Group Project #1

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Part 1: Frankie

Create 1st grade variable

```
classroom <- classroom %>% mutate(Math1 = mathkind + mathgain)
```

Random Intercepts for classroom, nested in schools UMM

We begin our analysis by looking at the UMM with random intercepts for schools and classrooms, i.e. :

$$Math1st_{ijk} = \beta_{0ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

where i represents students, j represents classrooms and k represents schools.$\zeta_k \sim N(0, \sigma^2_\zeta), \eta_{jk} \sim N(0, \sigma^2_\eta)$, and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, all are independent of each other

```
model1 <- lmer(Math1~(1|schoolid/classid),data=classroom)
summary(model1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: Math1 ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.46   9.244
##  schoolid         (Intercept)  280.68  16.754
##  Residual                     1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  522.540      2.037   256.6
```

$$ICC_{class} = \frac{85.46}{1146.8 + 280.68 + 85.46} \approx .056$$

$$ICC_{school} = \frac{280.68}{1146.8 + 280.68 + 85.46} \approx .186$$

We hence find, from the fit summary above, that the equation for our model is:

$$Math1st_{ijk} = 522.54 + \zeta_k + \eta_{jk} + \epsilon_{ijk}$$

$\zeta_k \sim N(0, 280.68), \eta_{jk} \sim N(0, 85.46)$, and $\epsilon_{ijk} \sim N(0, 1146.80)$, all are independent of each other

Model with School Level Predictors Added

We then add all the school level predictors (that is, "housepov") and report below the model fit :

```
model2 <- lmer(Math1~housepov+(1|schoolid/classid),data=classroom)
summary(model2)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula: Math1 ~ housepov + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11927.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1142 -0.6011 -0.0350  0.5600  3.8154
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   82.36   9.075
##  schoolid         (Intercept)  250.93  15.841
##  Residual                     1146.95  33.867
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error      df t value Pr(>|t|)
## (Intercept)  531.294      3.341 102.810 159.024   <2e-16 ***
## housepov     -45.783     14.236 111.060  -3.216   0.0017 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr)
## housepov -0.810
```

```
anova(model1, model2, refit = F)
```

```
## Data: classroom
## Models:
## object: Math1 ~ (1 | schoolid/classid)
## ..1: Math1 ~ housepov + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## object  4 11953 11973 -5972.3    11945
## ..1     5 11937 11963 -5963.7    11927 17.186      1   3.39e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Report the changes in the variances of the random effects:

Change in σ^2_ζ: decreased to 250.93 from 280.63 σ^2_η decreases to 82.36 from 85.46 σ^2_ϵ slightly increases to 1146.95 from 1146.8

The LRT has a p-value of almost zero, $p = 3.39e-05$, thus we reject the H_0: coefficient on Housepov = 0 at $\alpha = 0.05$. That is, we find evidence that it makes sense to include the school level predictor, housepov.

Model with all Class Level Predictors Added

We now re-run the model after including all the classroom level predictors, that is "mathknow", "yearstea", "mathprep", and report the model fit.

```
model3 <- lmer(Math1~housepov+mathknow+yearstea+mathprep+
                 (1|schoolid/classid),data=classroom)
summary(model3)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                      1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29853    5.20496 228.86000 102.268  < 2e-16 ***
## housepov    -41.62116   14.08835 109.83000  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.07000   1.765  0.07883 .
## yearstea      0.06193    0.14717 223.77000   0.421  0.67432
## mathprep     -0.75440    1.42809 203.21000  -0.528  0.59790
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst
## housepov -0.568
## mathknow -0.052  0.082
## yearstea -0.264  0.077  0.030
## mathprep -0.666  0.032  0.004 -0.175
```

creating reducted dataset taking away missing data

The variable of interest *Mathknow* includes some missing values. The model for which we have reported the summary above therefore removes the observations for which missing data is present.

To be able to compare Model 2 (with school level predictors) with Model 3 (with both school level and classroom level predictors), we removed from the dataset students that had missing values, creating a reduced dataset. This left us with a sample of 1081 students. We then re-run model 2 on this reducted dataset and compared it to Model 3.

```
classroom_red = na.omit(classroom)
model2_red <- lmer(Math1~housepov+(1|schoolid/classid),data=classroom_red)
model3_red <- lmer(Math1~housepov+mathknow+yearstea+mathprep+
                     (1|schoolid/classid),data=classroom_red)

summary(model3_red)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##    Data: classroom_red
##
## REML criterion at convergence: 10821
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5552 -0.6118 -0.0311  0.5863  3.8315
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   94.36   9.714
##  schoolid         (Intercept)  223.31  14.943
##  Residual                      1136.43  33.711
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error        df t value Pr(>|t|)
## (Intercept) 532.29853    5.20496 228.86000 102.268  < 2e-16 ***
## housepov    -41.62116   14.08835 109.83000  -2.954  0.00383 **
## mathknow      2.55143    1.44530 231.07000   1.765  0.07883 .
## yearstea      0.06193    0.14717 223.77000   0.421  0.67432
## mathprep     -0.75440    1.42809 203.21000  -0.528  0.59790
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst
## housepov -0.568
## mathknow -0.052  0.082
## yearstea -0.264  0.077  0.030
## mathprep -0.666  0.032  0.004 -0.175
```

```
anova(model2_red, model3_red, refit = F)
```

```
## Data: classroom_red
```

```
## Models:
## object: Math1 ~ housepov + (1 | schoolid/classid)
## ..1: Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## object  5 10838 10862 -5413.8    10828
## ..1     8 10837 10877 -5410.5    10821 6.5771      3    0.08667 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Change in σ^2_ϵ and σ^2_η: σ^2_ϵ decreased to 1136.43, σ^2_η increased to 94.36; $\sigma^2_\zeta = 223.31$

A possible reason why ϵ decreased in this model, but not η is that adding the classroom level predictors makes it so that more of the overall variation is explained by "structured" variation (that is, related to the fact that students are in different classrooms) rather than by unstructured (ϵ), so that the latter decreases. However, we also have to note that in this case we are using the reduced dataset, so that some of the changes may be due to the fact that we are using two slightly different datasets.

The anova test comparing the school level predictor to the model with the classroom predictors has a p-value 0.087, so we fail to reject the null hypothesis at our $\alpha = 0.05$ and conclude that adding classroom level predictors is not necessary, as it does not signficantly improve the model.

