---
title: 'Group Project #1'
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output: pdf_document

Part 1: Frankie

Create 1st grade variable

```
classroom <- classroom %>% mutate(Math1 = mathkind + mathgain)
```

Random Intercepts for classroom, nested in schools UMM

We begin our analysis by looking at the UMM with random intercepts for schools and classrooms, i.e. :

$$Math1st_{ijk} = \beta_{0ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$\zeta_k \sim N(0, \sigma^2_\zeta), \eta_{jk} \sim N(0, \sigma^2_\eta)$, and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, all are independent of each other

```
model1 <- lmer(Math1~(1|schoolid/classid),data=classroom)
summary(model1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.46   9.244
##  schoolid         (Intercept)  280.68  16.754
##  Residual                     1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.406   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

$$ICC_{class} = \frac{85.46}{1146.8 + 280.68 + 85.46} \approx .056$$

$$ICC_{school} = \frac{280.68}{1146.8 + 280.68 + 85.46} \approx .186$$

We hence find, from the fit summary above, that the equation for our model is:

$$Math1st_{ijk} = 522.54 + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$\zeta_k \sim N(0, 280.68), \eta_{jk} \sim N(0, 85.46)$, and $\epsilon_{ijk} \sim N(0, 1146.80)$, all are independent of each other

Model with School Level Predictors Added

We then add all the school level predictors (that is, "housepov") and report below the model fit :

```
model2 <- lmer(Math1~housepov+(1|schoolid/classid),data=classroom)
summary(model2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ housepov + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   82.36   9.075
##  schoolid         (Intercept)  250.93  15.841
##  Residual                     1146.95  33.867
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.294      3.341 102.809 159.024   <2e-16 ***
## housepov     -45.783     14.236 111.063  -3.216   0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.810
```

```
anova(model1, model2, refit = F)
```

```
## Data: classroom
## Models:
## model1: Math1 ~ (1 | schoolid/classid)
## model2: Math1 ~ housepov + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model1  4 11953 11973 -5972.3    11945
## model2  5 11937 11963 -5963.7    11927 17.186      1   3.39e-05 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Report the changes in the variances of the random effects:

Change in σ^2_ζ: decreased to 250.93 from 280.63 σ^2_η decreases to 82.36 from 85.46 σ^2_ϵ slightly increases to 1146.95 from 1146.8

The ANOVA/LRT has a p-value of almost zero, $p = 3.39e-05$, thus we reject the H_0 at our $\alpha = 0.05$. That is, we find evidence that it makes sense to include the school level predictor, housepov.

Model with all Class Level Predictors Added

We now re-run the model after including all the classroom level predictors, that is "mathknow", "yearstea", "mathprep", and report the model fit.

```
model3 <- lmer(Math1~housepov+mathknow+yearstea+mathprep+
                 (1|schoolid/classid),data=classroom)
summary(model3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                      1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29853    5.20496 228.85764 102.268  < 2e-16 ***
## housepov    -41.62116   14.08835 109.83227  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.06566   1.765  0.07883 .
## yearstea      0.06193    0.14717 223.76582   0.421  0.67432
## mathprep     -0.75440    1.42809 203.20767  -0.528  0.59790
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst
## housepov -0.568
## mathknow -0.052  0.082
## yearstea -0.264  0.077  0.030
## mathprep -0.666  0.032  0.004 -0.175
```

creating reducted dataset taking away missing data

The variable of interest "mathknown" includes some missing values. The model for which we have reported the summary above therefore removes the observations for which missing data is present.

To be able to compare Model 2 (with school level predictors) with Model 3 (with both school level and classroom level predictors), we removed from the dataset students that had missing values, creating a reduced dataset. This left us with a sample of 1081 students. We then re-run model 2 on this reducted dataset and compared it to Model 3.

```
classroom_red = na.omit(classroom)
model2_red <- lmer(Math1~housepov+(1|schoolid/classid),data=classroom_red)
model3_red <- lmer(Math1~housepov+mathknow+yearstea+mathprep+
                     (1|schoolid/classid),data=classroom_red)

summary(model3_red)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##    Data: classroom_red
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                      1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29853    5.20496 228.85764 102.268  < 2e-16 ***
## housepov    -41.62116   14.08835 109.83227  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.06566   1.765  0.07883 .
## yearstea      0.06193    0.14717 223.76582   0.421  0.67432
## mathprep     -0.75440    1.42809 203.20767  -0.528  0.59790
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst
## housepov -0.568
## mathknow -0.052  0.082
## yearstea -0.264  0.077  0.030
## mathprep -0.666  0.032  0.004 -0.175
```

```
anova(model2_red, model3_red, refit = F)
```

```
## Data: classroom_red
```

```
## Models:
## model2_red: Math1 ~ housepov + (1 | schoolid/classid)
## model3_red: Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##            Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model2_red  5 10838 10862 -5413.8    10828
## model3_red  8 10837 10877 -5410.5    10821 6.5771      3    0.08667 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Change in σ^2_ϵ and σ^2_η: σ^2_ϵ decreased to 1136.43, σ^2_η increased to 94.36; $\sigma^2_\zeta = 223.31$

A possible reason why ϵ decreased in this model, but not η is that adding the classroom level predictors makes it so that more of the overall variation is explained by "structured" variation (that is, related to the fact that students are in different classrooms) rather than by unstructured (ϵ), so that the latter decreases. However, we also have to note that in this case we are using the reduced dataset, so that some of the changes may be due to the fact that we are using two slightly different datasets.

The anova test comparing the school level predictor to the model with the classroom predictors has a p-value 0.087, so we fail to reject the null hypothesis at our $\alpha = 0.05$ and conclude that adding classroom level predictors is not necessary, as it does not signficantly improve the model.

Add all student-level predictors

We now include all the student level predictors in our model:

```
model4 <- lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                 ses+(1|schoolid/classid),data=classroom)
summary(model4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                     1064.95  32.63
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38922 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.87774  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
```

```
## mathprep      -0.27705    1.37583  205.27157  -0.201    0.841
## sex           -1.21419    2.09483 1022.42136  -0.580    0.562
## minority     -16.18678    3.02605  704.47889  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

