

# Multi-Level Models: Final Project

*Bianca Brusco, Clare Clingain, Kaushik Mohan, & Frankie Wunschel*

All four parts of the Group Project are compiled in this PDF. Here is how the group split up the work. Bookmark tabs have been created in the PDF for ease of finding each member's section.

-Part 1: Frankie

-Part 2: Clare

-Part 3: Bianca

-Part 4: Kaushik

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## Part 1: Frankie

### Create 1st grade variable

```
classroom <- classroom %>% mutate(Math1 = mathkind + mathgain)
```

### Random Intercepts for classroom, nested in schools UMM

We begin our analysis by looking at the UMM with random intercepts for schools and classrooms, i.e. :

$$Math1st_{ijk} = \beta_{0ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

where  $i$  represents students,  $j$  represents classrooms and  $k$  represents schools.  $\zeta_k \sim N(0, \sigma_\zeta^2)$ ,  $\eta_{jk} \sim N(0, \sigma_\eta^2)$ , and  $\epsilon_{ijk} \sim N(0, \sigma_\epsilon^2)$ , all are independent of each other

```
model1 <- lmer(Math1 ~ (1|schoolid/classid), data=classroom)
summary(model1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##   Groups                Name             Variance Std.Dev.
## classid:schoolid (Intercept)    85.46      9.244
## schoolid         (Intercept)  280.68     16.754
## Residual                        1146.80    33.864
## Number of obs: 1190, groups: classid:schoolid, 312; schoolid, 107
```

```
##
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.406   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$ICC_{class} = \frac{85.46}{1146.8 + 280.68 + 85.46} \approx .056$$

$$ICC_{school} = \frac{280.68}{1146.8 + 280.68 + 85.46} \approx .186$$

We hence find, from the fit summary above, that the equation for our model is:

$$Math1st_{ijk} = 522.54 + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$\zeta_k \sim N(0, 280.68)$ ,  $\eta_{jk} \sim N(0, 85.46)$ , and  $\epsilon_{ijk} \sim N(0, 1146.80)$ , all are independent of each other

## Model with School Level Predictors Added

We then add all the school level predictors (that is, “housepov”) and report below the model fit :

```
model2 <- lmer(Math1~housepov+(1|schoolid/classid),data=classroom)
summary(model2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ housepov + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
## Groups           Name              Variance Std.Dev.
## classid:schoolid (Intercept)    82.36     9.075
## schoolid         (Intercept)   250.93    15.841
## Residual                        1146.95    33.867
## Number of obs: 1190, groups: classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.294      3.341 102.809  159.024   <2e-16 ***
## housepov     -45.783     14.236 111.063   -3.216    0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr)
## housepov -0.810
```

```
anova(model1, model2, refit = F)
```

```
## Data: classroom
## Models:
## model1: Math1 ~ (1 | schoolid/classid)
## model2: Math1 ~ housepov + (1 | schoolid/classid)
##           Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## model1    4 11953 11973 -5972.3   11945
## model2    5 11937 11963 -5963.7   11927 17.186     1 3.39e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Report the changes in the variances of the random effects:

Change in  $\sigma_\zeta^2$ : decreased to 250.93 from 280.63  $\sigma_\eta^2$  decreases to 82.36 from 85.46  $\sigma_\epsilon^2$  slightly increases to 1146.95 from 1146.8

The LRT has a p-value of almost zero,  $p = 3.39e - 05$ , thus we reject the  $H_0$ : coefficient on Housepov = 0 at  $\alpha = 0.05$ . That is, we find evidence that it makes sense to include the school level predictor, housepov.

## Model with all Class Level Predictors Added

We now re-run the model after including all the classroom level predictors, that is “mathknow”, “yearstea”, “mathprep”, and report the model fit.

```
model3 <- lmer(Math1~housepov+mathknow+yearstea+mathprep+
               (1|schoolid/classid),data=classroom)
summary(model3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
## Groups           Name             Variance Std.Dev.
## classid:schoolid (Intercept)    94.36    9.714
## schoolid         (Intercept)   223.31   14.943
## Residual                        1136.43   33.711
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept) 532.29853    5.20496 228.85764 102.268 < 2e-16 ***
## housepov    -41.62116   14.08835 109.83227  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.06566   1.765  0.07883 .
## yearstea      0.06193    0.14717 223.76582   0.421  0.67432
## mathprep     -0.75440    1.42809 203.20767  -0.528  0.59790
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst
## houspv -0.568
## mathknow -0.052  0.082
## yearstea -0.264  0.077  0.030
## mathprep -0.666  0.032  0.004 -0.175
```

## creating reduced dataset taking away missing data

The variable of interest *Mathknow* includes some missing values. The model for which we have reported the summary above therefore removes the observations for which missing data is present.

To be able to compare Model 2 (with school level predictors) with Model 3 (with both school level and classroom level predictors), we removed from the dataset students that had missing values, creating a reduced dataset. This left us with a sample of 1081 students. We then re-run model 2 on this reduced dataset and compared it to Model 3.

```
classroom_red = na.omit(classroom)
model2_red <- lmer(Math1~housepv+(1|schoolid/classid),data=classroom_red)
model3_red <- lmer(Math1~housepv+mathknow+yearstea+mathprep+
                    (1|schoolid/classid),data=classroom_red)

summary(model3_red)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepv + mathknow + yearstea + mathprep + (1 | schoolid/classid)
## Data: classroom_red
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
## Groups          Name          Variance Std.Dev.
## classid:schoolid (Intercept)    94.36   9.714
## schoolid        (Intercept)   223.31  14.943
## Residual                        1136.43  33.711
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error    df t value Pr(>|t|)
## (Intercept)  532.29853    5.20496 228.85764 102.268 < 2e-16 ***
## housepv      -41.62116   14.08835 109.83227  -2.954  0.00383 **
## mathknow       2.55143    1.44530 231.06566   1.765  0.07883 .
## yearstea       0.06193    0.14717 223.76582   0.421  0.67432
## mathprep      -0.75440    1.42809 203.20767  -0.528  0.59790
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst
## housepv -0.568
## mathknow -0.052  0.082
## yearstea -0.264  0.077  0.030
## mathprep -0.666  0.032  0.004 -0.175
anova(model2_red, model3_red, refit = F)

## Data: classroom_red
## Models:
## model2_red: Math1 ~ housepv + (1 | schoolid/classid)
## model3_red: Math1 ~ housepv + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##      Df    AIC    BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## model2_red  5 10838 10862 -5413.8    10828
## model3_red  8 10837 10877 -5410.5    10821 6.5771      3    0.08667 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Change in  $\sigma_\epsilon^2$  and  $\sigma_\eta^2$ :  $\sigma_\epsilon^2$  decreased to 1136.43,  $\sigma_\eta^2$  increased to 94.36;  $\sigma_\zeta^2 = 223.31$

A possible reason why  $\epsilon$  decreased in this model, but not  $\eta$  is that adding the classroom level predictors makes it so that more of the overall variation is explained by “structured” variation (that is, related to the fact that students are in different classrooms) rather than by unstructured ( $\epsilon$ ), so that the latter decreases. However, we also have to note that in this case we are using the reduced dataset, so that some of the changes may be due to the fact that we are using two slightly different datasets.

The anova test comparing the school level predictor to the model with the classroom predictors has a p-value 0.087, so we fail to reject the null hypothesis at our  $\alpha = 0.05$  and conclude that adding classroom level predictors is not necessary, as it does not significantly improve the model.

## Add all student-level predictors

We now include all the student level predictors in our model:

```
model4 <- lmer(Math1~housepv+mathknow+yearstea+mathprep+sex+minority+
               ses+(1|schoolid/classid),data=classroom)
summary(model4)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid/classid)
##      Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##      Groups                Name             Variance Std.Dev.
```

```

## classid:schoolid (Intercept) 93.89 9.69
## schoolid (Intercept) 169.45 13.02
## Residual 1064.95 32.63
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 539.63042 5.31210 275.38922 101.585 < 2e-16 ***
## housepov -17.64847 13.21757 113.87774 -1.335 0.184
## mathknow 1.35004 1.39168 234.49776 0.970 0.333
## yearstea 0.01129 0.14141 226.80899 0.080 0.936
## mathprep -0.27705 1.37583 205.27157 -0.201 0.841
## sex -1.21419 2.09483 1022.42136 -0.580 0.562
## minority -16.18678 3.02605 704.47889 -5.349 1.20e-07 ***
## ses 10.05075 1.54484 1066.56223 6.506 1.18e-10 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) houspv mthknw yearst mthprp sex minrty
## housepov -0.451
## mathknow -0.083 0.058
## yearstea -0.259 0.071 0.029
## mathprep -0.631 0.038 0.004 -0.172
## sex -0.190 -0.007 0.007 0.016 -0.006
## minority -0.320 -0.178 0.115 0.024 0.001 -0.011
## ses -0.121 0.082 -0.007 -0.028 0.053 0.020 0.162

```

We test this new block compared to the model with both school-level and classroom level predictors.

```
anova(model3, model4, refit = F)
```

```

## Data: classroom
## Models:
## model3: Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
## model4: Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
## model4: ses + (1 | schoolid/classid)
## Df AIC BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## model3 8 10837 10877 -5410.5 10821
## model4 11 10752 10806 -5364.8 10730 91.446 3 < 2.2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

The LRT test between this two models has a p-value  $< 2.2 \times 10^{-16}$ . Therefore, at our  $\alpha = 0.05$ , we reject the null hypothesis and conclude that adding this block of predictors is justified.

Changes in variance components :

$\sigma_\epsilon^2$  decreased to 1064.95,  $\sigma_\eta^2$  decreased to 93.89, and  $\sigma_\zeta^2$  decreased to 169.45.

We note that adding student-level predictors leads to a decrease in the overall variance of the model. By “controlling” for student-related variables, we also explain the between schools, as students with similar attributes might be similar across schools, hence reducing the overall variance of  $\zeta$ .

The final model, with all school level, classroom level, and student level predictors, is:

$$Math1st_{ijk} = 539.63 + \zeta_k + \eta_{jk} + \epsilon_{ijk} - 17.65 * Housepov_k + 1.35 * Mathknow_{jk} +$$

$$0.01 * YearsTea_{jk} - 0.27 * Mathprep_{jk} - 0.19 * sex_{ijk} - 0.32 * minority_{ijk} - 0.12 * ses_{ijk}$$

With:

$\zeta_k \sim N(0, \sigma_\zeta^2)$ ,  $\eta_{jk} \sim N(0, \sigma_\eta^2)$ , and  $\epsilon_{ijk} \sim N(0, \sigma_\epsilon^2)$ , all are independent of each other

From the model fit above therefore we find that the fitted model is:

$$Math1st_{ijk} = \beta_{0ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk} + \beta_1 Housepov_k + \beta_2 Mathknow_{jk} + \beta_3 YearsTea_{jk} + \beta_4 Mathprep_{jk} + \beta_5 sex_{ijk} + \beta_6 minority_{ijk} + \beta_7 ses_{ijk}$$

With:

$\zeta_k \sim N(0, 169.45)$ ,  $\eta_{jk} \sim N(0, 93.89)$ , and  $\epsilon_{ijk} \sim N(0, 1064.95)$ , all are independent of each other.

## Random Slope for Teacher-level predictor varying at school-level

We try adding a random slope for each teacher level predictor (varying at the school level; one by one - not all together).

### MATHKNOW

```
rst.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
            ses+(1+mathknow||schoolid)+(1|classid),data=classroom)
summary(rst.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + mathknow || schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups      Name                Variance Std.Dev.
## classid     (Intercept) 9.389e+01 9.690e+00
## schoolid    mathknow    4.260e-11 6.527e-06
## schoolid.1 (Intercept) 1.694e+02 1.302e+01
## Residual                    1.065e+03 3.263e+01
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38921 101.585 < 2e-16 ***
## housepov     -17.64847   13.21757  113.87774  -1.335  0.184
## mathknow       1.35004    1.39168  234.49776   0.970  0.333
## yearstea       0.01129    0.14141  226.80899   0.080  0.936
## mathprep      -0.27705    1.37583  205.27156  -0.201  0.841
## sex           -1.21419    2.09483 1022.42136  -0.580  0.562
```

```
## minority      -16.18678      3.02605  704.47889  -5.349 1.20e-07 ***
## ses           10.05075      1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow  -0.083  0.058
## yearstea  -0.259  0.071  0.029
## mathprep  -0.631  0.038  0.004 -0.172
## sex       -0.190 -0.007  0.007  0.016 -0.006
## minority  -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses       -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
ranova(rst.1,refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid) + (0 + mathknow | schoolid) + (1 | classid)
##              npar logLik  AIC    LRT Df
## <none>              12 -5364.8 10754
## (1 | schoolid)        11 -5376.5 10775 23.410  1
## mathknow in (0 + mathknow | schoolid)  11 -5364.8 10752  0.000  1
## (1 | classid)         11 -5368.1 10758  6.741  1
##              Pr(>Chisq)
## <none>
## (1 | schoolid)        1.309e-06 ***
## mathknow in (0 + mathknow | schoolid)  0.999999
## (1 | classid)         0.009422 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for *MATHKNOW* at a school level as the p value = 1 for the Chi-square test is not significant at  $\alpha = 0.05$ .

## YEARSTEA

```
rst.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
      ses+(1+yearstea||schoolid)+(1|classid),data=classroom)
summary(rst.2)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + yearstea || schoolid) + (1 | classid)
##      Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min      1Q  Median      3Q      Max
## -3.8485 -0.6149 -0.0323  0.5980  3.6600
```



```
##
## Random effects:
## Groups      Name      Variance Std.Dev.
## classid     (Intercept) 9.266e+01 9.62593
## schoolid    yearstea    9.669e-03 0.09833
## schoolid.1  (Intercept) 1.685e+02 12.97894
## Residual                    1.065e+03 32.63452
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.60060    5.30865   266.34157 101.645 < 2e-16 ***
## housepov     -17.71727   13.21854   113.56407  -1.340   0.183
## mathknow      1.33198    1.39177   234.33551   0.957   0.340
## yearstea      0.01124    0.14193   122.38000   0.079   0.937
## mathprep     -0.26633    1.37610   204.91605  -0.194   0.847
## sex          -1.21077    2.09476  1022.22247  -0.578   0.563
## minority     -16.16833    3.02641   702.64837  -5.342 1.24e-07 ***
## ses          10.04529    1.54490  1066.09768   6.502 1.21e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.450
## mathknow     -0.082  0.057
## yearstea     -0.258  0.070  0.028
## mathprep     -0.632  0.037  0.003 -0.172
## sex          -0.190 -0.007  0.006  0.015 -0.006
## minority     -0.320 -0.179  0.115  0.023  0.001 -0.010
## ses          -0.121  0.082 -0.007 -0.027  0.053  0.020  0.162

ranova(rst.2, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid) + (0 + yearstea | schoolid) + (1 | classid)
##              npar logLik  AIC    LRT Df
## <none>              12 -5364.8 10754
## (1 | schoolid)        11 -5374.7 10771 19.8301 1
## yearstea in (0 + yearstea | schoolid)  11 -5364.8 10752 0.0070 1
## (1 | classid)         11 -5367.7 10757 5.9158 1
##              Pr(>Chisq)
## <none>
## (1 | schoolid)          8.464e-06 ***
## yearstea in (0 + yearstea | schoolid)  0.93342
## (1 | classid)          0.01501 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for *YEARSTEA* at a school level as the p value = 0.93 for the Chi-square test is not significant at  $\alpha = 0.05$ .

