CHAPTER 8 The Complete Response of RL and RC Circuits

IN THIS CHAPTER

8.1 Introduction

In this chapter, we consider the response of *RL* and *RC* circuits to abrupt changes. The abrupt change might be a change to the circuit, as when a switch opens or closes. Alternately, the abrupt change might be a change to the input to the circuit, as when the voltage of a voltage source is a discontinuous function of time.

RL and *RC* circuits are called first-order circuits. In this chapter, we will do the following:

- Develop vocabulary that will help us talk about the response of a first-order circuit.
- Analyze first-order circuits with inputs that are constant after some particular time, t_0.
- Introduce the notion of a stable circuit and use it to identify stable first-order circuits.
- Analyze first-order circuits that experience more than one abrupt change.
- Introduce the step function and use it to determine the step response of a first-order circuit.
- Analyze first-order circuits with inputs that are not constant.

8.2 First-Order Circuits

Circuits that contain capacitors and inductors can be represented by differential equations. The order of the differential equation is usually equal to the number of capacitors plus the number of inductors in the circuit.

Circuits that contain only one inductor and no capacitors or only one capacitor and no inductors can be represented by a first-order differential equation. These circuits are called **first-order circuits**.

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Thévenin and Norton equivalent circuits simplify the analysis of first-order circuits by showing that all first-order circuits are equivalent to one of two simple first-order circuits. Figure 8.2-1 shows how this is accomplished. In Figure 8.2-1*a*, a first-order circuit is partitioned into two parts. One part is the single capacitor or inductor that we expect to find in a first-order circuit. The other part is the rest of the circuit—everything except that capacitor or inductor. The next step, shown in Figure 8.2-1*b*, depends on whether the energy storage element is a capacitor or an inductor. If it is a capacitor, then the rest of the circuit is replaced by its Thévenin equivalent circuit. The result is a simple first-order circuit—a series circuit consisting of a voltage source, a resistor, and a capacitor. On the other hand, if the energy storage element is an inductor, then the rest of the circuit is replaced by its Norton equivalent circuit. The result is another simple first-order circuit—a parallel circuit consisting of a current source, a resistor, and an inductor. Indeed, all first-order circuits are equivalent to one of these two simple first-order circuits.

Consider the first-order circuit shown in Figure 8.2-2*a*. The input to this circuit is the voltage $v_s(t)$. The output, or response, of this circuit is the voltage across the capacitor. This circuit is at steady state before the switch is closed at time $t = 0$. Closing the switch disturbs this circuit. Eventually, the disturbance dies out and the circuit is again at steady state. The steady-state condition with the switch closed will probably be different from the steady-state condition with the switch open. Figure 8.2-2*b* shows a plot of the capacitor voltage versus time.

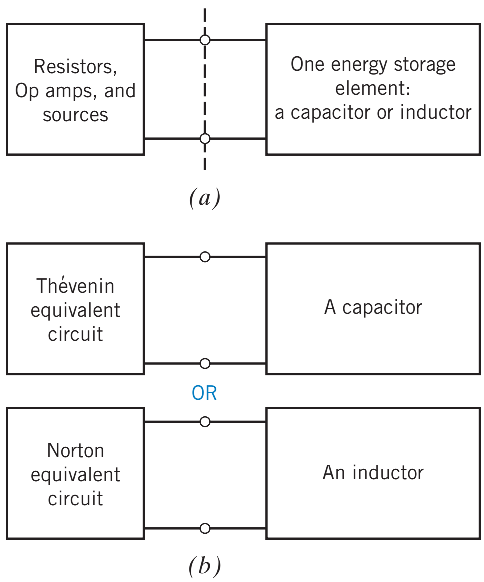

FIGURE 8.2-1 A plan for analyzing first-order circuits. (*a*) First, separate the energy storage element from the rest of the circuit. (*b*) Next, replace the circuit connected to a capacitor by its Thévenin equivalent circuit or replace the circuit connected to an inductor by its Norton equivalent circuit.

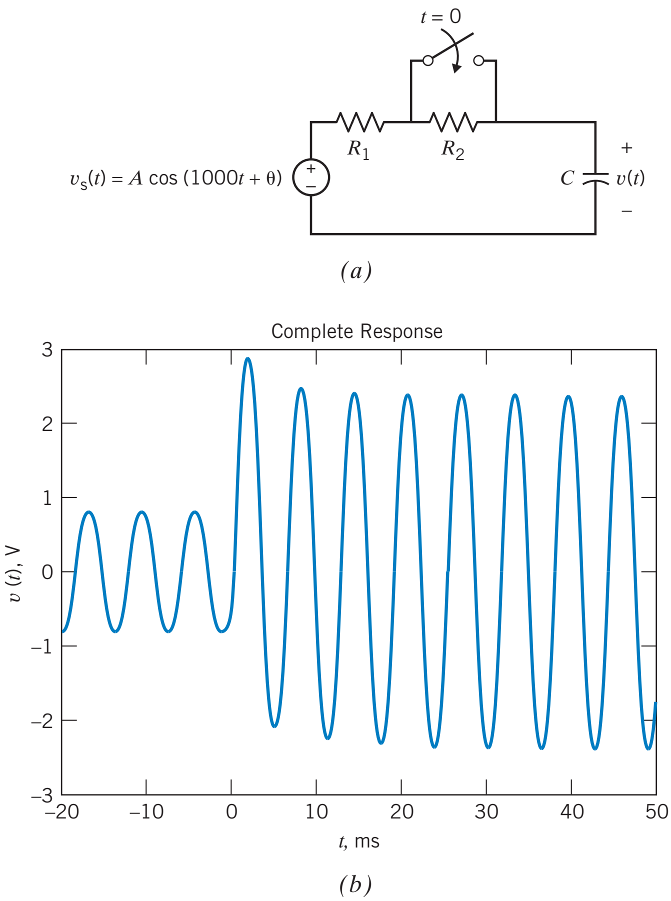

FIGURE 8.2-2 (*a*) A circuit and (*b*) its complete response.

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When the input to a circuit is sinusoidal, the steady-state response is also sinusoidal. Furthermore, the frequency of the response sinusoid must be the same as the frequency of the input sinusoid. The circuit shown in Figure 8.2-2*a* is at steady state before the switch is closed. The steady-state capacitor voltage will be

$$v(t) = B\cos(1000t + \phi),\ t < 0 \tag{8.2-1}$$

The switch closes at time $t = 0$. The value of the capacitor voltage at the time the switch closes is

$$v(0) = B\cos(\phi),\ t = 0 \tag{8.2-2}$$

After the switch closes, the response will consist of two parts: a transient part that eventually dies out and a steady-state part. The steady-state part of the response will be sinusoidal and will have the frequency of the input. For a first-order circuit, the transient part of the response is exponential. Indeed, we consider first-order circuits separately to take advantage of the simple form of the transient response of these circuits. After the switch is closed, the capacitor voltage is

$$v(t) = Ke^{-t/\tau} + M\cos(1000t + \delta) \tag{8.2-3}$$

Notice that $Ke^{-t/\tau}$ goes to zero as t becomes large. This is the transient part of the response, which dies out, leaving the steady-state response, $M\cos(1000t + \delta)$.

As a matter of vocabulary, the "transient part of the response" is frequently shortened to the **transient response**, and the "steady-state part of the response" is shortened to the "steady-state response." The response, $v(t)$, given by Eq. 8.2-3, is called the **complete response** to contrast it with the transient and steady-state responses.

$$\text{complete response} = \text{transient response} + \text{steady-state response}$$

(The term *transient response* is used in two different ways by electrical engineers. Sometimes it refers to the "transient part of the complete response," and at other times, it refers to a complete response, which includes a transient part. In particular, PSpice uses the term *transient response* to refer to the complete response. This can be confusing, so the term *transient response* must be used carefully.)

In general, the complete response of a first-order circuit can be represented as the sum of two parts, the **natural response** and the **forced response**:

$$\text{complete response} = \text{natural response} + \text{forced response}$$

The natural response is the general solution of the differential equation representing the first-order circuit, when the input is set to zero. The forced response is a particular solution of the differential equation representing the circuit.

The complete response of a first-order circuit will depend on an initial condition, usually a capacitor voltage or an inductor current at a particular time. Let t_0 denote the time at which the initial condition is given. The natural response of a first-order circuit will be of the form

$$\text{natural response} = Ke^{-(t-t_0)/\tau}$$

When $t_0 = 0$, then

$$\text{natural response} = Ke^{-t/\tau}$$

The constant K in the natural response depends on the initial condition, for example, the capacitor voltage at time t_0.

In this chapter, we will consider three cases. In these cases, the input to the circuit after the disturbance will be (1) a constant, for example,

$$v_s(t) = V_0$$

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or (2) an exponential, for example,

$$v_s(t) = V_0 e^{-t/\tau}$$

or (3) a sinusoid, for example,

$$v_s(t) = V_0 \cos(\omega t + \theta)$$

These three cases are special because the forced response will have the same form as the input. For example, in Figure 8.2-2, both the forced response and the input are sinusoidal, and the frequency of the forced response is the same as the frequency of the input. For other inputs, the forced response may not have the same form as the input. For example, when the input is a square wave, the forced response is not a square wave.

When the input is a constant or a sinusoid, the forced response is also called the steady-state response, and the natural response is called the transient response.

Here is our plan for finding the complete response of first-order circuits:

Step 1: Find the forced response before the disturbance. Evaluate this response at time $t = t_0$ to obtain the initial condition of the energy storage element.

Step 2: Find the forced response after the disturbance.

Step 3: Add the natural response $= Ke^{-t/\tau}$ to the forced response to get the complete response. Use the initial condition to evaluate the constant K.

8.3 The Response of a First-Order Circuit to a Constant Input

In this section, we find the complete response of a first-order circuit when the input to the circuit is constant after time t_0. Figure 8.3-1 illustrates this situation. In Figure 8.3-1*a*, we find a first-order circuit that contains a single capacitor and no inductors. This circuit is at steady state before the switch closes, disturbing the steady state. The time at which steady state is disturbed is denoted as t_0. In Figure 8.3-1*a*, $t_0 = 0$. Closing the switch removes the resistor R_1 from the circuit. (A closed switch is modeled by a short circuit. A short circuit in parallel with a resistor is equivalent to a short circuit.) After the switch closes, the circuit can be represented as shown in Figure 8.3-1*b*. In Figure 8.3-1*b*, the part of the circuit that is connected to the capacitor has been replaced by its Thévenin equivalent circuit. Therefore,

$$V_{oc} = \frac{R_3}{R_2 + R_3} V_s \quad \text{and} \quad R_t = \frac{R_2 R_3}{R_2 + R_3}$$

Let's represent the circuit in Figure 8.3-1*b* by a differential equation. The capacitor current is given by

$$i(t) = C \frac{d}{dt} v(t)$$

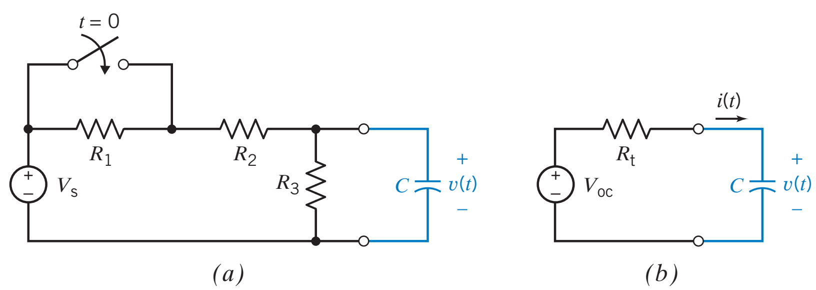

FIGURE 8.3-1 (*a*) A first-order circuit containing a capacitor. (*b*) After the switch closes, the circuit connected to the capacitor is replaced by its Thévenin equivalent circuit.

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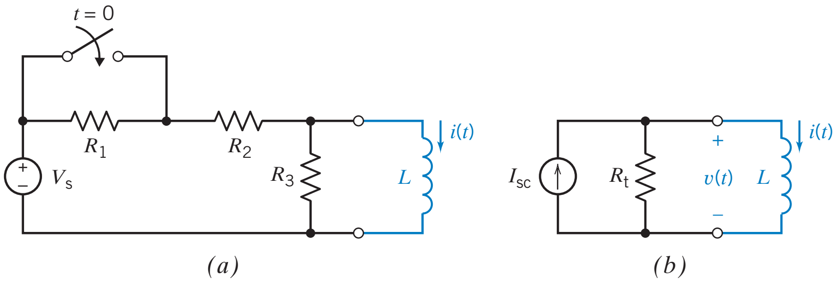

FIGURE 8.3-2 (*a*) A first-order circuit containing an inductor. (*b*) After the switch closes, the circuit connected to the inductor is replaced by its Norton equivalent circuit.

The same current, $i(t)$, passes through the resistor. Apply KVL to Figure 8.3-1*b* to get

$$V_{oc} = R_t i(t) + v(t) = R_t\left(C\frac{d}{dt}v(t)\right) + v(t)$$

Therefore,

$$\frac{d}{dt}v(t) + \frac{v(t)}{R_t C} = \frac{V_{oc}}{R_t C} \tag{8.3-1}$$

The highest-order derivative in this equation is first order, so this is a first-order differential equation.

Next, let's turn our attention to the circuit shown in Figure 8.3-2*a*. This circuit contains a single inductor and no capacitors. This circuit is at steady state before the switch closes at time $t_0 = 0$, disturbing the steady state. After the switch closes, the circuit can be represented as shown in Figure 8.3-2*b*. In Figure 8.3-2*b*, the part of the circuit that is connected to the inductor has been replaced by its Norton equivalent circuit. We calculate

$$I_{sc} = \frac{V_s}{R_2} \quad \text{and} \quad R_t = \frac{R_2 R_3}{R_2 + R_3}$$

Let's represent the circuit in Figure 8.3-2*b* by a differential equation. The inductor voltage is given by

$$v(t) = L\frac{d}{dt}i(t)$$

The voltage $v(t)$ appears across the resistor. Apply KCL to the top node in Figure 8.3-2*b* to get

$$I_{sc} = \frac{v(t)}{R_t} + i(t) = \frac{L\frac{d}{dt}i(t)}{R_t} + i(t)$$

Therefore,

$$\frac{d}{dt}i(t) + \frac{R_t}{L}i(t) = \frac{R_t}{L}I_{sc} \tag{8.3-2}$$

As before, this is a first-order differential equation.

Equations 8.3-1 and 8.3-2 have the same form. That is,

$$\frac{d}{dt}x(t) + \frac{x(t)}{\tau} = K \tag{8.3-3}$$

The parameter τ is called the time constant. We will solve this differential equation by separating the variables and integrating. Then we will use the solution of Eq. 8.3-3 to obtain solutions of Eqs. 8.3-1 and 8.3-2.

We may rewrite Eq. 8.3-3 as

$$\frac{dx}{dt} = \frac{K\tau - x}{\tau}$$

or, separating the variables,

$$\frac{dx}{x - K\tau} = -\frac{dt}{\tau}$$

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Forming the indefinite integral, we have

$$\int \frac{dx}{x - K\tau} = -\frac{1}{\tau}\int dt + D$$

where D is a constant of integration. Performing the integration, we have

$$\ln(x - K\tau) = -\frac{t}{\tau} + D$$

Solving for x gives

$$x(t) = K\tau + Ae^{-t/\tau}$$

where $A = e^D$, which is determined from the initial condition, $x(0)$. To find A, let $t = 0$. Then

$$x(0) = K\tau + Ae^{-0/\tau} = K\tau + A$$

or

$$A = x(0) - K\tau$$

Therefore, we obtain

$$x(t) = K\tau + [x(0) - K\tau]e^{-t/\tau} \tag{8.3-4}$$

Because

$$x(\infty) = \lim_{t\to\infty} x(t) = K\tau$$

Equation 8.3-4 can be written as

$$\boxed{x(t) = x(\infty) + [x(0) - x(\infty)]e^{-t/\tau}}$$

Taking the derivative of $x(t)$ with respect to t leads to a procedure for measuring or calculating the time constant:

$$\frac{d}{dt}x(t) = -\frac{1}{\tau}[x(0) - x(\infty)]e^{-t/\tau}$$

Now let $t = 0$ to get

$$\left.\frac{d}{dt}x(t)\right|_{t=0} = -\frac{1}{\tau}[x(0) - x(\infty)]$$

or

$$\boxed{\tau = \frac{x(\infty) - x(0)}{\left.\frac{d}{dt}x(t)\right|_{t=0}}} \tag{8.3-5}$$

Figure 8.3-3 shows a plot of $x(t)$ versus t. We can determine the values of (1) the slope of the plot at time $t = 0$, (2) the initial value of $x(t)$, and (3) the final value of $x(t)$ from this plot. Equation 8.3-5 can be used to calculate the time constant from these values. Equivalently, Figure 8.3-3 shows how to measure the time constant from a plot of $x(t)$ versus t.

Next, we apply these results to the RC circuit in Figure 8.3-1. Comparing Eqs. 8.3-1 and 8.3-3, we see that

$$x(t) = v(t),\ \tau = R_t C,\ \text{ and } K = \frac{V_{oc}}{R_t C}$$

Making these substitutions in Eq. 8.3-4 gives

$$\boxed{v(t) = V_{oc} + (v(0) - V_{oc})e^{-t/(R_t C)}} \tag{8.3-6}$$

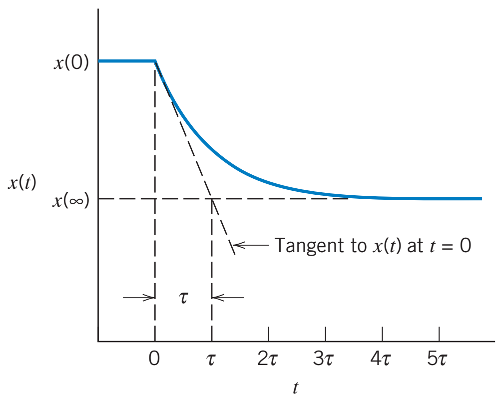

FIGURE 8.3-3 A graphical technique for measuring the time constant of a first-order circuit.

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The second term on the right-hand side of Eq. 8.3-6 dies out as t increases. This is the transient or natural response. At $t = 0$, $e^{-0} = 1$. Letting $t = 0$ in Eq. 8.3-6 gives $v(0) = v(0)$, as required. When $t = 5\tau$, $e^{-5} = 0.0067 \approx 0$, so at time $t = 5\tau$, the capacitor voltage will be

$$v(5\tau) = 0.9933\ V_{oc} + 0.0067\ v(0) \approx V_{oc}$$

This is the steady-state or forced response. The forced response is of the same form, a constant, as the input to the circuit. The sum of the natural and forced responses is the complete response:

$$\text{complete response} = v(t), \quad \text{forced response} = V_{oc}$$

and

$$\text{natural response} = (v(0) - V_{oc})e^{-t/(R_t C)}$$

Next, compare Eqs. 8.3-2 and 8.3-3 to find the solution of the *RL* circuit in Figure 8.3-2. We see that

$$x(t) = i(t), \quad \tau = \frac{L}{R_t}, \quad \text{and} \quad K = \frac{L}{R_t} I_{sc}$$

Making these substitutions in Eq. 8.3-4 gives

$$\boxed{i(t) = I_{sc} + (i(0) - I_{sc})e^{-(R_t/L)t}} \tag{8.3-7}$$

Again, the complete response is the sum of the forced (steady-state) response and the transient (natural) response:

$$\text{complete response} = i(t), \quad \text{forced response} = I_{sc}$$

and

$$\text{natural response} = (i(0) - I_{sc})e^{-(R_t/L)t}$$

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EXAMPLE 8.3-1 First-Order Circuit with a Capacitor

Find the capacitor voltage after the switch opens in the circuit shown in Figure 8.3-4*a*. What is the value of the capacitor voltage 50 ms after the switch opens?

Solution

The 2-volt voltage source forces the capacitor voltage to be 2 volts until the switch opens. Because the capacitor voltage cannot change instantaneously, the capacitor voltage will be 2 volts immediately after the switch opens. Therefore, the initial condition is

$$v(0) = 2\ \text{V}$$

Figure 8.3-4*b* shows the circuit after the switch opens. Comparing this circuit to the *RC* circuit in Figure 8.3-1*b*, we see that

$$R_t = 10\ \text{k}\Omega \quad \text{and} \quad V_{oc} = 8\ \text{V}$$

The time constant for this first-order circuit containing a capacitor is

$$\tau = R_t C = \left(10 \times 10^3\right)\left(2 \times 10^{-6}\right) = 20 \times 10^{-3} = 20\ \text{ms}$$

Substituting these values into Eq. 8.3-6 gives

$$v(t) = 8 - 6e^{-t/20}\ \text{V} \tag{8.3-8}$$

where t has units of ms. To find the voltage 50 ms after the switch opens, let $t = 50$. Then,

$$v(50) = 8 - 6e^{-50/20} = 7.51\ \text{V}$$

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Figure 8.3-4*c* shows a plot of the capacitor voltage as a function of time.

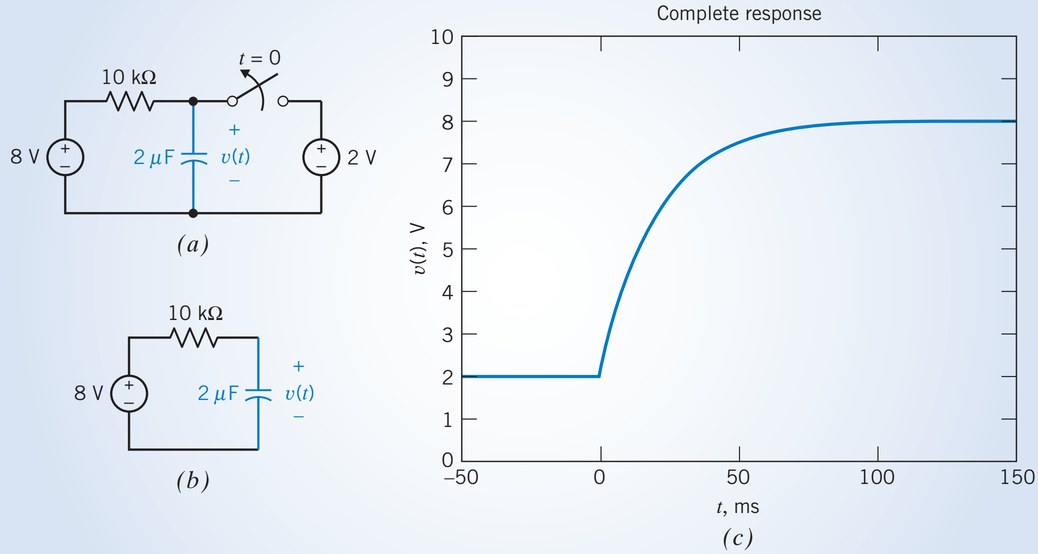

FIGURE 8.3-4 (*a*) A first-order circuit and (*b*) an equivalent circuit that is valid after the switch opens. (*c*) A plot of the complete response, $v(t)$, given in Eq. 8.3-8.

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EXAMPLE 8.3-2 First-Order Circuit with an Inductor

Find the inductor current after the switch closes in the circuit shown in Figure 8.3-5*a*. How long will it take for the inductor current to reach 2 mA?

