# MLM Project Part 4

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## Part 4: Kaushik

### Reload the data and make person-period file

A reduced dataset was used for this analysis since there is missing data for one of the variables of interest. We acknowledge that this is not ideal in practice.

```
#re-read data
classroom <- read.csv("classroom.csv")
classroom2 <- na.omit(classroom)
#new variables
classroom2 <- classroom2 %>% mutate(math0 = mathkind) %>% mutate(math1 = mathkind+mathgain)
#reshape the data
class_pp <- reshape(classroom2, varying = c("math0", "math1"), v.names = "math", timevar = "year",
times = c(0, 1), direction = "long")
```

### Baseline model: unconditional growth model

$$MATH_{tijk} = b_0 + \delta_{0ijk} + (b_1 + \zeta_{1k})TIME_{tijk} + \zeta_{0k} + \epsilon_{tijk}$$

where $t$ represents occasion (in this case, year/grade), $i$ represents students, $j$ represents classrooms and $k$ represents schools. $\delta_{0ijk} \sim N(0, \sigma^2_{\delta_0})$, $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0})$, $\zeta_{1k} \sim N(0, \sigma^2_{\zeta_1})$ and $\epsilon_{ijk} \sim N(0, \sigma^2_{\epsilon})$ all independent of each other except for $\zeta_{0k}$ and $\zeta_{1k}$ having a correlation $\rho_{\zeta_0\zeta_1}$.

```
ugm <- lmer(math ~ year + (year|schoolid) + (1|childid), data=class_pp)
summary(ugm)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: math ~ year + (year | schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 21391.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.6737 -0.4699  0.0038  0.4683  3.4882
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  childid  (Intercept) 749.0    27.37
##  schoolid (Intercept) 373.5    19.33
##           year        112.4    10.60    -0.53
##  Residual             547.8    23.41
## Number of obs: 2162, groups:  childid, 1081; schoolid, 105
##
```

```
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  465.257      2.241 101.265   207.6   <2e-16 ***
## year          58.006      1.491  95.409    38.9   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##      (Intr)
## year -0.486
```

### Add student, classroom and school level fixed effects

$MATH_{tijk} = b_0+\delta_{0ijk}+(b_1+\zeta_{1k})TIME_{tijk}+b_2SEX_{ijk}+b_3SES_{ijk}+b_4MINORITY_{ijk}+b_5YEARSTEA_{jk}+b_6MATHKNOW$

where $t$ represents occasion (in this case, year/grade), $i$ represents students, $j$ represents classrooms and $k$ represents schools. $\delta_{0ijk} \sim N(0, \sigma^2_{\delta_0})$, $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0})$, $\zeta_{1k} \sim N(0, \sigma^2_{\zeta_1})$ and $\epsilon_{ijk} \sim N(0, \sigma^2_{\epsilon})$ all independent of each other except for $\zeta_{0k}$ and $\zeta_{1k}$ having a correlation $\rho_{\zeta_0\zeta_1}$.

```
fit2 <- lmer(math ~ year + sex + ses + minority + yearstea + mathknow + mathprep + housepov + (year|scho
summary(fit2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math ~ year + sex + ses + minority + yearstea + mathknow + mathprep +
##     housepov + (year | schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 21275.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.6812 -0.4784  0.0040  0.4712  3.4245
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  childid  (Intercept) 689.5    26.26
##  schoolid (Intercept) 249.2    15.79
##           year        114.2    10.69    -0.53
##  Residual             547.4    23.40
## Number of obs: 2162, groups:  childid, 1081; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  483.44962    4.79270  369.63678 100.872  < 2e-16 ***
## year          57.90494    1.49801   95.33745  38.655  < 2e-16 ***
## sex           -0.52171    1.95332 1033.75341  -0.267    0.789
## ses            9.59015    1.43816 1071.21739   6.668 4.14e-11 ***
## minority     -16.01167    2.82859  705.01716  -5.661 2.19e-08 ***
## yearstea       0.02304    0.11846  876.23254   0.194    0.846
## mathknow      -0.22237    1.17471  768.51612  -0.189    0.850
## mathprep      -1.08389    1.14195  932.69685  -0.949    0.343
## housepov     -18.17699   12.40599  108.12089  -1.465    0.146
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) year   sex    ses    minrty yearst mthknw mthprp
## year     -0.204
## sex      -0.194 -0.001
## ses      -0.120 -0.001  0.028
## minority -0.333  0.004 -0.007  0.157
## yearstea -0.240  0.001  0.018 -0.035  0.021
## mathknow -0.085 -0.001  0.002 -0.012  0.106  0.038
## mathprep -0.578 -0.002 -0.014  0.054  0.003 -0.182 -0.001
## housepov -0.463  0.004 -0.007  0.080 -0.178  0.067  0.060  0.036
```

