

## Part 1: Frankie

### Create 1st grade variable

```
classroom <- classroom %>% mutate(Math1 = mathkind + mathgain)
```

### Random Intercepts for classroom, nested in schools UMM

```
model1 <- lmer(Math1 ~ (1 | schoolid/classid), data = classroom)
summary(model1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
## Groups              Name             Variance Std.Dev.
## classid:schoolid (Intercept)    85.46    9.244
## schoolid          (Intercept)   280.68   16.754
## Residual                        1146.80   33.864
## Number of obs: 1190, groups: classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.406   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$ICC_{class} = \frac{85.46}{1146.8 + 280.68 + 85.46} \approx .056$$

$$ICC_{school} = \frac{280.68}{1146.8 + 280.68 + 85.46} \approx .186$$

$$Math1st_{ijk} = \beta_{0ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$\zeta_k \sim N(0, \sigma_\zeta^2)$ ,  $\eta_{jk} \sim N(0, \sigma_\eta^2)$ , and  $\epsilon_{ijk} \sim N(0, \sigma_\epsilon^2)$ , all are independent of each other

## Model with School Level Predictors Added

```
model2 <- lmer(Math1~housepov+(1|schoolid/classid),data=classroom)
summary(model2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ housepov + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
## Groups             Name             Variance Std.Dev.
## classid:schoolid (Intercept)    82.36    9.075
## schoolid          (Intercept)  250.93   15.841
## Residual                        1146.95   33.867
## Number of obs: 1190, groups: classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)   531.294      3.341 102.809 159.024 <2e-16 ***
## housepov      -45.783     14.236 111.063  -3.216  0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr)
## housepov -0.810
```

```
anova(model1, model2, refit = F)
```

```
## Data: classroom
## Models:
## model1: Math1 ~ (1 | schoolid/classid)
## model2: Math1 ~ housepov + (1 | schoolid/classid)
##      Df   AIC   BIC logLik deviance Chisq Chi Df Pr(>Chisq)
## model1  4 11953 11973 -5972.3   11945
## model2  5 11937 11963 -5963.7   11927 17.186      1 3.39e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Change in  $\sigma_{\zeta}^2$ : decreased to 250.93 from 280.63  $\sigma_{\eta}^2$  decreases to 82.36 from 85.46  $\sigma_{\epsilon}^2$  slightly increases to 1146.95 from 1146.8

The ANOVA/LRT has a pvalue of almost zero, 3.39e-05, thus we reject the  $H_0$  at our  $\alpha = 0.05$  and meaning that it makes sense to include the school level predictor, housepov.

## Model with all Class Level Predictors Added

```
model3 <- lmer(Math1~housepov+mathknow+yearstea+mathprep+(1|schoolid/classid),data=classroom)
summary(model3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
## Groups           Name          Variance Std.Dev.
## classid:schoolid (Intercept)   94.36    9.714
## schoolid         (Intercept)  223.31   14.943
## Residual                        1136.43  33.711
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  532.29853    5.20496 228.85764 102.268 < 2e-16 ***
## housepov     -41.62116   14.08835 109.83227  -2.954  0.00383 **
## mathknow       2.55143    1.44530 231.06566   1.765  0.07883 .
## yearstea       0.06193    0.14717 223.76582   0.421  0.67432
## mathprep      -0.75440    1.42809 203.20767  -0.528  0.59790
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst
## housepov -0.568
## mathknow -0.052  0.082
## yearstea -0.264  0.077  0.030
## mathprep -0.666  0.032  0.004 -0.175
```

## creating reduced dataset taking away missing data

To deal with the missing data, we removed those students. This left us with a sample of 1081 students.

```
classroom_red = na.omit(classroom)
model2_red <- lmer(Math1~housepov+(1|schoolid/classid),data=classroom_red)
model3_red <- lmer(Math1~housepov+mathknow+yearstea+mathprep+(1|schoolid/classid),data=classroom_red)
anova(model2_red, model3_red, refit = F)
```

```
## Data: classroom_red
## Models:
## model2_red: Math1 ~ housepov + (1 | schoolid/classid)
## model3_red: Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
```

```
##           Df    AIC    BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model2_red  5 10838 10862 -5413.8    10828
## model3_red  8 10837 10877 -5410.5    10821 6.5771      3    0.08667 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Change in  $\sigma_\epsilon^2$  and  $\sigma_\eta^2$ :  $\sigma_\epsilon^2$  decreased to 1136.43,  $\sigma_\eta^2$  increased to 94.36;  $\sigma_\zeta^2 = 223.31$

The reason epsilon was reduced but eta was not is because the new model explains what is happening at a student level, but not at a classroom level. In addition adding the classroom level predictors makes it so that more of the overall variation is explained by “structured” variation rather than by unstructured ( $\epsilon$ ) May increase because of sample decrease (missing data) –

The anova test comparing the school level predictor to the model with the classroom predictors has a p-value 0.087, so we fail to reject the null hypothesis at our  $\alpha = 0.05$  and thus though borderline to significance, it still concludes that the models are not different so adding the classroom level predictors isn’t necessary.

## Add all student-level predictors

```
model4 <- lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1|schoolid/classid),data=classroom)
summary(model4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid/classid)
##      Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##      Groups             Name             Variance Std.Dev.
## classid:schoolid (Intercept)    93.89     9.69
## schoolid         (Intercept)   169.45    13.02
## Residual                        1064.95    32.63
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210   275.38922 101.585 < 2e-16 ***
## housepov     -17.64847   13.21757   113.87774  -1.335    0.184
## mathknow       1.35004    1.39168   234.49776   0.970    0.333
## yearstea       0.01129    0.14141   226.80899   0.080    0.936
## mathprep      -0.27705    1.37583   205.27157  -0.201    0.841
## sex           -1.21419    2.09483  1022.42136  -0.580    0.562
## minority     -16.18678    3.02605   704.47889  -5.349 1.20e-07 ***
## ses           10.05075    1.54484  1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

We test this new block compared to the model with just school level predictors as the classroom level predictors were not significant.

```
model4_red <- lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1|schoolid/classid),data=
anova(model2_red, model4_red, refit = F)
```

```
## Data: classroom_red
## Models:
## model2_red: Math1 ~ housepov + (1 | schoolid/classid)
## model4_red: Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
## model4_red:      ses + (1 | schoolid/classid)
##          Df   AIC   BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
## model2_red  5 10838 10862 -5413.8    10828
## model4_red 11 10752 10806 -5364.8    10730 98.023      6 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
anova(model3, model4, refit = F)
```

```
## Data: classroom
## Models:
## model3: Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
## model4: Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
## model4:      ses + (1 | schoolid/classid)
##          Df   AIC   BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
## model3   8 10837 10877 -5410.5    10821
## model4  11 10752 10806 -5364.8    10730 91.446      3 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$\sigma_\epsilon^2$  decreased to 1064.95,  $\sigma_\eta^2$  decreased to 93.89, and  $\sigma_\zeta^2$  decreased to 169.45.

School level may drop because students may be similar within schools but different between schools, or the fact that math know directly effects school level effects, better schools tend to have better teachers

$$\hat{math1st}_{ijk} = \beta_{0ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk} + \beta_1 Housepov_k + \beta_2 Mathknow_{jk} + \beta_3 YearsTea_{jk} + \beta_4 Mathprep_{jk} + \beta_5 sex_{ijk} + \beta_6 minority_{ijk}$$

The anova test comparing the model with school and classroom level predictors to the model with almost all the predictors has a p-value that is approximately zero at  $< 2.2e-16$ , so we reject  $H_0$  and conclude that it makes sense to include student level predictors. Moreover, the Chi-Sq test comparing the model with just school level predictors to the model with almost all predictors has a p-value  $< 2.2e-16$ , so we conclude that the model with student level predictors (as a block) improves compared to the model with only school-level predictors both somewhat reiterating the other.

## Random Slope for Teacher-level predictor varying at school-level