## Mathprep

```
rst.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
ses+(1+mathprep||schoolid)+(1|classid),data=classroom)
summary(rst.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + mathprep || schoolid) + (1 | classid)
##      Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##      Groups      Name      Variance Std.Dev.
##      classid    (Intercept)   93.89   9.69
##      schoolid   mathprep       0.00   0.00
##      schoolid.1 (Intercept) 169.45 13.02
##      Residual                1064.95 32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38917 101.585 < 2e-16 ***
## housepov     -17.64847   13.21758  113.87771  -1.335  0.184
## mathknow       1.35004    1.39168  234.49776   0.970  0.333
## yearstea       0.01129    0.14141  226.80899   0.080  0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201  0.841
## sex           -1.21419    2.09483 1022.42137  -0.580  0.562
## minority     -16.18678    3.02605  704.47892  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.451
## mathknow    -0.083  0.058
## yearstea    -0.259  0.071  0.029
## mathprep    -0.631  0.038  0.004 -0.172
## sex         -0.190 -0.007  0.007  0.016 -0.006
## minority    -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses         -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rst.3, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
```

```
##      ses + (1 | schoolid) + (0 + mathprep | schoolid) + (1 | classid)
##                                npar logLik  AIC    LRT Df
## <none>                        12 -5364.8 10754
## (1 | schoolid)                 11 -5371.6 10765 13.6179  1
## mathprep in (0 + mathprep | schoolid) 11 -5364.8 10752 0.0000  1
## (1 | classid)                  11 -5368.3 10759  7.1357  1
##                                Pr(>Chisq)
## <none>
## (1 | schoolid)                  0.000224 ***
## mathprep in (0 + mathprep | schoolid) 1.000000
## (1 | classid)                  0.007556 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for *MATHPREP* at a school level as the p value = 1 for the Chi-square test is not significant at  $\alpha = 0.05$ .

**Question:** Why is a random slope on housepov a bad idea?

**Answer:** There is only one data point per school, so we do not have enough information to calculate the slope for each school.

## Allowing correlations with random intercepts

### ONE BY ONE

Again, we add random slopes for each teacher-level predictor varying at the school level, but this time by allowing them to be correlated with the random intercepts.

#### MATHKNOW

```
rstc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
              ses+(1+mathknow|schoolid)+(1|classid),data=classroom)
summary(rstc.1)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + mathknow | schoolid) + (1 | classid)
##      Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8581 -0.6131 -0.0324  0.5969  3.6603
##
## Random effects:
##  Groups   Name                Variance Std.Dev. Corr
##  classid  (Intercept)  9.394e+01  9.69205
##  schoolid (Intercept) 1.693e+02 13.01223
##          mathknow     8.596e-04  0.02932 1.00
## Residual                1.065e+03 32.63393
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
```

```
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.64037    5.31212  275.37948 101.587 < 2e-16 ***
## housepov    -17.64148   13.21274  103.97679  -1.335  0.185
## mathknow     1.35459    1.39203  214.63820   0.973  0.332
## yearstea     0.01114    0.14141  226.85277   0.079  0.937
## mathprep    -0.27753    1.37601  201.27912  -0.202  0.840
## sex         -1.21329    2.09485 1021.79964  -0.579  0.563
## minority    -16.19376    3.02609  703.81038  -5.351 1.18e-07 ***
## ses         10.04788    1.54488 1062.12341   6.504 1.20e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.082  0.057
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.173
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162

ranova(rstc.1, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + mathknow | schoolid) + (1 | classid)
##
##               npar logLik   AIC   LRT Df
## <none>              13 -5364.8 10756
## mathknow in (1 + mathknow | schoolid)  11 -5364.8 10752 0.0003  2
## (1 | classid)              12 -5368.1 10760 6.6768  1
##
##               Pr(>Chisq)
## <none>
## mathknow in (1 + mathknow | schoolid)  0.999840
## (1 | classid)              0.009767 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for math knowledge at a school level as the p value = 1.00 for the Chi-square test is not significant at  $\alpha = 0.05$ .

## YEARSTEA

```
rstc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
              ses+(1+yearstea|schoolid)+(1|classid),data=classroom)
summary(rstc.2)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + yearstea | schoolid) + (1 | classid)
##      Data: classroom
```

```

##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.7462 -0.6036 -0.0290  0.6041  3.8449
##
## Random effects:
##      Groups   Name      Variance Std.Dev. Corr
##      classid  (Intercept)  37.9283  6.1586
##      schoolid (Intercept) 366.1148 19.1341
##      yearstea          0.5523  0.7432 -0.78
##      Residual          1066.4510 32.6566
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  538.95245    5.48825  222.69673  98.201 < 2e-16 ***
## housepov     -17.13994   13.45959  119.63687  -1.273  0.205
## mathknow       1.04635    1.34381  209.72527   0.779  0.437
## yearstea       0.02204    0.15766   75.76696   0.140  0.889
## mathprep       0.05046    1.34549  190.82671   0.038  0.970
## sex           -1.33553    2.08774 1024.45936  -0.640  0.523
## minority      -16.44555    2.99655  669.50401  -5.488 5.77e-08 ***
## ses            10.15038    1.53873 1062.66131   6.597 6.62e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.455
## mathknow     -0.085  0.049
## yearstea     -0.370  0.084  0.012
## mathprep     -0.606  0.050  0.014 -0.139
## sex          -0.184 -0.004  0.008  0.009 -0.004
## minority     -0.305 -0.169  0.122  0.032 -0.007 -0.012
## ses          -0.119  0.079 -0.001 -0.019  0.049  0.022  0.168

```

```

ranova(rstc.2,refit=F)

```

```

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + yearstea | schoolid) + (1 | classid)
##              npar logLik   AIC    LRT Df
## <none>              13 -5361.8 10750
## yearstea in (1 + yearstea | schoolid)  11 -5364.8 10752 5.8254  2
## (1 | classid)              12 -5362.3 10749 0.9028  1
##              Pr(>Chisq)
## <none>
## yearstea in (1 + yearstea | schoolid)    0.05433 .
## (1 | classid)              0.34202
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

There is no need for the random slope for yearstea at a school level as the p value = 0.054 for the Chi-square test is not significant at  $\alpha = 0.05$ .

## MATHPREP

```
rstc.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
              ses+(1+mathprep|schoolid)+(1|classid),data=classroom)
summary(rstc.3)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##       ses + (1 + mathprep | schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10724.7
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8542 -0.6034 -0.0221  0.5915  3.6475
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## classid (Intercept) 78.46 8.858
## schoolid (Intercept) 552.76 23.511
## mathprep 15.89 3.986 -1.00
## Residual 1064.26 32.623
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 538.60855 5.60813 159.88774 96.041 < 2e-16 ***
## housepov -14.01306 12.88689 116.05900 -1.087 0.279
## mathknow 1.29884 1.37194 229.68146 0.947 0.345
## yearstea -0.02586 0.13949 223.50098 -0.185 0.853
## mathprep 0.04074 1.34845 139.04228 0.030 0.976
## sex -1.16759 2.08697 1023.15084 -0.559 0.576
## minority -16.46422 2.99524 663.67316 -5.497 5.52e-08 ***
## ses 10.14166 1.53961 1060.93421 6.587 7.04e-11 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) houspv mthknw yearst mthprp sex minrty
## housepov -0.461
## mathknow -0.071 0.027
## yearstea -0.260 0.089 0.049
## mathprep -0.692 0.107 0.012 -0.155
## sex -0.183 0.003 0.002 0.023 -0.008
## minority -0.275 -0.187 0.107 0.025 -0.035 -0.013
## ses -0.121 0.095 -0.001 -0.033 0.061 0.024 0.161
ranova(rstc.3, refit=F)

## ANOVA-like table for random-effects: Single term deletions
```

```
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + mathprep | schoolid) + (1 | classid)
##
##               npar logLik   AIC   LRT Df
## <none>                13 -5362.3 10751
## mathprep in (1 + mathprep | schoolid)  11 -5364.8 10752 4.8144  2
## (1 | classid)                12 -5364.9 10754 5.0971  1
##
##               Pr(>Chisq)
## <none>
## mathprep in (1 + mathprep | schoolid)  0.09007 .
## (1 | classid)                0.02397 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for mathprep at a school level as the p value = 0.09 for the Chi-square test is not significant at  $\alpha = 0.05$ .

**Question:** Anything unusual about the variances? Why might this have occurred? (hint: what did you add to the model?)

**Answer:** We note that the model did not estimate the correlation parameter correctly for the models with random slopes for mathknown and mathprepr. Indeed, with a correlation of respectively 1 and -1 with the random intercept, the parameter is a linear function of the variance component for the slope. This could be due to the fact that there is not enough classrooms in the schools (as we are adding random effects at the school levels, for classroom level predictors), so that there is not enough degrees of freedom, nor enough variation among the variables of interest, to calculate all the parameters required in the model. Obtaining a correlation of 1 and -1 should warn us of the fact that the models generated should not be trusted.

Why is the correlation between random intercept and slope then calculated for yearstea? This could be due to the fact that this variable has a larger range, so that it can be more robustly estimated for some of the schools and the correlation between random slope and intercept then estimated more accurately even for schools with few classes.

## Random slopes for student-level predictors varying at classroom level

We now repeat the exercise by adding student level predictors, varying at the classroom level.

### ONE BY ONE

#### SEX

```
rss.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
            ses+(1+sex||classid)+(1|schoolid),data=classroom)
summary(rss.1)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + sex || classid) + (1 | schoolid)
##      Data: classroom
##
## REML criterion at convergence: 10729.5
##
```

```
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##      Groups      Name      Variance Std.Dev.
## classid  (Intercept)   93.89   9.69
## classid.1 sex           0.00   0.00
## schoolid (Intercept) 169.45  13.02
## Residual              1064.95  32.63
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210   275.38920 101.585 < 2e-16 ***
## housepov     -17.64847   13.21757   113.87773  -1.335  0.184
## mathknow       1.35004    1.39168   234.49776   0.970  0.333
## yearstea       0.01129    0.14141   226.80899   0.080  0.936
## mathprep      -0.27705    1.37583   205.27157  -0.201  0.841
## sex           -1.21419    2.09483  1022.42137  -0.580  0.562
## minority     -16.18678    3.02605   704.47890  -5.349 1.20e-07 ***
## ses           10.05075    1.54484  1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.451
## mathknow     -0.083  0.058
## yearstea     -0.259  0.071  0.029
## mathprep     -0.631  0.038  0.004 -0.172
## sex          -0.190 -0.007  0.007  0.016 -0.006
## minority     -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses          -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162

ranova(rss.1, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | classid) + (0 + sex | classid) + (1 | schoolid)
##              npar logLik   AIC      LRT Df Pr(>Chisq)
## <none>              12 -5364.8 10754
## (1 | classid)         11 -5368.0 10758  6.4894  1    0.01085 *
## sex in (0 + sex | classid) 11 -5364.8 10752  0.0000  1    1.00000
## (1 | schoolid)        11 -5377.1 10776 24.7881  1   6.399e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for sex at the classroom level, as the p value = 1 for the Chi-square test is not significant at  $\alpha = 0.05$ .

## MINORITY



```

rss.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
            ses+(1+minority||classid)+(1|schoolid),data=classroom)
summary(rss.2)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##       ses + (1 + minority || classid) + (1 | schoolid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups      Name                Variance Std.Dev.
## classid     (Intercept)         93.89    9.69
## classid.1 minority              0.00    0.00
## schoolid    (Intercept)        169.45   13.02
## Residual                    1064.95   32.63
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210   275.38919 101.585 < 2e-16 ***
## housepov     -17.64847   13.21758   113.87772  -1.335  0.184
## mathknow       1.35004    1.39168   234.49776   0.970  0.333
## yearstea       0.01129    0.14141   226.80899   0.080  0.936
## mathprep      -0.27705    1.37583   205.27157  -0.201  0.841
## sex           -1.21419    2.09483  1022.42137  -0.580  0.562
## minority     -16.18678    3.02605   704.47891  -5.349 1.20e-07 ***
## ses           10.05075    1.54484  1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.451
## mathknow    -0.083  0.058
## yearstea    -0.259  0.071  0.029
## mathprep    -0.631  0.038  0.004 -0.172
## sex         -0.190 -0.007  0.007  0.016 -0.006
## minority    -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses         -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162

ranova(rss.2, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +

```

```
##      ses + (1 | classid) + (0 + minority | classid) + (1 | schoolid)
##                                npar logLik  AIC    LRT Df
## <none>                        12 -5364.8 10754
## (1 | classid)                  11 -5367.3 10757  5.1497  1
## minority in (0 + minority | classid) 11 -5364.8 10752  0.0000  1
## (1 | schoolid)                 11 -5377.1 10776 24.7881  1
##                                Pr(>Chisq)
## <none>
## (1 | classid)                   0.02325 *
## minority in (0 + minority | classid) 1.00000
## (1 | schoolid)                  6.399e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for minority at the classroom level, as the p value = 1 for the Chi-square test is not significant at  $\alpha = 0.05$ .

## SES

```
rss.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
              ses+(1+ses||classid)+(1|schoolid),data=classroom)
summary(rss.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + ses || classid) + (1 | schoolid)
##      Data: classroom
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##      Groups      Name      Variance Std.Dev.
## classid  (Intercept)    87.11   9.333
## classid.1 ses         49.60   7.043
## schoolid (Intercept)  171.02  13.077
## Residual                1043.44  32.302
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641  274.46487 101.710 < 2e-16 ***
## housepov     -17.50879   13.21775  113.44869  -1.325  0.188
## mathknow       1.36796    1.38563  229.40646   0.987  0.325
## yearstea       0.01103    0.14117  226.97687   0.078  0.938
## mathprep      -0.27938    1.37171  204.89340  -0.204  0.839
## sex           -1.37733    2.09334 1022.81818  -0.658  0.511
## minority     -16.29362    3.02464  703.33762  -5.387 9.78e-08 ***
## ses           10.14363    1.64248  176.39739   6.176 4.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.082  0.058
## yearstea -0.259  0.070  0.029
## mathprep -0.631  0.040  0.005 -0.172
## sex      -0.190 -0.007  0.006  0.014 -0.005
## minority -0.321 -0.180  0.111  0.025  0.002 -0.011
## ses      -0.108  0.081  0.002 -0.026  0.050  0.020  0.145

ranova(rss.3, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | classid) + (0 + ses | classid) + (1 | schoolid)
##               npar logLik   AIC      LRT Df Pr(>Chisq)
## <none>                12 -5364.0 10752
## (1 | classid)          11 -5366.9 10756  5.9221  1    0.01495 *
## ses in (0 + ses | classid) 11 -5364.8 10752  1.5969  1    0.20634
## (1 | schoolid)         11 -5376.6 10775 25.2710  1  4.982e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for ses at the classroom level, as the p value = 0.206 for the Chi-square test is not significant at  $\alpha = 0.05$ .

**Question:** why is this a bad idea to include a classroom-level variable with random slopes at classroom-level?

**Answer:** Because all of the observations for a class will be the same, so we will not be able to compute the classroom slopes for each classroom (as we will only have one point).

## Allowing for correlations with random intercepts

### ONE BY ONE

#### SEX

```
rssc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
              ses+(1+sex|classid)+(1|schoolid),data=classroom)
summary(rssc.1)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + sex | classid) + (1 | schoolid)
## Data: classroom
##
## REML criterion at convergence: 10729
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.7565 -0.6134 -0.0307  0.5916  3.7116
```

```
##
## Random effects:
##   Groups   Name      Variance Std.Dev. Corr
##   classid (Intercept) 130.07  11.41
##           sex          31.36   5.60   -0.67
##   schoolid (Intercept) 169.85  13.03
##   Residual          1056.41  32.50
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  5.400e+02  5.332e+00  2.723e+02 101.285 < 2e-16 ***
## housepov     -1.829e+01  1.323e+01  1.145e+02  -1.382   0.170
## mathknow      1.306e+00  1.391e+00  2.315e+02   0.939   0.349
## yearstea      3.087e-03  1.416e-01  2.270e+02   0.022   0.983
## mathprep     -3.459e-01  1.374e+00  2.014e+02  -0.252   0.801
## sex          -1.197e+00  2.122e+00  2.160e+02  -0.564   0.573
## minority     -1.619e+01  3.028e+00  7.042e+02  -5.347 1.21e-07 ***
## ses           1.010e+01  1.544e+00  1.065e+03   6.539 9.62e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.452
## mathknow -0.085  0.060
## yearstea -0.258  0.072  0.029
## mathprep -0.628  0.040  0.005 -0.174
## sex      -0.203 -0.005  0.003  0.015 -0.008
## minority -0.321 -0.178  0.116  0.024  0.003 -0.009
## ses      -0.123  0.083 -0.005 -0.027  0.054  0.020  0.164

ranova(rssc.1, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + sex | classid) + (1 | schoolid)
##               npar logLik   AIC      LRT Df Pr(>Chisq)
## <none>                13 -5364.5 10755
## sex in (1 + sex | classid)  11 -5364.8 10752  0.5003  2    0.7787
## (1 | schoolid)            12 -5377.0 10778 24.8912  1  6.066e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for sex at the classroom level, as the p value = 0.779 for the Chi-square test is not significant at  $\alpha = 0.05$ .