We test this new block compared to the model with both school-level and classroom level predictors.

```
anova(model3, model4, refit = F)
```

```
## Data: classroom
## Models:
## model3: Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
## model4: Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
## model4:     ses + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## model3  8 10837 10877 -5410.5    10821
## model4 11 10752 10806 -5364.8    10730 91.446      3  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The LRT test between this two models has a p-value $< 2.2 * 10^{-16}$. Therefore, at our $\alpha = 0.05$, we reject the null hypothesis and conclude that adding this black of predictors is justified.

Changes in variance components :

σ^2_ϵ decreased to 1064.95, σ^2_η decreased to 93.89, and σ^2_ζ decreased to 169.45.

We note that adding student-level predictors leads to a decrease in the overall variance of the model. By "controlling" for student-related variables, we also explain the between schools, as students with similar attributes might be similar across schools, hence reducing the overall variance of ζ.

The final model, with all school level, classroom level, and student level predictors, is:

$$Math1st_{ijk} = 539.63 + \zeta_k + \eta_{jk} + \epsilon_{ijk} - 17.65 * Housepov_k + 1.35 * Mathknow_{jk} +$$
$$0.01 * YearsTea_{jk} - 0.27 * Mathprep_{jk} - 0.19 * sex_{ijk} + -0.32 * minority_{ijk} - 0.12 * ses_{ijk}$$

With:

$\zeta_k \sim N(0, \sigma^2_\zeta), \eta_{jk} \sim N(0, \sigma^2_\eta)$, and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, all are independent of each other

From the model fit above therefore we find that the fitted model is:

$$Math1st_{ijk} = \beta_{0ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk} + \beta_1 Housepov_k + \beta_2 Mathknow_{jk} +$$
$$\beta_3 YearsTea_{jk} + \beta_4 Mathprep_{jk} + \beta_5 sex_{ijk} + \beta_6 minority_{ijk} + \beta_7 ses_{ijk}$$

With:

$\zeta_k \sim N(0, 169.45), \eta_{jk} \sim N(0, 93.89)$, and $\epsilon_{ijk} \sim N(0, 1064.95)$, all are independent of each other.

Random Slope for Teacher-level predictor varying at school-level

We try adding a random slope for each teacher level predictor (varying at the school level; one by one - not all together).

mathknown

```
rst.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+mathknow||schoolid)+(1|classid),data=classroom)
summary(rst.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathknow || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.389e+01 9.690e+00
##  schoolid   mathknow    4.260e-11 6.527e-06
##  schoolid.1 (Intercept) 1.694e+02 1.302e+01
##  Residual               1.065e+03 3.263e+01
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38921 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.87774  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
## mathprep      -0.27705    1.37583  205.27156  -0.201    0.841
## sex           -1.21419    2.09483 1022.42136  -0.580    0.562
## minority     -16.18678    3.02605  704.47889  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
```

```
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rst.1,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + mathknow | schoolid) + (1 | classid)
##                                      npar  logLik   AIC    LRT Df
## <none>                                 12 -5364.8 10754
## (1 | schoolid)                         11 -5376.5 10775 23.410  1
## mathknow in (0 + mathknow | schoolid)  11 -5364.8 10752  0.000  1
## (1 | classid)                          11 -5368.1 10758  6.741  1
##                                      Pr(>Chisq)
## <none>
## (1 | schoolid)                        1.309e-06 ***
## mathknow in (0 + mathknow | schoolid)  0.999999
## (1 | classid)                         0.009422 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for math knowledge at a school level as the p value $= 0.99$ for the Chi-square test is not signifcant at $\alpha = 0.05$.

yearstea

```
rst.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+yearstea||schoolid)+(1|classid),data=classroom)
summary(rst.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + yearstea || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8485 -0.6149 -0.0323  0.5980  3.6600
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.266e+01  9.62593
##  schoolid   yearstea    9.669e-03  0.09833
##  schoolid.1 (Intercept) 1.685e+02 12.97894
##  Residual               1.065e+03 32.63452
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
```

```
## (Intercept)  539.60060    5.30865  266.34157 101.645  < 2e-16 ***
## housepov     -17.71727   13.21854  113.56407  -1.340    0.183
## mathknow       1.33198    1.39177  234.33551   0.957    0.340
## yearstea       0.01124    0.14193  122.38000   0.079    0.937
## mathprep      -0.26633    1.37610  204.91605  -0.194    0.847
## sex           -1.21077    2.09476 1022.22247  -0.578    0.563
## minority     -16.16833    3.02641  702.64837  -5.342 1.24e-07 ***
## ses           10.04529    1.54490 1066.09768   6.502 1.21e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.450
## mathknow -0.082  0.057
## yearstea -0.258  0.070  0.028
## mathprep -0.632  0.037  0.003 -0.172
## sex      -0.190 -0.007  0.006  0.015 -0.006
## minority -0.320 -0.179  0.115  0.023  0.001 -0.010
## ses      -0.121  0.082 -0.007 -0.027  0.053  0.020  0.162
```

```
ranova(rst.2, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + yearstea | schoolid) + (1 | classid)
##                                        npar  logLik   AIC     LRT Df
## <none>                                   12 -5364.8 10754
## (1 | schoolid)                           11 -5374.7 10771 19.8301  1
## yearstea in (0 + yearstea | schoolid)    11 -5364.8 10752  0.0070  1
## (1 | classid)                            11 -5367.7 10757  5.9158  1
##                                        Pr(>Chisq)
## <none>
## (1 | schoolid)                          8.464e-06 ***
## yearstea in (0 + yearstea | schoolid)     0.93342
## (1 | classid)                            0.01501 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for "yeartea" at a school level as the p value $= 0.93$ for the Chi-square test is not signifcant at $\alpha = 0.05$.

```
rst.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+mathprep||schoolid)+(1|classid),data=classroom)
summary(rst.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathprep || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
```

```
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   93.89   9.69
##  schoolid   mathprep       0.00   0.00
##  schoolid.1 (Intercept)  169.45  13.02
##  Residual               1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38917 101.585  < 2e-16 ***
## housepov     -17.64847   13.21758  113.87771  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201    0.841
## sex           -1.21419    2.09483 1022.42137  -0.580    0.562
## minority     -16.18678    3.02605  704.47892  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rst.3, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + mathprep | schoolid) + (1 | classid)
##                                        npar  logLik   AIC     LRT Df
## <none>                                   12 -5364.8 10754
## (1 | schoolid)                           11 -5371.6 10765 13.6179  1
## mathprep in (0 + mathprep | schoolid)    11 -5364.8 10752  0.0000  1
## (1 | classid)                            11 -5368.3 10759  7.1357  1
##                                        Pr(>Chisq)
## <none>
## (1 | schoolid)                           0.000224 ***
## mathprep in (0 + mathprep | schoolid)    1.000000
## (1 | classid)                            0.007556 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for "housepov" at a school level as the p value = 1 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Question: Why housepov bad idea?

