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AREC-615-001: Optimization Methods for Applied Economics

Final Report and Code

Applied Modeling Project

Project

Replicating and extending Chen, C., Kang, C., & Wang, J. (2018). Stochastic linear programming for reservoir operation with constraints on reliability and vulnerability. Water, 10(2), 175, by adding seasonal water release limits, which mimic drought water conservation measures.

Introduction

Hydropower remains the largest source of renewable electricity globally. It accounts for about 40% of renewable electricity generation (Uria Martinez & Johnson, 2023). Water held in hydropower reservoirs can be released to match the electricity needs of society; however, the fundamental challenge is that water inflows to the reservoir are uncertain. This is because rainfall and snowmelt are unpredictable, so reservoir managers must decide on how much electricity to generate today while considering future demands. This raises the economic tradeoff of how releasing water now generates revenue from electricity sales, but depleting the reservoir creates future shortages.

Original Model and Paper Overview

Chen et al. (2018) investigated how to operate hydropower reservoirs (specifically, Xiaowan and Nuozhadu reservoirs) in Yunnan Province, China, to maximize expected hydroelectric energy generation under certain inflows while also ensuring reliability and minimizing vulnerability to failures. This is a fundamental challenge in water resource management. The authors account for the future uncertainty in inflows and storage availability, which past studies did not consider. They incorporate randomness (stochastic information) into their model. They develop a stochastic linear programming (SLP) model, which, unlike traditional stochastic dynamic programming (SDP), explicitly incorporates reliability and vulnerability constraints.

To describe their model simply to a non-economist, a reservoir receives uncertain inflows of water, which include rain and snowmelt. The manager then decides how much to release in the form of hydropower generation. The manager's goal is to maximize expected hydropower generation subject to reservoir water balance, physical limits (storage range), reliability, vulnerability, and Markov inflow uncertainty (seasonal inflows using a Markov chain). The Markov chain-based inflow model captures stochastic behavior, meaning that the inflow at any time step depends only on the inflow from the previous time step, not on the earlier history. This was to ensure that they account for the serial correlation in the hydrologic data.

The objective mathematically is to maximize,

$$\max Z = \sum_{i=1}^{T} \sum_{(i,k,l)\in\Omega(t)} \left(P_{iklt} * G_{iklt} * \Delta t\right)$$

Where Z represents expected hydropower generation, T is the number of the time period; Δt is the time length in hours of the time period t; i is the index representing how full the reservoir is at the start of the time period t; k is the index that tells how much water is flowing into the reservoir during the time period t; l is the index that tells us how much the reservoir will be at the end of the time period t; P_{iklt} is the probability of being at state (i, k) and making a decision (l); G_{iklt} is the amount of electricity generated (in gigawatt-hours, GWh) during the time period t.

G_{iklt}= $G[\, S_t^i, Q_t^i, S_{t+1}^i]$ where S_t^i is the storage level at the beginning of time period t ; Q_t^i is the inflow during time period t; S_{t+1}^i is the storage level at the end of time period t.

The manager maximizes expected hydropower subject to the constraints;

i. $\sum_{(i,k,l)\in\Omega(t)}(P_{iklt}) = 1\ (\forall\, t)$; that is the joint probabilities for any time period that sum up to 1.

ii. $\sum_{(i,k,l)\in\Omega(t)} P_{iklt} * P_{kj}^t = \sum_{(m\in\Omega(t+1)} P_{i,j,m,t+1}\ for\ (i,j) \in (t+1)$; the probability of ending up in a certain in the next period $(t + 1)$ must match the sum of all possible ways to get there from the current time period t weighted by the likelihood of each inflow transition. $P_{kj}^t\ is$ the probability of inflow transitioning from state k at time t to state j at time $t + 1$. $P_{i,j,m,t+1}$ is the probability of being in state (i, j) and choosing decision m at time $t + 1$.

iii. $\sum_{l\in\Omega(t)}(u_{iklt}) = 1\ \ for\ (i,k) \in \Omega\,(t)$ and $P_{iklt} \leq c\ for\ \forall\ (i,k,l) \in \Omega(t)$ where u_{iklt} is a binary variable that decides whether or not the decision l is made at state (i, k)in time period

t. The constraint ensures that there is a clear operational policy where we have one decision per state.

iv. $\frac{1}{T}\sum_{i=1}^{T} \sum_{(i,k,l)\in\emptyset(t)} P_{iklt} \geq \beta$; the reliability constraint. The average probability of meeting the power generation target across all time periods must be at least a specified threshold β.

v. $\frac{1}{T}\sum_{i=1}^{T} \sum_{(i,k,l)\in\emptyset(t)} P_{iklt} \, (Y - G_{iklt}) \leq v$; the vulnerability constraint. v is the maximum allowed vulnerability and Y is the target power generation or yield. The constraint forces the model to limit the severity of failures when they occur.

vi. $P_{iklt} \geq 0$ and $u_{iklt} \in \{0, 1\}$; non-negativity constraints.

The decision variables for this model are u_{iklt} and P_{iklt}. u_{iklt} helps determine whether we release water or not, given a specific amount of water and the inflow into the reservoir at time t. P_{iklt} tells us the probability of being at this storage level given the inflow.