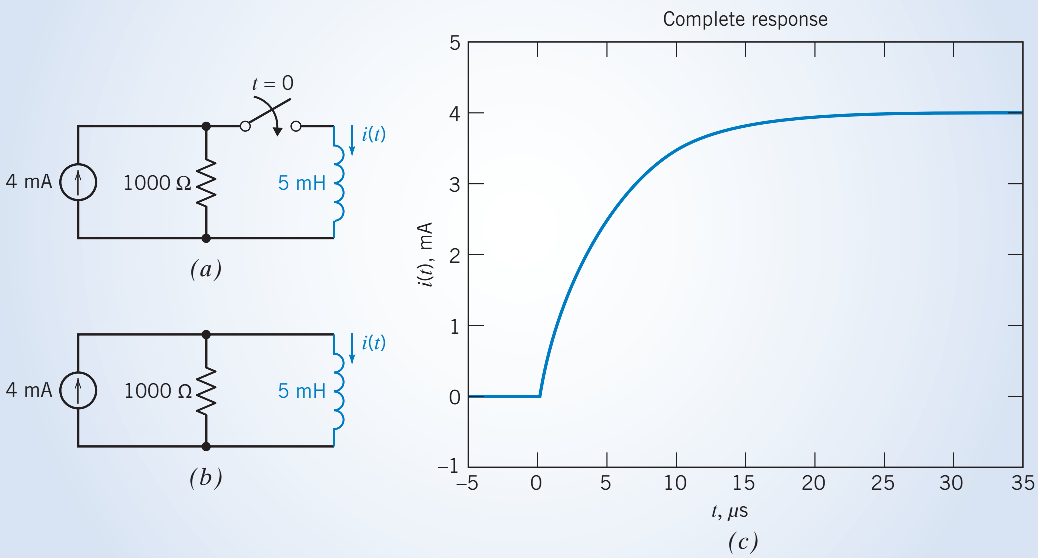

FIGURE 8.3-5 (*a*) A first-order circuit and (*b*) an equivalent circuit that is valid after the switch closes. (*c*) A plot of the complete response, $i(t)$, given by Eq. 8.3-9.

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Solution

The inductor current will be 0 A until the switch closes. Because the inductor current cannot change instantaneously, it will be 0 A immediately after the switch closes. Therefore, the initial condition is

$$i(0) = 0$$

Figure 8.3-5*b* shows the circuit after the switch closes. Comparing this circuit to the *RL* circuit in Figure 8.3-2*b*, we see that

$$R_t = 1000\ \Omega \quad \text{and} \quad I_{sc} = 4\ \text{mA}$$

The time constant for this first-order circuit containing an inductor is

$$\tau = \frac{L}{R_t} = \frac{5 \times 10^{-3}}{1000} = 5 \times 10^{-6} = 5\ \mu\text{s}$$

Substituting these values into Eq. 8.3-7 gives

$$i(t) = 4 - 4e^{-t/5}\ \text{mA} \tag{8.3-9}$$

where t has units of microseconds. To find the time when the current reaches 2 mA, substitute $i(t) = 2$ mA. Then

$$2 = 4 - 4e^{-t/5}\ \text{mA}$$

Solving for t gives

$$t = -5 \times \ln\left(\frac{2-4}{-4}\right) = 3.47\ \mu\text{s}$$

Figure 8.3-5*c* shows a plot of the inductor current as a function of time.

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EXAMPLE 8.3-3 First-Order Circuit

INTERACTIVE EXAMPLE

The switch in Figure 8.3-6*a* has been open for a long time, and the circuit has reached steady state before the switch closes at time $t = 0$. Find the capacitor voltage for $t \geq 0$.

Solution

The switch has been open for a long time before it closes at time $t = 0$. The circuit will have reached steady state before the switch closes. Because the input to this circuit is a constant, all the element currents and voltages will be constant when the circuit is at steady state. In particular, the capacitor voltage will be constant. The capacitor current will be

$$i(t) = C\frac{d}{dt}v(t) = C\frac{d}{dt}(\text{a constant}) = 0$$

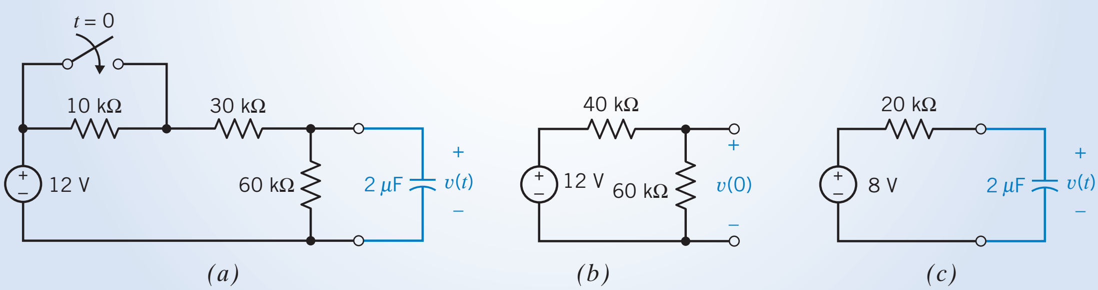

FIGURE 8.3-6 (*a*) A first-order circuit. The equivalent circuit for (*b*) $t < 0$ and (*c*) $t > 0$.

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The capacitor voltage is unknown, but the capacitor current is zero. In other words, the capacitor acts like an open circuit when the input is constant and the circuit is at steady state. (By a similar argument, inductors act like short circuits when the input is constant and the circuit is at steady state.)

Figure 8.3-6*b* shows the appropriate equivalent circuit while the switch is open. An open switch acts like an open circuit; thus, the 10-kΩ and 30-kΩ resistors are in series. They have been replaced by an equivalent 40-kΩ resistor. The input to the circuit is a constant (12 volts), and the circuit is at steady state; therefore, the capacitor acts like an open circuit. The voltage across this open circuit is the capacitor voltage. Because we are interested in the initial condition, the capacitor voltage has been labeled as $v(0)$. Analyzing the circuit in Figure 8.3-6*b* using voltage division gives

$$v(0) = \frac{60 \times 10^3}{40 \times 10^3 + 60 \times 10^3} 12 = 7.2 \text{ V}$$

Figure 8.3-6*c* shows the appropriate equivalent circuit after the switch closes. Closing the switch shorts out the 10-kΩ resistor, removing it from the circuit. (A short circuit in parallel with any resistor is equivalent to a short circuit.) The part of the circuit that is connected to the capacitor has been replaced by its Thévenin equivalent circuit. After the switch is closed,

$$V_{\text{oc}} = \frac{60 \times 10^3}{30 \times 10^3 + 60 \times 10^3} 12 = 8 \text{ V}$$

and

$$R_{\text{t}} = \frac{30 \times 10^3 \times 60 \times 10^3}{30 \times 10^3 + 60 \times 10^3} = 20 \times 10^3 = 20 \text{ k}\Omega$$

and the time constant is

$$\tau = R_{\text{t}} \times C = \left(20 \times 10^3\right) \times \left(2 \times 10^{-6}\right) = 40 \times 10^{-3} = 40 \text{ ms}$$

Substituting these values into Eq. 8.3-6 gives

$$v(t) = 8 - 0.8e^{-t/40} \text{ V}$$

where t has units of ms.

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EXAMPLE 8.3-4 First-Order Circuit

INTERACTIVE EXAMPLE

The switch in Figure 8.3-7*a* has been open for a long time, and the circuit has reached steady state before the switch closes at time $t = 0$. Find the inductor current for $t \geq 0$.

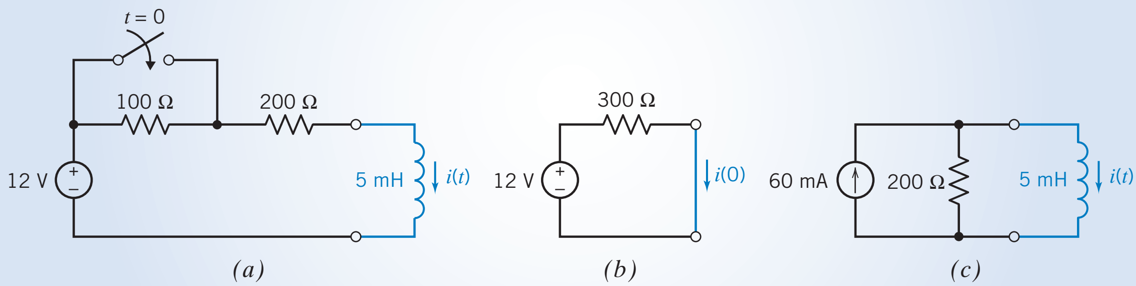

FIGURE 8.3-7 (*a*) A first-order circuit. The equivalent circuit for (*b*) $t < 0$ and (*c*) $t > 0$.

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Solution

Figure 8.3-7*b* shows the appropriate equivalent circuit while the switch is open. The 100-Ω and 200-Ω resistors are in series and have been replaced by an equivalent 300-Ω resistor. The input to the circuit is a constant (12 volts), and the circuit is at steady state; therefore, the inductor acts like a short circuit. The current in this short circuit is the inductor current. Because we are interested in the initial condition, the initial inductor current has been labeled as $i(0)$. This current can be calculated using Ohm's law:

$$i(0) = \frac{12}{300} = 40 \text{ mA}$$

Figure 8.3-7*c* shows the appropriate equivalent circuit after the switch closes. Closing the switch shorts out the 100-Ω resistor, removing it from the circuit. The part of the circuit that is connected to the inductor has been replaced by its Norton equivalent circuit. After the switch is closed,

$$I_{sc} = \frac{12}{200} = 60 \text{ mA} \quad \text{and} \quad R_t = 200\ \Omega$$

and the time constant is

$$\tau = \frac{L}{R_t} = \frac{5 \times 10^{-3}}{200} = 25 \times 10^{-6} = 25\ \mu\text{s}$$

Substituting these values into Eq. 8.3-7 gives

$$i(t) = 60 - 20e^{-t/25} \text{ mA}$$

where t has units of microseconds.

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EXAMPLE 8.3-5 First-Order Circuit

The circuit in Figure 8.3-8*a* is at steady state before the switch opens. Find the current $i(t)$ for $t > 0$.

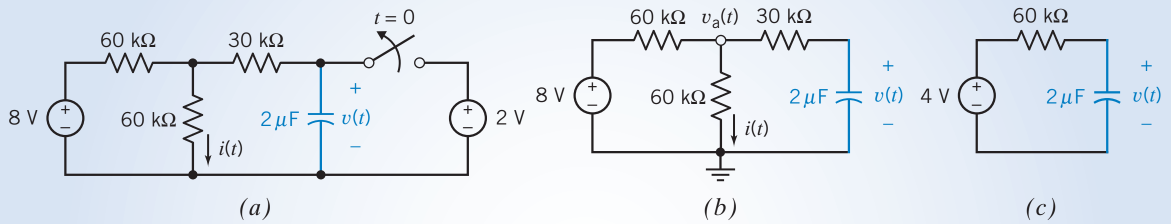

FIGURE 8.3-8 (*a*) A first-order circuit, (*b*) the circuit after the switch opens, and (*c*) the equivalent circuit after the switch opens.

Solution

The response or output of a circuit can be any element current or voltage. Frequently, the response is not the capacitor voltage or inductor current. In Figure 8.3-8*a*, the response is the current $i(t)$ in a resistor rather than the capacitor voltage. In this case, two steps are required to solve the problem. First, find the capacitor voltage using the methods already described in this chapter. Once the capacitor voltage is known, write node or mesh equations to express the response in terms of the input and the capacitor voltage.

First we find the capacitor voltage. Before the switch opens, the capacitor voltage is equal to the voltage of the 2-volt source. The initial condition is

$$v(0) = 2 \text{ V}$$

Figure 8.3-8*b* shows the circuit as it will be after the switch is opened. The part of the circuit connected to the capacitor has been replaced by its Thévenin equivalent circuit in Figure 8.3-8*c*. The parameters of the Thévenin

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equivalent circuit are

$$V_{oc} = \frac{60 \times 10^3}{60 \times 10^3 + 60 \times 10^3} 8 = 4\text{ V}$$

and
$$R_t = 30 \times 10^3 + \frac{60 \times 10^3 \times 60 \times 10^3}{60 \times 10^3 + 60 \times 10^3} = 60 \times 10^3 = 60\text{ k}\Omega$$

The time constant is

$$\tau = R_t \times C = \left(60 \times 10^3\right) \times \left(2 \times 10^{-6}\right) = 120 \times 10^{-3} = 120\text{ ms}$$

Substituting these values into Eq. 8.3-6 gives

$$v(t) = 4 - 2e^{-t/120}\text{ V}$$

where t has units of ms.

Now that the capacitor voltage is known, we return to the circuit in Figure 8.3-8*b*. Notice that the node voltage at the middle node at the top of the circuit has been labeled as $v_a(t)$. The node equation corresponding to this node is

$$\frac{v_a(t) - 8}{60 \times 10^3} + \frac{v_a(t)}{60 \times 10^3} + \frac{v_a(t) - v(t)}{30 \times 10^3} = 0$$

Substituting the expression for the capacitor voltage gives

$$\frac{v_a(t) - 8}{60 \times 10^3} + \frac{v_a(t)}{60 \times 10^3} + \frac{v_a(t) - \left(4 - 2e^{-t/120}\right)}{30 \times 10^3} = 0$$

or
$$v_a(t) - 8 + v_a(t) + 2\left[v_a(t) - \left(4 - 2e^{-t/120}\right)\right] = 0$$

Solving for $v_a(t)$, we get

$$v_a(t) = \frac{8 + 2\left(4 - 2e^{-t/120}\right)}{4} = 4 - e^{-t/120}\text{ V}$$

Finally, we calculate $i(t)$ using Ohm's law:

$$i(t) = \frac{v_a(t)}{60 \times 10^3} = \frac{4 - e^{-t/120}}{60 \times 10^3} = 66.7 - 16.7e^{-t/120}\ \mu\text{A}$$

where t has units of ms.

EXAMPLE 8.3-6 First-Order Circuit with $t_0 \neq 0$

Find the capacitor voltage after the switch opens in the circuit shown in Figure 8.3-9*a*. What is the value of the capacitor voltage 50 ms after the switch opens?

Solution

This example is similar to Example 8.3-1. The difference between the two examples is the time at which the switch opens. The switch opens at time $t = 0$ in Example 8.3-1 and at time $t = 50$ ms $= 0.05$ s in this example.

The 2-volt voltage source forces the capacitor voltage to be 2 volts until the switch opens. Consequently,

$$v(t) = 2\text{ V} \quad \text{for} \quad t \leq 0.05\text{ s}$$

In particular, the initial condition is

$$v(0.05) = 2\text{ V}$$

Figure 8.3-9*b* shows the circuit after the switch opens. Comparing this circuit to the *RC* circuit in Figure 8.3-1*b*,

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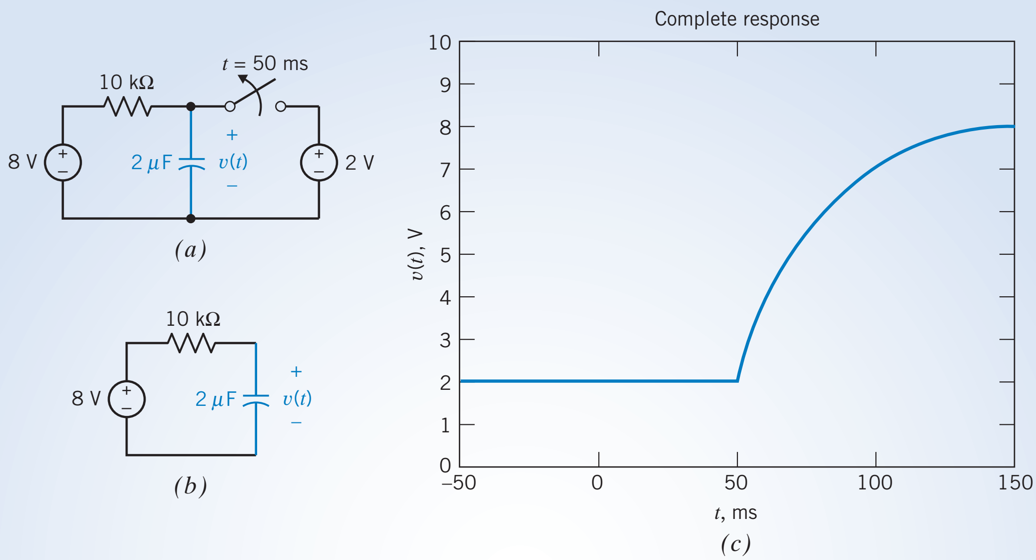

FIGURE 8.3-9 (*a*) A first-order circuit and (*b*) an equivalent circuit that is valid after the switch opens. (*c*) A plot of the complete response, $v(t)$, given by Eq. 8.3-10.

we see that

$$R_t = 10\ \text{k}\Omega \quad \text{and} \quad V_{oc} = 8\ \text{V}$$

The time constant for this first-order circuit containing a capacitor is

$$\tau = R_t C = 0.020\ \text{s}$$

A plot of the capacitor voltage in this example will have the same shape as did the plot of the capacitor voltage in Example 8.3-1, but the capacitor voltage in this example will be delayed by 50 ms because the switch opened 50 ms later. To account for this delay, we replace t by $t - 50$ ms in the equation that represents the capacitor voltage. Consequently, the voltage of the capacitor in this example is given by

$$v(t) = 8 - 6e^{-(t-50)/20}\ \text{V} \tag{8.3-10}$$

where t has units of ms. (Compare Eq. 8.3-8 and 8.3-10.) To find the voltage 50 ms after the switch opens, let $t = 100$ ms. Then,

$$v(100) = 8 - 6e^{-(100-50)/20} = 7.51\ \text{V}$$

The value of the capacitor voltage 50 ms after the switch opens is the same here as it was in Example 8.3-1. Figure 8.3-9*c* shows a plot of the capacitor voltage as a function of time. As expected, this plot is a delayed copy of the plot shown in Figure 8.3-4*c*.

EXAMPLE 8.3-7 First-Order Circuit with $t_0 \neq 0$

Find the inductor current after the switch closes in the circuit shown in Figure 8.3-10*a*. How long will it take for the inductor current to reach 2 mA?

Solution

This example is similar to Example 8.3-2. The difference between the two examples is the time at which the switch closes. The switch closes at time $t = 0$ in Example 8.3-2 and at time $t = 10\ \mu$s in this example.

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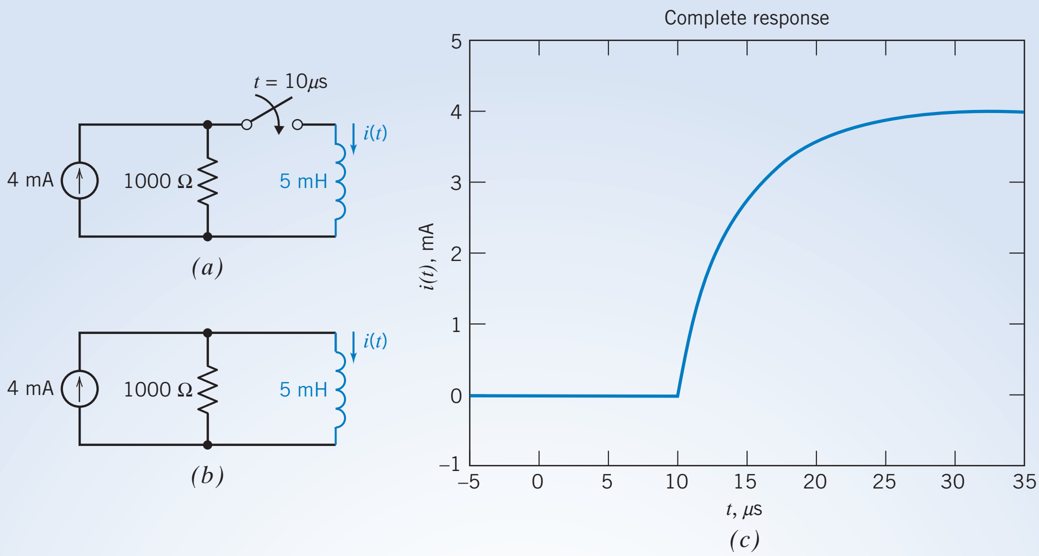

FIGURE 8.3-10 (*a*) A first-order circuit and (*b*) an equivalent circuit that is valid after the switch closes. (*c*) A plot of the complete response, $i(t)$, given by Eq. 8.3-11.

The inductor current will be 0 A until the switch closes. Because the inductor current cannot change instantaneously, it will be 0 A immediately after the switch closes. Therefore, the initial condition is

$$i(10\ \mu\text{s}) = 0\ \text{A}$$

Figure 8.3-10*b* shows the circuit after the switch closes. Comparing this circuit to the *RL* circuit in Figure 8.3-2*b*, we see that

$$R_t = 1000\ \Omega \quad \text{and} \quad I_{sc} = 4\ \text{mA}$$

The time constant for this first-order circuit containing an inductor is

$$\tau = \frac{L}{R_t} = \frac{5 \times 10^{-3}}{1000} = 5 \times 10^{-6} = 5\ \mu\text{s}$$

A plot of the inductor current in this example will have the same shape as did the plot of the inductor current in Example 8.3-2, but the inductor current in this example will be delayed by 10 μs because the switch closed 10 μs later. To account for this delay, we replace t by $t-10$ μs in the equation that represents the inductor current. Consequently, the current of the inductor in this example is given by

$$i(t) = 4 - 4e^{-(t-10)/5}\ \text{mA} \tag{8.3-11}$$

where t has units of microseconds. (Compare Eq. 8.3-9 and 8.3-11.) To find the time when the current reaches 2 mA, substitute $i(t) = 2$ mA. Then

$$2 = 4 - 4e^{-(t-10)/5}\ \text{mA}$$

Solving for t gives

$$t = -5 \times \ln\left(\frac{2-4}{-4}\right) + 10 = 13.47\ \mu\text{s}$$

Because the switch closes at time 10 μs, an additional time of 3.47 μs after the switch closes is required for the value of the current to reach 2 mA. Figure 8.3-10*c* shows a plot of the inductor current as a function of time. As expected, this plot is a delayed copy of the plot shown in Figure 8.3-5*c*.

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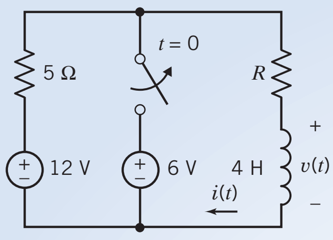
EXAMPLE 8.3-8 Exponential Response of a First-Order Circuit

Figure 8.3-11*a* shows a plot of the voltage across the inductor in Figure 8.3-11*b*.



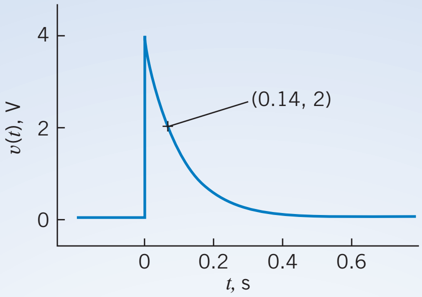


FIGURE 8.3-11 (*a*) A first-order circuit and (*b*) a plot of the inductor voltage.

(a) Determine the equation that represents the inductor voltage as a function of time.
(b) Determine the value of the resistance *R*.
(c) Determine the equation that represents the inductor current as a function of time.

Solution

(a) The inductor voltage is represented by an equation of the form

$$v(t) = \begin{cases} D & \text{for } t < 0 \\ E + F\,e^{-at} & \text{for } t \geq 0 \end{cases}$$

where D, E, F, and a are unknown constants. The constants D, E, and F are described by

$$D = v(t) \quad \text{when } t < 0, \qquad E = \lim_{t\to\infty} v(t), \quad \text{and } E + F = \lim_{t\to 0+} v(t)$$

From the plot, we see that

$$D = 0,\ E = 0, \quad \text{and} \quad E + F = 4\ \text{V}$$

Consequently,
$$v(t) = \begin{cases} 0 & \text{for } t < 0 \\ 4e^{-at} & \text{for } t \geq 0 \end{cases}$$

To determine the value of a, we pick a time when the circuit is not at steady state. One such point is labeled on the plot in Figure 8.3-11. We see $v\,(0.14) = 2$ V; that is, the value of the voltage is 2 volts at time 0.14 seconds. Substituting these into the equation for $v(t)$ gives

$$2 = 4e^{-a(0.14)} \quad \Rightarrow \quad a = \frac{\ln(0.5)}{-0.14} = 5$$

Consequently,
$$v(t) = \begin{cases} 0 & \text{for } t < 0 \\ 4e^{-5t} & \text{for } t \geq 0 \end{cases}$$

(b) Figure 8.3-12*a* shows the circuit immediately after the switch opens. In Figure 8.3-12*b*, the part of the circuit connected to the inductor has been replaced by its Thévenin equivalent circuit.
The time constant of the circuit is given by

$$\tau = \frac{L}{R_t} = \frac{4}{R + 5}$$

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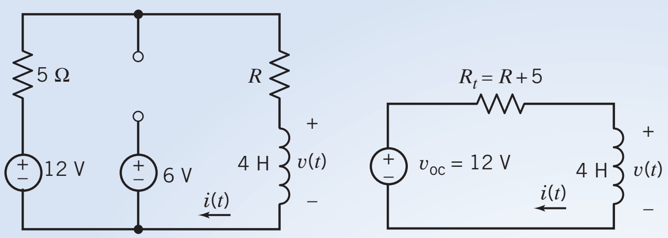

(a) (b)

FIGURE 8.3-12 (*a*) The first-order circuit after the switch opens. (*b*) An equivalent circuit.

