B.Eng. Thesis
Bachelor of Engineering

DTU Compute
Department of Applied Mathematics and Computer Science

Supply Chains and Predictability in Financial Markets

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DTU 2018

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Summary

This thesis investigates the possibility to predict an industry's monthly returns by using other industries. The data sets used contains different industries monthly returns. The analysis is made by using a VAR(1) model to explore the linear inter-dependencies of stock market returns. The predictors of the model will be restricted, such that only statistically significant variables will be present. This will be done for small windows of the entire time series. This makes it possible to see how significance of lagged monthly industry returns will vary over time. The most promising models will be chosen by accuracy and skill score tests. Finally the last models selected will be the ones to conclude if any predictability between the industries can be found.

The conclusion of this investigation is that there are predictability between industries. Some industries predictive power will vary over time, while other will remain fairly consistent at being statistically significant for certain industries.

Preface

This bachelor thesis was prepared at the department of Applied Mathematics and Computer Science at the Technical University of Denmark in fulfilment of the requirements for acquiring a bachelor degree in Mathematics and Technology.

DTU, June 20, 2018

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Acknowledgements

We would like to offer our special thanks to our project supervisor Lasse Engbo Christiansen for his valuable and constructive feedback, his never ending patience and cheerful encouragement during the entire development of this thesis. He never failed to inspire us with interesting topics, helpful discussions and by broadening our understandings within our field of study.

Contents

CHAPTER 1

Introduction

The predictability of stock market returns has been a researched topic in finance for a long time. Both contributions in empirical and theoretical financial research have tried to explain predictability across markets. This has led to an emerging consensus among economists, that stock market returns do contain an important predictable component [1], [2]. Predictability of returns is therefore a hot topic within the financial world.

One of the reasons predictability of market returns is of interest, is that it will assist practitioners to develop portfolio strategies to enhance profits [3]. This has driven a strong desire to investigate predictability in market returns and have been the base for several papers. The techniques used to investigate predictability varies from using deep learning [4], [5], machine learning [6], [7], and time series analysis [8], [9], to simplistic bets that a markets current movement will continue in the same direction as its previous movement [10]. Several different predictors have shown to have an amount of predictive power. Predictors such as market returns, dividend yield, price/earning ratios, interest rates and even sunshine has been shown to be significantly correlated with market returns [11].

In this thesis the focus will be on market returns, where it is investigated if they can be used to predict other markets. The idea is that different markets acts as supply chains to each other. This connection will be exploited and the problem will be approached statistically using time series analysis. An attempt of identifying statistically significant variables to predict one industry's returns, by the use of other industries, will be made.

To approach the problem a vector autoregressive (VAR) model will be used, as presented in the original paper by T. Bjerring [12]. This will allow linear interdependencies among multiple different time series to be captured. In contrast to Bjerring's work, of looking at the entire time series, this thesis will instead look at the significance of each predictor in different sub-periods. This is to find significant predictors across multiple periods and will show how the predictors varies over time. To address the problem five different data sets will be used. Four of them contain monthly returns from different industry sectors. The fifth one contains well known market predictors and are included in this thesis as an attempt to find alternative explanations for why some industries might predict other industries returns. The goal is to identify commonly used predictors for each of the industries and evaluate the created models. The structure, of the remainder of the paper, will be as follows:

- *Chapter 2:* Introduction to the used data sets; the pre-processing and important choices made for them.
- *Chapter 3:* The theory behind and the methods used to understand the results.
- *Chapter 4:* Presentation of the found results.
- *Chapter 5:* Discussion and future work.
- *Chapter 6:* A conclusion and wrap up of the results.

CHAPTER 2
Descriptive Statistics

The focus of this chapter will be on getting to know the data. Five data sets are used and therefore this section is divided into several sections focusing on the different data sets. First an investigating of four different industry portfolios are made, containing monthly returns provided by Kenneth R. French's website [13]. Lastly an investigation is made of a data set containing well known market predictors provided by Amit Goyal's website [14].

2.1 Industry Portfolios

The four different data sets that will be used contains 5, 10, 17 and 48 industry portfolios. Each portfolio are derived from the same amount of data, but are divided into a different amount of variables. Hence 5, 10, 17 and 48 variables. This makes the sets resemble each other and means that the industry portfolios containing more variables will be more specific. The sets are made by assigning the stocks from NYSE, AMEX and NASDAQ to an industry portfolio, based on what industry area the stock is within [13]. Hence it should be kept in mind that the results found in this thesis are for American stock markets.

2.1.1 5 Industry Portfolios

The first data set to be investigated is the 5 Industry Portfolio containing monthly returns from 5 industries. The data are structured as follows:

- `Date`: Date for observation.
- `Cnsmr`, *Consumer*: Consumer Durables, NonDurables, Wholesale, Retail, and Some Services (Laundries, Repair Shops).
- `Manuf`, *Manufacturing*: Manufacturing, Energy, and Utilities.
- `HiTec`, *High Tech*: Business Equipment, Telephone and Television Transmission.
- `Hlth`, *Health*: Healthcare, Medical Equipment, and Drugs.

- `Other`: Mines, Construction, Transportation, Hotels, Bus Service, Entertainment, Finance.

The above groups are sorted by using the Standard Industrial Classification (SIC) codes [15]. See [13] for data references.

The portfolio is preprocessed by looking for missing data and no such are found. To get an overview of the data, the time series can be seen below:

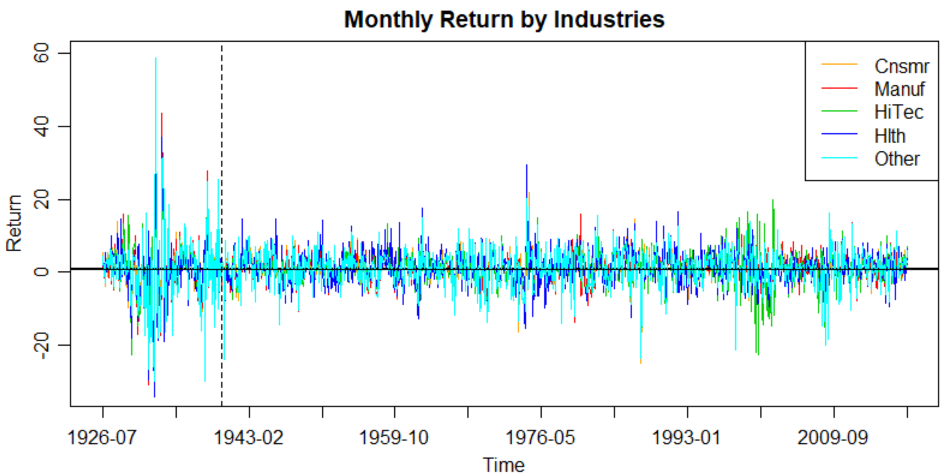


Figure 2.1: The 5 industries monthly market returns from 1926-07 to 2018-01. The dashed line indicates the cutoff of the data to be removed, the period 1929-1939.