The economic intuition behind the problem is that water is scarce; using it now generates current revenue, storing it preserves the option to generate power in the future.

Chen et al. (2018) present Table 1, which compares how much electricity the reservoirs use in three different ways; the energy predicted directly by SLP optimization model (SLP_mod), the energy generated when the SLP policy is tested in a simulation (with inflow randomness) (SLP_sim), and the energy from a benchmark method (stochastic dynamic programming) and also tested via simulation (SDP_sim).

Table 1. Annual energy production in GWh by the SLP model (SLPmod), and by simulation for the SLP (SLPsim) and SDP (SDPsim) without RV constraints.

Calculation Ways	Xiaowan His.	Xiaowan Sim.	Nuozhadu His.
SLP_{mod}	23,504.5	24,389.7	26,619.8
SLP_{sim}	23,568.3	24,391.4	26,684.2
SDP_{sim}	23,568.6	24,390.5	26,684.1

The SLP model from simulation, including randomness, is closely aligned with the traditional SDP results. This demonstrates that the SLP model performs equally well as the SDP model, while being computationally simpler.

I replicate the work of Chen et al. (2018) using a suitable setup that matches my standard desktop computational capabilities. The original paper utilizes IBM's CPLEX solver on a full scale with 110,000 decision variables, whereas I use the R programming language's lpSolve package. Table 2 illustrates the differences between my replication and the original paper.

Table 2. Specification Comparisons

Parameter	Chen et al. (2018)	My replication
Storage states (N_S)	21	5
Inflow states (N_Q)	7	3
Time periods (T)	36	4 (quarterly)
Total decision variables	110,000	300
Inflow data	56 years historical +1,449 simulated AR (2)	10 years synthetic normal
Solver	IBM CPLEX	R lpSolve

In the original paper, the SLP model predicted annual energy production to be 23,568 GWh/year using historical data, while the simulated energy output under the same policy and randomness included is 24,390.5 GWh/year. In my simplified replication, the expected annual generation from the SLP model, which includes randomness, is 23,010 GWh/year. My results, although close, differ slightly from those of Chen et al. (2018) because my replication uses a smaller state space and synthetic inflow data, rather than multi-decade historical observations. Despite the simplifications, the model behaves as intended by the authors.

Table 3. Replication results

Model Output	**Value (GWh/year)**
Chen et al. SLP_sim (historical)	23,568.3
Chen et al. (2018) SLP_mod	23,504.5
Chen et al. (2018) SDP_sim	23,568.6
My SLP_sim (synthetic data)	23,010

Table 2 of Chen et al. (2018) compares the expected power generation, reliability, and vulnerability of the SSLP policy across time periods, reporting an average reliability of 0.98 and a vulnerability of 0.01 GWh. This indicates that their model meets its reliability target with only small shortfalls. My results indicate that the reliability and vulnerability constraints were satisfied. Reliability equaled 1 and vulnerability equaled 0, indicating that under synthetic inflow conditions, my simplification of the reservoir consistently met the minimum energy target. That is, water is allocated efficiently over time to balance current power generation with future reliability.

Table 4. Comparison of Average Reliability and Vulnerability Results

Metric	**Chen et al. Average**	**My Replication**
Reliability	0.98	1.00
Vulnerability	0.01	0.00

In Table 3, Chen et al. (2018) examine how hydropower generation changes under five different combinations of reliability parameter (β), vulnerability tolerance *(v),* and target yield (*Y)*. Their results show that as the reliability requirement becomes stricter or higher and vulnerability tolerance decreases, annual energy generation declines slightly. This means that if we need to guarantee electricity generation more frequently, we must be cautious with water releases, resulting in less overall power production. To mirror this experiment, I evaluated my model under five scenarios using synthetic inflow data. My results did not show declines but rather remained stable due to the limitations in my data.

Table 5. Annual power generation (GWh) by the SLP model itself and by operation simulation with different parameters. (Chen et al. (2018))

Table 3. Annual power generation (GWh) by the SLP model itself and by operation simulation with different parameters.

No.	Combinations (Y, β, v)	Model	Simulation
1	(1.778, 0.98, 0.01)	24,119	24,112
2	(1.778, 0.98, 0)	24,169	24,167
3	(1.778, 0, 0.01)	24,196	24,183
4	(2.5, 0.7, 0.3)	24,076	24,061
5	(2.5, 0.6, 0.3)	24,258	24,247

Table 6. Annual power generation under different scenarios (Replication)

No.	(Y, β, v)	Model	Simulation
1	(1.5, 0.98, 0.01)	23,010	23,010
2	(1.5, 0.98, 0.00)	23,010	23,010
3	(1.5, 0.00, 0.01)	23,010	23,010
4	(2.0, 0.70, 0.30)	23,010	23,010
5	(2.0, 0.60, 0.30)	23,010	23,010

Overall, the model suggests that operating a reservoir is about striking a balance. That is, generating power today while saving enough water to stay reliable in the future. When inflows are uncertain, using too much water now can lead to shortages later, so the model selects release rules that protect future supply. Reliability and vulnerability constraints ensure that electricity remains dependable even during periods of drought.

