Multi-Level Models: Final Project

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All four parts of the Group Project are compiled in this PDF. Here is how the group split up the work. Bookmark tabs have been created in the PDF for ease of finding each member's section.

-Part 1: Frankie

-Part 2: Clare

-Part 3: Bianca

-Part 4: Kaushik

Part 1: Frankie

Create 1st grade variable

```
classroom <- classroom %>% mutate(Math1 = mathkind + mathgain)
```

Random Intercepts for classroom, nested in schools UMM

We begin our analysis by looking at the UMM with random intercepts for schools and classrooms, i.e. :

$$Math1st_{ijk} = \beta_{0ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

where i represents students, j represents classrooms and k represents schools.$\zeta_k \sim N(0, \sigma^2_\zeta), \eta_{jk} \sim N(0, \sigma^2_\eta)$, and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, all are independent of each other

```
model1 <- lmer(Math1~(1|schoolid/classid),data=classroom)
summary(model1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.46   9.244
##  schoolid         (Intercept)  280.68  16.754
##  Residual                     1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
```

```
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.406   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$ICC_{class} = \frac{85.46}{1146.8 + 280.68 + 85.46} \approx .056$$
$$ICC_{school} = \frac{280.68}{1146.8 + 280.68 + 85.46} \approx .186$$

We hence find, from the fit summary above, that the equation for our model is:

$$Math1st_{ijk} = 522.54 + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$\zeta_k \sim N(0, 280.68), \eta_{jk} \sim N(0, 85.46)$, and $\epsilon_{ijk} \sim N(0, 1146.80)$, all are independent of each other

Model with School Level Predictors Added

We then add all the school level predictors (that is, “housepov”) and report below the model fit :

```r
model2 <- lmer(Math1~housepov+(1|schoolid/classid),data=classroom)
summary(model2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ housepov + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   82.36   9.075
##  schoolid         (Intercept)  250.93  15.841
##  Residual                     1146.95  33.867
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.294      3.341 102.809 159.024   <2e-16 ***
## housepov     -45.783     14.236 111.063  -3.216   0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.810
```

```r
anova(model1, model2, refit = F)
```

```
## Data: classroom
## Models:
## model1: Math1 ~ (1 | schoolid/classid)
## model2: Math1 ~ housepov + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model1  4 11953 11973 -5972.3    11945
## model2  5 11937 11963 -5963.7    11927 17.186      1   3.39e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Report the changes in the variances of the random effects:

Change in σ_ζ^2: decreased to 250.93 from 280.63 σ_η^2 decreases to 82.36 from 85.46 σ_ϵ^2 slightly increases to 1146.95 from 1146.8

The LRT has a p-value of almost zero, $p = 3.39e-05$, thus we reject the H_0: coefficient on Housepov = 0 at $\alpha = 0.05$. That is, we find evidence that it makes sense to include the school level predictor, housepov.

Model with all Class Level Predictors Added

We now re-run the model after including all the classroom level predictors, that is "mathknow", "yearstea", "mathprep", and report the model fit.

```r
model3 <- lmer(Math1~housepov+mathknow+yearstea+mathprep+
                 (1|schoolid/classid),data=classroom)
summary(model3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                     1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29853    5.20496 228.85764 102.268  < 2e-16 ***
## housepov    -41.62116   14.08835 109.83227  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.06566   1.765  0.07883 .
## yearstea      0.06193    0.14717 223.76582   0.421  0.67432
## mathprep     -0.75440    1.42809 203.20767  -0.528  0.59790
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst
## housepov -0.568
## mathknow -0.052  0.082
## yearstea -0.264  0.077  0.030
## mathprep -0.666  0.032  0.004 -0.175
```

creating reducted dataset taking away missing data

The variable of interest *Mathknow* includes some missing values. The model for which we have reported the summary above therefore removes the observations for which missing data is present.

To be able to compare Model 2 (with school level predictors) with Model 3 (with both school level and classroom level predictors), we removed from the dataset students that had missing values, creating a reduced dataset. This left us with a sample of 1081 students. We then re-run model 2 on this reducted dataset and compared it to Model 3.

```r
classroom_red = na.omit(classroom)
model2_red <- lmer(Math1~housepov+(1|schoolid/classid),data=classroom_red)
model3_red <- lmer(Math1~housepov+mathknow+yearstea+mathprep+
                     (1|schoolid/classid),data=classroom_red)

summary(model3_red)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##    Data: classroom_red
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                     1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29853    5.20496 228.85764 102.268  < 2e-16 ***
## housepov    -41.62116   14.08835 109.83227  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.06566   1.765  0.07883 .
## yearstea      0.06193    0.14717 223.76582   0.421  0.67432
## mathprep     -0.75440    1.42809 203.20767  -0.528  0.59790
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst
## housepov -0.568
## mathknow -0.052  0.082
## yearstea -0.264  0.077  0.030
## mathprep -0.666  0.032  0.004 -0.175
```

```
anova(model2_red, model3_red, refit = F)
```

```
## Data: classroom_red
## Models:
## model2_red: Math1 ~ housepov + (1 | schoolid/classid)
## model3_red: Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##            Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model2_red  5 10838 10862 -5413.8    10828
## model3_red  8 10837 10877 -5410.5    10821 6.5771      3    0.08667 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Change in σ^2_ϵ and σ^2_η: σ^2_ϵ decreased to 1136.43, σ^2_η increased to 94.36; $\sigma^2_\zeta = 223.31$

A possible reason why ϵ decreased in this model, but not η is that adding the classroom level predictors makes it so that more of the overall variation is explained by "structured" variation (that is, related to the fact that students are in different classrooms) rather than by unstructured (ϵ), so that the latter decreases. However, we also have to note that in this case we are using the reduced dataset, so that some of the changes may be due to the fact that we are using two slightly different datasets.

The anova test comparing the school level predictor to the model with the classroom predictors has a p-value 0.087, so we fail to reject the null hypothesis at our $\alpha = 0.05$ and conclude that adding classroom level predictors is not necessary, as it does not signficantly improve the model.

Add all student-level predictors

We now include all the student level predictors in our model:

```
model4 <- lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                 ses+(1|schoolid/classid),data=classroom)
summary(model4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
```

```
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                      1064.95  32.63
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38922 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.87774  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201    0.841
## sex           -1.21419    2.09483 1022.42136  -0.580    0.562
## minority     -16.18678    3.02605  704.47889  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

We test this new block compared to the model with both school-level and classroom level predictors.

```
anova(model3, model4, refit = F)
```

```
## Data: classroom
## Models:
## model3: Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
## model4: Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
## model4:     ses + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model3  8 10837 10877 -5410.5    10821
## model4 11 10752 10806 -5364.8    10730 91.446      3  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The LRT test between this two models has a p-value $< 2.2 * 10^{-16}$. Therefore, at our $\alpha = 0.05$, we reject the null hypothesis and conclude that adding this block of predictors is justified.

Changes in variance components :

σ_ϵ^2 decreased to 1064.95, σ_η^2 decreased to 93.89, and σ_ζ^2 decreased to 169.45.

We note that adding student-level predictors leads to a decrease in the overall variance of the model. By "controlling" for student-related variables, we also explain the between schools, as students with similar attributes might be similar across schools, hence reducing the overall variance of ζ.

The final model, with all school level, classroom level, and student level predictors, is:

$$Math1st_{ijk} = 539.63 + \zeta_k + \eta_{jk} + \epsilon_{ijk} - 17.65 * Housepov_k + 1.35 * Mathknow_{jk} +$$

$$0.01 * YearsTea_{jk} - 0.27 * Mathprep_{jk} - 0.19 * sex_{ijk} + -0.32 * minority_{ijk} - 0.12 * ses_{ijk}$$

With:

$\zeta_k \sim N(0, \sigma^2_\zeta), \eta_{jk} \sim N(0, \sigma^2_\eta)$, and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, all are independent of each other

From the model fit above therefore we find that the fitted model is:

$$Math1st_{ijk} = \beta_{0ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk} + \beta_1 Housepov_k + \beta_2 Mathknow_{jk} +$$
$$\beta_3 YearsTea_{jk} + \beta_4 Mathprep_{jk} + \beta_5 sex_{ijk} + \beta_6 minority_{ijk} + \beta_7 ses_{ijk}$$

With:

$\zeta_k \sim N(0, 169.45), \eta_{jk} \sim N(0, 93.89)$, and $\epsilon_{ijk} \sim N(0, 1064.95)$, all are independent of each other.

Random Slope for Teacher-level predictor varying at school-level

We try adding a random slope for each teacher level predictor (varying at the school level; one by one - not all together).

MATHKNOW

```
rst.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+mathknow||schoolid)+(1|classid),data=classroom)
summary(rst.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathknow || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.389e+01 9.690e+00
##  schoolid   mathknow    4.260e-11 6.527e-06
##  schoolid.1 (Intercept) 1.694e+02 1.302e+01
##  Residual               1.065e+03 3.263e+01
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##                Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38921 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.87774  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
## mathprep      -0.27705    1.37583  205.27156  -0.201    0.841
## sex           -1.21419    2.09483 1022.42136  -0.580    0.562
```

```
## minority      -16.18678    3.02605  704.47889  -5.349 1.20e-07 ***
## ses            10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```r
ranova(rst.1,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + mathknow | schoolid) + (1 | classid)
##                                      npar  logLik   AIC    LRT Df
## <none>                                 12 -5364.8 10754
## (1 | schoolid)                         11 -5376.5 10775 23.410  1
## mathknow in (0 + mathknow | schoolid)   11 -5364.8 10752  0.000  1
## (1 | classid)                          11 -5368.1 10758  6.741  1
##                                      Pr(>Chisq)
## <none>
## (1 | schoolid)                        1.309e-06 ***
## mathknow in (0 + mathknow | schoolid)   0.999999
## (1 | classid)                          0.009422 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for $MATHKNOW$ at a school level as the p value = 1 for the Chi-square test is not signifcant at $\alpha = 0.05$.

YEARSTEA

```r
rst.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+yearstea||schoolid)+(1|classid),data=classroom)
summary(rst.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + yearstea || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8485 -0.6149 -0.0323  0.5980  3.6600
```

```
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.266e+01  9.62593
##  schoolid   yearstea    9.669e-03  0.09833
##  schoolid.1 (Intercept) 1.685e+02 12.97894
##  Residual               1.065e+03 32.63452
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.60060    5.30865  266.34157 101.645  < 2e-16 ***
## housepov     -17.71727   13.21854  113.56407  -1.340    0.183
## mathknow       1.33198    1.39177  234.33551   0.957    0.340
## yearstea       0.01124    0.14193  122.38000   0.079    0.937
## mathprep      -0.26633    1.37610  204.91605  -0.194    0.847
## sex           -1.21077    2.09476 1022.22247  -0.578    0.563
## minority     -16.16833    3.02641  702.64837  -5.342 1.24e-07 ***
## ses           10.04529    1.54490 1066.09768   6.502 1.21e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.450
## mathknow -0.082  0.057
## yearstea -0.258  0.070  0.028
## mathprep -0.632  0.037  0.003 -0.172
## sex      -0.190 -0.007  0.006  0.015 -0.006
## minority -0.320 -0.179  0.115  0.023  0.001 -0.010
## ses      -0.121  0.082 -0.007 -0.027  0.053  0.020  0.162
```

```r
ranova(rst.2, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + yearstea | schoolid) + (1 | classid)
##                                      npar  logLik   AIC     LRT Df
## <none>                                 12 -5364.8 10754
## (1 | schoolid)                         11 -5374.7 10771 19.8301  1
## yearstea in (0 + yearstea | schoolid)   11 -5364.8 10752  0.0070  1
## (1 | classid)                          11 -5367.7 10757  5.9158  1
##                                      Pr(>Chisq)
## <none>
## (1 | schoolid)                        8.464e-06 ***
## yearstea in (0 + yearstea | schoolid)    0.93342
## (1 | classid)                          0.01501 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for *YEARSTEA* at a school level as the p value = 0.93 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Mathprep

```r
rst.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+mathprep||schoolid)+(1|classid),data=classroom)
summary(rst.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathprep || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   93.89   9.69
##  schoolid   mathprep       0.00   0.00
##  schoolid.1 (Intercept)  169.45  13.02
##  Residual               1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 539.63042    5.31210  275.38917 101.585  < 2e-16 ***
## housepov    -17.64847   13.21758  113.87771  -1.335    0.184
## mathknow      1.35004    1.39168  234.49776   0.970    0.333
## yearstea      0.01129    0.14141  226.80899   0.080    0.936
## mathprep     -0.27705    1.37583  205.27157  -0.201    0.841
## sex          -1.21419    2.09483 1022.42137  -0.580    0.562
## minority    -16.18678    3.02605  704.47892  -5.349 1.20e-07 ***
## ses          10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```r
ranova(rst.3, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
```

```
##     ses + (1 | schoolid) + (0 + mathprep | schoolid) + (1 | classid)
##                                    npar  logLik   AIC     LRT Df
## <none>                               12 -5364.8 10754
## (1 | schoolid)                       11 -5371.6 10765 13.6179  1
## mathprep in (0 + mathprep | schoolid)   11 -5364.8 10752  0.0000  1
## (1 | classid)                        11 -5368.3 10759  7.1357  1
##                                    Pr(>Chisq)
## <none>
## (1 | schoolid)                       0.000224 ***
## mathprep in (0 + mathprep | schoolid)   1.000000
## (1 | classid)                        0.007556 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for *MATHPREP* at a school level as the p value = 1 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Question: Why is a random slope on housepov a bad idea?

Answer: There is only one data point per school, so we do not have enough information to calculate the slope for each school.

Allowing correlations with random intercepts

ONE BY ONE

Again, we add random slopes for each teacher-level predictor varying at the school level, but this time by allowing them to be correlated with the random intercepts.

