Part3-B

Bianca Brusco

4/28/2018

Part 3: Bianca

Create person-period file

```
#re-read data
classroom2 <- na.omit(classroom)
#new variables
classroom2 <- classroom2 %>% mutate(math0 = mathkind) %>% mutate(math1 = mathkind+mathgain)
#reshape the data
class_pp <- reshape(classroom2, varying = c("math0", "math1"), v.names = "math", timevar = "year",
times = c(0, 1), direction = "long")
```

Note: we ignore classroom in this analysis but keep it in the notation.

Initial longitudinal model

We fit a model with math as outcome, and fixed effect for time trend (year), as well as random intercept for school.

The equation for the model below:

$$Math_{tijk} = b_0 + \zeta_{0k} + b_1 * Time_{tijk} + \epsilon_{tijk}$$

where $\zeta_{0k} \sim N(0, \sigma^2_{zeta0})$ and $\epsilon_{tijk} \sim N(0, \sigma^2_{\epsilon})$

We refer to this as Model 0.

Below the model fit:

```
fit1 <- lmer(math ~ year + (1|schoolid), data = class_pp)
summary(fit1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 | schoolid)
##    Data: class_pp
##
## REML criterion at convergence: 21794.2
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.0693 -0.6031  0.0030  0.6321  3.7529
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  schoolid (Intercept)  337     18.36
##  Residual             1288     35.89
```

```
## Number of obs: 2162, groups:  schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)    
## (Intercept)  465.053      2.136  133.580  217.76   <2e-16 ***
## year          57.844      1.544 2055.557   37.47   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.361
```

Add child-level random intercept

To the previous model, we now add random intercepts for child:

$$Math_{tijk} = b_0 + \delta_{0tijk} + \zeta_{0k} + b_1 * Time_{tijk} + \epsilon_{tijk}$$

where $\delta_{0tijk} \sim N(0, \sigma^2_{\delta_0}), \zeta_{0k} \sim N(0, \sigma^2_{zeta0})$ and $\epsilon_{tijk} \sim N(0, \sigma^2_{\epsilon})$ independetly of one another.

We refer to this as M1.

```r
fit2 <- lmer(math ~ year + (1|schoolid/childid), data = class_pp)
summary(fit2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 | schoolid/childid)
##    Data: class_pp
##
## REML criterion at convergence: 21425.2
##
## Scaled residuals: 
##     Min      1Q  Median      3Q     Max 
## -4.7233 -0.4827  0.0125  0.4922  3.4892 
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  childid:schoolid (Intercept) 722.0    26.87   
##  schoolid         (Intercept) 293.2    17.12   
##  Residual                     602.2    24.54   
## Number of obs: 2162, groups:  childid:schoolid, 1081; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)    
## (Intercept)  465.288      2.057  116.653   226.2   <2e-16 ***
## year          57.844      1.056 1080.000    54.8   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.257
```

In model 0 the variance $\sigma^2_{\zeta_0} = 377$ and in model 1 $\sigma^2_{\zeta_0} = 293.2$.
In model 0 , the varaince for $\sigma^2_{\epsilon} = 1288$ and in model 1 $\sigma^2_{\epsilon_0} = 602.2$. We note that including child-level variation leads to a decrease in the variance of both the random effects.

Compute Pseudo-R^2

Compute a pseudo R^2 relating the between school variation and ignoring between students in the same school.

We calculate this as :

$$\frac{\sigma^2_{\zeta_0}(M_0) - \sigma^2_{\zeta_0}(M_1)}{\sigma^2_{\zeta_0}(M_0)} = \frac{377 - 293.2}{377} = 0.22$$

The between-school variance is reduced by 22% (or 'explained') with the introduction of student random effect.

Does the total variation stay about the same?

```
tot_m0 = 337 + 1288
tot_m1 = 722 + 293.2 + 602.2
paste("Tot variance for model 0 : ", tot_m0)
```

```
## [1] "Tot variance for model 0 :  1625"
```

```
paste("Tot variance for model 1: ", tot_m1)
```

```
## [1] "Tot variance for model 1:  1617.4"
```

There is only a slightly decrease in the total variance between Model 0 and Model1.

Add a random slope for time trend

We now add a random slope (ζ_1) for time trend within schools.

$$Math_{tijk} = b_0 + \delta_{0tijk} + \zeta_{0k} + (b_1 + \zeta_{1k}) * Time_{tijk} + \epsilon_{tijk}$$

where $\delta_{0tijk} \sim N(0, \sigma^2_{\delta_0})$, $\quad \zeta_{0k} \sim N(0, \sigma^2_{zeta0})$, $\quad \zeta_{1k} \sim N(0, \sigma^2_{zeta0})$ and $\epsilon_{tijk} \sim N(0, \sigma^2_{\epsilon})$ – each independetly of one another.

