# Part3-B

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## Part 3: Bianca

### Create person-period file

In this part of the project, no variables of interested have missing observation. Therefore, the full dataset is used.

```
#new variables
classroom2 <- classroom2 %>% mutate(math0 = mathkind) %>% mutate(math1 = mathkind+mathgain)
#reshape the data
class_pp <- reshape(classroom2, varying = c("math0", "math1"), v.names = "math", timevar = "year",
times = c(0, 1), direction = "long")
```

Note: we ignore classroom in this analysis but keep it in the notation.

### Initial longitudinal model

We fit a model with math as outcome, and fixed effect for time trend (year), as well as random intercept for school.

The equation for the model below:

$$Math_{tijk} = b_0 + \zeta_{0k} + b_1 * Time_{tijk} + \epsilon_{tijk}$$

where $\zeta_{0k} \sim N(0, \sigma^2_{zeta0})$ and $\epsilon_{tijk} \sim N(0, \sigma^2_{\epsilon})$

We refer to this as Model 0.

Below the model fit:

```
fit1 <- lmer(math ~ year + (1|schoolid), data = class_pp)
summary(fit1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 | schoolid)
##    Data: class_pp
##
## REML criterion at convergence: 23951.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.2833 -0.6084  0.0037  0.6329  3.7761
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  schoolid (Intercept)  348.7   18.67
##  Residual             1268.4   35.62
```

```
## Number of obs: 2380, groups:  schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)    
## (Intercept)  464.932      2.116  132.154  219.73   <2e-16 ***
## year          57.566      1.460 2270.855   39.43   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.345
```

## Add child-level random intercept

To the previous model, we now add random intercepts for child:

$$Math_{tijk} = b_0 + \delta_{0ijk} + \zeta_{0k} + b_1 * Time_{tijk} + \epsilon_{tijk}$$

where $\delta_{0tijk} \sim N(0, \sigma^2_{\delta_0}), \zeta_{0k} \sim N(0, \sigma^2_{zeta0})$ and $\epsilon_{tijk} \sim N(0, \sigma^2_{\epsilon})$ independetly of one another.

We refer to this as M1.

```r
fit2 <- lmer(math ~ year + (1|schoolid/childid), data = class_pp)
summary(fit2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 | schoolid/childid)
##    Data: class_pp
##
## REML criterion at convergence: 23554.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max 
## -4.7492 -0.4811  0.0085  0.4881  3.4957 
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  childid:schoolid (Intercept) 702.0    26.50   
##  schoolid         (Intercept) 307.5    17.54   
##  Residual                     599.1    24.48   
## Number of obs: 2380, groups:  childid:schoolid, 1190; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)    
## (Intercept)  465.118      2.042  117.023  227.74   <2e-16 ***
## year          57.566      1.003 1189.000   57.37   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.246
```

In model 0 the variance $\sigma^2_{\zeta_0} = 348.7$ and in model 1 $\sigma^2_{\zeta_0} = 307.5$.
In model 0 , the varaince for $\sigma^2_{\epsilon} = 1268.4$ and in model 1 $\sigma^2_{\epsilon_0} = 599.1$. We note that including child-level variation leads to a decrease in the variance of both the random effects.

## Compute Pseudo-R^2

Compute a pseudo R^2 relating the between school variation and ignoring between students in the same school.

We calculate this as :

$$\frac{\sigma^2_{\zeta_0}(M_0) - \sigma^2_{\zeta_0}(M_1)}{\sigma^2_{\zeta_0}(M_0)} = \frac{348.7 - 307.5}{348.7} = 0.12$$

The between-school variance is reduced by 12% ( or 'explained') with the introduction of student random effect.

**Does the total variation stay about the same?**

```
tot_m0 = 348.7 + 1268.4
tot_m1 = 702 + 307.5 + 599.1
paste("Tot variance for model 0 : ", tot_m0)
```

```
## [1] "Tot variance for model 0 :  1617.1"
```

```
paste("Tot variance for model 1: ", tot_m1)
```

```
## [1] "Tot variance for model 1:  1608.6"
```

There is only a slightly decrease in the total variance between Model 0 and Model1.

## Add a random slope for time trend

We now add a random slope ($\zeta_1$) for time trend within schools.

$$Math_{tijk} = b_0 + \delta_{0ijk} + \zeta_{0k} + (b_1 + \zeta_{1k}) * Time_{tijk} + \epsilon_{tijk}$$

where $\delta_{0tijk} \sim N(0, \sigma^2_{\delta_{0ijk}})$, $\quad \zeta_{0k} \sim N(0, \sigma^2_{\zeta_0})$, $\quad \zeta_{1k} \sim N(0, \sigma^2_{\zeta_0})$ and $\epsilon_{tijk} \sim N(0, \sigma^2_{\epsilon})$ – each independetly of one another.