MATHKNOW

```r
rstc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+mathknow|schoolid)+(1|classid),data=classroom)
summary(rstc.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathknow | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6131 -0.0324  0.5969  3.6603
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept) 9.394e+01  9.69205
##  schoolid (Intercept) 1.693e+02 13.01223
##           mathknow    8.596e-04  0.02932 1.00
##  Residual             1.065e+03 32.63393
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
```

```
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)    
## (Intercept)  539.64037    5.31212  275.37948 101.587  < 2e-16 ***
## housepov     -17.64148   13.21274  103.97679  -1.335    0.185    
## mathknow       1.35459    1.39203  214.63820   0.973    0.332    
## yearstea       0.01114    0.14141  226.85277   0.079    0.937    
## mathprep      -0.27753    1.37601  201.27912  -0.202    0.840    
## sex           -1.21329    2.09485 1021.79964  -0.579    0.563    
## minority     -16.19376    3.02609  703.81038  -5.351 1.18e-07 ***
## ses           10.04788    1.54488 1062.12341   6.504 1.20e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451                                          
## mathknow -0.082  0.057                                   
## yearstea -0.259  0.071  0.029                            
## mathprep -0.631  0.038  0.004 -0.173                     
## sex      -0.190 -0.007  0.007  0.016 -0.006              
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011       
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```r
ranova(rstc.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathknow | schoolid) + (1 | classid)
##                                        npar  logLik   AIC    LRT Df
## <none>                                   13 -5364.8 10756          
## mathknow in (1 + mathknow | schoolid)   11 -5364.8 10752 0.0003  2
## (1 | classid)                            12 -5368.1 10760 6.6768  1
##                                        Pr(>Chisq)   
## <none>                                              
## mathknow in (1 + mathknow | schoolid)   0.999840   
## (1 | classid)                            0.009767 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for math knowledge at a school level as the p value $= 1.00$ for the Chi-square test is not signifcant at $\alpha = 0.05$.

YEARSTEA

```r
rstc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+yearstea|schoolid)+(1|classid),data=classroom)
summary(rstc.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + yearstea | schoolid) + (1 | classid)
##    Data: classroom
```

```
##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7462 -0.6036 -0.0290  0.6041  3.8449
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept)   37.9283  6.1586
##  schoolid (Intercept)  366.1148 19.1341
##           yearstea       0.5523  0.7432  -0.78
##  Residual             1066.4510 32.6566
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.95245    5.48825  222.69673  98.201  < 2e-16 ***
## housepov     -17.13994   13.45959  119.63687  -1.273    0.205
## mathknow       1.04635    1.34381  209.72527   0.779    0.437
## yearstea       0.02204    0.15766   75.76696   0.140    0.889
## mathprep       0.05046    1.34549  190.82671   0.038    0.970
## sex           -1.33553    2.08774 1024.45936  -0.640    0.523
## minority     -16.44555    2.99655  669.50401  -5.488 5.77e-08 ***
## ses           10.15038    1.53873 1062.66131   6.597 6.62e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.455
## mathknow -0.085  0.049
## yearstea -0.370  0.084  0.012
## mathprep -0.606  0.050  0.014 -0.139
## sex      -0.184 -0.004  0.008  0.009 -0.004
## minority -0.305 -0.169  0.122  0.032 -0.007 -0.012
## ses      -0.119  0.079 -0.001 -0.019  0.049  0.022  0.168
```

```
ranova(rstc.2,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + yearstea | schoolid) + (1 | classid)
##                                    npar  logLik   AIC    LRT Df
## <none>                               13 -5361.8 10750
## yearstea in (1 + yearstea | schoolid)   11 -5364.8 10752 5.8254  2
## (1 | classid)                        12 -5362.3 10749 0.9028  1
##                                    Pr(>Chisq)
## <none>
## yearstea in (1 + yearstea | schoolid)    0.05433 .
## (1 | classid)                          0.34202
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for yearstea at a school level as the p value = 0.054 for the Chi-square test is not signifcant at $\alpha = 0.05$.

MATHPREP

```
rstc.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+mathprep|schoolid)+(1|classid),data=classroom)
summary(rstc.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathprep | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8542 -0.6034 -0.0221  0.5915  3.6475
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   78.46   8.858
##  schoolid (Intercept)  552.76  23.511
##           mathprep      15.89   3.986   -1.00
##  Residual             1064.26  32.623
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.60855    5.60813  159.88774  96.041  < 2e-16 ***
## housepov     -14.01306   12.88689  116.05900  -1.087    0.279
## mathknow       1.29884    1.37194  229.68146   0.947    0.345
## yearstea      -0.02586    0.13949  223.50098  -0.185    0.853
## mathprep       0.04074    1.34845  139.04228   0.030    0.976
## sex           -1.16759    2.08697 1023.15084  -0.559    0.576
## minority     -16.46422    2.99524  663.67316  -5.497 5.52e-08 ***
## ses           10.14166    1.53961 1060.93421   6.587 7.04e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.461
## mathknow -0.071  0.027
## yearstea -0.260  0.089  0.049
## mathprep -0.692  0.107  0.012 -0.155
## sex      -0.183  0.003  0.002  0.023 -0.008
## minority -0.275 -0.187  0.107  0.025 -0.035 -0.013
## ses      -0.121  0.095 -0.001 -0.033  0.061  0.024  0.161
```

```
ranova(rstc.3, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
```

```
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathprep | schoolid) + (1 | classid)
##                                    npar  logLik   AIC    LRT Df
## <none>                               13 -5362.3 10751
## mathprep in (1 + mathprep | schoolid)   11 -5364.8 10752 4.8144  2
## (1 | classid)                        12 -5364.9 10754 5.0971  1
##                                    Pr(>Chisq)
## <none>
## mathprep in (1 + mathprep | schoolid)    0.09007 .
## (1 | classid)                          0.02397 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for mathprep at a school level as the p value = 0.09 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Question: Anything unusual about the variances? Why might this have occurred? (hint: what did you add to the model?)

Answer: We note that the model did not estimate the correlation parameter correctly for the models with random slopes for mathknown and mathprepr. Indeed, with a correaltion of respectively 1 and -1 with the random intercept, the parameter is a linear function of the variance component for the slope. This could be due to the fact that there is not enough classrooms in the schools (as we are adding random effects at the school levels, for classroom level predictors), so that there is not enough degrees of freedom, nor enough variation among the variables of interest, to calculate all the parameters required in the model. Obtaining a correlation of 1 and -1 should warn us of the fact that the models generated should not be trusted.
Why is the correlation between random intercept and slope then calculated for yearstea? This could be due to the fact that this variable has a larger range, so that it can be more robustly estimated for some of the schools and the correlation between random slope and intercept then estimated more accurately even for schools with few classes.

Random slopes for student-level predictors varying at classroom level

We now repeat the exercise by adding student level predictors, varying at the classroom level.

ONE BY ONE

SEX

```r
rss.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+sex||classid)+(1|schoolid),data=classroom)
summary(rss.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex || classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
```

```
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   93.89   9.69
##  classid.1 sex            0.00   0.00
##  schoolid  (Intercept)  169.45  13.02
##  Residual              1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38920 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.87773  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201    0.841
## sex           -1.21419    2.09483 1022.42137  -0.580    0.562
## minority     -16.18678    3.02605  704.47890  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```r
ranova(rss.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | classid) + (0 + sex | classid) + (1 | schoolid)
##                            npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                       12 -5364.8 10754
## (1 | classid)                11 -5368.0 10758  6.4894  1    0.01085 *
## sex in (0 + sex | classid)   11 -5364.8 10752  0.0000  1    1.00000
## (1 | schoolid)               11 -5377.1 10776 24.7881  1  6.399e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for sex at the classroom level, as the p value = 1 for the Chi-square test is not signifcant at $\alpha = 0.05$.

MINORITY

```r
rss.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+minority||classid)+(1|schoolid),data=classroom)
summary(rss.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority || classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   93.89   9.69
##  classid.1 minority       0.00   0.00
##  schoolid  (Intercept)  169.45  13.02
##  Residual              1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept) 539.63042    5.31210  275.38919 101.585  < 2e-16 ***
## housepov    -17.64847   13.21758  113.87772  -1.335    0.184
## mathknow      1.35004    1.39168  234.49776   0.970    0.333
## yearstea      0.01129    0.14141  226.80899   0.080    0.936
## mathprep     -0.27705    1.37583  205.27157  -0.201    0.841
## sex          -1.21419    2.09483 1022.42137  -0.580    0.562
## minority    -16.18678    3.02605  704.47891  -5.349 1.20e-07 ***
## ses          10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```r
ranova(rss.2, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
```

```
##     ses + (1 | classid) + (0 + minority | classid) + (1 | schoolid)
##                                    npar  logLik   AIC     LRT Df
## <none>                               12 -5364.8 10754
## (1 | classid)                        11 -5367.3 10757  5.1497  1
## minority in (0 + minority | classid)   11 -5364.8 10752  0.0000  1
## (1 | schoolid)                       11 -5377.1 10776 24.7881  1
##                                    Pr(>Chisq)
## <none>
## (1 | classid)                         0.02325 *
## minority in (0 + minority | classid)    1.00000
## (1 | schoolid)                      6.399e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for minority at the classroom level, as the p value = 1 for the Chi-square test is not signifcant at $\alpha = 0.05$.

SES

```r
rss.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+ses||classid)+(1|schoolid),data=classroom)
summary(rss.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses || classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   87.11   9.333
##  classid.1 ses           49.60   7.043
##  schoolid  (Intercept)  171.02  13.077
##  Residual              1043.44  32.302
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept) 539.71226    5.30641  274.46487 101.710  < 2e-16 ***
## housepov    -17.50879   13.21775  113.44869  -1.325    0.188
## mathknow      1.36796    1.38563  229.40646   0.987    0.325
## yearstea      0.01103    0.14117  226.97687   0.078    0.938
## mathprep     -0.27938    1.37171  204.89340  -0.204    0.839
## sex          -1.37733    2.09334 1022.81818  -0.658    0.511
## minority    -16.29362    3.02464  703.33762  -5.387 9.78e-08 ***
## ses          10.14363    1.64248  176.39739   6.176 4.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.082  0.058
## yearstea -0.259  0.070  0.029
## mathprep -0.631  0.040  0.005 -0.172
## sex      -0.190 -0.007  0.006  0.014 -0.005
## minority -0.321 -0.180  0.111  0.025  0.002 -0.011
## ses      -0.108  0.081  0.002 -0.026  0.050  0.020  0.145
```

```
ranova(rss.3, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | classid) + (0 + ses | classid) + (1 | schoolid)
##                            npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                       12 -5364.0 10752
## (1 | classid)                11 -5366.9 10756  5.9221  1    0.01495 *
## ses in (0 + ses | classid)   11 -5364.8 10752  1.5969  1    0.20634
## (1 | schoolid)               11 -5376.6 10775 25.2710  1  4.982e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for ses at the classroom level, as the p value = 0.206 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Question: why is this a bad idea to include a classroom-level variable with random slopes at classroom-level?

Answer: Because all of the observations for a class will be the same, so we will not be able to compute the classroom slopes for each classroom (as we will only have one point).

Allowing for correlations with random intercepts

ONE BY ONE

SEX

```
rssc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+sex|classid)+(1|schoolid),data=classroom)
summary(rssc.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7565 -0.6134 -0.0307  0.5916  3.7116
```

```
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  130.07  11.41
##           sex           31.36   5.60    -0.67
##  schoolid (Intercept)  169.85  13.03
##  Residual             1056.41  32.50
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  5.400e+02  5.332e+00  2.723e+02 101.285  < 2e-16 ***
## housepov    -1.829e+01  1.323e+01  1.145e+02  -1.382    0.170
## mathknow     1.306e+00  1.391e+00  2.315e+02   0.939    0.349
## yearstea     3.087e-03  1.416e-01  2.270e+02   0.022    0.983
## mathprep    -3.459e-01  1.374e+00  2.014e+02  -0.252    0.801
## sex         -1.197e+00  2.122e+00  2.160e+02  -0.564    0.573
## minority    -1.619e+01  3.028e+00  7.042e+02  -5.347 1.21e-07 ***
## ses          1.010e+01  1.544e+00  1.065e+03   6.539 9.62e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.452
## mathknow -0.085  0.060
## yearstea -0.258  0.072  0.029
## mathprep -0.628  0.040  0.005 -0.174
## sex      -0.203 -0.005  0.003  0.015 -0.008
## minority -0.321 -0.178  0.116  0.024  0.003 -0.009
## ses      -0.123  0.083 -0.005 -0.027  0.054  0.020  0.164
```

```r
ranova(rssc.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | classid) + (1 | schoolid)
##                           npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                      13 -5364.5 10755
## sex in (1 + sex | classid)   11 -5364.8 10752  0.5003  2     0.7787
## (1 | schoolid)              12 -5377.0 10778 24.8912  1  6.066e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for sex at the classroom level, as the p value = 0.779 for the Chi-square test is not signifcant at $\alpha = 0.05$.

MINORITY

```r
rssc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+minority|classid)+(1|schoolid),data=classroom)
summary(rssc.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
```

```
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10726.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9037 -0.6221 -0.0295  0.6033  3.4574
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  225.4   15.01
##           minority     171.3   13.09    -0.82
##  schoolid (Intercept)  157.4   12.55
##  Residual             1045.3   32.33
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.73594    5.38023  270.70509 100.318  < 2e-16 ***
## housepov     -17.34698   12.91268  103.34670  -1.343    0.182
## mathknow       1.45702    1.39355  234.04713   1.046    0.297
## yearstea      -0.01636    0.14285  234.25121  -0.115    0.909
## mathprep      -0.13520    1.37018  203.97000  -0.099    0.921
## sex           -1.01012    2.08966 1015.73461  -0.483    0.629
## minority     -16.48614    3.21756  183.20472  -5.124 7.55e-07 ***
## ses            9.89350    1.54595 1062.82882   6.400 2.33e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.435
## mathknow -0.079  0.061
## yearstea -0.265  0.080  0.038
## mathprep -0.618  0.037 -0.006 -0.171
## sex      -0.188 -0.009  0.009  0.015 -0.005
## minority -0.368 -0.171  0.108  0.025 -0.004 -0.009
## ses      -0.117  0.085  0.001 -0.023  0.051  0.021  0.149
```

```r
ranova(rssc.2)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | classid) + (1 | schoolid)
##                                     npar  logLik   AIC     LRT Df
## <none>                                13 -5363.2 10752
## minority in (1 + minority | classid)   11 -5364.8 10752  3.1967  2
## (1 | schoolid)                        12 -5373.2 10770 20.1422  1
##                                     Pr(>Chisq)
```

```
## <none>
## minority in (1 + minority | classid)     0.2022
## (1 | schoolid)                        7.189e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for minority at the classroom level, as the p value = 0.202 for the Chi-square test is not signifcant at $\alpha = 0.05$.