From Figure 2.1 it is seen that the data has a greater variance around the first years compared to the rest of the data. After further investigation this can partly be explained by the great depression (1929-1939) [16]. To get more stable data, it is decided that all the data before January 1940 are to be removed which is marked by the dashed line in the figure.

Figure 2.1 shows that even when removing the first part of the data, there will still be periods of increased variance. These disturbances could be from events of turmoil such as 9/11 (2001) [17], the finance crisis (2008) [18] and Black Monday (1987) [19]. The time series seems to be stationary for most of its entirety, but there are subperiods of heteroscedastic variance. This is however something just kept in mind for now and no transformations are done.

With the data removed, leaving data from January 1940 to January 2018, the summary statistics is presented below:

Table 2.1: A summary statistics of the 5 Industry Portfolio for data between 1940-01 to 2018-01.

	Date	Cnsmr	Manuf	HiTec	Hlth	Other
Min.	1940-01	-25.020	-22.4500	-22.6200	-21.21	-24.010
1st Qu.	1959-07	-1.470	-1.5200	-1.6300	-1.84	-1.820
Median	1979-01	1.230	1.2700	1.2100	1.10	1.380
Mean	1978-12	1.033	0.9964	0.9667	1.12	0.999
3rd Qu.	1998-07	3.720	3.7300	3.9200	3.99	3.910
Max.	2018-01	21.740	17.2800	19.9800	29.52	20.220

In Table 2.1 it is seen that the different industries contain values within the same range and with mean values close to each other, hence no normalisation is needed. Further on by comparing this table with the time series figure, Figure 2.1, it is clear by the table that all the industries peaks high and low at some point. It is therefore expected that the data set contains outliers.

From Figure 2.1 it can further on be seen that the data seems correlated. This is investigated by observing a correlation figure seen below:

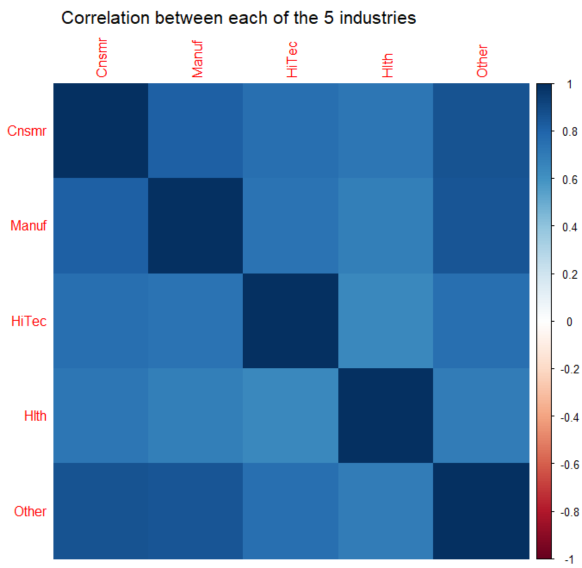


Figure 2.2: Correlation between the 5 industries monthly returns.

It is clear that the industries are highly positive correlated. This means that e.g. when the return of one industry increases, an increase in other industries is often to be expected.

Finally it is investigated if the data is normally distributed by looking at Q-Q-plots:

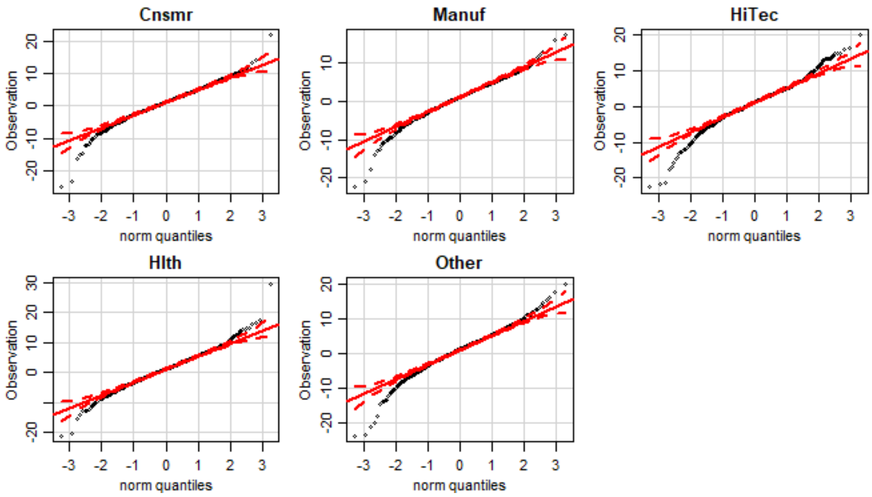


Figure 2.3: Quantile-Quantile plot of the observations for each industry.

It is clear that the data does not seem to be normally distributed[1]. Heavy tails are seen, which is due to the outliers from the volatile periods as was seen in the time series figure, Figure 2.1.

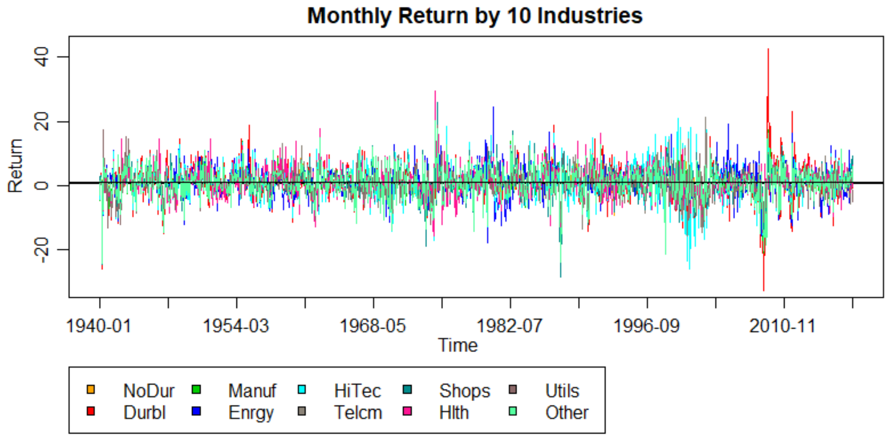
2.1.2 10, 17 & 48 Industry Portfolios

The same analysis, as made in section 2.1.1 for the 5 Industry Portfolio, are made for the data sets containing 10, 17 and 48 Industry Portfolios. The data structures can be found on Kenneth R. French's website [13].