## MINORITY

```
rssc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
              ses+(1+minority|classid)+(1|schoolid),data=classroom)
summary(rssc.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
```

```

## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority | classid) + (1 | schoolid)
## Data: classroom
##
## REML criterion at convergence: 10726.3
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.9037 -0.6221 -0.0295  0.6033  3.4574
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## classid (Intercept) 225.4 15.01
##      minority 171.3 13.09 -0.82
## schoolid (Intercept) 157.4 12.55
## Residual 1045.3 32.33
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 539.73594 5.38023 270.70509 100.318 < 2e-16 ***
## housepov -17.34698 12.91268 103.34670 -1.343 0.182
## mathknow 1.45702 1.39355 234.04713 1.046 0.297
## yearstea -0.01636 0.14285 234.25121 -0.115 0.909
## mathprep -0.13520 1.37018 203.97000 -0.099 0.921
## sex -1.01012 2.08966 1015.73461 -0.483 0.629
## minority -16.48614 3.21756 183.20472 -5.124 7.55e-07 ***
## ses 9.89350 1.54595 1062.82882 6.400 2.33e-10 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) houspv mthknw yearst mthprp sex minrty
## housepov -0.435
## mathknow -0.079 0.061
## yearstea -0.265 0.080 0.038
## mathprep -0.618 0.037 -0.006 -0.171
## sex -0.188 -0.009 0.009 0.015 -0.005
## minority -0.368 -0.171 0.108 0.025 -0.004 -0.009
## ses -0.117 0.085 0.001 -0.023 0.051 0.021 0.149
ranova(rssc.2)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority | classid) + (1 | schoolid)
## npar logLik AIC LRT Df
## <none> 13 -5363.2 10752
## minority in (1 + minority | classid) 11 -5364.8 10752 3.1967 2
## (1 | schoolid) 12 -5373.2 10770 20.1422 1
## Pr(>Chisq)

```

```
## <none>
## minority in (1 + minority | classid)      0.2022
## (1 | schoolid)                          7.189e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for minority at the classroom level, as the p value = 0.202 for the Chi-square test is not significant at  $\alpha = 0.05$ .

## SES

```
rssc.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
              ses+(1+ses|classid)+(1|schoolid),data=classroom)
summary(rssc.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##       ses + (1 + ses | classid) + (1 | schoolid)
## Data: classroom
##
## REML criterion at convergence: 10725.7
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.5688 -0.6004 -0.0316  0.5959  3.6176
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## classid (Intercept) 86.06 9.277
##       ses 44.09 6.640 0.75
## schoolid (Intercept) 173.16 13.159
## Residual 1048.32 32.378
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept) 539.52093    5.26665 269.59234 102.441 < 2e-16 ***
## housepov    -16.28994   13.13445 111.28619  -1.240  0.217
## mathknow      1.37996    1.37294 222.43201   1.005  0.316
## yearstea      0.01605    0.14080 227.59545   0.114  0.909
## mathprep     -0.37734    1.34603 182.84309  -0.280  0.780
## sex          -1.32178    2.08794 1017.08508  -0.633  0.527
## minority    -16.09272    3.03497 717.66470  -5.302 1.52e-07 ***
## ses          10.05535    1.64507 171.13536   6.112 6.44e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst mthprp sex  minrty
## housepov -0.450
## mathknow -0.078 0.059
## yearstea -0.266 0.074 0.030
## mathprep -0.625 0.036 -0.001 -0.165
## sex      -0.186 -0.009 0.007 0.013 -0.009
```

```
## minority -0.325 -0.181 0.108 0.021 0.004 -0.014
## ses      -0.084 0.078 0.015 -0.024 0.056 0.022 0.142
```

```
ranova(rssc.3)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + ses | classid) + (1 | schoolid)
##               npar logLik   AIC      LRT Df Pr(>Chisq)
## <none>                13 -5362.8 10752
## ses in (1 + ses | classid) 11 -5364.8 10752 3.8395 2      0.1466
## (1 | schoolid)           12 -5375.8 10776 26.0221 1 3.375e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for minority at the classroom level, as the p value = 0.147 for the Chi-square test is not significant at  $\alpha = 0.05$ .

## Random slopes for student-level predictors varying at school level

### ONE BY ONE

#### Sex

```
rss.4 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
              ses+(1+sex||schoolid)+(1|classid),data=classroom)
summary(rss.4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + sex || schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8578 -0.6110 -0.0259  0.5922  3.5557
##
## Random effects:
## Groups      Name                Variance Std.Dev.
## classid     (Intercept)         96.08    9.802
## schoolid    sex                 35.83    5.986
## schoolid.1  (Intercept)        161.63   12.713
## Residual                    1054.36   32.471
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.43517    5.30740  272.54946 101.638 < 2e-16 ***
## housepov     -16.77661   13.22881  112.39593  -1.268  0.207
```

```
## mathknow      1.40067      1.39464 234.45882    1.004    0.316
## yearstea      0.01448      0.14163 226.44519    0.102    0.919
## mathprep     -0.27193      1.38010 205.78503   -0.197    0.844
## sex          -1.33534      2.18746 138.08788   -0.610    0.543
## minority     -16.16536      3.02861 704.25758   -5.338 1.27e-07 ***
## ses           9.98477      1.54243 1058.27875    6.473 1.46e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst mthprp sex    minrty
## housepv -0.449
## mathknow -0.081  0.055
## yearstea -0.259  0.070  0.028
## mathprep -0.633  0.036  0.004 -0.172
## sex      -0.179 -0.010  0.007  0.013 -0.004
## minority -0.320 -0.178  0.114  0.024  0.001 -0.015
## ses      -0.120  0.081 -0.007 -0.029  0.052  0.020  0.161

ranova(rss.4, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid) + (0 + sex | schoolid) + (1 | classid)
##              npar logLik   AIC     LRT Df Pr(>Chisq)
## <none>              12 -5364.4 10753
## (1 | schoolid)       11 -5374.4 10771 19.9994  1  7.747e-06 ***
## sex in (0 + sex | schoolid) 11 -5364.8 10752  0.6137  1  0.433392
## (1 | classid)        11 -5368.2 10758  7.4171  1  0.006461 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated sex random slope at a school level is insignificant with a p value of .433.

### Minority

```
rss.5 <-lmer(Math1~housepv+mathknow+yearstea+mathprep+sex+minority+
      ses+(1+minority||schoolid)+(1|classid),data=classroom)
summary(rss.5)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority || schoolid) + (1 | classid)
##      Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
```



```

## Groups      Name      Variance Std.Dev.
## classid    (Intercept)  93.89   9.69
## schoolid   minority      0.00   0.00
## schoolid.1 (Intercept) 169.45  13.02
## Residual                1064.95 32.63
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38919 101.585 < 2e-16 ***
## housepov     -17.64847   13.21758  113.87772  -1.335  0.184
## mathknow      1.35004    1.39168  234.49776   0.970  0.333
## yearstea      0.01129    0.14141  226.80899   0.080  0.936
## mathprep     -0.27705    1.37583  205.27157  -0.201  0.841
## sex          -1.21419    2.09483 1022.42137  -0.580  0.562
## minority     -16.18678    3.02605  704.47892  -5.349 1.20e-07 ***
## ses          10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
ranova(rss.5,refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid) + (0 + minority | schoolid) + (1 | classid)
##              npar logLik  AIC    LRT Df
## <none>              12 -5364.8 10754
## (1 | schoolid)        11 -5375.2 10772 20.8586 1
## minority in (0 + minority | schoolid) 11 -5364.8 10752 0.0000 1
## (1 | classid)         11 -5368.3 10759 7.1357 1
##              Pr(>Chisq)
## <none>
## (1 | schoolid)        4.945e-06 ***
## minority in (0 + minority | schoolid) 1.000000
## (1 | classid)         0.007556 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

The uncorrelated minority random slope at school level is insignificant with a pvalue of 1.0.

## SES

```

rss.6 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
      ses+(1+ses||schoolid)+(1|classid),data=classroom)

```

```
summary(rss.6) #IS SIG
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + ses || schoolid) + (1 | classid)
##      Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##      Groups      Name      Variance Std.Dev.
##      classid    (Intercept)  88.56   9.411
##      schoolid    ses         72.50   8.515
##      schoolid.1 (Intercept) 167.98 12.961
##      Residual                1035.12 32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917 270.54314 102.126 < 2e-16 ***
## housepov     -16.94564   13.21116 112.82496  -1.283  0.202
## mathknow       1.35576    1.38459 232.19983   0.979  0.329
## yearstea       0.03079    0.14052 223.94305   0.219  0.827
## mathprep      -0.19801    1.35994 198.59419  -0.146  0.884
## sex           -1.40185    2.08170 1011.28944  -0.673  0.501
## minority     -16.52525    3.02189 700.06637  -5.469 6.32e-08 ***
## ses           9.78982     1.82217  79.01645   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.451
## mathknow    -0.079  0.056
## yearstea    -0.260  0.070  0.028
## mathprep    -0.628  0.041  0.002 -0.172
## sex         -0.190 -0.007  0.006  0.018 -0.007
## minority    -0.323 -0.180  0.110  0.024  0.001 -0.010
## ses         -0.091  0.076  0.006 -0.019  0.042  0.017  0.124
```

```
ranova(rss.6,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid) + (0 + ses | schoolid) + (1 | classid)
##              npar  logLik   AIC     LRT Df Pr(>Chisq)
```

```
## <none> 12 -5362.4 10749
## (1 | schoolid) 11 -5374.6 10771 24.2924 1 8.276e-07 ***
## ses in (0 + ses | schoolid) 11 -5364.8 10752 4.6972 1 0.03021 *
## (1 | classid) 11 -5365.7 10753 6.5177 1 0.01068 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated ses random slope at school level is significant with a p value of .03.

## Allowing for correlations with random intercepts

### ONE BY ONE

#### Sex

```
rssc.4 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
              ses+(1+sex|schoolid)+(1|classid),data=classroom)
summary(rssc.4)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + sex | schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10727.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8048 -0.6095 -0.0222  0.5969  3.5525
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## classid (Intercept) 97.34 9.866
## schoolid (Intercept) 206.33 14.364
## sex 84.08 9.170 -0.43
## Residual 1041.76 32.276
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 5.399e+02 5.363e+00 2.626e+02 100.661 < 2e-16 ***
## housepov -1.742e+01 1.325e+01 1.136e+02 -1.314 0.191
## mathknow 1.379e+00 1.396e+00 2.364e+02 0.988 0.324
## yearstea 6.876e-03 1.418e-01 2.277e+02 0.048 0.961
## mathprep -2.796e-01 1.378e+00 2.061e+02 -0.203 0.839
## sex -1.340e+00 2.301e+00 8.742e+01 -0.582 0.562
## minority -1.642e+01 3.027e+00 7.076e+02 -5.425 7.96e-08 ***
## ses 9.928e+00 1.540e+00 1.055e+03 6.448 1.72e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
```

```
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepv -0.449
## mathknow -0.082  0.060
## yearstea -0.258  0.072  0.027
## mathprep -0.627  0.038  0.004 -0.172
## sex      -0.222 -0.003  0.006  0.014 -0.005
## minority -0.319 -0.178  0.114  0.024  0.004 -0.011
## ses      -0.121  0.083 -0.006 -0.028  0.053  0.018  0.163
ranova(rssc.4, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + sex | schoolid) + (1 | classid)
##              npar logLik   AIC   LRT Df Pr(>Chisq)
## <none>              13 -5363.8 10754
## sex in (1 + sex | schoolid) 11 -5364.8 10752 1.8631 2  0.393952
## (1 | classid)              12 -5367.6 10759 7.6414 1  0.005704 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated sex random slope at school-level is insignificant with a pvalue of .394.

### Minority

```
rssc.5 <-lmer(Math1~housepv+mathknow+yearstea+mathprep+sex+minority+
              ses+(1+minority|schoolid)+(1|classid),data=classroom)
summary(rssc.5)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority | schoolid) + (1 | classid)
##      Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##      Groups   Name                Variance Std.Dev. Corr
##      classid  (Intercept)         86.69    9.311
##      schoolid (Intercept)        381.20   19.524
##              minority            343.13   18.524  -0.83
##      Residual                    1039.39   32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399 < 2e-16 ***
## housepv      -1.606e+01  1.257e+01  9.999e+01  -1.277   0.204
```

```
## mathknow      1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## yearstea     -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathprep     -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex          -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority     -1.638e+01  3.896e+00  5.824e+01  -4.203  9.17e-05 ***
## ses           9.431e+00  1.543e+00  1.063e+03   6.111  1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) housepv mthknw yearst mthprp sex    minrty
## housepv -0.394
## mathknow -0.078  0.061
## yearstea -0.253  0.091  0.024
## mathprep -0.576  0.037 -0.002 -0.167
## sex      -0.172 -0.013  0.010  0.014 -0.005
## minority -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses      -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113

ranova(rssc.5,refit=F) #sig

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority | schoolid) + (1 | classid)
##
##              npar logLik  AIC    LRT Df
## <none>              13 -5358.8 10744
## minority in (1 + minority | schoolid)  11 -5364.8 10752 11.967  2
## (1 | classid)              12 -5361.8 10748  6.077  1
##
##              Pr(>Chisq)
## <none>
## minority in (1 + minority | schoolid)    0.00252 **
## (1 | classid)              0.01370 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated minority random slope at school-level is significant with a pvalue of .003.

## SES

```
rssc.6 <-lmer(Math1~housepv+mathknow+yearstea+mathprep+sex+minority+
              ses+(1+ses|schoolid)+(1|classid),data=classroom)
summary(rssc.6)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + ses | schoolid) + (1 | classid)
##      Data: classroom
##
## REML criterion at convergence: 10724.4
##
## Scaled residuals:
##      Min        1Q    Median        3Q        Max
```

```

## -3.5646 -0.6166 -0.0264 0.5888 3.7073
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## classid (Intercept) 86.57 9.305
## schoolid (Intercept) 171.18 13.083
## ses 73.37 8.565 0.19
## Residual 1035.90 32.185
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 538.72222 5.27647 271.13405 102.099 < 2e-16 ***
## housepov -15.89873 13.15393 111.71410 -1.209 0.229
## mathknow 1.26025 1.38201 230.89932 0.912 0.363
## yearstea 0.03617 0.14002 220.42247 0.258 0.796
## mathprep -0.21697 1.35642 197.10752 -0.160 0.873
## sex -1.40436 2.08074 1011.40322 -0.675 0.500
## minority -16.26699 3.03580 668.91517 -5.358 1.16e-07 ***
## ses 9.72646 1.82985 78.36218 5.315 9.75e-07 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) houspv mthknw yearst mthprp sex minrty
## housepov -0.449
## mathknow -0.077 0.057
## yearstea -0.259 0.073 0.028
## mathprep -0.627 0.039 0.001 -0.172
## sex -0.188 -0.009 0.005 0.017 -0.008
## minority -0.325 -0.182 0.108 0.021 0.002 -0.011
## ses -0.062 0.070 0.007 -0.021 0.045 0.018 0.117
ranova(rssc.6,refit=F) #not sig

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
## ses + (1 + ses | schoolid) + (1 | classid)
## npar logLik AIC LRT Df Pr(>Chisq)
## <none> 13 -5362.2 10750
## ses in (1 + ses | schoolid) 11 -5364.8 10752 5.1385 2 0.07659 .
## (1 | classid) 12 -5365.3 10755 6.2117 1 0.01269 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

The correlated ses random slope at school-level is not significant with a p-value of .08.

**Question: Report unusual changes in variance.**

**Answer:** Perhaps most striking is the change in variance for the random slope term on minority. Previously, it was 0. However, it jumps to 343.13 in the correlated model. The variance for the random slope term on SES also increases, but the correlated random slope is not a significant addition to our model according to the rand test results.