Add all student-level predictors

We now include all the student level predictors in our model:

```
model4 <- lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                 ses+(1|schoolid/classid),data=classroom)
summary(model4)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                     1064.95  32.63
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
```

```
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.30000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.50000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

We test this new block compared to the model with both school-level and classroom level predictors.

```
anova(model3, model4, refit = F)
```

```
## Data: classroom
## Models:
## object: Math1 ~ housepov + mathknow + yearstea + mathprep + (1 | schoolid/classid)
## ..1: Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
## ..1:     ses + (1 | schoolid/classid)
##        Df   AIC   BIC  logLik deviance  Chisq Chi Df Pr(>Chisq)
## object  8 10837 10877 -5410.5    10821
## ..1    11 10752 10806 -5364.8    10730 91.446      3  < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The LRT test between this two models has a p-value $< 2.2 * 10^{-16}$. Therefore, at our $\alpha = 0.05$, we reject the null hypothesis and conclude that adding this block of predictors is justified.

Changes in variance components :

σ^2_ϵ decreased to 1064.95, σ^2_η decreased to 93.89, and σ^2_ζ decreased to 169.45.

We note that adding student-level predictors leads to a decrease in the overall variance of the model. By "controlling" for student-related variables, we also explain the between schools, as students with similar attributes might be similar across schools, hence reducing the overall variance of ζ.

The final model, with all school level, classroom level, and student level predictors, is:

$$Math1st_{ijk} = 539.63 + \zeta_k + \eta_{jk} + \epsilon_{ijk} - 17.65 * Housepov_k + 1.35 * Mathknow_{jk} +$$
$$0.01 * YearsTea_{jk} - 0.27 * Mathprep_{jk} - 0.19 * sex_{ijk} + -0.32 * minority_{ijk} - 0.12 * ses_{ijk}$$

With:

$\zeta_k \sim N(0, \sigma^2_\zeta), \eta_{jk} \sim N(0, \sigma^2_\eta)$, and $\epsilon_{ijk} \sim N(0, \sigma^2_\epsilon)$, all are independent of each other

From the model fit above therefore we find that the fitted model is:

$$Math1st_{ijk} = \beta_{0ijk} + \zeta_k + \eta_{jk} + \epsilon_{ijk} + \beta_1 Housepov_k + \beta_2 Mathknow_{jk} +$$
$$\beta_3 YearsTea_{jk} + \beta_4 Mathprep_{jk} + \beta_5 sex_{ijk} + \beta_6 minority_{ijk} + \beta_7 ses_{ijk}$$

With:

$\zeta_k \sim N(0, 169.45), \eta_{jk} \sim N(0, 93.89)$, and $\epsilon_{ijk} \sim N(0, 1064.95)$, all are independent of each other.

Random Slope for Teacher-level predictor varying at school-level

We try adding a random slope for each teacher level predictor (varying at the school level; one by one - not all together).

MATHKNOW

```
rst.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+mathknow||schoolid)+(1|classid),data=classroom)
summary(rst.1)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + ((1 | schoolid) + (0 + mathknow | schoolid)) + (1 |
##     classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.389e+01 9.690e+00
##  schoolid   mathknow    4.260e-11 6.527e-06
##  schoolid.1 (Intercept) 1.694e+02 1.302e+01
##  Residual               1.065e+03 3.263e+01
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.50000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.60000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
```

```
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
rand(rst.1,refit=F)
```

```
## Analysis of Random effects Table:
##                     Chi.sq Chi.DF p.value
## schoolid          2.34e+01      1   1e-06 ***
## mathknow:schoolid 1.82e-12      1   1.000
## classid           6.74e+00      1   0.009 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for *MATHKNOW* at a school level as the p value = 1 for the Chi-square test is not signifcant at $\alpha = 0.05$.

YEARSTEA

```
rst.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+yearstea||schoolid)+(1|classid),data=classroom)
summary(rst.2)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + ((1 | schoolid) + (0 + yearstea | schoolid)) + (1 |
##     classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8485 -0.6149 -0.0323  0.5980  3.6600
##
## Random effects:
##  Groups     Name        Variance  Std.Dev.
##  classid    (Intercept) 9.266e+01  9.62593
##  schoolid   yearstea    9.669e-03  0.09833
##  schoolid.1 (Intercept) 1.685e+02 12.97894
##  Residual               1.065e+03 32.63452
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.60060    5.30865  266.30000 101.645  < 2e-16 ***
## housepov     -17.71727   13.21854  113.60000  -1.340    0.183
## mathknow       1.33198    1.39177  234.30000   0.957    0.340
## yearstea       0.01124    0.14193  122.40000   0.079    0.937
## mathprep      -0.26633    1.37610  204.90000  -0.194    0.847
## sex           -1.21077    2.09476 1022.20000  -0.578    0.563
## minority     -16.16833    3.02641  702.60000  -5.342 1.24e-07 ***
## ses           10.04529    1.54490 1066.00000   6.502 1.21e-10 ***
```

```
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) housepv mthknw yearst mthprp sex    minrty
## housepov -0.450
## mathknow -0.082  0.057
## yearstea -0.258  0.070  0.028
## mathprep -0.632  0.037  0.003 -0.172
## sex      -0.190 -0.007  0.006  0.015 -0.006
## minority -0.320 -0.179  0.115  0.023  0.001 -0.010
## ses      -0.121  0.082 -0.007 -0.027  0.053  0.020  0.162
```

```
rand(rst.2, refit=F)
```

```
## Analysis of Random effects Table:
##                     Chi.sq Chi.DF p.value
## schoolid          19.83009      1   8e-06 ***
## yearstea:schoolid  0.00698      1    0.93
## classid            5.91580      1    0.02 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for *YEARSTEA* at a school level as the p value = 0.93 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Mathprep

```
rst.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+mathprep||schoolid)+(1|classid),data=classroom)
summary(rst.3)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + ((1 | schoolid) + (0 + mathprep | schoolid)) + (1 |
##     classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   93.89   9.69
##  schoolid   mathprep       0.00   0.00
##  schoolid.1 (Intercept)  169.45  13.02
##  Residual                1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
```

```
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21758  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.50000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.60000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
rand(rst.3, refit=F)
```

```
## Analysis of Random effects Table:
##                   Chi.sq Chi.DF p.value
## schoolid           13.62      1   2e-04 ***
## mathprep:schoolid   0.00      1   1.000
## classid             7.14      1   0.008 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for *MATHPREP* at a school level as the p value = 1 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Question: Why is a random slope on housepov a bad idea?

Answer: There is only one data point per school, so we do not have enough information to calculate the slope for each school.

Allowing correlations with random intercepts

ONE BY ONE

Again, we add random slopes for each teacher-level predictor varying at the school level, but this time by allowing them to be correlated with the random intercepts.

MATHKNOW