We refer to this as Model 2

We run the model and report the fit:

```
fit3 = lmer(math ~ year + (1 + year|| schoolid) + (1|childid), data = class_pp)
summary(fit3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 + year || schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 21403.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
```

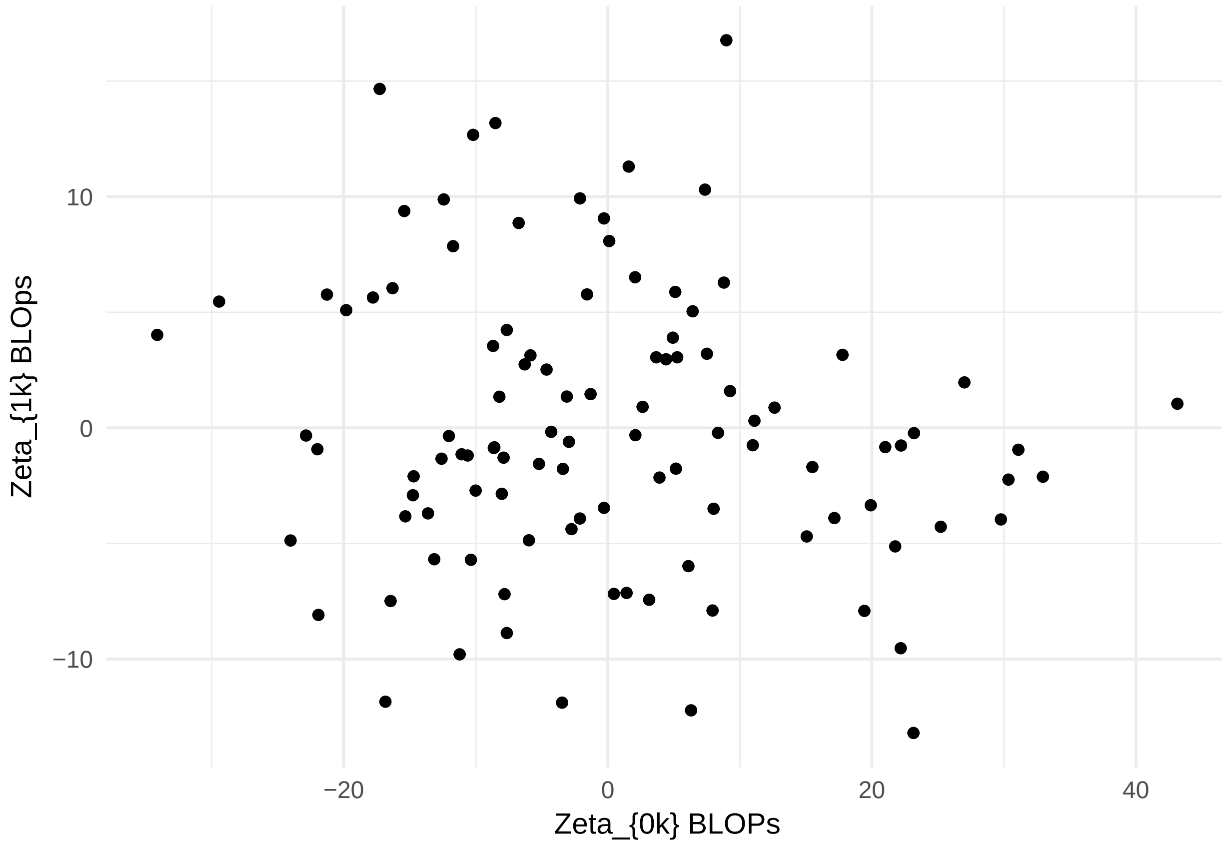
```
## -4.7461 -0.4788  0.0119  0.4719  3.5976
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  childid    (Intercept) 745.36   27.301
##  schoolid   year         85.96    9.272
##  schoolid.1 (Intercept) 315.69   17.768
##  Residual               554.92   23.557
## Number of obs: 2162, groups:  childid, 1081; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.257      2.108 108.631  220.73   <2e-16 ***
## year          57.751      1.402 101.342   41.18   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.188
```

Generate the BLUPs for this model (Model 2)

Examine then whether the independence between zeta0 and zeta1 is reflected in a scatterplot of these two sets of effects.

```r
pp_ranefs <- ranef(fit3)

if (vanillaR) {
plot(pp_ranefs$schoolid[,2],pp_ranefs$schoolid[,1])
}else{
ggplot(pp_ranefs$schoolid, aes(x = pp_ranefs$schoolid[,2], y = pp_ranefs$schoolid[,1] )) +
geom_point() + labs(x = "Zeta_{0k} BLOPs", y = "Zeta_{1k} BLOps") + theme_minimal()
}
```



From the plot, the BLOPs for ζ_{0k} and for ζ_{1k} appear uncorrelated, reflecting the way in which the model was built.