```
rst.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+mathknow||schoolid)+(1|classid)
summary(rst.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##       ses + (1 + mathknow || schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups      Name                Variance Std.Dev.
## classid     (Intercept) 9.389e+01 9.690e+00
## schoolid    mathknow    4.260e-11 6.527e-06
## schoolid.1 (Intercept) 1.694e+02 1.302e+01
## Residual                    1.065e+03 3.263e+01
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38921 101.585 < 2e-16 ***
## housepov     -17.64847   13.21757  113.87774  -1.335  0.184
## mathknow      1.35004    1.39168  234.49776   0.970  0.333
## yearstea      0.01129    0.14141  226.80899   0.080  0.936
## mathprep     -0.27705    1.37583  205.27156  -0.201  0.841
## sex          -1.21419    2.09483 1022.42136  -0.580  0.562
## minority     -16.18678    3.02605  704.47889  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rst.1,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
```

```
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid) + (0 + mathknow | schoolid) + (1 | classid)
##
##              npar  logLik   AIC    LRT Df
## <none>              12 -5364.8 10754
## (1 | schoolid)      11 -5376.5 10775 23.410 1
## mathknow in (0 + mathknow | schoolid) 11 -5364.8 10752 0.000 1
## (1 | classid)       11 -5368.1 10758 6.741 1
##
##              Pr(>Chisq)
## <none>
## (1 | schoolid)      1.309e-06 ***
## mathknow in (0 + mathknow | schoolid) 0.999999
## (1 | classid)       0.009422 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is not a need for the random slope for math knowledge at a school level as the p value is not significant at .999

```
rst.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+yearstea||schoolid)+(1|classid)|
summary(rst.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + yearstea || schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8485 -0.6149 -0.0323  0.5980  3.6600
##
## Random effects:
## Groups      Name                Variance Std.Dev.
## classid     (Intercept) 9.266e+01  9.62593
## schoolid    yearstea    9.669e-03  0.09833
## schoolid.1  (Intercept) 1.685e+02 12.97894
## Residual                    1.065e+03 32.63452
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.60060    5.30865 266.34157 101.645 < 2e-16 ***
## housepov     -17.71727   13.21854 113.56407  -1.340  0.183
## mathknow       1.33198    1.39177 234.33551   0.957  0.340
## yearstea       0.01124    0.14193 122.38000   0.079  0.937
## mathprep      -0.26633    1.37610 204.91605  -0.194  0.847
## sex           -1.21077    2.09476 1022.22247  -0.578  0.563
## minority     -16.16833    3.02641 702.64837  -5.342 1.24e-07 ***
## ses           10.04529    1.54490 1066.09768   6.502 1.21e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst mthprp sex    minrty
## housepv -0.450
## mathknow -0.082  0.057
## yearstea -0.258  0.070  0.028
## mathprep -0.632  0.037  0.003 -0.172
## sex      -0.190 -0.007  0.006  0.015 -0.006
## minority -0.320 -0.179  0.115  0.023  0.001 -0.010
## ses      -0.121  0.082 -0.007 -0.027  0.053  0.020  0.162
```

```
ranova(rst.2, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid) + (0 + yearstea | schoolid) + (1 | classid)
##
##      npar  logLik   AIC     LRT Df
## <none>          12 -5364.8 10754
## (1 | schoolid)    11 -5374.7 10771 19.8301 1
## yearstea in (0 + yearstea | schoolid) 11 -5364.8 10752 0.0070 1
## (1 | classid)     11 -5367.7 10757 5.9158 1
##
##      Pr(>Chisq)
## <none>
## (1 | schoolid)      8.464e-06 ***
## yearstea in (0 + yearstea | schoolid) 0.93342
## (1 | classid)       0.01501 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There seems to be no need for for the random slope for years teaching at a school level as the p value is insignificant at .933

```
rst.3 <-lmer(Math1~housepv+mathknow+yearstea+mathprep+sex+minority+ses+(1+mathprep||schoolid)+(1|classid)|Data=classroom)
summary(rst.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + mathprep || schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups      Name                Variance Std.Dev.
## classid     (Intercept)         93.89    9.69
## schoolid    mathprep              0.00    0.00
## schoolid.1  (Intercept)        169.45   13.02
## Residual                    1064.95   32.63
## Number of obs: 1081, groups: classid, 285; schoolid, 105
```



```
##
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38917 101.585 < 2e-16 ***
## housepov     -17.64847   13.21758  113.87771  -1.335  0.184
## mathknow      1.35004    1.39168  234.49776   0.970  0.333
## yearstea      0.01129    0.14141  226.80899   0.080  0.936
## mathprep     -0.27705    1.37583  205.27157  -0.201  0.841
## sex          -1.21419    2.09483 1022.42137  -0.580  0.562
## minority     -16.18678    3.02605  704.47892  -5.349 1.20e-07 ***
## ses          10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
ranova(rst.3, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid) + (0 + mathprep | schoolid) + (1 | classid)
##           npar logLik  AIC    LRT Df
## <none>          12 -5364.8 10754
## (1 | schoolid)    11 -5371.6 10765 13.6179 1
## mathprep in (0 + mathprep | schoolid) 11 -5364.8 10752 0.0000 1
## (1 | classid)     11 -5368.3 10759 7.1357 1
##           Pr(>Chisq)
## <none>
## (1 | schoolid)          0.000224 ***
## mathprep in (0 + mathprep | schoolid) 1.000000
## (1 | classid)          0.007556 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There seems to be no need for for the random slope for math prep at a school level as the p value is insignificant at 1.00

**Question:** Why housepov bad idea?

**Answer:** There is only one data point per school, so we cannot have a random slope since we can't even calculate a slope.

## Allowing correlations with random intercepts

### ONE BY ONE

```
rstc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+mathknow|schoolid)+(1|classid)
summary(rstc.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + mathknow | schoolid) + (1 | classid)
##      Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8581 -0.6131 -0.0324  0.5969  3.6603
##
## Random effects:
##      Groups      Name      Variance Std.Dev. Corr
##      classid (Intercept) 9.394e+01  9.69205
##      schoolid (Intercept) 1.693e+02 13.01223
##              mathknow    8.596e-04  0.02932  1.00
##      Residual              1.065e+03 32.63393
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.64037    5.31212   275.37948 101.587 < 2e-16 ***
## housepov     -17.64148   13.21274   103.97679  -1.335    0.185
## mathknow       1.35459    1.39203   214.63820   0.973    0.332
## yearstea       0.01114    0.14141   226.85277   0.079    0.937
## mathprep      -0.27753    1.37601   201.27912  -0.202    0.840
## sex           -1.21329    2.09485  1021.79964  -0.579    0.563
## minority     -16.19376    3.02609   703.81038  -5.351 1.18e-07 ***
## ses            10.04788    1.54488  1062.12341   6.504 1.20e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthkwn yearst mthprp sex    minrty
## housepov    -0.451
## mathknow     -0.082  0.057
## yearstea     -0.259  0.071  0.029
## mathprep     -0.631  0.038  0.004 -0.173
## sex          -0.190 -0.007  0.007  0.016 -0.006
## minority     -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses          -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rstc.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
```

```
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + mathknow | schoolid) + (1 | classid)
##
##               npar logLik   AIC   LRT Df
## <none>                13 -5364.8 10756
## mathknow in (1 + mathknow | schoolid)  11 -5364.8 10752 0.0003  2
## (1 | classid)                12 -5368.1 10760 6.6768  1
##
##               Pr(>Chisq)
## <none>
## mathknow in (1 + mathknow | schoolid)  0.999840
## (1 | classid)                0.009767 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated math knowledge is insignificant and seems to add no value to the model.

yearstea

