---
title: 'Group Project #1'
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output: pdf_document

Part 1: Frankie

Create 1st grade variable

```
classroom <- classroom %>% mutate(Math1 = mathkind + mathgain)
```

Random Intercepts for classroom, nested in schools UMM

```
model1 <- lmer(Math1~(1|schoolid/classid),data=classroom)
summary(model1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.46   9.244
##  schoolid         (Intercept)  280.68  16.754
##  Residual                     1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.410   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$ICC_{class} = \frac{85.46}{1146.8 + 280.68 + 85.46} \approx .056$$

$$ICC_{school} = \frac{280.68}{1146.8 + 280.68 + 85.46} \approx .186$$

$$Math1st_{ijk} = \beta_{0ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$\zeta_k \sim N(0, \sigma^2_\zeta), \eta_{jk} \sim N(0, \sigma^2_\eta)$, and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, all are independent of each other

Model with School Level Predictors Added

```
model2 <- lmer(Math1~housepov+(1|schoolid/classid),data=classroom)
summary(model2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ housepov + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   82.36   9.075
##  schoolid         (Intercept)  250.93  15.841
##  Residual                      1146.95  33.867
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.294      3.341 102.810 159.024   <2e-16 ***
## housepov     -45.783     14.236 111.060  -3.216   0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.810
```

```
anova(model1, model2, refit = F)
```

```
## Data: classroom
## Models:
## model1: Math1 ~ (1 | schoolid/classid)
## model2: Math1 ~ housepov + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model1  4 11953 11973 -5972.3    11945
## model2  5 11937 11963 -5963.7    11927 17.186      1   3.39e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Change in σ^2_ζ: decreased to 250.93 from 280.63 σ^2_η decreases to 82.36 from 85.46 σ^2_ϵ slightly increases to 1146.95 from 1146.8

The ANOVA/LRT has a pvalue of almost zero, 3.39e-05, thus we reject the H_0 at our $\alpha = 0.05$ and meaning that it makes sense to include the school level predictor, housepov.

Model with all Class Level Predictors Added

```
model3 <- lmer(Math1~housepov+mathknow+yearstea+mathprep+(1|schoolid/classid),data=classroom)
summary(model3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                      1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29853    5.20496 228.86000 102.268  < 2e-16 ***
## housepov    -41.62116   14.08835 109.83000  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.07000   1.765  0.07883 .
## yearstea      0.06193    0.14717 223.77000   0.421  0.67432
## mathprep     -0.75440    1.42809 203.21000  -0.528  0.59790
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst
## housepov -0.568
## mathknow -0.052  0.082
## yearstea -0.264  0.077  0.030
## mathprep -0.666  0.032  0.004 -0.175
```

creating reducted dataset taking away missing data

```
classroom_red = na.omit(classroom)
model2_red <- lmer(Math1~housepov+(1|schoolid/classid),data=classroom_red)
model3_red <- lmer(Math1~housepov+mathknow+yearstea+mathprep+(1|schoolid/classid),data=classroom_red)
anova(model2_red, model3_red, refit = F)
```

```
## Data: classroom_red
## Models:
## model2_red: Math1 ~ housepov + (1 | schoolid/classid)
## model3_red: Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##            Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
```

```
## model2_red  5 10838 10862 -5413.8    10828
## model3_red  8 10837 10877 -5410.5    10821 6.5771      3    0.08667 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Change in σ^2_ϵ and σ^2_η: σ^2_ϵ decreased to 1136.43, σ^2_η increased to 94.36; $\sigma^2_\zeta = 223.31$

The reason epsilon was reduced but eta was not is because the new model explains what is happening at a student level, but not at a classroom level. In addition adding the classroom level predictors makes it so that more of the overall variation is explained by "structured" variation rather than by unstructured (ϵ) May increase because of sample decrease (missing data) –

The anova test comparing the school level predictor to the model with the classroom predictors has a p-value 0.087, so we fail to reject the null hypothesis at our $\alpha = 0.05$ and thus though boarderline to signifcance, it still concludes that the models are not different so adding the classroom level predictors isn't neccesary.

Add all student-level predictors

```
model4 <- lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1|schoolid/classid),data=cla
summary(model4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                      1064.95  32.63
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.40000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.60000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Correlation of Fixed Effects:
##          (Intr) housepv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

We test this new block compared to the model with just school level predictors as the classroom level predictors were not siginificant.

```
model4_red <- lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1|schoolid/classid),data=

anova(model2_red, model4_red, refit = F)
```

```
## Data: classroom_red
## Models:
## model2_red: Math1 ~ housepov + (1 | schoolid/classid)
## model4_red: Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
## model4_red:     ses + (1 | schoolid/classid)
##            Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model2_red  5 10838 10862 -5413.8    10828
## model4_red 11 10752 10806 -5364.8    10730 98.023      6  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(model3, model4, refit = F)
```

```
## Data: classroom
## Models:
## model3: Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
## model4: Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
## model4:     ses + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model3  8 10837 10877 -5410.5    10821
## model4 11 10752 10806 -5364.8    10730 91.446      3  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

σ^2_ϵ decreased to 1064.95, σ^2_η decreased to 93.89, σ^2_ζ decreased to 169.45

School level may drop because sstudents may be similar within schools but different between schools, or the fact that math know directly effects school level effects, better schools tend to have better teachers

$$\hat{math1st}_{ijk} = \beta_{0ijk}+\zeta_k+\eta_{jk}+\epsilon_{ijk}+\beta_1 Housepov_k+\beta_2 Mathknow_{jk}+\beta_3 YearsTea_{jk}+\beta_4 Mathprep_{jk}+\beta_5 sex_{ijk}+\beta_6 minority_{ijk}$$

The anova test comparing the model with school and classroom level predictors to the model with almost all the predictors has a p-value that is approximately zero at $<$ 2.2e-16, so we reject H_0 and conclude that it makes sense to include student level predictors. Moreover, the Chi-Sq test comparing the model with just school level predictors to the model with almost all predictors has a p-value $<$ 2.2e-16, so we conclude that the model with student level predictors (as a block) improves compared to the model with only school-level predictors both somewhat reiterating the other.

Random Slope for Teacher-level predictor varying at school-level

