\pi\big(\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\big)$	$\cos(\omega_0 t) \overset{\mathcal{FT}}{\leftrightarrow} \pi\big(\delta(\omega+\omega_0)+\delta(\omega-\omega_0)\big)$
$u(t+a)-u(t-a) \overset{\mathcal{FT}}{\leftrightarrow} 2\dfrac{\sin \omega a}{\omega}$	$\dfrac{\sin \omega_0 t}{\pi t} \overset{\mathcal{FT}}{\leftrightarrow} u(\omega+\omega_0)-u(\omega-\omega_0)$

$e^{at}u(t) \overset{\mathcal{FT}}{\leftrightarrow} \dfrac{1}{j\omega - a}$	$te^{at}u(t) \overset{\mathcal{FT}}{\leftrightarrow} \dfrac{1}{(j\omega-a)^2}$	$e^{a\lvert t\rvert} \overset{\mathcal{FT}}{\leftrightarrow} \dfrac{-2a}{a^2+\omega^2}$	$x(t) * y(t) \overset{\mathcal{FT}}{\leftrightarrow} X(\omega)Y(\omega)$

Discrete-time Fourier transform stuff

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}\, d\Omega = x[n] \overset{\mathcal{FT}}{\leftrightarrow} X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

$$ax[n] + by[n] \overset{\mathcal{FT}}{\leftrightarrow} aX\big(e^{j\Omega}\big) + bY\big(e^{j\Omega}\big)$$

$$e^{j\Omega_0 n}x[n+n_0] \overset{\mathcal{FT}}{\leftrightarrow} e^{j\Omega t_0}X\big(e^{j(\Omega-\Omega_0)}\big)$$

$\delta[n]$	1
$\sum_{k=-\infty}^{\infty} \delta[n-kN]$	$\frac{2\pi}{N}\sum_{k=-\infty}^{\infty} \delta(\omega - \frac{2\pi}{N}k)$
$u[n]$	$\pi\delta(\omega) + \frac{1}{1-e^{-j\omega}}$
1	$2\pi\delta(\omega)$
$sgn[n]$	$\frac{2}{1-e^{-j\omega}}$
$e^{j\omega_0 n}$	$2\pi\delta(\omega-\omega_0)$
$\cos(\omega_0 n)$	$\pi(\delta(\omega+\omega_0)+\delta(\omega-\omega_0))$
$\sin(\omega_0 n)$	$j\pi(\delta(\omega+\omega_0)-\delta(\omega-\omega_0))$

$\begin{cases} 1 \text{ for } -N \le n \le N \\ 0 \text{ otherwise} \end{cases}$	$\frac{\sin(\omega\frac{(2N+1)}{2})}{\sin(\frac{\omega}{2})}$
$\frac{\sin(an)}{\pi n},\ 0 < a \le \pi$	$\begin{cases} 1 \text{ for } \lvert\omega\rvert < a \\ 0 \text{ for } a < \lvert\omega\rvert \le \pi \end{cases}$
$a^n u[n]\ ,\ \lvert a\rvert < 1$	$\frac{1}{1-ae^{-j\omega}}$
$(n+1)a^n u[n],\ \lvert a\rvert < 1$	$\frac{1}{(1-ae^{-j\omega})^2}$
$a^{\lvert n\rvert},\ \lvert a\rvert < 1$	$\frac{1-a^2}{1-2a\cos(\omega)+a^2}$
$a^n \sin(\omega_0 n)u[n],\ \lvert a\rvert < 1$	$\frac{(a)e^{-j\omega}\sin(\omega_0)}{1-2(a)e^{-j\omega}\cos(\omega_0)+(a)^2e^{-j2\omega}}$
$a^n \cos(\omega_0 n)u[n],\ \lvert a\rvert < 1$	$\frac{1-(a)e^{-j\omega}\cos(\omega_0)}{1-2(a)e^{-j\omega}\cos(\omega_0)+(a)^2e^{-j2\omega}}$