Answer: There is only one data point per school, so we do not have enough information to calculate the slope for each school.

Allowing correlations with random intercepts

ONE BY ONE

Again, we add random slopes for each teacher-level predictor varying at the school level, but this time by allowing them to be correlated with the random intercepts.

mathknown

```
rstc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+mathknow|schoolid)+(1|classid),data=classroom)
summary(rstc.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathknow | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6131 -0.0324  0.5969  3.6603
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept) 9.394e+01  9.69205
##  schoolid (Intercept) 1.693e+02 13.01223
##           mathknow    8.596e-04  0.02932 1.00
##  Residual             1.065e+03 32.63393
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.64037    5.31212  275.37948 101.587  < 2e-16 ***
## housepov     -17.64148   13.21274  103.97679  -1.335    0.185
## mathknow       1.35459    1.39203  214.63820   0.973    0.332
## yearstea       0.01114    0.14141  226.85277   0.079    0.937
## mathprep      -0.27753    1.37601  201.27912  -0.202    0.840
## sex           -1.21329    2.09485 1021.79964  -0.579    0.563
## minority     -16.19376    3.02609  703.81038  -5.351 1.18e-07 ***
## ses           10.04788    1.54488 1062.12341   6.504 1.20e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.082  0.057
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.173
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rstc.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathknow | schoolid) + (1 | classid)
##                                        npar  logLik   AIC    LRT Df
## <none>                                   13 -5364.8 10756
## mathknow in (1 + mathknow | schoolid)    11 -5364.8 10752 0.0003  2
## (1 | classid)                            12 -5368.1 10760 6.6768  1
##                                        Pr(>Chisq)
## <none>
## mathknow in (1 + mathknow | schoolid)    0.999840
## (1 | classid)                            0.009767 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for math knowledge at a school level as the p value $= 0.99$ for the Chi-square test is not signifcant at $\alpha = 0.05$.

yearstea

```
rstc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+yearstea|schoolid)+(1|classid),data=classroom)
summary(rstc.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + yearstea | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7462 -0.6036 -0.0290  0.6041  3.8449
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept)   37.9283  6.1586
##  schoolid (Intercept)  366.1148 19.1341
##           yearstea       0.5523  0.7432  -0.78
##  Residual             1066.4510 32.6566
```

```
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.95245    5.48825  222.69673  98.201  < 2e-16 ***
## housepov     -17.13994   13.45959  119.63687  -1.273    0.205
## mathknow       1.04635    1.34381  209.72527   0.779    0.437
## yearstea       0.02204    0.15766   75.76696   0.140    0.889
## mathprep       0.05046    1.34549  190.82671   0.038    0.970
## sex           -1.33553    2.08774 1024.45936  -0.640    0.523
## minority     -16.44555    2.99655  669.50401  -5.488 5.77e-08 ***
## ses           10.15038    1.53873 1062.66131   6.597 6.62e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.455
## mathknow -0.085  0.049
## yearstea -0.370  0.084  0.012
## mathprep -0.606  0.050  0.014 -0.139
## sex      -0.184 -0.004  0.008  0.009 -0.004
## minority -0.305 -0.169  0.122  0.032 -0.007 -0.012
## ses      -0.119  0.079 -0.001 -0.019  0.049  0.022  0.168
```

```
ranova(rstc.2,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + yearstea | schoolid) + (1 | classid)
##                                       npar  logLik   AIC    LRT Df
## <none>                                  13 -5361.8 10750
## yearstea in (1 + yearstea | schoolid)   11 -5364.8 10752 5.8254  2
## (1 | classid)                           12 -5362.3 10749 0.9028  1
##                                       Pr(>Chisq)
## <none>
## yearstea in (1 + yearstea | schoolid)    0.05433 .
## (1 | classid)                            0.34202
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for yearstea at a school level as the p value $= 0.054$ for the Chi-square test is not signifcant at $\alpha = 0.05$.

mathprep

```
rstc.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+mathprep|schoolid)+(1|classid),data=classroom)
summary(rstc.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
```

```
##     ses + (1 + mathprep | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8542 -0.6034 -0.0221  0.5915  3.6475
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   78.46   8.858
##  schoolid (Intercept)  552.76  23.511
##           mathprep      15.89   3.986   -1.00
##  Residual             1064.26  32.623
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.60855    5.60813  159.88774  96.041  < 2e-16 ***
## housepov     -14.01306   12.88689  116.05900  -1.087    0.279
## mathknow       1.29884    1.37194  229.68146   0.947    0.345
## yearstea      -0.02586    0.13949  223.50098  -0.185    0.853
## mathprep       0.04074    1.34845  139.04228   0.030    0.976
## sex           -1.16759    2.08697 1023.15084  -0.559    0.576
## minority     -16.46422    2.99524  663.67316  -5.497 5.52e-08 ***
## ses           10.14166    1.53961 1060.93421   6.587 7.04e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.461
## mathknow -0.071  0.027
## yearstea -0.260  0.089  0.049
## mathprep -0.692  0.107  0.012 -0.155
## sex      -0.183  0.003  0.002  0.023 -0.008
## minority -0.275 -0.187  0.107  0.025 -0.035 -0.013
## ses      -0.121  0.095 -0.001 -0.033  0.061  0.024  0.161
```

```
ranova(rstc.3, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathprep | schoolid) + (1 | classid)
##                                       npar  logLik   AIC    LRT Df
## <none>                                  13 -5362.3 10751
## mathprep in (1 + mathprep | schoolid)   11 -5364.8 10752 4.8144  2
## (1 | classid)                           12 -5364.9 10754 5.0971  1
##                                       Pr(>Chisq)
## <none>
## mathprep in (1 + mathprep | schoolid)    0.09007 .
## (1 | classid)                            0.02397 *
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for mathprep at a school level as the p value = 0.09 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Question: Anything unusual about the variances? Why might this have occurred? (hint: what did you add to the model?)

Answer: We note that the model did not estimate the correlation parameter correctly for the models with random slopes for mathknown and mathprepr. Indeed, with a correaltion of respectively 1 and -1 with the random intercept, the parameter is a linear function of the variance component for the slope. This could be due to the fact that there is not enough classrooms in the schools (as we are adding random effects at the school levels, for classroom level predictors), so that there is not enough degrees of freedom, nor enough variation among the variables of interest, to calculate all the parameters required in the model. Obtaining a correlation of 1 and -1 should warn us of the fact that the models generated should not be trusted.
Why is the correlation between random intercept and slope then calculated for yearstea? This could be due to the fact that this variable has a larger range, so that it can be more robustly estimated for some of the schools and the correlation between random slope and intercept then estimated more accurately even for schools with few classes.

Random slopes for student-level predictors varying at classroom level

We now repeat the exercise by adding student level predictors, varying at the classroom level.

ONE BY ONE

sex