```
rstc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+mathknow|schoolid)+(1|classid),data=classroom)
summary(rstc.1)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
```

```
##     ses + (1 + mathknow | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8581 -0.6131 -0.0324  0.5969  3.6603
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept) 9.394e+01  9.69205
##  schoolid (Intercept) 1.693e+02 13.01223
##           mathknow    8.596e-04  0.02932 1.00
##  Residual             1.065e+03 32.63393
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.64037    5.31212  275.40000 101.587  < 2e-16 ***
## housepov     -17.64148   13.21274  104.00000  -1.335    0.185
## mathknow       1.35459    1.39203  214.60000   0.973    0.332
## yearstea       0.01114    0.14141  226.90000   0.079    0.937
## mathprep      -0.27753    1.37601  201.30000  -0.202    0.840
## sex           -1.21329    2.09485 1021.70000  -0.579    0.563
## minority     -16.19376    3.02609  703.80000  -5.351 1.18e-07 ***
## ses           10.04788    1.54488 1062.10000   6.504 1.20e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.082  0.057
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.173
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
rand(rstc.1, refit=F)
```

```
## Analysis of Random effects Table:
##                     Chi.sq Chi.DF p.value
## mathknow:schoolid 0.000321      2    1.00
## classid           6.676842      1    0.01 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for math knowledge at a school level as the p value = 1.00 for the Chi-square test is not signifcant at $\alpha = 0.05$.

YEARSTEA

```
rstc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+yearstea|schoolid)+(1|classid),data=classroom)
```

```
summary(rstc.2)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + yearstea | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10723.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7462 -0.6036 -0.0290  0.6041  3.8449
##
## Random effects:
##  Groups   Name        Variance  Std.Dev. Corr
##  classid  (Intercept)   37.9283  6.1586
##  schoolid (Intercept)  366.1148 19.1341
##           yearstea       0.5523  0.7432  -0.78
##  Residual             1066.4510 32.6566
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.95245    5.48825  222.70000  98.201  < 2e-16 ***
## housepov     -17.13994   13.45959  119.60000  -1.273    0.205
## mathknow       1.04635    1.34381  209.70000   0.779    0.437
## yearstea       0.02204    0.15766   75.80000   0.140    0.889
## mathprep       0.05046    1.34549  190.80000   0.038    0.970
## sex           -1.33553    2.08774 1024.60000  -0.640    0.523
## minority     -16.44555    2.99655  669.50000  -5.488 5.77e-08 ***
## ses           10.15038    1.53873 1062.80000   6.597 6.62e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.455
## mathknow -0.085  0.049
## yearstea -0.370  0.084  0.012
## mathprep -0.606  0.050  0.014 -0.139
## sex      -0.184 -0.004  0.008  0.009 -0.004
## minority -0.305 -0.169  0.122  0.032 -0.007 -0.012
## ses      -0.119  0.079 -0.001 -0.019  0.049  0.022  0.168
```

```
rand(rstc.2,refit=F)
```

```
## Analysis of Random effects Table:
##                    Chi.sq Chi.DF p.value
## yearstea:schoolid  5.825      2    0.05 .
## classid            0.903      1    0.34
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for yearstea at a school level as the p value = 0.054 for the Chi-square test is not signifcant at $\alpha = 0.05$.

MATHPREP

```
rstc.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+mathprep|schoolid)+(1|classid),data=classroom)
summary(rstc.3)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + mathprep | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8542 -0.6034 -0.0221  0.5915  3.6475
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   78.46   8.858
##  schoolid (Intercept)  552.76  23.511
##           mathprep      15.89   3.986   -1.00
##  Residual             1064.26  32.623
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.60855    5.60813  159.90000  96.041  < 2e-16 ***
## housepov     -14.01306   12.88689  116.10000  -1.087    0.279
## mathknow       1.29884    1.37194  229.70000   0.947    0.345
## yearstea      -0.02586    0.13949  223.50000  -0.185    0.853
## mathprep       0.04074    1.34845  139.00000   0.030    0.976
## sex           -1.16759    2.08697 1023.10000  -0.559    0.576
## minority     -16.46422    2.99524  663.70000  -5.497 5.52e-08 ***
## ses           10.14166    1.53961 1060.80000   6.587 7.04e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.461
## mathknow -0.071  0.027
## yearstea -0.260  0.089  0.049
## mathprep -0.692  0.107  0.012 -0.155
## sex      -0.183  0.003  0.002  0.023 -0.008
## minority -0.275 -0.187  0.107  0.025 -0.035 -0.013
## ses      -0.121  0.095 -0.001 -0.033  0.061  0.024  0.161
```

```
rand(rstc.3, refit=F)
```

```
## Analysis of Random effects Table:
```

```
##                   Chi.sq Chi.DF p.value
## mathprep:schoolid   4.81      2    0.09 .
## classid             5.10      1    0.02 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for mathprep at a school level as the p value = 0.09 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Question: Anything unusual about the variances? Why might this have occurred? (hint: what did you add to the model?)

Answer: We note that the model did not estimate the correlation parameter correctly for the models with random slopes for mathknown and mathprepr. Indeed, with a correaltion of respectively 1 and -1 with the random intercept, the parameter is a linear function of the variance component for the slope. This could be due to the fact that there is not enough classrooms in the schools (as we are adding random effects at the school levels, for classroom level predictors), so that there is not enough degrees of freedom, nor enough variation among the variables of interest, to calculate all the parameters required in the model. Obtaining a correlation of 1 and -1 should warn us of the fact that the models generated should not be trusted.
Why is the correlation between random intercept and slope then calculated for yearstea? This could be due to the fact that this variable has a larger range, so that it can be more robustly estimated for some of the schools and the correlation between random slope and intercept then estimated more accurately even for schools with few classes.

Random slopes for student-level predictors varying at classroom level

We now repeat the exercise by adding student level predictors, varying at the classroom level.