SES

```r
rssc.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+ses|classid)+(1|schoolid),data=classroom)
summary(rssc.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10725.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5688 -0.6004 -0.0316  0.5959  3.6176
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.06   9.277
##           ses           44.09   6.640   0.75
##  schoolid (Intercept)  173.16  13.159
##  Residual             1048.32  32.378
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  539.52093    5.26665  269.59234 102.441  < 2e-16 ***
## housepov     -16.28994   13.13445  111.28619  -1.240    0.217
## mathknow       1.37996    1.37294  222.43201   1.005    0.316
## yearstea       0.01605    0.14080  227.59545   0.114    0.909
## mathprep      -0.37734    1.34603  182.84309  -0.280    0.780
## sex           -1.32178    2.08794 1017.08508  -0.633    0.527
## minority     -16.09272    3.03497  717.66470  -5.302 1.52e-07 ***
## ses           10.05535    1.64507  171.13536   6.112 6.44e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.450
## mathknow -0.078  0.059
## yearstea -0.266  0.074  0.030
## mathprep -0.625  0.036 -0.001 -0.165
## sex      -0.186 -0.009  0.007  0.013 -0.009
```

```
## minority -0.325 -0.181  0.108  0.021  0.004 -0.014
## ses      -0.084  0.078  0.015 -0.024  0.056  0.022  0.142
```

```r
ranova(rssc.3)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | classid) + (1 | schoolid)
##                              npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                         13 -5362.8 10752
## ses in (1 + ses | classid)    11 -5364.8 10752  3.8395  2     0.1466
## (1 | schoolid)                 12 -5375.8 10776 26.0221  1  3.375e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for minority at the classroom level, as the p value = 0.147 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Random slopes for student-level predictors varying at school level

ONE BY ONE

Sex

```r
rss.4 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+sex||schoolid)+(1|classid),data=classroom)
summary(rss.4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8578 -0.6110 -0.0259  0.5922  3.5557
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   96.08   9.802
##  schoolid   sex           35.83   5.986
##  schoolid.1 (Intercept)  161.63  12.713
##  Residual               1054.36  32.471
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 539.43517    5.30740 272.54946 101.638  < 2e-16 ***
## housepov    -16.77661   13.22881 112.39593  -1.268    0.207
```

```
## mathknow       1.40067    1.39464  234.45882   1.004    0.316
## yearstea       0.01448    0.14163  226.44519   0.102    0.919
## mathprep      -0.27193    1.38010  205.78503  -0.197    0.844
## sex           -1.33534    2.18746  138.08788  -0.610    0.543
## minority     -16.16536    3.02861  704.25758  -5.338 1.27e-07 ***
## ses            9.98477    1.54243 1058.27875   6.473 1.46e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.081  0.055
## yearstea -0.259  0.070  0.028
## mathprep -0.633  0.036  0.004 -0.172
## sex      -0.179 -0.010  0.007  0.013 -0.004
## minority -0.320 -0.178  0.114  0.024  0.001 -0.015
## ses      -0.120  0.081 -0.007 -0.029  0.052  0.020  0.161
```

```r
ranova(rss.4, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + sex | schoolid) + (1 | classid)
##                             npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                        12 -5364.4 10753
## (1 | schoolid)                11 -5374.4 10771 19.9994  1  7.747e-06 ***
## sex in (0 + sex | schoolid)   11 -5364.8 10752  0.6137  1   0.433392
## (1 | classid)                 11 -5368.2 10758  7.4171  1   0.006461 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated sex random slope at a school level is insignifcant with a p value of .433.

Minority

```r
rss.5 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+minority||schoolid)+(1|classid),data=classroom)
summary(rss.5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
```

```
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   93.89   9.69
##  schoolid   minority       0.00   0.00
##  schoolid.1 (Intercept)  169.45  13.02
##  Residual               1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38919 101.585  < 2e-16 ***
## housepov     -17.64847   13.21758  113.87772  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201    0.841
## sex           -1.21419    2.09483 1022.42137  -0.580    0.562
## minority     -16.18678    3.02605  704.47892  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```r
ranova(rss.5,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + minority | schoolid) + (1 | classid)
##                                  npar  logLik   AIC     LRT Df
## <none>                             12 -5364.8 10754
## (1 | schoolid)                     11 -5375.2 10772 20.8586  1
## minority in (0 + minority | schoolid)   11 -5364.8 10752  0.0000  1
## (1 | classid)                      11 -5368.3 10759  7.1357  1
##                                  Pr(>Chisq)
## <none>
## (1 | schoolid)                    4.945e-06 ***
## minority in (0 + minority | schoolid)   1.000000
## (1 | classid)                      0.007556 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated minority random slope at school level is insignificant with a pvalue of 1.0.

SES

```r
rss.6 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+ses||schoolid)+(1|classid),data=classroom)
```

```r
summary(rss.6) #IS SIG
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   88.56   9.411
##  schoolid   ses           72.50   8.515
##  schoolid.1 (Intercept)  167.98  12.961
##  Residual               1035.12  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917  270.54314 102.126  < 2e-16 ***
## housepov     -16.94564   13.21116  112.82496  -1.283    0.202
## mathknow       1.35576    1.38459  232.19983   0.979    0.329
## yearstea       0.03079    0.14052  223.94305   0.219    0.827
## mathprep      -0.19801    1.35994  198.59419  -0.146    0.884
## sex           -1.40185    2.08170 1011.28944  -0.673    0.501
## minority     -16.52525    3.02189  700.06637  -5.469 6.32e-08 ***
## ses            9.78982    1.82217   79.01645   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.079  0.056
## yearstea -0.260  0.070  0.028
## mathprep -0.628  0.041  0.002 -0.172
## sex      -0.190 -0.007  0.006  0.018 -0.007
## minority -0.323 -0.180  0.110  0.024  0.001 -0.010
## ses      -0.091  0.076  0.006 -0.019  0.042  0.017  0.124
```

```r
ranova(rss.6,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + ses | schoolid) + (1 | classid)
##                              npar  logLik   AIC     LRT Df Pr(>Chisq)
```

```
## <none>                        12 -5362.4 10749
## (1 | schoolid)                11 -5374.6 10771 24.2924  1  8.276e-07 ***
## ses in (0 + ses | schoolid)   11 -5364.8 10752  4.6972  1    0.03021 *
## (1 | classid)                 11 -5365.7 10753  6.5177  1    0.01068 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated ses random slope at school level is signifcant with a p value of .03.

Allowing for correlations with random intercepts

ONE BY ONE

Sex

```r
rssc.4 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+sex|schoolid)+(1|classid),data=classroom)
summary(rssc.4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8048 -0.6095 -0.0222  0.5969  3.5525
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   97.34   9.866
##  schoolid (Intercept)  206.33  14.364
##           sex           84.08   9.170   -0.43
##  Residual             1041.76  32.276
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.399e+02  5.363e+00  2.626e+02 100.661  < 2e-16 ***
## housepov    -1.742e+01  1.325e+01  1.136e+02  -1.314    0.191
## mathknow     1.379e+00  1.396e+00  2.364e+02   0.988    0.324
## yearstea     6.876e-03  1.418e-01  2.277e+02   0.048    0.961
## mathprep    -2.796e-01  1.378e+00  2.061e+02  -0.203    0.839
## sex         -1.340e+00  2.301e+00  8.742e+01  -0.582    0.562
## minority    -1.642e+01  3.027e+00  7.076e+02  -5.425 7.96e-08 ***
## ses          9.928e+00  1.540e+00  1.055e+03   6.448 1.72e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
```

```
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.082  0.060
## yearstea -0.258  0.072  0.027
## mathprep -0.627  0.038  0.004 -0.172
## sex      -0.222 -0.003  0.006  0.014 -0.005
## minority -0.319 -0.178  0.114  0.024  0.004 -0.011
## ses      -0.121  0.083 -0.006 -0.028  0.053  0.018  0.163
```

```
ranova(rssc.4, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | schoolid) + (1 | classid)
##                               npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                          13 -5363.8 10754
## sex in (1 + sex | schoolid)     11 -5364.8 10752 1.8631  2   0.393952
## (1 | classid)                   12 -5367.6 10759 7.6414  1   0.005704 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated sex random slope at school-level is insignificant with a pvalue of .394.

Minority

```
rssc.5 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+minority|schoolid)+(1|classid),data=classroom)
summary(rssc.5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.69   9.311
##  schoolid (Intercept)  381.20  19.524
##           minority     343.13  18.524   -0.83
##  Residual             1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##                Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  9.999e+01  -1.277    0.204
```

```
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.824e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.394
## mathknow -0.078  0.061
## yearstea -0.253  0.091  0.024
## mathprep -0.576  0.037 -0.002 -0.167
## sex      -0.172 -0.013  0.010  0.014 -0.005
## minority -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses      -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

```
ranova(rssc.5,refit=F) #sig
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | schoolid) + (1 | classid)
##                                     npar  logLik   AIC    LRT Df
## <none>                                13 -5358.8 10744
## minority in (1 + minority | schoolid)   11 -5364.8 10752 11.967  2
## (1 | classid)                         12 -5361.8 10748  6.077  1
##                                     Pr(>Chisq)
## <none>
## minority in (1 + minority | schoolid)    0.00252 **
## (1 | classid)                          0.01370 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated minority random slope at school-level is significant with a pvalue of .003.

SES

```
rssc.6 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+ses|schoolid)+(1|classid),data=classroom)
summary(rssc.6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
```

```
## -3.5646 -0.6166 -0.0264  0.5888  3.7073
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.57   9.305
##  schoolid (Intercept)  171.18  13.083
##           ses           73.37   8.565   0.19
##  Residual             1035.90  32.185
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  538.72222    5.27647  271.13405 102.099  < 2e-16 ***
## housepov     -15.89873   13.15393  111.71410  -1.209    0.229
## mathknow       1.26025    1.38201  230.89932   0.912    0.363
## yearstea       0.03617    0.14002  220.42247   0.258    0.796
## mathprep      -0.21697    1.35642  197.10752  -0.160    0.873
## sex           -1.40436    2.08074 1011.40322  -0.675    0.500
## minority     -16.26699    3.03580  668.91517  -5.358 1.16e-07 ***
## ses            9.72646    1.82985   78.36218   5.315 9.75e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.077  0.057
## yearstea -0.259  0.073  0.028
## mathprep -0.627  0.039  0.001 -0.172
## sex      -0.188 -0.009  0.005  0.017 -0.008
## minority -0.325 -0.182  0.108  0.021  0.002 -0.011
## ses      -0.062  0.070  0.007 -0.021  0.045  0.018  0.117
```

```
ranova(rssc.6,refit=F) #not sig
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | schoolid) + (1 | classid)
##                             npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                        13 -5362.2 10750
## ses in (1 + ses | schoolid)   11 -5364.8 10752 5.1385  2    0.07659 .
## (1 | classid)                 12 -5365.3 10755 6.2117  1    0.01269 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated ses random slope at school-level is not significant with a p-value of .08.

Question: Report unusual changes in variance.

Answer: Perhaps most striking is the change in variance for the random slope term on minority. Previously, it was 0. However, it jumps to 343.13 in the correlated model. The variance for the random slope term on SES also increases, but the correlated random slope is not a significant addition to our model according to the rand test results.

Complex model

Take two predictors that had sig random slopes and add to model, test for need of one conditional on the other

-Minority is sig for correlated

-Ses is sig for uncorrelated

```
complex <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                 ses+(0+ses|schoolid)+(1+minority|schoolid)+(1|classid),data=classroom)
summary(complex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##  Groups     Name        Variance Std.Dev. Corr
##  classid    (Intercept)   80.63   8.979
##  schoolid   (Intercept)  404.54  20.113
##             minority     336.04  18.332   -0.84
##  schoolid.1 ses           74.93   8.656
##  Residual               1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.74621  95.160  < 2e-16 ***
## housepov     -15.32111   12.49443   99.25865  -1.226    0.223
## mathknow       1.67475    1.35000  221.33588   1.241    0.216
## yearstea       0.02102    0.13657  213.65672   0.154    0.878
## mathprep      -0.23546    1.31730  191.22014  -0.179    0.858
## sex           -1.03871    2.06951 1010.41144  -0.502    0.616
## minority     -16.72884    3.90720   55.41065  -4.282 7.43e-05 ***
## ses            9.19654    1.82272   82.48814   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
```

```
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

```r
ranova(complex, refit=F)
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -1.2e-04
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##                                       npar  logLik   AIC     LRT Df
## <none>                                  14 -5356.2 10740
## ses in (0 + ses | schoolid)             14 -5358.8 10746  5.1200  0
## minority in (1 + minority | schoolid)   12 -5362.4 10749 12.3899  2
## (1 | classid)                           13 -5358.9 10744  5.3724  1
##                                       Pr(>Chisq)
## <none>
## ses in (0 + ses | schoolid)
## minority in (1 + minority | schoolid)    0.00204 **
## (1 | classid)                            0.02046 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Question: Is the more complex model (with both random slopes in it) justified?

Answer: The complex model is justified since the rand test shows that the random slopes are both statistically significant at the 0.05 level, the only question revolves around statistical significance justifying compared to the Bayesian approach that would push for a simpler model.

The equation for the complex model is given by the following:

$Math1st_{ijk} = \beta_0 + \beta_1 * housepov_k + \beta_2 * mathknow_{jk} + \beta_3 * yearstea_{jk} + \beta_4 * mathprep_{jk} + \beta_5 * sex_{ijk} + \beta_{6k} * ses_{ijk} + \beta_{7k} * minority_{ijk} + \zeta_{0k} + \zeta_{6k} + \zeta_{7k} + eta_{jk} + \epsilon_{ijk}$

where $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0}), \zeta_{6k} \sim N(0, \sigma^2_{\zeta_6}), \zeta_{7k} \sim N(0, \sigma^2_{\zeta_7}), \eta_{jk} \sim N(0, \sigma^2_{\eta})$, and $\epsilon_{ijk} \sim N(0, \sigma^2_{\epsilon})$, all independent of each other.

```r
summary(model1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)  85.46    9.244
##  schoolid         (Intercept) 280.68   16.754
```

```
##  Residual                     1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.406   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Question: For UCM, write down: V_C, V_S, V_E for the three variance components (simply the estimates). Think of them as possibly varying with a covariate, though.

Answer: For the UCM, $V_C = 85.46$, $V_S = 280.68$, and $V_E = 1146.80$

```r
summary(model4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                     1064.95  32.63
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38922 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.87774  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201    0.841
## sex           -1.21419    2.09483 1022.42136  -0.580    0.562
## minority     -16.18678    3.02605  704.47889  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
```

```
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

Question: For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are: V_C, V_S, V_E?

Answer: For the most complicated fixed effects model with only random intercepts, $V_C = 93.89$, $V_S = 169.45$, and $V_E = 1064.95$.

Question: By what fraction did these each decrease with the new predictors in the model?

Answer: V_C increased by $\frac{93.89}{85.46} \sim 1.10$ times. V_S decreased by $\frac{169.45}{280.68} \sim 0.60$ times. V_E decreased by $\frac{1064.95}{1146.80} \sim 0.93$ times.

```r
summary(rss.6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   88.56   9.411
##  schoolid   ses           72.50   8.515
##  schoolid.1 (Intercept)  167.98  12.961
##  Residual               1035.12  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917  270.54314 102.126  < 2e-16 ***
## housepov     -16.94564   13.21116  112.82496  -1.283    0.202
## mathknow       1.35576    1.38459  232.19983   0.979    0.329
## yearstea       0.03079    0.14052  223.94305   0.219    0.827
## mathprep      -0.19801    1.35994  198.59419  -0.146    0.884
## sex           -1.40185    2.08170 1011.28944  -0.673    0.501
## minority     -16.52525    3.02189  700.06637  -5.469 6.32e-08 ***
## ses            9.78982    1.82217   79.01645   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.079  0.056
## yearstea -0.260  0.070  0.028
## mathprep -0.628  0.041  0.002 -0.172
## sex      -0.190 -0.007  0.006  0.018 -0.007
```

```
## minority -0.323 -0.180  0.110  0.024  0.001 -0.010
## ses      -0.091  0.076  0.006 -0.019  0.042  0.017  0.124
```

Question: Now consider the model with a random slope in ses. What are: V_C, $V_S(ses = 0)$, V_E ? We need to list 'ses=0' here, or we don't know how to use the slope variance

Answer: For the model with a random slope in ses at the school level, $V_C = 88.56$, $V_S(ses = 0) = 167.98$, and $V_E = 1035.12$.