```
rst.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+mathknow||schoolid)+(1|class
summary(rst.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathknow || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.389e+01 9.690e+00
##  schoolid   mathknow    4.260e-11 6.527e-06
##  schoolid.1 (Intercept) 1.694e+02 1.302e+01
##  Residual               1.065e+03 3.263e+01
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.40000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.60000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rst.1,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
```

```
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + mathknow | schoolid) + (1 | classid)
##                                     npar  logLik   AIC    LRT Df
## <none>                                12 -5364.8 10754
## (1 | schoolid)                        11 -5376.5 10775 23.410  1
## mathknow in (0 + mathknow | schoolid)   11 -5364.8 10752  0.000  1
## (1 | classid)                         11 -5368.1 10758  6.741  1
##                                     Pr(>Chisq)
## <none>
## (1 | schoolid)                       1.309e-06 ***
## mathknow in (0 + mathknow | schoolid)   0.999999
## (1 | classid)                         0.009422 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is not a need for the random slope for math knowledge at a school level as the p value is not signifcant at .999

```
rst.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+yearstea||schoolid)+(1|class
summary(rst.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + yearstea || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8485 -0.6149 -0.0323  0.5980  3.6600
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.266e+01  9.62593
##  schoolid   yearstea    9.669e-03  0.09833
##  schoolid.1 (Intercept) 1.685e+02 12.97894
##  Residual               1.065e+03 32.63452
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.60060    5.30865  266.30000 101.645  < 2e-16 ***
## housepov     -17.71727   13.21854  113.60000  -1.340    0.183
## mathknow       1.33198    1.39177  234.30000   0.957    0.340
## yearstea       0.01124    0.14193  122.40000   0.079    0.937
## mathprep      -0.26633    1.37610  204.90000  -0.194    0.847
## sex           -1.21077    2.09476 1022.20000  -0.578    0.563
## minority     -16.16833    3.02641  702.60000  -5.342 1.24e-07 ***
## ses           10.04529    1.54490 1066.10000   6.502 1.21e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Correlation of Fixed Effects:
##          (Intr) housepv mthknw yearst mthprp sex    minrty
## housepov -0.450
## mathknow -0.082  0.057
## yearstea -0.258  0.070  0.028
## mathprep -0.632  0.037  0.003 -0.172
## sex      -0.190 -0.007  0.006  0.015 -0.006
## minority -0.320 -0.179  0.115  0.023  0.001 -0.010
## ses      -0.121  0.082 -0.007 -0.027  0.053  0.020  0.162
```

```
ranova(rst.2, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + yearstea | schoolid) + (1 | classid)
##                                       npar  logLik   AIC     LRT Df
## <none>                                  12 -5364.8 10754
## (1 | schoolid)                          11 -5374.7 10771 19.8301  1
## yearstea in (0 + yearstea | schoolid)   11 -5364.8 10752  0.0070  1
## (1 | classid)                           11 -5367.7 10757  5.9158  1
##                                       Pr(>Chisq)
## <none>
## (1 | schoolid)                         8.464e-06 ***
## yearstea in (0 + yearstea | schoolid)    0.93342
## (1 | classid)                           0.01501 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There seems to be no need for for the random slope for years teaching at a school level as the p value is insignificant at .933

```
rst.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+mathprep||schoolid)+(1|class
summary(rst.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathprep || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   93.89   9.69
##  schoolid   mathprep       0.00   0.00
##  schoolid.1 (Intercept)  169.45  13.02
##  Residual               1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
```

```
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21758  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.40000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.60000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rst.3, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + mathprep | schoolid) + (1 | classid)
##                                       npar  logLik   AIC     LRT Df
## <none>                                  12 -5364.8 10754
## (1 | schoolid)                          11 -5371.6 10765 13.6179  1
## mathprep in (0 + mathprep | schoolid)   11 -5364.8 10752  0.0000  1
## (1 | classid)                           11 -5368.3 10759  7.1357  1
##                                       Pr(>Chisq)
## <none>
## (1 | schoolid)                          0.000224 ***
## mathprep in (0 + mathprep | schoolid)   1.000000
## (1 | classid)                           0.007556 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There seems to be no need for for the random slope for math prep at a school level as the p value is insignificant at 1.00

Question: Why housepov bad idea?

Answer: There is only one data point per school, so we cannot have a random slope since we can't even calculate a slope.