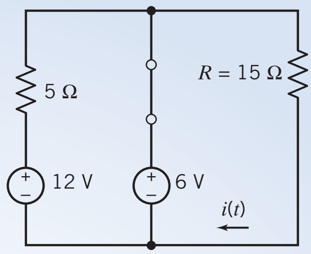

FIGURE 8.3-13 The first-order circuit before the switch opens.

Also, the time constant is related to the exponent in $v(t)$ by $-5t = -\frac{t}{\tau}$. Consequently,

$$5 = \frac{1}{\tau} = \frac{R+5}{4} \quad \Rightarrow \quad R = 15\ \Omega$$

(c) The inductor current is related to the inductor voltage by

$$i(t) = \frac{1}{L}\int_0^t v(\tau)d\tau + i(0)$$

Figure 8.3-13 shows the circuit before the switch opens. The closed switch is represented by a short circuit. The circuit is at steady state, and the voltage sources have constant voltages, so the inductor acts like a short circuit. The inductor current is given by

$$i(t) = \frac{6}{15} = 0.4\ \text{A}$$

In particular, $i(0-) = 0.4$ A. The current in an inductor is continuous, so $i(0+) = i(0-)$. Consequently,

$$i(0) = 0.4\ \text{A}$$

Returning to the equation for the inductor current, after the switch opens, we have

$$i(t) = \frac{1}{4}\int_0^t 4e^{-5\tau}d\tau + 0.4 = \frac{1}{-5}\left(e^{-5t} - 1\right) + 0.4 = 0.6 - 0.2e^{-5t}$$

In summary,

$$i(t) = \begin{cases} 0.4 & \text{for } t < 0 \\ 0.6 - 0.2e^{-5t} & \text{for } t \geq 0 \end{cases}$$

EXERCISE 8.3-1 The circuit shown in Figure E 8.3-1 is at steady state before the switch closes at time $t = 0$. Determine the capacitor voltage $v(t)$ for $t \geq 0$.

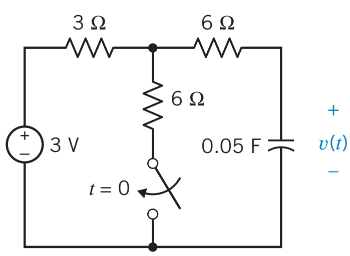

FIGURE E 8.3-1

Answer: $v(t) = 2 + e^{-2.5t}$ V for $t > 0$

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EXERCISE 8.3-2 The circuit shown in Figure E 8.3-2 is at steady state before the switch closes at time $t = 0$. Determine the inductor current $i(t)$ for $t > 0$.

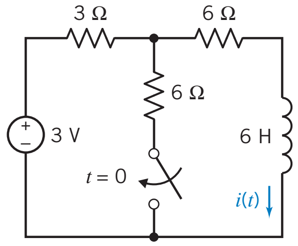

FIGURE E 8.3-2

Answer: $i(t) = \dfrac{1}{4} + \dfrac{1}{12} e^{-1.33t}$ A for $t > 0$

8.4 Sequential Switching

Often, circuits contain several switches that are not switched at the same time. For example, a circuit may have two switches where the first switch changes state at time $t = 0$ and the second switch closes at $t = 1$ ms.

> **Sequential switching** occurs when a circuit contains two or more switches that change state at different instants.

Circuits with sequential switching can be solved using the methods described in the previous sections, based on the fact that inductor currents and capacitor voltages do not change instantaneously.

As an example of sequential switching, consider the circuit shown in Figure 8.4-1*a*. This circuit contains two switches—one that changes state at time $t = 0$ and a second that closes at $t = 1$ ms. Suppose this circuit has reached steady state before the switch changes state at time $t = 0$. Figure 8.4-1*b* shows the equivalent circuit that is appropriate for $t < 0$. Because the circuit is at steady state and the input is constant, the inductor acts like a short circuit and the current in this short circuit is the

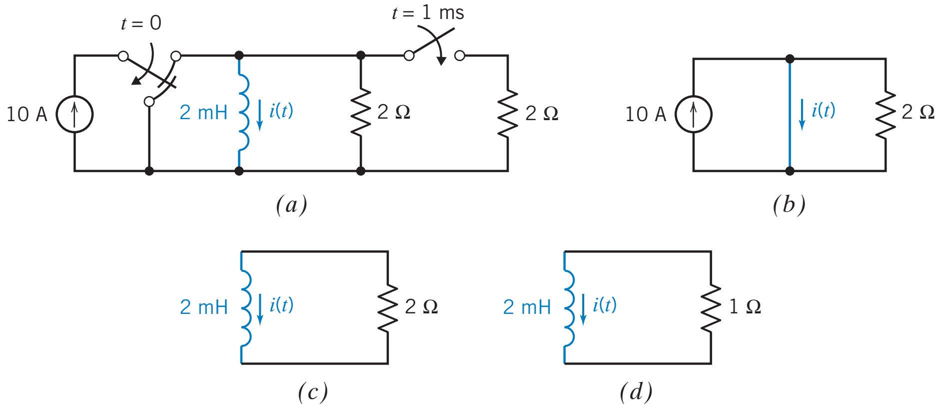

FIGURE 8.4-1 (*a*) A circuit with sequential switching. (*b*) The equivalent circuit before $t = 0$. (*c*) The equivalent circuit for $0 < t < 1$ ms. (*d*) The equivalent circuit after $t = 1$ ms.

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inductor current. The short circuit forces the voltage across the resistor to be zero, so the current in the resistor is also zero. As a result, all of the source current flows in the short circuit and

$$i(t) = 10\text{ A} \quad t < 0$$

The inductor current will be 10 A immediately before the switch changes state at time $t = 0$. We express this as

$$i(0^-) = 10\text{ A}$$

Because the inductor current does not change instantaneously, the inductor current will also be 10 A immediately after the switch changes state. That is,

$$i(0^+) = 10\text{ A}$$

This is the initial condition that is used to calculate the inductor current after $t = 0$. Figure 8.4-1*c* shows the equivalent circuit that is appropriate after one switch changes state at time $t = 0$ and before the other switch closes at time $t = 1$ ms. We see that the Norton equivalent of the part of the circuit connected to the inductor has the parameters

$$I_{sc} = 0\text{ A} \quad \text{and} \quad R_t = 2\ \Omega$$

The time constant of this first-order circuit is

$$\tau = \frac{L}{R_t} = \frac{2 \times 10^{-3}}{2} = 1 \times 10^{-3} = 1\text{ ms}$$

The inductor current is

$$i(t) = i(0)e^{-t/\tau} = 10e^{-t}\text{ A}$$

for $0 < t < 1$ ms. Notice that t has units of ms. Immediately before the other switch closes at time $t = 1$ ms, the inductor current will be

$$i(1^-) = 10e^{-1} = 3.68\text{ A}$$

Because the inductor current does not change instantaneously, the inductor current will also be 3.68 A immediately after the switch changes state. That is,

$$i(1^+) = 3.68\text{ A}$$

This is the initial condition that is used to calculate the inductor current after the switch closes at time $t = 1$ ms. Figure 8.4-1*d* shows the appropriate equivalent circuit. We see that the Norton equivalent of the part of the circuit connected to the inductor has the parameters

$$I_{sc} = 0\text{ A} \quad \text{and} \quad R_t = 1\ \Omega$$

The time constant of this first-order circuit is

$$\tau = \frac{L}{R_t} = \frac{2 \times 10^{-3}}{1} = 2 \times 10^{-3} = 2\text{ ms}$$

The inductor current is

$$i(t) = i(t_0)e^{-(t-t_0)/\tau} = 3.68e^{-(t-1)/2}\text{ A}$$

for 1 ms $< t$. Once again, t has units of ms. Also, t_0 denotes the time when the switch changes state—1 ms in this example.

Figure 8.4-2 shows a plot of the inductor current. The time constant changes when the second switch closes. As a result, the slope of the plot changes at $t = 1$ ms. Immediately before the switch closes, the slope is -3.68 A/ms. Immediately after the switch closes, the slope becomes $-3.68/2$ A/ms.

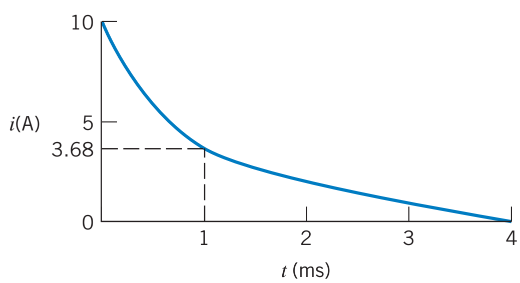

FIGURE 8.4-2 Current waveform for $t \geq 0$. The exponential has a different time constant for $0 \leq t < t_1$ and for $t \geq t_1$ where $t_1 = 1$ ms.

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8.5 Stability of First-Order Circuits

We have shown that the natural response of a first-order circuit is

$$x_n(t) = Ke^{-t/\tau}$$

and that the complete response is the sum of the natural and forced responses:

$$x(t) = x_n(t) + x_f(t)$$

When $\tau > 0$, the natural response vanishes as $t \to 0$, leaving the forced response. In this case, the circuit is said to be *stable*. When $\tau < 0$, the natural response grows without bound as $t \to 0$. The forced response becomes negligible, compared to the natural response. The circuit is said to be *unstable*. When a circuit is stable, the forced response depends on the input to the circuit. That means that the forced response contains information about the input. When the circuit is unstable, the forced response is negligible, and this information is lost. In practice, the natural response of an unstable circuit is not unbounded. This response will grow until something happens to change the circuit. Perhaps that change will be saturation of an op amp or of a dependent source. Perhaps that change will be the destruction of a circuit element. In most applications, the behavior of unstable circuits is undesirable and is to be avoided.

How can we design first-order circuits to be stable? Recalling that $\tau = R_t C$ or $\tau = L/R_t$, we see that

> $R_t > 0$ is required to make a first-order circuit stable.

This condition will always be satisfied whenever the part of the circuit connected to the capacitor or inductor consists of only resistors and independent sources. Such circuits are guaranteed to be stable. In contrast, a first-order circuit that contains op amps or dependent sources may be unstable.

EXAMPLE 8.5-1 Response of an Unstable First-Order Circuit

The first-order circuit shown in Figure 8.5-1*a* is at steady state before the switch closes at $t = 0$. This circuit contains a dependent source and so may be unstable. Find the capacitor voltage $v(t)$ for $t > 0$.

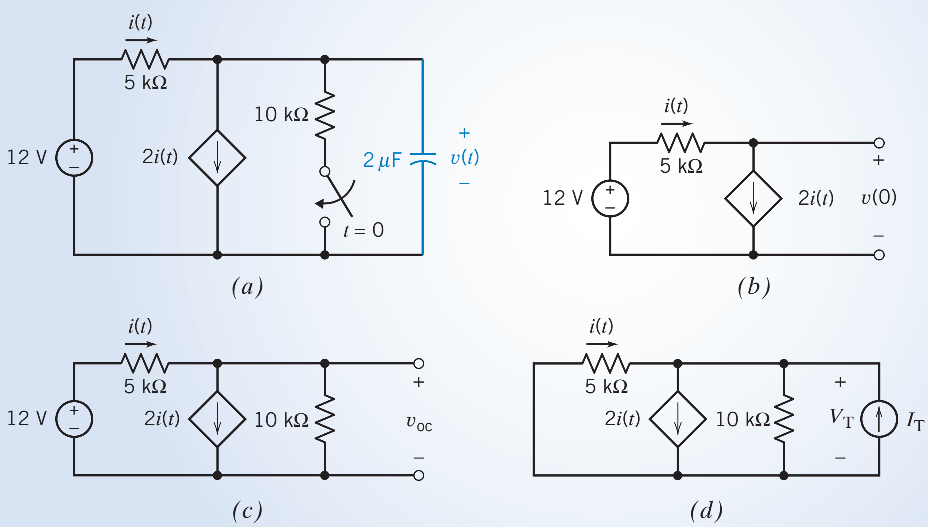

FIGURE 8.5-1 (*a*) A first-order circuit containing a dependent source. (*b*) The circuit used to calculate the initial condition. (*c*) The circuit used to calculate V_{oc}. (*d*) The circuit used to calculate R_t.

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Solution

The input to the circuit is a constant, so the capacitor acts like an open circuit at steady state. We calculate the initial condition from the circuit in Figure 8.5-1*b*. Applying KCL to the top node of the dependent current source, we get

$$-i + 2i = 0$$

Therefore, $i = 0$. Consequently, there is no voltage drop across the resistor, and

$$v(0) = 12 \text{ V}$$

Next, we determine the Thévenin equivalent circuit for the part of the circuit connected to the capacitor. This requires two calculations. First, calculate the open-circuit voltage, using the circuit in Figure 8.5-1*c*. Writing a KVL equation for the loop consisting of the two resistors and the voltage source, we get

$$12 = \left(5 \times 10^3\right) \times i + \left(10 \times 10^3\right) \times (i - 2i)$$

Solving for the current, we find

$$i = -2.4 \text{ mA}$$

Applying Ohm's law to the 10-kΩ resistor, we get

$$V_{\text{oc}} = \left(10 \times 10^3\right) \times (i - 2i) = 24 \text{ V}$$

Now calculate the Thévenin resistance using the circuit shown in Figure 8.5-1*d*. Apply KVL to the loop consisting of the two resistors to get

$$0 = \left(5 \times 10^3\right) \times i + \left(10 \times 10^3\right) \times (I_{\text{T}} + i - 2i)$$

Solving for the current,

$$i = 2I_{\text{T}}$$

Applying Ohm's law to the 10-kΩ resistor, we get

$$V_{\text{T}} = 10 \times 10^3 \times (I_{\text{T}} + i - 2i) = -10 \times 10^3 \times I_{\text{T}}$$

The Thévenin resistance is given by

$$R_{\text{t}} = \frac{V_{\text{T}}}{I_{\text{T}}} = -10 \text{ k}\Omega$$

The time constant is

$$\tau = R_{\text{t}}C = -20 \text{ ms}$$

This circuit is unstable. The complete response is

$$v(t) = 24 - 12\, e^{t/20}$$

The capacitor voltage *decreases* from $v(0) = 12$ V rather than *increasing* toward $v_{\text{f}} = 24$ V. Notice that

$$v(\infty) = \lim_{t\to\infty} v(t) = -\infty$$

It's not appropriate to refer to the forced response as a steady-state response when the circuit is unstable.

EXAMPLE 8.5-2 Designing First-Order Circuits to be Stable

The circuit considered in Example 8.5-1 has been redrawn in Figure 8.5-2*a*, with the gain of the dependent source represented by the variable *B*. What restrictions must be placed on the gain of the dependent source to ensure that it is stable? Design this circuit to have a time constant of +20 ms.

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Solution

Figure 8.5-2*b* shows the circuit used to calculate R_t. Applying KVL to the loop consisting of the two resistors,

$$5 \times 10^3 \times i + V_T = 0$$

Solving for the current gives

$$i = -\frac{V_T}{5 \times 10^3}$$

Applying KCL to the top node of the dependent source, we get

$$-i + Bi + \frac{V_T}{10 \times 10^3} - I_T = 0$$

Combining these equations, we get

$$\left(\frac{1-B}{5 \times 10^3} + \frac{1}{10 \times 10^3}\right) V_T - I_T = 0$$

The Thévenin resistance is given by

$$R_t = \frac{V_T}{I_T} = -\frac{10 \times 10^3}{2B - 3}$$

The condition $B < 3/2$ is required to ensure that R_t is positive and the circuit is stable.

To obtain a time constant of +20 ms requires

$$R_t = \frac{\tau}{C} = \frac{20 \times 10^{-3}}{2 \times 10^{-6}} = 10 \times 10^3 = 10\,\text{k}\Omega$$

which in turn requires

$$10 \times 10^3 = -\frac{10 \times 10^3}{2B - 3}$$

Therefore $B = 1$. This suggests that we can fix the unstable circuit by decreasing the gain of the dependent source from 2 A/A to 1 A/A.

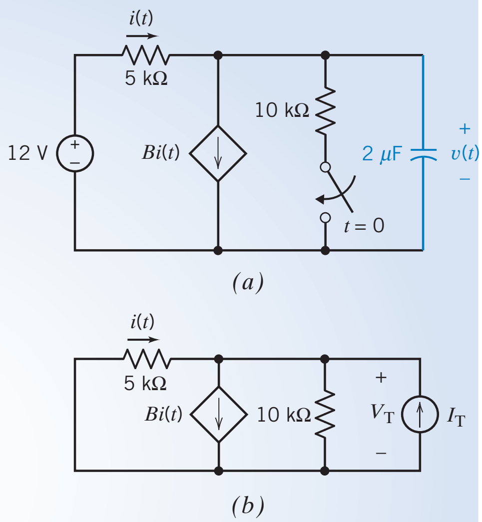

FIGURE 8.5-2 (*a*) A first-order circuit containing a dependent source. (*b*) The circuit used to calculate the Thévenin resistance of the part of the circuit connected to the capacitor.

8.6 *The Unit Step Source*

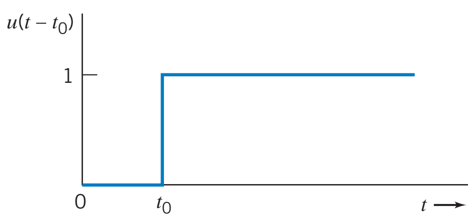

FIGURE 8.6-1 Unit step forcing function, $u(t - t_0)$.

The unit step function provides a convenient way to represent an abrupt change in a voltage or current.

We define the *unit step function* as a function of time that is zero for $t < t_0$ and unity for $t > t_0$. At $t = t_0$, the value changes from zero to one. We represent the unit step function by $u(t - t_0)$, where

$$u(t - t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t > t_0 \end{cases} \quad (8.6\text{-}1)$$

The value of $u(t - t_0)$ is not defined at $t = t_0$, where it switches instantaneously from a value of zero to one. The unit step function is shown in Figure 8.6-1. We will often consider $t_0 = 0$.

The unit step function is dimensionless. To represent a voltage that changes abruptly from one constant value to another constant value at time $t = t_0$, we can write

$$v(t) = A + B\,u(t - t_0)$$

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which indicates that
$$v(t) = \begin{cases} A & t < t_0 \\ A + B & t > t_0 \end{cases}$$

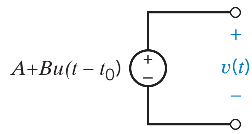

FIGURE 8.6-2 Symbol for a voltage source having a voltage that changes abruptly at time $t = t_0$.

where A and B have units of Volt. Figure 8.6-2 shows a voltage source having this voltage.

It is worth noting that $u(-t)$ indicates that we have a value of 1 for $t < 0$, so that

$$u(-t) = \begin{cases} 1 & t < 0 \\ 0 & t > 0 \end{cases}$$

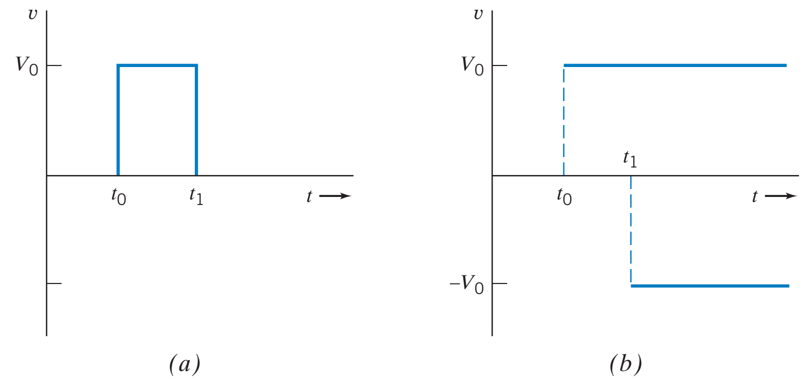

FIGURE 8.6-3 (*a*) Rectangular voltage pulse. (*b*) Two-step voltage waveforms that yield the voltage pulse.

Let us consider the *pulse* source

$$v(t) = \begin{cases} 0 & t < t_0 \\ V_0 & t_0 < t < t_1 \\ 0 & t_1 < t \end{cases}$$

which is shown in Figure 8.6-3*a*. As shown in Figure 8.6-3*b*, the pulse can be obtained from two-step voltage sources, the first of value V_0 occurring at $t = t_0$ and the second equal to $-V_0$ occurring at $t = t_1$. Thus, the two-step sources of magnitude V_0 shown in Figure 8.6-4 will yield the desired pulse. We have $v(t) = V_0u(t - t_0) - V_0u(t - t_1)$ to provide the pulse. Notice how easy it is to use two-step function symbols to represent this pulse source. The pulse is said to have a duration of $(t_1 - t_0)$ s.

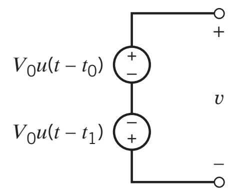

FIGURE 8.6-4 Two-step voltage sources that yield a rectangular voltage pulse $v(t)$ with a magnitude of V_0 and a duration of $(t_1 - t_0)$ where $t_0 < t_1$.

A **pulse signal** has a constant nonzero value for a time duration of $\Delta_t = t_1 - t_0$.

We recognize that the unit step function is an ideal model. No real element can switch instantaneously. However, if the switching time is very short compared to the time constant of the circuit, we can approximate the switching as instantaneous.

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EXAMPLE 8.6-1 First-Order Circuit

INTERACTIVE EXAMPLE

Figure 8.6-5 shows a first-order circuit. The input to the circuit is the voltage of the voltage source, $v_s(t)$. The output is the current of the inductor, $i_o(t)$. Determine the output of this circuit when the input is $v_s(t) = 4 - 8u(t)$ V.

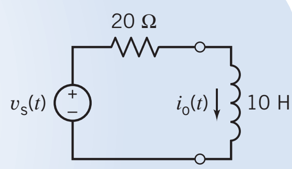

FIGURE 8.6-5 The circuit considered in Example 8.6-1.

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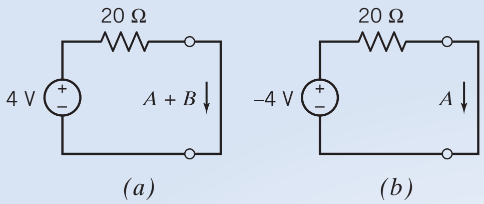

FIGURE 8.6-6 Circuits used to calculate the steady-state response (*a*) before $t = 0$ and (*b*) after $t = 0$.

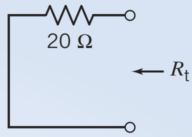

FIGURE 8.6-7 The circuit used to calculate R_t.

Solution

The value of the input is one constant, 4 V, before time $t = 0$ and a different constant, -4 V, after time $t = 0$. The response of the first-order circuit to the change in the value of the input will be

$$i_o(t) = A + Be^{-at} \quad \text{for} \quad t > 0 \tag{8.6-2}$$

where the values of the three constants A, B, and a are to be determined.

The values of A and B are determined from the steady-state responses of this circuit before and after the input changes value. Figures 8.6-6*a*,*b* show the circuits used to calculate those steady-state responses. Figures 8.6-6*a*,*b* require some explanation.

Inductors act like short circuits when the input is constant and the circuit is at steady state. Consequently, the inductor is replaced by a short circuit in Figure 8.6-6*a* and in Figure 8.6-6*b*.

The value of the inductor current at time $t = 0$ will be equal to the steady-state inductor current before the input changes. At time $t = 0$, the output current is

$$i_o(0) = A + Be^{-a(0)} = A + B$$

Consequently, the inductor current is labeled as $A + B$ in Figure 8.6-6*a*.

The value of the inductor current at time $t = \infty$ will be equal to the steady-state inductor current after the input changes. At time $t = \infty$, the output current is

$$i_o(\infty) = A + Be^{-a(\infty)} = A$$

Consequently, the inductor current is labeled as A in Figure 8.6-6*b*.