Question: What are: $V_S(ses = -0.50)$, $V_S(ses = +0.5)$?

Answer: In this model, in which the random slope for SES is uncorrelated with the random school-level intercept, $V_S(ses = -0.50) = 167.98 + (-.5)^2 72.50 + 2(-.5)0 * \sqrt{167.98} * \sqrt{72.50} = 186.105$, and $V_S(ses = +0.5) = 167.98 + (.5)^2 72.50 + 2 * (.5)0 * \sqrt{167.98} * \sqrt{72.50} = 186.105$

```r
summary(rssc.5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.69   9.311
##  schoolid (Intercept)  381.20  19.524
##           minority     343.13  18.524   -0.83
##  Residual             1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  9.999e+01  -1.277    0.204
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.824e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.394
## mathknow -0.078  0.061
## yearstea -0.253  0.091  0.024
## mathprep -0.576  0.037 -0.002 -0.167
```

```
## sex      -0.172 -0.013  0.010  0.014 -0.005
## minority -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses      -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

Question: Now consider the model with a random slope in minority. What are: V_C, $V_S(minority = 0)$, V_E? We need to list 'minority=0' here, or we don't know how to use the slope variance

Answer: For the model with a random slope in minority at the school level, $V_C = 86.69$, $V_S(minority = 0)$ $= 381.20$, and $V_E = 1039.39$.

Question: What are: $V_S(minority = 0.25)$, $V_S(minority = +0.50)$, $V_S(minority = +0.75)$?

Answer: In this model, in which the random slope for minority is correlated with the random school-level, intercept, $V_S(minority = 0.25) = 381.20 + (0.25)^2 343.13 + 2(0.25)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 252.5549$,

$V_S(minority = +0.50) = 381.20 + (0.50)^2 343.13 + 2(0.50)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 166.801$, and

$V_S(minority = +0.75) = 381.20 + (0.25)^2 343.13 + 2(0.25)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 123.9384$.

```r
summary(complex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##  Groups     Name        Variance Std.Dev. Corr
##  classid    (Intercept)   80.63   8.979
##  schoolid   (Intercept)  404.54  20.113
##             minority     336.04  18.332   -0.84
##  schoolid.1 ses           74.93   8.656
##  Residual               1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##                Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.74621  95.160  < 2e-16 ***
## housepov     -15.32111   12.49443   99.25865  -1.226    0.223
## mathknow       1.67475    1.35000  221.33588   1.241    0.216
## yearstea       0.02102    0.13657  213.65672   0.154    0.878
## mathprep      -0.23546    1.31730  191.22014  -0.179    0.858
## sex           -1.03871    2.06951 1010.41144  -0.502    0.616
## minority     -16.72884    3.90720   55.41065  -4.282 7.43e-05 ***
## ses            9.19654    1.82272   82.48814   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
```

```
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

Question: Now consider the model with a random slope in ses & minority. What are: V_C, $V_S(minority = 0, ses = 0)$, V_E? We need to list 'ses=0, minority=0' here, or we don't know how to use the slope variance.

Answer: For the model with a random slope in ses & minority, $V_C = 80.63$, $V_S(minority = 0, ses = 0) = 404.54$, and $V_E = 1009.73$.

Question: What are: $V_S(ses = 0, minority = 0.50)$, $V_S(ses = 0.50, minority = 0)$, $V_S(ses = 0.50, minority = 0.50)$?

Answer: In this model, in which the random slope for ses is uncorrelated with the random intercept, but the random slope for minority is correlated with the random intercept,

$V_S(ses = 0, minority = 0.50) = 404.54 + (0)^2 * 74.93 + (0.50)^2 * 336.04 + 2 * 404.54 * 74.93 + 2 * (0.50)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 182.5268$,

$V_S(ses = 0.50, minority = 0) = 404.54 + (0.50)^2 * 74.93 + (0)^2 * 336.04 + 2 * 0.5 * 404.54 * 74.93 + 2 * (0)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 423.2725$

$V_S(ses = 0.50, minority = 0.50) = 404.54 + (0.50)^2 * 74.93 + (0.50)^2 * 336.04 + 2 * 0.5 * 404.54 * 74.93 + 2 * (0.50)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 201.2593$

Question: In the last model, what is a "likely" (+/- 1 sd) range for η_{0jk}

Answer: For the complex model, the "likely" range for η_{0jk} is $(-8.979, 8.979)$.

Question: Can we make a similar statement about ζ_{0k}?

Answer: We cannot make a similar statement for ζ_{0k} since it is correlated with ζ_{2k} on *Minority*.

Question: If you had a large value for η_{0jk}, would you expect a large or small or "any" value for: the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority?

Answer: There is no correlation between η_{0jk} (classroom-level intercept) and the school-level random slopes ζ_{1k} and ζ_{2k} on *SES* and *MINORITY*. Therefore, we would not expect a large value of η_{0jk} to have any effect on the two random slope terms as they are independent.

Question: If you had a large value for ζ_{0k}, would you expect a large or small or "any" value for: the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority (discuss each separately)?

Answer: ζ_{1k} could be any value due to the lack of correlation with ζ_{0k}

Answer: While ζ_{2k} would be small given a large value of ζ_{0k} because of the negative correlation between the two variables.

Part 2: Clare

Running initial model

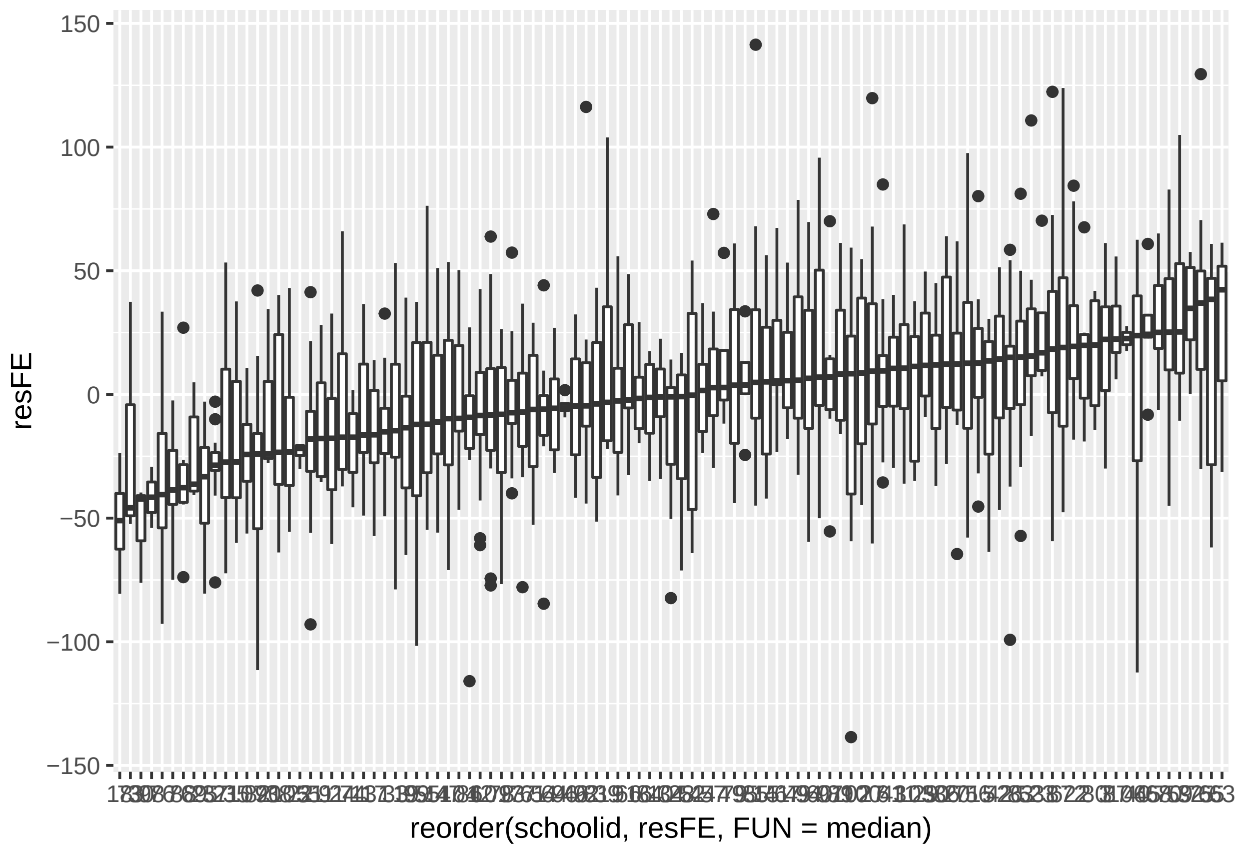
The initial model was run on a smaller dataset with 1081 observations due to missing data. School-level and classroom-level random intercepts are included in the model.

```r
#remove missing data -- not ideal, but have to do it for this analysis
classroom <- classroom %>% mutate(Math1st = mathkind + mathgain)
classroom2 <- na.omit(classroom)
#model
new1 <-lmer(Math1st~housepov+mathknow+yearstea+mathprep+sex
            +minority+ses+(1|schoolid)+(1|classid),data=classroom2)
```

Residual that removes only the "fixed effects"

Below we calculate the residuals that removes only the fixed effects. The boxplot of the residuals shows that there is great variation between schools and that there is a steady linear trend to the residuals, suggesting dependence.

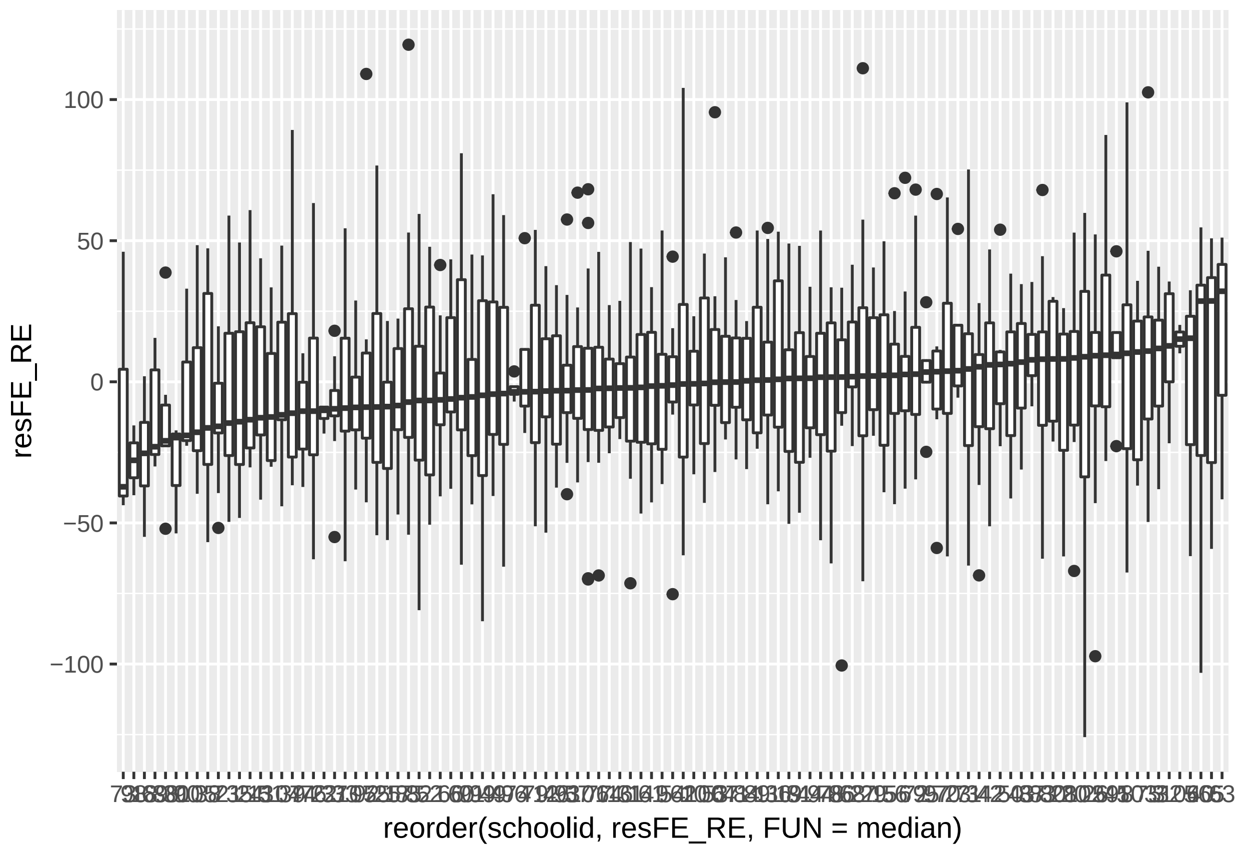
```r
#predicted scores
pred.yhat <- predict(new1,re.form=~0)

#residual
resFE <- classroom2$Math1st-pred.yhat

#show that it's not independent
if (vanillaR) {
ord <- order(unlist(tapply(resFE, classroom2$schoolid, median)))
boxplot(split(resFE, classroom2$schoolid)[ord])
} else {
ggplot(classroom2, aes(x = reorder(schoolid, resFE, FUN = median), y = resFE)) +
geom_boxplot()
}
```



Residuals for BLUPs random effects

The residuals for the BLUPs random effects are calculated below. The boxplot reveals a similar dependency to the previous plot, though not as pronounced. There doesn't seem to be as high a correlation as there is in the other residuals plot.