ONE BY ONE

SEX

```
rss.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+sex||classid)+(1|schoolid),data=classroom)
summary(rss.1)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + ((1 | classid) + (0 + sex | classid)) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   93.89   9.69
##  classid.1 sex            0.00   0.00
##  schoolid  (Intercept)  169.45  13.02
```

```
##  Residual              1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21757  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.40000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.50000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
rand(rss.1, refit=F)
```

```
## Analysis of Random effects Table:
##             Chi.sq Chi.DF p.value
## classid       6.49      1    0.01 *
## sex:classid   0.00      1    1.00
## schoolid     24.79      1   6e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for sex at the classroom level, as the p value = 1 for the Chi-square test is not signifcant at $\alpha = 0.05$.

MINORITY

```
rss.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+minority||classid)+(1|schoolid),data=classroom)
summary(rss.2)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + ((1 | classid) + (0 + minority | classid)) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
```

```
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   93.89   9.69
##  classid.1 minority       0.00   0.00
##  schoolid  (Intercept)  169.45  13.02
##  Residual              1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21758  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.30000  -0.580    0.562
## minority     -16.18678    3.02605  704.40000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.40000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
rand(rss.2, refit=F)
```

```
## Analysis of Random effects Table:
##                   Chi.sq Chi.DF p.value
## classid             5.15      1    0.02 *
## minority:classid    0.00      1    1.00
## schoolid           24.79      1   6e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for minority at the classroom level, as the p value = 1 for the Chi-square test is not signifcant at $\alpha = 0.05$.

SES

```
rss.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+ses||classid)+(1|schoolid),data=classroom)
summary(rss.3)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
```

```
##     ses + ((1 | classid) + (0 + ses | classid)) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10727.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7163 -0.6032 -0.0331  0.5855  3.6840
##
## Random effects:
##  Groups    Name        Variance Std.Dev.
##  classid   (Intercept)   87.11   9.333
##  classid.1 ses           49.60   7.043
##  schoolid  (Intercept)  171.02  13.077
##  Residual              1043.44  32.302
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.71226    5.30641  274.50000 101.710  < 2e-16 ***
## housepov     -17.50879   13.21775  113.40000  -1.325    0.188
## mathknow       1.36796    1.38563  229.40000   0.987    0.325
## yearstea       0.01103    0.14117  227.00000   0.078    0.938
## mathprep      -0.27938    1.37171  204.90000  -0.204    0.839
## sex           -1.37733    2.09334 1022.80000  -0.658    0.511
## minority     -16.29362    3.02464  703.30000  -5.387 9.78e-08 ***
## ses           10.14363    1.64248  176.40000   6.176 4.41e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.082  0.058
## yearstea -0.259  0.070  0.029
## mathprep -0.631  0.040  0.005 -0.172
## sex      -0.190 -0.007  0.006  0.014 -0.005
## minority -0.321 -0.180  0.111  0.025  0.002 -0.011
## ses      -0.108  0.081  0.002 -0.026  0.050  0.020  0.145
```

```
rand(rss.3, refit=F)
```

```
## Analysis of Random effects Table:
##             Chi.sq Chi.DF p.value
## classid       5.92      1    0.01 *
## ses:classid   1.60      1    0.21
## schoolid     25.27      1   5e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the random slope for ses at the classroom level, as the p value = 0.206 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Question: why is this a bad idea to include a classroom-level variable with random slopes at classroom-level?

Answer: Because all of the observations for a class will be the same, so we will not be able to compute the

classroom slopes for each classroom (as we will only have one point).

Allowing for correlations with random intercepts

ONE BY ONE

SEX

```
rssc.1 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+sex|classid)+(1|schoolid),data=classroom)
summary(rssc.1)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10729
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.7565 -0.6134 -0.0307  0.5916  3.7116
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  130.07  11.41
##           sex           31.36   5.60    -0.67
##  schoolid (Intercept)  169.85  13.03
##  Residual             1056.41  32.50
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.400e+02  5.332e+00  2.723e+02 101.285  < 2e-16 ***
## housepov    -1.829e+01  1.323e+01  1.145e+02  -1.382    0.170
## mathknow     1.306e+00  1.391e+00  2.315e+02   0.939    0.349
## yearstea     3.087e-03  1.416e-01  2.270e+02   0.022    0.983
## mathprep    -3.459e-01  1.374e+00  2.014e+02  -0.252    0.801
## sex         -1.197e+00  2.122e+00  2.160e+02  -0.564    0.573
## minority    -1.619e+01  3.028e+00  7.042e+02  -5.347 1.21e-07 ***
## ses          1.010e+01  1.544e+00  1.065e+03   6.539 9.62e-11 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.452
## mathknow -0.085  0.060
## yearstea -0.258  0.072  0.029
## mathprep -0.628  0.040  0.005 -0.174
## sex      -0.203 -0.005  0.003  0.015 -0.008
## minority -0.321 -0.178  0.116  0.024  0.003 -0.009
```

```
## ses      -0.123  0.083 -0.005 -0.027  0.054  0.020  0.164
```

```
rand(rssc.1, refit=F)
```

```
## Analysis of Random effects Table:
##             Chi.sq Chi.DF p.value
## sex:classid    0.5      2     0.8
## schoolid      24.9      1   6e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for sex at the classroom level, as the p value $= 0.779$ for the Chi-square test is not signifcant at $\alpha = 0.05$.

MINORITY

```
rssc.2 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+minority|classid)+(1|schoolid),data=classroom)
summary(rssc.2)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10726.3
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.9037 -0.6221 -0.0295  0.6033  3.4574
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)  225.4   15.01
##           minority     171.3   13.09    -0.82
##  schoolid (Intercept)  157.4   12.55
##  Residual             1045.3   32.33
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.73594    5.38023  270.70000 100.318  < 2e-16 ***
## housepov     -17.34698   12.91268  103.30000  -1.343    0.182
## mathknow       1.45702    1.39355  234.00000   1.046    0.297
## yearstea      -0.01636    0.14285  234.30000  -0.115    0.909
## mathprep      -0.13520    1.37018  204.00000  -0.099    0.921
## sex           -1.01012    2.08966 1015.70000  -0.483    0.629
## minority     -16.48614    3.21756  183.20000  -5.124 7.55e-07 ***
## ses            9.89350    1.54595 1062.80000   6.400 2.33e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) housepv mthknw yearst mthprp sex    minrty
```

```
## housepov -0.435
## mathknow -0.079  0.061
## yearstea -0.265  0.080  0.038
## mathprep -0.618  0.037 -0.006 -0.171
## sex      -0.188 -0.009  0.009  0.015 -0.005
## minority -0.368 -0.171  0.108  0.025 -0.004 -0.009
## ses      -0.117  0.085  0.001 -0.023  0.051  0.021  0.149
```

```
rand(rssc.2)
```

```
## Analysis of Random effects Table:
##                   Chi.sq Chi.DF p.value
## minority:classid    3.2      2     0.2
## schoolid           20.1      1   7e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for minority at the classroom level, as the p value = 0.202 for the Chi-square test is not signifcant at $\alpha = 0.05$.

SES