Analysis of the circuit in Figure 8.6-6*a* gives

$$A + B = 0.2 \text{ A}$$

Analysis of the circuit in Figure 8.6-6*b* gives

$$A = -0.2 \text{ A}$$

Therefore,

$$B = 0.4 \text{ A}$$

The value of the constant a in Eq. 8.6-2 is determined from the time constant, τ, which in turn is calculated from the values of the inductance L and of the Thévenin resistance, R_t, of the circuit connected to the inductor.

$$\frac{1}{a} = \tau = \frac{L}{R_t}$$

Figure 8.6-7 shows the circuit used to calculate R_t. It is seen from Figure 8.6-7 that

$$R_t = 20\ \Omega$$

Therefore,

$$a = \frac{20}{10} = 2\,\frac{1}{\text{s}}$$

(The time constant is $\tau = 10/20 = 0.5$ s.) Substituting the values of A, B, and a into Eq. 8.6-2 gives

$$i_o(t) = \begin{cases} 0.2 \text{ A} & \text{for } t \le 0 \\ -0.2 + 0.4\, e^{-2t} \text{ A} & \text{for } t \ge 0 \end{cases}$$

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EXAMPLE 8.6-2 First-Order Circuit

INTERACTIVE EXAMPLE

Figure 8.6-8 shows a first-order circuit. The input to the circuit is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the capacitor, $v_o(t)$. Determine the output of this circuit when the input is $v_s(t) = 7 - 14u(t)$ V.

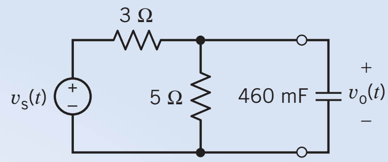

FIGURE 8.6-8 The circuit considered in Example 8.6-2.

Solution

The value of the input is one constant, 7 V, before time $t = 0$ and a different constant, -7 V, after time $t = 0$. The response of the first-order circuit to the change in the value of the input will be

$$v_o(t) = A + Be^{-at} \quad \text{for} \quad t > 0 \tag{8.6-3}$$

where the values of the three constants A, B, and a are to be determined.

The values of A and B are determined from the steady-state responses of this circuit before and after the input changes value. Figures 8.6-9*a*, *b* show the circuits used to calculate those steady-state responses. Figures 8.6-9*a*, *b* require some explanation.

Capacitors act like open circuits when the input is constant and the circuit is at steady state. Consequently, the capacitor is replaced by an open circuit in Figure 8.6-9*a* and in Figure 8.6-9*b*.

The value of the capacitor voltage at time $t = 0$ will be equal to the steady-state capacitor voltage before the input changes. At time $t = 0$, the output voltage is

$$v_o(0) = A + Be^{-a(0)} = A + B$$

Consequently, the capacitor voltage is labeled as $A + B$ in Figure 8.6-9*a*.

The value of the capacitor voltage at time $t = \infty$ will be equal to the steady-state capacitor voltage after the input changes. At time $t = \infty$, the output voltage is

$$v_o(\infty) = A + Be^{-a(\infty)} = A$$

Consequently, the capacitor voltage is labeled as A in Figure 8.6-9*b*.

Apply the voltage division rule to the circuit in Figure 8.6-9*a* to get

$$A + B = \frac{5}{3+5} \times 7 = 4.38 \text{ V}$$

Apply the voltage division rule to the circuit in Figure 8.6-9*b* to get

$$A = \frac{5}{3+5} \times (-7) = -4.38 \text{ V}$$

Therefore,

$$B = 8.76 \text{ V}$$

The value of the constant a in Eq. 8.6-3 is determined from the time constant τ, which in turn is calculated from the values of the capacitance C and of the Thévenin resistance R_t of the circuit connected to the capacitor:

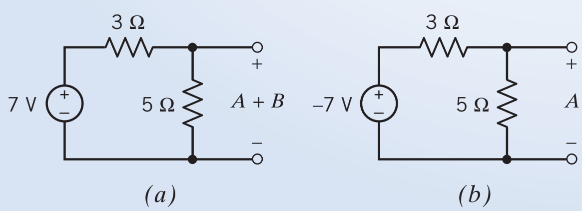

FIGURE 8.6-9 Circuits used to calculate the steady-state response (*a*) before $t = 0$ and (*b*) after $t = 0$.

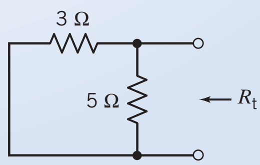

FIGURE 8.6-10 The circuit used to calculate R_t.

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$$\frac{1}{a} = \tau = R_t C$$

Figure 8.6-10 shows the circuit used to calculate R_t. It is seen from Figure 8.6-10 that

$$R_t = \frac{(5)(3)}{5+3} = 1.875\ \Omega$$

Therefore,

$$a = \frac{1}{(1.875)\left(460 \times 10^{-3}\right)} = 1.16\frac{1}{\mathrm{s}}$$

(The time constant is $\tau = (1.875)(460 \times 10^{-3}) = 0.86$ s.) Substituting the values of A, B, and a into Eq. 8.6-3 gives

$$v_o(t) = \begin{cases} -4.38\ \mathrm{V} & \text{for } t \le 0 \\ -4.38 + 8.76\, e^{-1.16\, t}\ \mathrm{V} & \text{for } t \ge 0 \end{cases}$$

8.7 The Response of a First-Order Circuit to a Nonconstant Source

In the previous sections, we wisely used the fact that the forced response to a constant source will be a constant itself. It now remains to determine what the response will be when the forcing function is not a constant.

The differential equation described by an *RL* or *RC* circuit is represented by the general form

$$\frac{dx(t)}{dt} + ax(t) = y(t) \tag{8.7-1}$$

where $y(t)$ is a constant only when we have a constant-current or constant-voltage source and where $a = 1/\tau$ is the reciprocal of the time constant.

In this section, we introduce the *integrating factor method*, which consists of multiplying Eq. 8.7-1 by a factor that makes the left-hand side a perfect derivative, and then integrating both sides.

Consider the derivative of a product of two terms such that

$$\frac{d}{dt}\left(xe^{at}\right) = \frac{dx}{dt}e^{at} + axe^{at} = \left(\frac{dx}{dt} + ax\right)e^{at} \tag{8.7-2}$$

The term within the parentheses on the right-hand side of Eq. 8.7-2 is exactly the form on the left-hand side of Eq. 8.7-1.

Therefore, if we multiply both sides of Eq. 8.7-1 by e^{at}, the left-hand side of the equation can be represented by the perfect derivative, $d(xe^{at})/dt$. Carrying out these steps, we show that

$$\left(\frac{dx}{dt} + ax\right)e^{at} = ye^{at}$$

or

$$\frac{d}{dt}\left(xe^{at}\right) = ye^{at}$$

Integrating both sides of the second equation, we have

$$xe^{at} = \int ye^{at}\,dt + K$$

where K is a constant of integration. Therefore, solving for $x(t)$, we multiply by e^{-at} to obtain

$$x = e^{-at}\int ye^{at}\,dt + Ke^{-at} \tag{8.7-3}$$

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When the source is a constant so that $y(t) = M$, we have

$$x = e^{-at}M\int e^{at}dt + Ke^{-at} = \frac{M}{a} + Ke^{-at} = x_f + x_n$$

where the natural response is $x_n = Ke^{-at}$ and the forced response is $x_f = M/a$, a constant.

Now consider the case in which $y(t)$, the forcing function, is not a constant. Considering Eq. 8.7-3, we see that the natural response remains $x_n = Ke^{-at}$. However, the forced response is

$$x_f = e^{-at}\int y(t)e^{at}\,dt$$

Thus, the forced response will be dictated by the form of $y(t)$. Let us consider the case in which $y(t)$ is an exponential function so that $y(t) = e^{bt}$. We assume that $(a + b)$ is not equal to zero. Then we have

$$x_f = e^{-at}\int e^{bt}e^{at}dt = e^{-at}\int e^{(a+b)t}dt = \frac{1}{a+b}e^{-at}e^{(a+b)} = \frac{e^{bt}}{a+b} \tag{8.7-4}$$

Therefore, the forced response of an *RL* or *RC* circuit to an exponential forcing function is of the same form as the forcing function itself. When $a + b$ is not equal to zero, we assume that the forced response will be of the same form as the forcing function itself, and we try to obtain the relationship that will be satisfied under those conditions.

EXAMPLE 8.7-1 First-Order Circuit with Nonconstant Source

Find the current i for the circuit of Figure 8.7-1*a* for $t > 0$ when

$$v_s = 10e^{-2t}u(t)\ \text{V}$$

Assume the circuit is in steady state at $t = 0^-$.

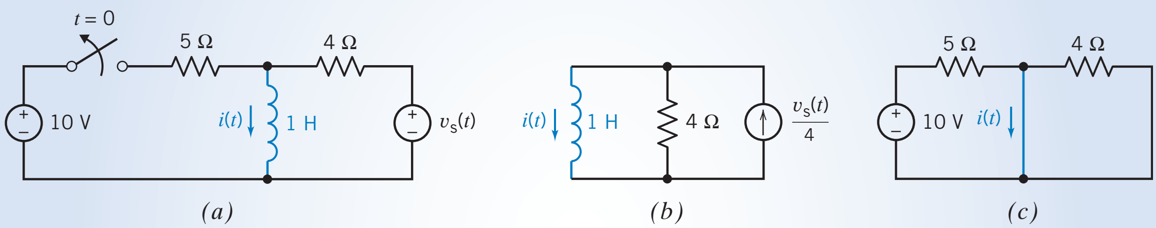

FIGURE 8.7-1 (*a*) A circuit with a nonconstant source, (*b*) the appropriate equivalent circuit after the switch opens, and (*c*) the appropriate equivalent circuit before the switch opens.

Solution

Because the forcing function is an exponential, we expect an exponential for the forced response i_f. Therefore, we expect i_f to be

$$i_f = Be^{-2t}$$

for $t \geq 0$. Writing KVL around the right-hand mesh, we have

$$L\frac{di}{dt} + Ri = v_s$$

or

$$\frac{di}{dt} + 4i = 10e^{-2t}$$

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for $t > 0$. Substituting $i_f = Be^{-2t}$, we have

$$-2Be^{-2t} + 4Be^{-2t} = 10e^{-2t}$$

or

$$(-2B + 4B)e^{-2t} = 10e^{-2t}$$

Hence, $B = 5$ and

$$i_f = 5e^{-2t}$$

The natural response can be obtained by considering the circuit shown in Figure 8.7-1*b*. This is the equivalent circuit that is appropriate after the switch has opened. The part of the circuit that is connected to the inductor has been replaced by its Norton equivalent circuit. The natural response is

$$i_n = Ae^{-(R_t/L)t} = Ae^{-4t}$$

The complete response is

$$i = i_n + i_f = Ae^{-4t} + 5e^{-2t}$$

The constant A can be determined from the value of the inductor current at time $t = 0$. The initial inductor current $i(0)$ can be obtained by considering the circuit shown in Figure 8.7-1*c*. This is the equivalent circuit that is appropriate before the switch opens. Because $v_s(t) = 0$ for $t < 0$ and a zero voltage source is a short circuit, the voltage source at the right side of the circuit has been replaced by a short circuit. Also, because the circuit is at steady state before the switch opens and the only input is the constant 10-volt source, the inductor acts like a short circuit. The current in the short circuit that replaces the inductor is the initial condition $i(0)$. From Figure 8.7-1*c*,

$$i(0) = \frac{10}{5} = 2\text{ A}$$

Therefore, at $t = 0$,

$$i(0) = Ae^{-4\times 0} + 5e^{-2\times 0} = A + 5$$

or

$$2 = A + 5$$

or $A = -3$. Therefore,

$$i = \left(-3e^{-4t} + 5e^{-2t}\right)\text{ A} \quad t > 0$$

The voltage source of Example 8.7-1 is a decaying exponential of the form

$$v_s = 10e^{-2t}u(t)\text{ V}$$

This source is said to be *aperiodic* (nonperiodic). A periodic source is one that repeats itself exactly after a fixed length of time. Thus, the signal $f(t)$ is *periodic* if there is a number T such that for all t

$$f(t + T) = f(t) \tag{8.7-5}$$

The smallest positive number T that satisfies Eq. 8.7-5 is called the *period*. The period defines the duration of one complete cycle of $f(t)$. Thus, any source for which there is no value of T satisfying Eq. 8.7-5 is said to be aperiodic. An example of a periodic source is 10 sin $2t$, which we consider in Example 8.7-2. The period of this sinusoidal source is π s.

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EXAMPLE 8.7-2 First-Order Circuit with Nonconstant Source

Find the response $v(t)$ for $t > 0$ for the circuit of Figure 8.7-2*a*. The initial voltage $v(0) = 0$, and the current source is $i_s = (10 \sin 2t)u(t)$ A.

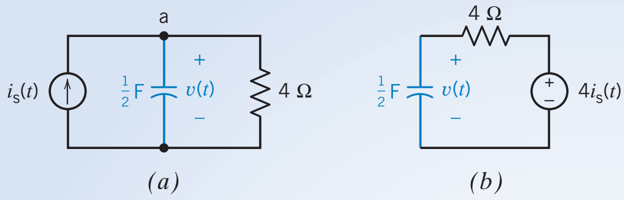

FIGURE 8.7-2 (*a*) A circuit with a nonconstant source. (*b*) The equivalent circuit for $t > 0$.

Solution

Because the forcing function is a sinusoidal function, we expect that v_f is of the same form. Writing KCL at node a, we obtain

$$C\frac{dv}{dt} + \frac{v}{R} = i_s$$

or

$$0.5\frac{dv}{dt} + \frac{v}{4} = 10 \sin 2t \tag{8.7-6}$$

for $t > 0$. We assume that v_f will consist of the sinusoidal function sin $2t$ and its derivatives.

Examining Eq. 8.7-6, $v_f/4$ plus $0.5\, dv_f/dt$ must equal 10 sin $2t$. However, $d(\sin 2t)/dt = 2 \cos 2t$. Therefore, the trial v_f needs to contain both sin $2t$ and cos $2t$ terms. Thus, we try the proposed solution

$$v_f = A \sin 2t + B \cos 2t$$

The derivative of v_f is then

$$\frac{dv_f}{dt} = 2A \cos 2t - 2B \sin 2t$$

Substituting v_f and dv_f/dt into Eq. 8.7-6, we obtain

$$(A \cos 2t - B \sin 2t) + \frac{1}{4}(A \sin 2t + B \cos 2t) = 10 \sin 2t$$

Therefore, equating sin $2t$ terms and cos $2t$ terms, we obtain

$$\left(\frac{A}{4} - B\right) = 10 \quad \text{and} \quad \left(A + \frac{B}{4}\right) = 0$$

Solving for A and B, we obtain

$$A = \frac{40}{17} \quad \text{and} \quad B = \frac{-160}{17}$$

Consequently,

$$v_f = \frac{40}{17} \sin 2t - \frac{160}{17} \cos 2t$$

It is necessary that v_f be made up of sin $2t$ and cos $2t$ because the solution has to satisfy the differential equation. Of course, the derivative of sin ωt is ω cos ωt.

The natural response can be obtained by considering the circuit shown in Figure 8.7-2*b*. This is the equivalent circuit that is appropriate for $t > 0$. The part of the circuit connected to the capacitor has been replaced by its Thévenin equivalent circuit. The natural response is

$$v_n = De^{-t/(R_t C)} = De^{-t/2}$$

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The complete response is then

$$v = v_{\mathrm{n}} + v_{\mathrm{f}} = De^{-t/2} + \frac{40}{17}\sin 2t - \frac{160}{17}\cos 2t$$

Because $v(0) = 0$, we obtain at $t = 0$

$$0 = D - \frac{160}{17}$$

or

$$D = \frac{160}{17}$$

Then the complete response is

$$v = \left(\frac{160}{17}e^{-t/2} + \frac{40}{17}\sin 2t - \frac{160}{17}\cos 2t\right)\mathrm{V}$$

Table 8.7-1 Forced Response to a Forcing Function

FORCING FUNCTION, $y(t)$	FORCED RESPONSE, $x_{\mathrm{f}}(t)$
1. Constant	
$y(t) = M$	$x_{\mathrm{f}} = N$, a constant
2. Exponential	
$y(t) = Me^{-bt}$	$x_{\mathrm{f}} = Ne^{-bt}$
3. Sinusoid	
$y(t) = M\sin(\omega t + \theta)$	$x_{\mathrm{f}} = A\sin\omega t + B\cos\omega t$

A special case for the forced response of a circuit may occur when the forcing function is a damped exponential when we have $y(t) = e^{-bt}$. Referring back to Eq. 8.7-4, we can show that

$$x_{\mathrm{f}} = \frac{e^{-bt}}{a - b}$$

when $y(t) = e^{-bt}$. Note that here we have e^{-bt} whereas we used e^{bt} for Eq. 8.7-4. For the special case when $a = b$, we have $a - b = 0$, and this form of the response is indeterminate. For the special case, we must use $x_{\mathrm{f}} = te^{-bt}$ as the forced response. The solution x_{f} for the forced response when $a = b$ will satisfy the original differential Eq. (8.7-1). Thus, when the natural response already contains a term of the same form as the forcing function, we need to multiply the assumed form of the forced response by t.

The forced response to selected forcing functions is summarized in Table 8.7-1. We note that if a circuit is linear, at steady state, and excited by a single sinusoidal source having frequency ω, then all the element currents and voltages are sinusoids having frequency ω.

EXERCISE 8.7-1 The electrical power plant for the orbiting space station shown in Figure E 8.7-1*a* uses photovoltaic cells to store energy in batteries. The charging circuit is modeled by the circuit shown in Figure E 8.7-1*b*, where $v_s = 10\sin 20t$ V. If $v(0^-) = 0$, find $v(t)$ for $t > 0$.

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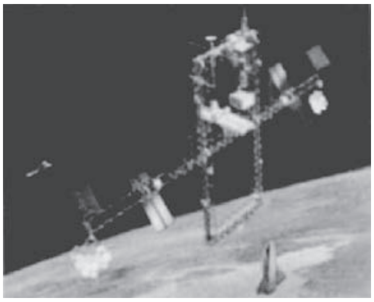
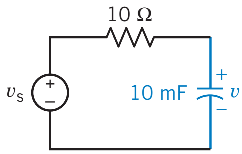


(a)



(b)

Courtesy of the National Aeronautics and Space Administration

FIGURE E 8.7-1 (*a*) The NASA space station design shows the longer habitable modules that would house an orbiting scientific laboratory. (*b*) The circuit for energy storage for the laboratories.

Answer: $v = 4\,e^{-10t} - 4\cos 20t + 2\sin 20t$ V

8.8 Differential Operators

In this section, we introduce the differential operator s.

An **operator** is a symbol that represents a mathematical operation. We can define a differential *operator* s such that

$$sx = \frac{dx}{dt} \quad \text{and} \quad s^2x = \frac{d^2x}{dt^2}$$

Thus, the operator s denotes differentiation of the variable with respect to time. The utility of the operator s is that it can be treated as an algebraic quantity. This permits the replacement of differential equations with algebraic equations, which are easily handled.

Use of the s operator is particularly attractive when higher-order differential equations are involved. Then we use the s operator, so that

$$s^n x = \frac{d^n x}{dt^n} \quad \text{for } n \geq 0$$

We assume that $n = 0$ represents no differentiation, so that

$$s^0 = 1$$

which implies $s^0 x = x$.

Because integration is the inverse of differentiation, we define

$$\frac{1}{s}x = \int_{-\infty}^{t} x\,d\tau \tag{8.8-1}$$

The operator $1/s$ must be shown to satisfy the usual rules of algebraic manipulations. Of these rules, the commutative multiplication property presents the only difficulty. Thus, we require

$$s \cdot \frac{1}{s} = \frac{1}{s} \cdot s = 1 \tag{8.8-2}$$

Is this true for the operator s? First, we examine Eq. 8.8-1. Multiplying Eq. 8.8-1 by s yields

$$s \cdot \frac{1}{s}x = \frac{d}{dt}\int_{-\infty}^{t} x\,d\tau$$

or

$$x = x$$

as required. Now we try the reverse order by multiplying sx by the integration operator to obtain

$$\frac{1}{s}sx = \int_{-\infty}^{t} \frac{dx}{d\tau}\,d\tau = x(t) - x(-\infty)$$

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Therefore,
$$\frac{1}{s}sx = x$$
only when $x(-\infty) = 0$. From a physical point of view, we require that all capacitor voltages and inductor currents be zero at $t = -\infty$. Then the operator $1/s$ can be said to satisfy Eq. 8.8-2 and can be manipulated as an ordinary algebraic quantity.

Differential operators can be used to find the natural solution of a differential equation. For example, consider the first-order differential equation
$$\frac{d}{dt}x(t) + ax(t) = by(t) \tag{8.8-3}$$
The natural solution of this differential equation is
$$x_n(t) = Ke^{st} \tag{8.8-4}$$
The homogeneous form of a differential equation is obtained by setting the forcing function equal to zero. The forcing function in Eq. 8.8-3 is $y(t)$. The homogeneous form of this equation is
$$\frac{d}{dt}x(t) + ax(t) = 0 \tag{8.8-5}$$
To see that $x_n(t)$ is a solution of the homogeneous form of the differential equation, we substitute Eq. 8.8-4 into Eq. 8.8-5.
$$\frac{d}{dt}(Ke^{st}) + a(Ke^{st}) = sKe^{st} + aKe^{st} = 0$$
To obtain the parameter s in Eq. 8.8-4, replace d/dt in Eq. 8.8-5 by the differential operator s. This results in
$$sx + ax = (s + a)x = 0 \tag{8.8-6}$$
This equation has two solutions: $x = 0$ and $s = -a$. The solution $x = 0$ isn't useful, so we use the solution $s = -a$. Substituting this solution into Eq. 8.8-4 gives
$$x_n(t) = Ke^{-at}$$
This is the same expression for the natural response that we obtained earlier in this chapter by other methods. That's reassuring but not new. Differential operators will be quite useful when we analyze circuits that are represented by second- and higher-order differential equations.

8.9 Using PSpice to Analyze First-Order Circuits

To use PSpice to analyze a first-order circuit, we do the following:

1. Draw the circuit in the OrCAD Capture workspace.
2. Specify a Time Domain (Transient) simulation.
3. Run the simulation.
4. Plot the simulation results.

Time domain analysis is most interesting for circuits that contain capacitors or inductors or both. PSpice provides parts representing capacitors and inductors in the ANALOG parts library. The part name for the capacitor is C. The part properties that are of the most interest are the capacitance and the initial condition, both of which are specified using the OrCAD Capture property editor. (The initial condition of a capacitor is the value of the capacitor voltage at time $t = 0$.) The part name for the inductor is L. The inductance and the initial condition of the inductor are specified using the property editor. (The initial condition of an inductor is the value of the inductor current at time $t = 0$.)