## Complex model

Take two predictors that had sig random slopes and add to model, test for need of one conditional on the other

-Minority is sig for correlated

-Ses is sig for uncorrelated

```
complex <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(0+ses|schoolid)+(1+minority|schoolid)+(1|classid),data=classroom)
summary(complex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##       ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##       (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
## Groups      Name                Variance Std.Dev. Corr
## classid     (Intercept)         80.63    8.979
## schoolid     (Intercept)       404.54   20.113
##              minority          336.04   18.332  -0.84
## schoolid.1 ses                74.93    8.656
## Residual                    1009.73   31.776
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.74621  95.160 < 2e-16 ***
## housepov     -15.32111   12.49443   99.25865  -1.226  0.223
## mathknow       1.67475    1.35000  221.33588   1.241  0.216
## yearstea       0.02102    0.13657  213.65672   0.154  0.878
## mathprep      -0.23546    1.31730  191.22014  -0.179  0.858
## sex           -1.03871    2.06951 1010.41144  -0.502  0.616
## minority     -16.72884    3.90720   55.41065  -4.282 7.43e-05 ***
## ses            9.19654    1.82272   82.48814   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.395
## mathknow    -0.072  0.060
## yearstea    -0.254  0.093  0.024
## mathprep    -0.568  0.040 -0.004 -0.166
## sex         -0.170 -0.014  0.010  0.017 -0.005
```

```
## minority -0.509 -0.149 0.092 0.027 -0.003 -0.013
## ses      -0.080 0.083 0.006 -0.011 0.041 0.020 0.087
ranova(complex, refit=F)

## Warning: Model failed to converge with 1 negative eigenvalue: -1.2e-04
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##      (1 | classid)
##
##               npar  logLik   AIC     LRT Df
## <none>              14 -5356.2 10740
## ses in (0 + ses | schoolid)      14 -5358.8 10746  5.1200  0
## minority in (1 + minority | schoolid) 12 -5362.4 10749 12.3899  2
## (1 | classid)              13 -5358.9 10744  5.3724  1
##
##               Pr(>Chisq)
## <none>
## ses in (0 + ses | schoolid)
## minority in (1 + minority | schoolid) 0.00204 **
## (1 | classid)              0.02046 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

**Question:** Is the more complex model (with both random slopes in it) justified?

**Answer:** The complex model is justified since the rand test shows that the random slopes are both statistically significant at the 0.05 level, the only question revolves around statistical significance justifying compared to the Bayesian approach that would push for a simpler model.

The equation for the complex model is given by the following:

$$Math1st_{ijk} = \beta_0 + \beta_1 * housepov_k + \beta_2 * mathknow_{jk} + \beta_3 * yearstea_{jk} + \beta_4 * mathprep_{jk} + \beta_5 * sex_{ijk} + \beta_6 * ses_{ijk} + \beta_7 * minority_{ijk} + \zeta_{0k} + \zeta_{6k} + \zeta_{7k} + \eta_{jk} + \epsilon_{ijk}$$

where  $\zeta_{0k} \sim N(0, \sigma_{\zeta_0}^2)$ ,  $\zeta_{6k} \sim N(0, \sigma_{\zeta_6}^2)$ ,  $\zeta_{7k} \sim N(0, \sigma_{\zeta_7}^2)$ ,  $\eta_{jk} \sim N(0, \sigma_{\eta}^2)$ , and  $\epsilon_{ijk} \sim N(0, \sigma_{\epsilon}^2)$ , all independent of each other.

---

```
summary(model11)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ (1 | schoolid/classid)
##      Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##      Groups              Name             Variance Std.Dev.
## classid:schoolid (Intercept)      85.46      9.244
## schoolid         (Intercept)    280.68    16.754
```



```
## Residual                      1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.406   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

**Question:** For UCM, write down:  $V_C$ ,  $V_S$ ,  $V_E$  for the three variance components (simply the estimates). Think of them as possibly varying with a covariate, though.

**Answer:** For the UCM,  $V_C = 85.46$ ,  $V_S = 280.68$ , and  $V_E = 1146.80$

```
summary(model4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##       ses + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups           Name          Variance Std.Dev.
## classid:schoolid (Intercept)   93.89    9.69
## schoolid         (Intercept)  169.45   13.02
## Residual                        1064.95  32.63
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38922  101.585 < 2e-16 ***
## housepov     -17.64847   13.21757  113.87774   -1.335   0.184
## mathknow       1.35004    1.39168  234.49776    0.970   0.333
## yearstea       0.01129    0.14141  226.80899    0.080   0.936
## mathprep      -0.27705    1.37583  205.27157   -0.201   0.841
## sex           -1.21419    2.09483 1022.42136   -0.580   0.562
## minority     -16.18678    3.02605  704.47889   -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223    6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
```

```
## minority -0.320 -0.178 0.115 0.024 0.001 -0.011
## ses      -0.121 0.082 -0.007 -0.028 0.053 0.020 0.162
```

**Question:** For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are:  $V_C$ ,  $V_S$ ,  $V_E$ ?

**Answer:** For the most complicated fixed effects model with only random intercepts,  $V_C = 93.89$ ,  $V_S = 169.45$ , and  $V_E = 1064.95$ .

**Question:** By what fraction did these each decrease with the new predictors in the model?

**Answer:**  $V_C$  increased by  $\frac{93.89}{85.46} \sim 1.10$  times.  $V_S$  decreased by  $\frac{169.45}{280.68} \sim 0.60$  times.  $V_E$  decreased by  $\frac{1064.95}{1146.80} \sim 0.93$  times.

```
summary(rss.6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + ses || schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
## Groups      Name                Variance Std.Dev.
## classid     (Intercept)         88.56    9.411
## schoolid    ses                 72.50    8.515
## schoolid.1  (Intercept)       167.98   12.961
## Residual                    1035.12   32.173
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917  270.54314 102.126 < 2e-16 ***
## housepov     -16.94564   13.21116  112.82496  -1.283  0.202
## mathknow       1.35576    1.38459  232.19983   0.979  0.329
## yearstea       0.03079    0.14052  223.94305   0.219  0.827
## mathprep      -0.19801    1.35994  198.59419  -0.146  0.884
## sex           -1.40185    2.08170 1011.28944  -0.673  0.501
## minority     -16.52525    3.02189  700.06637  -5.469 6.32e-08 ***
## ses           9.78982     1.82217   79.01645   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.451
## mathknow     -0.079  0.056
## yearstea     -0.260  0.070  0.028
## mathprep     -0.628  0.041  0.002 -0.172
## sex          -0.190 -0.007  0.006  0.018 -0.007
```

```
## minority -0.323 -0.180 0.110 0.024 0.001 -0.010
## ses      -0.091 0.076 0.006 -0.019 0.042 0.017 0.124
```

**Question:** Now consider the model with a random slope in ses. What are:  $V_C$ ,  $V_S(\text{ses} = 0)$ ,  $V_E$  ? We need to list 'ses=0' here, or we don't know how to use the slope variance

**Answer:** For the model with a random slope in ses at the school level,  $V_C = 88.56$ ,  $V_S(\text{ses} = 0) = 167.98$ , and  $V_E = 1035.12$ .

**Question:** What are:  $V_S(\text{ses} = -0.50)$ ,  $V_S(\text{ses} = +0.5)$  ?

**Answer:** In this model, in which the random slope for SES is uncorrelated with the random school-level intercept,  $V_S(\text{ses} = -0.50) = 167.98 + (-.5)^2 72.50 + 2(-.5)0 * \sqrt{167.98} * \sqrt{72.50} = 186.105$ , and  $V_S(\text{ses} = +0.5) = 167.98 + (.5)^2 72.50 + 2 * (.5)0 * \sqrt{167.98} * \sqrt{72.50} = 186.105$

```
summary(rssc.5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority | schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## classid (Intercept) 86.69 9.311
## schoolid (Intercept) 381.20 19.524
##      minority 343.13 18.524 -0.83
## Residual 1039.39 32.240
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept) 5.395e+02 5.655e+00 1.731e+02 95.399 < 2e-16 ***
## housepov -1.606e+01 1.257e+01 9.999e+01 -1.277 0.204
## mathknow 1.632e+00 1.359e+00 2.248e+02 1.201 0.231
## yearstea -4.368e-03 1.376e-01 2.172e+02 -0.032 0.975
## mathprep -2.918e-01 1.335e+00 1.981e+02 -0.218 0.827
## sex -8.628e-01 2.084e+00 1.022e+03 -0.414 0.679
## minority -1.638e+01 3.896e+00 5.824e+01 -4.203 9.17e-05 ***
## ses 9.431e+00 1.543e+00 1.063e+03 6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst mthprp sex  minrty
## housepov -0.394
## mathknow -0.078 0.061
## yearstea -0.253 0.091 0.024
## mathprep -0.576 0.037 -0.002 -0.167
```

```
## sex      -0.172 -0.013  0.010  0.014 -0.005
## minority -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses      -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

**Question:** Now consider the model with a random slope in minority. What are:  $V_C$ ,  $V_S(\text{minority} = 0)$ ,  $V_E$ ? We need to list 'minority=0' here, or we don't know how to use the slope variance

**Answer:** For the model with a random slope in minority at the school level,  $V_C = 86.69$ ,  $V_S(\text{minority} = 0) = 381.20$ , and  $V_E = 1039.39$ .

**Question:** What are:  $V_S(\text{minority} = 0.25)$ ,  $V_S(\text{minority} = +0.50)$ ,  $V_S(\text{minority} = +0.75)$ ?

**Answer:** In this model, in which the random slope for minority is correlated with the random school-level, intercept,  $V_S(\text{minority} = 0.25) = 381.20 + (0.25)^2 343.13 + 2(0.25)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 252.5549$ ,

$V_S(\text{minority} = +0.50) = 381.20 + (0.50)^2 343.13 + 2(0.50)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 166.801$ , and

$V_S(\text{minority} = +0.75) = 381.20 + (0.75)^2 343.13 + 2(0.75)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 123.9384$ .

`summary(complex)`

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##      (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
## Groups      Name                Variance Std.Dev. Corr
## classid     (Intercept)         80.63    8.979
## schoolid    (Intercept)        404.54   20.113
##              minority          336.04   18.332  -0.84
## schoolid.1 ses                 74.93    8.656
## Residual                    1009.73   31.776
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.74621  95.160 < 2e-16 ***
## housepov     -15.32111   12.49443   99.25865  -1.226  0.223
## mathknow       1.67475    1.35000  221.33588   1.241  0.216
## yearstea       0.02102    0.13657  213.65672   0.154  0.878
## mathprep      -0.23546    1.31730  191.22014  -0.179  0.858
## sex           -1.03871    2.06951 1010.41144  -0.502  0.616
## minority     -16.72884    3.90720   55.41065  -4.282 7.43e-05 ***
## ses           9.19654    1.82272   82.48814   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
```

```
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

**Question:** Now consider the model with a random slope in ses & minority. What are:  $V_C$ ,  $V_S(\text{minority} = 0, \text{ses} = 0)$ ,  $V_E$ ? We need to list ‘ses=0, minority=0’ here, or we don’t know how to use the slope variance.

**Answer:** For the model with a random slope in ses & minority,  $V_C = 80.63$ ,  $V_S(\text{minority} = 0, \text{ses} = 0) = 404.54$ , and  $V_E = 1009.73$ .

**Question:** What are:  $V_S(\text{ses} = 0, \text{minority} = 0.50)$ ,  $V_S(\text{ses} = 0.50, \text{minority} = 0)$ ,  $V_S(\text{ses} = 0.50, \text{minority} = 0.50)$ ?

**Answer:** In this model, in which the random slope for ses is uncorrelated with the random intercept, but the random slope for minority is correlated with the random intercept,

$$V_S(\text{ses} = 0, \text{minority} = 0.50) = 404.54 + (0)^2 * 74.93 + (0.50)^2 * 336.04 + 2 * 404.54 * 74.93 + 2 * (0.50)(-0.83)\sqrt{404.54 * 336.04} = 182.5268,$$

$$V_S(\text{ses} = 0.50, \text{minority} = 0) = 404.54 + (0.50)^2 * 74.93 + (0)^2 * 336.04 + 2 * 0.5 * 404.54 * 74.93 + 2 * (0)(-0.83)\sqrt{404.54 * 336.04} = 423.2725$$

$$V_S(\text{ses} = 0.50, \text{minority} = 0.50) = 404.54 + (0.50)^2 * 74.93 + (0.50)^2 * 336.04 + 2 * 0.5 * 404.54 * 74.93 + 2 * (0.50)(-0.83)\sqrt{404.54 * 336.04} = 201.2593$$

**Question:** In the last model, what is a “likely” (+/- 1 sd) range for  $\eta_{0jk}$

**Answer:** For the complex model, the “likely” range for  $\eta_{0jk}$  is  $(-8.979, 8.979)$ .

**Question:** Can we make a similar statement about  $\zeta_{0k}$ ?

**Answer:** We cannot make a similar statement for  $\zeta_{0k}$  since it is correlated with  $\zeta_{2k}$  on *Minority*.

**Question:** If you had a large value for  $\eta_{0jk}$ , would you expect a large or small or “any” value for: the two random slope terms,  $\zeta_{1k}$  and  $\zeta_{2k}$  for ses and minority?

**Answer:** There is no correlation between  $\eta_{0jk}$  (classroom-level intercept) and the school-level random slopes  $\zeta_{1k}$  and  $\zeta_{2k}$  on *SES* and *MINORITY*. Therefore, we would not expect a large value of  $\eta_{0jk}$  to have any effect on the two random slope terms as they are independent.

**Question:** If you had a large value for  $\zeta_{0k}$ , would you expect a large or small or “any” value for: the two random slope terms,  $\zeta_{1k}$  and  $\zeta_{2k}$  for ses and minority (discuss each separately)?

**Answer:**  $\zeta_{1k}$  could be any value due to the lack of correlation with  $\zeta_{0k}$

**Answer:** While  $\zeta_{2k}$  would be small given a large value of  $\zeta_{0k}$  because of the negative correlation between the two variables.

## Part 2: Clare

### Running initial model

The initial model was run on a smaller dataset with 1081 observations due to missing data. School-level and classroom-level random intercepts are included in the model.

```

#remove missing data -- not ideal, but have to do it for this analysis
classroom <- classroom %>% mutate(Math1st = mathkind + mathgain)
classroom2 <- na.omit(classroom)
#model
new1 <- lmer(Math1st~housepov+mathknow+yearstea+mathprep+sex
             +minority+ses+(1|schoolid)+(1|classid),data=classroom2)

```

## Residual that removes only the “fixed effects”

Below we calculate the residuals that removes only the fixed effects. The boxplot of the residuals shows that there is great variation within schools and that there is a steady linear trend to the residuals, suggesting dependence.

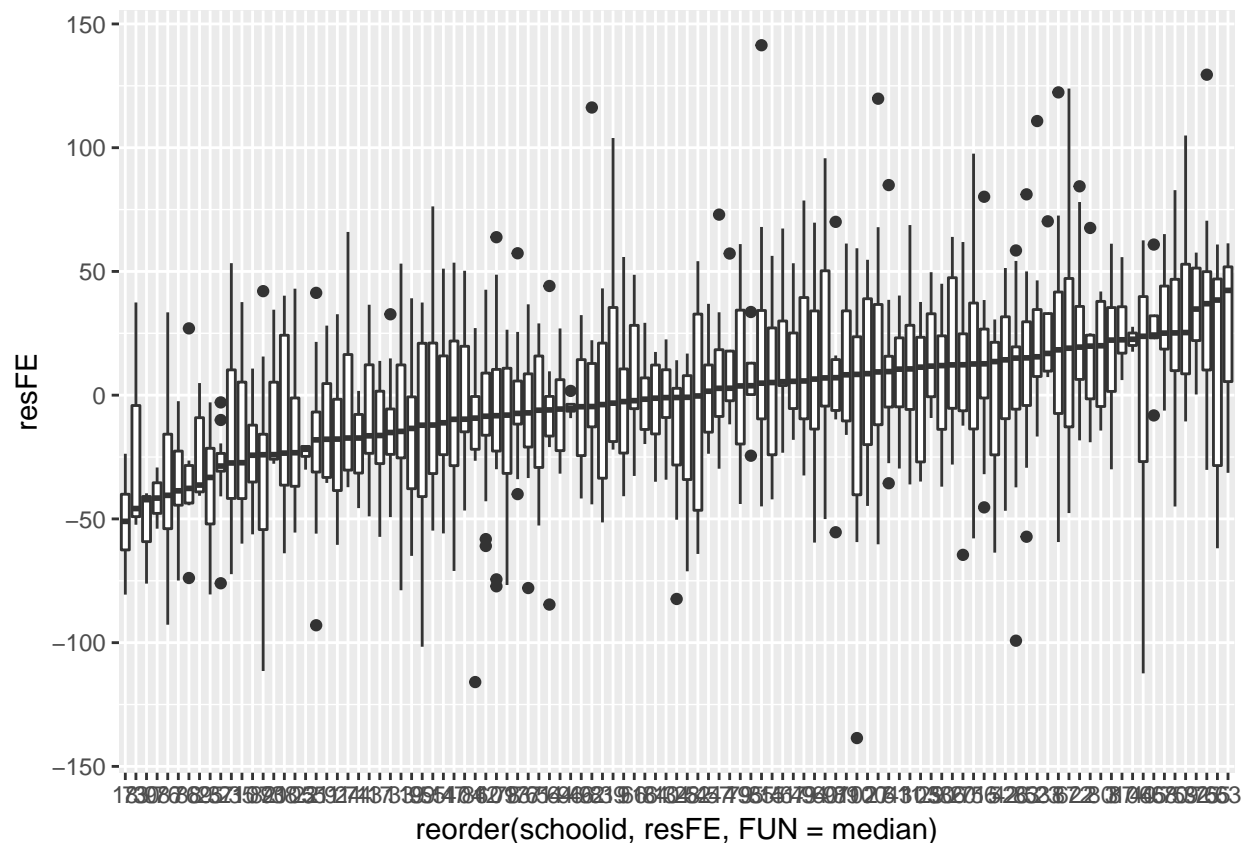
```