```
rssc.3 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+ses|classid)+(1|schoolid),data=classroom)
summary(rssc.3)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | classid) + (1 | schoolid)
##    Data: classroom
##
## REML criterion at convergence: 10725.7
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5688 -0.6004 -0.0316  0.5959  3.6176
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.06   9.277
##           ses           44.09   6.640   0.75
##  schoolid (Intercept)  173.16  13.159
##  Residual             1048.32  32.378
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.52093    5.26665  269.60000 102.441  < 2e-16 ***
## housepov     -16.28994   13.13445  111.30000  -1.240    0.217
## mathknow       1.37996    1.37294  222.40000   1.005    0.316
## yearstea       0.01605    0.14080  227.60000   0.114    0.909
## mathprep      -0.37734    1.34603  182.80000  -0.280    0.780
## sex           -1.32178    2.08794 1017.10000  -0.633    0.527
## minority     -16.09272    3.03497  717.70000  -5.302 1.52e-07 ***
```

```
## ses           10.05535    1.64507  171.10000   6.112 6.44e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.450
## mathknow -0.078  0.059
## yearstea -0.266  0.074  0.030
## mathprep -0.625  0.036 -0.001 -0.165
## sex      -0.186 -0.009  0.007  0.013 -0.009
## minority -0.325 -0.181  0.108  0.021  0.004 -0.014
## ses      -0.084  0.078  0.015 -0.024  0.056  0.022  0.142
```

```
rand(rssc.3)
```

```
## Analysis of Random effects Table:
##             Chi.sq Chi.DF p.value
## ses:classid   3.84      2     0.1
## schoolid     26.02      1   3e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

There is no need for the (correlated) random slope for minority at the classroom level, as the p value = 0.147 for the Chi-square test is not signifcant at $\alpha = 0.05$.

Random slopes for student-level predictors varying at school level

ONE BY ONE

Sex

```
rss.4 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+sex||schoolid)+(1|classid),data=classroom)
summary(rss.4)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + ((1 | schoolid) + (0 + sex | schoolid)) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10728.9
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8578 -0.6110 -0.0259  0.5922  3.5557
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   96.08   9.802
##  schoolid   sex           35.83   5.986
##  schoolid.1 (Intercept)  161.63  12.713
```

```
##  Residual               1054.36  32.471
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.43517    5.30740  272.50000 101.638  < 2e-16 ***
## housepov     -16.77661   13.22881  112.40000  -1.268    0.207
## mathknow       1.40067    1.39464  234.50000   1.004    0.316
## yearstea       0.01448    0.14163  226.40000   0.102    0.919
## mathprep      -0.27193    1.38010  205.80000  -0.197    0.844
## sex           -1.33534    2.18746  138.10000  -0.610    0.543
## minority     -16.16536    3.02861  704.30000  -5.338 1.27e-07 ***
## ses            9.98477    1.54243 1058.30000   6.473 1.46e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.081  0.055
## yearstea -0.259  0.070  0.028
## mathprep -0.633  0.036  0.004 -0.172
## sex      -0.179 -0.010  0.007  0.013 -0.004
## minority -0.320 -0.178  0.114  0.024  0.001 -0.015
## ses      -0.120  0.081 -0.007 -0.029  0.052  0.020  0.161
```

```
rand(rss.4, refit=F)
```

```
## Analysis of Random effects Table:
##              Chi.sq Chi.DF p.value
## schoolid     19.999      1   8e-06 ***
## sex:schoolid  0.614      1   0.433
## classid       7.417      1   0.006 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated sex random slope at a school level is insignifcant with a p value of .433.

Minority

```
rss.5 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+minority||schoolid)+(1|classid),data=classroom)
summary(rss.5)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + ((1 | schoolid) + (0 + minority | schoolid)) + (1 |
##     classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
```

```
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   93.89   9.69
##  schoolid   minority       0.00   0.00
##  schoolid.1 (Intercept)  169.45  13.02
##  Residual               1064.95  32.63
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
## housepov     -17.64847   13.21758  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.40000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.50000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

```
rand(rss.5,refit=F)
```

```
## Analysis of Random effects Table:
##                    Chi.sq Chi.DF p.value
## schoolid            20.86      1   5e-06 ***
## minority:schoolid    0.00      1   1.000
## classid              7.14      1   0.008 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated minority random slope at school level is insignificant with a pvalue of 1.0.

SES

```
rss.6 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+ses||schoolid)+(1|classid),data=classroom)
summary(rss.6) #IS SIG
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
```

```
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + ((1 | schoolid) + (0 + ses | schoolid)) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   88.56   9.411
##  schoolid   ses           72.50   8.515
##  schoolid.1 (Intercept)  167.98  12.961
##  Residual               1035.12  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917  270.50000 102.126  < 2e-16 ***
## housepov     -16.94564   13.21116  112.80000  -1.283    0.202
## mathknow       1.35576    1.38459  232.20000   0.979    0.329
## yearstea       0.03079    0.14052  223.90000   0.219    0.827
## mathprep      -0.19801    1.35994  198.60000  -0.146    0.884
## sex           -1.40185    2.08170 1011.30000  -0.673    0.501
## minority     -16.52525    3.02189  700.10000  -5.469 6.32e-08 ***
## ses            9.78982    1.82217   79.00000   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.079  0.056
## yearstea -0.260  0.070  0.028
## mathprep -0.628  0.041  0.002 -0.172
## sex      -0.190 -0.007  0.006  0.018 -0.007
## minority -0.323 -0.180  0.110  0.024  0.001 -0.010
## ses      -0.091  0.076  0.006 -0.019  0.042  0.017  0.124
```

```
rand(rss.6,refit=F)
```

```
## Analysis of Random effects Table:
##              Chi.sq Chi.DF p.value
## schoolid      24.29      1   8e-07 ***
## ses:schoolid   4.70      1    0.03 *
## classid        6.52      1    0.01 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The uncorrelated ses random slope at school level is signifcant with a p value of .03.