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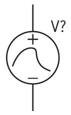
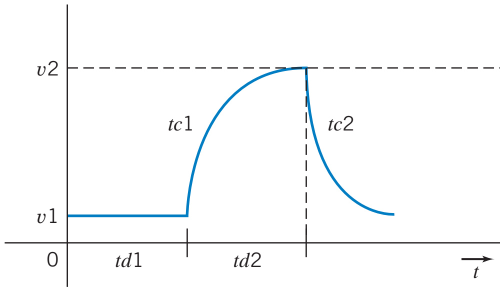
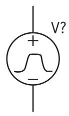
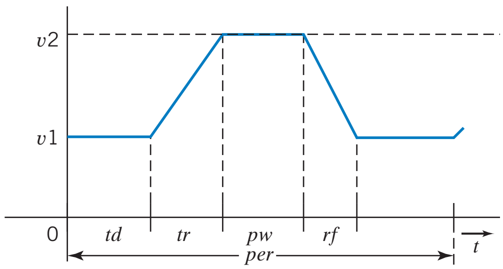
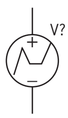
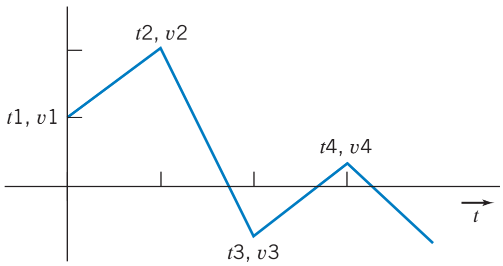
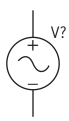
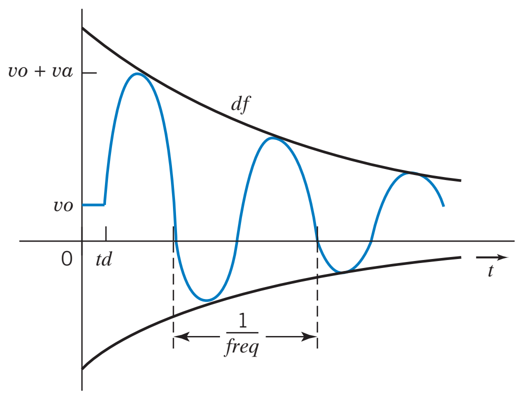
Table 8.9-1 PSpice Voltage Sources for Transient Response Simulations

NAME	SYMBOL		VOLTAGE WAVEFORM
VEXP	V1 = V2 = TD1 = TC1 = TD2 = TC2 =		
VPULSE	V1 = V2 = TD = TR = TF = PW = PER =		
VPWL			
VSIN	VOFF = VAMPL = FREQ =		

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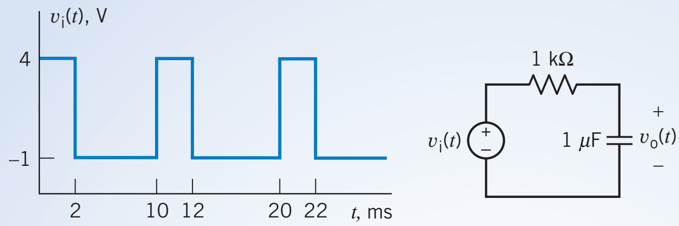
The voltage and current sources that represent time-varying inputs are provided in the SOURCE parts library. Table 8.9-1 summarizes these voltage sources. The voltage waveform describes the shape of the voltage source voltage as a function of time. Each voltage waveform is described using a series of parameters. For example, the voltage of an exponential source VEXP is described using *v*l, *v*2, *td*l, *td*2, *tc*1, and *tc*2. The parameters of the voltage sources in Table 8.9-1 are specified using the property editor.

EXAMPLE 8.9-1 Using PSpice to Analyze First-Order Circuits

The input to the circuit shown in Figure 8.9-1*a* is the voltage source voltage, $v_i(t)$, shown in Figure 8.9-l*a*. The output, or response, of the circuit is the voltage across the capacitor, $v_o(t)$. Use PSpice to plot the response of this circuit.



(*a*) (*b*)

FIGURE 8.9-1 An *RC* circuit (*b*) with a pulse input (*a*).

Solution

We begin by drawing the circuit in the OrCAD workspace as shown in Figure 8.9-2 (see Appendix A). The voltage source is a VPULSE part (see the second row of Table 8.9-1). Figure 8.9-l*a* shows $v_i(t)$ making the transition from -1 V to 4 V instantaneously. Zero is not an acceptable value for the parameters *tr* or *tf*. Choosing a very small value for *tr* and *tf* will make the transitions appear to be instantaneous when using a time scale that shows a period of the input waveform. In this example, the period of the input waveform is 10 ms, so 1 ns is a reasonable choice for the values of *tr* and *tf*.

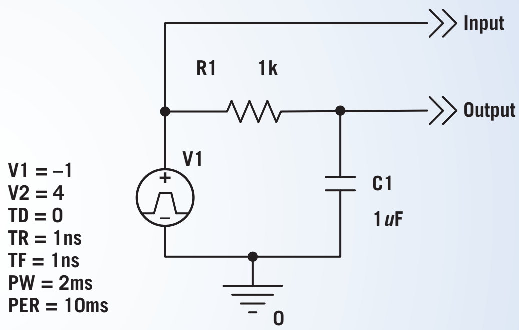


FIGURE 8.9-2 The circuit of Figure 8.9-1 as drawn in the OrCAD workspace.

It's convenient to set *td*, the delay before the periodic part of the waveform, to zero. Then the values of *v*l and *v*2 are -1 and 4, respectively. The value of *pw* is the length of time that $v_i(t) = v2 = 4$ V, so $pw = 2$ ms in this example. The pulse input is a periodic function of time. The value of *per* is the period of the pulse function, 10 ms.

The circuit shown in Figure 8.9-1*b* does not have a ground node. PSpice requires that all circuits have a ground node, so it is necessary to select a ground node. Figure 8.9-2 shows that the bottom node has been selected to be the ground node.

We will perform a Time Domain (Transient) simulation. (Select PSpice\New Simulation Profile from the OrCAD Capture menu bar; then choose Time Domain (Transient) from the Analysis Type drop-down list. The simulation starts at time zero and ends at the Run to Time. Specify the Run to Time as 20 ms to run the simulation for two full periods of the input waveform. Select the Skip The Initial Transient Bias Point Calculation (SKJPBP) check box.) Select PSpice\Run from the OrCAD Capture menu bar to run the simulation.

After a successful Time Domain (Transient) simulation, OrCAD Capture will automatically open a Schematics window. Select Trace/Add Trace to pop up the Add Traces dialog box. Add the traces V(OUTPUT) and V(INPUT). Figure 8.9-3 shows the resulting plot after removing the grid and labeling some points.

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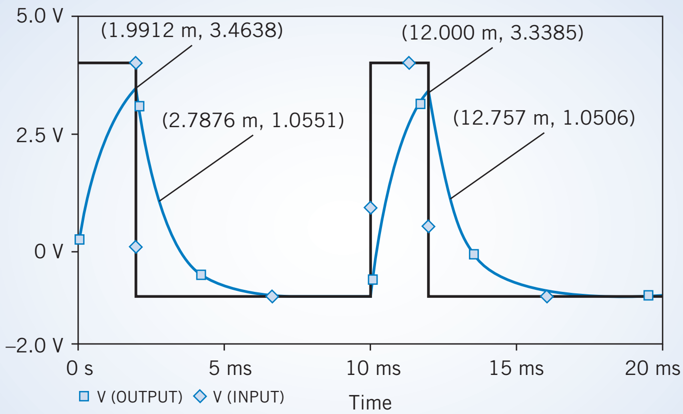

FIGURE 8.9-3 The response of the *RC* circuit to the pulse input.

8.10 *How Can We Check . . . ?*

Engineers are frequently called upon to check that a solution to a problem is indeed correct. For example, proposed solutions to design problems must be checked to confirm that all of the specifications have been satisfied. In addition, computer output must be reviewed to guard against data-entry errors, and claims made by vendors must be examined critically.

Engineering students are also asked to check the correctness of their work. For example, occasionally just a little time remains at the end of an exam. It is useful to be able to quickly identify those solutions that need more work.

The following examples illustrate techniques useful for checking the solutions of the sort of problems discussed in this chapter.

EXAMPLE 8.10-1 How Can We Check the Response of a First-Order Circuit?

Consider the circuit and corresponding transient response shown in Figure 8.10-1. **How can we check** whether the transient response is correct? Three things need to be verified: the initial voltage, $v_o(t_0)$; the final voltage, $v_o(\infty)$; and the time constant, τ.

Solution

Consider first the initial voltage, $v_o(t_0)$. (In this example, $t_0 = 10\ \mu s$.) Before time $t_0 = 10\ \mu s$, the switch is closed and has been closed long enough for the circuit to reach steady state, that is, for any transients to have died out. To calculate $v_o(t_0)$, we simplify the circuit in two ways. First, replace the switch with a short circuit because the switch is closed. Second, replace the inductor with a short circuit because inductors act like short circuits when all the inputs are constants and the circuit is at steady state. The resulting circuit is shown in Figure 8.10-2*a*. After replacing the parallel 300-Ω and 600-Ω resistors by the equivalent 200-Ω resistor, the initial voltage is calculated using voltage division as

$$v_o(t_0) = \frac{200}{200+200} 8 = 4\text{ V}$$

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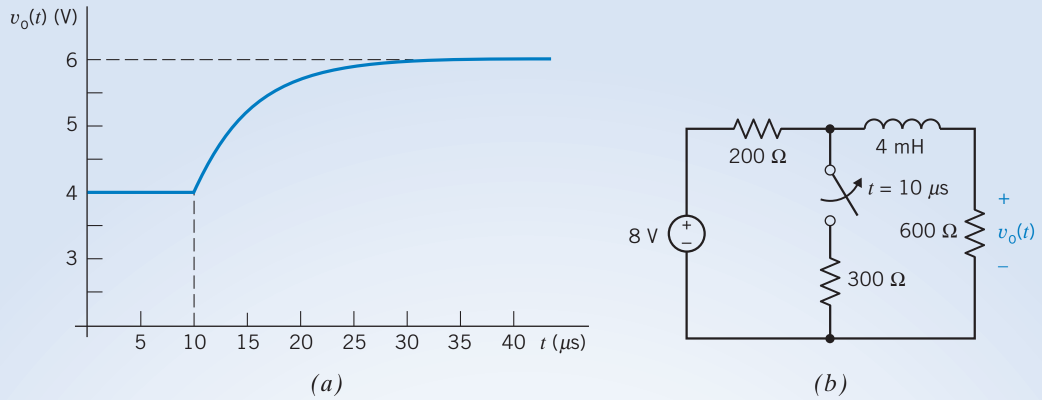

FIGURE 8.10-1 (*a*) A transient response and (*b*) the corresponding circuit.

Next consider the final voltage, $v_o(\infty)$. In this case, the switch is open and the circuit has reached steady state. Again, the circuit is simplified in two ways. The switch is replaced with an open circuit because the switch is open. The inductor is replaced by a short circuit because inductors act like short circuits when all the inputs are constants and the circuit is at steady state. The simplified circuit is shown in Figure 8.10-2*b*. The final voltage is calculated using voltage division as

$$v_o(\infty) = \frac{600}{200 + 600} 8 = 6 \text{ V}$$

The time constant is calculated from the circuit shown in Figure 8.10-2*c*. This circuit has been simplified by setting the input to zero (a zero voltage source acts like a short circuit) and replacing the switch by an open circuit. The time constant is

$$\tau = \frac{L}{R_t} = \frac{4 \times 10^{-3}}{200 + 600} = 5 \times 10^{-6} = 5\ \mu\text{s}$$

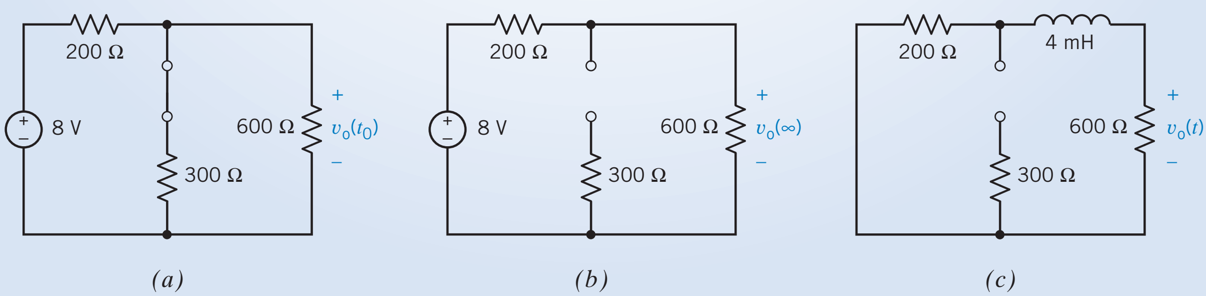

FIGURE 8.10-2 Circuits used to calculate the (*a*) initial voltage, (*b*) final voltage, and (*c*) time constant.

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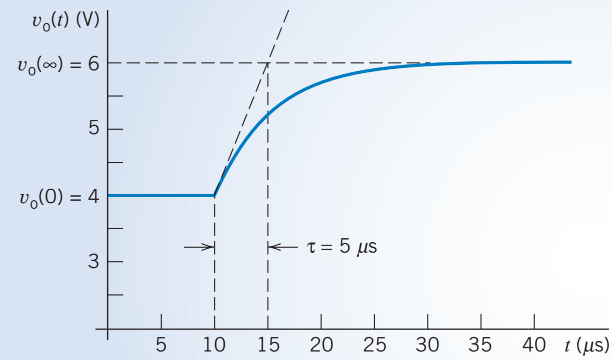

FIGURE 8.10-3 Interpretation of the transient response.

Figure 8.10-3 shows how the initial voltage, final voltage, and time constant can be determined from the plot of the transient response. (Recall that a procedure for determining the time constant graphically was illustrated in Figure 8.3-3.) Because the values of $v_o(t_0)$, $v_o(\infty)$, and τ obtained from the transient response are the same as the values obtained by analyzing the circuit, we conclude that the transient response is indeed correct.

EXAMPLE 8.10-2 How Can We Check the Response of a First-Order Circuit?

Consider the circuit and corresponding transient response shown in Figure 8.10-4. **How can we check** whether the transient response is correct? Four things need to be verified: the steady-state capacitor voltage when the switch is open, the steady-state capacitor voltage when the switch is closed, the time constant when the switch is open, and the time constant when the switch is closed.

Solution

Figure 8.10-5*a* shows the circuit used to calculate the steady-state capacitor voltage when the switch is open. The circuit has been simplified in two ways. First, the switch has been replaced with an open circuit. Second, the capacitor has been replaced with an open circuit because capacitors act like open circuits when all the inputs are constants and the circuit is at steady state. The steady-state capacitor voltage is calculated using voltage division as

$$v_c(\infty) = \frac{60}{60 + 30 + 150} 12 = 3 \text{ V}$$

Figure 8.10-5*b* shows the circuit used to calculate the steady-state capacitor voltage

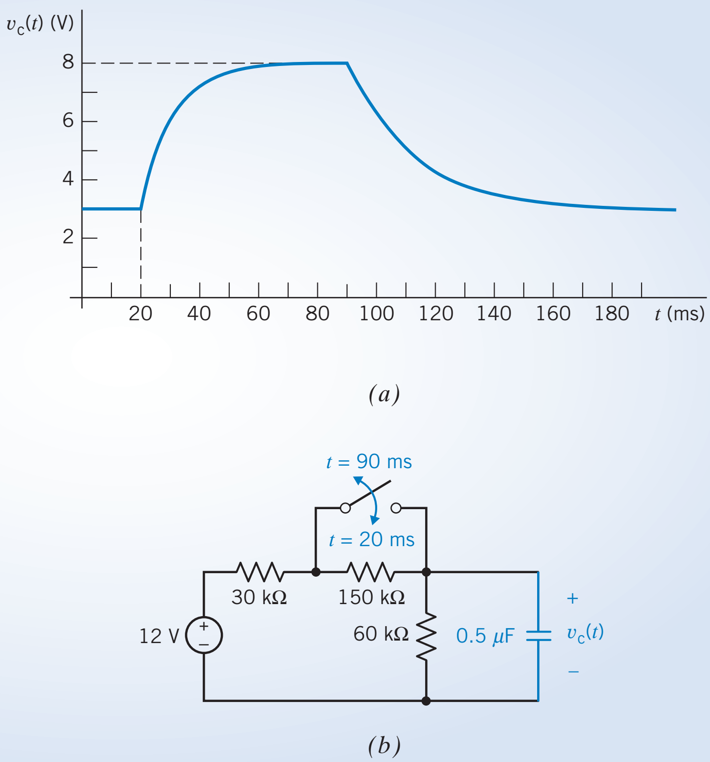

FIGURE 8.10-4 (*a*) A transient response and (*b*) the corresponding circuit.

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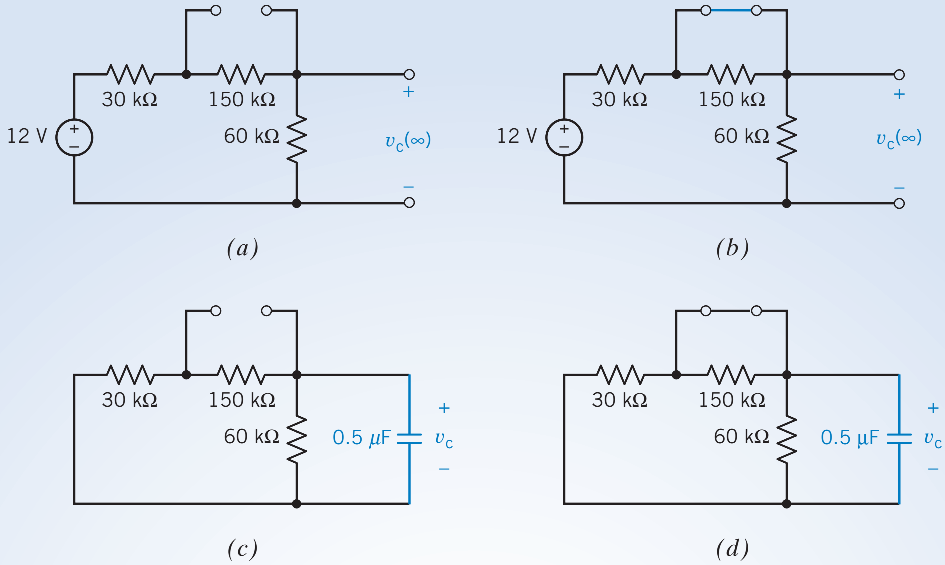

FIGURE 8.10-5 Circuits used to calculate (*a*) the steady-state voltage when the switch is open, (*b*) the steady-state voltage when the switch is closed, (*c*) the time constant when the switch is open, and (*d*) the time constant when the switch is closed.

when the switch is closed. Again, this circuit has been simplified in two ways. First, the switch has been replaced with a short circuit. Second, the capacitor has been replaced with an open circuit. The steady-state capacitor voltage is calculated using voltage division as

$$v_c(\infty) = \frac{60}{60+30}12 = 8\text{ V}$$

Figure 8.10-5*c* shows the circuit used to calculate the time constant when the switch is open. This circuit has been simplified in two ways. First, the switch has been replaced with an open circuit. Second, the input has been set to zero (a zero voltage source acts like a short circuit). Notice that 180 kΩ in parallel with 60 kΩ is equivalent to 45 kΩ. The time constant is

$$\tau = \left(45\times 10^3\right)\cdot\left(0.5\times 10^{-6}\right) = 22.5\times 10^{-3} = 22.5\text{ ms}$$

Figure 8.10-5*d* shows the circuit used to calculate the time constant when the switch is closed. The switch has been replaced with a short circuit, and the input has been set to zero. Notice that 30 kΩ in parallel with 60 kΩ is equivalent to 20 kΩ. The time constant is

$$\tau = \left(20\times 10^3\right)\cdot\left(0.5\times 10^{-6}\right) = 10^{-2} = 10\text{ ms}$$

Having done these calculations, we expect the capacitor voltage to be 3 V until the switch closes at $t = 20$ ms. The capacitor voltage will then increase exponentially to 8 V, with a time constant equal to 10 ms. The capacitor voltage will remain 8 V until the switch opens at $t = 90$ ms. The capacitor voltage will then decrease exponentially to 3 V, with a time constant equal to 22.5 ms. Figure 8.10-6 shows that the transient response satisfies this description. We conclude that the transient response is correct.

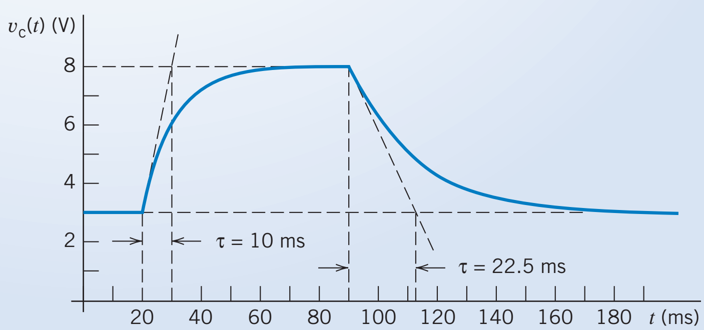

FIGURE 8.10-6 Interpretation of the transient response.

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8.11 DESIGN EXAMPLE A Computer and Printer

It is frequently necessary to connect two pieces of electronic equipment together so that the output from one device can be used as the input to another device. For example, this situation occurs when a printer is connected to a computer, as shown in Figure 8.11-1*a*. This situation is represented more generally by the circuit shown in Figure 8.11-1*b*. The driver sends a signal through the cable to the receiver. Let us replace the driver, cable, and receiver with simple models. Model the driver as a voltage source, the cable as an *RC* circuit, and the receiver as an open circuit. The values of resistance and capacitance used to model the cable will depend on the length of the cable. For example, when RG58 coaxial cable is used,

$$R = r \cdot \ell \text{ where } r = 0.54 \frac{\Omega}{\text{m}}$$

and

$$C = c \cdot \ell \text{ where } c = 88 \frac{\text{pF}}{\text{m}}$$

and ℓ is the length of the cable in meters, Figure 8.11-1*c* shows the equivalent circuit.

Suppose that the circuits connected by the cable are digital circuits. The driver will send 1's and 0's to the receiver. These 1's and 0's will be represented by voltages. The output of the driver will be one voltage, V_{OH}, to represent logic 1 and another voltage, V_{OL}, to represent a logic 0. For example, one popular type of logic, called TTL logic, uses $V_{\text{OH}} = 2.4$ V and $V_{\text{OL}} = 0.4$ V. (TTL stands for transistor–transistor logic.) The receiver uses two different voltages, V_{IH} and V_{IL}, to represent 1's and 0's. (This is done to provide noise immunity, but that is another story.) The receiver will interpret its input, v_{b}, to be a logic 1 whenever $v_{\text{b}} > V_{\text{IH}}$ and to be a logic 0 whenever $v_{\text{b}} < V_{\text{IL}}$. (Voltages between V_{IH} and V_{IL} will occur only during transitions between logic 1 and logic 0. These voltages will sometimes be interpreted as logic 1 and other times as logic 0.) TTL logic uses $V_{\text{IH}} = 2.0$ V and $V_{\text{IL}} = 0.8$ V.

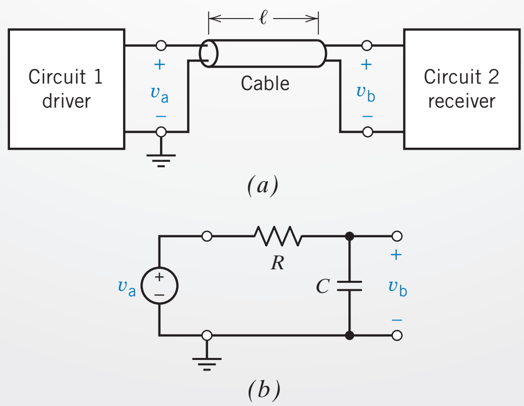

FIGURE 8.11-1 (*a*) Two circuits connected by a cable. (*b*) An equivalent circuit.

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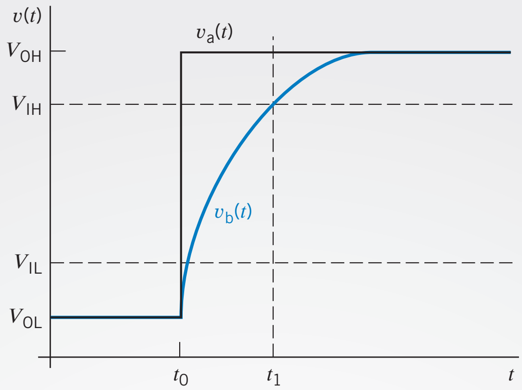

FIGURE 8.11-2 Voltages that occur during a transition from a logic 0 to a logic 1.

Figure 8.11-2 shows what happens when the driver output changes from logic 0 to logic 1. Before time t_0,

$$v_a = V_{OL} \quad \text{and} \quad v_b < V_{IL} \quad \text{for} \quad t < t_0$$

In words, a logic 0 is sent and received. The driver output switches to V_{OH} at time t_0. The receiver input v_b makes this transition more slowly. Not until time t_1 does the receiver input become large enough to be interpreted as a logic 1. That is,

$$v_b > V_{IH} \quad \text{for} \quad t > t_1$$

The time that it takes for the receiver to recognize the transition from logic 0 to logic 1

$$\Delta t = t_1 - t_0$$

is called the delay. This delay is important because it puts a limit on how fast 1's and 0's can be sent from the driver to the receiver. To ensure that the 1's and 0's are received reliably, each 1 and each 0 must last at least Δt. The rate at which 1's and 0's are sent from the driver to the receiver is inversely proportional to the delay.