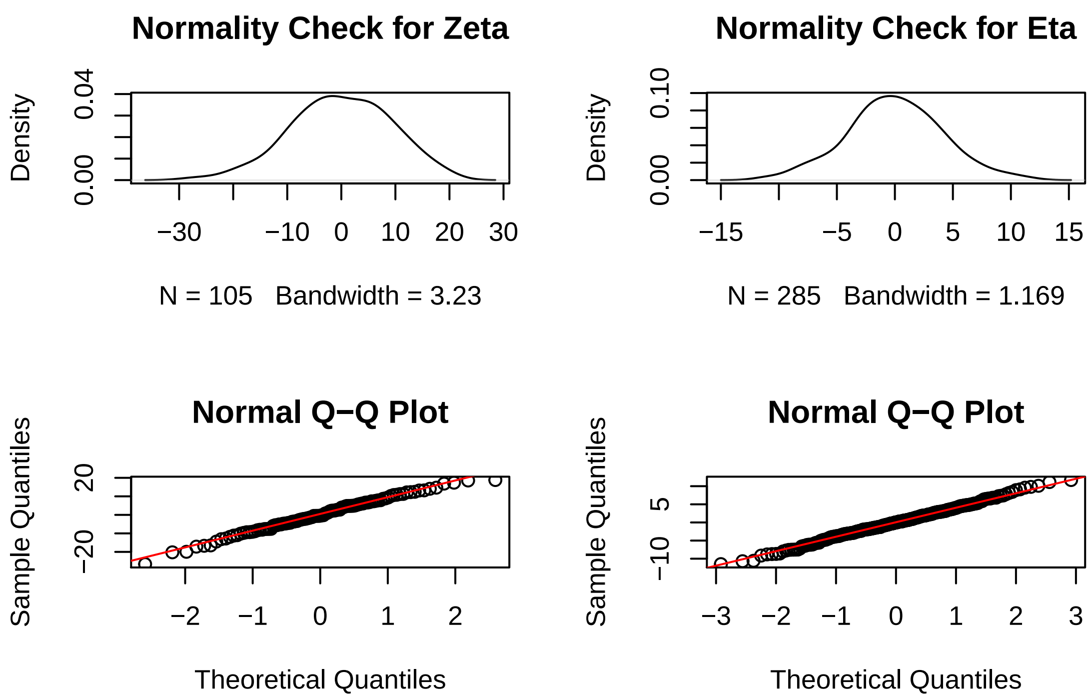
```
#getting predicted zeta_0 and eta_0
ranefs <- ranef(new1)
zeta0 <- ranefs$schoolid[,1]
eta0 <- ranefs$classid[,1]
#indexing
idx.sch <- match(classroom2$schoolid, sort(unique(classroom2$schoolid)))
idx.cls <- match(classroom2$classid, sort(unique(classroom2$classid)))
classroom2$zeta0 <- zeta0[idx.sch]
classroom2$eta0 <- eta0[idx.cls]
#now subtract all from outcome
resFE_RE <- classroom2$Math1st-pred.yhat-classroom2$zeta0-classroom2$eta0
#show that it's not independent, but much less correlated than resFE
if (vanillaR) {
ord <- order(unlist(tapply(resFE_RE, classroom2$schoolid, median)))
boxplot(split(resFE_RE, classroom2$schoolid)[ord])
}else{
ggplot(classroom2, aes(x = reorder(schoolid, resFE_RE, FUN = median), y = resFE_RE)) +
geom_boxplot()
}
```



Examining BLUPs for normality

To examine the BLUPs for mormality, density plots and Q-Q plots were constructed. Both ζ_0 and η_0 appear to be normal, with a few poossible outliers near the tails.

```
par(mfrow=c(2,2))
plot(density(zeta0), main ="Normality Check for Zeta")
plot(density(eta0), main = "Normality Check for Eta")
#looking good
qqnorm(zeta0);qqline(zeta0,col=2)
qqnorm(eta0);qqline(eta0,col=2)
```

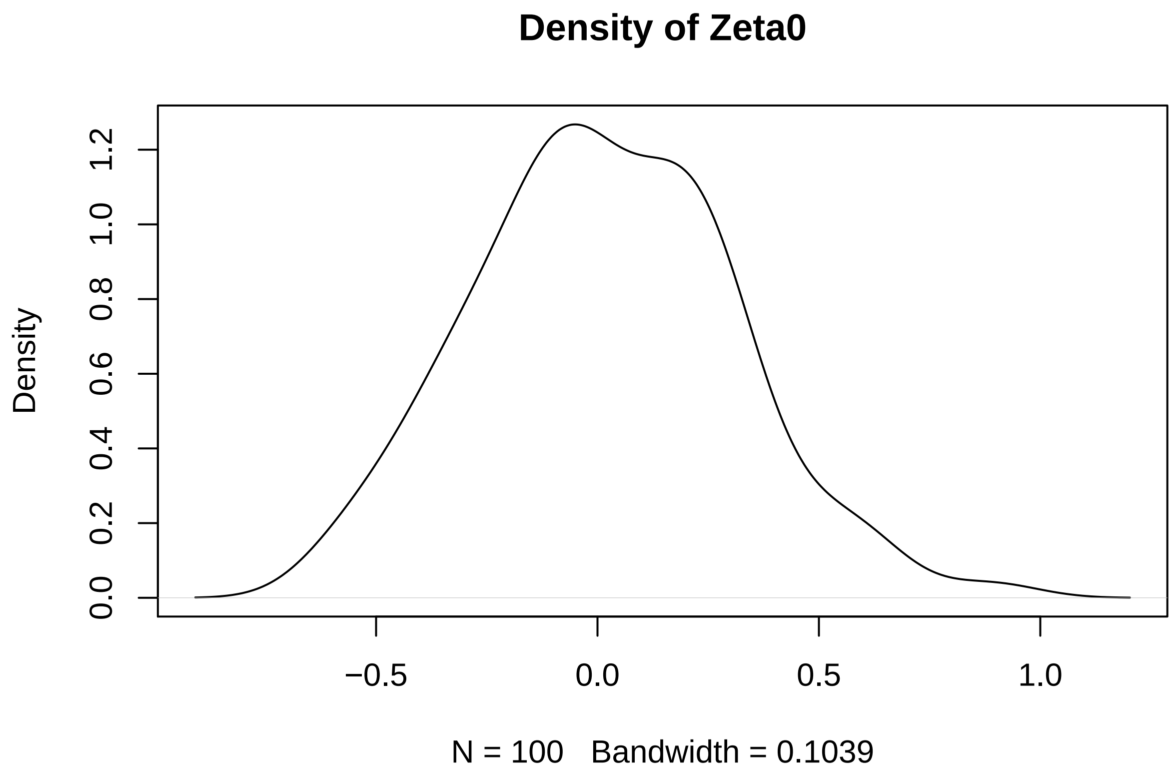



```
#looking good
```

Simulation

Below is a simulation based on the H0 being true, and a $\sigma_\epsilon = 1$. We find that the potential estimate is very close to 0, which we would expect since our $\sigma^2_{\zeta_0}$ has a "true" value of 0.

```
set.seed(10314)
school.sim <- matrix(1,10,100)
means <- NULL
for (i in 1:100){
school.sim[,i] <- rnorm(10,mean=0, sd=1)
means[i] <- mean(school.sim[,i])
}
plot(density(means), main = "Density of Zeta0")
```




```r
#we see the density is concentrated around 0
paste("A potential estimate of sigma_{zeta_0} is ",mean(means))
```

```
## [1] "A potential estimate of sigma_{zeta_0} is  0.0142117878263361"
```

New Complex Model

We now include a correlated random slope at the school-level for minority.

```r
classroom <- read.csv("classroom.csv")
classroom <- classroom %>% mutate(Math1st = mathkind+mathgain)
classroom2 <- na.omit(classroom)
newcomplex <-lmer(Math1st~housepov+mathknow+yearstea+mathprep+sex+minority+ses+
                    (minority|schoolid)+(1|classid),data=classroom2)
summary(newcomplex)
```

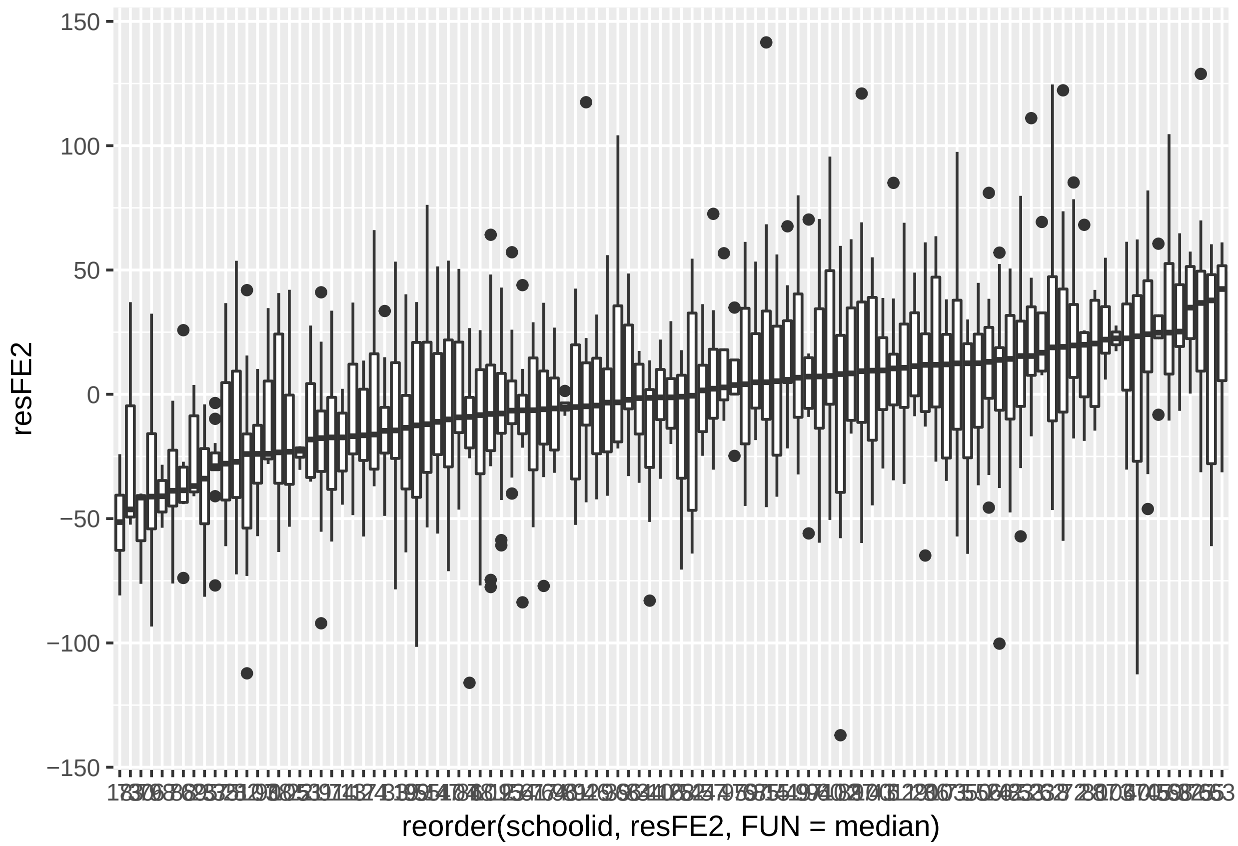
```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1st ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (minority | schoolid) + (1 | classid)
##    Data: classroom2
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
```

```
##      Min       1Q   Median       3Q      Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.69   9.311
##  schoolid (Intercept)  381.20  19.524
##           minority     343.13  18.524   -0.83
##  Residual             1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  9.999e+01  -1.277    0.204
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.824e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.394
## mathknow -0.078  0.061
## yearstea -0.253  0.091  0.024
## mathprep -0.576  0.037 -0.002 -0.167
## sex      -0.172 -0.013  0.010  0.014 -0.005
## minority -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses      -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

Manually calculate residuals for fixed effects

In the new model, we see a similar pattern of dependency. There is a general positive, linear trend to the residuals, and there is heterogeneity of variance across and within schools. These findings all suggest dependence.

```
#predicted scores
pred.yhat2 <- predict(newcomplex,re.form=~0)

#residual
resFE2 <- classroom2$Math1st-pred.yhat2

#show that it's not independent
if (vanillaR) {
ord <- order(unlist(tapply(resFE2, classroom2$schoolid, median)))
boxplot(split(resFE2, classroom2$schoolid)[ord])
} else {
ggplot(classroom2, aes(x = reorder(schoolid, resFE2, FUN = median), y = resFE2)) +
geom_boxplot()
}
```

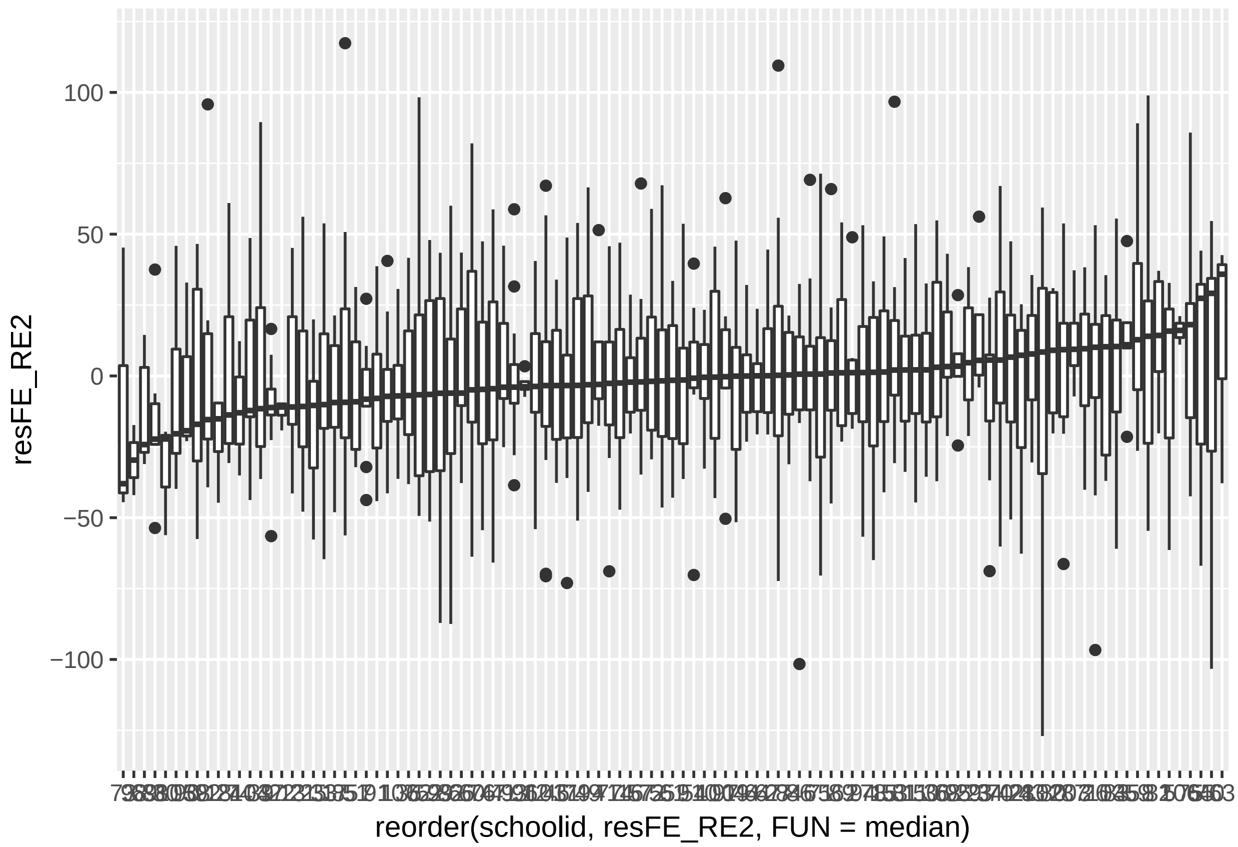


Residuals from BLUPs random effects

The residuals from the BLUPs random effects are calculated below. The boxplot of the residuals appears to be only slightly correlated, partly due to the uptake near the final set of schools on the x-axis. Although the correlation of the residuals is probably near 0, there is still enough variation within schools, and enough of a correlation in the data to suggest dependence.