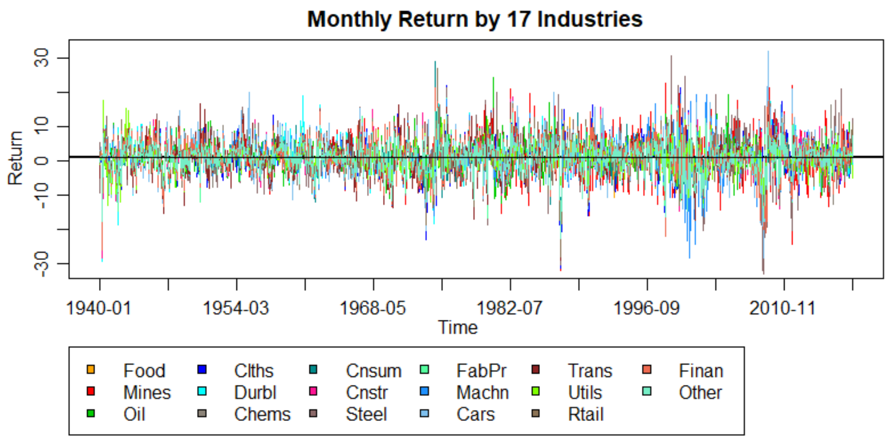
The portfolios are preprocessed by looking for missing data. It can be concluded that only the data set containing 48 industries contains missing data. Therefore all the data before the last discovered missing data point, in the 48 Industry Portfolio, are removed. This results in removing all the data until July 1969.

For the other two sets, all the data before January 1940 will be removed based on the investigation made for the 5 Industry Portfolio, Section 2.1.1. A time series plot of the Industry Portfolios can be seen below:

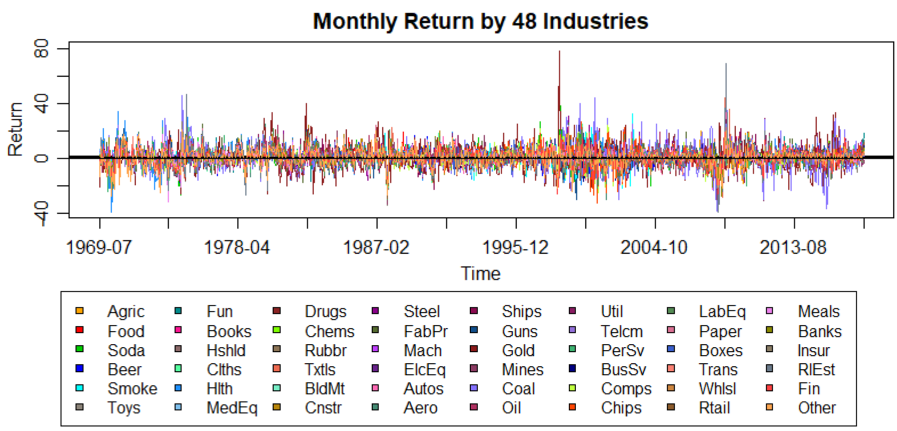
[1] Note that the black dots should be within the red confidence interval to be classified as normal distributed. Please see section 3.8.3 for further information.



(a) The 10 industries monthly returns from 1940-01 to 2018-01.



(b) The 17 industries monthly returns from 1940-01 to 2018-01.



(c) The 48 industries monthly returns from 1969-07 to 2018-01.

Figure 2.4: The monthly returns for the 10, 17 and 48 Industry Portfolios.

It can be seen in Figure 2.4 that the three data sets show the same characteristics as for the 5 Industry Portfolio. A positive mean close to zero and they all contain volatile periods, which means the variance is heteroscedastic. These periods can partly be explained by the events of turmoil described in section 2.1.1 and as argued then, no transformations to decrease these volatile periods are made. A summary statistic for these three data sets can further on be found in Appendix A.1. From this it seems like the attributes within the data sets are ranging more or less between the same intervals and no normalisation is needed.

Due to that the time series of the portfolios are so much alike, as seen in Figure 2.1 and Figure 2.4, and that the portfolios are derived from the same data, the correlation figures and Q-Q-plots are omitted. The 10, 17 and 48 portfolios are expected to show the same results as the 5 Industry Portfolio. Thus the data shows correlation between the attributes and they are not normally distributed.

2.2 Predictor Data

The Predictor data set contains 17 attributes. However, in this report the focus will be on only three of the attributes, plus the time attribute `Date`. This gives the following data structure:

- `Date`: Date for observation.
- `Index`: S&P 500 index prices from CRSP's month-end values.
- `D12`: The market dividend.
- `Infl`: Inflation- the consumer price index.

See [14] for reference. These attributes are chosen by imitating the work done by T. Bjerring [12]. Bjerrring has made a few changes to these attributes and the same changes will be made. The changes will be introduced below.

The market dividend, `D12`, will be recalculated to the *dividend yield*, which is given by the difference between the logarithm of dividends and the logarithm of lagged market prices [12].

The market index, `Index`, will also be changed. Instead of looking at the market index, this will be recalculated to the *market returns* [20], which is found by:

$$Return_t = \frac{Index_t - Index_{t-1}}{Index_{t-1}}. \tag{2.1}$$

From this point onward, when referred to the Predictor Data set, it means the reduced data set containing `Infl`, the `Dividend Yield` and the lagged `Market Returns`.

The predictor data set ranges from year 1871 to the end of 2016. Thus when using the Predictor data all data sets will be cropped to match each other, hence ranging from January 1940 to December 2016. Further on no missing data can be found in the predictor data set for the entire ranging period.

The predictor data set contains well known market predictors and are included in this thesis as an attempt to find alternative explanations for why some industries might forecast other industries returns. The data set is used by including it in the model as exogenous variables (predictors).

