Crank-Nicolson :

$$\begin{cases} u_{n+1} = u_n + \dfrac{h}{2}\left(v_n + v_{n+1}\right) \\ v_{n+1} = \left(1-h\right) v_n - h v_{n+1} - \dfrac{h}{2}\left(\nabla E\left(u_n\right) + \nabla E\left(u_{n+1}\right)\right) \end{cases} . \tag{1}$$

Heun :

$$\begin{cases} \tilde{u}_{n+1} = u_n + h v_n \\ \tilde{v}_{n+1} = (1-2h) v_n - h \nabla E(u_n) \\ u_{n+1} = u_n + \dfrac{h}{2}\left(v_n + \tilde{v}_{n+1}\right) \\ v_{n+1} = (1-h) v_n - h \tilde{v}_{n+1} - \dfrac{h}{2}\left(\nabla E\left(u_n\right) + \nabla E\left(\tilde{u}_{n+1}\right)\right) \end{cases} . \tag{2}$$

Polyak :

$$\begin{cases} u_{n+1} = u_n + h v_n \\ v_{n+1} = (1-2h) v_n - h \nabla E(u_{n+1}) \end{cases} . \tag{3}$$

Dynamical System for the splitting-based algorithms

$$\left\{\begin{array}{rcl}\dot{u} & = & v,\\ \dot{v} & = & -2v-\nabla E\left(u\right),\end{array}\right.$$

Two subproblems, one being linear, dissipative and explicitly solvable while the second is nonlinear and conservative, namely

$$\left\{\begin{array}{rcl}\dot{u} & = & 0,\\ \dot{v} & = & -2v,\end{array}\right. \tag{4}$$

and

$$\left\{\begin{array}{rcl}\dot{u} & = & v,\\ \dot{v} & = & -\nabla E\left(u\right),\end{array}\right. \tag{5}$$

Strang Splitting :

$$\left\{\begin{array}{rcl}u_{n+1} & = & u_n+\frac{h}{2}(cv_{n+1}+c^{-1}v_n)\\ cv_{n+1} & = & c^{-1}v_n-h\int\limits_0^1\nabla E_n^{n+1}\end{array}\right. \tag{6}$$

where $\int\limits_0^1\nabla E_n^{n+1}:=\int\limits_0^1\nabla E((1-\alpha)u_n+\alpha u_{n+1})d\alpha$

Strang Splitting - Predictor Corrector :

$$\begin{cases}u_{n+1}=u_n+\dfrac{h}{c}v_n-\dfrac{h^2}{2}\nabla E\left(u_n+\dfrac{h}{c}v_n\right)\\ v_{n+1}=\dfrac{1}{c^2}v_n-\dfrac{h}{c}\displaystyle\int_0^1\nabla E\left(u_n+\alpha\dfrac{h}{c}\right)d\alpha\end{cases} \tag{7}$$

Now, we present the basic idea behind the new algorithm (7). For the conservative subproblem (5), we have the following predictor-corrector scheme :

$$
\begin{cases}
\tilde{u}_{n+1} = u_n + hv_n \\
\tilde{v}_{n+1} = v_n - h\nabla E(\tilde{u}_{n+1}) \\
u_{n+1} = u_n + \dfrac{h}{2}(v_n + \tilde{v}_{n+1}) \\
v_{n+1} = v_n - h\displaystyle\int_0^1 \nabla E\left(\left(1-\alpha\right)u_n + \alpha\tilde{u}_{n+1}\right)d\alpha
\end{cases}
. \tag{8}
$$

Furthermore, we have the AVF predictor corrector :

$$
\begin{cases}
u_{n+1} = u_n + hv_n - \dfrac{h^2}{2}\nabla E(u_n + hv_n) \\
v_{n+1} = v_n - h\displaystyle\int_0^1 \nabla E\left(u_n + \alpha v_n\right)d\alpha
\end{cases}
. \tag{9}
$$