```
rss.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+sex||classid)+(1|schoolid),data=classroom)
summary(rss.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex || classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   93.89   9.69
##  classid.1 sex            0.00   0.00
##  schoolid  (Intercept)  169.45  13.02
##  Residual              1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
```

```
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38920 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.87773  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201    0.841
## sex           -1.21419    2.09483 1022.42137  -0.580    0.562
## minority     -16.18678    3.02605  704.47890  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rss.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | classid) + (0 + sex | classid) + (1 | schoolid)
##                             npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                        12 -5364.8 10754
## (1 | classid)                 11 -5368.0 10758  6.4894  1    0.01085 *
## sex in (0 + sex | classid)    11 -5364.8 10752  0.0000  1    1.00000
## (1 | schoolid)                11 -5377.1 10776 24.7881  1  6.399e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for sex at the classroom level, as the p value $= 1$ for the Chi-square test is not signifcant at $\alpha = 0.05$.

minority

```
rss.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+minority||classid)+(1|schoolid),data=classroom)
summary(rss.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority || classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
```

```
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   93.89   9.69
##  classid.1 minority       0.00   0.00
##  schoolid  (Intercept)  169.45  13.02
##  Residual              1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38919 101.585  < 2e-16 ***
## housepov     -17.64847   13.21758  113.87772  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201    0.841
## sex           -1.21419    2.09483 1022.42137  -0.580    0.562
## minority     -16.18678    3.02605  704.47891  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rss.2, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | classid) + (0 + minority | classid) + (1 | schoolid)
##                                      npar  logLik   AIC     LRT Df
## <none>                                 12 -5364.8 10754
## (1 | classid)                          11 -5367.3 10757  5.1497  1
## minority in (0 + minority | classid)   11 -5364.8 10752  0.0000  1
## (1 | schoolid)                         11 -5377.1 10776 24.7881  1
##                                     Pr(>Chisq)
## <none>
## (1 | classid)                         0.02325 *
## minority in (0 + minority | classid)  1.00000
## (1 | schoolid)                       6.399e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for minority at the classroom level, as the p value = 1 for the Chi-square test is not signifcant at $\alpha = 0.05$.

SES

```
rss.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+ses||classid)+(1|schoolid),data=classroom)
summary(rss.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses || classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   87.11   9.333
##  classid.1 ses           49.60   7.043
##  schoolid  (Intercept)  171.02  13.077
##  Residual              1043.44  32.302
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641  274.46487 101.710  < 2e-16 ***
## housepov     -17.50879   13.21775  113.44869  -1.325    0.188
## mathknow       1.36796    1.38563  229.40646   0.987    0.325
## yearstea       0.01103    0.14117  226.97687   0.078    0.938
## mathprep      -0.27938    1.37171  204.89340  -0.204    0.839
## sex           -1.37733    2.09334 1022.81818  -0.658    0.511
## minority     -16.29362    3.02464  703.33762  -5.387 9.78e-08 ***
## ses           10.14363    1.64248  176.39739   6.176 4.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.082  0.058
## yearstea -0.259  0.070  0.029
## mathprep -0.631  0.040  0.005 -0.172
## sex      -0.190 -0.007  0.006  0.014 -0.005
## minority -0.321 -0.180  0.111  0.025  0.002 -0.011
## ses      -0.108  0.081  0.002 -0.026  0.050  0.020  0.145
```

```
ranova(rss.3, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
```

```
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | classid) + (0 + ses | classid) + (1 | schoolid)
##                            npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                       12 -5364.0 10752
## (1 | classid)                11 -5366.9 10756  5.9221  1    0.01495 *
## ses in (0 + ses | classid)   11 -5364.8 10752  1.5969  1    0.20634
## (1 | schoolid)               11 -5376.6 10775 25.2710  1  4.982e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for ses at the classroom level, as the p value = 0.206 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Question: why is this a bad idea to include a classroom-level variable with random slopes at classroom-level?

Answer: Because all of the observations for a class will be the same, so we will not be able to compute the classroom slopes for each classroom (as we will only have one point).

Allowing for correlations with random intercepts

ONE BY ONE

sex

```
rssc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+sex|classid)+(1|schoolid),data=classroom)
summary(rssc.1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7565 -0.6134 -0.0307  0.5916  3.7116
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  130.07  11.41
##           sex           31.36   5.60    -0.67
##  schoolid (Intercept)  169.85  13.03
##  Residual              1056.41  32.50
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.400e+02  5.332e+00  2.723e+02 101.285  < 2e-16 ***
## housepov    -1.829e+01  1.323e+01  1.145e+02  -1.382    0.170
```

```
## mathknow     1.306e+00  1.391e+00  2.315e+02   0.939    0.349
## yearstea     3.087e-03  1.416e-01  2.270e+02   0.022    0.983
## mathprep    -3.459e-01  1.374e+00  2.014e+02  -0.252    0.801
## sex         -1.197e+00  2.122e+00  2.160e+02  -0.564    0.573
## minority    -1.619e+01  3.028e+00  7.042e+02  -5.347 1.21e-07 ***
## ses          1.010e+01  1.544e+00  1.065e+03   6.539 9.62e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.452
## mathknow -0.085  0.060
## yearstea -0.258  0.072  0.029
## mathprep -0.628  0.040  0.005 -0.174
## sex      -0.203 -0.005  0.003  0.015 -0.008
## minority -0.321 -0.178  0.116  0.024  0.003 -0.009
## ses      -0.123  0.083 -0.005 -0.027  0.054  0.020  0.164
```

```
ranova(rssc.1, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | classid) + (1 | schoolid)
##                            npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                       13 -5364.5 10755
## sex in (1 + sex | classid)   11 -5364.8 10752  0.5003  2     0.7787
## (1 | schoolid)               12 -5377.0 10778 24.8912  1  6.066e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for sex at the classroom level, as the p value $= 0.779$ for the Chi-square test is not signifcant at $\alpha = 0.05$.

Minority