To get an overview of the data set, the three attributes are plotted and shown below:

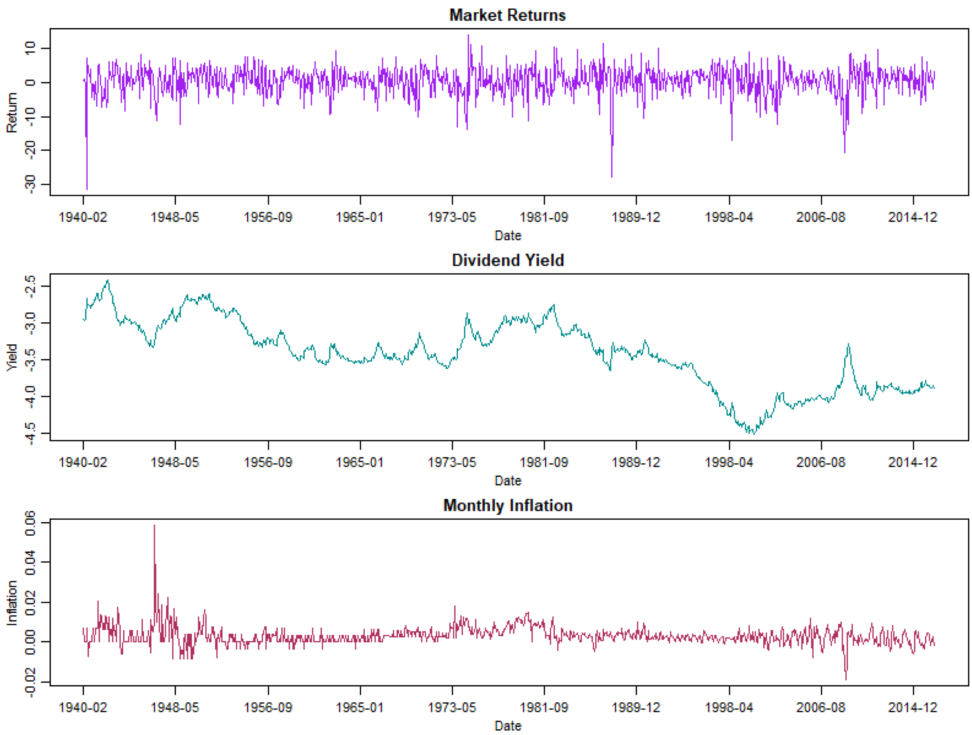


Figure 2.5: Figures for each of the attributes used in the predictor data set.

Looking at Figure 2.5 it is clear that a differencing is needed for the `Dividend Yield`, which seems to be very non-stationary. Thus a first order differencing is made. The results of the differencing can be seen below:

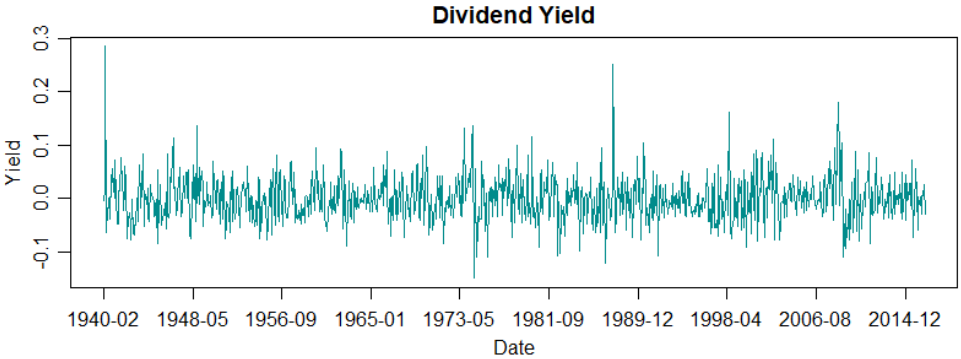


Figure 2.6: The differenced Dividend Yield. The data is now stationary.

With the differencing in Figure 2.6 it can be seen that the *Dividend Yield* now looks more stationary, just as the other two variables `Market Returns` and `Monthly Inflation` in Figure 2.5. Further on for all the attributes the variance seems to be heteroscedastic, with a few volatile periods as for the industry portfolios. The Predictor data set is now ready for use.

CHAPTER 3
Statistical Analysis

Different methods are used to analyse the predictability of the financial industries. In this chapter the theory of the methods will be introduced and explained.

3.1 Vector Autoregressive Model

The analysis will be based on a vector autoregressive (VAR) model. A response variable in the VAR model is not only dependent on its predecessors in time, but is also dependent on past values of other variables. This will allow linear inter-dependencies among multiple different time series to be captured, which are assumed to be dependent [12].

The vector autoregressive model (VAR(p), with p being the order) is a pure multivariate autoregressive model and is given by:

$$\mathbf{Y}_t = \boldsymbol{\phi}_1 \mathbf{Y}_{t-1} + \cdots + \boldsymbol{\phi}_p \mathbf{Y}_{t-p} + \boldsymbol{e}_t, \tag{3.1}$$

where $\mathbf{Y}_t$ is a $K \times 1$ vector of response variables, and $\boldsymbol{e}_t$ assigns an error term of the same dimension [21]. The error term follows the assumptions of being white noise, are serially uncorrelated and with time invariant covariance matrix $\boldsymbol{\Sigma}$ [12]. Further on $\boldsymbol{\phi}_1, \cdots, \boldsymbol{\phi}_p$ denotes the coefficient matrices and are of dimensions $K \times K$ [21]. Using short notation the model can be written linearly as:

$$\mathbf{Y}_t^T = \mathbf{X}_t^T \boldsymbol{\theta} + \boldsymbol{e}_t^T, \tag{3.2}$$

where $\mathbf{X}_t^T = [\mathbf{Y}_{t-1}^T, ..., \mathbf{Y}_{t-p}^T]$ and $\boldsymbol{\theta} = [\boldsymbol{\phi}_1^T, ..., \boldsymbol{\phi}_p^T]$, as given in [22]. Note that these notations will be used throughout this Chapter.

In addition a *constant* and a *trend* can be included as deterministic regressors, as well as *exogenous variables* with their corresponding coefficients [23].

To calculate the VAR process the `vars` package, [23], in the software `R`, [24], will be used.

3.1.1 Order of the VAR Model

As mentioned, this thesis is derived upon the work of T. Bjerring, [12], where it is assumed that the VAR process is an adequate model for the used time series data. Bjerring uses a first order VAR process, VAR(1). However, before using this, this is to be verified by trying to identify the optimal lag order.

Modelling of multivariate time series are similar, in a lot of ways, to methods used for univariate models [22]. Therefore the principles from *Table 6.1* in [22] is used to give a qualified guess of an appropriate model order by looking at the structure of the autocorrelation function (ACF), the partial autocorrelation function (PACF) and the cross correlation function (CCF), [22]. However to calculate these, the built in functions `acf`, `pacf` and `ccf` of the software `R`, [24], will be used. Therefore no further depth will be taken into how to calculate these functions.