Allowing correlations with random intercepts

ONE BY ONE

```
rstc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+mathknow|schoolid)+(1|classi
summary(rstc.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathknow | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6131 -0.0324  0.5969  3.6603
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept) 9.394e+01  9.69205
##  schoolid (Intercept) 1.693e+02 13.01223
##           mathknow    8.596e-04  0.02932 1.00
##  Residual             1.065e+03 32.63393
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.64037    5.31212  275.40000 101.587  < 2e-16 ***
## housepov     -17.64148   13.21274  104.00000  -1.335    0.185
## mathknow       1.35459    1.39203  214.60000   0.973    0.332
## yearstea       0.01114    0.14141  226.90000   0.079    0.937
## mathprep      -0.27753    1.37601  201.30000  -0.202    0.840
## sex           -1.21329    2.09485 1021.80000  -0.579    0.563
## minority     -16.19376    3.02609  703.80000  -5.351 1.18e-07 ***
## ses           10.04788    1.54488 1062.10000   6.504 1.20e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.082  0.057
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.173
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rstc.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
```

```
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathknow | schoolid) + (1 | classid)
##                                       npar  logLik   AIC    LRT Df
## <none>                                  13 -5364.8 10756
## mathknow in (1 + mathknow | schoolid)   11 -5364.8 10752 0.0003  2
## (1 | classid)                           12 -5368.1 10760 6.6768  1
##                                       Pr(>Chisq)
## <none>
## mathknow in (1 + mathknow | schoolid)   0.999840
## (1 | classid)                           0.009767 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated math knowledge is insignificant and seems to add no value to the model

yearstea

```
rstc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+yearstea|schoolid)+(1|class
summary(rstc.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + yearstea | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7462 -0.6036 -0.0290  0.6041  3.8449
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept)   37.9283  6.1586
##  schoolid (Intercept)  366.1148 19.1341
##           yearstea       0.5523  0.7432  -0.78
##  Residual             1066.4510 32.6566
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.95245    5.48825  222.70000  98.201  < 2e-16 ***
## housepov     -17.13994   13.45959  119.60000  -1.273    0.205
## mathknow       1.04635    1.34381  209.70000   0.779    0.437
## yearstea       0.02204    0.15766   75.80000   0.140    0.889
## mathprep       0.05046    1.34549  190.80000   0.038    0.970
## sex           -1.33553    2.08774 1024.50000  -0.640    0.523
## minority     -16.44555    2.99655  669.50000  -5.488 5.77e-08 ***
## ses           10.15038    1.53873 1062.70000   6.597 6.62e-11 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.455
## mathknow -0.085  0.049
## yearstea -0.370  0.084  0.012
## mathprep -0.606  0.050  0.014 -0.139
## sex      -0.184 -0.004  0.008  0.009 -0.004
## minority -0.305 -0.169  0.122  0.032 -0.007 -0.012
## ses      -0.119  0.079 -0.001 -0.019  0.049  0.022  0.168
```

```
ranova(rstc.2,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + yearstea | schoolid) + (1 | classid)
##                                       npar  logLik   AIC    LRT Df
## <none>                                  13 -5361.8 10750
## yearstea in (1 + yearstea | schoolid)   11 -5364.8 10752 5.8254  2
## (1 | classid)                           12 -5362.3 10749 0.9028  1
##                                       Pr(>Chisq)
## <none>
## yearstea in (1 + yearstea | schoolid)    0.05433 .
## (1 | classid)                            0.34202
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

This correlated random slope for years teaching is right on the cusp of signicance and should be observed further in attempts to understand its need for adding it to the model it has a p value of .0543

mathprep

```
rstc.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+mathprep|schoolid)+(1|class
summary(rstc.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathprep | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8542 -0.6034 -0.0221  0.5915  3.6475
##
## Random effects:
```

```
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   78.46   8.858
##  schoolid (Intercept)  552.76  23.511
##           mathprep      15.89   3.986   -1.00
##  Residual             1064.26  32.623
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.60855    5.60813  159.90000  96.041  < 2e-16 ***
## housepov     -14.01306   12.88689  116.10000  -1.087    0.279
## mathknow       1.29884    1.37194  229.70000   0.947    0.345
## yearstea      -0.02586    0.13949  223.50000  -0.185    0.853
## mathprep       0.04074    1.34845  139.00000   0.030    0.976
## sex           -1.16759    2.08697 1023.20000  -0.559    0.576
## minority     -16.46422    2.99524  663.70000  -5.497 5.52e-08 ***
## ses           10.14166    1.53961 1060.90000   6.587 7.04e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.461
## mathknow -0.071  0.027
## yearstea -0.260  0.089  0.049
## mathprep -0.692  0.107  0.012 -0.155
## sex      -0.183  0.003  0.002  0.023 -0.008
## minority -0.275 -0.187  0.107  0.025 -0.035 -0.013
## ses      -0.121  0.095 -0.001 -0.033  0.061  0.024  0.161
```

```
ranova(rstc.3, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathprep | schoolid) + (1 | classid)
##                                       npar  logLik   AIC    LRT Df
## <none>                                  13 -5362.3 10751
## mathprep in (1 + mathprep | schoolid)   11 -5364.8 10752 4.8144  2
## (1 | classid)                           12 -5364.9 10754 5.0971  1
##                                       Pr(>Chisq)
## <none>
## mathprep in (1 + mathprep | schoolid)    0.09007 .
## (1 | classid)                            0.02397 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated math prep is just a bit to high with a pvalue of .09 thus it is insignificant and seems to add no value to the model

Question: Anything unusual about the variances? Why might this have occurred? (hint: what did you add to the model?)

Answer: The random slope for mathknow greatly increases in the second model, which is probably due to its correlation with the random intercept at the school-level.

There seems to be an issue with the model as the slope and intercept correlation is negative one, this could be due to the sample sizes of the classrooms as some only have a single observation

B. note: Also – weird that correlation among slope and intercept is prefectly -1. This looks like model is not fitting properly – probably because some classrooms have very few observations so not all parameters are estimated.

Random slopes for student-level predictors varying at classroom level

ONE BY ONE

sex

```
rss.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+sex||classid)+(1|schoolid),da
summary(rss.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex || classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   93.89   9.69
##  classid.1 sex            0.00   0.00
##  schoolid  (Intercept)  169.45  13.02
##  Residual              1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.40000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.60000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
```