```
#getting predicted zeta_0 and eta_0
ranefs2 <- ranef(newcomplex)
zeta0c <- ranefs2$schoolid[,1]
eta0c <- ranefs2$classid[,1]
zeta1c <- ranefs2$schoolid[,2]
#indexing
idx.sch <- match(classroom2$schoolid, sort(unique(classroom2$schoolid)))
idx.cls <- match(classroom2$classid, sort(unique(classroom2$classid)))
classroom2$zeta0c <- zeta0c[idx.sch]
classroom2$eta0c <- eta0c[idx.cls]
classroom2$zeta1c <- zeta1c[idx.sch]
#now subtract all from outcome
resFE_RE2 <- classroom2$Math1st-pred.yhat-classroom2$zeta0c-classroom2$eta0c-(classroom2$minority*class
#show that it's not independent, but much less correlated than resFE
if (vanillaR) {
ord <- order(unlist(tapply(resFE_RE2, classroom2$schoolid, median)))
boxplot(split(resFE_RE2, classroom2$schoolid)[ord])
```

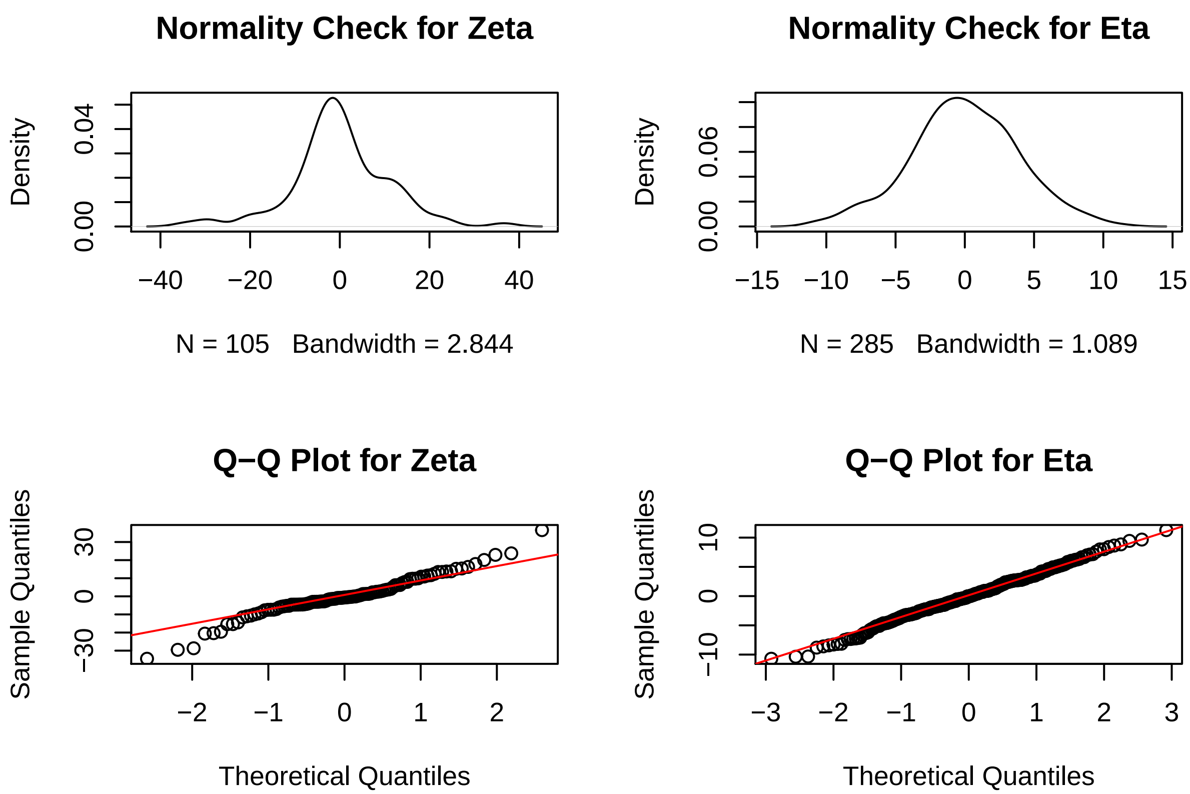
```r
}else{
ggplot(classroom2, aes(x = reorder(schoolid, resFE_RE2, FUN = median), y = resFE_RE2)) +
geom_boxplot()
}
```



Examining Normality of BLUPs

Below we examine the normality of ζ_0 and η_0. The density and Q-Q plots for η_0 suggest normality, with a possibility of a few outliers near the tails. The normality of ζ_0 is more questionable. The tails do not appear to fit a normal distribution.

```r
par(mfrow=c(2,2))
plot(density(zeta0c), main ="Normality Check for Zeta")
plot(density(eta0c), main = "Normality Check for Eta")
# eta looks pretty normal
#zeta not so much
qqnorm(zeta0c, main = "Q-Q Plot for Zeta");qqline(zeta0c,col=2)
qqnorm(eta0c, main = "Q-Q Plot for Eta");qqline(eta0c,col=2)
```



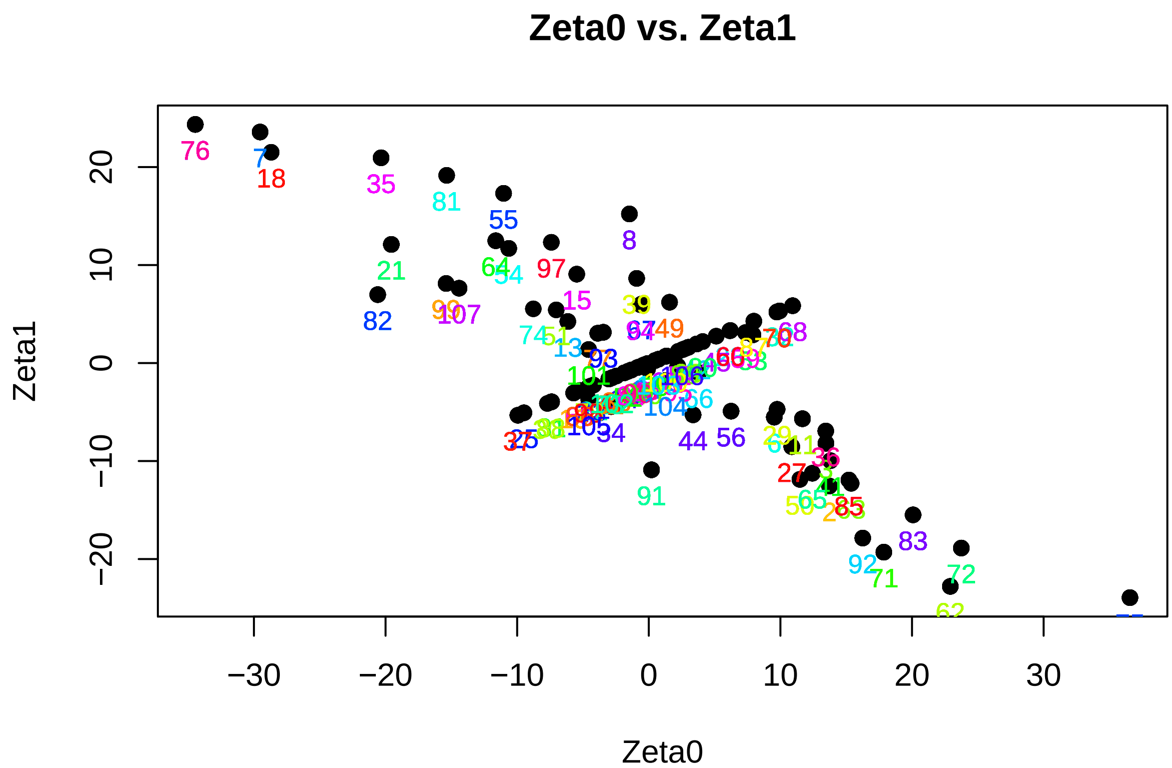

```
#zeta looking iffy, but with a few possible outliers
#eta good too, with few outliers.
```

Plotting ζ_0 versus ζ_1

The correlation between ζ_0 and ζ_1 in the output is -0.83. The graph below suggests a moderate negative trend, but there are some outliers that do not support this trend. Rather, they seem to be positively related.

*Note: the labels were put in rainbow in order to better discern their locations.

```
plot(classroom2$zeta0c,classroom2$zeta1c, main = "Zeta0 vs. Zeta1",
     ylab = "Zeta1",xlab = "Zeta0", pch=19)
 text(classroom2$zeta0c,classroom2$zeta1c, labels = classroom2$schoolid,
      cex = 0.8, col = rainbow(100), pos = 1)
```



Tracking down outliers

The outliers from the plots above can be tracked down by examining the data points via their IDs.

```
classroom2$zeta0c[classroom2$schoolid==45][[1]]/classroom2$zeta1c[classroom2$schoolid==45][[1]]
```

```
## [1] 1.868107
```

```
classroom2$zeta0c[classroom2$schoolid==68][[1]]/classroom2$zeta1c[classroom2$schoolid==68][[1]]
```

```
## [1] 1.868107
```

```
classroom2$zeta0c[classroom2$schoolid==30][[1]]/classroom2$zeta1c[classroom2$schoolid==30][[1]]
```

```
## [1] 1.868107
```

There seems to be a trend here that the zeta0/zeta1 ratio is = 1.868107, so let's filter it out.

```
outliers <- classroom2 %>% filter(round(zeta0c/zeta1c,6)==1.868107) %>%
  select(zeta0c,zeta1c,schoolid,minority)
```

Now let's make sure the IDs from the plot are showing up here.

```
unique(outliers$schoolid)
```

```
##  [1]   1   4   5   9  10  12  14  16  17  19  20  22  23  24  25  26  28
## [18]  30  31  32  33  37  38  42  43  45  46  47  52  57  60  61  68  69
## [35]  70  73  78  79  80  84  86  87  88  89  90  96  98 100 102 103 106
```

They are! Now what's going on with minority (ζ_1)?

```
table(outliers$minority)
```

```
##
##   1
## 455
```

All the students are minorities!

It seems like the (perfectly) positive trend in the data is being driven by schools in which all the students are minorities. That is, in schools in which there are only minority students, all other factors held equal, there is a boost in math scores in 1st grade for minority students. In a way, being in a totally minority school is a "protective" factor for minority students.

Part 3: Bianca

Create person-period file

In this part of the project, no variables of interested have missing observation. Therefore, the full dataset is used.

```
#new variables
classroom2 <- classroom %>% mutate(math0 = mathkind) %>% mutate(math1 = mathkind+mathgain)
#reshape the data
class_pp <- reshape(classroom2, varying = c("math0", "math1"), v.names = "math", timevar = "year",
times = c(0, 1), direction = "long")
```

Note: we ignore classroom in this analysis but keep it in the notation.

Initial longitudinal model

We fit a model with math as outcome, and fixed effect for time trend (year), as well as random intercept for school.

The equation for the model below:

$$Math_{tijk} = b_0 + \zeta_{0k} + b_1 * Time_{tijk} + \epsilon_{tijk}$$

where $\zeta_{0k} \sim N(0, \sigma^2_{zeta0})$ and $\epsilon_{tijk} \sim N(0, \sigma^2_{\epsilon})$

We refer to this as Model 0.

Below the model fit:

```
fit1 <- lmer(math ~ year + (1|schoolid), data = class_pp)
summary(fit1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 | schoolid)
##    Data: class_pp
##
## REML criterion at convergence: 23951.7
##
```

```
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.2833 -0.6084  0.0037  0.6329  3.7761
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  schoolid (Intercept)  348.7   18.67
##  Residual             1268.4   35.62
## Number of obs: 2380, groups:  schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)
## (Intercept)  464.932      2.116  132.154  219.73   <2e-16 ***
## year          57.566      1.460 2270.855   39.43   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.345
```

Add child-level random intercept

To the previous model, we now add random intercepts for child:

$$Math_{tijk} = b_0 + \delta_{0ijk} + \zeta_{0k} + b_1 * Time_{tijk} + \epsilon_{tijk}$$

where $\delta_{0tijk} \sim N(0, \sigma^2_{\delta_0}), \zeta_{0k} \sim N(0, \sigma^2_{\zeta_0})$ and $\epsilon_{tijk} \sim N(0, \sigma^2_{\epsilon})$ independetly of one another.

We refer to this as M1.

```r
fit2 <- lmer(math ~ year + (1|schoolid/childid), data = class_pp)
summary(fit2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 | schoolid/childid)
##    Data: class_pp
##
## REML criterion at convergence: 23554.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7492 -0.4811  0.0085  0.4881  3.4957
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  childid:schoolid (Intercept) 702.0    26.50
##  schoolid         (Intercept) 307.5    17.54
##  Residual                     599.1    24.48
## Number of obs: 2380, groups:  childid:schoolid, 1190; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error       df t value Pr(>|t|)
```

```
## (Intercept)  465.118      2.042  117.023  227.74   <2e-16 ***
## year          57.566      1.003 1189.000   57.37   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.246
```

In model 0 the variance $\sigma^2_{\zeta_0} = 348.7$ and in model 1 $\sigma^2_{\zeta_0} = 307.5$.
In model 0 , the varaince $\sigma^2_{\epsilon} = 1268.4$ and in model 1 $\sigma^2_{\epsilon} = 599.1$. We note that including child-level variation leads to a decrease in the variance of both the random effects.

Compute Pseudo-R^2

Compute a pseudo R^2 relating the between school variation and ignoring between students in the same school.

We calculate this as :

$$\frac{\sigma^2_{\zeta_0}(M_0) - \sigma^2_{\zeta_0}(M_1)}{\sigma^2_{\zeta_0}(M_0)} = \frac{348.7 - 307.5}{348.7} = 0.12$$

The between-school variance is reduced by 12% (or 'explained') with the introduction of student random effect.

Does the total variation stay about the same?