#predicted scores
pred.yhat <- predict(new1,re.form=~0)

#residual
resFE <- classroom2$Math1st-pred.yhat

#show that it's not independent
if (vanillaR) {
  ord <- order(unlist(tapply(resFE, classroom2$schoolid, median)))
  boxplot(split(resFE, classroom2$schoolid)[ord])
} else {
  ggplot(classroom2, aes(x = reorder(schoolid, resFE, FUN = median), y = resFE)) +
  geom_boxplot()
}

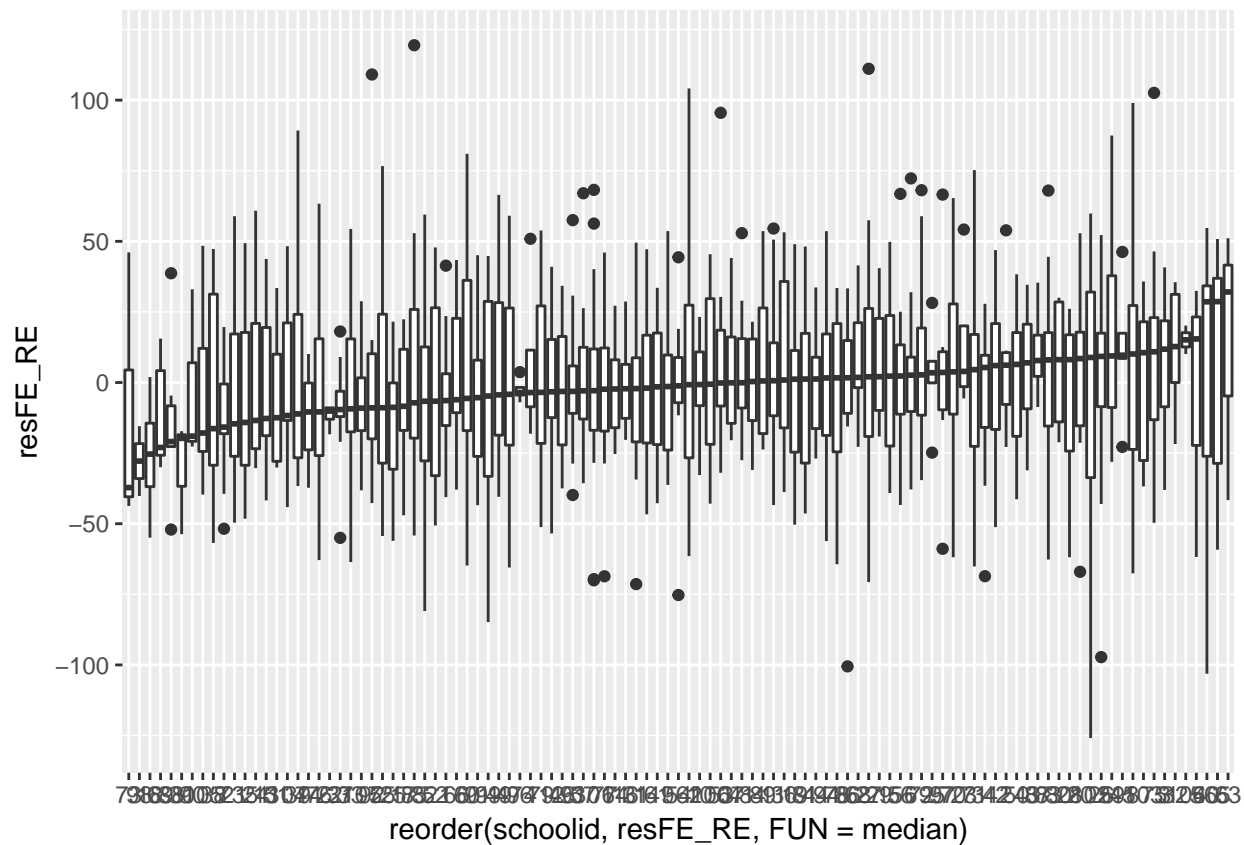
```



## Residuals for BLUPs random effects

The residuals for the BLUPs random effects are calculated below. The boxplot reveals a similar dependency to the previous plot, though not as pronounced. There doesn't seem to be as high a correlation as there is in the other residuals plot.

```
#getting predicted zeta_0 and eta_0
ranefs <- ranef(new1)
zeta0 <- ranefs$schoolid[,1]
eta0 <- ranefs$classid[,1]
#indexing
idx.sch <- match(classroom2$schoolid, sort(unique(classroom2$schoolid)))
idx.cls <- match(classroom2$classid, sort(unique(classroom2$classid)))
classroom2$zeta0 <- zeta0[idx.sch]
classroom2$eta0 <- eta0[idx.cls]
#now subtract all from outcome
resFE_RE <- classroom2$Math1st-pred.yhat-classroom2$zeta0-classroom2$eta0
#show that it's not independent, but much less correlated than resFE
if (vanillaR) {
  ord <- order(unlist(tapply(resFE_RE, classroom2$schoolid, median)))
  boxplot(split(resFE_RE, classroom2$schoolid)[ord])
}else{
  ggplot(classroom2, aes(x = reorder(schoolid, resFE_RE, FUN = median), y = resFE_RE)) +
  geom_boxplot()
}
```

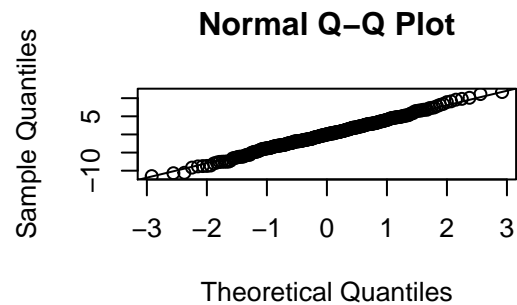
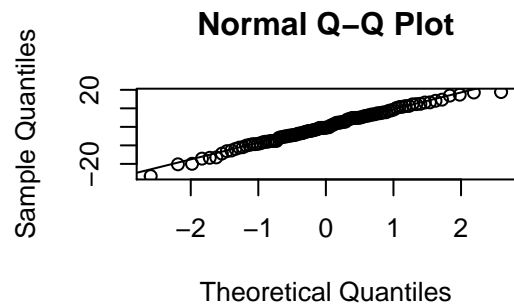
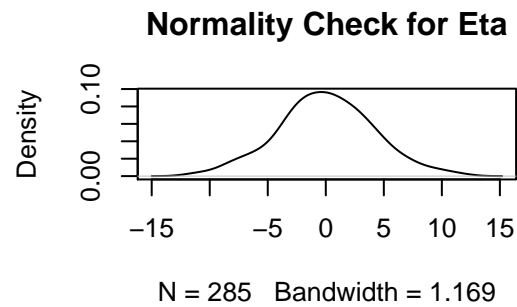
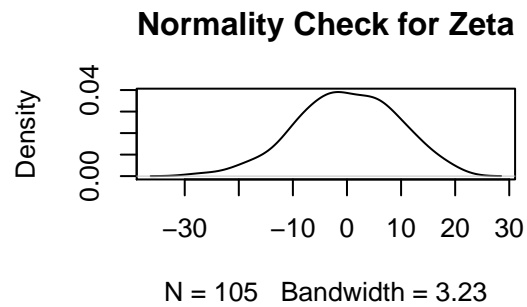


## Examining BLUPs for normality

To examine the BLUPs for normality, density plots and Q-Q plots were constructed. Both  $\text{zeta}_0$  and  $\text{eta}_0$  appear to be normal, with a few possible outliers near the tails.

```
par(mfrow=c(2,2))
plot(density(zeta0), main = "Normality Check for Zeta")
plot(density(eta0), main = "Normality Check for Eta")
#looking good
qqnorm(zeta0);qqline(zeta0)
qqnorm(eta0);qqline(eta0)
```





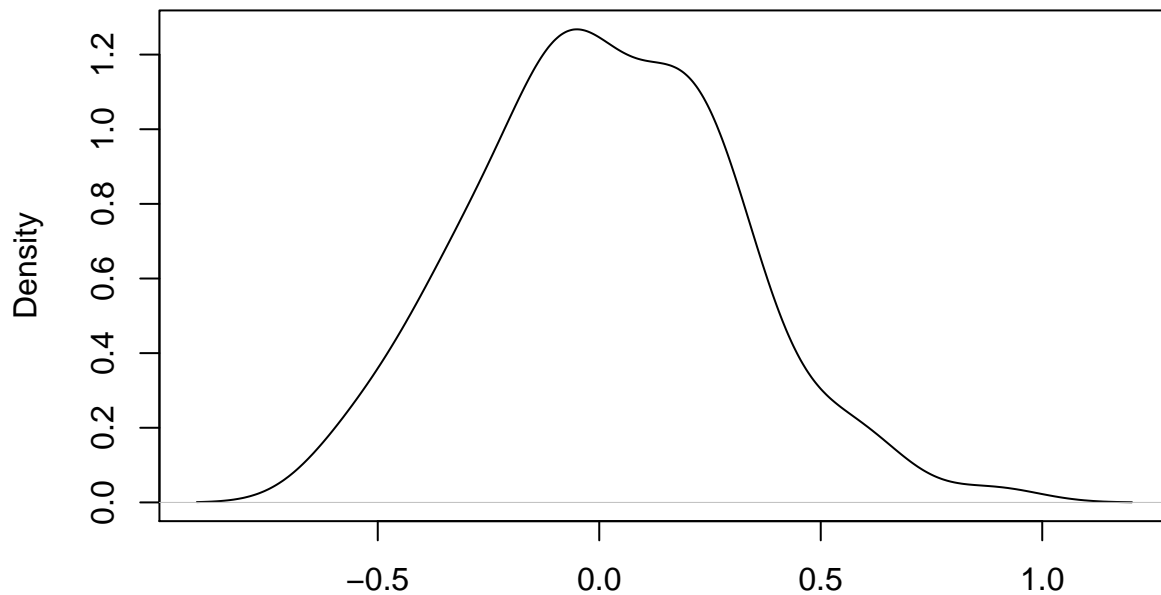
*#looking good*

## Simulation

Below is a simulation based on the  $H_0$  being true, and a  $\sigma_\epsilon = 1$ . We find that the potential estimate is very close to 0, which we would expect since our  $\sigma_{\zeta_0}^2$  has a “true” value of 0.

```
set.seed(10314)
school.sim <- matrix(1,10,100)
means <- NULL
for (i in 1:100){
  school.sim[,i] <- rnorm(10,mean=0, sd=1)
  means[i] <- mean(school.sim[,i])
}
plot(density(means), main = "Density of Zeta0")
```

## Density of Zeta0



N = 100 Bandwidth = 0.1039

```
#we see the density is concentrated around 0
paste("A potential estimate of sigma_{zeta_0} is ",mean(means))

## [1] "A potential estimate of sigma_{zeta_0} is  0.0142117878263361"
```

## New Complex Model

We now include a correlated random slope at the school-level for minority.

```
classroom <- read.csv("classroom.csv")
classroom <- classroom %>% mutate(Math1st = mathkind+mathgain)
classroom2 <- na.omit(classroom)
newcomplex <- lmer(Math1st~housepov+mathknow+yearstea+mathprep+sex+minority+ses+
                    (minority|schoolid)+(1|classid),data=classroom2)
summary(newcomplex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1st ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (minority | schoolid) + (1 | classid)
##      Data: classroom2
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
```

```
##      Min      1Q  Median      3Q      Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##   Groups   Name      Variance Std.Dev.  Corr
##   classid (Intercept)  86.69   9.311
##   schoolid (Intercept) 381.20  19.524
##             minority    343.13  18.524  -0.83
##   Residual              1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399 < 2e-16 ***
## housepov     -1.606e+01  1.257e+01  9.999e+01  -1.277   0.204
## mathknow      1.632e+00  1.359e+00  2.248e+02   1.201   0.231
## yearstea     -4.368e-03  1.376e-01  2.172e+02  -0.032   0.975
## mathprep     -2.918e-01  1.335e+00  1.981e+02  -0.218   0.827
## sex          -8.628e-01  2.084e+00  1.022e+03  -0.414   0.679
## minority     -1.638e+01  3.896e+00  5.824e+01  -4.203 9.17e-05 ***
## ses           9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.394
## mathknow  -0.078  0.061
## yearstea  -0.253  0.091  0.024
## mathprep  -0.576  0.037 -0.002 -0.167
## sex        -0.172 -0.013  0.010  0.014 -0.005
## minority  -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses        -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

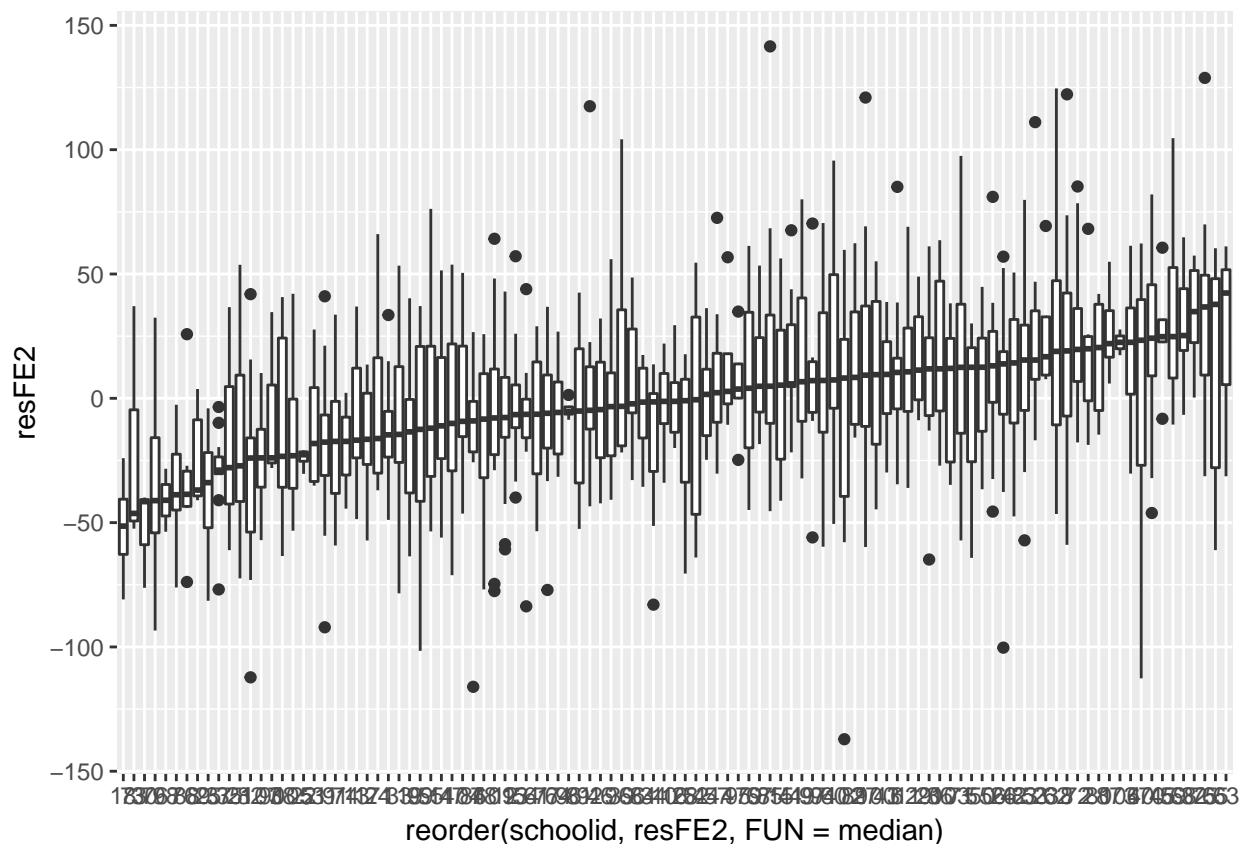
## Manually calculate residuals for fixed effects