```
rstc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+yearstea|schoolid)+(1|classid),
summary(rstc.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + yearstea | schoolid) + (1 | classid)
##      Data: classroom
##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.7462 -0.6036 -0.0290  0.6041  3.8449
##
## Random effects:
##      Groups   Name                Variance Std.Dev. Corr
##      classid  (Intercept)         37.9283  6.1586
##      schoolid (Intercept)        366.1148 19.1341
##              yearstea             0.5523  0.7432 -0.78
##      Residual                    1066.4510 32.6566
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  538.95245    5.48825  222.69673  98.201 < 2e-16 ***
## housepov     -17.13994   13.45959  119.63687  -1.273  0.205
## mathknow       1.04635    1.34381  209.72527   0.779  0.437
## yearstea       0.02204    0.15766   75.76696   0.140  0.889
## mathprep       0.05046    1.34549  190.82671   0.038  0.970
## sex          -1.33553    2.08774 1024.45936  -0.640  0.523
## minority     -16.44555    2.99655  669.50401  -5.488 5.77e-08 ***
## ses           10.15038    1.53873 1062.66131   6.597 6.62e-11 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) housepv mthknw yearst mthprp sex    minrty
## housepv -0.455
## mathknow -0.085  0.049
## yearstea -0.370  0.084  0.012
## mathprep -0.606  0.050  0.014 -0.139
## sex      -0.184 -0.004  0.008  0.009 -0.004
## minority -0.305 -0.169  0.122  0.032 -0.007 -0.012
## ses      -0.119  0.079 -0.001 -0.019  0.049  0.022  0.168
ranova(rstc.2,refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + yearstea | schoolid) + (1 | classid)
##
##              npar logLik   AIC   LRT Df
## <none>              13 -5361.8 10750
## yearstea in (1 + yearstea | schoolid)  11 -5364.8 10752 5.8254  2
## (1 | classid)              12 -5362.3 10749 0.9028  1
##
##              Pr(>Chisq)
## <none>
## yearstea in (1 + yearstea | schoolid)    0.05433 .
## (1 | classid)              0.34202
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

This correlated random slope for years teaching is right on the cusp of significance and should be observed further in attempts to understand its need for adding it to the model it has a p value of .0543.

## mathprep

```
rstc.3 <-lmer(Math1~housepv+mathknow+yearstea+mathprep+sex+minority+ses+(1+mathprep|schoolid)+(1|classid),
summary(rstc.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + mathprep | schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10724.7
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8542 -0.6034 -0.0221  0.5915  3.6475
##
## Random effects:
## Groups   Name                Variance Std.Dev. Corr
```

```

## classid (Intercept) 78.46 8.858
## schoolid (Intercept) 552.76 23.511
## mathprep 15.89 3.986 -1.00
## Residual 1064.26 32.623
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 538.60855 5.60813 159.88774 96.041 < 2e-16 ***
## housepov -14.01306 12.88689 116.05900 -1.087 0.279
## mathknow 1.29884 1.37194 229.68146 0.947 0.345
## yearstea -0.02586 0.13949 223.50098 -0.185 0.853
## mathprep 0.04074 1.34845 139.04228 0.030 0.976
## sex -1.16759 2.08697 1023.15084 -0.559 0.576
## minority -16.46422 2.99524 663.67316 -5.497 5.52e-08 ***
## ses 10.14166 1.53961 1060.93421 6.587 7.04e-11 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) houspv mthknw yearst mthprp sex minrty
## housepov -0.461
## mathknow -0.071 0.027
## yearstea -0.260 0.089 0.049
## mathprep -0.692 0.107 0.012 -0.155
## sex -0.183 0.003 0.002 0.023 -0.008
## minority -0.275 -0.187 0.107 0.025 -0.035 -0.013
## ses -0.121 0.095 -0.001 -0.033 0.061 0.024 0.161
ranova(rstc.3, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
## ses + (1 + mathprep | schoolid) + (1 | classid)
## npar logLik AIC LRT Df
## <none> 13 -5362.3 10751
## mathprep in (1 + mathprep | schoolid) 11 -5364.8 10752 4.8144 2
## (1 | classid) 12 -5364.9 10754 5.0971 1
## Pr(>Chisq)
## <none>
## mathprep in (1 + mathprep | schoolid) 0.09007 .
## (1 | classid) 0.02397 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

The correlated math prep is just a bit too high with a pvalue of .09, thus it is insignificant and seems to add no value to the model.

**Question:** Anything unusual about the variances? Why might this have occurred? (hint: what did you add to the model?)

**Answer:** The random slope for mathknow greatly increases in the second model, which is probably due to its correlation with the random intercept at the school-level.

There seems to be an issue with the model as the slope and intercept correlation is negative one, this could

be due to the sample sizes of the classrooms as some only have a single observation.

## Random slopes for student-level predictors varying at classroom level

### ONE BY ONE

sex

```
rss.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+sex||classid)+(1|schoolid),data=classroom,REML=FALSE)
summary(rss.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##       ses + (1 + sex || classid) + (1 | schoolid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups      Name                Variance Std.Dev.
## classid     (Intercept)         93.89    9.69
## classid.1 sex                   0.00     0.00
## schoolid    (Intercept)        169.45   13.02
## Residual                    1064.95   32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38920 101.585 < 2e-16 ***
## housepov     -17.64847   13.21757  113.87773  -1.335  0.184
## mathknow       1.35004    1.39168  234.49776   0.970  0.333
## yearstea       0.01129    0.14141  226.80899   0.080  0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201  0.841
## sex           -1.21419    2.09483 1022.42137  -0.580  0.562
## minority     -16.18678    3.02605  704.47890  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.451
## mathknow    -0.083  0.058
## yearstea    -0.259  0.071  0.029
## mathprep    -0.631  0.038  0.004 -0.172
## sex         -0.190 -0.007  0.007  0.016 -0.006
## minority    -0.320 -0.178  0.115  0.024  0.001 -0.011
```

```
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rss.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
```

```
##
```

```
## Model:
```

```
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
```

```
##      ses + (1 | classid) + (0 + sex | classid) + (1 | schoolid)
```

```
##                                     npar logLik   AIC      LRT Df Pr(>Chisq)
```

```
## <none>                             12 -5364.8 10754
```

```
## (1 | classid)                       11 -5368.0 10758  6.4894  1    0.01085 *
```

```
## sex in (0 + sex | classid)          11 -5364.8 10752  0.0000  1    1.00000
```

```
## (1 | schoolid)                      11 -5377.1 10776 24.7881  1   6.399e-07 ***
```

```
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Sex random slope with class is insignificant with a p value of 1.