Suppose two TTL circuits are connected using RG58 coaxial cable. What restriction must be placed on the length of the cable to ensure that the delay, Δt, is less than 2 ns?

Describe the Situation and the Assumptions

The voltage $v_b(t)$ is the capacitor voltage of an *RC* circuit. The *RC* circuit is at steady state just before time t_0.

The input to the *RC* circuit is $v_a(t)$. Before time t_0, $v_a(t) = V_{OL} = 0.4$ V. At time t_0, $v_a(t)$ changes abruptly. After time t_0, $v_a(t) = V_{OH} = 2.4$ V.

Before time t_0, $v_b(t) = V_{OL} = 0.4$ V. After time t_0, $v_b(t)$ increases exponentially. Eventually, $v_b(t) = V_{OH} = 2.4$ V.

The time constant of the *RC* circuit is

$$\tau = R \cdot C = rc\ell^2 = 47.52 \times 10^{-2} \cdot \ell^2$$

where ℓ is the cable length in meters.

State the Goal

Calculate the maximum value of the cable length ℓ for which $v_b > V_{IH} = 2.0$ V by time $t = t_0 + \Delta t$, where $\Delta t = 2$ ns.

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Generate a Plan

Calculate the voltage $v_b(t)$ in Figure 8.11-1*b*. The voltage $v_b(t)$ will depend on the length of the cable, ℓ, because the time constant of the *RC* circuit is a function of ℓ. Set $v_b = V_{IH}$ at time $t = t_0 + \Delta t$. Solve the resulting equation for the length of the cable.

Act on the Plan

Using the notation introduced in this chapter,

$$v_b(0) = V_{OL} = 0.4\ \text{V}$$
$$v_b(\infty) = V_{OH} = 2.4\ \text{V}$$

and

$$\tau = 47.52 \times 10^{-12} \cdot \ell^2$$

Using Eq. 8.3-6, we express the voltage $v_b(t)$ as

$$v_b(t) = V_{OH} + (V_{OL} - V_{OH})e^{-(t-t_0)/\tau}$$

The capacitor voltage v_b will be equal to V_{IH} at time $t_1 = t_0 + \Delta t$, so

$$V_{IH} = V_{OH} + (V_{OL} - V_{OH})e^{-\Delta t/\tau}$$

Solving for the delay, Δt, gives

$$\Delta t = -\tau \ln\left[\frac{V_{IH} - V_{OH}}{V_{OL} - V_{OH}}\right] = -47.52 \times 10^{-12} \cdot \ell^2 \cdot \ln\left[\frac{V_{IH} - V_{OH}}{V_{OL} - V_{OH}}\right]$$

In this case,

$$\ell = \sqrt{\frac{-\Delta t}{47.52 \times 10^{-12} \cdot \ln\left[\frac{V_{IH} - V_{OH}}{V_{OL} - V_{OH}}\right]}}$$

and, therefore,

$$\ell = \sqrt{\frac{-2 \cdot 10^{-9}}{47.52 \times 10^{-12} \cdot \ln\left[\frac{2.0 - 2.4}{0.4 - 2.4}\right]}} = 5.11\ \text{m} = 16.8\ \text{ft}$$

Verify the Proposed Solution

When $\ell = 5.11$ m, then

$$R = 0.54 \times 5.11 = 2.76\ \Omega$$

and

$$C = \left(88 \times 10^{-12}\right) \times 5.11 = 450\ \text{pF}$$

so

$$\tau = 2.76 \times \left(450 \times 10^{-12}\right) = 1.24\ \text{ns}$$

Finally,

$$\Delta t = -1.24 \times 10^{-9} \times \ln\left[\frac{2.0 - 2.4}{0.4 - 2.4}\right] = 1.995\ \text{ns}$$

Because $\Delta t < 2$ ns, the specifications have been satisfied but with no margin for error.

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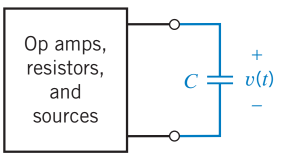
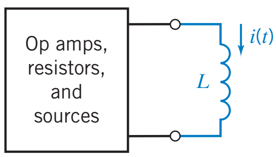
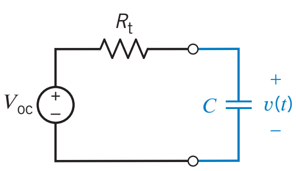
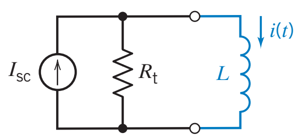
8.12 SUMMARY

- Voltages and currents can be used to encode, store, and process information. When a voltage or current is used to represent information, that voltage or current is called a signal. Electric circuits that process that information are called signal-processing circuits.
- Circuits that contain energy-storing elements, that is, capacitors and inductors, are represented by differential equations rather than by algebraic equations. Analysis of these circuits requires the solution of differential equations.
- In this chapter, we restricted our attention to first-order circuits. First-order circuits contain one energy storage element and are represented by first-order differential equations, which are reasonably easy to solve. We solved first-order differential equations, using the method called separation of variables.
- The *complete response* of a circuit is the sum of the *natural response* and *the forced response*. The natural response is the general solution of the differential equation that represents the circuit when the input is set to zero. The forced response is the particular solution of the differential equation representing the circuit.
- The complete response can be separated into the *transient response* and the *steady-state response*. The transient response vanishes with time, leaving the steady-state response. When the input to the circuit is either a constant or a sinusoid, the steady-state response can be used as the forced response.
- The term *transient response* sometimes refers to the "transient part of the complete response" and other times to a complete response that includes a transient part. In particular, PSpice uses the term *transient response* to refer to the complete response. Because this can be confusing, the term must be used carefully.
- The *step response* of a circuit is the response when the input is equal to a unit step function and all the initial conditions of the circuit are equal to zero.
- We used Thévenin and Norton equivalent circuits to reduce the problem of analyzing any first-order circuit to the problem of analyzing one of two simple first-order circuits. One of the simple first-order circuits is a series circuit consisting of a voltage source, a resistor, and a capacitor. The other is a parallel circuit consisting of a current source, a resistor, and an inductor. Table 8.12-1 summarizes the equations used to determine the complete response of a first-order circuit.
- The parameter τ in the first-order differential equation $\frac{d}{dt}x(t) + \frac{x(t)}{\tau} = K$ is called the time constant. The time constant τ is the time for the response of a first-order circuit to complete 63 percent of the transition from initial value to final value.
- Stability is a property of well-behaved circuits. It is easy to tell whether a first-order circuit is stable. A first-order circuit is stable if, and only if, its time constant is not negative, that is, $\tau \geq 0$.

Table 8.12-1 Summary of First-Order Circuits

FIRST-ORDER CIRCUIT CONTAINING A CAPACITOR	FIRST-ORDER CIRCUIT CONTAINING AN INDUCTOR
	
Replace the circuit consisting of op amps, resistors, and sources by its Thévenin equivalent circuit:	Replace the circuit consisting of op amps, resistors, and sources by its Norton equivalent circuit:
	
The capacitor voltage is: $v(t) = V_{oc} + (v(0) - V_{oc})e^{-t/\tau}$ where the time constant τ is $\tau = R_t C$ and the initial condition $v(0)$ is the capacitor voltage at time $t = 0$.	The inductor current is $i(t) = I_{sc} + (i(0) - I_{sc})e^{-t/\tau}$ where the time constant τ is $\tau = \frac{L}{R_t}$ and the initial condition $i(0)$ is the inductor current at time $t = 0$.

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PROBLEMS

⊕ Problem available in WileyPLUS at instructor's discretion.

Section 8.3 The Response of a First-Order Circuit to a Constant Input

P 8.3-1 ⊕ The circuit shown in Figure P 8.3-1 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the voltage across the capacitor, $v(t)$. Determine $v(t)$ for $t > 0$.

Answer: $v(t) = 6 - 2e^{-1.33t}$ V for $t > 0$

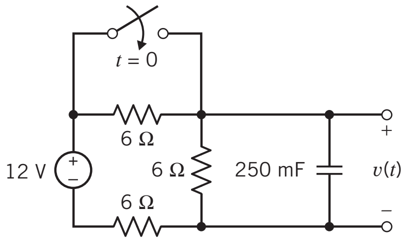

Figure P 8.3-1

P 8.3-2 ⊕ The circuit shown in Figure P 8.3-2 is at steady state before the switch opens at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the current in the inductor, $i(t)$. Determine $i(t)$ for $t > 0$.

Answer: $i(t) = 1 + e^{-0.5t}$ A for $t > 0$

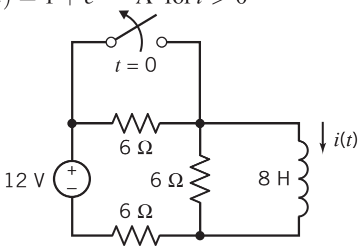

Figure P 8.3-2

P 8.3-3 ⊕ The circuit shown in Figure P 8.3-3 is at steady state before the switch closes at time $t = 0$. Determine the capacitor voltage $v(t)$ for $t > 0$.

Answer: $v(t) = -6 + 18e^{-6.67t}$ V for $t > 0$

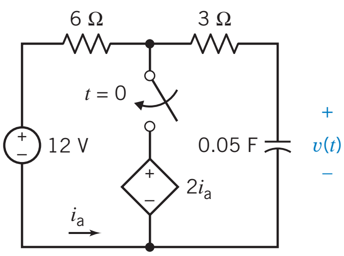

Figure P 8.3-3

P 8.3-4 ⊕ The circuit shown in Figure P 8.3-4 is at steady state before the switch closes at time $t = 0$. Determine the inductor current $i(t)$ for $t > 0$.

Answer: $i(t) = -2 + \frac{10}{3}e^{-0.5t}$ A for $t > 0$

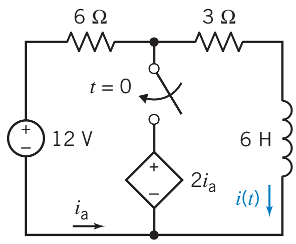

Figure P 8.3-4

P 8.3-5 ⊕ The circuit shown in Figure P 8.3-5 is at steady state before the switch opens at time $t = 0$. Determine the voltage $v_o(t)$ for $t > 0$.

Answer: $v_o(t) = 10 - 5e^{-12.5t}$ V for $t > 0$

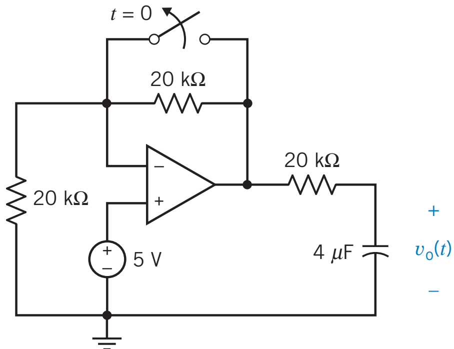

Figure P 8.3-5

P 8.3-6 The circuit shown in Figure P 8.3-6 is at steady state before the switch opens at time $t = 0$. Determine the voltage $v_o(t)$ for $t > 0$.

Answer: $v_o(t) = 5e^{-4000t}$ V for $t > 0$

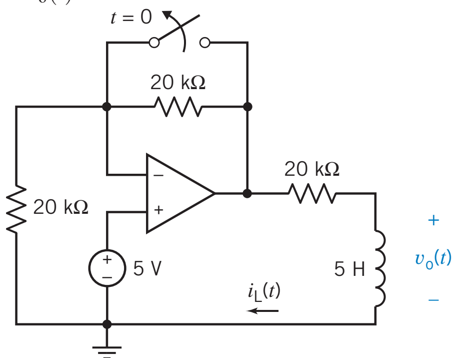

Figure P 8.3-6

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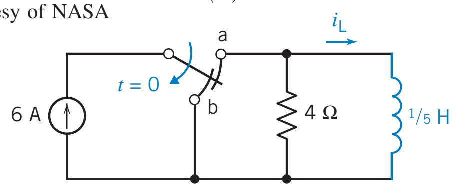
P 8.3-7 Figure P 8.3-7*a* shows astronaut Dale Gardner using the manned maneuvering unit to dock with the spinning *Westar VI* satellite on November 14, 1984. Gardner used a large tool called the apogee capture device (ACD) to stabilize the satellite and capture it for recovery, as shown in Figure P 8.3-7*a*. The ACD can be modeled by the circuit of Figure P 8.3-7*b*. Find the inductor current i_L for $t > 0$.

Answer: $i_L(t) = 6e^{-20t}$ A

(a)

Courtesy of NASA



(b)

Figure P 8.3-7 (*a*) Astronaut Dale Gardner using the manned maneuvering unit to dock with the *Westar VI* satellite. (*b*) Model of the apogee capture device. Assume that the switch has been in position for a long time at $t = 0^-$.

P 8.3-8 The circuit shown in Figure P 8.3-8 is at steady state before the switch opens at time $t = 0$. The input to the circuit is the voltage of the voltage source, V_s. This voltage source is a dc voltage source; that is, V_s is a constant. The output of this circuit is the voltage across the capacitor, $v_o(t)$. The output voltage is given by

$$v_o(t) = 2 + 8e^{-0.5t} \text{ V} \quad \text{for } t > 0$$

Determine the values of the input voltage V_s, the capacitance C, and the resistance R.

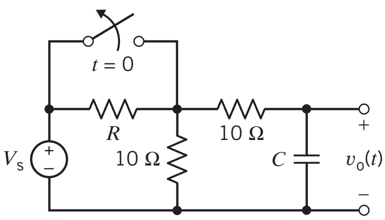


Figure P 8.3-8

P 8.3-9 The circuit shown in Figure P 8.3-9 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the voltage of the voltage source, 24 V. The output of this circuit, the voltage across the 3-Ω resistor, is given by

$$v_o(t) = 6 - 3e^{-0.35t} \text{ V} \quad \text{when } t > 0$$

Determine the value of the inductance L and of the resistances R_1 and R_2.

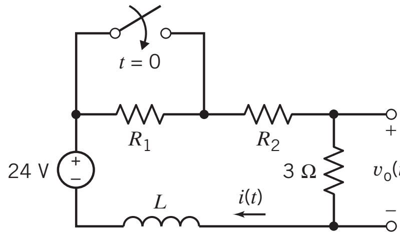


Figure P 8.3-9

P 8.3-10 A security alarm for an office building door is modeled by the circuit of Figure P 8.3-10. The switch represents the door interlock, and v is the alarm indicator voltage. Find $v(t)$ for $t > 0$ for the circuit of Figure P 8.3-10. The switch has been closed for a long time at $t = 0^-$.

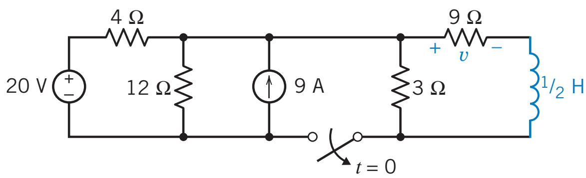


Figure P 8.3-10 A security alarm circuit.

P 8.3-11 The voltage $v(t)$ in the circuit shown in Figure P 8.3-11 is given by

$$v(t) = 8 + 4e^{-2t} \text{ V} \quad \text{for } t > 0$$

Determine the values of R_1, R_2, and C.

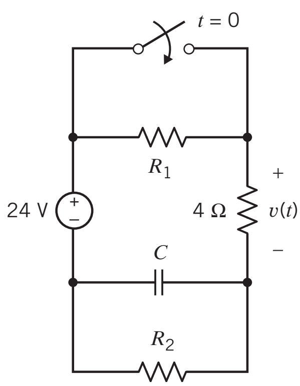


Figure P 8.3-11

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P 8.3-12 ⊕ The circuit shown in Figure P 8.3-12 is at steady state when the switch opens at time $t = 0$. Determine $i(t)$ for $t \geq 0$.

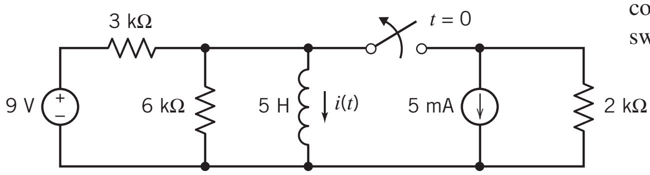

Figure P 8.3-12

P 8.3-13 ⊕ The circuit shown in Figure P 8.3-13 is at steady state when the switch opens at time $t = 0$. Determine $v(t)$ for $t \geq 0$.

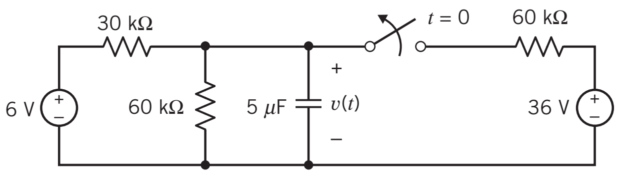

Figure P 8.3-13

P 8.3-14 ⊕ The circuit shown in Figure P 8.3-14 is at steady state when the switch closes at time $t = 0$. Determine $i(t)$ for $t \geq 0$.

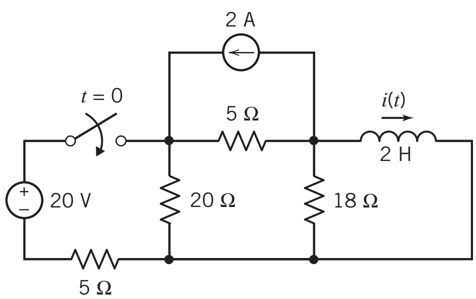

Figure P 8.3-14

P 8.3-15 ⊕ The circuit in Figure P 8.3-15 is at steady state before the switch closes. Find the inductor current after the switch closes.

Hint: $i(0) = 0.1$ A, $I_{sc} = 0.3$ A, $R_t = 40\ \Omega$

Answer: $i(t) = 0.3 - 0.2e^{-2t}$ A $\quad t \geq 0$

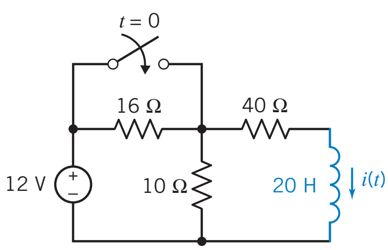

Figure P 8.3-15

P 8.3-16 Consider the circuit shown in Figure P 8.3-16. ***(a)*** Determine the time constant τ and the steady-state capacitor voltage when the switch is open. ***(b)*** Determine the time constant τ and the steady-state capacitor voltage when the switch is closed.

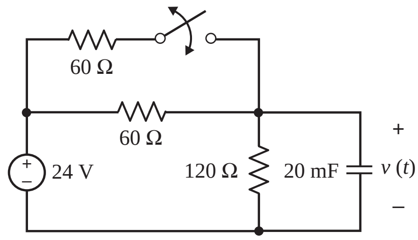

Figure P 8.3-16

P 8.3-17 The circuit shown in Figure P 8.3-17 is at steady state before the switch closes. The response of the circuit is the voltage $v(t)$. Find $v(t)$ for $t > 0$.

Hint: After the switch closes, the inductor current is $i(t) = 0.2\,(1 - e^{-1.8t})$ A

Answer: $v(t) = 8 + e^{-1.8t}$ V

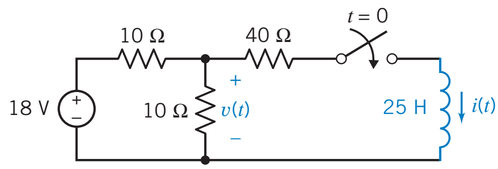

Figure P 8.3-17

P 8.3-18 ⊕ The circuit shown in Figure P 8.3-18 is at steady state before the switch closes. The response of the circuit is the voltage $v(t)$. Find $v(t)$ for $t > 0$.

Answer: $v(t) = 37.5 - 97.5e^{-6400t}$ V

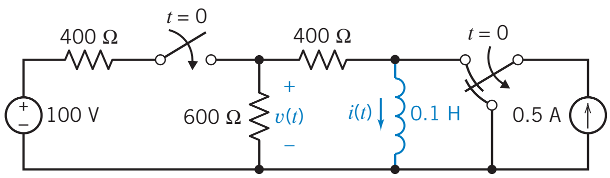

Figure P 8.3-18

P 8.3-19 ⊕ The circuit shown in Figure P 8.3-19 is at steady state before the switch closes. Find $v(t)$ for $t \geq 0$.

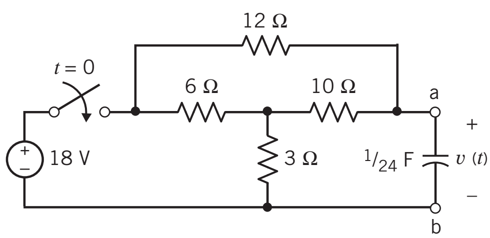

Figure P 8.3-19

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P 8.3-20 ⊕ The circuit shown in Figure P 8.3-20 is at steady state before the switch closes. Determine $i(t)$ for $t \geq 0$.

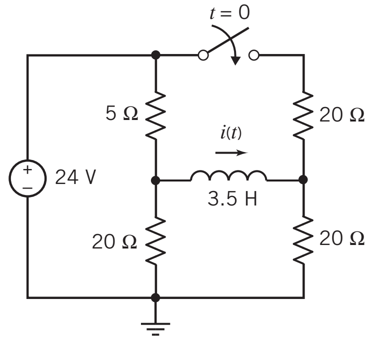


Figure P 8.3-20

P 8.3-21 The circuit in Figure P 8.3-21 is at steady state before the switch closes. Determine an equation that represents the capacitor voltage after the switch closes.

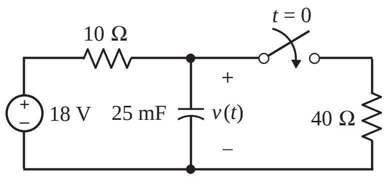


Figure P 8.3-21

P 8.3-22 The circuit shown in Figure P 8.3-22 is at steady state when the switch closes at time $t = 0$. Determine $i(t)$ for $t \geq 0$.

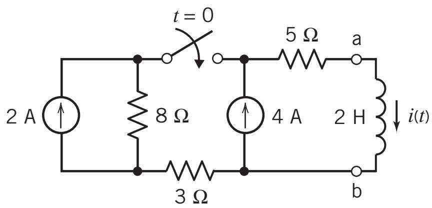


Figure P 8.3-22

P 8.3-23 The circuit in Figure P 8.3-23 is at steady state before the switch closes. Determine an equation that represents the inductor current after the switch closes.

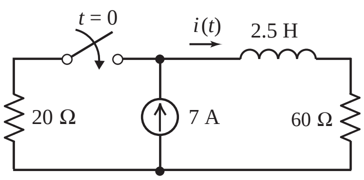


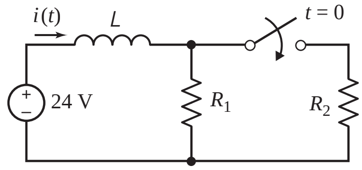
Figure P 8.3-23

P 8.3-24 Consider the circuit shown in Figure P 8.3-24*a* and corresponding plot of the inductor current shown in Figure P 8.3-24*b*. Determine the values of L, R_1, and R_2.

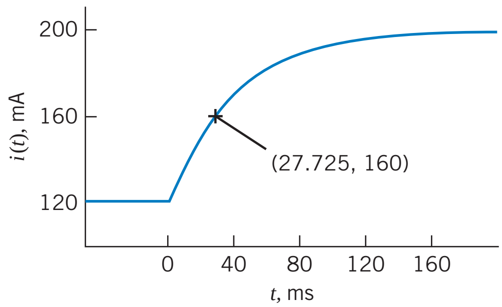
Hint: Use the plot to determine values of D, E, F, and a such that the inductor current can be represented as

$$i(t) = \begin{cases} D & \text{for } t \leq 0 \\ E + Fe^{-at} & \text{for } t \geq 0 \end{cases}$$

Answers: $L = 4.8$ H, $R_1 = 200\ \Omega$, and $R_2 = 300\ \Omega$



(*a*)



(*b*)

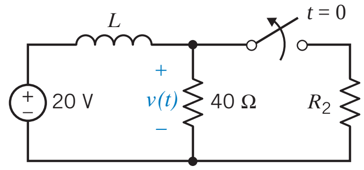
Figure P 8.3-24

P 8.3-25 Consider the circuit shown in Figure P 8.3-25*a* and corresponding plot of the voltage across the 40-Ω resistor shown in Figure P 8.3-25*b*. Determine the values of L and R_2.