In the new model, we see a similar pattern of dependency. There is a general positive, linear trend to the residuals, and there is heterogeneity of variance across and within schools. These findings all suggest dependence.

```
#predicted scores
pred.yhat2 <- predict(newcomplex,re.form=-0)

#residual
resFE2 <- classroom2$Math1st-pred.yhat2

#show that it's not independent
if (vanillaR) {
  ord <- order(unlist(tapply(resFE2, classroom2$schoolid, median)))
  boxplot(split(resFE2, classroom2$schoolid)[ord])
} else {
  ggplot(classroom2, aes(x = reorder(schoolid, resFE2, FUN = median), y = resFE2)) +
  geom_boxplot()
}
```



## Residuals from BLUPs random effects

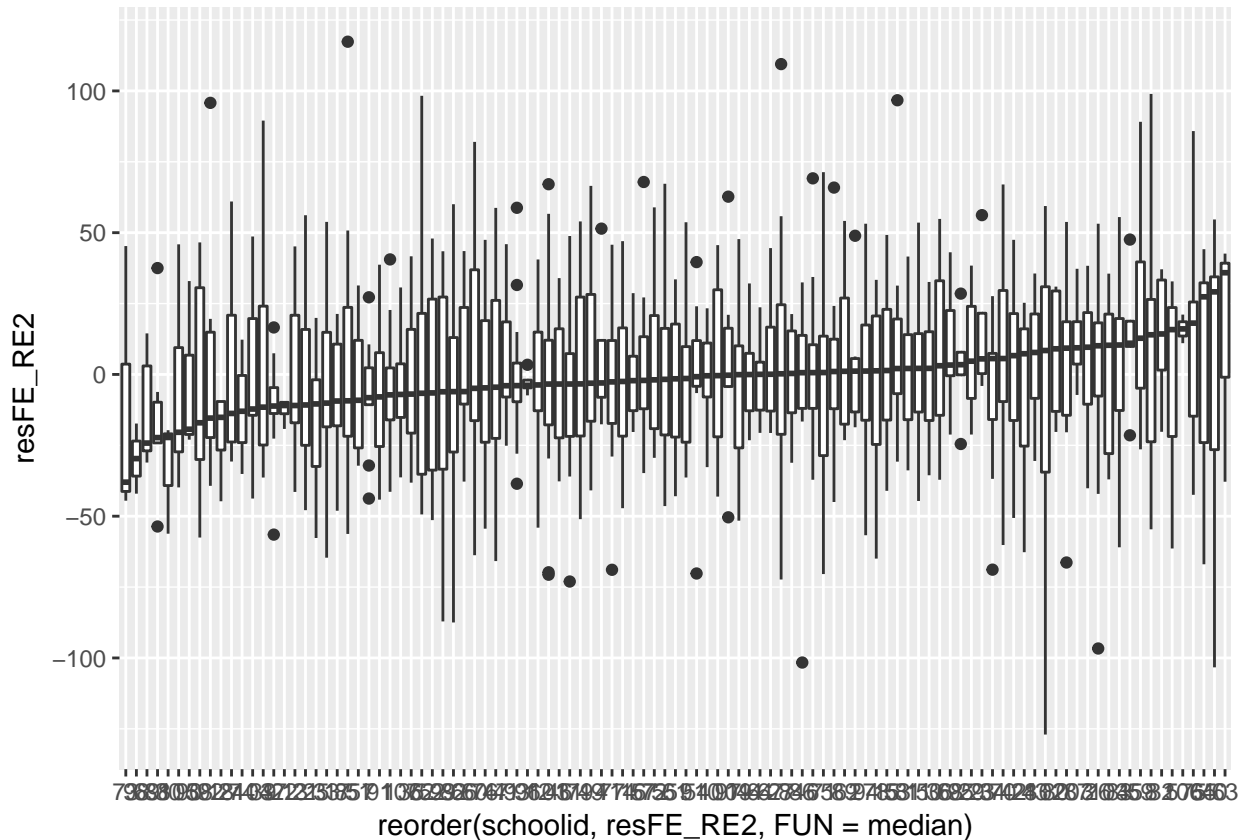
The residuals from the BLUPs random effects are calculated below. The boxplot of the residuals appears to be only slightly correlated, partly due to the uptake near the final set of schools on the x-axis. Although the correlation of the residuals is probably near 0, there is still enough variation within schools, and enough of a correlation in the data to suggest dependence.

```
#getting predicted zeta_0 and eta_0
ranefs2 <- ranef(newcomplex)
zeta0c <- ranefs2$schoolid[,1]
eta0c <- ranefs2$classid[,1]
zeta1c <- ranefs2$schoolid[,2]
#indexing
idx.sch <- match(classroom2$schoolid, sort(unique(classroom2$schoolid)))
idx.cls <- match(classroom2$classid, sort(unique(classroom2$classid)))
classroom2$zeta0c <- zeta0c[idx.sch]
classroom2$eta0c <- eta0c[idx.cls]
classroom2$zeta1c <- zeta1c[idx.sch]
#now subtract all from outcome
resFE_RE2 <- classroom2$Math1st-pred.yhat-classroom2$zeta0c-classroom2$eta0c-(classroom2$minority*classroom2$zeta1c)
#show that it's not independent, but much less correlated than resFE
if (vanillaR) {
  ord <- order(unlist(tapply(resFE_RE2, classroom2$schoolid, median)))
  boxplot(split(resFE_RE2, classroom2$schoolid)[ord])
}
```

```

}else{
  ggplot(classroom2, aes(x = reorder(schoolid, resFE_RE2, FUN = median), y = resFE_RE2)) +
    geom_boxplot()
}

```



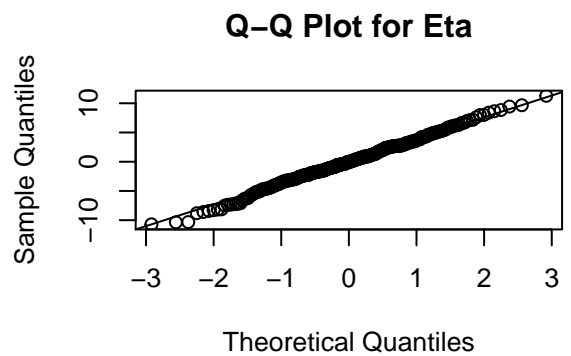
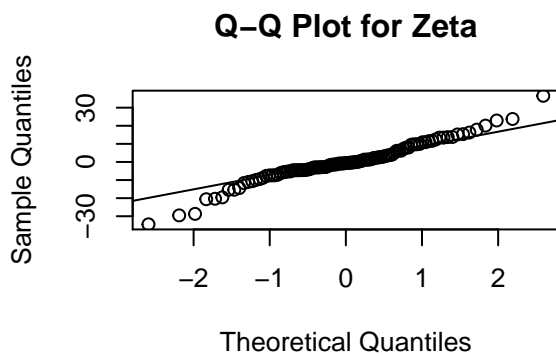
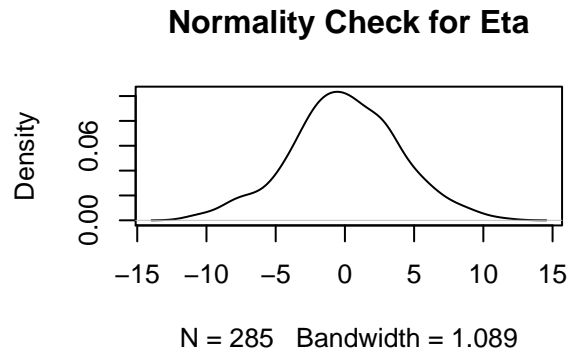
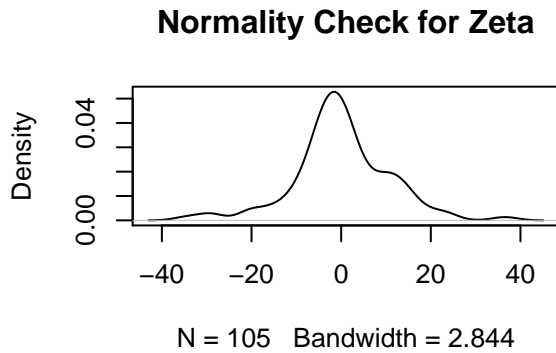
## Examining Normality of BLUPs

Below we examine the normality of  $\zeta_0$  and  $\eta_0$ . The density and Q-Q plots for  $\eta_0$  suggest normality, with a possibility of a few outliers near the tails. The normality of  $\zeta_0$  is more questionable. The tails do not appear to fit a normal distribution.

```

par(mfrow=c(2,2))
plot(density(zeta0c), main = "Normality Check for Zeta")
plot(density(eta0c), main = "Normality Check for Eta")
# eta looks pretty normal
# zeta not so much
qqnorm(zeta0c, main = "Q-Q Plot for Zeta");qqline(zeta0c)
qqnorm(eta0c, main = "Q-Q Plot for Eta");qqline(eta0c)

```



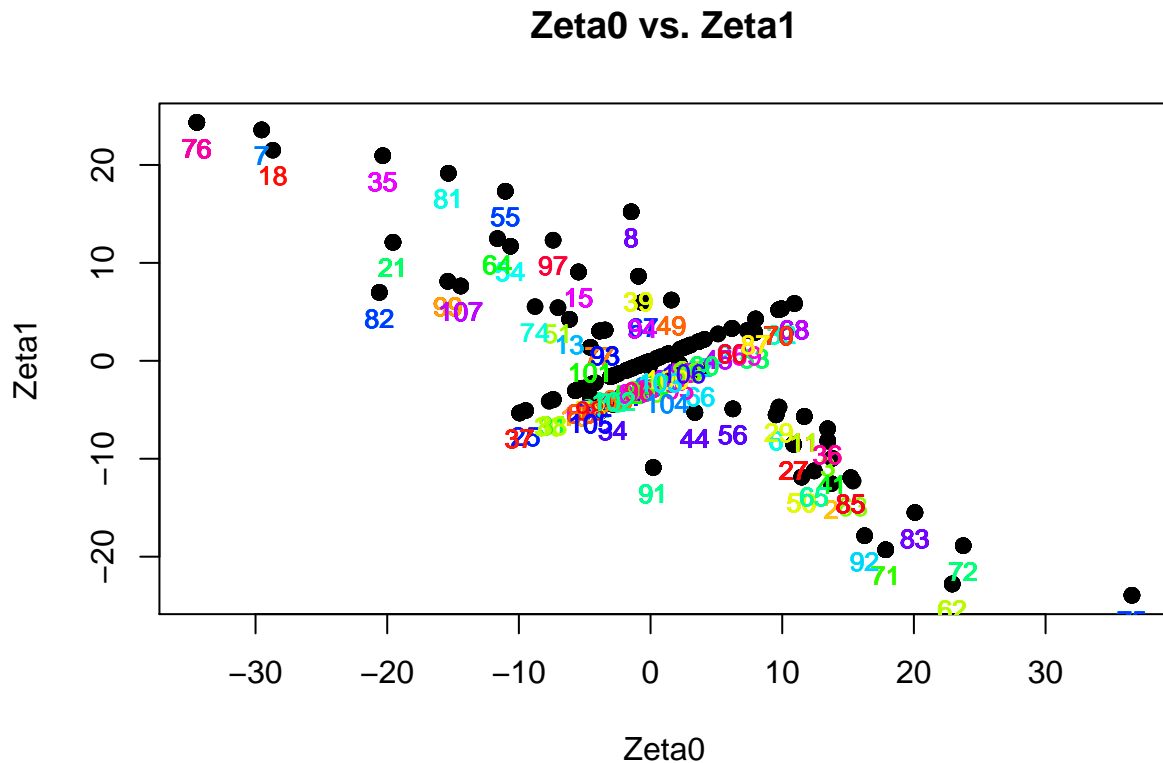
*#zeta looking iffy, but with a few possible outliers  
#eta good too, with few outliers.*

### Plotting $\zeta_0$ versus $\zeta_1$

The correlation between  $\zeta_0$  and  $\zeta_1$  in the output is -0.83. The graph below suggests a moderate negative trend, but there are some outliers that do not support this trend. Rather, they seem to be positively related.

\*Note: the labels were put in rainbow in order to better discern their locations.

```
plot(classroom2$zeta0c,classroom2$zeta1c, main = "Zeta0 vs. Zeta1",
     ylab = "Zeta1",xlab = "Zeta0", pch=19)
text(classroom2$zeta0c,classroom2$zeta1c, labels = classroom2$schoolid,
     cex = 0.8, col = rainbow(100), pos = 1)
```



## Tracking down outliers

The outliers from the plots above can be tracked down by examining the data points via their IDs.

```
classroom2$zeta0c[classroom2$schoolid==45][[1]]/classroom2$zeta1c[classroom2$schoolid==45][[1]]
```

```
## [1] 1.868107
```

```
classroom2$zeta0c[classroom2$schoolid==68][[1]]/classroom2$zeta1c[classroom2$schoolid==68][[1]]
```

```
## [1] 1.868107
```

```
classroom2$zeta0c[classroom2$schoolid==30][[1]]/classroom2$zeta1c[classroom2$schoolid==30][[1]]
```

```
## [1] 1.868107
```

```
#there seems to be a trend here that the zeta0/zeta1 ratio is > 3, so let's filter it out
```

```
outliers <- classroom2 %>% filter(round(zeta0c/zeta1c,6)==1.868107) %>% select(zeta0c,zeta1c,schoolid,m)
```

```
#now let's make sure the IDs from the plot are showing up here
```

```
unique(outliers$schoolid)
```

```
## [1] 1 4 5 9 10 12 14 16 17 19 20 22 23 24 25 26 28
```

```
## [18] 30 31 32 33 37 38 42 43 45 46 47 52 57 60 61 68 69
```

```
## [35] 70 73 78 79 80 84 86 87 88 89 90 96 98 100 102 103 106
```

```
#They are! Now what's going on with minority?
```

```
table(outliers$minority)
```

```
##
```

```
## 1
## 455
tapply(outliers$minority, INDEX = outliers$schoolid, FUN = sum)

## 1 4 5 9 10 12 14 16 17 19 20 22 23 24 25 26 28 30
## 8 4 6 6 10 24 15 6 8 11 12 4 5 8 7 15 10 3
## 31 32 33 37 38 42 43 45 46 47 52 57 60 61 68 69 70 73
## 16 9 18 11 6 18 4 5 10 10 9 8 13 11 16 5 19 3
## 78 79 80 84 86 87 88 89 90 96 98 100 102 103 106
## 12 12 7 8 7 10 6 4 3 4 2 6 11 8 2

#The values match -- all students are minorities!
```

It seems like the (perfectly) positive trend in the data is being driven by schools in which all the students are minorities. That is, in schools in which there are only minority students, all other factors held equal, there is a boost in math scores in 1st grade for minority students. In a way, being in a totally minority school is a “protective” factor for minority students.

## Part 3: Bianca

### Create person-period file

In this part of the project, no variables of interested have missing observation. Therefore, the full dataset is used.

```
#new variables
classroom2 <- classroom2 %>% mutate(math0 = mathkind) %>% mutate(math1 = mathkind+mathgain)
#reshape the data
class_pp <- reshape(classroom2, varying = c("math0", "math1"), v.names = "math", timevar = "year",
times = c(0, 1), direction = "long")
```

Note: we ignore classroom in this analysis but keep it in the notation.

### Initial longitudinal model

We fit a model with math as outcome, and fixed effect for time trend (year), as well as random intercept for school.

The equation for the model below:

$$Math_{tijk} = b_0 + \zeta_{0k} + b_1 * Time_{tijk} + \epsilon_{tijk}$$

where  $\zeta_{0k} \sim N(0, \sigma_{\zeta_{eta0}}^2)$  and  $\epsilon_{tijk} \sim N(0, \sigma_{\epsilon}^2)$

We refer to this as Model 0.

Below the model fit:

```
fit1 <- lmer(math ~ year + (1|schoolid), data = class_pp)
summary(fit1)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
```



```
## Formula: math ~ year + (1 | schoolid)
## Data: class_pp
##
## REML criterion at convergence: 21794.2
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -5.0693 -0.6031  0.0030  0.6321  3.7529
##
## Random effects:
## Groups Name Variance Std.Dev.
## schoolid (Intercept) 337 18.36
## Residual 1288 35.89
## Number of obs: 2162, groups: schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 465.053 2.136 133.580 217.76 <2e-16 ***
## year 57.844 1.544 2055.557 37.47 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr)
## year -0.361
```

## Add child-level random intercept

To the previous model, we now add random intercepts for child:

$$Math_{tijk} = b_0 + \delta_{0ijk} + \zeta_{0k} + b_1 * Time_{tijk} + \epsilon_{tijk}$$

where  $\delta_{0tijk} \sim N(0, \sigma_{\delta_0}^2)$ ,  $\zeta_{0k} \sim N(0, \sigma_{\zeta_0}^2)$  and  $\epsilon_{tijk} \sim N(0, \sigma_{\epsilon}^2)$  independently of one another.