minority

```
rss.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+minority||classid)+(1|schoolid), data=classroom)
summary(rss.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
```

```
## lmerModLmerTest]
```

```
## Formula:
```

```
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
```

```
##      ses + (1 + minority || classid) + (1 | schoolid)
```

```
##      Data: classroom
```

```
##
```

```
## REML criterion at convergence: 10729.5
```

```
##
```

```
## Scaled residuals:
```

```
##      Min       1Q   Median       3Q      Max
```

```
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
```

```
##
```

```
## Random effects:
```

```
##      Groups      Name      Variance Std.Dev.
```

```
##      classid  (Intercept)    93.89    9.69
```

```
##      classid.1 minority      0.00    0.00
```

```
##      schoolid (Intercept)  169.45   13.02
```

```
##      Residual              1064.95   32.63
```

```
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
```

```
##
```

```
## Fixed effects:
```

```
##      Estimate Std. Error      df t value Pr(>|t|)
```

```
## (Intercept)  539.63042    5.31210  275.38919 101.585 < 2e-16 ***
```

```
## housepov     -17.64847   13.21758  113.87772  -1.335    0.184
```

```
## mathknow      1.35004    1.39168  234.49776   0.970    0.333
```

```
## yearstea      0.01129    0.14141  226.80899   0.080    0.936
```

```
## mathprep     -0.27705    1.37583  205.27157  -0.201    0.841
```

```
## sex          -1.21419    2.09483 1022.42137  -0.580    0.562
```

```
## minority     -16.18678    3.02605  704.47891  -5.349 1.20e-07 ***
```

```
## ses          10.05075      1.54484 1066.56223    6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
ranova(rss.1, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | classid) + (0 + sex | classid) + (1 | schoolid)
##               npar logLik   AIC    LRT Df Pr(>Chisq)
## <none>                12 -5364.8 10754
## (1 | classid)           11 -5368.0 10758  6.4894 1    0.01085 *
## sex in (0 + sex | classid) 11 -5364.8 10752  0.0000 1    1.00000
## (1 | schoolid)          11 -5377.1 10776 24.7881 1   6.399e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Sex random slope with class id is insignificant with a p value of 1.0.

## SES

```
rss.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+ses||classid)+(1|schoolid),data=classroom)
summary(rss.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + ses || classid) + (1 | schoolid)
##      Data: classroom
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##      Groups      Name              Variance Std.Dev.
##      classid   (Intercept)         87.11    9.333
##      classid.1 ses              49.60     7.043
##      schoolid  (Intercept)       171.02   13.077
```



```
## Residual                1043.44  32.302
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641   274.46487 101.710 < 2e-16 ***
## housepov     -17.50879   13.21775   113.44869  -1.325   0.188
## mathknow       1.36796    1.38563   229.40646   0.987   0.325
## yearstea       0.01103    0.14117   226.97687   0.078   0.938
## mathprep      -0.27938    1.37171   204.89340  -0.204   0.839
## sex          -1.37733    2.09334  1022.81818  -0.658   0.511
## minority     -16.29362    3.02464   703.33762  -5.387 9.78e-08 ***
## ses           10.14363    1.64248   176.39739   6.176 4.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.451
## mathknow    -0.082  0.058
## yearstea    -0.259  0.070  0.029
## mathprep    -0.631  0.040  0.005 -0.172
## sex         -0.190 -0.007  0.006  0.014 -0.005
## minority    -0.321 -0.180  0.111  0.025  0.002 -0.011
## ses         -0.108  0.081  0.002 -0.026  0.050  0.020  0.145

ranova(rss.3, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | classid) + (0 + ses | classid) + (1 | schoolid)
##              npar logLik   AIC      LRT Df Pr(>Chisq)
## <none>              12 -5364.0 10752
## (1 | classid)         11 -5366.9 10756  5.9221  1    0.01495 *
## ses in (0 + ses | classid) 11 -5364.8 10752  1.5969  1    0.20634
## (1 | schoolid)        11 -5376.6 10775 25.2710  1  4.982e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

With a p-value of .206 ses is insignificant from an uncorrelated random slope at classroom level.

**Question:** why is this a bad idea to include a classroom-level variable with random slopes at classroom-level?

**Answer:** It may not explain much variance due to the fact that it seems somewhat redundant.

## Allowing for correlations with random intercepts

### ONE BY ONE

Sex

```
rssc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+sex|classid)+(1|schoolid),d
summary(rssc.1)
```

```

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + sex | classid) + (1 | schoolid)
##      Data: classroom
##
## REML criterion at convergence: 10729
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.7565 -0.6134 -0.0307  0.5916  3.7116
##
## Random effects:
##      Groups      Name      Variance Std.Dev. Corr
##      classid (Intercept) 130.07  11.41
##              sex         31.36   5.60  -0.67
##      schoolid (Intercept) 169.85  13.03
##      Residual          1056.41  32.50
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  5.400e+02  5.332e+00  2.723e+02 101.285 < 2e-16 ***
## housepov     -1.829e+01  1.323e+01  1.145e+02  -1.382   0.170
## mathknow      1.306e+00  1.391e+00  2.315e+02   0.939   0.349
## yearstea      3.087e-03  1.416e-01  2.270e+02   0.022   0.983
## mathprep     -3.459e-01  1.374e+00  2.014e+02  -0.252   0.801
## sex          -1.197e+00  2.122e+00  2.160e+02  -0.564   0.573
## minority     -1.619e+01  3.028e+00  7.042e+02  -5.347 1.21e-07 ***
## ses           1.010e+01  1.544e+00  1.065e+03   6.539 9.62e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.452
## mathknow     -0.085  0.060
## yearstea     -0.258  0.072  0.029
## mathprep     -0.628  0.040  0.005 -0.174
## sex          -0.203 -0.005  0.003  0.015 -0.008
## minority     -0.321 -0.178  0.116  0.024  0.003 -0.009
## ses          -0.123  0.083 -0.005 -0.027  0.054  0.020  0.164
ranova(rssc.1, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + sex | classid) + (1 | schoolid)
##              npar logLik   AIC      LRT Df Pr(>Chisq)
## <none>              13 -5364.5 10755

```

```
## sex in (1 + sex | classid)    11 -5364.8 10752  0.5003  2      0.7787
## (1 | schoolid)                12 -5377.0 10778 24.8912  1  6.066e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated random slope is insignificant with a p value of .779.

## Minority

```
rssc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+minority|classid)+(1|schoolid),
summary(rssc.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority | classid) + (1 | schoolid)
##      Data: classroom
##
## REML criterion at convergence: 10726.3
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.9037 -0.6221 -0.0295  0.6033  3.4574
##
## Random effects:
##      Groups      Name      Variance Std.Dev. Corr
##      classid (Intercept) 225.4    15.01
##      minority      171.3    13.09   -0.82
##      schoolid (Intercept) 157.4    12.55
##      Residual      1045.3    32.33
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.73594   5.38023  270.70509 100.318 < 2e-16 ***
## housepov     -17.34698  12.91268  103.34670  -1.343   0.182
## mathknow       1.45702   1.39355  234.04713   1.046   0.297
## yearstea     -0.01636   0.14285  234.25121  -0.115   0.909
## mathprep     -0.13520   1.37018  203.97000  -0.099   0.921
## sex          -1.01012   2.08966 1015.73461  -0.483   0.629
## minority     -16.48614   3.21756  183.20472  -5.124 7.55e-07 ***
## ses           9.89350   1.54595 1062.82882   6.400 2.33e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.435
## mathknow    -0.079  0.061
## yearstea    -0.265  0.080  0.038
## mathprep    -0.618  0.037 -0.006 -0.171
## sex         -0.188 -0.009  0.009  0.015 -0.005
```

```
## minority -0.368 -0.171 0.108 0.025 -0.004 -0.009
## ses      -0.117 0.085 0.001 -0.023 0.051 0.021 0.149
```

```
ranova(rssc.2)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority | classid) + (1 | schoolid)
##
##               npar  logLik   AIC      LRT Df
## <none>                13 -5363.2 10752
## minority in (1 + minority | classid)  11 -5364.8 10752  3.1967  2
## (1 | schoolid)                12 -5373.2 10770 20.1422  1
##
##               Pr(>Chisq)
## <none>
## minority in (1 + minority | classid)    0.2022
## (1 | schoolid)                7.189e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated random slope for minority is insignificant with a p value of .202.