```
rssc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+minority|classid)+(1|schoolid),data=classroom)
summary(rssc.2)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10726.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9037 -0.6221 -0.0295  0.6033  3.4574
##
## Random effects:
```

```
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  225.4   15.01
##           minority     171.3   13.09    -0.82
##  schoolid (Intercept)  157.4   12.55
##  Residual             1045.3   32.33
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.73594    5.38023  270.70509 100.318  < 2e-16 ***
## housepov     -17.34698   12.91268  103.34670  -1.343    0.182
## mathknow       1.45702    1.39355  234.04713   1.046    0.297
## yearstea      -0.01636    0.14285  234.25121  -0.115    0.909
## mathprep      -0.13520    1.37018  203.97000  -0.099    0.921
## sex           -1.01012    2.08966 1015.73461  -0.483    0.629
## minority     -16.48614    3.21756  183.20472  -5.124 7.55e-07 ***
## ses            9.89350    1.54595 1062.82882   6.400 2.33e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.435
## mathknow -0.079  0.061
## yearstea -0.265  0.080  0.038
## mathprep -0.618  0.037 -0.006 -0.171
## sex      -0.188 -0.009  0.009  0.015 -0.005
## minority -0.368 -0.171  0.108  0.025 -0.004 -0.009
## ses      -0.117  0.085  0.001 -0.023  0.051  0.021  0.149
```

```
ranova(rssc.2)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | classid) + (1 | schoolid)
##                                      npar  logLik   AIC     LRT Df
## <none>                                 13 -5363.2 10752
## minority in (1 + minority | classid)   11 -5364.8 10752  3.1967  2
## (1 | schoolid)                         12 -5373.2 10770 20.1422  1
##                                      Pr(>Chisq)
## <none>
## minority in (1 + minority | classid)     0.2022
## (1 | schoolid)                        7.189e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for minority at the classroom level, as the p value = 0.202 for the Chi-square test is not signifcant at $\alpha = 0.05$.

SES

```
rssc.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+ses|classid)+(1|schoolid),data=classroom)
summary(rssc.3)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10725.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5688 -0.6004 -0.0316  0.5959  3.6176
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.06   9.277
##           ses           44.09   6.640   0.75
##  schoolid (Intercept)  173.16  13.159
##  Residual             1048.32  32.378
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.52093    5.26665  269.59234 102.441  < 2e-16 ***
## housepov     -16.28994   13.13445  111.28619  -1.240    0.217
## mathknow       1.37996    1.37294  222.43201   1.005    0.316
## yearstea       0.01605    0.14080  227.59545   0.114    0.909
## mathprep      -0.37734    1.34603  182.84309  -0.280    0.780
## sex           -1.32178    2.08794 1017.08508  -0.633    0.527
## minority     -16.09272    3.03497  717.66470  -5.302 1.52e-07 ***
## ses           10.05535    1.64507  171.13536   6.112 6.44e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.450
## mathknow -0.078  0.059
## yearstea -0.266  0.074  0.030
## mathprep -0.625  0.036 -0.001 -0.165
## sex      -0.186 -0.009  0.007  0.013 -0.009
## minority -0.325 -0.181  0.108  0.021  0.004 -0.014
## ses      -0.084  0.078  0.015 -0.024  0.056  0.022  0.142
```

```
ranova(rssc.3)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | classid) + (1 | schoolid)
##                             npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                        13 -5362.8 10752
## ses in (1 + ses | classid)    11 -5364.8 10752  3.8395  2     0.1466
## (1 | schoolid)                12 -5375.8 10776 26.0221  1  3.375e-07 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for minority at the classroom level, as the p value = 0.147 for the Chi-square test is not signifcant at $\alpha = 0.05$.

B: – reviewed up to here.

Random slopes for student-level predictors varying at school level

ONE BY ONE

Sex

```
rss.4 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+sex||schoolid)+(1|classid),data=classroom)
summary(rss.4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8578 -0.6110 -0.0259  0.5922  3.5557
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   96.08   9.802
##  schoolid   sex           35.83   5.986
##  schoolid.1 (Intercept)  161.63  12.713
##  Residual               1054.36  32.471
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.43517    5.30740  272.54946 101.638  < 2e-16 ***
## housepov     -16.77661   13.22881  112.39593  -1.268    0.207
## mathknow       1.40067    1.39464  234.45882   1.004    0.316
## yearstea       0.01448    0.14163  226.44519   0.102    0.919
## mathprep      -0.27193    1.38010  205.78503  -0.197    0.844
## sex           -1.33534    2.18746  138.08788  -0.610    0.543
## minority     -16.16536    3.02861  704.25758  -5.338 1.27e-07 ***
## ses            9.98477    1.54243 1058.27875   6.473 1.46e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.081  0.055
## yearstea -0.259  0.070  0.028
## mathprep -0.633  0.036  0.004 -0.172
## sex      -0.179 -0.010  0.007  0.013 -0.004
## minority -0.320 -0.178  0.114  0.024  0.001 -0.015
## ses      -0.120  0.081 -0.007 -0.029  0.052  0.020  0.161
```

```
ranova(rss.4, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + sex | schoolid) + (1 | classid)
##                              npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                         12 -5364.4 10753
## (1 | schoolid)                 11 -5374.4 10771 19.9994  1  7.747e-06 ***
## sex in (0 + sex | schoolid)    11 -5364.8 10752  0.6137  1   0.433392
## (1 | classid)                  11 -5368.2 10758  7.4171  1   0.006461 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated sex random slope at a school level is insignifcant with a p value of .433.