```
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rss.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | classid) + (0 + sex | classid) + (1 | schoolid)
##                             npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                        12 -5364.8 10754
## (1 | classid)                 11 -5368.0 10758  6.4894  1    0.01085 *
## sex in (0 + sex | classid)    11 -5364.8 10752  0.0000  1    1.00000
## (1 | schoolid)                11 -5377.1 10776 24.7881  1  6.399e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Sex random slope with class is insignificant with a p value of 1

minority

```
rss.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+minority||classid)+(1|schooli
summary(rss.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority || classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   93.89   9.69
##  classid.1 minority       0.00   0.00
##  schoolid  (Intercept)  169.45  13.02
##  Residual              1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
```

```
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21758  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.40000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.60000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rss.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | classid) + (0 + sex | classid) + (1 | schoolid)
##                             npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                        12 -5364.8 10754
## (1 | classid)                 11 -5368.0 10758  6.4894  1    0.01085 *
## sex in (0 + sex | classid)    11 -5364.8 10752  0.0000  1    1.00000
## (1 | schoolid)                11 -5377.1 10776 24.7881  1  6.399e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Sex random slope with class id is insignificant with a p value of 1.0

SES

```
rss.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+ses||classid)+(1|schoolid),da
summary(rss.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses || classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   87.11   9.333
##  classid.1 ses           49.60   7.043
##  schoolid  (Intercept)  171.02  13.077
##  Residual              1043.44  32.302
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641  274.50000 101.710  < 2e-16 ***
## housepov     -17.50879   13.21775  113.40000  -1.325    0.188
## mathknow       1.36796    1.38563  229.40000   0.987    0.325
## yearstea       0.01103    0.14117  227.00000   0.078    0.938
## mathprep      -0.27938    1.37171  204.90000  -0.204    0.839
## sex           -1.37733    2.09334 1022.80000  -0.658    0.511
## minority     -16.29362    3.02464  703.30000  -5.387 9.78e-08 ***
## ses           10.14363    1.64248  176.40000   6.176 4.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.082  0.058
## yearstea -0.259  0.070  0.029
## mathprep -0.631  0.040  0.005 -0.172
## sex      -0.190 -0.007  0.006  0.014 -0.005
## minority -0.321 -0.180  0.111  0.025  0.002 -0.011
## ses      -0.108  0.081  0.002 -0.026  0.050  0.020  0.145
```

```
ranova(rss.3, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | classid) + (0 + ses | classid) + (1 | schoolid)
##                             npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                        12 -5364.0 10752
## (1 | classid)                 11 -5366.9 10756  5.9221  1    0.01495 *
## ses in (0 + ses | classid)    11 -5364.8 10752  1.5969  1    0.20634
## (1 | schoolid)                11 -5376.6 10775 25.2710  1  4.982e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

With a p-value of .206 ses is insignificant from an uncorrelated random slope at classroom level

Question: why is this a bad idea to include a classroom-level variable with random slopes at classroom-level?

Answer: It may not explain much variance due to the fact that it seems somewhat redundant.

Allowing for correlations with random intercepts

ONE BY ONE

sex

```
rssc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+sex|classid)+(1|schoolid),da
summary(rssc.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7565 -0.6134 -0.0307  0.5916  3.7116
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  130.07  11.41
##           sex           31.36   5.60    -0.67
##  schoolid (Intercept)  169.85  13.03
##  Residual              1056.41  32.50
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.400e+02  5.332e+00  2.723e+02 101.285  < 2e-16 ***
## housepov    -1.829e+01  1.323e+01  1.145e+02  -1.382    0.170
## mathknow     1.306e+00  1.391e+00  2.315e+02   0.939    0.349
## yearstea     3.087e-03  1.416e-01  2.270e+02   0.022    0.983
## mathprep    -3.459e-01  1.374e+00  2.014e+02  -0.252    0.801
## sex         -1.197e+00  2.122e+00  2.160e+02  -0.564    0.573
## minority    -1.619e+01  3.028e+00  7.042e+02  -5.347 1.21e-07 ***
## ses          1.010e+01  1.544e+00  1.065e+03   6.539 9.62e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.452
## mathknow -0.085  0.060
## yearstea -0.258  0.072  0.029
## mathprep -0.628  0.040  0.005 -0.174
## sex      -0.203 -0.005  0.003  0.015 -0.008
## minority -0.321 -0.178  0.116  0.024  0.003 -0.009
## ses      -0.123  0.083 -0.005 -0.027  0.054  0.020  0.164
```

```
ranova(rssc.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | classid) + (1 | schoolid)
##                             npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                        13 -5364.5 10755
## sex in (1 + sex | classid)    11 -5364.8 10752  0.5003  2     0.7787
## (1 | schoolid)                12 -5377.0 10778 24.8912  1  6.066e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated random slope is insignificant with a p value of .779

minority

```
rssc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+minority|classid)+(1|schooli
summary(rssc.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10726.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9037 -0.6221 -0.0295  0.6033  3.4574
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  225.4   15.01
##           minority     171.3   13.09    -0.82
##  schoolid (Intercept)  157.4   12.55
##  Residual             1045.3   32.33
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.73594    5.38023  270.70000 100.318  < 2e-16 ***
## housepov     -17.34698   12.91268  103.30000  -1.343    0.182
## mathknow       1.45702    1.39355  234.00000   1.046    0.297
## yearstea      -0.01636    0.14285  234.30000  -0.115    0.909
## mathprep      -0.13520    1.37018  204.00000  -0.099    0.921
## sex           -1.01012    2.08966 1015.70000  -0.483    0.629
## minority     -16.48614    3.21756  183.20000  -5.124 7.55e-07 ***
## ses            9.89350    1.54595 1062.80000   6.400 2.33e-10 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.435
## mathknow -0.079  0.061
## yearstea -0.265  0.080  0.038
## mathprep -0.618  0.037 -0.006 -0.171
## sex      -0.188 -0.009  0.009  0.015 -0.005
## minority -0.368 -0.171  0.108  0.025 -0.004 -0.009
## ses      -0.117  0.085  0.001 -0.023  0.051  0.021  0.149
```