Allowing for correlations with random intercepts

ONE BY ONE

Sex

```
rssc.4 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(1+sex|schoolid)+(1|classid),data=classroom)
summary(rssc.4)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + sex | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10727.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8048 -0.6095 -0.0222  0.5969  3.5525
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   97.34   9.866
##  schoolid (Intercept)  206.33  14.364
##           sex           84.08   9.170   -0.43
##  Residual             1041.76  32.276
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.399e+02  5.363e+00  2.626e+02 100.661  < 2e-16 ***
## housepov    -1.742e+01  1.325e+01  1.136e+02  -1.314    0.191
## mathknow     1.379e+00  1.396e+00  2.364e+02   0.988    0.324
## yearstea     6.876e-03  1.418e-01  2.277e+02   0.048    0.961
## mathprep    -2.796e-01  1.378e+00  2.061e+02  -0.203    0.839
## sex         -1.340e+00  2.301e+00  8.740e+01  -0.582    0.562
## minority    -1.642e+01  3.027e+00  7.076e+02  -5.425 7.96e-08 ***
## ses          9.928e+00  1.540e+00  1.055e+03   6.448 1.72e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.082  0.060
## yearstea -0.258  0.072  0.027
## mathprep -0.627  0.038  0.004 -0.172
## sex      -0.222 -0.003  0.006  0.014 -0.005
## minority -0.319 -0.178  0.114  0.024  0.004 -0.011
## ses      -0.121  0.083 -0.006 -0.028  0.053  0.018  0.163
```

```
rand(rssc.4, refit=F)
```

```
## Analysis of Random effects Table:
##              Chi.sq Chi.DF p.value
## sex:schoolid   1.86      2   0.394
## classid        7.64      1   0.006 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated sex random slope at school-level is insignificant with a pvalue of .394.

Minority

```
rssc.5 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+minority|schoolid)+(1|classid),data=classroom)
summary(rssc.5)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.69   9.311
##  schoolid (Intercept)  381.20  19.524
##           minority     343.13  18.524   -0.83
##  Residual             1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  1.000e+02  -1.277    0.204
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.820e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
```

```
## housepov -0.394
## mathknow -0.078  0.061
## yearstea -0.253  0.091  0.024
## mathprep -0.576  0.037 -0.002 -0.167
## sex      -0.172 -0.013  0.010  0.014 -0.005
## minority -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses      -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

```
rand(rssc.5,refit=F) #sig
```

```
## Analysis of Random effects Table:
##                   Chi.sq Chi.DF p.value
## minority:schoolid  11.97      2   0.003 **
## classid             6.08      1   0.014 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated minority random slope at school-level is significant with a pvalue of .003.

SES

```
rssc.6 <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
               ses+(1+ses|schoolid)+(1|classid),data=classroom)
summary(rssc.6)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + ses | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.5646 -0.6166 -0.0264  0.5888  3.7073
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.57   9.305
##  schoolid (Intercept)  171.18  13.083
##           ses           73.37   8.565   0.19
##  Residual             1035.90  32.185
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  538.72222    5.27647  271.10000 102.099  < 2e-16 ***
## housepov     -15.89873   13.15393  111.70000  -1.209    0.229
## mathknow       1.26025    1.38201  230.90000   0.912    0.363
## yearstea       0.03617    0.14002  220.40000   0.258    0.796
## mathprep      -0.21697    1.35642  197.10000  -0.160    0.873
## sex           -1.40436    2.08074 1011.40000  -0.675    0.500
```

```
## minority     -16.26699    3.03580  668.90000  -5.358 1.16e-07 ***
## ses            9.72646    1.82985   78.40000   5.315 9.75e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.449
## mathknow -0.077  0.057
## yearstea -0.259  0.073  0.028
## mathprep -0.627  0.039  0.001 -0.172
## sex      -0.188 -0.009  0.005  0.017 -0.008
## minority -0.325 -0.182  0.108  0.021  0.002 -0.011
## ses      -0.062  0.070  0.007 -0.021  0.045  0.018  0.117
```

```
rand(rssc.6,refit=F) #not sig
```

```
## Analysis of Random effects Table:
##              Chi.sq Chi.DF p.value
## ses:schoolid   5.14      2    0.08 .
## classid        6.21      1    0.01 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The correlated ses random slope at school-level is not significant with a p-value of .08.

Question: Report unusual changes in variance.

Answer: Perhaps most striking is the change in variance for the random slope term on minority. Previously, it was 0. However, it jumps to 343.13 in the correlated model. The variance for the random slope term on SES also increases, but the correlated random slope is not a significant addition to our model according to the rand test results.

Complex model

Take two predictors that had sig random slopes and add to model, test for need of one conditional on the other

-Minority is sig for correlated

-Ses is sig for uncorrelated

```
complex <-lmer(Math1~housepov+mathknow+yearstea+mathprep+sex+minority+
                ses+(0+ses|schoolid)+(1+minority|schoolid)+(1|classid),data=classroom)
summary(complex)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
```

```
##     Min      1Q  Median      3Q     Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##  Groups     Name        Variance Std.Dev. Corr
##  classid    (Intercept)   80.63   8.979
##  schoolid   (Intercept)  404.54  20.113
##             minority     336.04  18.332   -0.84
##  schoolid.1 ses           74.93   8.656
##  Residual                1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.70000  95.160  < 2e-16 ***
## housepov     -15.32111   12.49443   99.30000  -1.226    0.223
## mathknow       1.67475    1.35000  221.30000   1.241    0.216
## yearstea       0.02102    0.13657  213.70000   0.154    0.878
## mathprep      -0.23546    1.31730  191.20000  -0.179    0.858
## sex           -1.03871    2.06951 1010.30000  -0.502    0.616
## minority     -16.72884    3.90720   55.40000  -4.282 7.43e-05 ***
## ses            9.19654    1.82272   82.50000   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

```
rand(complex, refit=F)
```

```
## Analysis of Random effects Table:
##                   Chi.sq Chi.DF p.value
## ses:schoolid        5.12      1    0.02 *
## minority:schoolid  36.68      3   5e-08 ***
## classid             5.37      1    0.02 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Question: Is the more complex model (with both random slopes in it) justified?

Answer: The complex model is justified since the rand test shows that the random slopes are both statistically significant at the 0.05 level, the only question revolves around statistical significance justifying compared to the Bayesian approach that would push for a simpler model.