```r
tot_m0 = 348.7 + 1268.4
tot_m1 = 702 + 307.5 + 599.1
paste("Tot variance for model 0 : ", tot_m0)
```

```
## [1] "Tot variance for model 0 :  1617.1"
```

```r
paste("Tot variance for model 1: ", tot_m1)
```

```
## [1] "Tot variance for model 1:  1608.6"
```

There is only a slightly decrease in the total variance between Model 0 and Model1.

Add a random slope for time trend

We now add a random slope (ζ_{1k}) for time trend between schools.

$$Math_{tijk} = b_0 + \delta_{0ijk} + \zeta_{0k} + (b_1 + \zeta_{1k}) * Time_{tijk} + \epsilon_{tijk}$$

where $\delta_{0tijk} \sim N(0, \sigma^2_{\delta_0})$, $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0})$, $\zeta_{1k} \sim N(0, \sigma^2_{\zeta_1})$ and $\epsilon_{tijk} \sim N(0, \sigma^2_{\epsilon})$ – each independetly of one another.

We refer to this as Model 2

We run the model and report the fit:

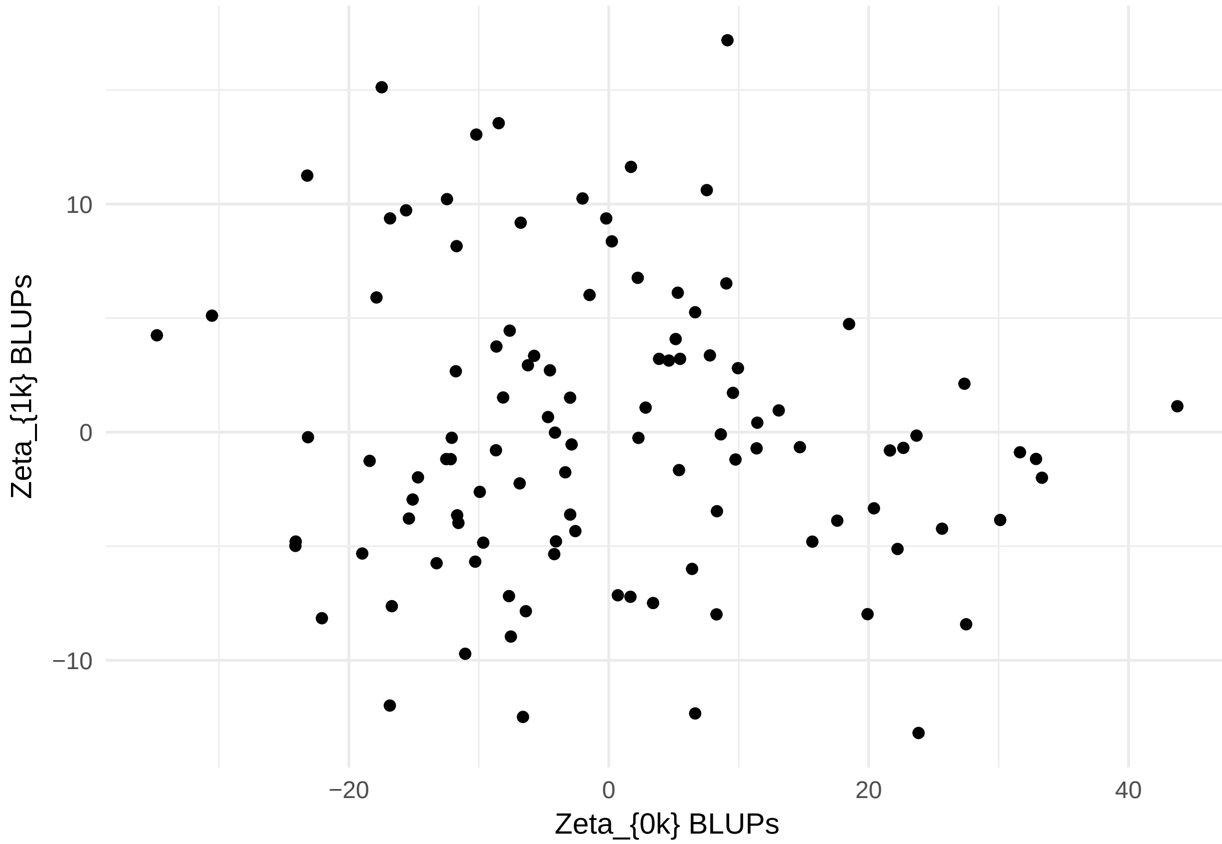
```r
fit3 = lmer(math ~ year + (1 + year|| schoolid) + (1|childid), data = class_pp)
summary(fit3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 + year || schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 23529.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7665 -0.4721  0.0139  0.4686  3.6080
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  childid    (Intercept) 725.13   26.928
##  schoolid   year         88.67    9.417
##  schoolid.1 (Intercept) 324.79   18.022
##  Residual               552.21   23.499
## Number of obs: 2380, groups:  childid, 1190; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.087      2.081 109.954  223.44   <2e-16 ***
## year          57.499      1.370  99.917   41.97   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.178
```

Generate the BLUPs for this model (Model 2)

Examine then whether the independence between ζ_0 and ζ_1 is reflected in a scatterplot of these two sets of effects.

```r
pp_ranefs <- ranef(fit3)

if (vanillaR) {
plot(pp_ranefs$schoolid[,2],pp_ranefs$schoolid[,1])
}else{
ggplot(pp_ranefs$schoolid, aes(x = pp_ranefs$schoolid[,2], y = pp_ranefs$schoolid[,1] )) +
geom_point() + labs(x = "Zeta_{0k} BLUPs", y = "Zeta_{1k} BLUPs") + theme_minimal()
}
```



From the plot, the BLUPs for ζ_{0k} and for ζ_{1k} appear uncorrelated, reflecting the way in which the model was built. In the BLUPs, we have a correlation of:

```
cor(pp_ranefs$schoolid[,2],pp_ranefs$schoolid[,1])
```

```
## [1] -0.1118599
```

```
corrtestp = cor.test(pp_ranefs$schoolid[,2],pp_ranefs$schoolid[,1],
                     method = "pearson")$p.value

paste("P-value for pearson test for correlatio:", round(corrtestp,3))
```

```
## [1] "P-value for pearson test for correlatio: 0.251"
```

That is, between the slope random effects and the ranom intercept blups, there is a very small negative correlation, which is not significantly different from 0 – which we see from the plot and would expect from how we have specified the model.

Heteroscedasticity in the random effects

Question: What are: $V_S(year = 0)$, $V_S(year = 1)$?

The model we are considering is :

$Math_{tijk} = b_0 + \delta_{0tijk} + \zeta_{0k} + (b_1 + \zeta_{1k})Time_{tijk} + \epsilon_{tijk}$

So we have that (in this model, in which we are forcing correlation of 0 between slope and intercept):

- $V_S(year = 0) = \sigma^2_{\zeta_0} = 324.79$

- $V_S(year = 1) = \sigma^2_{\zeta_0} + (1^2) * \sigma^2_{\zeta_1} = 88.67 + 324.79 = 413.46$

Run model separately by year

We now examine what happens if we run the model separately by year. Do we get the same estimates for the variance between schools?

```
class_year0 = class_pp[class_pp$year == 0,]


# Run model for year 0
fit4 = lmer(math ~ (1 | schoolid), data = class_year0)
summary(fit4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ (1 | schoolid)
##    Data: class_year0
##
## REML criterion at convergence: 12085.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.8223 -0.5749  0.0005  0.6454  3.6237
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
##  schoolid (Intercept)  364.3   19.09
##  Residual             1344.5   36.67
## Number of obs: 1190, groups:  schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error     df t value Pr(>|t|)
## (Intercept)   465.23       2.19 103.20   212.4   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
# Run modelfor year 1
class_year1 = class_pp[class_pp$year == 1,]
fit5 = lmer(math ~ (1 | schoolid), data = class_year1)
summary(fit5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ (1 | schoolid)
##    Data: class_year1
##
## REML criterion at convergence: 11950.8
##
## Scaled residuals:
##    Min     1Q Median     3Q    Max
## -5.291 -0.612 -0.005  0.613  3.793
##
## Random effects:
##  Groups   Name        Variance Std.Dev.
```

```
##  schoolid (Intercept)  306.8   17.52
##  Residual             1205.0   34.71
## Number of obs: 1190, groups:  schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.698      2.027 103.069   257.8   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

In this case, for the Year 0 Model, we get an estimated $\hat{\sigma}^2_{\zeta_{0k}} = 364.3$, while for Year 1 Model, we have $\hat{\sigma}^2_{\zeta_{0k}} = 306.8$. We note that these estimates are different from the ones computed above.

Allow for correlation

We now allow for correlation between the random effects for the intercept and the slope. We call this Model 3.

```r
fit6 = lmer(math ~ year + (1 + year| schoolid) + (1|childid), data = class_pp)
summary(fit6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (1 + year | schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 23520.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.7030 -0.4686  0.0066  0.4669  3.5142
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  childid  (Intercept) 728.0    26.98
##  schoolid (Intercept) 370.6    19.25
##           year        109.1    10.44    -0.45
##  Residual             547.0    23.39
## Number of obs: 2380, groups:  childid, 1190; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.099      2.188 102.919  212.60   <2e-16 ***
## year          57.668      1.440  94.575   40.04   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.439
```

Correlation beteween ζ_0 and ζ_1 = -0.45.

To test whether the correlation is statistically significant, we can compare Model 2 with Model 3 using an anova test.

```r
anova(fit3,fit6, refit = F)
```

```
## Data: class_pp
## Models:
## fit3: math ~ year + (1 + year || schoolid) + (1 | childid)
## fit6: math ~ year + (1 + year | schoolid) + (1 | childid)
##      Df   AIC   BIC logLik deviance  Chisq Chi Df Pr(>Chisq)
## fit3  6 23541 23576 -11764    23529
## fit6  7 23534 23575 -11760    23520 8.8241      1   0.002973 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

With a p-value $p = 0.003$, we reject the null hypothesis at $\alpha = 0.05$ significance level, and conclude that there correlation term is statistically significant.

So we have that (in this model where we are allowing for correlation between slope and intercept):

- $V_S(year = 0) = \sigma^2_{\zeta_0} = 370.6$
- $V_S(year = 1) = \sigma^2_{\zeta_0} + \sigma^2_{\zeta_1} + 2\rho_{\zeta_0\zeta_1}\sigma_{\zeta_0}\sigma_{\zeta_1} = 370.6 + 109.1 - 2 * 0.45 * \sqrt{370.6}\sqrt{109.1} = 298.72$

These estimates are a lot closer to the school variances that result from fitting the models for the two years separately (in which we have σ^2_ζ respectively be 364.3 for year 0 and 306.8 for year 1).

Therefore, it seems that the model that allows for correlation between the two random effects has a better fit than the one forcing that correlation to be 0.

Part 4: Kaushik

Reload the data and make person-period file

A reduced dataset was used for this analysis since there is missing data for one of the variables of interest. We acknowledge that this is not ideal in practice.

```r
#re-read data
classroom <- read.csv("classroom.csv")
classroom2 <- na.omit(classroom)
#new variables
classroom2 <- classroom2 %>% mutate(math0 = mathkind) %>% mutate(math1 = mathkind+mathgain)
#reshape the data
class_pp <- reshape(classroom2, varying = c("math0", "math1"), v.names = "math", timevar = "year",
times = c(0, 1), direction = "long")
```

Baseline model: unconditional growth model

$$MATH_{tijk} = b_0 + \delta_{0ijk} + (b_1 + \zeta_{1k})TIME_{tijk} + \zeta_{0k} + \epsilon_{tijk}$$

where t represents occasion (in this case, year/grade), i represents students, j represents classrooms and k represents schools. $\delta_{0ijk} \sim N(0, \sigma^2_{\delta_0})$, $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0})$, $\zeta_{1k} \sim N(0, \sigma^2_{\zeta_1})$ and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$ all independent of each other except for ζ_{0k} and ζ_{1k} having a correlation $\rho_{\zeta_0\zeta_1}$.

```r
ugm <- lmer(math ~ year + (year|schoolid) + (1|childid), data=class_pp)
summary(ugm)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (year | schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 21391.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.6737 -0.4699  0.0038  0.4683  3.4882
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  childid  (Intercept) 749.0    27.37
##  schoolid (Intercept) 373.5    19.33
##           year        112.4    10.60    -0.53
##  Residual             547.8    23.41
## Number of obs: 2162, groups:  childid, 1081; schoolid, 105
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.257      2.241 101.265   207.6   <2e-16 ***
## year          58.006      1.491  95.409    38.9   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.486
```

Add student, classroom and school level fixed effects

$MATH_{tijk} = b_0 + \delta_{0ijk} + (b_1 + \zeta_{1k})TIME_{tijk} + b_2 SES_{ijk} + b_3 SEX_{ijk} + b_4 MINORITY_{ijk} + b_5 YEARSTEA_{jk} + b_6 MATHKNOW_{jk} + b_7 MATHPREP_{jk} + b_8 HOUSEPOV_k + \zeta_{0k} + \epsilon_{tijk}$

where t represents occasion (in this case, year/grade), i represents students, j represents classrooms and k represents schools. $\delta_{0ijk} \sim N(0, \sigma^2_{\delta_0})$, $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0})$, $\zeta_{1k} \sim N(0, \sigma^2_{\zeta_1})$ and $\epsilon_{ijk} \sim N(0, \sigma^2_{\epsilon})$ all independent of each other except for ζ_{0k} and ζ_{1k} having a correlation $\rho_{\zeta_0\zeta_1}$.

```r
fit2 <- lmer(math ~ year + sex + ses + minority + yearstea + mathknow + mathprep + housepov + (year|sch
summary(fit2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math ~ year + sex + ses + minority + yearstea + mathknow + mathprep +
##     housepov + (year | schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 21275.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.6812 -0.4784  0.0040  0.4712  3.4245
```