## SES

```
rssc.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+ses|classid)+(1|schoolid),d
summary(rssc.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + ses | classid) + (1 | schoolid)
##      Data: classroom
##
## REML criterion at convergence: 10725.7
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.5688 -0.6004 -0.0316  0.5959  3.6176
##
## Random effects:
##      Groups   Name                Variance Std.Dev. Corr
##      classid  (Intercept)         86.06    9.277
##              ses                 44.09    6.640   0.75
##      schoolid (Intercept)       173.16   13.159
##      Residual                1048.32   32.378
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.52093    5.26665  269.59234 102.441 < 2e-16 ***
## housepov     -16.28994   13.13445  111.28619  -1.240   0.217
## mathknow       1.37996    1.37294  222.43201   1.005   0.316
```

```
## yearstea      0.01605    0.14080  227.59545    0.114    0.909
## mathprep     -0.37734    1.34603  182.84309   -0.280    0.780
## sex          -1.32178    2.08794 1017.08508   -0.633    0.527
## minority     -16.09272    3.03497  717.66470   -5.302 1.52e-07 ***
## ses          10.05535    1.64507  171.13536    6.112 6.44e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) housepv mthknw yearst mthprp sex    minrty
## housepv -0.450
## mathknow -0.078  0.059
## yearstea -0.266  0.074  0.030
## mathprep -0.625  0.036 -0.001 -0.165
## sex      -0.186 -0.009  0.007  0.013 -0.009
## minority -0.325 -0.181  0.108  0.021  0.004 -0.014
## ses      -0.084  0.078  0.015 -0.024  0.056  0.022  0.142
ranova(rssc.3)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + ses | classid) + (1 | schoolid)
##              npar logLik   AIC      LRT Df Pr(>Chisq)
## <none>              13 -5362.8 10752
## ses in (1 + ses | classid)  11 -5364.8 10752  3.8395  2    0.1466
## (1 | schoolid)            12 -5375.8 10776 26.0221  1  3.375e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated random slope for ses is insignificant with a pvalue of .147.

## Random slopes for student-level predictors varying at school level

### ONE BY ONE

#### Sex

```
rss.4 <-lmer(Math1~housepv+mathknow+yearstea+mathprep+sex+minority+ses+(1+sex||schoolid)+(1|classid),d
summary(rss.4)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepv + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + sex || schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
```

```
## -3.8578 -0.6110 -0.0259 0.5922 3.5557
##
## Random effects:
## Groups Name Variance Std.Dev.
## classid (Intercept) 96.08 9.802
## schoolid sex 35.83 5.986
## schoolid.1 (Intercept) 161.63 12.713
## Residual 1054.36 32.471
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
## Estimate Std. Error df t value Pr(>|t|)
## (Intercept) 539.43517 5.30740 272.54946 101.638 < 2e-16 ***
## housepov -16.77661 13.22881 112.39593 -1.268 0.207
## mathknow 1.40067 1.39464 234.45882 1.004 0.316
## yearstea 0.01448 0.14163 226.44519 0.102 0.919
## mathprep -0.27193 1.38010 205.78503 -0.197 0.844
## sex -1.33534 2.18746 138.08788 -0.610 0.543
## minority -16.16536 3.02861 704.25758 -5.338 1.27e-07 ***
## ses 9.98477 1.54243 1058.27875 6.473 1.46e-10 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
## (Intr) houspv mthknw yearst mthprp sex minrty
## housepov -0.449
## mathknow -0.081 0.055
## yearstea -0.259 0.070 0.028
## mathprep -0.633 0.036 0.004 -0.172
## sex -0.179 -0.010 0.007 0.013 -0.004
## minority -0.320 -0.178 0.114 0.024 0.001 -0.015
## ses -0.120 0.081 -0.007 -0.029 0.052 0.020 0.161
ranova(rss.4, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
## ses + (1 | schoolid) + (0 + sex | schoolid) + (1 | classid)
## npar logLik AIC LRT Df Pr(>Chisq)
## <none> 12 -5364.4 10753
## (1 | schoolid) 11 -5374.4 10771 19.9994 1 7.747e-06 ***
## sex in (0 + sex | schoolid) 11 -5364.8 10752 0.6137 1 0.433392
## (1 | classid) 11 -5368.2 10758 7.4171 1 0.006461 **
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated sex random slope at a school level is insignificant with a p value of .433.

## Minority

```
rss.5 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+minority||schoolid)+(1|classid)|
summary(rss.5)
```

```

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority || schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups      Name      Variance Std.Dev.
## classid     (Intercept)  93.89   9.69
## schoolid    minority      0.00   0.00
## schoolid.1  (Intercept) 169.45  13.02
## Residual                1064.95 32.63
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38919 101.585 < 2e-16 ***
## housepov     -17.64847   13.21758  113.87772  -1.335  0.184
## mathknow       1.35004    1.39168  234.49776   0.970  0.333
## yearstea       0.01129    0.14141  226.80899   0.080  0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201  0.841
## sex           -1.21419    2.09483 1022.42137  -0.580  0.562
## minority     -16.18678    3.02605  704.47892  -5.349 1.20e-07 ***
## ses            10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
ranova(rss.5,refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid) + (0 + minority | schoolid) + (1 | classid)
##              npar logLik  AIC    LRT Df
## <none>              12 -5364.8 10754
## (1 | schoolid)       11 -5375.2 10772 20.8586 1
## minority in (0 + minority | schoolid) 11 -5364.8 10752 0.0000 1

```

```
## (1 | classid)                11 -5368.3 10759  7.1357  1
##                               Pr(>Chisq)
## <none>
## (1 | schoolid)              4.945e-06 ***
## minority in (0 + minority | schoolid)  1.000000
## (1 | classid)                0.007556 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated minority random slope at school level is insignificant with a pvalue of 1.0.

## SES

```
rss.6 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+ses||schoolid)+(1|classid),d
summary(rss.6) #IS SIG
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##       ses + (1 + ses || schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##  Groups      Name      Variance Std.Dev.
##  classid    (Intercept)  88.56    9.411
##  schoolid    ses         72.50    8.515
##  schoolid.1 (Intercept) 167.98   12.961
##  Residual                1035.12  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917   270.54314 102.126 < 2e-16 ***
## housepov     -16.94564   13.21116   112.82496  -1.283   0.202
## mathknow      1.35576    1.38459   232.19983   0.979   0.329
## yearstea      0.03079    0.14052   223.94305   0.219   0.827
## mathprep     -0.19801    1.35994   198.59419  -0.146   0.884
## sex          -1.40185    2.08170  1011.28944  -0.673   0.501
## minority     -16.52525    3.02189   700.06637  -5.469 6.32e-08 ***
## ses           9.78982    1.82217    79.01645   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.451
```