The equation for the complex model is given by the following:

$Math1st_{ijk} = \beta_0 + \beta_1 * housepov_k + \beta_2 * mathknow_{jk} + \beta_3 * yearstea_{jk} + \beta_4 * mathprep_{jk} + \beta_5 * sex_{ijk} + \beta_{6k} * ses_{ijk} + \beta_{7k} * minority_{ijk} + \zeta_{0k} + \zeta_{6k} + \zeta_{7k} + eta_{jk} + \epsilon_{ijk}$

where $\zeta_{0k} \sim N(0, \sigma^2_{\zeta_0}), \zeta_{6k} \sim N(0, \sigma^2_{\zeta_6}), \zeta_{7k} \sim N(0, \sigma^2_{\zeta_7}), \eta_{jk} \sim N(0, \sigma^2_{\eta})$, and $\epsilon_{ijk} \sim N(0, \sigma^2_{\epsilon})$, all independent

of each other.

```
summary(model1)
```

```
## Linear mixed model fit by REML ['lmerMod']
## Formula: Math1 ~ (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 11944.6
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -5.1872 -0.6174 -0.0204  0.5821  3.8339
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   85.46   9.244
##  schoolid         (Intercept)  280.68  16.754
##  Residual                      1146.80  33.864
## Number of obs: 1190, groups:  classid:schoolid, 312; schoolid, 107
##
## Fixed effects:
##             Estimate Std. Error t value
## (Intercept)  522.540      2.037   256.6
```

V_C, V_S, and V_E **Question:** For UCM, write down: V_C, V_S, V_E for the three variance components (simply the estimates). Think of them as possibly varying with a covariate, though.

Answer: For the UCM, $V_C = 85.46$, $V_S = 280.68$, and $V_E = 1146.80$

```
summary(model4)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 | schoolid/classid)
##    Data: classroom
##
## REML criterion at convergence: 10729.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8580 -0.6134 -0.0321  0.5971  3.6598
##
## Random effects:
##  Groups           Name        Variance Std.Dev.
##  classid:schoolid (Intercept)   93.89   9.69
##  schoolid         (Intercept)  169.45  13.02
##  Residual                      1064.95  32.63
## Number of obs: 1081, groups:  classid:schoolid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.63042    5.31210  275.40000 101.585  < 2e-16 ***
```

```
## housepov     -17.64847   13.21757  113.90000  -1.335    0.184
## mathknow       1.35004    1.39168  234.50000   0.970    0.333
## yearstea       0.01129    0.14141  226.80000   0.080    0.936
## mathprep      -0.27705    1.37583  205.30000  -0.201    0.841
## sex           -1.21419    2.09483 1022.30000  -0.580    0.562
## minority     -16.18678    3.02605  704.50000  -5.349 1.20e-07 ***
## ses           10.05075    1.54484 1066.50000   6.506 1.18e-10 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.083  0.058
## yearstea -0.259  0.071  0.029
## mathprep -0.631  0.038  0.004 -0.172
## sex      -0.190 -0.007  0.007  0.016 -0.006
## minority -0.320 -0.178  0.115  0.024  0.001 -0.011
## ses      -0.121  0.082 -0.007 -0.028  0.053  0.020  0.162
```

Question: For the most complicated (all fixed effects) random INTERCEPTS ONLY model, what are: V_C, V_S, V_E?

Answer: For the most complicated fixed effects model with only random intercepts, $V_C = 93.89$, $V_S = 169.45$, and $V_E = 1064.95$.

Question: By what fraction did these each decrease with the new predictors in the model?

Answer: V_C increased by $\frac{93.89}{85.46} \sim 1.10$ times. V_S decreased by $\frac{169.45}{280.68} \sim 0.60$ times. V_E decreased by $\frac{1064.95}{1146.80} \sim 0.93$ times.

```
summary(rss.6)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + ((1 | schoolid) + (0 + ses | schoolid)) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10724.8
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6138 -0.6185 -0.0290  0.5798  3.7130
##
## Random effects:
##  Groups     Name        Variance Std.Dev.
##  classid    (Intercept)   88.56   9.411
##  schoolid   ses           72.50   8.515
##  schoolid.1 (Intercept)  167.98  12.961
##  Residual                1035.12  32.173
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.13751    5.27917  270.50000 102.126  < 2e-16 ***
```

```
## housepov     -16.94564   13.21116  112.80000  -1.283    0.202
## mathknow       1.35576    1.38459  232.20000   0.979    0.329
## yearstea       0.03079    0.14052  223.90000   0.219    0.827
## mathprep      -0.19801    1.35994  198.60000  -0.146    0.884
## sex           -1.40185    2.08170 1011.30000  -0.673    0.501
## minority     -16.52525    3.02189  700.10000  -5.469 6.32e-08 ***
## ses            9.78982    1.82217   79.00000   5.373 7.62e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.451
## mathknow -0.079  0.056
## yearstea -0.260  0.070  0.028
## mathprep -0.628  0.041  0.002 -0.172
## sex      -0.190 -0.007  0.006  0.018 -0.007
## minority -0.323 -0.180  0.110  0.024  0.001 -0.010
## ses      -0.091  0.076  0.006 -0.019  0.042  0.017  0.124
```

Question: Now consider the model with a random slope in ses. What are: V_C, $V_S(ses = 0)$, V_E ? We need to list 'ses=0' here, or we don't know how to use the slope variance

Answer: For the model with a random slope in ses at the school level, $V_C = 88.56$, $V_S(ses = 0) = 167.98$, and $V_E = 1035.12$.

Question: What are: $V_S(ses = -0.50)$, $V_S(ses = +0.5)$?

Answer: In this model, in which the random slope for SES is uncorrelated with the random school-level intercept, $V_S(ses = -0.50) = 167.98 + (-.5)^2 72.50 + 2(-.5)0 * \sqrt{167.98} * \sqrt{72.50} = 186.105$, and $V_S(ses = +0.5) = 167.98 + (.5)^2 72.50 + 2 * (.5)0 * \sqrt{167.98} * \sqrt{72.50} = 186.105$

```
summary(rssc.5)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (1 + minority | schoolid) + (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10717.5
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.8952 -0.6358 -0.0345  0.6129  3.6444
##
## Random effects:
##  Groups   Name        Variance Std.Dev. Corr
##  classid  (Intercept)   86.69   9.311
##  schoolid (Intercept)  381.20  19.524
##           minority     343.13  18.524   -0.83
##  Residual             1039.39  32.240
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##              Estimate Std. Error         df t value Pr(>|t|)
```

```
## (Intercept)  5.395e+02  5.655e+00  1.731e+02  95.399  < 2e-16 ***
## housepov    -1.606e+01  1.257e+01  1.000e+02  -1.277    0.204
## mathknow     1.632e+00  1.359e+00  2.248e+02   1.201    0.231
## yearstea    -4.368e-03  1.376e-01  2.172e+02  -0.032    0.975
## mathprep    -2.918e-01  1.335e+00  1.981e+02  -0.218    0.827
## sex         -8.628e-01  2.084e+00  1.022e+03  -0.414    0.679
## minority    -1.638e+01  3.896e+00  5.820e+01  -4.203 9.17e-05 ***
## ses          9.431e+00  1.543e+00  1.063e+03   6.111 1.39e-09 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.394
## mathknow -0.078  0.061
## yearstea -0.253  0.091  0.024
## mathprep -0.576  0.037 -0.002 -0.167
## sex      -0.172 -0.013  0.010  0.014 -0.005
## minority -0.494 -0.157  0.099  0.027 -0.002 -0.014
## ses      -0.105  0.089 -0.005 -0.021  0.052  0.024  0.113
```