Minority

```
rss.5 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+minority||schoolid)+(1|classid),data=classroom)
summary(rss.5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   93.89   9.69
##  schoolid   minority       0.00   0.00
##  schoolid.1 (Intercept)  169.45  13.02
##  Residual               1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
```

```
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38919 101.585  < 2e-16 ***
## housepov     -17.64847   13.21758  113.87772  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201    0.841
## sex           -1.21419    2.09483 1022.42137  -0.580    0.562
## minority     -16.18678    3.02605  704.47892  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
ranova(rss.5,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + minority | schoolid) + (1 | classid)
##                                      npar  logLik   AIC     LRT Df
## <none>                                 12 -5364.8 10754
## (1 | schoolid)                         11 -5375.2 10772 20.8586  1
## minority in (0 + minority | schoolid)  11 -5364.8 10752  0.0000  1
## (1 | classid)                          11 -5368.3 10759  7.1357  1
##                                      Pr(>Chisq)
## <none>
## (1 | schoolid)                        4.945e-06 ***
## minority in (0 + minority | schoolid)   1.000000
## (1 | classid)                          0.007556 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated minority random slope at school level is insignificant with a pvalue of 1.0.

SES

```
rss.6 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+ses||schoolid)+(1|classid),data=classroom)
summary(rss.6) #IS SIG
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
```

```
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   88.56   9.411
##  schoolid   ses           72.50   8.515
##  schoolid.1 (Intercept)  167.98  12.961
##  Residual               1035.12  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917  270.54314 102.126  < 2e-16 ***
## housepov     -16.94564   13.21116  112.82496  -1.283    0.202
## mathknow       1.35576    1.38459  232.19983   0.979    0.329
## yearstea       0.03079    0.14052  223.94305   0.219    0.827
## mathprep      -0.19801    1.35994  198.59419  -0.146    0.884
## sex           -1.40185    2.08170 1011.28944  -0.673    0.501
## minority     -16.52525    3.02189  700.06637  -5.469 6.32e-08 ***
## ses            9.78982    1.82217   79.01645   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.079  0.056
## yearstea -0.260  0.070  0.028
## mathprep -0.628  0.041  0.002 -0.172
## sex      -0.190 -0.007  0.006  0.018 -0.007
## minority -0.323 -0.180  0.110  0.024  0.001 -0.010
## ses      -0.091  0.076  0.006 -0.019  0.042  0.017  0.124
```

```
ranova(rss.6,refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid) + (0 + ses | schoolid) + (1 | classid)
##                             npar  logLik   AIC     LRT Df Pr(>Chisq)
## <none>                        12 -5362.4 10749
## (1 | schoolid)                11 -5374.6 10771 24.2924  1  8.276e-07 ***
## ses in (0 + ses | schoolid)   11 -5364.8 10752  4.6972  1    0.03021 *
## (1 | classid)                 11 -5365.7 10753  6.5177  1    0.01068 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated ses random slope at school level is signifcant with a p value of .03.

Allowing for correlations with random intercepts

ONE BY ONE

Sex

```
rssc.4 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+sex|schoolid)+(1|classid),data=classroom)
summary(rssc.4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8048 -0.6095 -0.0222  0.5969  3.5525
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   97.34   9.866
##  schoolid (Intercept)  206.33  14.364
##           sex           84.08   9.170   -0.43
##  Residual             1041.76  32.276
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.399e+02  5.363e+00  2.626e+02 100.661  < 2e-16 ***
## housepov    -1.742e+01  1.325e+01  1.136e+02  -1.314    0.191
## mathknow     1.379e+00  1.396e+00  2.364e+02   0.988    0.324
## yearstea     6.876e-03  1.418e-01  2.277e+02   0.048    0.961
## mathprep    -2.796e-01  1.378e+00  2.061e+02  -0.203    0.839
## sex         -1.340e+00  2.301e+00  8.742e+01  -0.582    0.562
## minority    -1.642e+01  3.027e+00  7.076e+02  -5.425 7.96e-08 ***
## ses          9.928e+00  1.540e+00  1.055e+03   6.448 1.72e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.082  0.060
## yearstea -0.258  0.072  0.027
## mathprep -0.627  0.038  0.004 -0.172
## sex      -0.222 -0.003  0.006  0.014 -0.005
```

```
## minority -0.319 -0.178  0.114  0.024  0.004 -0.011
## ses      -0.121  0.083 -0.006 -0.028  0.053  0.018  0.163
```

```
ranova(rssc.4, refit=F)
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | schoolid) + (1 | classid)
##                              npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                         13 -5363.8 10754
## sex in (1 + sex | schoolid)    11 -5364.8 10752 1.8631  2   0.393952
## (1 | classid)                  12 -5367.6 10759 7.6414  1   0.005704 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated sex random slope at school-level is insignificant with a pvalue of .394.

Minority

```
rssc.5 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+minority|schoolid)+(1|classid),data=classroom)
summary(rssc.5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.69   9.311
##  schoolid (Intercept)  381.20  19.524
##           minority     343.13  18.524   -0.83
##  Residual             1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  9.999e+01  -1.277    0.204
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
```

```
## minority    -1.638e+01  3.896e+00  5.824e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.394
## mathknow -0.078  0.061
## yearstea -0.253  0.091  0.024
## mathprep -0.576  0.037 -0.002 -0.167
## sex      -0.172 -0.013  0.010  0.014 -0.005
## minority -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses      -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

```
ranova(rssc.5,refit=F) #sig
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | schoolid) + (1 | classid)
##                                       npar  logLik   AIC    LRT Df
## <none>                                  13 -5358.8 10744
## minority in (1 + minority | schoolid)   11 -5364.8 10752 11.967  2
## (1 | classid)                           12 -5361.8 10748  6.077  1
##                                       Pr(>Chisq)
## <none>
## minority in (1 + minority | schoolid)    0.00252 **
## (1 | classid)                            0.01370 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated minority random slope at school-level is significant with a pvalue of .0025.

SES

```
rssc.6 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+ses|schoolid)+(1|classid),data=classroom)
summary(rssc.6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5646 -0.6166 -0.0264  0.5888  3.7073
##
```

```
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.57   9.305
##  schoolid (Intercept)  171.18  13.083
##           ses           73.37   8.565   0.19
##  Residual             1035.90  32.185
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.72222    5.27647  271.13405 102.099  < 2e-16 ***
## housepov     -15.89873   13.15393  111.71410  -1.209    0.229
## mathknow       1.26025    1.38201  230.89932   0.912    0.363
## yearstea       0.03617    0.14002  220.42247   0.258    0.796
## mathprep      -0.21697    1.35642  197.10752  -0.160    0.873
## sex           -1.40436    2.08074 1011.40322  -0.675    0.500
## minority     -16.26699    3.03580  668.91517  -5.358 1.16e-07 ***
## ses            9.72646    1.82985   78.36218   5.315 9.75e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.077  0.057
## yearstea -0.259  0.073  0.028
## mathprep -0.627  0.039  0.001 -0.172
## sex      -0.188 -0.009  0.005  0.017 -0.008
## minority -0.325 -0.182  0.108  0.021  0.002 -0.011
## ses      -0.062  0.070  0.007 -0.021  0.045  0.018  0.117
```

```
ranova(rssc.6,refit=F) #not sig
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | schoolid) + (1 | classid)
##                              npar  logLik   AIC    LRT Df Pr(>Chisq)
## <none>                         13 -5362.2 10750
## ses in (1 + ses | schoolid)    11 -5364.8 10752 5.1385  2    0.07659 .
## (1 | classid)                  12 -5365.3 10755 6.2117  1    0.01269 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated ses random slope at school-level is Very close to significance but not quite there with a p-value of .0766.