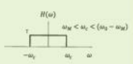
Discrete Fourier transform stuff

$$\frac{1}{N}\sum_{k=0}^{N-1} X[k]e^{\frac{1}{N}2\pi jkn} = x[n] \overset{\mathcal{FT}}{\leftrightarrow} X[k] = \sum_{n=0}^{N-1} x[n]e^{-\frac{1}{N}2\pi jkn}\,;\, N = \text{sample count}$$

Sampling & reconstruction stuff

In frequency domain, we multiply $X_s(\omega)$ by $H(\omega)$ with amplitude T and bandwidth ω_c with $\omega_M < \omega_c < (\omega_0 - \omega_M)$, to obtain $X_r(\omega)$.

$$H(\omega) = \begin{cases} T, & \lvert\omega\rvert < \omega_c \\ 0, & otherwise \end{cases}$$



$H(\omega)$ is a lowpass filter.

Thus, in time domain, $x_r(t)$ can be reconstructed from $x_s(t)$ with an ideal low-pass filter with pass-band gain T and cut-off frequency located between ω_M and $(\omega_0 - \omega_M)$. The reconstructed signal will exactly equal $x(t)$.

For simplicity, we set ω_c as the average of ω_M and $(\omega_0 - \omega_M)$,

$$\omega_c = \frac{\omega_0}{2} = \frac{\pi}{T}$$

We can obtain the time domain signal $h(t)$ by taking the inverse Fourier transform of $H(\omega)$

$$\begin{aligned} h(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} H(\omega)e^{j\omega t}d\omega \\ &= \frac{1}{2\pi}\int_{-\pi/T}^{\pi/T} Te^{j\omega t}d\omega \\ &= \frac{Te^{j\omega t}}{j2\pi t}\Bigg|_{\omega=-\pi/T}^{\pi/T} \\ &= \frac{T\sin(\frac{\pi t}{T})}{\pi t} = sinc(\frac{\pi}{T}t) \end{aligned}$$

where $sinc(t) = \sin(t)/(t)$

In time domain, we know the multiplication in frequency domain corresponding to the convolution in time domain.

Assume a low-pass filter in time-domain is $h(t)$, thus we have

$$\begin{aligned} x_r(t) &= \int_{-\infty}^{\infty} x_s(\tau)h(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty}\left(\sum_{n=-\infty}^{\infty} x(nT)\delta(\tau-nT)\right)h(t-\tau)d\tau \\ &= \int_{-\infty}^{\infty}\left(\sum_{n=-\infty}^{\infty} x[n]\delta(\tau-nT)\right)h(t-\tau)d\tau \\ &= \sum_{n=-\infty}^{\infty} x[n]h(t-nT) \end{aligned}$$

$$\boxed{x_s(t) = \sum_{n=-\infty}^{\infty} x(nT)\delta(t-nT)}$$

$$\boxed{x_r(t) = \sum_{n=-\infty}^{\infty} x[n]h(t-nT)}$$

It describes how to generate a continuous curve using the sample values $x[n]$.

Trigonometric identities

$\sin^2 x + \cos^2 x = 1$	$\sec^2 x - \tan^2 x = 1$	$\csc^2 x - \cot^2 x = 1$

$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
$\sin 2x = 2\sin x \cos x$	$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$; $\tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$	$\tan x = \frac{\sin 2x}{1+\cos 2x} = \frac{1-\cos 2x}{\sin 2x} = \sqrt{\frac{1-\cos 2x}{1+\cos 2x}}$
$\sin 3x = 3\sin x - 4\sin^3 x$	$\cos 3x = 4\cos^3 x - 3\cos x$
$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$	$\sin x \pm \sin y = 2\sin\frac{x\pm y}{2}\cos\frac{x\mp y}{2}$
$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$	$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$
$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$	$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$