To further verify the order, appropriate order criteria are used. These criteria are the Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC), the Hannan-Quinn Criterion (HQ) and the Final Prediction Error (FPE), all given in [25]. The difference between the criteria are that:

- AIC - penalises each parameter by an equal factor of 2 [21].
- BIC - looks at the sample size where complicated models get penalised heavier [21].
- HQ - also looks at the sample size but are not giving as high penalty as BIC, for high model order, but more heavily than AIC [21].
- FPE - is essentially the same as AIC for moderate and large sample sizes, but may outperform AIC for very small sample sizes [26].

The aim of the criteria are to find the best model (in this case using different lag orders), where the best model is given by the lowest value of each criteria. These criteria can be found by the *VARselect* function from the `vars` package, [23], of the software `R`, [24]. No further depth will be taken into how to calculate these and the built in function are simply used.

3.1.2 Estimating VAR(p) Coefficients

To estimate the coefficients for the VAR(p) model, equation (3.2), an ordinary least squares (OLS) estimation is used. The optimal coefficients are then found by minimising the sum of the least square cost function, S, i.e. minimising the sum of squared errors, [22]:

$$\hat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} S(\boldsymbol{\theta}). \tag{3.3}$$

For linear models, such as equation (3.2), this is done by solving the normal equation with respect to $\boldsymbol{\theta}$, [22]:

$$\hat{\boldsymbol{\theta}} = (\mathbf{X}^T\mathbf{X})^{-1}\mathbf{X}^T\mathbf{Y}, \tag{3.4}$$

However, the coefficients estimates will be calculated with the `vars` package, [23], of the software `R`, [24].

The covariance of the VAR(p) model, can be found by the cross product of the model error, divided by the degrees of freedom of each equation in the VAR(p) process [27]:

$$\boldsymbol{\Sigma} = \frac{\mathbf{e}_t^T\mathbf{e}_t}{n_{df}}, \tag{3.5}$$

where $n_{df} = n - K$ with n being the number of observations and K being the number of parameters.

3.1.3 Prediction

The one step prediction to time $t+1$ for a VAR(p) model (equation (3.2)) is found by the conditional mean, [21]:

$$\begin{aligned}\hat{\mathbf{Y}}_{t+1|t} &= \mathbb{E}\left[\mathbf{Y}_{t+1}|\mathbf{Y}_t, \mathbf{Y}_{t-1}, ...\right] \\ &= \boldsymbol{\phi}_1\mathbf{Y}_t + \cdots + \boldsymbol{\phi}_p\mathbf{Y}_{t-p+1}.\end{aligned} \tag{3.6}$$

Further on, for longer horizons, $k = 2, 3, ...$, the prediction is given by, [21]:

$$\hat{\mathbf{Y}}_{t+k|t} = \boldsymbol{\phi}_1\hat{\mathbf{Y}}_{t+k-1|t} + \cdots + \boldsymbol{\phi}_p\hat{\mathbf{Y}}_{t+k-p|t}. \tag{3.7}$$

The *prediction error* $\mathbf{e}_{t+k|t}$ is then given by, [21]:

$$\mathbf{e}_{t+k|t} = \mathbf{Y}_{t+k} - \hat{\mathbf{Y}}_{t+k|t}. \tag{3.8}$$

The predictions will however be calculated with the `vars` package, [23], of the software `R`, [24], by using the function `predict`.

3.1.4 Variance of the Predictions

The covariance of the prediction error, equation (3.8), of the VAR model is given by, [22]:

$$\mathbf{V}(k) = \boldsymbol{\Sigma} + \boldsymbol{\Phi}_1\boldsymbol{\Sigma}\boldsymbol{\Phi}_1^T + \cdots + \boldsymbol{\Phi}_{k-1}\boldsymbol{\Sigma}\boldsymbol{\Phi}_{k-1}^T, \tag{3.9}$$

where $\boldsymbol{\Sigma}$ is the covariance of the VAR(p) model, see equation (3.5). Further on $\boldsymbol{\Phi}$ is the estimated coefficient matrices of the moving average representation of the stable VAR(p) process, [22], and is found by using the function `Phi` in the `vars` package, [23], of the software `R`, [24].

The covariance of the prediction error for a one-step prediction, $k = 1$, is then found to simply be, [22]:

$$\mathbf{V}(1) = \mathbf{\Sigma}. \tag{3.10}$$

A $100(1-\alpha)\%$ confidence interval of the predictions can further on be calculated as:

$$\hat{\mathbf{Y}}_{t+k} \pm t_{\alpha/2}(N-p)\sqrt{Var[e_{t+k}]}, \tag{3.11}$$

where $t_{\alpha/2}(n-K)$ is the $\alpha/2$ quantile in the t-distribution with $(n-K)$ being the degrees of freedom [22]. The confidence interval is provided by the `vars` package, [23], of the software `R`, [24], by using the function `predict`.

3.2 A Sliding Model

The time series span over a long time period. To find the most significant variables for each response variables of the VAR model, during different sub-periods (referred to as blocks), an iterative approach is used. The size of the block is initialised through the accuracy and skill of the predictions, further explained in section 3.5.

A full model will be fitted to each block and then all non-significant parameters will be discarded. This is done iteratively, where the block is translated a single month forward in time for each iteration, until all of the data has been processed. For a visual explanation, see Figure 3.1:

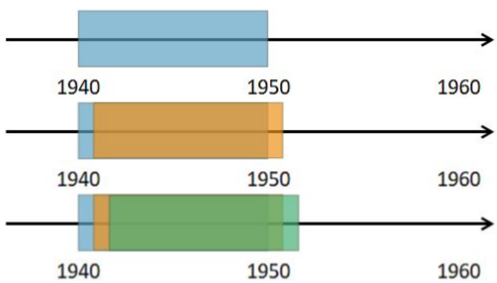


Figure 3.1: Visual explanation over how the sliding model works using a 10 years block. For each block a new VAR-model will be fitted and non-significant parameters removed. Note that the dimensions of this figure are not exact and that the blocks will only be translated one month at a time.

This approach will show how the significance of the variables varies over time. This will give an overview of what parameters often appear as statistically significant and are the most relevant to each response variable.

To deem a variable significant is done by using the `restrict` function in the `vars` package, [23], in the software `R`, [24]. It will simply fit a specified VAR(p) model,

$p = 1, 2, 3, ...$, and then check the t-value of each parameter. If the t-value for a parameter is lower than a specified threshold, the parameter will be insignificant and removed. A problem with this function is that it does not work if no variables are found significant. A modified version of the `restrict` function has therefore been made for the specific purpose of allowing no significant variables for the response variables.

The t-test assumes that the data is normally distributed, but when using large data sets the Central Limit Theorem makes the data asymptotically resemble a normal distribution. A t-test will therefore give an acceptable approximation of the p-value and inferences can still be made [28].