```
## mathknow -0.079 0.056
## yearstea -0.260 0.070 0.028
## mathprep -0.628 0.041 0.002 -0.172
## sex -0.190 -0.007 0.006 0.018 -0.007
## minority -0.323 -0.180 0.110 0.024 0.001 -0.010
## ses -0.091 0.076 0.006 -0.019 0.042 0.017 0.124

ranova(rss.6,refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid) + (0 + ses | schoolid) + (1 | classid)
##               npar logLik   AIC    LRT Df Pr(>Chisq)
## <none>                12 -5362.4 10749
## (1 | schoolid)         11 -5374.6 10771 24.2924 1 8.276e-07 ***
## ses in (0 + ses | schoolid) 11 -5364.8 10752 4.6972 1 0.03021 *
## (1 | classid)          11 -5365.7 10753 6.5177 1 0.01068 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated ses random slope at school level is significant with a p value of .03.

## Allowing for correlations with random intercepts

### ONE BY ONE

#### Sex

```
rssc.4 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+sex|schoolid)+(1|classid),d
summary(rssc.4)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + sex | schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10727.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8048 -0.6095 -0.0222  0.5969  3.5525
##
## Random effects:
## Groups   Name                Variance Std.Dev. Corr
## classid  (Intercept)         97.34    9.866
## schoolid (Intercept)        206.33   14.364
##          sex                 84.08    9.170  -0.43
## Residual                    1041.76   32.276
## Number of obs: 1081, groups: classid, 285; schoolid, 105
##
```

```
## Fixed effects:
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  5.399e+02  5.363e+00  2.626e+02 100.661 < 2e-16 ***
## housepov    -1.742e+01  1.325e+01  1.136e+02  -1.314   0.191
## mathknow     1.379e+00  1.396e+00  2.364e+02   0.988   0.324
## yearstea     6.876e-03  1.418e-01  2.277e+02   0.048   0.961
## mathprep    -2.796e-01  1.378e+00  2.061e+02  -0.203   0.839
## sex         -1.340e+00  2.301e+00  8.742e+01  -0.582   0.562
## minority    -1.642e+01  3.027e+00  7.076e+02  -5.425 7.96e-08 ***
## ses         9.928e+00  1.540e+00  1.055e+03   6.448 1.72e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.082  0.060
## yearstea -0.258  0.072  0.027
## mathprep -0.627  0.038  0.004 -0.172
## sex      -0.222 -0.003  0.006  0.014 -0.005
## minority -0.319 -0.178  0.114  0.024  0.004 -0.011
## ses      -0.121  0.083 -0.006 -0.028  0.053  0.018  0.163
ranova(rssc.4, refit=F)

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + sex | schoolid) + (1 | classid)
##           npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>           13 -5363.8 10754
## sex in (1 + sex | schoolid)  11 -5364.8 10752 1.8631  2   0.393952
## (1 | classid)           12 -5367.6 10759 7.6414  1   0.005704 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated sex random slope at school-level is insignificant with a pvalue of .394.

## Minority

```
rssc.5 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+minority|schoolid)+(1|classid),
summary(rssc.5)

## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority | schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
```

```

##      Min      1Q  Median      3Q      Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##   Groups   Name      Variance Std.Dev. Corr
##   classid (Intercept)  86.69   9.311
##   schoolid (Intercept) 381.20  19.524
##             minority   343.13  18.524  -0.83
##   Residual             1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399 < 2e-16 ***
## housepov     -1.606e+01  1.257e+01  9.999e+01  -1.277  0.204
## mathknow      1.632e+00  1.359e+00  2.248e+02   1.201  0.231
## yearstea     -4.368e-03  1.376e-01  2.172e+02  -0.032  0.975
## mathprep     -2.918e-01  1.335e+00  1.981e+02  -0.218  0.827
## sex          -8.628e-01  2.084e+00  1.022e+03  -0.414  0.679
## minority     -1.638e+01  3.896e+00  5.824e+01  -4.203  9.17e-05 ***
## ses           9.431e+00  1.543e+00  1.063e+03   6.111  1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.394
## mathknow  -0.078  0.061
## yearstea  -0.253  0.091  0.024
## mathprep  -0.576  0.037 -0.002 -0.167
## sex        -0.172 -0.013  0.010  0.014 -0.005
## minority  -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses        -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113

```

```

ranova(rssc.5,refit=F) #sig

```

```

## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority | schoolid) + (1 | classid)
##              npar logLik   AIC   LRT Df
## <none>              13 -5358.8 10744
## minority in (1 + minority | schoolid)  11 -5364.8 10752 11.967  2
## (1 | classid)              12 -5361.8 10748  6.077  1
##              Pr(>Chisq)
## <none>
## minority in (1 + minority | schoolid)    0.00252 **
## (1 | classid)              0.01370 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

```

The correlated minority random slope at school-level is significant with a pvalue of .0025.

## SES

```
rssc.6 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(1+ses|schoolid)+(1|classid),data=classroom,REML=F)
summary(rssc.6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + ses | schoolid) + (1 | classid)
##      Data: classroom
##
## REML criterion at convergence: 10724.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.5646 -0.6166 -0.0264  0.5888  3.7073
##
## Random effects:
##      Groups      Name      Variance Std.Dev. Corr
##      classid (Intercept)  86.57    9.305
##      schoolid (Intercept) 171.18   13.083
##              ses         73.37    8.565   0.19
##      Residual          1035.90   32.185
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  538.72222    5.27647  271.13405 102.099 < 2e-16 ***
## housepov     -15.89873   13.15393  111.71410  -1.209   0.229
## mathknow       1.26025    1.38201  230.89932   0.912   0.363
## yearstea       0.03617    0.14002  220.42247   0.258   0.796
## mathprep      -0.21697    1.35642  197.10752  -0.160   0.873
## sex           -1.40436    2.08074 1011.40322  -0.675   0.500
## minority     -16.26699    3.03580  668.91517  -5.358 1.16e-07 ***
## ses           9.72646    1.82985   78.36218   5.315 9.75e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.449
## mathknow    -0.077  0.057
## yearstea    -0.259  0.073  0.028
## mathprep    -0.627  0.039  0.001 -0.172
## sex         -0.188 -0.009  0.005  0.017 -0.008
## minority    -0.325 -0.182  0.108  0.021  0.002 -0.011
## ses         -0.062  0.070  0.007 -0.021  0.045  0.018  0.117
```

```
anova(rssc.6,refit=F) #not sig
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
```

```
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + ses | schoolid) + (1 | classid)
##                                npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                                13 -5362.2 10750
## ses in (1 + ses | schoolid)   11 -5364.8 10752 5.1385  2    0.07659 .
## (1 | classid)                 12 -5365.3 10755 6.2117  1    0.01269 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated ses random slope at school-level is Very close to significance but not quite there with a pvalue of .0766.

**Question:** Report unusual changes in variance.

**Answer:** Perhaps most striking is the change in variance for the random slope term on minority. Previously, it was 0. However, it jumps to 343.13 in the correlated model. The variance for the random slope term on SES also increases, but the correlated random slope is not a significant addition to our model according to the rand test results.