We refer to this as Model 2

We run the model and report the fit:

```
fit3 = lmer(math ~ year + (1 + year|| schoolid) + (1|childid), data = class_pp)
summary(fit3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 + year || schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 23529.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
```

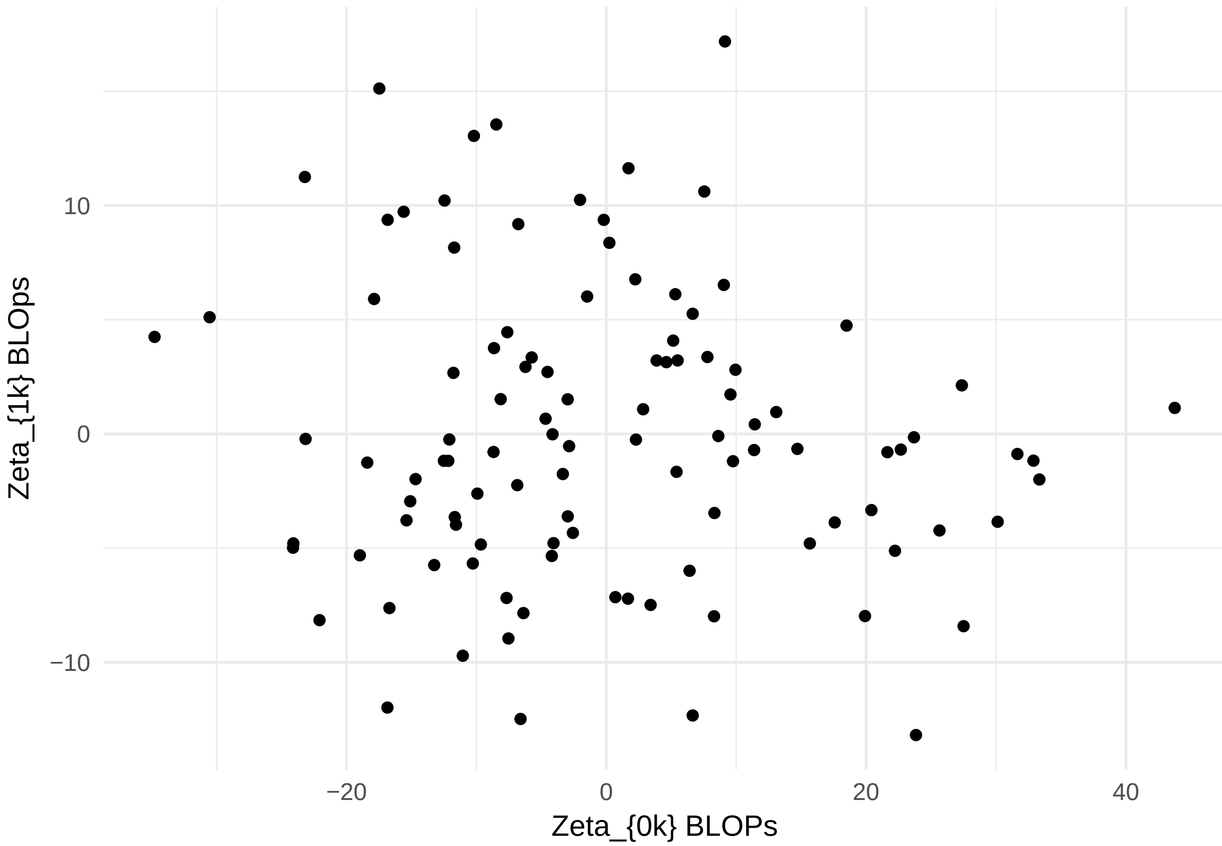
```
## -4.7665 -0.4721  0.0139  0.4686  3.6080
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  childid    (Intercept) 725.13   26.928
##  schoolid   year         88.67    9.417
##  schoolid.1 (Intercept) 324.79   18.022
##  Residual               552.21   23.499
## Number of obs: 2380, groups:  childid, 1190; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.087      2.081 109.954  223.44   <2e-16 ***
## year          57.499      1.370  99.917   41.97   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.178
```

### Generate the BLUPs for this model (Model 2)

Examine then whether the independence between zeta0 and zeta1 is reflected in a scatterplot of these two sets of effects.

```
pp_ranefs <- ranef(fit3)

if (vanillaR) {
plot(pp_ranefs$schoolid[,2],pp_ranefs$schoolid[,1])
}else{
ggplot(pp_ranefs$schoolid, aes(x = pp_ranefs$schoolid[,2], y = pp_ranefs$schoolid[,1] )) +
geom_point() + labs(x = "Zeta_{0k} BLOPs", y = "Zeta_{1k} BLOps") + theme_minimal()
}
```




From the plot, the BLOPs for $\zeta_{0k}$ and for $\zeta_{1k}$ appear uncorrelated, reflecting the way in which the model was built. In the BLUPS, we have a correlation of:

```
cor(pp_ranefs$schoolid[,2],pp_ranefs$schoolid[,1])
```

```
## [1] -0.1118599
```

```
corrtestp = cor.test(pp_ranefs$schoolid[,2],pp_ranefs$schoolid[,1],
                     method = "pearson")$p.value

paste("P-value for pearson test for correlatio:", round(corrtestp,3))
```

```
## [1] "P-value for pearson test for correlatio: 0.251"
```

That is, between the slope random effects and the ranom intercept blups, there is a very small negative correlation, which is not significantly different from 0 – which we see from the plot and would expect from how we have specified the model.