Question: Report unusual changes in variance.

Answer: Perhaps most striking is the change in variance for the random slope term on minority. Previously, it was 0. However, it jumps to 343.13 in the correlated model. The variance for the random slope term on SES also increases, but the correlated random slope is not a significant addition to our model according to the rand test results.

Complex model

Take two predictors that had sig random slopes and add to model, test for need of one conditional on the other

-Minority is sig for correlated

-Ses is sig for uncorrelated

```
complex <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                 ses+(0+ses|schoolid)+(1+minority|schoolid)+(1|classid),data=classroom)
summary(complex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##  Groups     Name        Variance Std.Dev. Corr
##  classid    (Intercept)   80.63   8.979
##  schoolid   (Intercept)  404.54  20.113
##             minority     336.04  18.332   -0.84
##  schoolid.1 ses           74.93   8.656
##  Residual                1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.74621  95.160  < 2e-16 ***
## housepov     -15.32111   12.49443   99.25865  -1.226    0.223
## mathknow       1.67475    1.35000  221.33588   1.241    0.216
## yearstea       0.02102    0.13657  213.65672   0.154    0.878
## mathprep      -0.23546    1.31730  191.22014  -0.179    0.858
## sex           -1.03871    2.06951 1010.41144  -0.502    0.616
## minority     -16.72884    3.90720   55.41065  -4.282 7.43e-05 ***
## ses            9.19654    1.82272   82.48814   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
```

```
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

```
ranova(complex, refit=F)
```

```
## Warning: Model failed to converge with 1 negative eigenvalue: -1.2e-04
```

```
## ANOVA-like table for random-effects: Single term deletions
##
## Model:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##                                       npar  logLik   AIC     LRT Df
## <none>                                  14 -5356.2 10740
## ses in (0 + ses | schoolid)             14 -5358.8 10746  5.1200  0
## minority in (1 + minority | schoolid)   12 -5362.4 10749 12.3899  2
## (1 | classid)                           13 -5358.9 10744  5.3724  1
##                                       Pr(>Chisq)
## <none>
## ses in (0 + ses | schoolid)
## minority in (1 + minority | schoolid)    0.00204 **
## (1 | classid)                            0.02046 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Question: Is the more complex model (with both random slopes in it) justified?

Answer: The complex model is justified since the rand test shows that the random slopes are both statistically significant at the 0.05 level, the only question revolves around statistical significance justifying compared to the Bayesian approach that would push for a simpler model.

The equation for the complex model is given by the following:

$Math1st_{ijk} = \beta_0 + \beta_1 * housepov_k + \beta_2 * mathknow_{jk} + \beta_3 * yearstea_{jk} + \beta_4 * mathprep_{jk} + \beta_5 * sex_{ijk} + \beta_{6k} * ses_{ijk} + \beta_{7k} * minority_{ijk} + \zeta_{0k} + \zeta_{6k} + \zeta_{7k} + eta_{jk} + \epsilon_{ijk}$

where $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0}), \zeta_{6k} \sim N(0, \sigma^2_{\zeta_6}), \zeta_{7k} \sim N(0, \sigma^2_{\zeta_7}), \eta_{jk} \sim N(0, \sigma^2_{\eta})$, and $\epsilon_{ijk} \sim N(0, \sigma^2_{\epsilon})$, all independent of each other.

```
summary(model1)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula: Math1 ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.46   9.244
##  schoolid         (Intercept)  280.68  16.754
```

```
##  Residual                      1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  522.540      2.037 104.406   256.6   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

V_C, V_S, and V_E **Question:** For UCM, write down: V_C, V_S, V_E for the three variance components (simply the estimates). Think of them as possibly varying with a covariate, though.

Answer: For the UCM, $V_C = 85.46$, $V_S = 280.68$, and $V_E = 1146.80$

```
summary(model4)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                      1064.95  32.63
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.38922 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.87774  -1.335    0.184
## mathknow       1.35004    1.39168  234.49776   0.970    0.333
## yearstea       0.01129    0.14141  226.80899   0.080    0.936
## mathprep      -0.27705    1.37583  205.27157  -0.201    0.841
## sex           -1.21419    2.09483 1022.42136  -0.580    0.562
## minority     -16.18678    3.02605  704.47889  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.56223   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
```

```
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

Question: For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are: V_C, V_S, V_E?

Answer: For the most complicated fixed effects model with only random intercepts, $V_C = 93.89$, $V_S = 169.45$, and $V_E = 1064.95$.

Question: By what fraction did these each decrease with the new predictors in the model?

Answer: V_C increased $\frac{93.89}{85.46}$

```
            $V_S$ decreased  $\frac{169.45}{280.68}$

            $V_E$ decreased $\frac{1064.95}{1146.80}$
```

```
summary(rss.6)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses || schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   88.56   9.411
##  schoolid   ses           72.50   8.515
##  schoolid.1 (Intercept)  167.98  12.961
##  Residual               1035.12  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917  270.54314 102.126  < 2e-16 ***
## housepov     -16.94564   13.21116  112.82496  -1.283    0.202
## mathknow       1.35576    1.38459  232.19983   0.979    0.329
## yearstea       0.03079    0.14052  223.94305   0.219    0.827
## mathprep      -0.19801    1.35994  198.59419  -0.146    0.884
## sex           -1.40185    2.08170 1011.28944  -0.673    0.501
## minority     -16.52525    3.02189  700.06637  -5.469 6.32e-08 ***
## ses            9.78982    1.82217   79.01645   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.079  0.056
```

```
## yearstea -0.260  0.070  0.028
## mathprep -0.628  0.041  0.002 -0.172
## sex      -0.190 -0.007  0.006  0.018 -0.007
## minority -0.323 -0.180  0.110  0.024  0.001 -0.010
## ses      -0.091  0.076  0.006 -0.019  0.042  0.017  0.124
```

Question: Now consider the model with a random slope in ses. What are: V_C, $V_S(ses = 0)$, V_E ? We need to list 'ses=0' here, or we don't know how to use the slope variance

Answer: For the model with a random slope in ses at the school level, $V_C = 88.56$, $V_S(ses = 0) = 167.98$, and $V_E = 1035.12$.