```
ranova(rssc.2)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | classid) + (1 | schoolid)
##                                     npar  logLik   AIC     LRT Df
## <none>                                13 -5363.2 10752
## minority in (1 + minority | classid)   11 -5364.8 10752  3.1967  2
## (1 | schoolid)                         12 -5373.2 10770 20.1422  1
##                                     Pr(>Chisq)
## <none>
## minority in (1 + minority | classid)     0.2022
## (1 | schoolid)                        7.189e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated random slope for minority is insignificant with a p value of .202

SES

```
rssc.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+ses|classid)+(1|schoolid),da
summary(rssc.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10725.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5688 -0.6004 -0.0316  0.5959  3.6176
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
```

```
##  classid  (Intercept)   86.06   9.277
##           ses           44.09   6.640   0.75
##  schoolid (Intercept)  173.16  13.159
##  Residual             1048.32  32.378
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.52093    5.26665  269.60000 102.441  < 2e-16 ***
## housepov     -16.28994   13.13445  111.30000  -1.240    0.217
## mathknow       1.37996    1.37294  222.40000   1.005    0.316
## yearstea       0.01605    0.14080  227.60000   0.114    0.909
## mathprep      -0.37734    1.34603  182.80000  -0.280    0.780
## sex           -1.32178    2.08794 1017.10000  -0.633    0.527
## minority     -16.09272    3.03497  717.70000  -5.302 1.52e-07 ***
## ses           10.05535    1.64507  171.10000   6.112 6.44e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.450
## mathknow -0.078  0.059
## yearstea -0.266  0.074  0.030
## mathprep -0.625  0.036 -0.001 -0.165
## sex      -0.186 -0.009  0.007  0.013 -0.009
## minority -0.325 -0.181  0.108  0.021  0.004 -0.014
## ses      -0.084  0.078  0.015 -0.024  0.056  0.022  0.142
```

```
ranova(rssc.3)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | classid) + (1 | schoolid)
##                             npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                        13 -5362.8 10752
## ses in (1 + ses | classid)   11 -5364.8 10752  3.8395  2     0.1466
## (1 | schoolid)               12 -5375.8 10776 26.0221  1  3.375e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated random slope for ses is insignificant with a pvalue of .147

Random slopes for student-level predictors varying at school level

ONE BY ONE

sex

```
rss.4 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+sex||schoolid)+(1|classid),da
summary(rss.4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8578 -0.6110 -0.0259  0.5922  3.5557
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   96.08   9.802
##  schoolid   sex           35.83   5.986
##  schoolid.1 (Intercept)  161.63  12.713
##  Residual               1054.36  32.471
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.43517    5.30740  272.50000 101.638  < 2e-16 ***
## housepov     -16.77661   13.22881  112.40000  -1.268    0.207
## mathknow       1.40067    1.39464  234.50000   1.004    0.316
## yearstea       0.01448    0.14163  226.40000   0.102    0.919
## mathprep      -0.27193    1.38010  205.80000  -0.197    0.844
## sex           -1.33534    2.18746  138.10000  -0.610    0.543
## minority     -16.16536    3.02861  704.30000  -5.338 1.27e-07 ***
## ses            9.98477    1.54243 1058.30000   6.473 1.46e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.081  0.055
## yearstea -0.259  0.070  0.028
## mathprep -0.633  0.036  0.004 -0.172
## sex      -0.179 -0.010  0.007  0.013 -0.004
## minority -0.320 -0.178  0.114  0.024  0.001 -0.015
## ses      -0.120  0.081 -0.007 -0.029  0.052  0.020  0.161
```

```
ranova(rss.4, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + sex | schoolid) + (1 | classid)
```

```
##                               npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                          12 -5364.4 10753
## (1 | schoolid)                  11 -5374.4 10771 19.9994  1  7.747e-06 ***
## sex in (0 + sex | schoolid)     11 -5364.8 10752  0.6137  1   0.433392
## (1 | classid)                   11 -5368.2 10758  7.4171  1   0.006461 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated ses random slope at a school level is insignifcant with a p value of .433

minority

```
rss.5 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+minority||schoolid)+(1|class
summary(rss.5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   93.89   9.69
##  schoolid   minority       0.00   0.00
##  schoolid.1 (Intercept)  169.45  13.02
##  Residual               1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21758  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.40000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.60000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
```

```
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rss.5,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + minority | schoolid) + (1 | classid)
##                                       npar  logLik   AIC     LRT Df
## <none>                                  12 -5364.8 10754
## (1 | schoolid)                          11 -5375.2 10772 20.8586  1
## minority in (0 + minority | schoolid)   11 -5364.8 10752  0.0000  1
## (1 | classid)                           11 -5368.3 10759  7.1357  1
##                                       Pr(>Chisq)
## <none>
## (1 | schoolid)                         4.945e-06 ***
## minority in (0 + minority | schoolid)   1.000000
## (1 | classid)                           0.007556 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

insignificant with a pvalue of 1.0

SES

```
rss.6 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+ses||schoolid)+(1|classid),da
summary(rss.6) #IS SIG
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   88.56   9.411
##  schoolid   ses           72.50   8.515
##  schoolid.1 (Intercept)  167.98  12.961
##  Residual               1035.12  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
```

