> # MANUFACTURED SOLUTION :

> restart :

> $Vr := (x, y) \rightarrow \mathrm{sqrt}\left(\frac{2 \cdot varK}{Nmod}\right) \cdot sum(\cos(\mathrm{phi}[j] + (K[j, 1] \cdot x + K[j, 2] \cdot y) \cdot (2 \cdot \mathrm{pi})), j = 1 ..Nmod);$

$$Vr := (x, y) \mapsto \sqrt{\frac{2\ varK}{Nmod}} \left(\sum_{j=1}^{Nmod} \cos\left(\phi_j + 2\left(K_{j,1} x + K_{j,2} y\right) \pi\right)\right) \tag{1}$$

> $K := (x, y) \rightarrow KMean \cdot \exp\left(-\frac{varK}{2}\right) \cdot \exp(Vr(x, y));$

$$K := (x, y) \mapsto KMean\, \mathrm{e}^{-\frac{varK}{2}}\, \mathrm{e}^{Vr(x, y)} \tag{2}$$

> $K(x, y)$

$$KMean\, \mathrm{e}^{-\frac{varK}{2}}\, \mathrm{e}^{\sqrt{2} \sqrt{\frac{varK}{Nmod}} \left(\sum_{j=1}^{Nmod} \cos\left(\phi_j + 2\left(K_{j,1} x + K_{j,2} y\right) \pi\right)\right)} \tag{3}$$

>

> $u := (x, y) \rightarrow 1 + \sin(2 \cdot x + y);$

$$u := (x, y) \mapsto 1 + \sin(2 x + y) \tag{4}$$

> $eq := diff(K(x, y) \cdot diff(u(x, y), x), x) + diff(K(x, y) \cdot diff(u(x, y), y), y);$

$$\begin{aligned} eq := {} & 2\, KMean\, \mathrm{e}^{-\frac{varK}{2}} \sqrt{2} \sqrt{\frac{varK}{Nmod}} \left(\sum_{j=1}^{Nmod} -2 \pi K_{j,1} \sin\left(\phi_j + 2\left(K_{j,1} x + K_{j,2} y\right) \pi\right)\right) \mathrm{e}^{\sqrt{2} \sqrt{\frac{varK}{Nmod}} \left(\sum_{j=1}^{Nmod} \cos\left(\phi_j + 2\left(K_{j,1} x + K_{j,2} y\right) \pi\right)\right)} \cos(2 x + y) \\ & - 5\, KMean\, \mathrm{e}^{-\frac{varK}{2}}\, \mathrm{e}^{\sqrt{2} \sqrt{\frac{varK}{Nmod}} \left(\sum_{j=1}^{Nmod} \cos\left(\phi_j + 2\left(K_{j,1} x + K_{j,2} y\right) \pi\right)\right)} \sin(2 x + y) \\ & + KMean\, \mathrm{e}^{-\frac{varK}{2}} \sqrt{2} \sqrt{\frac{varK}{Nmod}} \left(\sum_{j=1}^{Nmod} -2 \pi K_{j,2} \sin\left(\phi_j + 2\left(K_{j,1} x + K_{j,2} y\right) \pi\right)\right) \mathrm{e}^{\sqrt{2} \sqrt{\frac{varK}{Nmod}} \left(\sum_{j=1}^{Nmod} \cos\left(\phi_j + 2\left(K_{j,1} x + K_{j,2} y\right) \pi\right)\right)} \cos(2 x + y) \end{aligned} \tag{5}$$

> $simplify(\%)$

$$\begin{aligned} -4 \Bigg(& \left(\sum_{j=1}^{Nmod} K_{j,1} \sin\left(2 \pi x K_{j,1} + 2 \pi y K_{j,2} + \phi_j\right)\right) \cos(2 x + y) \sqrt{2} \sqrt{\frac{varK}{Nmod}}\, \pi \\ & + \frac{\cos(2 x + y) \left(\sum_{j=1}^{Nmod} K_{j,2} \sin\left(2 \pi x K_{j,1} + 2 \pi y K_{j,2} + \phi_j\right)\right) \sqrt{2} \sqrt{\frac{varK}{Nmod}}\, \pi}{2} \end{aligned} \tag{6}$$

$$+\frac{5\sin(2x+y)}{4}\Bigg)\, \mathrm{e}^{-\frac{varK}{2}+\sqrt{2}\sqrt{\frac{varK}{Nmod}}\left(\sum_{j=1}^{Nmod}\cos\left(2\pi x K_{j,1}+2\pi y K_{j,2}+\phi_j\right)\right)} KMean$$