## Complex model

Take two predictors that had sig random slopes and add to model, test for need of one conditional on the other

-Minority is sig for correlated

-Ses is sig for uncorrelated

```
complex <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+ses+(0+ses|schoolid)+(1+minority|
summary(complex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##      (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
## Groups      Name                Variance Std.Dev. Corr
## classid     (Intercept)         80.63    8.979
## schoolid    (Intercept)        404.54   20.113
##              minority           336.04   18.332  -0.84
## schoolid.1 ses                74.93    8.656
## Residual                    1009.73   31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
```

```
## (Intercept) 539.05335    5.66468  165.74621  95.160 < 2e-16 ***
## housepov   -15.32111   12.49443   99.25865  -1.226    0.223
## mathknow    1.67475    1.35000  221.33588   1.241    0.216
## yearstea    0.02102    0.13657  213.65672   0.154    0.878
## mathprep   -0.23546    1.31730  191.22014  -0.179    0.858
## sex        -1.03871    2.06951 1010.41144  -0.502    0.616
## minority   -16.72884    3.90720   55.41065  -4.282 7.43e-05 ***
## ses         9.19654    1.82272   82.48814   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
ranova(complex, refit=F)

## Warning: Model failed to converge with 1 negative eigenvalue: -1.2e-04
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##      (1 | classid)
##
##              npar  logLik   AIC      LRT Df
## <none>              14 -5356.2 10740
## ses in (0 + ses | schoolid)      14 -5358.8 10746  5.1200  0
## minority in (1 + minority | schoolid) 12 -5362.4 10749 12.3899  2
## (1 | classid)              13 -5358.9 10744  5.3724  1
##
##              Pr(>Chisq)
## <none>
## ses in (0 + ses | schoolid)
## minority in (1 + minority | schoolid) 0.00204 **
## (1 | classid) 0.02046 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

**Question:** Is the more complex model (with both random slopes in it) justified?

**Answer:** The complex model is justified since the rand test shows that the random slopes are both statistically significant at the 0.05 level, the only question revolves around statistical significance justifying compared to the Bayesian approach that would push for a simpler model.

The equation for the complex model is given by the following:

$$\widehat{Math1st}_{ijk} = \beta_0 + \beta_1 housepov_k + \beta_2 mathknow_{jk} + \beta_3 yearstea_{jk} + \beta_4 mathprep_{jk} + \beta_5 sex_{ijk} + \beta_6 ses_{ijk} + \beta_7 minority_{ijk} + \zeta_0$$

where  $\zeta_{0k} \sim N(0, \sigma_{\zeta_0}^2)$ ,  $\zeta_{6k} \sim N(0, \sigma_{\zeta_6}^2)$ ,  $\zeta_{7k} \sim N(0, \sigma_{\zeta_7}^2)$ ,  $\eta_{jk} \sim N(0, \sigma_{\eta}^2)$ , and  $\epsilon_{ijk} \sim N(0, \sigma_{\epsilon}^2)$ , all independent of each other.

---

```
summary(model1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
## Groups           Name          Variance Std.Dev.
## classid:schoolid (Intercept)   85.46   9.244
## schoolid         (Intercept)  280.68  16.754
## Residual                        1146.80  33.864
## Number of obs: 1190, groups: classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.406   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$V_C$ ,  $V_S$ , and  $V_E$  **Question:** For UCM, write down:  $V_C$ ,  $V_S$ ,  $V_E$  for the three variance components (simply the estimates). Think of them as possibly varying with a covariate, though.

**Answer:** For the UCM,  $V_C = 85.46$ ,  $V_S = 280.68$ , and  $V_E = 1146.80$

```
summary(model4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 | schoolid/classid)
## Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
## Groups           Name          Variance Std.Dev.
## classid:schoolid (Intercept)   93.89   9.69
## schoolid         (Intercept)  169.45  13.02
## Residual                        1064.95  32.63
## Number of obs: 1081, groups: classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
```

```
##           Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38922 101.585 < 2e-16 ***
## housepov    -17.64847   13.21757  113.87774  -1.335  0.184
## mathknow     1.35004    1.39168  234.49776   0.970  0.333
## yearstea     0.01129    0.14141  226.80899   0.080  0.936
## mathprep    -0.27705    1.37583  205.27157  -0.201  0.841
## sex         -1.21419    2.09483 1022.42136  -0.580  0.562
## minority    -16.18678    3.02605  704.47889  -5.349 1.20e-07 ***
## ses         10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##           (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

**Question:** For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are:  $V_C$ ,  $V_S$ ,  $V_E$ ?

**Answer:** For the most complicated fixed effects model with only random intercepts,  $V_C = 93.89$ ,  $V_S = 169.45$ , and  $V_E = 1064.95$ .

**Question:** By what fraction did these each decrease with the new predictors in the model?

**Answer:**  $V_C$  increased  $\frac{93.89}{85.46}$

$V_S$  decreased  $\frac{169.45}{280.68}$

$V_E$  decreased  $\frac{1064.95}{1146.80}$

```
summary(rss.6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##       ses + (1 + ses || schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
## Groups      Name             Variance Std.Dev.
## classid     (Intercept)      88.56    9.411
## schoolid    ses              72.50    8.515
## schoolid.1 (Intercept)     167.98   12.961
## Residual                    1035.12   32.173
```



```
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917  270.54314 102.126 < 2e-16 ***
## housepov    -16.94564   13.21116  112.82496  -1.283   0.202
## mathknow      1.35576    1.38459  232.19983   0.979   0.329
## yearstea      0.03079    0.14052  223.94305   0.219   0.827
## mathprep     -0.19801    1.35994  198.59419  -0.146   0.884
## sex          -1.40185    2.08170 1011.28944  -0.673   0.501
## minority     -16.52525    3.02189  700.06637  -5.469 6.32e-08 ***
## ses           9.78982    1.82217   79.01645   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.451
## mathknow    -0.079  0.056
## yearstea    -0.260  0.070  0.028
## mathprep    -0.628  0.041  0.002 -0.172
## sex         -0.190 -0.007  0.006  0.018 -0.007
## minority    -0.323 -0.180  0.110  0.024  0.001 -0.010
## ses         -0.091  0.076  0.006 -0.019  0.042  0.017  0.124
```

**Question:** Now consider the model with a random slope in ses. What are:  $V_C$ ,  $V_S(\text{ses} = 0)$ ,  $V_E$  ? We need to list 'ses=0' here, or we don't know how to use the slope variance

**Answer:** For the model with a random slope in ses at the school level,  $V_C = 88.56$ ,  $V_S(\text{ses} = 0) = 167.98$ , and  $V_E = 1035.12$ .

**Question:** What are:  $V_S(\text{ses} = -0.50)$ ,  $V_S(\text{ses} = +0.5)$  ?

**Answer:** In this model, in which the random slope for SES is uncorrelated with the random school-level intercept,  $V_S(\text{ses} = -0.50) = 167.98 + (-.5)^2 72.50 + 2(-.5)0.167.9872.50 = 186.105$ , and  $V_S(\text{ses} = +0.5) = 167.98 + (.5)^2 72.50 + 2 * (.5)0.167.98 * 72.50 = 186.105$

```
summary(rssc.5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (1 + minority | schoolid) + (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
## Groups   Name                Variance Std.Dev. Corr
## classid  (Intercept)         86.69    9.311
## schoolid (Intercept)       381.20   19.524
##          minority           343.13   18.524  -0.83
```

```
## Residual          1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399 < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  9.999e+01  -1.277   0.204
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201   0.231
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032   0.975
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218   0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414   0.679
## minority    -1.638e+01  3.896e+00  5.824e+01  -4.203 9.17e-05 ***
## ses         9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.394
## mathknow  -0.078  0.061
## yearstea  -0.253  0.091  0.024
## mathprep  -0.576  0.037 -0.002 -0.167
## sex       -0.172 -0.013  0.010  0.014 -0.005
## minority  -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses       -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

**Question:** Now consider the model with a random slope in minority. What are:  $V_C$ ,  $V_S(\text{minority} = 0)$ ,  $V_E$ ? We need to list ‘minority=0’ here, or we don’t know how to use the slope variance

**Answer:** For the model with a random slope in minority at the school level,  $V_C = 86.69$ ,  $V_S(\text{minority} = 0) = 381.20$ , and  $V_E = 1039.39$ .