3.3 Tabulator

The output from the sliding model is large. To keep track of which variables appear significant the most times, to each given response variable, a tabulator method is used. The tabulator will simply be based on counting the number of times a predictor appears significant for each response variable in the sliding model.

The tabulator will present all the models that appear based on a chosen threshold, which is set to be 5%. Thus a model must appear over 5% of the times for all the models generated by the sliding model. The number of models that appear over 5% of the times might however differ among the response variables. If it does, the maximum number of models will be used. In the case where there are not enough models appearing in the top 5% the number of models used are set to be 6 such that there indeed are a few models to compare.

3.4 Verification of Predictions

To compare the models in the tabulator, section 3.3, a process of assessing the quality of the predictions are made. Thus verifying the prediction will be done by comparing it against its corresponding observation of the true outcome, by looking at the skill and accuracy. Note that the skill and accuracy on several occasions will be compared by ranking these results among different models.

3.4.1 Verifying Continuous Forecast

To look at the accuracy of the model the sum of squared errors (SSE) and the mean squared error (MSE) will be used, [29]. The SSE is the squared difference between the forecast & observation pair and MSE is simply the average of the SSE:

$$MSE = \frac{SSE}{n} = \frac{1}{n}\sum_{t=1}^{n}(Y_t - \hat{Y}_t)^2, \tag{3.12}$$

where Y_t is the observation, $\hat{Y}_t$ the prediction and n the number of observations. Since the SSE and MSE is squared they only produces positive terms. Therefore only a perfect forecast will produce a MSE score of 0 [29].

The MSE will also be compared to a reference error, given by the classical stock market prediction of guessing that a markets movement right now will continue in the same direction [10], [29]. This gives the MSE:

$$MSE_{Ref} = \frac{1}{n-1}\sum_{t=2}^{n}(Y_t - Y_{t-1})^2. \tag{3.13}$$

Hence the reference term lets the last observation (Y_{t-1}) be the prediction of the given observation (Y_t). The reference is made to see if the model is better at predicting observation Y_t than the old observation is, Y_{t-1}.

3.4.2 Verifying Probabilistic Forecast

To measure the skill the continuous ranked probability score (CRPS), [30], will be used:

$$CRPS = \int_{-\infty}^{\infty}[F(y) - F_o(y)]^2 \mathrm{d}y, \tag{3.14}$$

where $F(y)$ is a cumulative distribution and

$$F_o(y) = \begin{cases} 0, & y < \text{observed value;} \\ 1, & y \geq \text{observed value,} \end{cases} \tag{3.15}$$

is a cumulative-probability step function also known as the Heaviside function, [30]. Just like the MSE the CRPS is usually averaged:

$$\overline{CRPS} = \frac{1}{n}\sum_{i=1}^{n} CRPS_i. \tag{3.16}$$

Since the CRPS has a negative orientation, smaller values are better. Therefore it will yield the smallest integrated squared difference for concentration of probability around the step function, which is located at the observed value, [30].

In this thesis it is assumed that the forecast distribution is Gaussian, with mean μ and variance σ^2, then the CRPS in equation (3.14) can be found by:

$$CRPS(\mathcal{N}(\mu,\sigma^2),y) = \sigma\left(\frac{y-\mu}{\sigma}\left[2\psi\left(\frac{y-\mu}{\sigma}\right)-1\right]+2\varphi\left(\frac{y-\mu}{\sigma}\right)-\frac{1}{\sqrt{\pi}}\right), \quad (3.17)$$

where $\psi(\bullet)$ and $\varphi(\bullet)$ are the CDF and PDF respectively of the standard Gaussian distribution [29].

3.5 Choosing Block Size

As commented in section 3.2 accuracy and skill is used to choose how many years each block should contain for the sliding model. The MSE and CRPS, section 3.4, will be calculated over several different block sizes, to see if an optimal sub-period for each block can be found. The ambition is to find the block size which minimises the MSE and CRPS. Only full years will be looked at.

3.6 Choosing Predictors

As explained in section 3.3 there will be a minimum of 6 models after the tabulator. From these models 3 models are to be chosen by investigating each response variables CRPS and SSE in 6 different categories; the *MSE*, the $\overline{CRPS}$, the *mean of ranks* for SSE and CRPS, and the *standard error* for a fitted LOESS regression, [31], of the ranks for SSE and CRPS. The mean ranks and the standard deviation for a fitted LOESS regression of the ranks refers to the ranking mentioned in section 3.4. Hence the MSE and CRPS will be ranked when compared between the models. The standard deviation for a fitted LOESS regression will be a measure the *consistency* of a model according to its SSE and CRPS.

For each of these 6 categories, the top 3 best models in each category will be collected. The models for each category will then be compared and the 3 models appearing most often will be chosen. In the case of a tie, hence there are several models appearing the same amount of times, the model that has a higher number of appearance in the tabulator (section 3.3) will be chosen.

3.7 Recursive Least Squares

In section 3.1 an ordinary least squares method was used to minimise the sum of the least square cost function. This might not be appropriate, as for dynamical models the assumptions of having a fixed model is not valid as the dynamical characteristics changes over time. This may be avoided by applying adaptive methods. The adaptive methods will allow for tuning of the parameters when new observations are introduced to the model. This is where the recursive least squares (RLS) will come into play, [22].

The RLS uses the same principles as the least squares, except that $\hat{\boldsymbol{\theta}}$ gets updated for each new observation. The RLS algorithm is given by following equations, [22]:

$$\hat{\boldsymbol{\theta}}_t = \hat{\boldsymbol{\theta}}_{t-1} + \mathbf{R}_t^{-1}\mathbf{X}_t\left[\mathbf{Y}_t - \mathbf{X}_t^T\hat{\boldsymbol{\theta}}_{t-1}\right]; \tag{3.18}$$

$$\mathbf{R}_t = \mathbf{R}_{t-1} + \mathbf{X}_t\mathbf{X}_t^T. \tag{3.19}$$

A problem with the algorithm is the inversion of $\mathbf{R}_t^{-1}$, which is often not possible due to singularity. Therefore the introduction of $\mathbf{P}_t$ and $\mathbf{K}_t$ is made such that, [22]:

$$\mathbf{P}_t = \mathbf{R}_t^{-1} = \mathbf{P}_{t-1} - \frac{\mathbf{P}_{t-1}\mathbf{X}_t\mathbf{X}_t^T\mathbf{P}_{t-1}\mathbf{X}_t}{1+\mathbf{X}_t^T\mathbf{P}_{t-1}\mathbf{X}_t}; \tag{3.20}$$

$$\mathbf{K}_t = \mathbf{R}_t^{-1}\mathbf{X}_t = \frac{\mathbf{P}_{t-1}\mathbf{X}_t}{1+\mathbf{X}_t^T\mathbf{P}_{t-1}\mathbf{X}_t}. \tag{3.21}$$