> $f := (x, y) \rightarrow eq;$

$$f := (x,y) \mapsto eq \tag{7}$$

> $f(x, y)$

$$2\, KMean\, \mathrm{e}^{-\frac{varK}{2}} \sqrt{2}\sqrt{\frac{varK}{Nmod}}\left(\sum_{j=1}^{Nmod} -2\,\pi K_{j,1}\sin\left(\phi_j+2\left(K_{j,1}x+yK_{j,2}\right)\pi\right)\right) \mathrm{e}^{\sqrt{2}\sqrt{\frac{varK}{Nmod}}\left(\sum_{j=1}^{Nmod}\cos\left(\phi_j+2\left(K_{j,1}x+yK_{j,2}\right)\pi\right)\right)} \cos(2x+y)$$
$$-5\, KMean\, \mathrm{e}^{-\frac{varK}{2}}\, \mathrm{e}^{\sqrt{2}\sqrt{\frac{varK}{Nmod}}\left(\sum_{j=1}^{Nmod}\cos\left(\phi_j+2\left(K_{j,1}x+yK_{j,2}\right)\pi\right)\right)} \sin(2x+y)$$
$$+KMean\, \mathrm{e}^{-\frac{varK}{2}} \sqrt{2}\sqrt{\frac{varK}{Nmod}}\left(\sum_{j=1}^{Nmod} -2\,\pi K_{j,2}\sin\left(\phi_j+2\left(K_{j,1}x+yK_{j,2}\right)\pi\right)\right) \mathrm{e}^{\sqrt{2}\sqrt{\frac{varK}{Nmod}}\left(\sum_{j=1}^{Nmod}\cos\left(\phi_j+2\left(K_{j,1}x+yK_{j,2}\right)\pi\right)\right)} \cos(2x+y) \tag{8}$$

> # *BOUNDARY CONDITIONS :*

> # *x from a to b ; y from c to d*

> $u(a, y) = 1 + \sin(2 \cdot a + y)$; # *Dirichlet 1*

$$1+\sin(2a+y) = 1+\sin(2a+y) \tag{9}$$

> $u(b, y) = 1 + \sin(2 \cdot b + y)$; # *Dirichlet 2*

$$1+\sin(2b+y) = 1+\sin(2b+y) \tag{10}$$

> $diff(u(x, y), y);$

$$\cos(2x+y) \tag{11}$$

> $N1 = \cos(2 \cdot x + c)$; # *Neumman 1*

$$N1 = \cos(2x+c) \tag{12}$$

> $N2 = \cos(2 \cdot x + d)$; # *Neumman 2*

$$N2 = \cos(2x+d) \tag{13}$$

> # *The derivative of K :*

> $diff(K(x, y), x);$

$$KMean\, \mathrm{e}^{-\frac{varK}{2}} \sqrt{2}\sqrt{\frac{varK}{Nmod}}\left(\sum_{j=1}^{Nmod} -2\,\pi K_{j,1}\sin\left(\phi_j+2\left(K_{j,1}x+yK_{j,2}\right)\pi\right)\right) \mathrm{e}^{\sqrt{2}\sqrt{\frac{varK}{Nmod}}\left(\sum_{j=1}^{Nmod}\cos\left(\phi_j+2\left(K_{j,1}x+yK_{j,2}\right)\pi\right)\right)} \tag{14}$$

> $diff(K(x, y), y);$

$$KMean\, e^{-\frac{varK}{2}} \sqrt{2} \sqrt{\frac{varK}{Nmod}} \left(\sum_{j=1}^{Nmod} -2\, \pi\, K_{j,\,2} \sin\left(\phi_j + 2\, \left(K_{j,\,1}\, x + y\, K_{j,\,2}\right) \pi\right) \right) e^{\sqrt{2} \sqrt{\frac{varK}{Nmod}} \left(\sum_{j=1}^{Nmod} \cos\left(\phi_j + 2\, \left(K_{j,\,1}\, x + y\, K_{j,\,2}\right) \pi\right) \right)} \tag{15}$$