**Question:** What are:  $V_S(\text{minority} = 0.25)$ ,  $V_S(\text{minority} = +0.50)$ ,  $V_S(\text{minority} = +0.75)$ ?

**Answer:** In this model, in which the random slope for minority is correlated with the random school-level, intercept,  $V_S(\text{minority} = 0.25) = 381.20 + (0.25)^2 343.13 + 2(0.25)(-0.83)\sqrt{381.20 * \sqrt{343.13}} = 252.5549$ ,

$V_S(\text{minority} = +0.50) = 381.20 + (0.50)^2 343.13 + 2(0.50)(-0.83)\sqrt{381.20 * \sqrt{343.13}} = 166.801$ , and

$V_S(\text{minority} = +0.75) = 381.20 + (0.75)^2 343.13 + 2(0.75)(-0.83)\sqrt{381.20 * \sqrt{343.13}} = 123.9384$ .

[summary\(complex\)](#)

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##      ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##      (1 | classid)
## Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##      Min      1Q  Median      3Q      Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
```

```
## Groups      Name      Variance Std.Dev. Corr
## classid    (Intercept)  80.63   8.979
## schoolid   (Intercept) 404.54  20.113
##            minority     336.04  18.332  -0.84
## schoolid.1 ses       74.93   8.656
## Residual                1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468   165.74621  95.160 < 2e-16 ***
## housepov     -15.32111   12.49443    99.25865  -1.226  0.223
## mathknow       1.67475    1.35000   221.33588   1.241  0.216
## yearstea       0.02102    0.13657   213.65672   0.154  0.878
## mathprep      -0.23546    1.31730   191.22014  -0.179  0.858
## sex           -1.03871    2.06951  1010.41144  -0.502  0.616
## minority     -16.72884    3.90720    55.41065  -4.282 7.43e-05 ***
## ses           9.19654     1.82272    82.48814   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.395
## mathknow     -0.072  0.060
## yearstea     -0.254  0.093  0.024
## mathprep     -0.568  0.040 -0.004 -0.166
## sex          -0.170 -0.014  0.010  0.017 -0.005
## minority     -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses          -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

**Question:** Now consider the model with a random slope in ses & minority. What are:  $V_C$ ,  $V_S(minority = 0, ses = 0)$ ,  $V_E$ ? We need to list 'ses=0, minority=0' here, or we don't know how to use the slope variance.

**Answer:** For the model with a random slope in ses & minority,  $V_C = 80.63$ ,  $V_S(minority = 0, ses = 0) = 404.54$ , and  $V_E = 1009.73$ .

**Question:** What are:  $V_S(ses = 0, minority = 0.50)$ ,  $V_S(ses = 0.50, minority = 0)$ ,  $V_S(ses = 0.50, minority = 0.50)$ ?

**Answer:** In this model, in which the random slope for ses is uncorrelated with the random intercept, but the random slope for minority is correlated with the random intercept,

$$V_S(ses = 0, minority = 0.50) = 404.54 + (0)^2 74.93 + (0.50)^2 336.04 + 200404.5474.93 + 2 * (0.50) (-0.83) \sqrt{404.54 * \sqrt{336.04}} = 182.5268,$$

$$V_S(ses = 0.50, minority = 0) = 404.54 + (0.50)^2 74.93 + (0)^2 336.04 + 20.500404.5474.93 + 2 * (0) (-0.83) \sqrt{404.54 * \sqrt{336.04}} = 423.2725$$

$$V_S(ses = 0.50, minority = 0.50) = 404.54 + (0.50)^2 74.93 + (0.50)^2 336.04 + 20.500404.5474.93 + 2 * (0.50) (-0.83) \sqrt{404.54 * \sqrt{336.04}} = 201.2593$$

```
summary(complex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
```

```

##      ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##      (1 | classid)
##      Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##      Min       1Q   Median       3Q      Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##      Groups      Name      Variance Std.Dev. Corr
##      classid    (Intercept)  80.63   8.979
##      schoolid   (Intercept) 404.54  20.113
##               minority     336.04  18.332  -0.84
##      schoolid.1 ses         74.93   8.656
##      Residual                1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468   165.74621  95.160 < 2e-16 ***
## housepov     -15.32111   12.49443    99.25865  -1.226  0.223
## mathknow       1.67475    1.35000   221.33588   1.241  0.216
## yearstea       0.02102    0.13657   213.65672   0.154  0.878
## mathprep      -0.23546    1.31730   191.22014  -0.179  0.858
## sex           -1.03871    2.06951  1010.41144  -0.502  0.616
## minority     -16.72884    3.90720    55.41065  -4.282 7.43e-05 ***
## ses           9.19654    1.82272    82.48814   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##              (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov    -0.395
## mathknow    -0.072  0.060
## yearstea    -0.254  0.093  0.024
## mathprep    -0.568  0.040 -0.004 -0.166
## sex         -0.170 -0.014  0.010  0.017 -0.005
## minority    -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses         -0.080  0.083  0.006 -0.011  0.041  0.020  0.087

```

**Question:** In the last model, what is a “likely” (+/- 1 sd) range for  $\eta_{0jk}$

**Answer:** For the complex model, the “likely” range for  $\eta_{0jk}$  is 71.651 to 89.609.

**Question:** Can we make a similar statement about  $\zeta_{0k}$ ?

**Answer:** Mathematically we can with a range of 384.427 to 424.653 though we can do this it doesn’t make much sense due to the correlated nature of this with the minority variable the values wouldn’t hold much meaning and are easily misinterpreted.

**Question:** If you had a large value for  $\eta_{0jk}$ , would you expect a large or small or “any” value for: the two random slope terms,  $\zeta_{1k}$  and  $\zeta_{2k}$  for ses and minority?

**Answer:** If you have a very large  $\eta_{0jk}$  you would expect a small value for  $\zeta_{1k}$  and  $\zeta_{2k}$  but the  $\zeta_{2k}$  would not be as small due to its negative correlation with our  $\zeta_{0k}$  which is effected by our eta value.

**Question:** If you had a large value for  $\zeta_{0k}$ , would you expect a large or small or “any” value for: the two random slope terms,  $\zeta_{1k}$  and  $\zeta_{2k}$  for ses and minority (discuss each separately)?

**Answer:** For  $\zeta_{1k}$  would increase in the same direction but it could be any value due to the lack of correlation, keeping in mind that  $\zeta_{0k}$  will create a ceiling effect of sorts for  $\zeta_{1k}$ . While  $\zeta_{2k}$  would be very small because of the correlation because of the two variables are negatively correlated.