These changes makes it possible to estimate and update $\hat{\boldsymbol{\theta}}$ as, [22]:

$$\hat{\boldsymbol{\theta}}_t = \hat{\boldsymbol{\theta}}_{t-1} + \mathbf{K}_t[e_t(\hat{\boldsymbol{\theta}}_{t-1})], \tag{3.22}$$

where

$$e_t(\hat{\boldsymbol{\theta}}_{t-1}) = Y_t - \mathbf{X}_t^T\hat{\boldsymbol{\theta}}_{t-1}. \tag{3.23}$$

The commonly applied starting values to the RLS are further on to let $\mathbf{P}_0 = \alpha\mathbf{I}$ and to use an arbitrary value for $\hat{\boldsymbol{\theta}}_0$, where $\mathbf{I}$ is the identity matrix with suitable dimensions and α is a large value [22].

3.7.1 Adaptive RLS

The forgetting factor, $0 < \lambda < 1$, reduces the impact of past observations that increases the SSE by giving the observations less weight. Therefore a small λ will make the model less sensitive to past observations. This can be used as a tool to combat the influence of outliers in the model. Including the forgetting factor is based on *equation (11.27)* and *(11.28)* in [22]. This means that the parameters for the RLS algorithm $\mathbf{P}_t$ (equation (3.20)) and $\mathbf{K}_t$ (equation (3.21)) gets updated as, [22]:

$$\mathbf{P}_t = \frac{1}{\lambda(t)}\left(\mathbf{P}_{t-1} - \frac{\mathbf{P}_{t-1}\mathbf{X}_t\mathbf{X}_t^T\mathbf{P}_{t-1}\mathbf{X}_t}{\lambda(t)+\mathbf{X}_t^T\mathbf{P}_{t-1}\mathbf{X}_t}\right); \tag{3.24}$$

$$\mathbf{K}_t = \frac{\mathbf{P}_{t-1}\mathbf{X}_t}{\lambda(t)+\mathbf{X}_t^T\mathbf{P}_{t-1}\mathbf{X}_t}. \tag{3.25}$$

Each weight is exponentially assigned to each previous value of the actual system, where $\lambda(t)$ are the exponential weights.

The forgetting factor, λ, is found by minimising the SSE's [1] one-step prediction errors, [22].

The ARLS are applied on the three chosen models mentioned in section 3.6. The first step is to calculate a forgetting factor, λ. When optimising the forgetting factor a burn-in period is needed. This period is chosen to be the same number of years as the block size calculated for the sliding model, section 3.5.

After the forgetting factor is found, the ARLS will be used to calculate the predictions over time. The results from the predictions for each of the three models will then be compared by their SSE and CRPS, and ranked accordingly. The variance of the predictions is found by using the weighted least squares estimation found at *equation (3.102)* in [22]. This will make it possible to see if one of the models are consistently predicting better than the others at different periods.

3.8 Residual Analysis

The VAR model assumes that the residuals resembles *white noise*, *are serially uncorrelated* and that the covariance matrix $\mathbf{\Sigma}$ is *time invariant* [12]. These assumptions should be fulfilled in an ideal world. To resemble white noise, the residuals should be *random*, *normally distributed* and they should have a *homoscedastic variance* [22]. Further on to check how the observations are weighted, by the least squares estimation, a Cook's distance will be looked at. This is done by using the `car` package, [32], in `R`, [24].

3.8.1 Randomness and Homoscedastic Variance

A simple way of checking randomness is by looking at the residuals over time. For the residuals to be random no clear patterns should be present in the residuals plot. Further on, an overview of whether or not the variance are homoscedastic can be seen in the same plot, [22]. To further emphasis if the variance are homoscedastic the residuals and the studentized residuals are plotted against the fitted values of the model.

To check if the residuals are random in a more precise way, a simple sign test can be used. It is expected that the residual on average will change sign every second time if it is random. Since it is assumed that the residuals are white noise, each change will be independent and follow a Bernoulli distribution. Therefore the residuals are

[1] Calculations for SSE can be found in section 3.4.1.

assumed to be random if 0.5 is included in the 95% confidence interval based on the Binomial distribution [22].

3.8.2 Correlation

To check if the residuals are serially correlated the ACF and PACF (presented in section 3.1.1) of the residuals will be used. When plotting the ACF and PACF no significant lags should be present to deem the residuals to be serially uncorrelated [22].

3.8.3 Normality

To check the normality of the residuals a Jarque-Bera test is performed. The test is based on the sample skewness and the sample kurtosis, which are variables that can be simplified as describing the symmetry and "tailedness" of the distribution [33]. The null hypothesis, H_0, is that the sample follows a normal distribution [33].

Furthermore a quantile-quantile plot (Q-Q-plot) is made of the residuals. A Q-Q-plot is made by plotting the quantiles of the residuals in ascending order against quantiles calculated from a theoretical distribution. This should result in a straight line, if the residuals comes from the same distribution as the theoretical one which in this case will be a normal distribution [34].

3.9 Inclusion of Predictor Data

As stated in section 2.2 the Predictor Data will be included to try to find alternative explanations for why some industries might forecast the subsequent returns of other industries. Furthermore an investigation will be made to see if the inclusion of this data set will have a positive impact on the predictions.

To conclude whether or not it makes sense to include the Predictor data set, three chosen criteria must be fulfilled:

1. The variables in the Predictor data set must appear to be significant.
2. The normality assumptions of the residuals should improve.
3. The MSE and CRPS should improve.

If the Predictor data set adheres to the above criteria, it will be included and/or deemed as an alternative explanation.

CHAPTER 4

Results

In this chapter the results of the methods implemented will be introduced for each industry portfolio. A thorough result section will be made for the 5 Industry Portfolio. For the 17 and 48 Industry Portfolios only the most relevant results will be shown. However since the results for the different portfolios are so much alike, the 10 Industry Portfolio are chosen to be omitted. A few of the results for the 10 Industry Portfolio can be found in Appendix D. Lastly, the results for including the Predictor data will be investigated.