## For year==0:

*what percent of between school differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BS} = \sigma^2_{\zeta_0} + 2 * year * \rho_{\zeta_0\zeta_1}\sigma_{\zeta_0}\sigma_{\zeta_1} + year^2 * \sigma^2_{\zeta_1}$$

$$V_{1BS}(year = 0) = \sigma^2_{\zeta_0} = 373.5$$

After adding fixed-effects:

$$V_{2BS} = \sigma^2_{\zeta_0} + 2 * year * \rho_{\zeta_0\zeta_1}\sigma_{\zeta_0}\sigma_{\zeta_1} + year^2 * \sigma^2_{\zeta_1}$$

$$V_{2BS}(year = 0) = \sigma^2_{\zeta_0} = 249.2$$

The percent difference in between-school variance for $year = 0$ is be given by:

$$\frac{V_{1BS} - V_{2BS}}{V_{1BS}} = \frac{373.5 - 249.2}{373.5} = 33.28\%$$

Model 2 explains 33.28% of the between-school variance for $year = 0$.

*what percent of between child differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BC}(year = 0) = \sigma^2_{\delta_0} = 749.0$$

After adding fixed effects:

$$V_{2BC}(year = 0) = \sigma^2_{\delta_0} = 689.5$$

The percent difference in between-child variance explained by the second model for $year = 0$ is given by:

$$\frac{V_{1BC} - V_{2BC}}{V_{1BC}} = \frac{749.0 - 689.5}{749.0} = 7.94\%$$

Model 2 explains 7.94% of the between-child variance for $year = 0$.

## For year==1:

*what percent of between school differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BS} = \sigma^2_{\zeta_0} + 2 * year * \rho_{\zeta_0\zeta_1}\sigma_{\zeta_0}\sigma_{\zeta_1} + year^2 * \sigma^2_{\zeta_1}$$

$$V_{1BS}(year = 1) = 373.5 + 2(-0.53)(19.33)(10.60) + 112.4 = 268.71$$

After adding fixed-effects:

$$V_{2BS} = \sigma^2_{\zeta_0} + 2 * year * \rho_{\zeta_0\zeta_1}\sigma_{\zeta_0}\sigma_{\zeta_1} + year^2 * \sigma^2_{\zeta_1}$$

$$V_{2BS}(year = 1) = 249.2 + 2(-0.53)(15.79)(10.69) + 114.2 = 184.48$$

The percent difference in between-school variance for $year = 0$ is be given by:

$$\frac{V_{1BS} - V_{2BS}}{V_{1BS}} = \frac{268.71 - 184.48}{268.71} = 31.35\%$$

Model 2 explains 31.35% of the between-school variance for $year = 1$.

*what percent of between child differences were explained as you go from the baseline to the second model?

For the baseline model:

$$V_{1BC}(year = 1) = \sigma^2_{\delta_0} = 749.0$$

After adding fixed effects:

$$V_{2BC}(year = 1) = \sigma^2_{\delta_0} = 689.5$$

The percent difference in between-child variance explained by the second model for $year = 1$ is given by:

$$\frac{V_{1BC} - V_{2BC}}{V_{1BC}} = \frac{749.0 - 689.5}{749.0} = 7.94\%$$

Model 2 explains 7.94% of the between-child variance for $year = 1$.

## Based on significance,

- what factors seem useful in describing ("explaining") differences between student outcomes?
- Point out the direction of the effect.

$SES$ and $MINORITY$ status are the significant fixed-effects terms in the model at $\alpha = 0.05$ implying that these terms (being in Level 1) help to explain the between-student variance conditional on the school.

The coefficient on $SES$ is positive meaning that two students in the same school and student-level random effect and all else equal, the one with the higher $SES$ has a higher Math score.

The coefficient on $MINORITY$ status is negative meaning that two students in the same school and student-level random effect and all else equal, the one who is classified as a Minority student has a lower Math score.