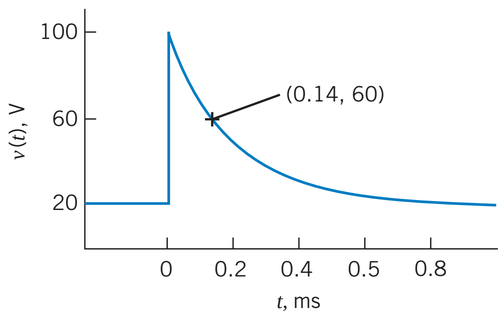
Hint: Use the plot to determine values of D, E, F, and a such that the voltage can be represented as

$$v(t) = \begin{cases} D & \text{for } t < 0 \\ E + Fe^{-at} & \text{for } t > 0 \end{cases}$$

Answers: $L = 8$ H and $R_2 = 10\ \Omega$.



(*a*)



(*b*)

Figure P 8.3-25

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P 8.3-26 Determine $v_o(t)$ for $t > 0$ for the circuit shown in Figure P 8.3-26.

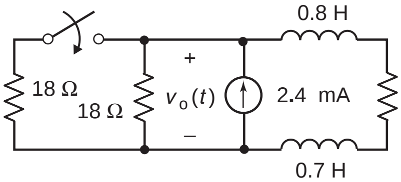

Figure P 8.3-26

P 8.3-27 ⊕ The circuit shown in Figure P 8.3-27 is at steady state before the switch closes at time $t = 0$. After the switch closes, the inductor current is given by

$$i(t) = 0.6 - 0.2e^{-5t}\text{ A} \quad \text{for } t \geq 0$$

Determine the values of R_1, R_2, and L.

Answers: $R_1 = 20\ \Omega$, $R_2 = 10\ \Omega$, and $L = 4$ H

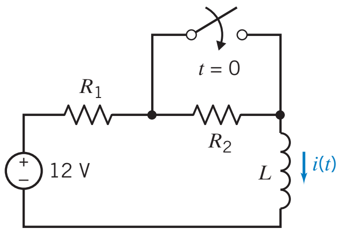

Figure P 8.3-27

P 8.3-28 ⊕ After time $t = 0$, a given circuit is represented by the circuit diagram shown in Figure P 8.3-28.

(a) Suppose that the inductor current is

$$i(t) = 21.6 + 28.4e^{-4t}\text{ mA} \quad \text{for } t \geq 0$$

Determine the values of R_1 and R_3.

(b) Suppose instead that $R_1 = 16\ \Omega$, $R_3 = 20\ \Omega$, and the initial condition is $i(0) = 10$ mA.
Determine the inductor current for $t \geq 0$.

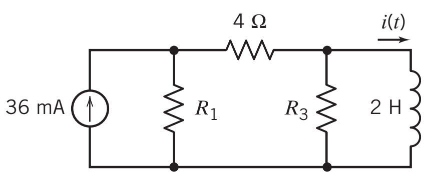

Figure P 8.3-28

P 8.3-29 ⊕ Consider the circuit shown in Figure P 8.3-29.

(a) Determine the time constant τ and the steady-state capacitor voltage $v(\infty)$ when the switch is open.

(b) Determine the time constant τ and the steady-state capacitor voltage $v(\infty)$ when the switch is closed.

Answers: **(a)** $\tau = 3$ s, and $v(\infty) = 24$ V; **(b)** $\tau = 2.25$ s, and $v(\infty) = 12$ V

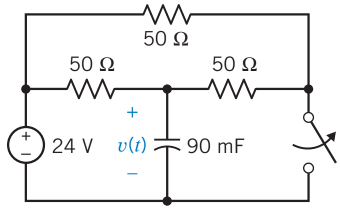

Figure P 8.3-29

Section 8.4 Sequential Switching

P 8.4-1 ⊕ The circuit shown in Figure P 8.4-1 is at steady state before the switch closes at time $t = 0$. The switch remains closed for 1.5 s and then opens. Determine the capacitor voltage $v(t)$ for $t > 0$.

Hint: Determine $v(t)$ when the switch is closed. Evaluate $v(t)$ at time $t = 1.5$ s to get $v(1.5)$. Use $v(1.5)$ as the initial condition to determine $v(t)$ after the switch opens again.

Answer: $v(t) = \begin{cases} 5 + 5e^{-5t}\text{ V} & \text{for } 0 < t < 1.5\text{ s} \\ 10 - 2.64e^{-2.5(t-1.5)}\text{ V} & \text{for } 1.5\text{ s} < t \end{cases}$

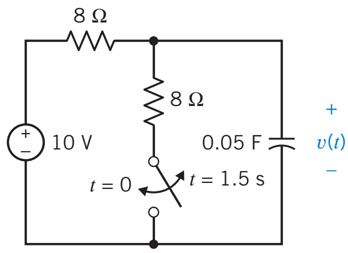

Figure P 8.4-1

P 8.4-2 ⊕ The circuit shown in Figure P 8.4-2 is at steady state before the switch closes at time $t = 0$. The switch remains closed for 1.5 s and then opens. Determine the inductor current $i(t)$ for $t > 0$.

Answer: $v(t) = \begin{cases} 2 + e^{-0.5t}\text{ A} & \text{for } 0 < t < 1.5\text{ s} \\ 3 - 0.53e^{-0.667(t-1.5)}\text{ A} & \text{for } 1.5\text{ s} < t \end{cases}$

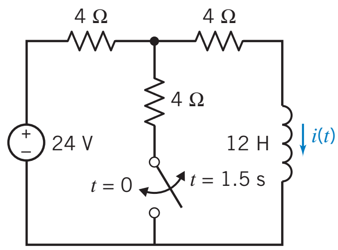

Figure P 8.4-2

P 8.4-3 Cardiac pacemakers are used by people to maintain regular heart rhythm when they have a damaged heart. The circuit of a pacemaker can be represented as shown in Figure P 8.4-3. The resistance of the wires, R, can be neglected because $R < 1$ mΩ. The heart's load resistance R_L is 1 kΩ. The first switch is activated at $t = t_0$, and the second switch is activated at $t_1 = t_0 + 10$ ms. This cycle is repeated every second. Find $v(t)$ for $t_0 \leq t \leq 1$. Note that it is easiest to consider $t_0 = 0$ for this

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calculation. The cycle repeats by switch 1 returning to position a and switch 2 returning to its open position.

Hint: Use $q = Cv$ to determine $v(0^-)$ for the 100-μF capacitor.

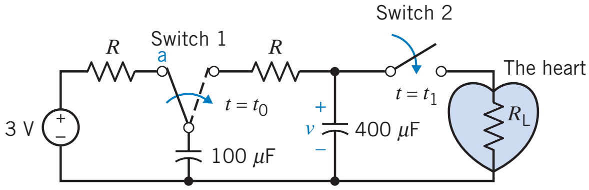

Figure P 8.4-3

P 8.4-4 An electronic flash on a camera uses the circuit shown in Figure P 8.4-4. Harold E. Edgerton invented the electronic flash in 1930. A capacitor builds a steady-state voltage and then discharges it as the shutter switch is pressed. The discharge produces a very brief light discharge. Determine the elapsed time t_1 to reduce the capacitor voltage to one-half of its initial voltage. Find the current $i(t)$ at $t = t_1$.

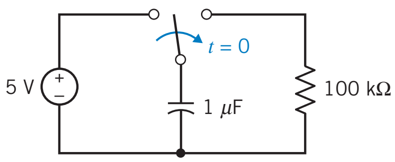

Figure P 8.4-4 Electronic flash circuit.

P 8.4-5 ⊕ The circuit shown in Figure P 8.4-5 is at steady state before the switch opens at $t = 0$. The switch remains open for 0.5 second and then closes. Determine $v(t)$ for $t \geq 0$.

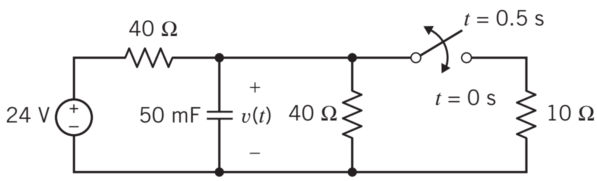

Figure P 8.4-5

Section 8.5 Stability of First-Order Circuits

P 8.5-1 ⊕ The circuit in Figure P 8.5-1 contains a current controlled voltage source. What restriction must be placed on the gain R of this dependent source to guarantee stability?

Answer: $R < 400\ \Omega$

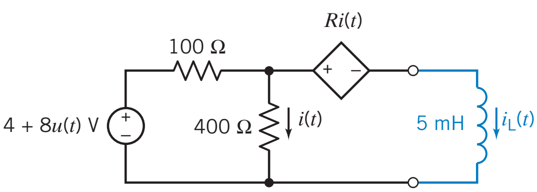

Figure P 8.5-1

P 8.5-2 The circuit in Figure P 8.5-2 contains a current-controlled current source. What restriction must be placed on the gain B of this dependent source to guarantee stability?

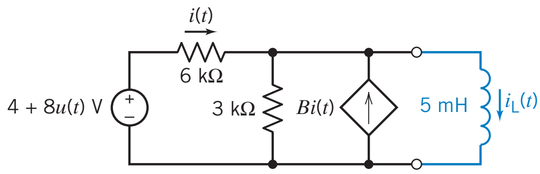

Figure P 8.5-2

Section 8.6 The Unit Step Source

P 8.6-1 ⊕ The input to the circuit shown in Figure P 8.6-1 is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the capacitor, $v_o(t)$. Determine the output of this circuit when the input is $v_s(t) = 8 - 15\ u(t)$ V.

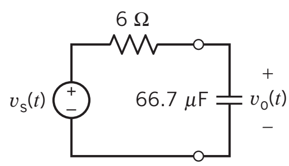

Figure P 8.6-1

P 8.6-2 ⊕ The input to the circuit shown in Figure P 8.6-2 is the voltage of the voltage source, $v_s(t)$. The output is the voltage across the capacitor, $v_o(t)$. Determine the output of this circuit when the input is $v_s(t) = 3 + 3\ u(t)$ V.

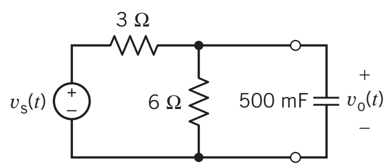

Figure P 8.6-2

P 8.6-3 ⊕ The input to the circuit shown in Figure P 8.6-3 is the voltage of the voltage source, $v_s(t)$. The output is the current in the inductor, $i_o(t)$. Determine the output of this circuit when the input is $v_s(t) = -7 + 13\ u(t)$ V.

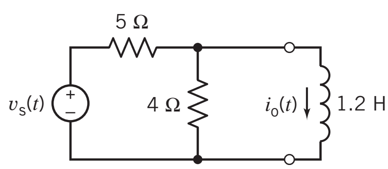

Figure P 8.6-3

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P 8.6-4 Determine $v_o(t)$ for $t > 0$ for the circuit shown in Figure P 8.6-4.

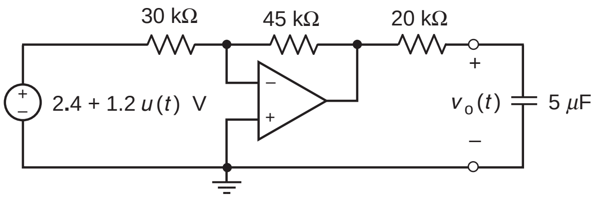

Figure P 8.6-4

P 8.6-5 The initial voltage of the capacitor of the circuit shown in Figure P 8.6-5 is zero. Determine the voltage $v(t)$ when the source is a pulse, described by

$$v_s = \begin{cases} 0 & t < 1 \text{ s} \\ 4 \text{ V} & 1 < t < 2 \text{ s} \\ 0 & t > 2 \text{ s} \end{cases}$$

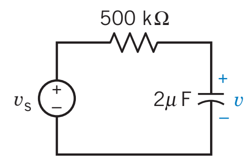

Figure P 8.6-5

P 8.6-6 Studies of an artificial insect are being used to understand the nervous system of animals. A model neuron in the nervous system of the artificial insect is shown in Figure P 8.6-6. A series of pulses, called synapses, is required. The switch generates a pulse by opening at $t = 0$ and closing at $t = 0.5$ s. Assume that the circuit is in steady state and that $v(0^-) = 10$ V. Determine the voltage $v(t)$ for $0 < t < 2$ s.

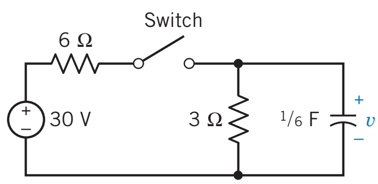

Figure P 8.6-6 Neuron circuit model.

P 8.6-7 Determine the voltage $v_o(t)$ in the circuit shown in Figure P 8.6-7.

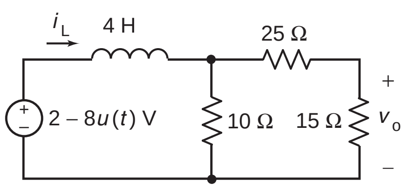

Figure P 8.6-7

P 8.6-8 Determine $v_c(t)$ for $t > 0$ for the circuit of Figure P 8.6-8.

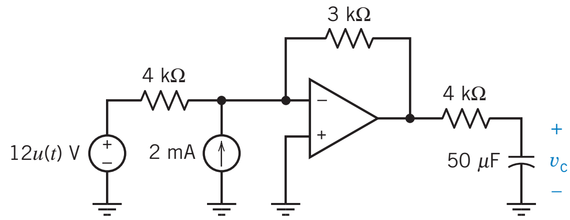

Figure P 8.6-8

P 8.6-9 The voltage source voltage in the circuit shown in Figure P 8.6-9 is

$$v_s(t) = 7 - 14u(t) \text{ V}$$

Determine $v(t)$ for $t > 0$.

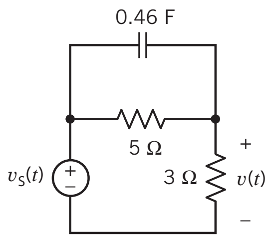

Figure P 8.6-9

P 8.6-10 Determine the voltage $v(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-10.

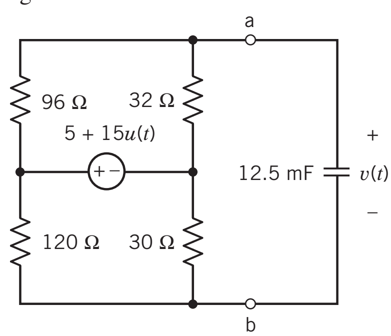

Figure P 8.6-10

P 8.6-11 The voltage source voltage in the circuit shown in Figure P 8.6-11 is

$$v_s(t) = 5 + 20u(t)$$

Determine $i(t)$ for $t \geq 0$.

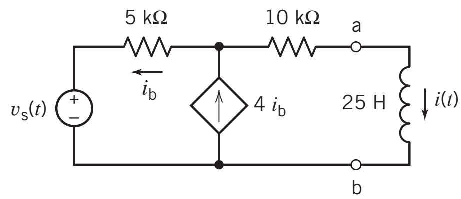

Figure P 8.6-11

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P 8.6-12 The voltage source voltage in the circuit shown in Figure P 8.6-12 is

$$v_s(t) = 12 - 6u(t)\text{ V}$$

Determine $v(t)$ for $t \geq 0$.

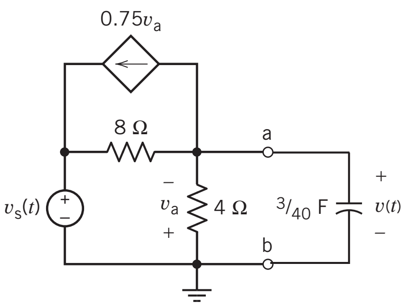

Figure P 8.6-12

P 8.6-13 Determine $i(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-13.

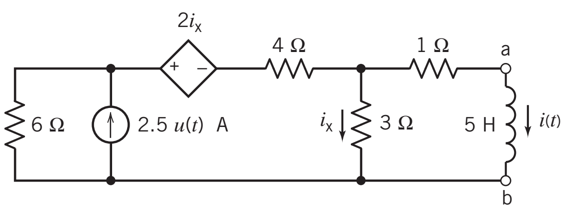

Figure P 8.6-13

P 8.6-14 Determine $i(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-14.

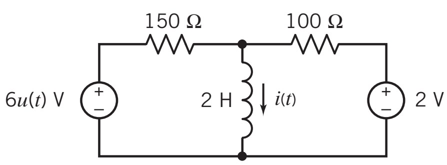

Figure P 8.6-14

P 8.6-15 Determine $v(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-15.

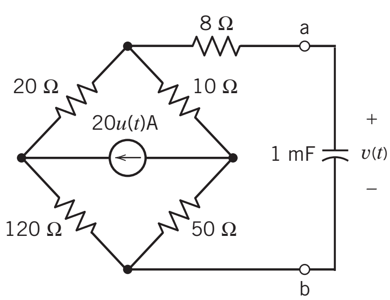

Figure P 8.6-15

P 8.6-16 Determine $v(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-16.

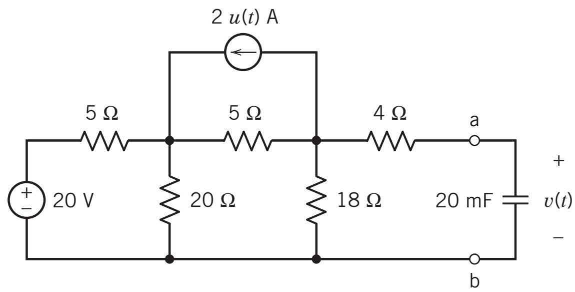

Figure P 8.6-16

P 8.6-17 Determine $i(t)$ for $t \geq 0$ for the circuit shown in Figure P 8.6-17.

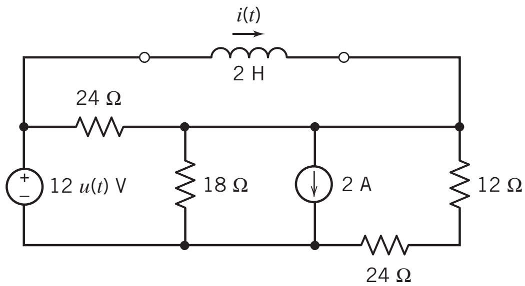

Figure P 8.6-17

P 8.6-18 The voltage source voltage in the circuit shown in Figure P 8.6-18 is

$$v_s(t) = 8 + 12u(t)\text{ V}$$

Determine $v(t)$ for $t \geq 0$.

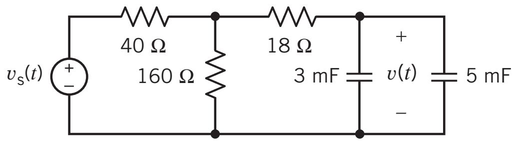

Figure P 8.6-18

P 8.6-19 Determine the current $i_o(t)$ in the circuit shown in Figure P 8.6-19.

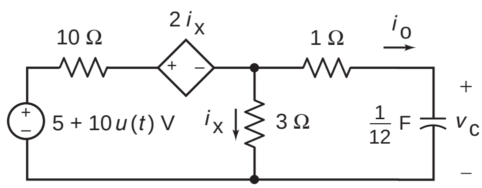

Figure P 8.6-19

P 8.6-20 The voltage source voltage in the circuit shown in Figure P 8.6-20 is

$$v_s(t) = 25u(t) - 10\text{ V}$$

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Determine $i(t)$ for $t \geq 0$.

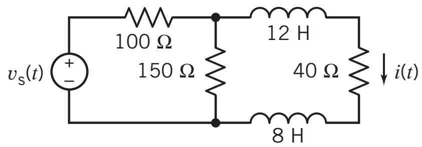

Figure P 8.6-20

P 8.6-21 The voltage source voltage in the circuit shown in Figure P 8.6-21 is

$$v_s(t) = 30 - 24u(t) \text{ V}$$

Determine $i(t)$ for $t \geq 0$.

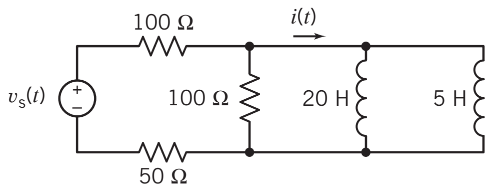

Figure P 8.6-21

P 8.6-22 The voltage source voltage in the circuit shown in Figure P 8.6-22 is

$$v_s(t) = 10 + 40u(t) \text{ V}$$

Determine $v(t)$ for $t \geq 0$.

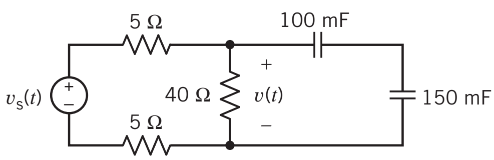

Figure P 8.6-22

P 8.6-23 Determine $v(t)$ for $t > 0$ for the circuit shown in Figure P 8.6-23.

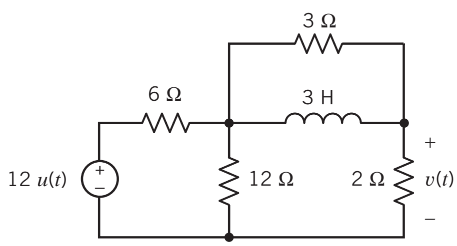

Figure P 8.6-23

P 8.6-24 The input to the circuit shown in Figure P 8.6-24 is the current source current

$$i_s(t) = 2 + 4u(t) \text{ A}$$

The output is the voltage $v(t)$. Determine $v(t)$ for $t > 0$.

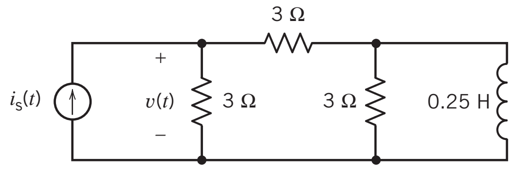

Figure P 8.6-24

P 8.6-25 The input to the circuit shown in Figure P 8.6-25 is the voltage source voltage

$$v_s = 6 + 6u(t)$$

The output is the voltage $v_o(t)$. Determine $v_o(t)$ for $t > 0$.

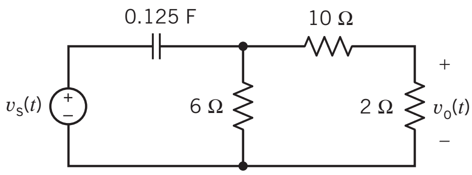

Figure P 8.6-25

P 8.6-26 Determine $v(t)$ for $t > 0$ for the circuit shown in Figure P 8.6-26.

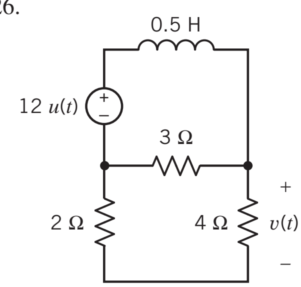

Figure P 8.6-26

P 8.6-27 When the input to the circuit shown in Figure P 8.6-27 is the voltage source voltage

$$v_s(t) = 3 - u(t) \text{ V}$$

The output is the voltage

$$v_o(t) = 10 + 5\,e^{-50t} \text{ V} \quad \text{for } t \geq 0$$

Determine the values of R_1 and R_2.

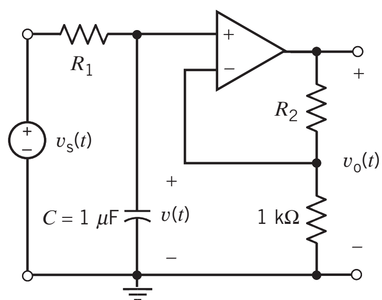

Figure P 8.6-27

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P 8.6-28 The time constant of a particular circuit is $\tau = 0.25$ s. In response to a step input, a capacitor voltage changes from -2.5 V to 4.2 V. How long did it take for the capacitor voltage to increase from -2.0 V to $+2.0$ V?