```
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917  270.50000 102.126  < 2e-16 ***
## housepov     -16.94564   13.21116  112.80000  -1.283    0.202
## mathknow       1.35576    1.38459  232.20000   0.979    0.329
## yearstea       0.03079    0.14052  223.90000   0.219    0.827
## mathprep      -0.19801    1.35994  198.60000  -0.146    0.884
## sex           -1.40185    2.08170 1011.30000  -0.673    0.501
## minority     -16.52525    3.02189  700.10000  -5.469 6.32e-08 ***
## ses            9.78982    1.82217   79.00000   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.079  0.056
## yearstea -0.260  0.070  0.028
## mathprep -0.628  0.041  0.002 -0.172
## sex      -0.190 -0.007  0.006  0.018 -0.007
## minority -0.323 -0.180  0.110  0.024  0.001 -0.010
## ses      -0.091  0.076  0.006 -0.019  0.042  0.017  0.124
```

```
ranova(rss.6,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + ses | schoolid) + (1 | classid)
##                             npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                        12 -5362.4 10749
## (1 | schoolid)                11 -5374.6 10771 24.2924  1  8.276e-07 ***
## ses in (0 + ses | schoolid)   11 -5364.8 10752  4.6972  1    0.03021 *
## (1 | classid)                 11 -5365.7 10753  6.5177  1    0.01068 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

is signifcant with a p value of .03

Allowing for correlations with random intercepts

ONE BY ONE

sex

```
rssc.4 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+sex|schoolid)+(1|classid),da
summary(rssc.4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
```

```
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8048 -0.6095 -0.0222  0.5969  3.5525
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   97.34   9.866
##  schoolid (Intercept)  206.33  14.364
##           sex           84.08   9.170   -0.43
##  Residual             1041.76  32.276
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.399e+02  5.363e+00  2.626e+02 100.661  < 2e-16 ***
## housepov    -1.742e+01  1.325e+01  1.136e+02  -1.314    0.191
## mathknow     1.379e+00  1.396e+00  2.364e+02   0.988    0.324
## yearstea     6.876e-03  1.418e-01  2.277e+02   0.048    0.961
## mathprep    -2.796e-01  1.378e+00  2.061e+02  -0.203    0.839
## sex         -1.340e+00  2.301e+00  8.740e+01  -0.582    0.562
## minority    -1.642e+01  3.027e+00  7.076e+02  -5.425 7.96e-08 ***
## ses          9.928e+00  1.540e+00  1.055e+03   6.448 1.72e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.082  0.060
## yearstea -0.258  0.072  0.027
## mathprep -0.627  0.038  0.004 -0.172
## sex      -0.222 -0.003  0.006  0.014 -0.005
## minority -0.319 -0.178  0.114  0.024  0.004 -0.011
## ses      -0.121  0.083 -0.006 -0.028  0.053  0.018  0.163
```

```
ranova(rssc.4, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | schoolid) + (1 | classid)
##                              npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                         13 -5363.8 10754
## sex in (1 + sex | schoolid)   11 -5364.8 10752 1.8631  2   0.393952
## (1 | classid)                 12 -5367.6 10759 7.6414  1   0.005704 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

insignificant with a pvalue of .394

minority

```
rssc.5 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+minority|schoolid)+(1|classi
summary(rssc.5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.69   9.311
##  schoolid (Intercept)  381.20  19.524
##           minority     343.13  18.524   -0.83
##  Residual             1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  1.000e+02  -1.277    0.204
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.820e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.394
## mathknow -0.078  0.061
## yearstea -0.253  0.091  0.024
## mathprep -0.576  0.037 -0.002 -0.167
## sex      -0.172 -0.013  0.010  0.014 -0.005
## minority -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses      -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

```
ranova(rssc.5,refit=F) #sig
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | schoolid) + (1 | classid)
##                                      npar  logLik   AIC    LRT Df
## <none>                                 13 -5358.8 10744
## minority in (1 + minority | schoolid)   11 -5364.8 10752 11.967  2
## (1 | classid)                          12 -5361.8 10748  6.077  1
##                                      Pr(>Chisq)
## <none>
## minority in (1 + minority | schoolid)    0.00252 **
## (1 | classid)                            0.01370 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

significant with a pvalue of .0025

SES

```
rssc.6 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+ses|schoolid)+(1|classid),da
summary(rssc.6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5646 -0.6166 -0.0264  0.5888  3.7073
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.57   9.305
##  schoolid (Intercept)  171.18  13.083
##           ses           73.37   8.565   0.19
##  Residual             1035.90  32.185
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.72222    5.27647  271.10000 102.099  < 2e-16 ***
## housepov     -15.89873   13.15393  111.70000  -1.209    0.229
## mathknow       1.26025    1.38201  230.90000   0.912    0.363
## yearstea       0.03617    0.14002  220.40000   0.258    0.796
## mathprep      -0.21697    1.35642  197.10000  -0.160    0.873
## sex           -1.40436    2.08074 1011.40000  -0.675    0.500
## minority     -16.26699    3.03580  668.90000  -5.358 1.16e-07 ***
```

```
## ses            9.72646    1.82985   78.40000   5.315 9.75e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.077  0.057
## yearstea -0.259  0.073  0.028
## mathprep -0.627  0.039  0.001 -0.172
## sex      -0.188 -0.009  0.005  0.017 -0.008
## minority -0.325 -0.182  0.108  0.021  0.002 -0.011
## ses      -0.062  0.070  0.007 -0.021  0.045  0.018  0.117
```

```
ranova(rssc.6,refit=F) #not sig
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | schoolid) + (1 | classid)
##                              npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                         13 -5362.2 10750
## ses in (1 + ses | schoolid)    11 -5364.8 10752 5.1385  2    0.07659 .
## (1 | classid)                  12 -5365.3 10755 6.2117  1    0.01269 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Very close to significance but not quite there with a pvalue of .0766

Question: Report unusual changes in variance.