```
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  childid  (Intercept) 689.5    26.26
##  schoolid (Intercept) 249.2    15.79
##           year        114.2    10.69    -0.53
##  Residual             547.4    23.40
## Number of obs: 2162, groups:  childid, 1081; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  483.44962    4.79270  369.63678 100.872  < 2e-16 ***
## year          57.90494    1.49801   95.33745  38.655  < 2e-16 ***
## sex           -0.52171    1.95332 1033.75341  -0.267    0.789
## ses            9.59015    1.43816 1071.21739   6.668 4.14e-11 ***
## minority     -16.01167    2.82859  705.01716  -5.661 2.19e-08 ***
## yearstea       0.02304    0.11846  876.23254   0.194    0.846
## mathknow      -0.22237    1.17471  768.51612  -0.189    0.850
## mathprep      -1.08389    1.14195  932.69685  -0.949    0.343
## housepov     -18.17699   12.40599  108.12089  -1.465    0.146
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) year   sex    ses    minrty yearst mthknw mthprp
## year     -0.204
## sex      -0.194 -0.001
## ses      -0.120 -0.001  0.028
## minority -0.333  0.004 -0.007  0.157
## yearstea -0.240  0.001  0.018 -0.035  0.021
## mathknow -0.085 -0.001  0.002 -0.012  0.106  0.038
## mathprep -0.578 -0.002 -0.014  0.054  0.003 -0.182 -0.001
## housepov -0.463  0.004 -0.007  0.080 -0.178  0.067  0.060  0.036
```

For year==0:

*what percent of between school differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BS} = \sigma^2_{\zeta_0} + 2 * year * \rho_{\zeta_0\zeta_1}\sigma_{\zeta_0}\sigma_{\zeta_1} + year^2 * \sigma^2_{\zeta_1}$$

$$V_{1BS}(year = 0) = \sigma^2_{\zeta_0} = 373.5$$

After adding fixed-effects:

$$V_{2BS} = \sigma^2_{\zeta_0} + 2 * year * \rho_{\zeta_0\zeta_1}\sigma_{\zeta_0}\sigma_{\zeta_1} + year^2 * \sigma^2_{\zeta_1}$$

$$V_{2BS}(year = 0) = \sigma^2_{\zeta_0} = 249.2$$

The percent difference in between-school variance for $year = 0$ is be given by:

$$\frac{V_{1BS} - V_{2BS}}{V_{1BS}} = \frac{373.5 - 249.2}{373.5} = 33.28\%$$

Model 2 explains 33.28% of the between-school variance for $year = 0$.

*what percent of between child differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BC}(year = 0) = \sigma^2_{\delta_0} = 749.0$$

After adding fixed effects:

$$V_{2BC}(year = 0) = \sigma^2_{\delta_0} = 689.5$$

The percent difference in between-child variance explained by the second model for $year = 0$ is given by:

$$\frac{V_{1BC} - V_{2BC}}{V_{1BC}} = \frac{749.0 - 689.5}{749.0} = 7.94\%$$

Model 2 explains 7.94% of the between-child variance for $year = 0$.

For year==1:

*what percent of between school differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BS} = \sigma^2_{\zeta_0} + 2 * year * \rho_{\zeta_0\zeta_1}\sigma_{\zeta_0}\sigma_{\zeta_1} + year^2 * \sigma^2_{\zeta_1}$$

$$V_{1BS}(year = 1) = 373.5 + 2(-0.53)(19.33)(10.60) + 112.4 = 268.71$$

After adding fixed-effects:

$$V_{2BS} = \sigma^2_{\zeta_0} + 2 * year * \rho_{\zeta_0\zeta_1}\sigma_{\zeta_0}\sigma_{\zeta_1} + year^2 * \sigma^2_{\zeta_1}$$

$$V_{2BS}(year = 1) = 249.2 + 2(-0.53)(15.79)(10.69) + 114.2 = 184.48$$

The percent difference in between-school variance for $year = 0$ is be given by:

$$\frac{V_{1BS} - V_{2BS}}{V_{1BS}} = \frac{268.71 - 184.48}{268.71} = 31.35\%$$

Model 2 explains 31.35% of the between-school variance for $year = 1$.

*what percent of between child differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BC}(year = 1) = \sigma^2_{\delta_0} = 749.0$$

After adding fixed effects:

$$V_{2BC}(year = 1) = \sigma^2_{\delta_0} = 689.5$$

The percent difference in between-child variance explained by the second model for $year = 1$ is given by:

$$\frac{V_{1BC} - V_{2BC}}{V_{1BC}} = \frac{749.0 - 689.5}{749.0} = 7.94\%$$

Model 2 explains 7.94% of the between-child variance for $year = 1$.

Based on significance,

- what factors seem useful in describing ("explaining") differences between student outcomes?
- Point out the direction of the effect.

SES and $MINORITY$ status are the significant fixed-effect terms in the model at $\alpha = 0.05$ implying that these terms (being in Level 1) help to explain the between-student variance conditional on the school.

The coefficient on SES is positive meaning that two students in the same school and conditional on the student-level random effect estimate (BLUP) and all else equal, the one with the higher SES has a higher Math score.

The coefficient on $MINORITY$ status is negative meaning that two students in the same school and conditional on the student-level random effect estimate (BLUP) and all else equal, the one who is classified as a Minority student has a lower Math score.

Add random slope for SES

$MATH_{tijk} = b_0 + \delta_{0ijk} + (b_1 + \zeta_{1k})TIME_{tijk} + (b_2 + \zeta_{2k})SES_{ijk} + b_3 SEX_{ijk} + b_4 MINORITY_{ijk} + b_5 YEARSTEA_{jk} + b_6 MATHKNOW_{jk} + b_7 MATHPREP_{jk} + b_8 HOUSEPOV_k + \epsilon_{tijk}$

where t represents occasion (in this case, year/grade), i represents students, j represents classrooms and k represents schools. $\delta_{0ijk} \sim N(0, \sigma^2_{\delta_0})$, $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0})$, $\zeta_{1k} \sim N(0, \sigma^2_{\zeta_1})$, $\zeta_{3k} \sim N(0, \sigma^2_{\zeta_3})$ and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$ all independent of each other except for ζ_{0k}, ζ_{1k} and ζ_{3k} could be correlated.

```
fit3 <- lmer(math ~ year + sex + ses + minority + yearstea + mathknow + mathprep + housepov + (ses+year
summary(fit3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math ~ year + sex + ses + minority + yearstea + mathknow + mathprep +
##     housepov + (ses + year | schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 21273.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.6663 -0.4808  0.0048  0.4722  3.4249
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  childid  (Intercept) 668.11   25.848
##  schoolid (Intercept) 251.26   15.851
##           ses          46.49    6.818   -0.03
##           year        114.87   10.718   -0.53  0.15
##  Residual             547.29   23.394
## Number of obs: 2162, groups:  childid, 1081; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  483.20541    4.79250  363.61574 100.825  < 2e-16 ***
## year          57.88435    1.50002   95.29594  38.589  < 2e-16 ***
## sex           -0.70446    1.94358 1016.33760  -0.362    0.717
## ses            9.32191    1.63892   69.42401   5.688 2.81e-07 ***
```

```
## minority     -16.26826    2.83356  658.65225  -5.741 1.43e-08 ***
## yearstea       0.03404    0.11840  876.03584   0.288    0.774
## mathknow      -0.28980    1.17539  774.87068  -0.247    0.805
## mathprep      -1.04672    1.13615  916.71569  -0.921    0.357
## housepov     -17.57739   12.45037  105.24186  -1.412    0.161
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) year   sex    ses    minrty yearst mthknw mthprp
## year     -0.205
## sex      -0.192 -0.001
## ses      -0.098  0.047  0.025
## minority -0.335  0.004 -0.007  0.125
## yearstea -0.240  0.002  0.020 -0.027  0.020
## mathknow -0.080 -0.001  0.001  0.001  0.100  0.037
## mathprep -0.575 -0.002 -0.015  0.045  0.002 -0.182 -0.003
## housepov -0.463  0.005 -0.007  0.074 -0.180  0.067  0.059  0.037
```

***is the estimated s.d. (square root of variance) of the random slope associated with SES large enough so that a value +/- 1 s.d. is sufficient to "cancel" (or flip the sign) the fixed effect for this predictor?**

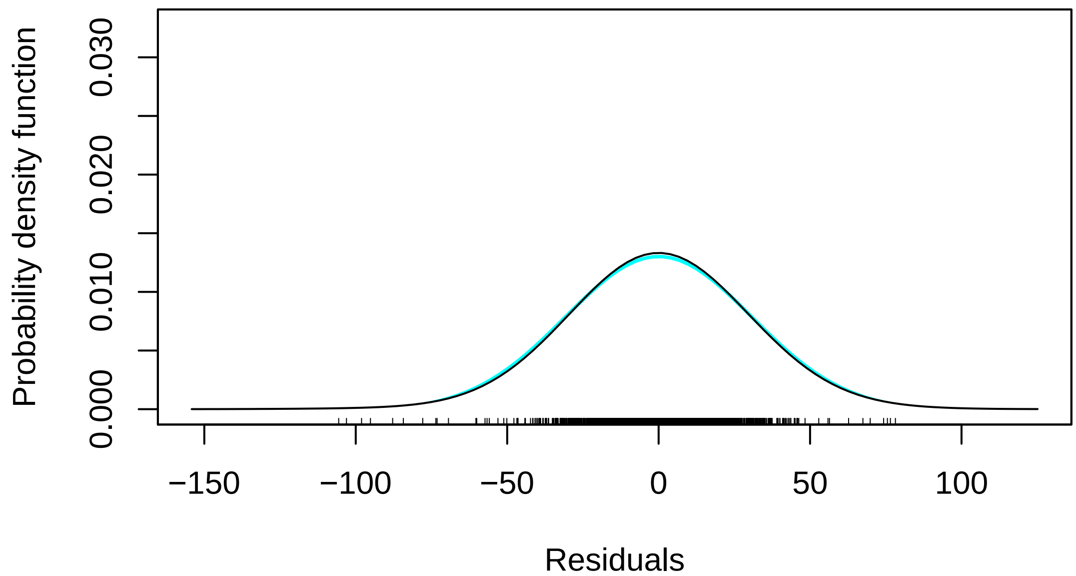
The estimated standard deviation of the random slope associated with SES is 6.818. We note that this is not large enough to "cancel" or flip the sign of the fixed-effect on SES within $+/-1$ standard deviation.

The majority of the values (middle 68%) for the fixed effect on SES is within the range $[(9.32191 - 6.818), (9.32191 + 6.818)] = [2.50391, 16.13991]$

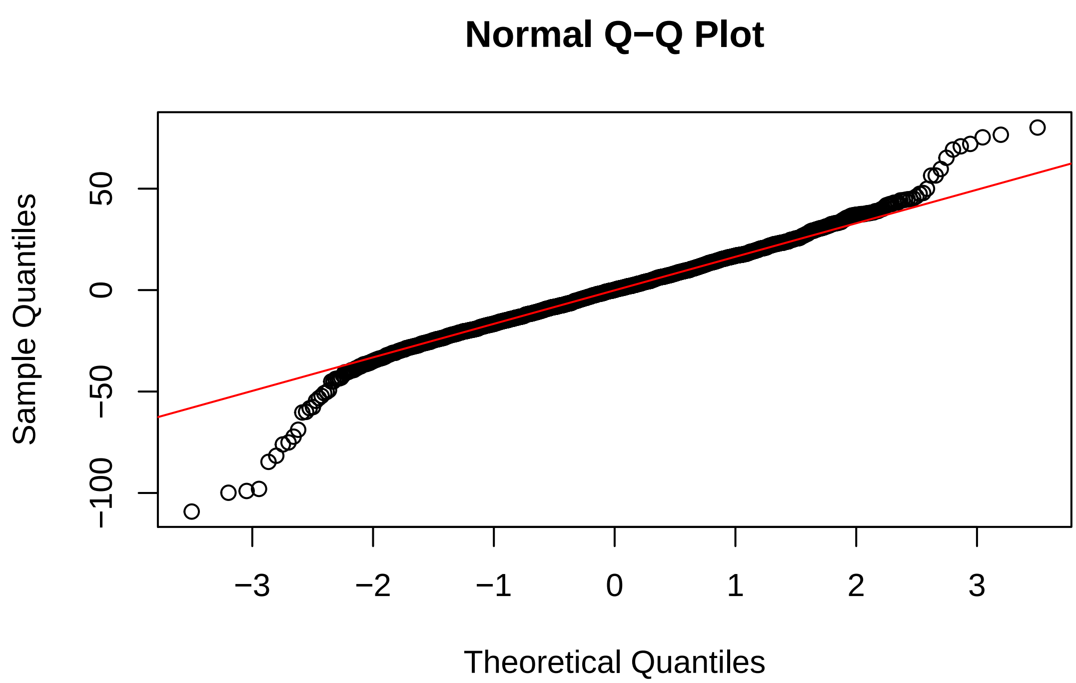
Residuals and Q-Q Plot

compute residuals in this final model. generate a qq plot and density (STATA: qnorm; kdensity ..., normal) Is there any reason to question the normality assumption?

```r
fit3.residuals <- residuals(fit3)
sm.density(fit3.residuals,model="Normal",xlab="Residuals")
```




```r
qqnorm(y=fit3.residuals)
qqline(y=fit3.residuals,col=2)
```

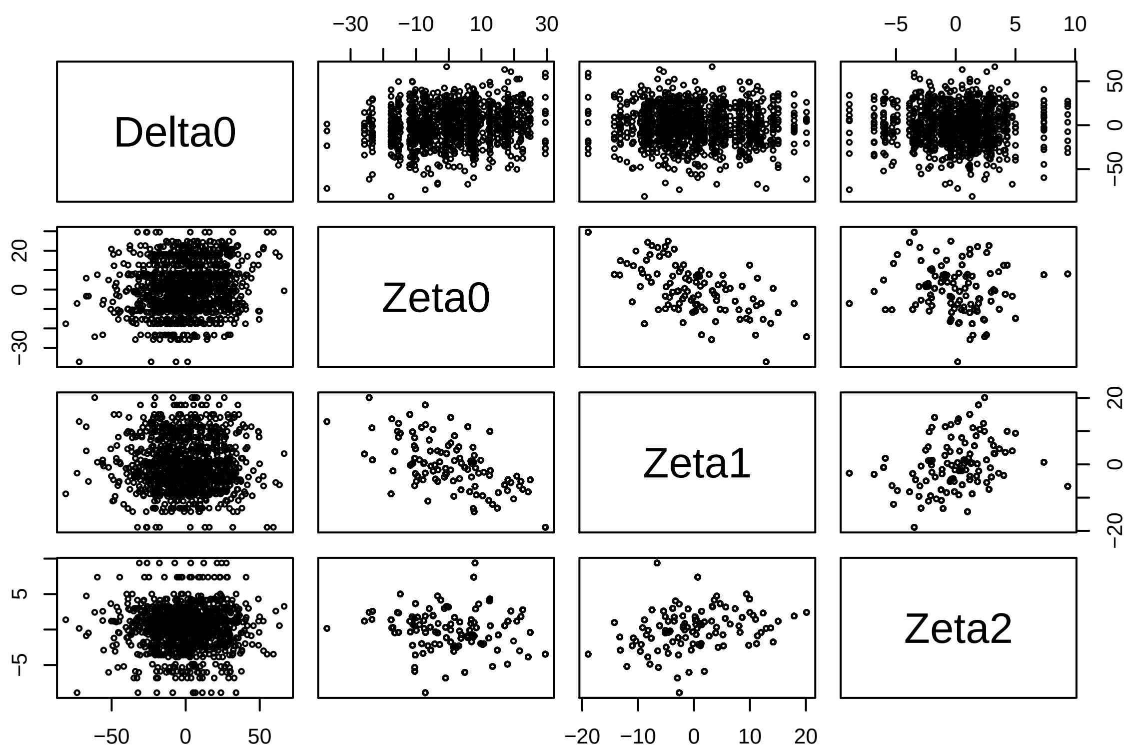


The residuals seem to have slightly heavier tails compared to a standard normal distribution. From the density plot, we also note that given the sample size, the peak is slightly higher than expected for a normal distribution. The distribution of residuals does not fall within the expected range given by the blue region although the deviation is minimal. Within a range of 4-sd (between +/- 2 sd), the Residual quantiles are seen to be linear compared to the Theoretical quantiles implying that the residuals are indeed quite normally distributed for the most part.

BLUPs for all 4 random effects & Scatter plots

generate an all pairs scatter plot matrix (4x4) of these * note whether or not you identify any concerns from these scatterplots.

```
ranefs <- ranef(fit3)
Delta0 <- ranefs$childid
idx.school <- match(classroom2$schoolid, sort(unique(classroom2$schoolid)))
Zeta0 <- ranefs$schoolid[idx.school,1]
Zeta1 <- ranefs$schoolid[idx.school,3]
Zeta2 <- ranefs$schoolid[idx.school,2]

ranefs <- data.frame(delta0=Delta0,zeta0=Zeta0,zeta1=Zeta1,zeta2=Zeta2)
colnames(ranefs) <- c("Delta0","Zeta0","Zeta1","Zeta2")
pairs(ranefs,cex=0.5)
```



We note that the distribution of the BLUPs for δ_0s are not homoscedastic particularly with respect to the ζ_0. We note that as ζ_0 is low, the δ_0 range is a bit lower and when ζ_0 is high, the range of δ_0 is slightly higher compared to the previous case. This possibly indicates that we are not fully capturing the between-school variation in the data.