### Heteroscedasticity in the random effects

**Question:** What are: $V_S(year = 0)$, $V_S(year = 1)$?

The model we are considering is :

$Math_{tijk} = b_0 + \delta_{0tijk} + \zeta_{0k} + (b_1 + \zeta_{1k})Time_{tijk} + \epsilon_{tijk}$

So we have that (in this model, in which we are forcing correlation of 0 between slope and intercept):

- $V_S(year = 0) = \sigma^2_{\zeta_{0k}} = 324.79$
- $V_S(year = 1) = \sigma^2_{\zeta_{0k}} + \sigma^2_{\zeta_{1k}} = 88.67 + 324.79 = 413.46$

## Run model separately by year

We now examine what happens if we run the model separately by year. Do we get the same estimates for the variance between schools?

```
class_year0 = class_pp[class_pp$year == 0,]


# Run model for year 0
fit4 = lmer(math ~ (1 | schoolid), data = class_year0)
summary(fit4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ (1 | schoolid)
##    Data: class_year0
##
## REML criterion at convergence: 12085.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.8223 -0.5749  0.0005  0.6454  3.6237
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  schoolid (Intercept)  364.3   19.09
##  Residual             1344.5   36.67
## Number of obs: 1190, groups:  schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error     df t value Pr(>|t|)
## (Intercept)   465.23       2.19 103.20   212.4   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Run modelfor year 1
class_year1 = class_pp[class_pp$year == 1,]
fit5 = lmer(math ~ (1 | schoolid), data = class_year1)
summary(fit5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ (1 | schoolid)
##    Data: class_year1
##
## REML criterion at convergence: 11950.8
##
## Scaled residuals:
##    Min     1Q Median     3Q    Max
## -5.291 -0.612 -0.005  0.613  3.793
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  schoolid (Intercept)  306.8   17.52
##  Residual             1205.0   34.71
```

```
## Number of obs: 1190, groups:  schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.698      2.027 103.069   257.8   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

In this case, for the Year 0 Model, we get an estimated $\hat{\sigma}^2_{\zeta_{0k}} = 364.3$, while for Year 1 Model, we have $\hat{\sigma}^2_{\zeta_{0k}} = 306.8$. We note that these estimates are different from the ones computed above.

## Allow for correlation

We now allow for correlation between the random effects for the intercept and the slope. We call this Model 3.

```
fit6 = lmer(math ~ year + (1 + year| schoolid) + (1|childid), data = class_pp)
summary(fit6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 + year | schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 23520.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7030 -0.4686  0.0066  0.4669  3.5142
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  childid  (Intercept) 728.0    26.98
##  schoolid (Intercept) 370.6    19.25
##           year        109.1    10.44    -0.45
##  Residual             547.0    23.39
## Number of obs: 2380, groups:  childid, 1190; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.099      2.188 102.919  212.60   <2e-16 ***
## year          57.668      1.440  94.575   40.04   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.439
```

Correlation beteween $\zeta_0$ and $\zeta_1$ = -0.45.

To test whether the correlation is statistically significant, we can compare Model 2 with Model 3 using an anova test.

```
anova(fit3,fit6, refit = F)
```

```
## Data: class_pp
```

```
## Models:
## fit3: math ~ year + (1 + year || schoolid) + (1 | childid)
## fit6: math ~ year + (1 + year | schoolid) + (1 | childid)
##      Df   AIC   BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
## fit3  6 23541 23576 -11764    23529
## fit6  7 23534 23575 -11760    23520 8.8241      1   0.002973 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

With a p-value $p = 0.003$, we reject the null hypothesis at $\alpha = 0.05$ significance level, and conclude that there correlation term is statistically significant.

So we have that (in this model where we are allowing for correlation between slope and intercept):

- $V_S(year = 0) = \sigma^2_{\zeta_{0k}} = 370.6$
- $V_S(year = 1) = \sigma^2_{\zeta_{0k}} + \sigma^2_{\zeta_{1k}} + 2\rho_{01}\sigma_{\zeta_{0k}}\sigma_{\zeta_{1k}} = 370.6 + 109.1 - 2 * 0.45 * \sqrt{370.6}\sqrt{109.1} = 298.72$

These estimates are a lot closer to the school variances that result from fitting the models for the two years separately ( in which we have $\sigma^2_\zeta$ respectively be 364.3 for year 0 and 306.8 for year 1.

Therefore, it seems that the model that allows for correlation between the two random effects has a better fit than the one forcing that correlation to be 0.