Answer: Perhaps most striking is the change in variance for the random slope term on minority. Previously, it was 0. However, it jumps to 343.13 in the correlated model. The variance for the random slope term on SES also increases, but the correlated random slope is not a significant addition to our model according to the rand test results.

Complex model

Take two predictors that had sig random slopes and add to model, test for need of one conditional on the other

minority is sig for correlated

ses is sig for uncorrelated

```
complex <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(0+ses|schoolid)+(1+minority|s
summary(complex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
```

```
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##  Groups     Name        Variance Std.Dev. Corr
##  classid    (Intercept)   80.63   8.979
##  schoolid   (Intercept)  404.54  20.113
##             minority     336.04  18.332   -0.84
##  schoolid.1 ses           74.93   8.656
##  Residual                1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.70000  95.160  < 2e-16 ***
## housepov     -15.32111   12.49443   99.30000  -1.226    0.223
## mathknow       1.67475    1.35000  221.30000   1.241    0.216
## yearstea       0.02102    0.13657  213.70000   0.154    0.878
## mathprep      -0.23546    1.31730  191.20000  -0.179    0.858
## sex           -1.03871    2.06951 1010.40000  -0.502    0.616
## minority     -16.72884    3.90720   55.40000  -4.282 7.43e-05 ***
## ses            9.19654    1.82272   82.50000   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

```
ranova(complex, refit=F)
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -1.2e-04
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##                                       npar  logLik   AIC     LRT Df
## <none>                                  14 -5356.2 10740
```

```
## ses in (0 + ses | schoolid)             14 -5358.8 10746  5.1200  0
## minority in (1 + minority | schoolid)   12 -5362.4 10749 12.3899  2
## (1 | classid)                           13 -5358.9 10744  5.3724  1
##                                      Pr(>Chisq)
## <none>
## ses in (0 + ses | schoolid)
## minority in (1 + minority | schoolid)    0.00204 **
## (1 | classid)                            0.02046 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Question: Is the more complex model (with both random slopes in it) justified?

Answer: The complex model is justified since the rand test shows that the random slopes are both statistically significant at the 0.05 level, the only question revolves around statistical significance justifying compared to the Bayesian approach that would push for a simpler model.

The equation for the complex model is given by the following:

$$\hat{Math1st}_{ijk} = \beta_0 + \beta_1 housepov_k + \beta_2 mathknow_{jk} + \beta_3 yearstea_{jk} + \beta_4 mathprep_{jk} + \beta_5 * sex_{ijk} + \beta_{6k} * ses_{ijk} + \beta_{7k} * minority_{ijk} + \zeta_0$$

where $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0}), \zeta_{6k} \sim N(0, \sigma^2_{\zeta_6}), \zeta_{7k} \sim N(0, \sigma^2_{\zeta_7}), \eta_{jk} \sim N(0, \sigma^2_{\eta})$, and $\epsilon_{ijk} \sim N(0, \sigma^2_{\epsilon})$, all independent of each other.

```
summary(model1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.46   9.244
##  schoolid         (Intercept)  280.68  16.754
##  Residual                      1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.410   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

V_C, V_S, and V_E **Question:** For UCM, write down: V_C, V_S, V_E for the three variance components (simply the estimates). Think of them as possibly varying with a covariate, though.

Answer: For the UCM, $V_C = 85.46$, $V_S = 280.68$, and $V_E = 1146.80$

```r
summary(model4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                      1064.95  32.63
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.40000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.60000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

Question: For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are: V_C, V_S, V_E?

Answer: For the most complicated fixed effects model with only random intercepts, $V_C = 93.89$, $V_S = 169.45$, and $V_E = 1064.95$.

Question: By what fraction did these each decrease with the new predictors in the model?

Answer: V_C increased $\frac{93.89}{85.46}$ V_S decreased $\frac{169.45}{280.68}$ V_E decreased $\frac{1064.95}{1146.80}$

```
summary(rss.6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   88.56   9.411
##  schoolid   ses           72.50   8.515
##  schoolid.1 (Intercept)  167.98  12.961
##  Residual               1035.12  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917  270.50000 102.126  < 2e-16 ***
## housepov     -16.94564   13.21116  112.80000  -1.283    0.202
## mathknow       1.35576    1.38459  232.20000   0.979    0.329
## yearstea       0.03079    0.14052  223.90000   0.219    0.827
## mathprep      -0.19801    1.35994  198.60000  -0.146    0.884
## sex           -1.40185    2.08170 1011.30000  -0.673    0.501
## minority     -16.52525    3.02189  700.10000  -5.469 6.32e-08 ***
## ses            9.78982    1.82217   79.00000   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.079  0.056
## yearstea -0.260  0.070  0.028
## mathprep -0.628  0.041  0.002 -0.172
## sex      -0.190 -0.007  0.006  0.018 -0.007
## minority -0.323 -0.180  0.110  0.024  0.001 -0.010
## ses      -0.091  0.076  0.006 -0.019  0.042  0.017  0.124
```

Question: Now consider the model with a random slope in ses. What are: V_C, $V_S(ses = 0)$, V_E ? We need to list 'ses=0' here, or we don't know how to use the slope variance

Answer: For the model with a random slope in ses at the school level, $V_C = 88.56$, $V_S(ses = 0) = 167.98$, and $V_E = 1035.12$.

Question: What are: $V_S(ses = -0.50)$, $V_S(ses = +0.5)$?