Question: Now consider the model with a random slope in minority. What are: V_C, $V_S(minority = 0)$, V_E? We need to list 'minority=0' here, or we don't know how to use the slope variance

Answer: For the model with a random slope in minority at the school level, $V_C = 86.69$, $V_S(minority = 0) = 381.20$, and $V_E = 1039.39$.

Question: What are: $V_S(minority = 0.25)$, $V_S(minority = +0.50)$, $V_S(minority = +0.75)$?

Answer: In this model, in which the random slope for minority is correlated with the random school-level, intercept, $V_S(minority = 0.25) = 381.20 + (0.25)^2 343.13 + 2(0.25)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 252.5549$,

$V_S(minority = +0.50) = 381.20 + (0.50)^2 343.13 + 2(0.50)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 166.801$, and

$V_S(minority = +0.75) = 381.20 + (0.25)^2 343.13 + 2(0.25)(-0.83)\sqrt{381.20} * \sqrt{343.13} = 123.9384$.

```
summary(complex)
```

```
## Linear mixed model fit by REML t-tests use Satterthwaite approximations
##   to degrees of freedom [lmerMod]
## Formula:
## Math1 ~ housepov + mathknow + yearstea + mathprep + sex + minority +
##     ses + (0 + ses | schoolid) + (1 + minority | schoolid) +
##     (1 | classid)
##    Data: classroom
##
## REML criterion at convergence: 10712.4
##
## Scaled residuals:
##     Min      1Q  Median      3Q     Max
## -3.6526 -0.6251 -0.0339  0.6050  3.6961
##
## Random effects:
##  Groups     Name        Variance Std.Dev. Corr
##  classid    (Intercept)   80.63   8.979
##  schoolid   (Intercept)  404.54  20.113
##             minority     336.04  18.332   -0.84
##  schoolid.1 ses           74.93   8.656
```

```
##  Residual               1009.73  31.776
## Number of obs: 1081, groups:  classid, 285; schoolid, 105
##
## Fixed effects:
##               Estimate Std. Error         df t value Pr(>|t|)
## (Intercept)  539.05335    5.66468  165.70000  95.160  < 2e-16 ***
## housepov     -15.32111   12.49443   99.30000  -1.226    0.223
## mathknow       1.67475    1.35000  221.30000   1.241    0.216
## yearstea       0.02102    0.13657  213.70000   0.154    0.878
## mathprep      -0.23546    1.31730  191.20000  -0.179    0.858
## sex           -1.03871    2.06951 1010.30000  -0.502    0.616
## minority     -16.72884    3.90720   55.40000  -4.282 7.43e-05 ***
## ses            9.19654    1.82272   82.50000   5.046 2.65e-06 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Correlation of Fixed Effects:
##          (Intr) houspv mthknw yearst mthprp sex    minrty
## housepov -0.395
## mathknow -0.072  0.060
## yearstea -0.254  0.093  0.024
## mathprep -0.568  0.040 -0.004 -0.166
## sex      -0.170 -0.014  0.010  0.017 -0.005
## minority -0.509 -0.149  0.092  0.027 -0.003 -0.013
## ses      -0.080  0.083  0.006 -0.011  0.041  0.020  0.087
```

Question: Now consider the model with a random slope in ses & minority. What are: V_C, $V_S(minority = 0, ses = 0)$, V_E? We need to list 'ses=0, minority=0' here, or we don't know how to use the slope variance.

Answer: For the model with a random slope in ses & minority, $V_C = 80.63$, $V_S(minority = 0, ses = 0) = 404.54$, and $V_E = 1009.73$.

Question: What are: $V_S(ses = 0, minority = 0.50)$, $V_S(ses = 0.50, minority = 0)$, $V_S(ses = 0.50, minority = 0.50)$?

Answer: In this model, in which the random slope for ses is uncorrelated with the random intercept, but the random slope for minority is correlated with the random intercept,

$V_S(ses = 0, minority = 0.50) = 404.54 + (0)^2 * 74.93 + (0.50)^2 * 336.04 + 2 * 404.54 * 74.93 + 2 * (0.50)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 182.5268$,

$V_S(ses = 0.50, minority = 0) = 404.54 + (0.50)^2 * 74.93 + (0)^2 * 336.04 + 2 * 0.5 * 404.54 * 74.93 + 2 * (0)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 423.2725$

$V_S(ses = 0.50, minority = 0.50) = 404.54 + (0.50)^2 * 74.93 + (0.50)^2 * 336.04 + 2 * 0.5 * 404.54 * 74.93 + 2 * (0.50)(-0.83)\sqrt{404.54} * \sqrt{336.04} = 201.2593$

Question: In the last model, what is a "likely" (+/- 1 sd) range for η_{0jk}

Answer: For the complex model, the "likely" range for η_{0jk} is $(-8.979, 8.979)$.

Question: Can we make a similar statement about ζ_{0k}?

Answer: We cannot make a similar statement for ζ_{0k} since it is correlated with ζ_{2k} on *Minority*.

Question: If you had a large value for η_{0jk}, would you expect a large or small or "any" value for: the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority?

Answer: There is no correlation between η_{0jk} (classroom-level intercept) and the school-level random slopes ζ_{1k} and ζ_{2k} on *SES* and *MINORITY*. Therefore, we would not expect a large value of η_{0jk} to have any effect

on the two random slope terms as they are independent.

Question: If you had a large value for ζ_{0k}, would you expect a large or small or "any" value for: the two random slope terms, ζ_{1k} and ζ_{2k} for ses and minority (discuss each separately)?

Answer: ζ_{1k} could be any value due to the lack of correlation with ζ_{0k}

Answer: While ζ_{2k} would be small given a large value of ζ_{0k} because of the negative correlation between the two variables.