Question: What are: $V_S(ses = -0.50)$, $V_S(ses = +0.5)$?

Answer: In this model, in which the random slope for SES is uncorrelated with the random school-level intercept, $V_S(ses = -0.50) = 167.98 + (-.5)^2 72.50 + 2(-.5)0167.9872.50 = 186.105$, and $V_S(ses = +0.5) = 167.98 + (.5)^2 72.50 + 2 * (.5)0167.98 * 72.50 = 186.105$

```
summary(rssc.5)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.69   9.311
##  schoolid (Intercept)  381.20  19.524
##           minority     343.13  18.524   -0.83
##  Residual             1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  9.999e+01  -1.277    0.204
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.824e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.394
```

```
## mathknow -0.078  0.061
## yearstea -0.253  0.091  0.024
## mathprep -0.576  0.037 -0.002 -0.167
## sex      -0.172 -0.013  0.010  0.014 -0.005
## minority -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses      -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

Question: Now consider the model with a random slope in minority. What are: V_C, $V_S(minority = 0)$, V_E? We need to list 'minority=0' here, or we don't know how to use the slope variance

Answer: For the model with a random slope in minority at the school level, $V_C = 86.69$, $V_S(minority = 0) = 381.20$, and $V_E = 1039.39$.

Question: What are: $V_S(minority = 0.25)$, $V_S(minority = +0.50)$, $V_S(minority = +0.75)$?

Answer: In this model, in which the random slope for minority is correlated with the random school-level, intercept, $V_S(minority = 0.25) = 381.20 + (0.25)^2 343.13 + 2(0.25)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 252.5549$,

$V_S(minority = +0.50) = 381.20 + (0.50)^2 343.13 + 2(0.50)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 166.801$, and

$V_S(minority = +0.75) = 381.20 + (0.25)^2 343.13 + 2(0.25)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 123.9384$.

```
summary(complex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##  Groups     Name        Variance Std.Dev. Corr
##  classid    (Intercept)   80.63   8.979
##  schoolid   (Intercept)  404.54  20.113
##             minority     336.04  18.332   -0.84
##  schoolid.1 ses           74.93   8.656
##  Residual                1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.74621  95.160  < 2e-16 ***
## housepov     -15.32111   12.49443   99.25865  -1.226    0.223
## mathknow       1.67475    1.35000  221.33588   1.241    0.216
## yearstea       0.02102    0.13657  213.65672   0.154    0.878
## mathprep      -0.23546    1.31730  191.22014  -0.179    0.858
## sex           -1.03871    2.06951 1010.41144  -0.502    0.616
## minority     -16.72884    3.90720   55.41065  -4.282 7.43e-05 ***
## ses            9.19654    1.82272   82.48814   5.046 2.65e-06 ***
## ---
```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

Question: Now consider the model with a random slope in ses & minority. What are: V_C, $V_S(minority = 0, ses = 0)$, V_E? We need to list 'ses=0, minority=0' here, or we don't know how to use the slope variance.

Answer: For the model with a random slope in ses & minority, $V_C = 80.63$, $V_S(minority = 0, ses = 0) = 404.54$, and $V_E = 1009.73$.

Question: What are: $V_S(ses = 0, minority = 0.50)$, $V_S(ses = 0.50, minority = 0)$, $V_S(ses = 0.50, minority = 0.50)$?

Answer: In this model, in which the random slope for ses is uncorrelated with the random intercept, but the random slope for minority is correlated with the random intercept,

$V_S(ses = 0, minority = 0.50) = 404.54 + (0)^2 74.93 + (0.50)^2 336.04 + 200404.5474.93 + 2 * (0.50)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 182.5268$,

$V_S(ses = 0.50, minority = 0) = 404.54 + (0.50)^2 74.93 + (0)^2 336.04 + 20.500404.5474.93 + 2 * (0)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 423.2725$

$V_S(ses = 0.50, minority = 0.50) = 404.54 + (0.50)^2 74.93 + (0.50)^2 336.04 + 20.500404.5474.93 + 2 * (0.50)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 201.2593$

```
summary(complex)
```

```
## Linear mixed model fit by REML. t-tests use Satterthwaite's method [
## lmerModLmerTest]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##  Groups     Name        Variance Std.Dev. Corr
##  classid    (Intercept)   80.63   8.979
##  schoolid   (Intercept)  404.54  20.113
##             minority     336.04  18.332   -0.84
##  schoolid.1 ses           74.93   8.656
##  Residual                1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
```

```
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.74621  95.160  < 2e-16 ***
## housepov     -15.32111   12.49443   99.25865  -1.226    0.223
## mathknow       1.67475    1.35000  221.33588   1.241    0.216
## yearstea       0.02102    0.13657  213.65672   0.154    0.878
## mathprep      -0.23546    1.31730  191.22014  -0.179    0.858
## sex           -1.03871    2.06951 1010.41144  -0.502    0.616
## minority     -16.72884    3.90720   55.41065  -4.282 7.43e-05 ***
## ses            9.19654    1.82272   82.48814   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

Question: In the last model, what is a "likely" (+/- 1 sd) range for η_{0jk}

Answer: For the complex model, the "likely" range for $\eta 0jk$ is 71.651 to 89.609.

Question: Can we make a similar statement about ζ_{0k}?

Answer: Mathmatically we can with a range of 384.427 to 424.653 though we can do this it doesn't make much sense due to the correlated nature of this with the minority variable the values wouldn't hold much meaning and are easily misinterpreted.

Question: If you had a large value for η_{0jk}, would you expect a large or small or "any" value for: the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority?

Answer: If you have a very large η_{0jk} you would expect a small value for ζ_{1k} and ζ_{2k} but the ζ_{2k} would not be as small due to its negative correlation with our ζ_{0k} which is effected by our eta value.

Question: If you had a large value for ζ_{0k}, would you expect a large or small or "any" value for: the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority (discuss each separately)?

Answer: For ζ_{1k} would increase in the same direction but it could be any value due to the lack of correlation, keeping in mind that ζ_{0k} will create a ceiling effect of sorts for ζ_{1k} . While ζ_{2k} would be very small because of the correlation because of the two variables are negatively correlated.