### Add random slope for SES

$MATH_{tijk} = b_0 + \delta_{0ijk} + (b_1 + \zeta_{1k})TIME_{tijk} + b_2 SEX_{ijk} + (b_3 + \zeta_{3k})SES_{ijk} + b_4 MINORITY_{ijk} + b_5 YEARSTEA_{jk} + b_6 MATH$

where $t$ represents occasion (in this case, year/grade), $i$ represents students, $j$ represents classrooms and $k$ represents schools. $\delta_{0ijk} \sim N(0, \sigma^2_{\delta_0})$, $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0})$, $\zeta_{1k} \sim N(0, \sigma^2_{\zeta_1})$, $\zeta_{3k} \sim N(0, \sigma^2_{\zeta_3})$ and $\epsilon_{ijk} \sim N(0, \sigma^2_{\epsilon})$ all independent of each other except for $\zeta_{0k}$, $\zeta_{1k}$ and $\zeta_{3k}$ could be correlated.

```
fit3 <- lmer(math ~ year + sex + ses + minority + yearstea + mathknow + mathprep + housepov + (ses+year
summary(fit3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## math ~ year + sex + ses + minority + yearstea + mathknow + mathprep +
##     housepov + (ses + year | schoolid) + (1 | childid)
##    Data: class_pp
##
## REML criterion at convergence: 21273.1
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -4.6663 -0.4808  0.0048  0.4722  3.4249
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  childid  (Intercept) 668.11   25.848
##  schoolid (Intercept) 251.26   15.851
##           ses          46.49    6.818   -0.03
##           year        114.87   10.718   -0.53  0.15
##  Residual             547.29   23.394
## Number of obs: 2162, groups:  childid, 1081; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept)  483.20541    4.79250  363.61574 100.825  < 2e-16 ***
## year          57.88435    1.50002   95.29594  38.589  < 2e-16 ***
## sex           -0.70446    1.94358 1016.33760  -0.362    0.717
## ses            9.32191    1.63892   69.42401   5.688 2.81e-07 ***
## minority     -16.26826    2.83356  658.65225  -5.741 1.43e-08 ***
## yearstea       0.03404    0.11840  876.03584   0.288    0.774
## mathknow      -0.28980    1.17539  774.87068  -0.247    0.805
## mathprep      -1.04672    1.13615  916.71569  -0.921    0.357
## housepov     -17.57739   12.45037  105.24186  -1.412    0.161
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) year   sex    ses    minrty yearst mthknw mthprp
## year     -0.205
## sex      -0.192 -0.001
## ses      -0.098  0.047  0.025
## minority -0.335  0.004 -0.007  0.125
## yearstea -0.240  0.002  0.020 -0.027  0.020
## mathknow -0.080 -0.001  0.001  0.001  0.100  0.037
```

```
## mathprep -0.575 -0.002 -0.015  0.045  0.002 -0.182 -0.003
## housepov -0.463  0.005 -0.007  0.074 -0.180  0.067  0.059  0.037
```

***is the estimated s.d. (square root of variance) of the random slope associated with SES large enough*** **so that a value +/- 1 s.d. is sufficient to "cancel" (or flip the sign) the fixed effect for this predictor?**

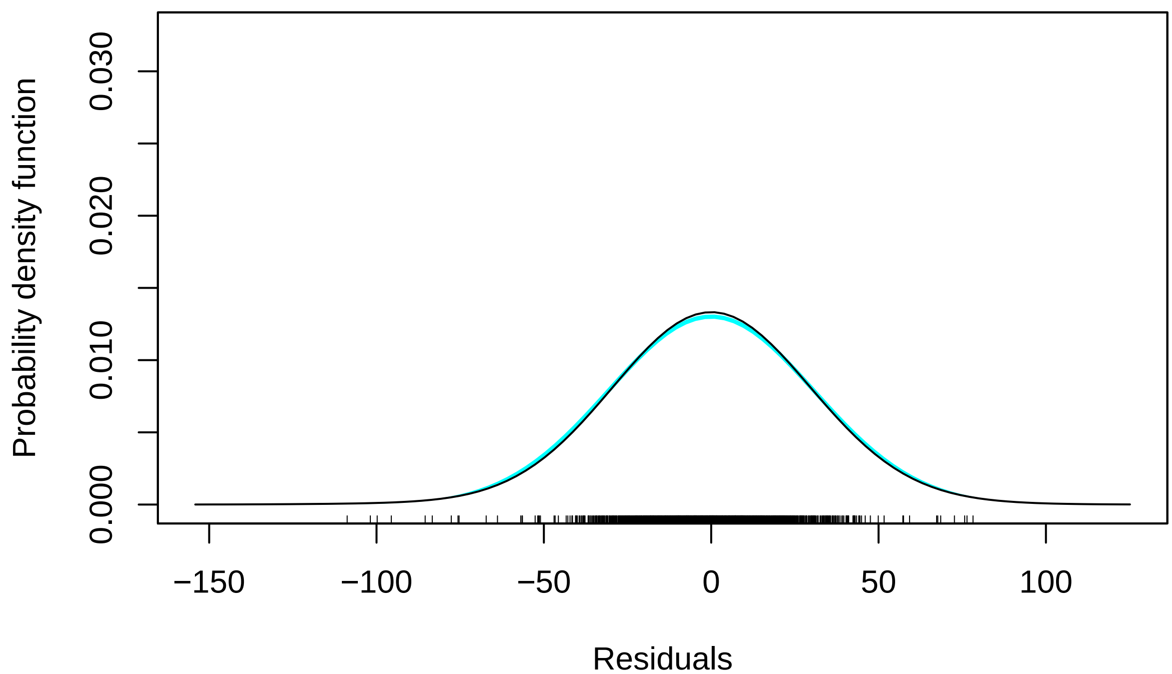
The estimated standard deviation of the random slope associated with SES is 6.818. We note that this is not large enough to "cancel" or flip the sign of the fixed-effect on SES within $+/-1$ standard deviation.