Heteroscedasticity in the random effects

Question: What are: $V_S(year = 0)$, $V_S(year = 1)$?

The model we are considering is :

$Math_{tijk} = b_0 + \delta_{0tijk} + \zeta_{0k} + (b_1 + \zeta_{1k})Time_{tijk} + \epsilon_{tijk}$

So we have that (in this model, in which we are forcing correlation of 0 between slope and intercept):

- $V_S(year = 0) = \sigma^2_{\zeta_{0k}} = 85.96$
- $V_S(year = 1) = \sigma^2_{\zeta_{0k}} + \sigma^2_{\zeta_{1k}} = 85.96 + 315.69 = 401.65$

Run model separately by year

We now examine what happens if we run the model separately by year. Do we get the same etimates for the variance between schools?

```
class_year0 = class_pp[class_pp$year == 0,]


# Run model for year 0
fit4 = lmer(math ~ (1 | schoolid), data = class_year0)
summary(fit4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ (1 | schoolid)
##    Data: class_year0
##
## REML criterion at convergence: 11017.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7416 -0.5655  0.0086  0.6286  3.5648
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  schoolid (Intercept)  364.1   19.08
##  Residual             1391.4   37.30
## Number of obs: 1081, groups:  schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.382      2.244 101.588   207.4   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Run modelfor year 1
class_year1 = class_pp[class_pp$year == 1,]
fit5 = lmer(math ~ (1 | schoolid), data = class_year1)
summary(fit5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ (1 | schoolid)
##    Data: class_year1
##
## REML criterion at convergence: 10851.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5824 -0.6158 -0.0063  0.6191  3.8063
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  schoolid (Intercept)  279.1   16.71
##  Residual             1202.6   34.68
## Number of obs: 1081, groups:  schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error    df t value Pr(>|t|)
## (Intercept)    523.2        2.0 101.9   261.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

In this case, for the Year 0 Model, we get an estimated $\hat{\sigma}^2_{\zeta_{0k}} = 364.1$, while for Year 1 Model, we have $\hat{\sigma}^2_{\zeta_{0k}} = 279.1$. We note that these estimates are different from the ones computed above.

Allow for correlation

We now allow for correlation between the random effects for the intercept and the slope. We call this Model 3.

```
fit6 = lmer(math ~ year + (1 + year| schoolid) + (1|childid), data = class_pp)
summary(fit6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 + year | schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 21391.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.6737 -0.4699  0.0038  0.4683  3.4882
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  childid  (Intercept) 749.0    27.37
##  schoolid (Intercept) 373.5    19.33
##           year        112.4    10.60    -0.53
##  Residual             547.8    23.41
## Number of obs: 2162, groups:  childid, 1081; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.257      2.241 101.265   207.6   <2e-16 ***
## year          58.006      1.491  95.409    38.9   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.486
```

Correlation beteween ζ_0 and ζ_1 = -0.53.

To test whether the correlation is statistically significant, we can compare Model 2 with Model 3 using an anova test.

```
anova(fit3,fit6, refit = F)
```

```
## Data: class_pp
## Models:
## fit3: math ~ year + (1 + year || schoolid) + (1 | childid)
## fit6: math ~ year + (1 + year | schoolid) + (1 | childid)
##      Df   AIC   BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
## fit3  6 21415 21449 -10702    21403
## fit6  7 21405 21445 -10696    21391 11.879      1  0.0005678 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

With a p-value $p = 0.0005678$, we reject the null hypothesis at $\alpha = 0.05$ significance level, and conclude that there correlation term is statistically significant.

So we have that (in this model where we are allowing for correlation between slope and intercept):

- $V_S(year = 0) = \sigma^2_{\zeta_{0k}} = 373.5$
- $V_S(year = 1) = \sigma^2_{\zeta_{0k}} + \sigma^2_{\zeta_{1k}} + 2\rho_{01}\sigma_{\zeta_{0k}}\sigma_{\zeta_{1k}} = 373.5 + 112.4 - 2 * 0.53 * \sqrt{373.5}\sqrt{112.4} = 268.7$

These estimates are a lot closer to the school variances that result from fitting the models for the two years separately (in which we have σ^2_ζ respectively be 364 for year 0 and 279 for year 1.

Therefore, it seemt that the model that allows for correlation between the two random effects has a better fit than the one forcing that correlation to be 0.