We refer to this as M1.

```
fit2 <- lmer(math ~ year + (1|schoolid/childid), data = class_pp)
summary(fit2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 | schoolid/childid)
## Data: class_pp
##
## REML criterion at convergence: 21425.2
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -4.7233 -0.4827  0.0125  0.4922  3.4892
##
## Random effects:
## Groups Name Variance Std.Dev.
## childid:schoolid (Intercept) 722.0 26.87
## schoolid (Intercept) 293.2 17.12
```

```
## Residual                                602.2    24.54
## Number of obs: 2162, groups:  childid:schoolid, 1081; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.288      2.057   116.653   226.2   <2e-16 ***
## year         57.844      1.056  1080.000    54.8   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.257
```

In model 0 the variance  $\sigma_{\zeta_0}^2 = 348.7$  and in model 1  $\sigma_{\zeta_0}^2 = 307.5$ .

In model 0, the variance for  $\sigma_\epsilon^2 = 1268.4$  and in model 1  $\sigma_{\epsilon_0}^2 = 599.1$ . We note that including child-level variation leads to a decrease in the variance of both the random effects.

## Compute Pseudo-R<sup>2</sup>

Compute a pseudo R<sup>2</sup> relating the between school variation and ignoring between students in the same school.

We calculate this as :

$$\frac{\sigma_{\zeta_0}^2(M_0) - \sigma_{\zeta_0}^2(M_1)}{\sigma_{\zeta_0}^2(M_0)} = \frac{348.7 - 307.5}{348.7} = 0.12$$

The between-school variance is reduced by 12% ( or ‘explained’) with the introduction of student random effect.

### Does the total variation stay about the same?

```
tot_m0 = 348.7 + 1268.4
tot_m1 = 702 + 307.5 + 599.1
paste("Tot variance for model 0 : ", tot_m0)

## [1] "Tot variance for model 0 : 1617.1"
paste("Tot variance for model 1: ", tot_m1)

## [1] "Tot variance for model 1: 1608.6"
```

There is only a slightly decrease in the total variance between Model 0 and Model1.

## Add a random slope for time trend

We now add a random slope ( $\zeta_1$ ) for time trend within schools.

$$Math_{tijk} = b_0 + \delta_{0ijk} + \zeta_{0k} + (b_1 + \zeta_{1k}) * Time_{tijk} + \epsilon_{tijk}$$

where  $\delta_{0tijk} \sim N(0, \sigma_{\delta_{0ijk}}^2)$ ,  $\zeta_{0k} \sim N(0, \sigma_{\zeta_0}^2)$ ,  $\zeta_{1k} \sim N(0, \sigma_{\zeta_1}^2)$  and  $\epsilon_{tijk} \sim N(0, \sigma_\epsilon^2)$  – each independently of one another.

We refer to this as Model 2

We run the model and report the fit:

```
fit3 = lmer(math ~ year + (1 + year || schoolid) + (1 | childid), data = class_pp)
summary(fit3)
```

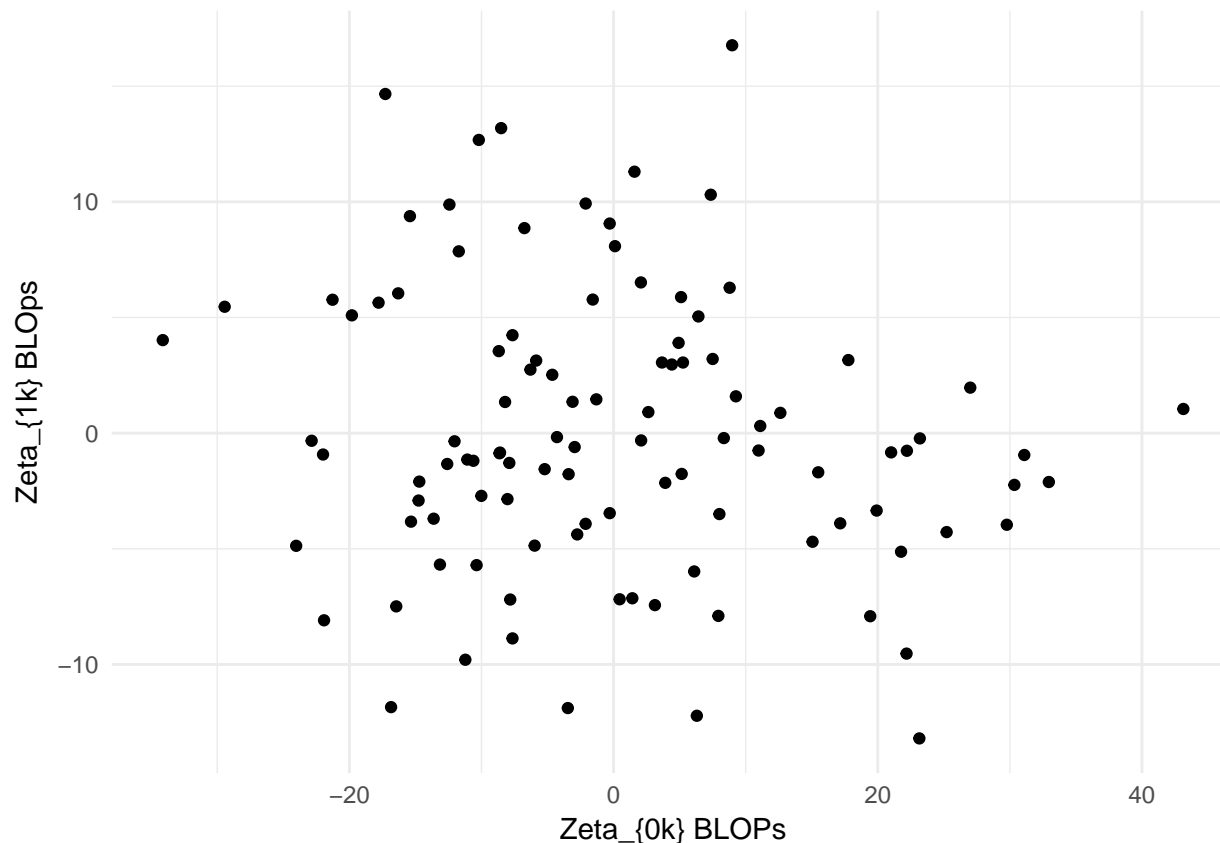
```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 + year || schoolid) + (1 | childid)
## Data: class_pp
##
## REML criterion at convergence: 21403.1
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -4.7461 -0.4788  0.0119  0.4719  3.5976
##
## Random effects:
## Groups      Name                Variance Std.Dev.
## childid     (Intercept)    745.36     27.301
## schoolid    year           85.96      9.272
## schoolid.1  (Intercept)   315.69     17.768
## Residual                    554.92     23.557
## Number of obs: 2162, groups:  childid, 1081; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.257      2.108 108.631  220.73   <2e-16 ***
## year         57.751       1.402 101.342   41.18   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.188
```

## Generate the BLUPs for this model (Model 2)

Examine then whether the independence between  $\text{zeta}_0$  and  $\text{zeta}_1$  is reflected in a scatterplot of these two sets of effects.

```
pp_ranefs <- ranef(fit3)

if (vanillaR) {
plot(pp_ranefs$schoolid[,2], pp_ranefs$schoolid[,1])
}else{
ggplot(pp_ranefs$schoolid, aes(x = pp_ranefs$schoolid[,2], y = pp_ranefs$schoolid[,1])) +
geom_point() + labs(x = "Zeta_{0k} BLOPs", y = "Zeta_{1k} BLOPs") + theme_minimal()
}
```



From the plot, the BLOPs for  $\zeta_{0k}$  and for  $\zeta_{1k}$  appear uncorrelated, reflecting the way in which the model was built. In the BLUPS, we have a correlation of:

```
cor(pp_ranefs$schoolid[,2],pp_ranefs$schoolid[,1])

## [1] -0.1526629

correstp = cor.test(pp_ranefs$schoolid[,2],pp_ranefs$schoolid[,1],
                    method = "pearson")$p.value

paste("P-value for pearson test for correlatio:", round(correstp,3))

## [1] "P-value for pearson test for correlatio: 0.12"
```

That is, between the slope random effects and the random intercept blups, there is a very small negative correlation, which is not significantly different from 0 – which we see from the plot and would expect from how we have specified the model.

## Heteroscedasticity in the random effects

**Question:** What are:  $V_S(\text{year} = 0)$ ,  $V_S(\text{year} = 1)$ ?

The model we are considering is :

$$Math_{tijk} = b_0 + \delta_{0tijk} + \zeta_{0k} + (b_1 + \zeta_{1k})Time_{tijk} + \epsilon_{tijk}$$

So we have that (in this model, in which we are forcing correlation of 0 between slope and intercept):

- $V_S(\text{year} = 0) = \sigma_{\zeta_{0k}}^2 = 324.79$

- $V_S(\text{year} = 1) = \sigma_{\zeta_{0k}}^2 + \sigma_{\zeta_{1k}}^2 = 88.67 + 324.79 = 413.46$

## Run model separately by year

We now examine what happens if we run the model separately by year. Do we get the same estimates for the variance between schools?

```
class_year0 = class_pp[class_pp$year == 0,]

# Run model for year 0
fit4 = lmer(math ~ (1 | schoolid), data = class_year0)
summary(fit4)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ (1 | schoolid)
## Data: class_year0
##
## REML criterion at convergence: 11017.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -4.7416 -0.5655  0.0086  0.6286  3.5648
##
## Random effects:
## Groups Name Variance Std.Dev.
## schoolid (Intercept) 364.1 19.08
## Residual 1391.4 37.30
## Number of obs: 1081, groups: schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 465.382 2.244 101.588 207.4 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

# Run model for year 1
class_year1 = class_pp[class_pp$year == 1,]
fit5 = lmer(math ~ (1 | schoolid), data = class_year1)
summary(fit5)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ (1 | schoolid)
## Data: class_year1
##
## REML criterion at convergence: 10851.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.5824 -0.6158 -0.0063  0.6191  3.8063
##
## Random effects:
## Groups Name Variance Std.Dev.
```

```
## schoolid (Intercept) 279.1 16.71
## Residual 1202.6 34.68
## Number of obs: 1081, groups: schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 523.2 2.0 101.9 261.6 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

In this case, for the Year 0 Model, we get an estimated  $\hat{\sigma}_{\zeta_{0k}}^2 = 364.3$ , while for Year 1 Model, we have  $\hat{\sigma}_{\zeta_{0k}}^2 = 306.8$ . We note that these estimates are different from the ones computed above.

## Allow for correlation

We now allow for correlation between the random effects for the intercept and the slope. We call this Model 3.

```
fit6 = lmer(math ~ year + (1 + year | schoolid) + (1 | childid), data = class_pp)
summary(fit6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 + year | schoolid) + (1 | childid)
## Data: class_pp
##
## REML criterion at convergence: 21391.3
##
## Scaled residuals:
## Min 1Q Median 3Q Max
## -4.6737 -0.4699 0.0038 0.4683 3.4882
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## childid (Intercept) 749.0 27.37
## schoolid (Intercept) 373.5 19.33
## year 112.4 10.60 -0.53
## Residual 547.8 23.41
## Number of obs: 2162, groups: childid, 1081; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 465.257 2.241 101.265 207.6 <2e-16 ***
## year 58.006 1.491 95.409 38.9 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr)
## year -0.486
```

Correlation between  $\zeta_0$  and  $\zeta_1 = -0.45$ .

To test whether the correlation is statistically significant, we can compare Model 2 with Model 3 using an anova test.

```
anova(fit3, fit6, refit = F)
```

```
## Data: class_pp
## Models:
## fit3: math ~ year + (1 + year || schoolid) + (1 | childid)
## fit6: math ~ year + (1 + year | schoolid) + (1 | childid)
##      Df    AIC    BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
## fit3  6 21415 21449 -10702    21403
## fit6  7 21405 21445 -10696    21391 11.879      1 0.0005678 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

With a p-value  $p = 0.003$ , we reject the null hypothesis at  $\alpha = 0.05$  significance level, and conclude that there correlation term is statistically significant.

So we have that (in this model where we are allowing for correlation between slope and intercept):

- $V_S(\text{year} = 0) = \sigma_{\zeta_{0k}}^2 = 370.6$
- $V_S(\text{year} = 1) = \sigma_{\zeta_{0k}}^2 + \sigma_{\zeta_{1k}}^2 + 2\rho_{01}\sigma_{\zeta_{0k}}\sigma_{\zeta_{1k}} = 370.6 + 109.1 - 2 * 0.45 * \sqrt{370.6}\sqrt{109.1} = 298.72$

These estimates are a lot closer to the school variances that result from fitting the models for the two years separately (in which we have  $\sigma_{\zeta}^2$  respectively be 364.3 for year 0 and 306.8 for year 1).

Therefore, it seems that the model that allows for correlation between the two random effects has a better fit than the one forcing that correlation to be 0.

## Part 4: Kaushik

### Reload the data and make person-period file

A reduced dataset was used for this analysis since there is missing data for one of the variables of interest. We acknowledge that this is not ideal in practice.

```
#re-read data
classroom <- read.csv("classroom.csv")
classroom2 <- na.omit(classroom)
#new variables
classroom2 <- classroom2 %>% mutate(math0 = mathkind) %>% mutate(math1 = mathkind+mathgain)
#reshape the data
class_pp <- reshape(classroom2, varying = c("math0", "math1"), v.names = "math", timevar = "year",
times = c(0, 1), direction = "long")
```

### Baseline model: unconditional growth model

$$MATH_{tijk} = b_0 + \delta_{0ijk} + (b_1 + \zeta_{1k})TIME_{tijk} + \zeta_{0k} + \epsilon_{tijk}$$

where  $t$  represents occasion (in this case, year/grade),  $i$  represents students,  $j$  represents classrooms and  $k$  represents schools.  $\delta_{0ijk} \sim N(0, \sigma_{\delta_0}^2)$ ,  $\zeta_{0k} \sim N(0, \sigma_{\zeta_0}^2)$ ,  $\zeta_{1k} \sim N(0, \sigma_{\zeta_1}^2)$  and  $\epsilon_{ijk} \sim N(0, \sigma_{\epsilon}^2)$  all independent of each other except for  $\zeta_{0k}$  and  $\zeta_{1k}$  having a correlation  $\rho_{\zeta_0\zeta_1}$ .

```
ugm <- lmer(math ~ year + (year|schoolid) + (1|childid), data=class_pp)
summary(ugm)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (year | schoolid) + (1 | childid)
## Data: class_pp
##
## REML criterion at convergence: 21391.3
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -4.6737 -0.4699  0.0038  0.4683  3.4882
##
## Random effects:
## Groups Name Variance Std.Dev. Corr
## childid (Intercept) 749.0 27.37
## schoolid (Intercept) 373.5 19.33
## year 112.4 10.60 -0.53
## Residual 547.8 23.41
## Number of obs: 2162, groups: childid, 1081; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 465.257 2.241 101.265 207.6 <2e-16 ***
## year 58.006 1.491 95.409 38.9 <2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr)
## year -0.486
```