Section 8.7 The Response of a First-Order Circuit to a Nonconstant Source

P 8.7-1 Find $v_c(t)$ for $t > 0$ for the circuit shown in Figure P 8.7-1 when $v_1 = 8e^{-5t}u(t)$ V. Assume the circuit is in steady state at $t = 0^-$.

Answer: $v_c(t) = 4e^{-9t} + 18e^{-5t}$ V

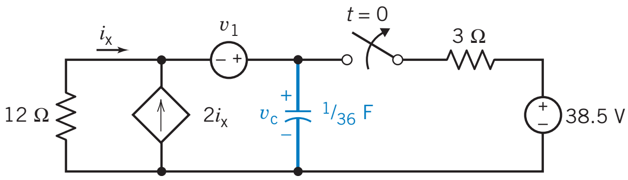

Figure P 8.7-1

P 8.7-2 Find $v(t)$ for $t > 0$ for the circuit shown in Figure P 8.7-2. Assume steady state at $t = 0^-$.

Answer: $v(t) = 20e^{-10t/3} - 12e^{-2t}$ V

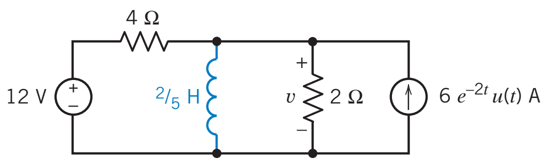

Figure P 8.7-2

P 8.7-3 Find $v_c(t)$ for $t > 0$ for the circuit shown in Figure P 8.7-3 when $i_s = [2 \cos 2t]\, u(t)$ mA.

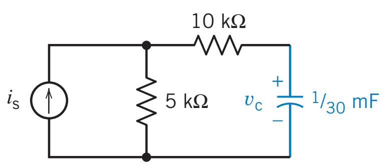

Figure P 8.7-3

P 8.7-4 Many have witnessed the use of an electrical megaphone for amplification of speech to a crowd. A model of a microphone and speaker is shown in Figure P 8.7-4*a*, and the circuit model is shown in Figure P 8.7-4*b*. Find $v(t)$ for $v_s = 10$ (sin $100t)u(t)$, which could represent a person whistling or singing a pure tone.

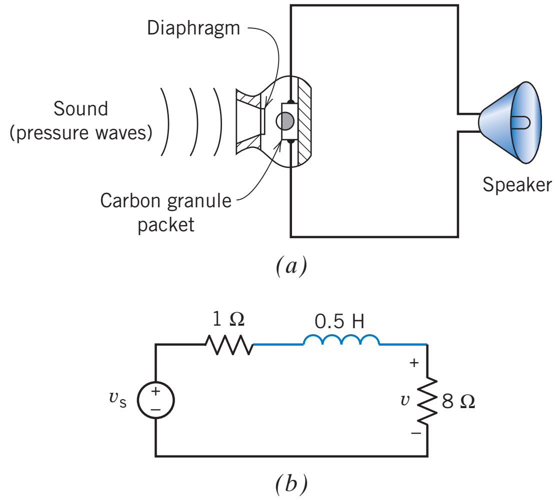

Figure P 8.7-4 Megaphone circuit.

P 8.7-5 A lossy integrator is shown in Figure P 8.7-5. The lossless capacitor of the ideal integrator circuit has been replaced with a model for the lossy capacitor, namely, a lossless capacitor in parallel with a 1-kΩ resistor. If $v_s = 15e^{-2t}u(t)$ V and $v_o(0) = 10$ V, find $v_o(t)$ for $t > 0$.

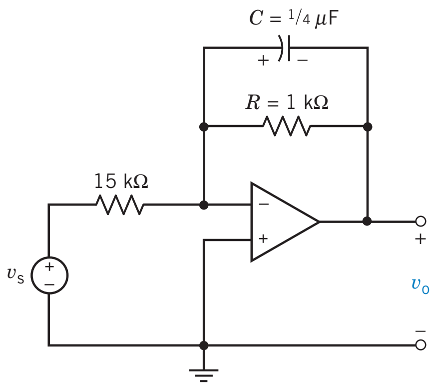

Figure P 8.7-5 Integrator circuit.

P 8.7-6 Determine $v(t)$ for the circuit shown in Figure P 8.7-6.

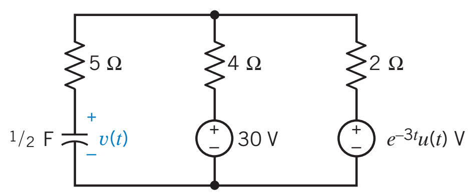

Figure P 8.7-6

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P 8.7-7 Determine $v(t)$ for the circuit shown in Figure P 8.7-7*a* when v_s varies as shown in Figure P 8.7-7*b*. The initial capacitor voltage is $v_c(0) = 0$.

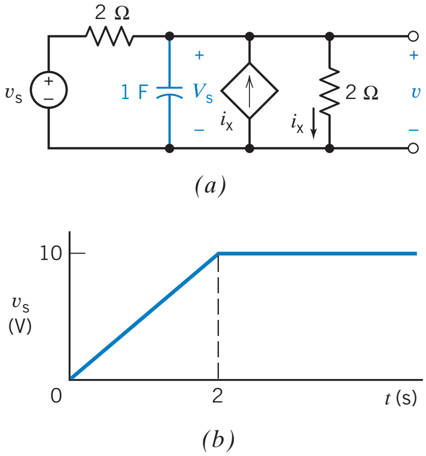

Figure P 8.7-7

P 8.7-8 The electron beam, which is used to draw signals on an oscilloscope, is moved across the face of a cathode-ray tube (CRT) by a force exerted on electrons in the beam. The basic system is shown in Figure P 8.7-8*a*. The force is created from a time-varying, ramp-type voltage applied across the vertical or the horizontal plates. As an example, consider the simple circuit of Figure P 8.7-8*b* for horizontal deflection in which the capacitance between the plates is C.

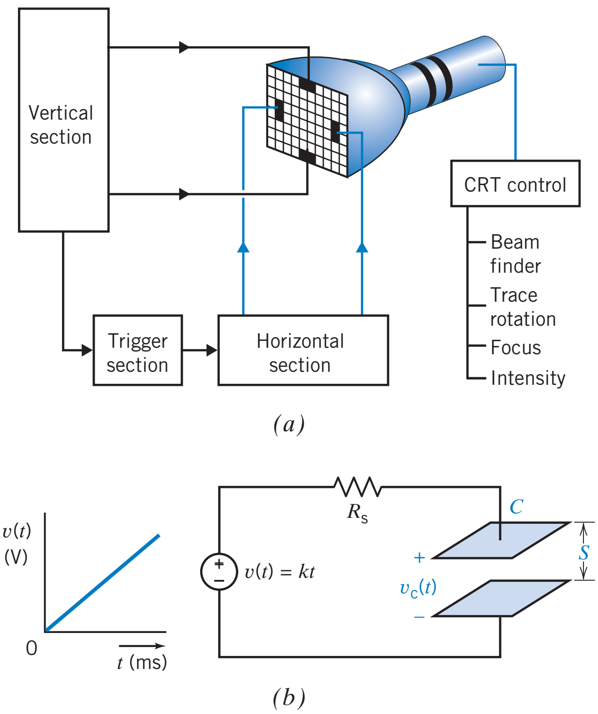

Figure P 8.7-8 Cathode-ray tube beam circuit.

Derive an expression for the voltage across the capacitance. If $v(t) = kt$ and $R_s = 625$ kΩ, $k = 1000$, and $C = 2000$ pF, compute v_c as a function of time. Sketch $v(t)$ and $v_c(t)$ on the same graph for time less than 10 ms. Does the voltage across the plates track the input voltage?

Section 8.10 How Can We Check . . . ?

P 8.10-1 Figure P 8.10-1 shows the transient response of a first-order circuit. This transient response was obtained using the computer program, PSpice. A point on this transient response has been labeled. The label indicates a time and the capacitor voltage at that time. Placing the circuit diagram on the plot suggests that the plot corresponds to the circuit. Verify that the plot does indeed represent the voltage of the capacitor in this circuit.

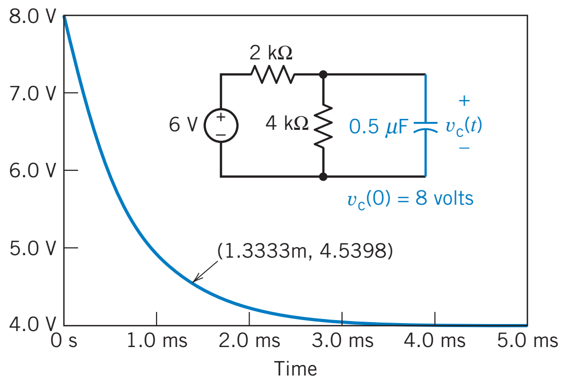

Figure P 8.10-1

P 8.10-2 Figure P 8.10-2 shows the transient response of a first-order circuit. This transient response was obtained using the computer program, PSpice. A point on this transient response has been labeled. The label indicates a time and the inductor current at that time. Placing the circuit diagram on the plot suggests that the plot corresponds to the circuit. Verify that the plot does indeed represent the current of the inductor in this circuit.

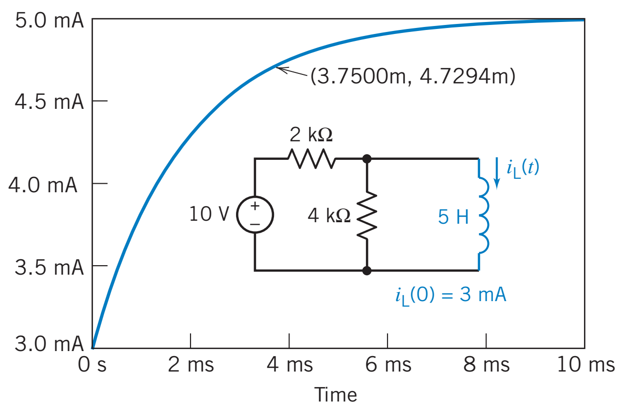

Figure P 8.10-2

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P 8.10-3 Figure P 8.10-3 shows the transient response of a first-order circuit. This transient response was obtained using the computer program, PSpice. A point on this transient response has been labeled. The label indicates a time and the inductor current at that time. Placing the circuit diagram on the plot suggests that the plot corresponds to the circuit. Specify that value of the inductance L required to cause the current of the inductor in this circuit to be accurately represented by this plot.

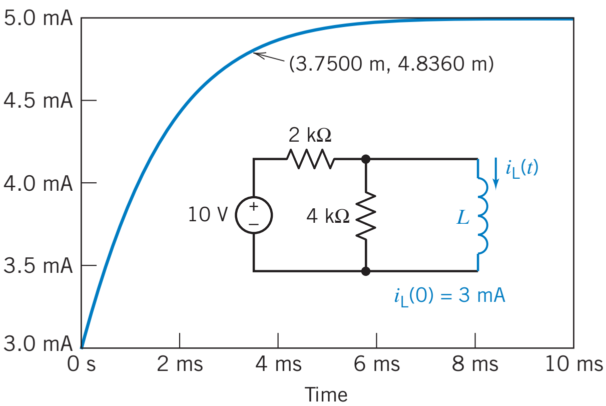

Figure P 8.10-3

P 8.10-4 Figure P 8.10-4 shows the transient response of a first-order circuit. This transient response was obtained using the computer program, PSpice. A point on this transient response has been labeled. The label indicates a time and the capacitor voltage at that time. Assume that this circuit has reached steady state before time $t = 0$. Placing the circuit diagram on the plot suggests that the plot corresponds to the circuit. Specify values of A, B, R_1, R_2, and C that cause the voltage across the capacitor in this circuit to be accurately represented by this plot.

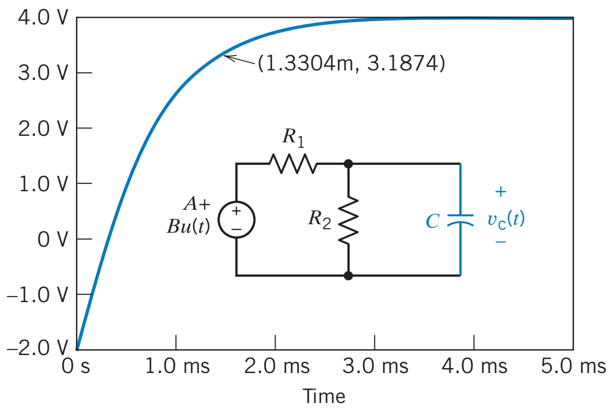

Figure P 8.10-4

PSpice Problems

SP 8-1 The input to the circuit shown in Figure SP 8-1 is the voltage of the voltage source, $v_i(t)$. The output is the voltage across the capacitor, $v_o(t)$. The input is the pulse signal specified graphically by the plot. Use PSpice to plot the output $v_o(t)$ as a function of t.

Hint: Represent the voltage source, using the PSpice part named VPULSE.

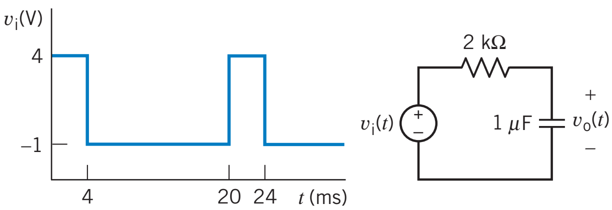

Figure SP 8-1

SP 8-2 The input to the circuit shown in Figure SP 8-2 is the voltage of the voltage source, $v_i(t)$. The output is the current in the inductor, $i_o(t)$. The input is the pulse signal specified graphically by the plot. Use PSpice to plot the output $i_o(t)$ as a function of t.

Hint: Represent the voltage source, using the PSpice part named VPULSE.

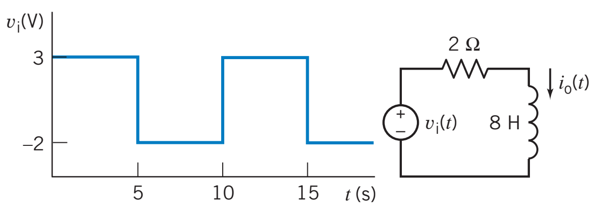

Figure SP 8-2

SP 8-3 The circuit shown in Figure SP 8-3 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the voltage of the voltage source, 12 V. The output of this circuit is the voltage across the capacitor, $v(t)$. Use PSpice to plot the output $v(t)$ as a function of t. Use the plot to obtain an analytic representation of $v(t)$ for $t > 0$.

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Hint: We expect $v(t) = A + Be^{-t/\tau}$ for $t > 0$, where A, B, and τ are constants to be determined.

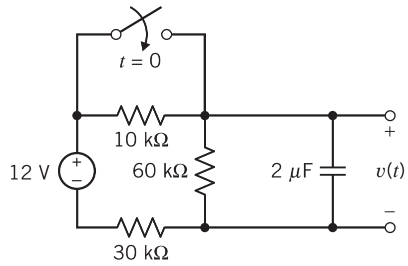

Figure SP 8-3

SP 8-4 The circuit shown in Figure SP 8-4 is at steady state before the switch closes at time $t = 0$. The input to the circuit is the current of the current source, 4 mA. The output of this circuit is the current in the inductor, $i(t)$. Use PSpice to plot the output $i(t)$ as a function of t. Use the plot to obtain an analytic representation of $i(t)$ for $t > 0$.

Hint: We expect $i(t) = A + Be^{-t/\tau}$ for $t > 0$, where A, B, and τ are constants to be determined.

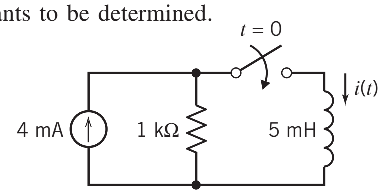

Figure SP 8-4

Design Problems

DP 8-1 Design the circuit in Figure DP 8-1 so that $v(t)$ makes the transition from $v(t) = 6$ V to $v(t) = 10$ V in 10 ms after the switch is closed. Assume that the circuit is at steady state before the switch is closed. Also assume that the transition will be complete after 5 time constants.

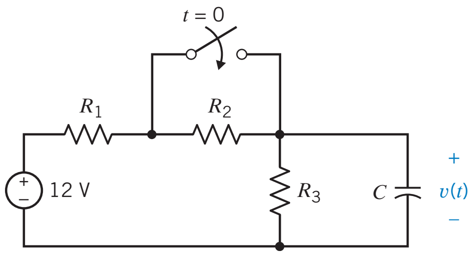

Figure DP 8-1

DP 8-2 Design the circuit in Figure DP 8-2 so that $i(t)$ makes the transition from $i(t) = 1$ mA to $i(t) = 4$ mA in 10 ms after the switch is closed. Assume that the circuit is at steady state before the switch is closed. Also assume that the transition will be complete after 5 time constants.

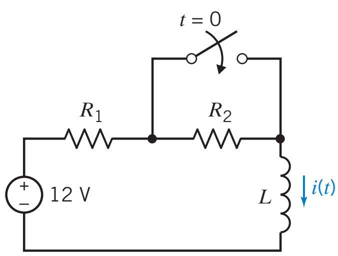

Figure DP 8-2

DP 8-3 The switch in Figure DP 8-3 closes at time 0, $2\Delta t$, $4\Delta t$, . . . $2k\Delta t$ and opens at times Δt, $3\Delta t$, $5\Delta t$, ... $(2k + 1)\Delta t$. When the switch closes, $v(t)$ makes the transition from $v(t) = 0$ V to $v(t) = 5$ V. Conversely, when the switch opens, $v(t)$ makes the transition from $v(t) = 5$ V to $v(t) = 0$ V. Suppose we require that $\Delta t = 5\tau$ so that one transition is complete before the next one begins. (a) Determine the value of C required so that $\Delta t = 1$ μs. (b) How large must Δt be when $C = 2$ μF?

Answer: (a) $C = 4$ pF; (b) $\Delta t = 0.5$s

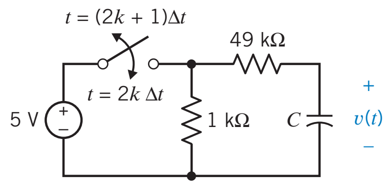

Figure DP 8-3

DP 8-4 The switch in Figure DP 8-3 closes at time 0, $2\Delta t$, $4\Delta t$, . . . $2k\Delta t$ and opens at times Δt, $3\Delta t$, $5\Delta t$, ... $(2k + 1)\Delta t$. When the switch closes, $v(t)$ makes the transition from $v(t) = 0$ V to $v(t) = 5$ V. Conversely, when the switch opens, $v(t)$ makes the transition from $v(t) = 5$ V to $v(t) = 0$ V. Suppose we require that one transition be 95 percent complete before the next one begins. (a) Determine the value of C required so that $\Delta t = 1$ μs. (b) How large must Δt be when $C = 2$ μF?

Hint: Show that $\Delta t = -\tau \ln(1 - k)$ is required for the transition to be 100 k percent complete.

Answer: (a) $C = 6.67$ pF; (b) $\Delta t = 0.3$ s

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DP 8-5 A laser trigger circuit is shown in Figure DP 8-5. To trigger the laser, we require 60 mA $< |i| <$ 180 mA for $0 < t <$ 200 μs. Determine a suitable value for R_1 and R_2.

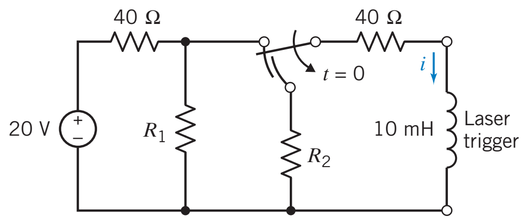


Figure DP 8-5 Laser trigger circuit.

DP 8-6 Fuses are used to open a circuit when excessive current flows (Wright, 1990). One fuse is designed to open when the power absorbed by R exceeds 10 W for 0.5 s. Consider the circuit shown in Figure DP 8-6. The input is given by $v_s = A[u(t) - u(t - 0.75)]$ V. Assume that $i_L(0^-) = 0$. Determine the largest value of A that will not cause the fuse to open.

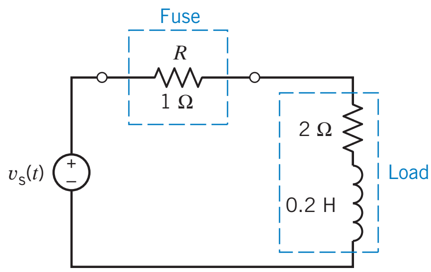
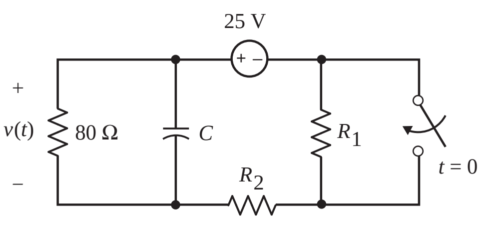
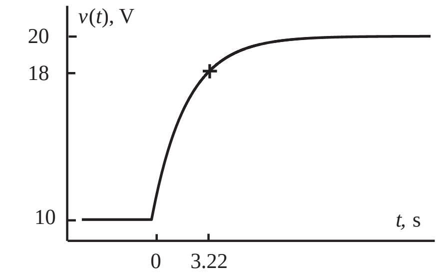


Figure DP 8-6 Fuse circuit.

DP 8-7 Design the circuit in Figure DP 8-7(a) to have the response in Figure DP 8-7(b) by specifying the values of C, R_1, and R_2.



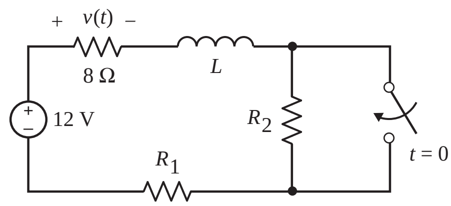
(a)



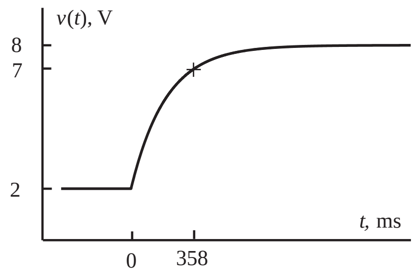
(b)

Figure DP 8-7

DP 8-8 Design the circuit in Figure DP 8-8(a) to have the response in Figure DP 8-8(b) by specifying the values of L, R_1, and R_2.



(a)



(b)

Figure DP 8-8

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DP 8-9 Design the circuit in Figure DP 8-9(*a*) to have the response in Figure DP 8-9(*b*) by specifying the values of C, R_1, and R_2.

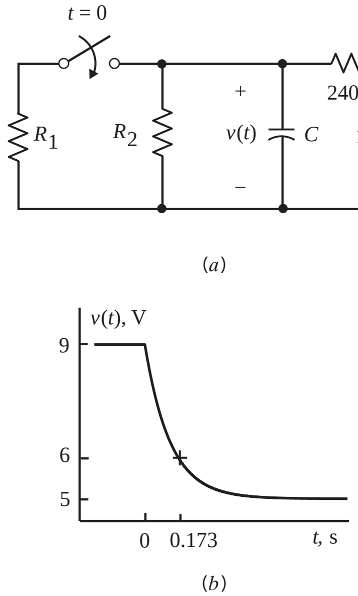

Figure DP 8-9

DP 8-10 Design the circuit in Figure DP 8-10(*a*) to have the response in Figure DP 8-10(*b*) by specifying the values of L, R_1, and R_2.

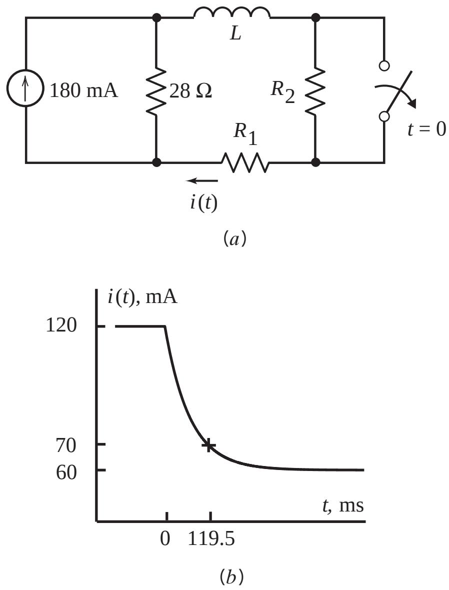

Figure DP 8-10

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