The majority of the values (middle 68% ) for the fixed effect on SES is within the range $[(9.32191 - 6.818), (9.32191 - 6.818)] = [2.50391, 16.13991]$

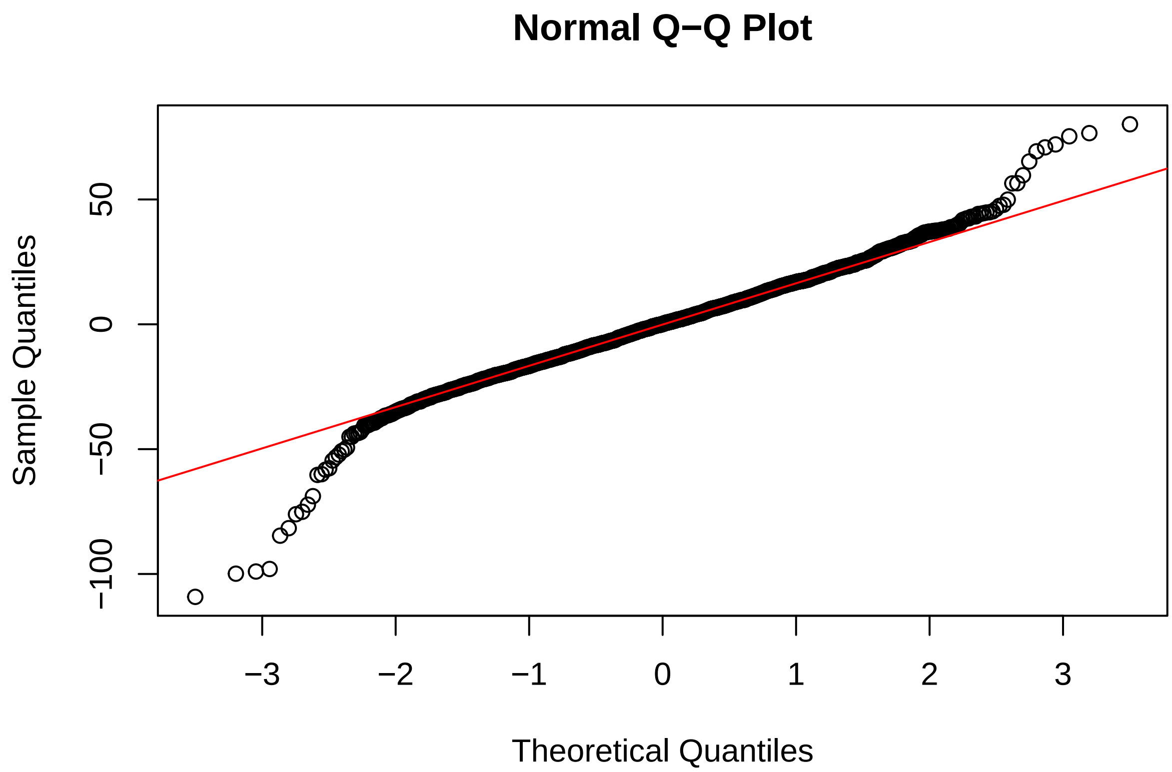
### Residuals and Q-Q Plot

*compute residuals in this final model. generate a qq plot and density (STATA: qnorm; kdensity . . . , normal)* Is there any reason to question the normality assumption?

```
fit3.residuals <- residuals(fit3)
sm.density(fit3.residuals,model="Normal",xlab="Residuals")
```




```
qqnorm(y=fit3.residuals)
qqline(y=fit3.residuals,col=2)
```