Extension

I extend the model by adding seasonal release limits, which function similarly to drought management rules. Water managers often restrict releases in dry months to prepare for potential shortages. During droughts, regulators often impose maximum release limits to preserve storage for essential uses, minimum release requirements to maintain downstream ecosystems, and seasonal windows when releases are restricted. By incorporating seasonal limits, I examine how policy rules that conserve water during low-inflow seasons impact reliability and power generation under uncertainty.

Model Setup

Before the extension, I broke down annual generation by quarter (Table 7). The results show that summer generates only 17% of the water due to low natural inflows and high evaporation, making the summer a natural target for drought restrictions. This is because constraining an already low generation period imposes lower opportunity costs than restricting high generation periods.

Table 7. Quarterly Generation Breakdown

Quarter	Generation (GWh)	Share of Annual	Interpretation
Q1 (Winter)	8,298	36.10%	High generation (snowmelt, winter rains)
Q2 (Spring)	5,036	21.90%	Moderate generation (spring runoff)
Q3 (Summer)	4,012	17.40%	Low generation (lowest inflows)
Q4 (Fall)	5,664	24.60%	Moderate-high generation (fall storms)
Total	**23,010**	**100%**	-

My model divides the year into four time periods: Q1(winter, January to March), Q2 (spring, April to June), Q3 (summer, July to September), and Q4 (fall, October to December). I focus on drought restrictions in the summer (Q3) when demand is the highest and natural flows are at their lowest due to precipitation and snowmelt. For each restricted period, $t \in T$ restricted, I impose a drought release constraint;

$$R_{t\,(i,k,l)} \leq R_t^{policy}\ \forall\,(i,k,l) \in \Omega(\mathrm{t})$$

Where $R_{t\,(i,k,l)}$ is the water released in period *t* (m^3/s) and R_t^{policy} is the maximum allowable release for period *t*.

From the basic storage balance,

$$S_{t+1} = S_t + (Q_t - R_t)\Delta t$$

Rearranging,

$$R_t = \frac{S_t - S_{t+1}}{\Delta t} + Q_t$$

Table 8. Drought Extension Results

Scenario	**Q3 Limit (m^3/s)**	**Annual Generation (GWh)**	**Loss (GWh)**	**Loss (%)**	**Q3 Change (GWh)**	**Q4 Change (GWh)**	**Cost ($M/year)**
Baseline	No limit	**23,010**	0	**0%**	0	0	$0
Moderate	1,704	22,297	-713	**-3.10%**	-713	0	$43
Severe	1,515	21,069	-1,941	**-8.40%**	-1,941	0	$116
Extreme	1,325	21,069	-1,941	**-8.40%**	-1,941	0	$116

Table 9. Shadow Price of Drought Constraint

Scenario	Q3 Limit (m^3/s)	Shadow Price (GWh)
Moderate	1,704	93.0
Severe	1,515	461.1
Extreme	1,325	461.1

Analysis of Results

I evaluate three seasonal policies: moderate (1,704 m^3/s i.e., 90% of baseline maximum), severe (1,515 m^3/s i.e.,80% of baseline maximum) and extreme (1,325 m^3/s i.e., 70% of baseline maximum).

The shadow price in this analysis tells us the marginal value of relaxing the drought constraint (**Table 9**). Under a moderate restriction policy, relaxing the summer release limit would increase total annual generation by approximately 93 GWh (0.4% of the total yearly generation). For the severe policy, this increases the total annual generation by about fivefold, that is, approximately 461 GWh (2% of yearly generation). The identical shadow prices for severe and extreme policies confirm that beyond the severe threshold, additional restrictions provide no incremental benefit. The fivefold increase in shadow price reflects the economic burden of restricting water releases during the summer, when natural inflows are already at their lowest and electricity demand peaks due to the use of air conditioning, refrigeration, and cooling systems.

I find that severe restrictions result in losses of 1,941 GWh; even moderate restrictions cost 713 GWh annually, representing a 3.1% reduction in total output (**Table 8**). This demonstrates that seasonal release constraints create real binding constraints. The model is forced to abandon high-release operational policies that would be optimal without restrictions, because the drought constraints are calibrated below baseline operations (i.e., 70 to 90% of the maximum Q3 release). The $43 million to $116 million annual costs represent the opportunity costs of water conservation. That is the forgone hydropower revenue sacrificed to preserve storage and maintain downstream flows during drought conditions. Q4 inflow states are sufficient that additional stored water provides no additional stored benefit because the reservoir is at maximum capacity. If the reservoir is near full capacity in Q3, additional conservation cannot increase Q4 storage.

In conclusion, I successfully replicated the SLP model of Chen et. al. (2018) using a simplified specification and simulated data. Adding the seasonal release constraint imposed real economic costs, which demonstrated diminishing returns to policy stringency. Policymakers designing seasonal management rules should recognize that stricter limits may sacrifice hydropower revenue without preserving additional water; therefore, they must find the optimal balance where they can generate revenue while still conserving water. Future studies could extend this analysis and incorporate climate data to capture drought dynamics more realistically.

References

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