Answer: In this model, in which the random slope for SES is uncorrelated with the random school-level intercept, $V_S(ses = -0.50) = 167.98 + (-.5)^2 72.50 + 2(-.5)0167.9872.50 = 186.105$, and $V_S(ses = +0.5) =$

$167.98 + (.5)^2 72.50 + 2 * (.5) 0167.98 * 72.50 = 186.105$

```
summary(rssc.5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.69   9.311
##  schoolid (Intercept)  381.20  19.524
##           minority     343.13  18.524   -0.83
##  Residual             1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  1.000e+02  -1.277    0.204
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.820e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.394
## mathknow -0.078  0.061
## yearstea -0.253  0.091  0.024
## mathprep -0.576  0.037 -0.002 -0.167
## sex      -0.172 -0.013  0.010  0.014 -0.005
## minority -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses      -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

Question: Now consider the model with a random slope in minority. What are: V_C, $V_S(minority = 0)$, V_E? We need to list 'minority=0' here, or we don't know how to use the slope variance

Answer: For the model with a random slope in minority at the school level, $V_C = 86.69$, $V_S(minority = 0) = 381.20$, and $V_E = 1039.39$.

Question: What are: $V_S(minority = 0.25)$, $V_S(minority = +0.50)$, $V_S(minority = +0.75)$?

Answer: In this model, in which the random slope for minority is correlated with the random school-level,

intercept, $V_S(minority = 0.25) = 381.20 + (0.25)^2 343.13 + 2(0.25)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 252.5549$,
$V_S(minority = +0.50) = 381.20 + (0.50)^2 343.13 + 2(0.50)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 166.801$, and
$V_S(minority = +0.75) = 381.20 + (0.25)^2 343.13 + 2(0.25)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 123.9384$.

```
summary(complex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##  Groups     Name        Variance Std.Dev. Corr
##  classid    (Intercept)   80.63   8.979
##  schoolid   (Intercept)  404.54  20.113
##             minority     336.04  18.332   -0.84
##  schoolid.1 ses           74.93   8.656
##  Residual                1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.70000  95.160  < 2e-16 ***
## housepov     -15.32111   12.49443   99.30000  -1.226    0.223
## mathknow       1.67475    1.35000  221.30000   1.241    0.216
## yearstea       0.02102    0.13657  213.70000   0.154    0.878
## mathprep      -0.23546    1.31730  191.20000  -0.179    0.858
## sex           -1.03871    2.06951 1010.40000  -0.502    0.616
## minority     -16.72884    3.90720   55.40000  -4.282 7.43e-05 ***
## ses            9.19654    1.82272   82.50000   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

Question: Now consider the model with a random slope in ses & minority. What are: V_C, $V_S(minority = 0, ses = 0)$, V_E? We need to list 'ses=0, minority=0' here, or we don't know how to use the slope variance.

Answer: For the model with a random slope in ses & minority, $V_C = 80.63$, $V_S(minority = 0, ses = 0) = 404.54$, and $V_E = 1009.73$.

Question: What are: $V_S(ses = 0, minority = 0.50)$, $V_S(ses = 0.50, minority = 0)$, $V_S(ses = 0.50, minority = 0.50)$?

Answer: In this model, in which the random slope for ses is uncorrelated with the random intercept, but the random slope for minority is correlated with the random intercept,

$V_S(ses = 0, minority = 0.50) = 404.54 + (0)^2 74.93 + (0.50)^2 336.04 + 200404.5474.93 + 2 * (0.50)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 182.5268$,

$V_S(ses = 0.50, minority = 0) = 404.54 + (0.50)^2 74.93 + (0)^2 336.04 + 20.500404.5474.93 + 2 * (0)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 423.2725$

$V_S(ses = 0.50, minority = 0.50) = 404.54 + (0.50)^2 74.93 + (0.50)^2 336.04 + 20.500404.5474.93 + 2 * (0.50)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 201.2593$

```
summary(complex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##  Groups     Name        Variance Std.Dev. Corr
##  classid    (Intercept)   80.63   8.979
##  schoolid   (Intercept)  404.54  20.113
##             minority     336.04  18.332   -0.84
##  schoolid.1 ses           74.93   8.656
##  Residual                1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.70000  95.160  < 2e-16 ***
## housepov     -15.32111   12.49443   99.30000  -1.226    0.223
## mathknow       1.67475    1.35000  221.30000   1.241    0.216
## yearstea       0.02102    0.13657  213.70000   0.154    0.878
## mathprep      -0.23546    1.31730  191.20000  -0.179    0.858
## sex           -1.03871    2.06951 1010.40000  -0.502    0.616
## minority     -16.72884    3.90720   55.40000  -4.282 7.43e-05 ***
## ses            9.19654    1.82272   82.50000   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
```

```
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

Question: In the last model, what is a "likely" (+/- 1 sd) range for η_{0jk}

Answer: For the complex model, the "likely" range for $\eta 0jk$ is 71.651 to 89.609.

Question: Can we make a similar statement about ζ_{0k}?

Answer: Mathmatically we can with a range of 384.427 to 424.653 though we can do this it doesn't make much sense due to the correlated nature of this with the minority variable the values wouldn't hold much meaning and are easily mis interpretted

Question: If you had a large value for η_{0jk}, would you expect a large or small or "any" value for: the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority?

Answer: If you have a very large η_{0jk} you would expect a small value for ζ_{1k} and ζ_{2k} but the ζ_{2k} would not be as small due to its negative correlation with our ζ_{0k} which is effected by our eta value

Question: If you had a large value for ζ_{0k}, would you expect a large or small or "any" value for: the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority (discuss each separately)?

Answer: For ζ_{1k} would increase in the same direction but it could be any value due to the lack of correlation, keeping in mind that ζ_{0k} will create a cieling effect of sorts for ζ_{1k} . While ζ_{2k} would be very small because of the correlation because of the two variables are negatively correlated