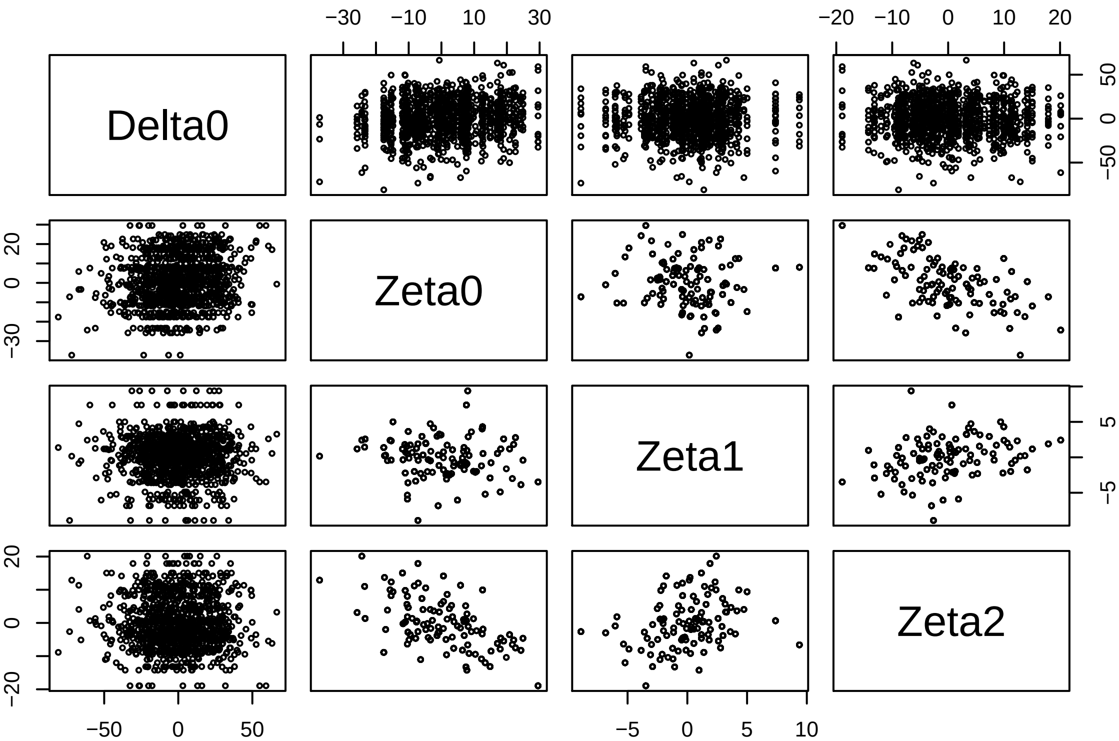

The residuals seem to have slightly heavier tails compared to a standard normal distribution. From the density plot, we also note that given the sample size, the peak is slightly higher than expected for a normal distribution. The distribution of residuals does not fall within the expected range given by the blue region although the deviation is minimal. Within a range of 4-sd (between +/- 2 sd), the Residual quantiles are seen to be linear compared to the Theoretical quantiles implying that the residuals are indeed quite normally distributed for the most part.

### BLUPs for all 4 random effects & Scatter plots

generate an all pairs scatter plot matrix (4x4) of these * note whether or not you identify any concerns from these scatterplots.

```
ranefs <- ranef(fit3)
Delta0 <- ranefs$childid
idx.school <- match(classroom2$schoolid, sort(unique(classroom2$schoolid)))
Zeta0 <- ranefs$schoolid[idx.school,1]
Zeta1 <- ranefs$schoolid[idx.school,2]
Zeta2 <- ranefs$schoolid[idx.school,3]

ranefs <- data.frame(delta0=Delta0,zeta0=Zeta0,zeta1=Zeta1,zeta2=Zeta2)
colnames(ranefs) <- c("Delta0","Zeta0","Zeta1","Zeta2")
pairs(ranefs,cex=0.5)
```



Delta0
Zeta0
Zeta1
Zeta2