## Add student, classroom and school level fixed effects

$$MATH_{tijk} = b_0 + \delta_{0ijk} + (b_1 + \zeta_{1k})TIME_{tijk} + b_2SEX_{ijk} + b_3SES_{ijk} + b_4MINORITY_{ijk} + b_5YEARSTEA_{jk} + b_6MATHKNOW_{jk} + b_7MATHPREP_{jk} + b_8HOUSEPOV_k + \zeta_{0k} + \epsilon_{tijk}$$

where  $t$  represents occasion (in this case, year/grade),  $i$  represents students,  $j$  represents classrooms and  $k$  represents schools.  $\delta_{0ijk} \sim N(0, \sigma_{\delta_0}^2)$ ,  $\zeta_{0k} \sim N(0, \sigma_{\zeta_0}^2)$ ,  $\zeta_{1k} \sim N(0, \sigma_{\zeta_1}^2)$  and  $\epsilon_{tijk} \sim N(0, \sigma_{\epsilon}^2)$  all independent of each other except for  $\zeta_{0k}$  and  $\zeta_{1k}$  having a correlation  $\rho_{\zeta_0\zeta_1}$ .

```
fit2 <- lmer(math ~ year + sex + ses + minority + yearstea + mathknow + mathprep + housepov + (year|sch
summary(fit2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math ~ year + sex + ses + minority + yearstea + mathknow + mathprep +
## housepov + (year | schoolid) + (1 | childid)
## Data: class_pp
##
## REML criterion at convergence: 21275.7
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -4.6812 -0.4784  0.0040  0.4712  3.4245
```



```

##
## Random effects:
##   Groups   Name      Variance Std.Dev.  Corr
##   childid  (Intercept) 689.5    26.26
##   schoolid (Intercept) 249.2    15.79
##   year      year      114.2    10.69   -0.53
##   Residual                547.4    23.40
## Number of obs: 2162, groups:  childid, 1081; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  483.44962   4.79270   369.63678 100.872 < 2e-16 ***
## year         57.90494   1.49801    95.33745  38.655 < 2e-16 ***
## sex          -0.52171   1.95332  1033.75341  -0.267  0.789
## ses           9.59015   1.43816  1071.21739   6.668 4.14e-11 ***
## minority     -16.01167   2.82859   705.01716 -5.661 2.19e-08 ***
## yearstea      0.02304   0.11846   876.23254   0.194  0.846
## mathknow      -0.22237   1.17471   768.51612  -0.189  0.850
## mathprep      -1.08389   1.14195   932.69685  -0.949  0.343
## housepov     -18.17699  12.40599   108.12089  -1.465  0.146
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) year    sex    ses    minrty yearst mthknw mthprp
## year      -0.204
## sex        -0.194 -0.001
## ses        -0.120 -0.001  0.028
## minority   -0.333  0.004 -0.007  0.157
## yearstea   -0.240  0.001  0.018 -0.035  0.021
## mathknow   -0.085 -0.001  0.002 -0.012  0.106  0.038
## mathprep   -0.578 -0.002 -0.014  0.054  0.003 -0.182 -0.001
## housepov   -0.463  0.004 -0.007  0.080 -0.178  0.067  0.060  0.036

```

**For year==0:**

\*what percent of between school differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BS} = \sigma_{\zeta_0}^2 + 2 * year * \rho_{\zeta_0 \zeta_1} \sigma_{\zeta_0} \sigma_{\zeta_1} + year^2 * \sigma_{\zeta_1}^2$$

$$V_{1BS}(year = 0) = \sigma_{\zeta_0}^2 = 373.5$$

After adding fixed-effects:

$$V_{2BS} = \sigma_{\zeta_0}^2 + 2 * year * \rho_{\zeta_0 \zeta_1} \sigma_{\zeta_0} \sigma_{\zeta_1} + year^2 * \sigma_{\zeta_1}^2$$

$$V_{2BS}(year = 0) = \sigma_{\zeta_0}^2 = 249.2$$

The percent difference in between-school variance for  $year = 0$  is be given by:

$$\frac{V_{1BS} - V_{2BS}}{V_{1BS}} = \frac{373.5 - 249.2}{373.5} = 33.28\%$$

Model 2 explains 33.28% of the between-school variance for  $year = 0$ .

\*what percent of between child differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BC}(year = 0) = \sigma_{\delta_0}^2 = 749.0$$

After adding fixed effects:

$$V_{2BC}(year = 0) = \sigma_{\delta_0}^2 = 689.5$$

The percent difference in between-child variance explained by the second model for  $year = 0$  is given by:

$$\frac{V_{1BC} - V_{2BC}}{V_{1BC}} = \frac{749.0 - 689.5}{749.0} = 7.94\%$$

Model 2 explains 7.94% of the between-child variance for  $year = 0$ .

### For year==1:

\*what percent of between school differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BS} = \sigma_{\zeta_0}^2 + 2 * year * \rho_{\zeta_0 \zeta_1} \sigma_{\zeta_0} \sigma_{\zeta_1} + year^2 * \sigma_{\zeta_1}^2$$

$$V_{1BS}(year = 1) = 373.5 + 2(-0.53)(19.33)(10.60) + 112.4 = 268.71$$

After adding fixed-effects:

$$V_{2BS} = \sigma_{\zeta_0}^2 + 2 * year * \rho_{\zeta_0 \zeta_1} \sigma_{\zeta_0} \sigma_{\zeta_1} + year^2 * \sigma_{\zeta_1}^2$$

$$V_{2BS}(year = 1) = 249.2 + 2(-0.53)(15.79)(10.69) + 114.2 = 184.48$$

The percent difference in between-school variance for  $year = 0$  is be given by:

$$\frac{V_{1BS} - V_{2BS}}{V_{1BS}} = \frac{268.71 - 184.48}{268.71} = 31.35\%$$

Model 2 explains 31.35% of the between-school variance for  $year = 1$ .

\*what percent of between child differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BC}(year = 1) = \sigma_{\delta_0}^2 = 749.0$$

After adding fixed effects:

$$V_{2BC}(year = 1) = \sigma_{\delta_0}^2 = 689.5$$

The percent difference in between-child variance explained by the second model for  $year = 1$  is given by:

$$\frac{V_{1BC} - V_{2BC}}{V_{1BC}} = \frac{749.0 - 689.5}{749.0} = 7.94\%$$

Model 2 explains 7.94% of the between-child variance for  $year = 1$ .

## Based on significance,

- what factors seem useful in describing (“explaining”) differences between student outcomes?
- Point out the direction of the effect.

*SES* and *MINORITY* status are the significant fixed-effects terms in the model at  $\alpha = 0.05$  implying that these terms (being in Level 1) help to explain the between-student variance conditional on the school.

The coefficient on *SES* is positive meaning that two students in the same school and student-level random effect and all else equal, the one with the higher *SES* has a higher Math score.

The coefficient on *MINORITY* status is negative meaning that two students in the same school and student-level random effect and all else equal, the one who is classified as a Minority student has a lower Math score.

## Add random slope for SES

$$MATH_{tijk} = b_0 + \delta_{0ijk} + (b_1 + \zeta_{1k})TIME_{tijk} + b_2SEX_{ijk} + (b_3 + \zeta_{3k})SES_{ijk} + b_4MINORITY_{ijk} + b_5YEARSTEAK_{jk} + b_6MATHKNOW_{jk} + b_7MATHPREP_{jk} + b_8HOUSEPOV_k + \epsilon_{tijk}$$

where  $t$  represents occasion (in this case, year/grade),  $i$  represents students,  $j$  represents classrooms and  $k$  represents schools.  $\delta_{0ijk} \sim N(0, \sigma_{\delta_0}^2)$ ,  $\zeta_{0k} \sim N(0, \sigma_{\zeta_0}^2)$ ,  $\zeta_{1k} \sim N(0, \sigma_{\zeta_1}^2)$ ,  $\zeta_{3k} \sim N(0, \sigma_{\zeta_3}^2)$  and  $\epsilon_{ijk} \sim N(0, \sigma_{\epsilon}^2)$  all independent of each other except for  $\zeta_{0k}$ ,  $\zeta_{1k}$  and  $\zeta_{3k}$  could be correlated.

```
fit3 <- lmer(math ~ year + sex + ses + minority + yearstea + mathknow + mathprep + housepov + (ses+year
summary(fit3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math ~ year + sex + ses + minority + yearstea + mathknow + mathprep +
##   housepov + (ses + year | schoolid) + (1 | childid)
##   Data: class_pp
##
## REML criterion at convergence: 21273.1
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -4.6663 -0.4808  0.0048  0.4722  3.4249
##
## Random effects:
##   Groups   Name                Variance Std.Dev. Corr
##   childid  (Intercept)         668.11   25.848
##   schoolid (Intercept)         251.26   15.851
##           ses                  46.49    6.818  -0.03
##           year                 114.87   10.718  -0.53  0.15
##   Residual                    547.29   23.394
## Number of obs: 2162, groups:  childid, 1081; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  483.20541    4.79250   363.61574 100.825 < 2e-16 ***
## year         57.88435    1.50002    95.29594  38.589 < 2e-16 ***
## sex          -0.70446    1.94358  1016.33760  -0.362  0.717
## ses           9.32191    1.63892    69.42401   5.688 2.81e-07 ***
## minority     -16.26826    2.83356   658.65225 -5.741 1.43e-08 ***
```

```

## yearstea      0.03404    0.11840  876.03584    0.288    0.774
## mathknow     -0.28980    1.17539  774.87068   -0.247    0.805
## mathprep     -1.04672    1.13615  916.71569   -0.921    0.357
## housepov     -17.57739   12.45037  105.24186   -1.412    0.161
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) year    sex    ses    minrty yearst mthknw mthprp
## year      -0.205
## sex       -0.192 -0.001
## ses       -0.098  0.047  0.025
## minority  -0.335  0.004 -0.007  0.125
## yearstea  -0.240  0.002  0.020 -0.027  0.020
## mathknow  -0.080 -0.001  0.001  0.001  0.100  0.037
## mathprep  -0.575 -0.002 -0.015  0.045  0.002 -0.182 -0.003
## housepov  -0.463  0.005 -0.007  0.074 -0.180  0.067  0.059  0.037

```

*is the estimated s.d. (square root of variance) of the random slope associated with SES large enough* so that a value  $\pm 1$  s.d. is sufficient to “cancel” (or flip the sign) the fixed effect for this predictor?

The estimated standard deviation of the random slope associated with SES is 6.818. We note that this is not large enough to “cancel” or flip the sign of the fixed-effect on SES within  $\pm 1$  standard deviation.

The majority of the values (middle 68% ) for the fixed effect on SES is within the range  $[(9.32191 - 6.818), (9.32191 + 6.818)] = [2.50391, 16.13991]$

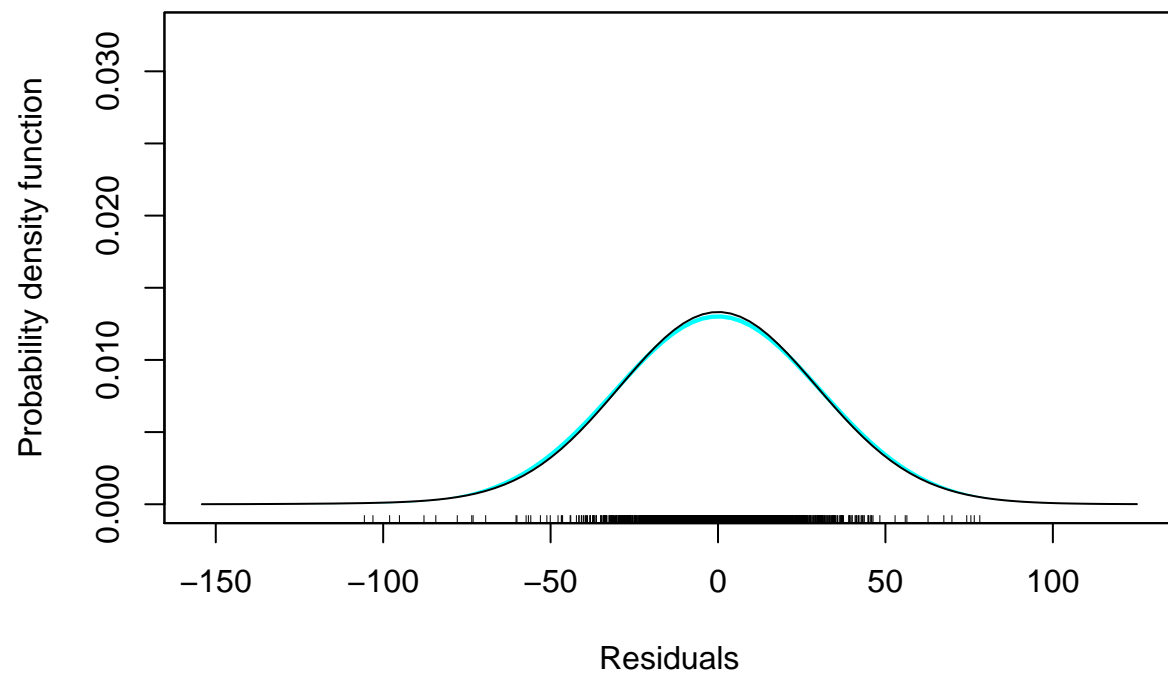
## Residuals and Q-Q Plot

*compute residuals in this final model. generate a qq plot and density (STATA: `qnorm`; `kdensity` ..., `normal`)*  
Is there any reason to question the normality assumption?

```

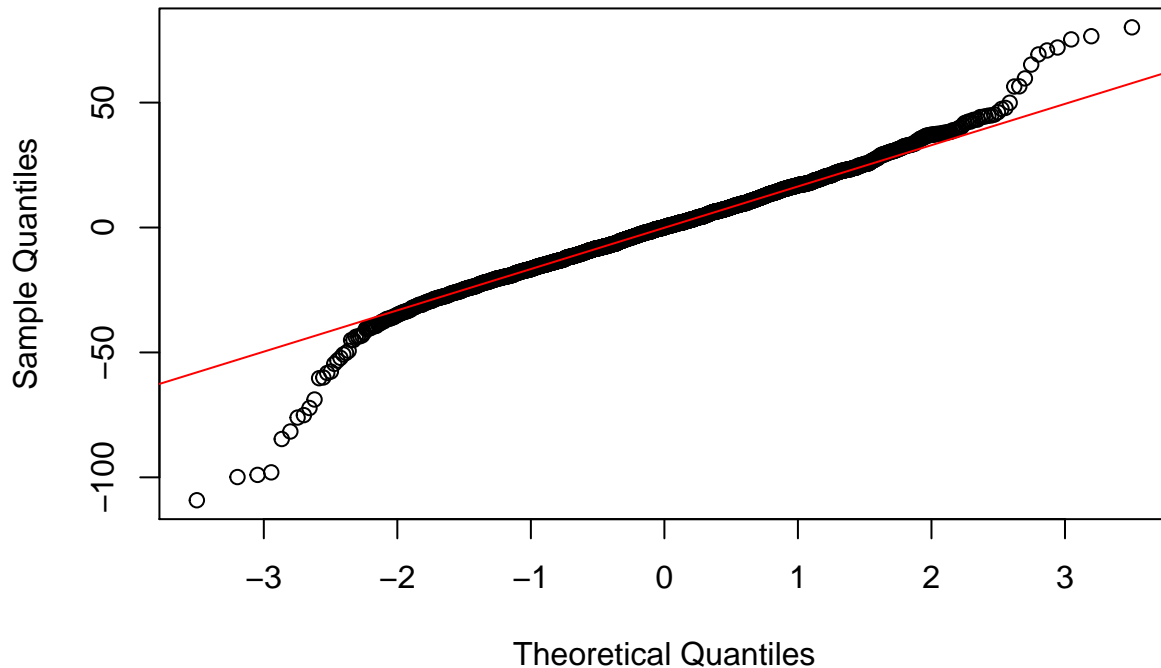
fit3.residuals <- residuals(fit3)
sm.density(fit3.residuals,model="Normal",xlab="Residuals")

```



```
qqnorm(y=fit3.residuals)  
qqline(y=fit3.residuals,col=2)
```

## Normal Q-Q Plot



The residuals seem to have slightly heavier tails compared to a standard normal distribution. From the density plot, we also note that given the sample size, the peak is slightly higher than expected for a normal distribution. The distribution of residuals does not fall within the expected range given by the blue region although the deviation is minimal. Within a range of 4-sd (between  $\pm 2$  sd), the Residual quantiles are seen to be linear compared to the Theoretical quantiles implying that the residuals are indeed quite normally distributed for the most part.

## BLUPs for all 4 random effects & Scatter plots

generate an all pairs scatter plot matrix (4x4) of these \* note whether or not you identify any concerns from these scatterplots.

```
ranefs <- ranef(fit3)
Delta0 <- ranefs$childid
idx.school <- match(classroom2$schoolid, sort(unique(classroom2$schoolid)))
Zeta0 <- ranefs$schoolid[idx.school,1]
Zeta1 <- ranefs$schoolid[idx.school,2]
Zeta2 <- ranefs$schoolid[idx.school,3]

ranefs <- data.frame(delta0=Delta0,zeta0=Zeta0,zeta1=Zeta1,zeta2=Zeta2)
colnames(ranefs) <- c("Delta0","Zeta0","Zeta1","Zeta2")
pairs(ranefs,cex=0.5)
```