4.1 Verification of Model Order

The identification of the optimal lag order is based on the methods introduced in section 3.1. The ACF, PACF, CCF and different information criteria are used. First the ACF of the 5 industry data set are presented below. A lag of 30 are used to identify if seasonality is present within the data:

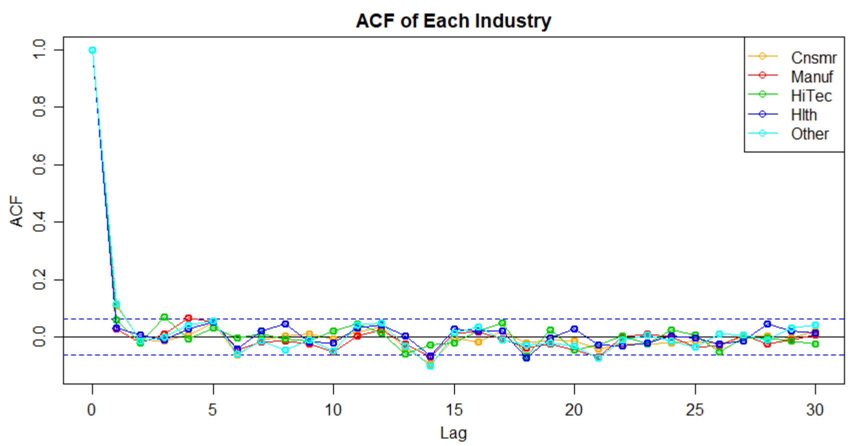


Figure 4.1: The ACF of the 5 Industry Portfolio. The most pronounced significant lags seem to be at lag 1 and lag 14. No clear indication of seasonality can be observed.

Looking at Figure 4.1 the ACF shows that only a few lags are found to be significant and no seasonality can be observed. To get a better idea of the order, the PACF is also shown:

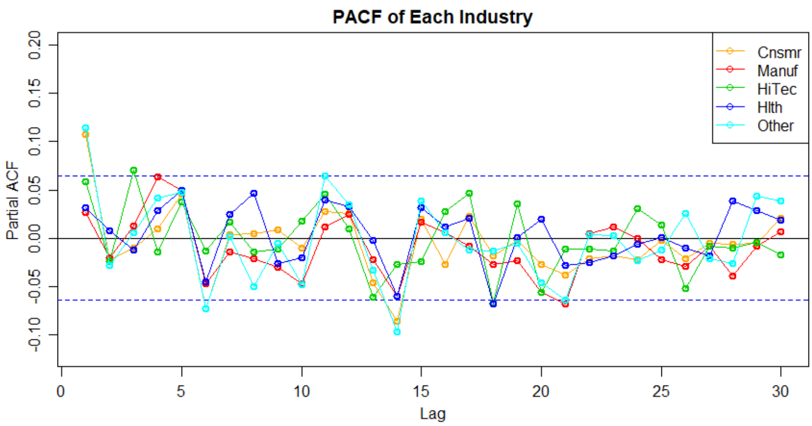


Figure 4.2: The PACF of the 5 Industry Portfolio. The PACF matches the ACF, with the significant lags at the same places as was found in the ACF.

Looking at Figure 4.2 the PACF shows that there are significant lags in order 1, but also scattered around for $k > 1$, like in the ACF. Lastly the CCF is displayed below:

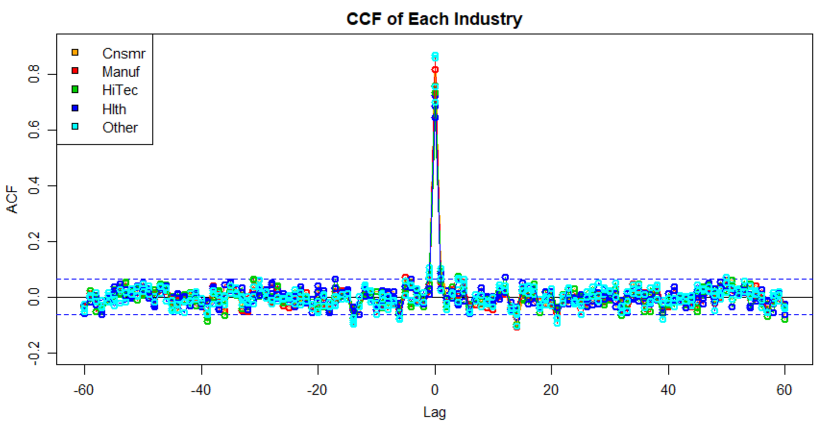


Figure 4.3: The CCF of the 5 Industry Portfolio. There are a lot of places with borderline significance. However significance for lag 1 and -1 can be seen. Note that the figure contains all unique combinations of the variables. Therefore 10 combinations can be observed for the 5 Industry Portfolio. A blue circle might be placed on a green line, which indicates that the CCF of `Hlth` and `HiTec` is evaluated. Admittedly this can be rather difficult to see in the figure.

Lastly Figure 4.3 shows that the CCF also seems to have several significant lags at different lag orders. Overall the ACF, PACF and CCF seems to have borderline significant information at different lags making it hard to determine an optimal lag order. Therefore to further investigate if the optimal lag order is 1, the different information criteria presented in section 3.1.1 are used on a model containing all 5 industries as predictors. An order up to $p = 5$ are used:

Table 4.1: Different criteria of a VAR(p) model. The lowest values are consistently found to be the model with order 1. The red colour indicates the minimum value.

p	1	2	3	4	5
AIC(p)	10.83	10.85	10.88	10.88	10.89
HQ(p)	10.93	11.00	11.08	11.13	11.19
SC(p)	11.09	11.24	11.40	11.54	11.67
FPE(p)	50304.92	51549.80	52945.50	53185.54	53502.50

From Table 4.1 the lowest values are found to be for the VAR(1) model. This verifies that a VAR(1) model indeed makes sense to use.

The same procedure is done for the 17 and 48 Industry Portfolios and it is found from the criteria that the optimal order is 1. The ACF, PACF and CCF are however again showing several different significant lags. The results can be found in Appendix B.1.

4.2 Results for the 5 Industry Portfolio

The first data set analysed is the *5 Industry Portfolio* which is specified in subsection 2.1.1.

4.2.1 The Sliding Model

Time series are dynamic in nature [21]. This means that the chosen predictors in the VAR model can be dependent on the time period used for estimation. With the sliding model the aim is to find consistent variables good at predicting different industry sectors, by going through a rolling window of observations throughout the time series as explained in section 3.2.

4.2.1.1 Block Size

As described in section 3.5 the optimal years used for each block in the sliding model are decided by looking at the MSE and CRPS for each response variable using different predictors. The focus will be purely based on the gain in the MSE and CRPS. The models used to find an optimal block size are the top four most consistent models found from the sliding model by increments of 1 year until the block size is 14 years. How the top four models are found is explained in section 3.3.

Using the 5 Industry Portfolio the MSE becomes as follows:

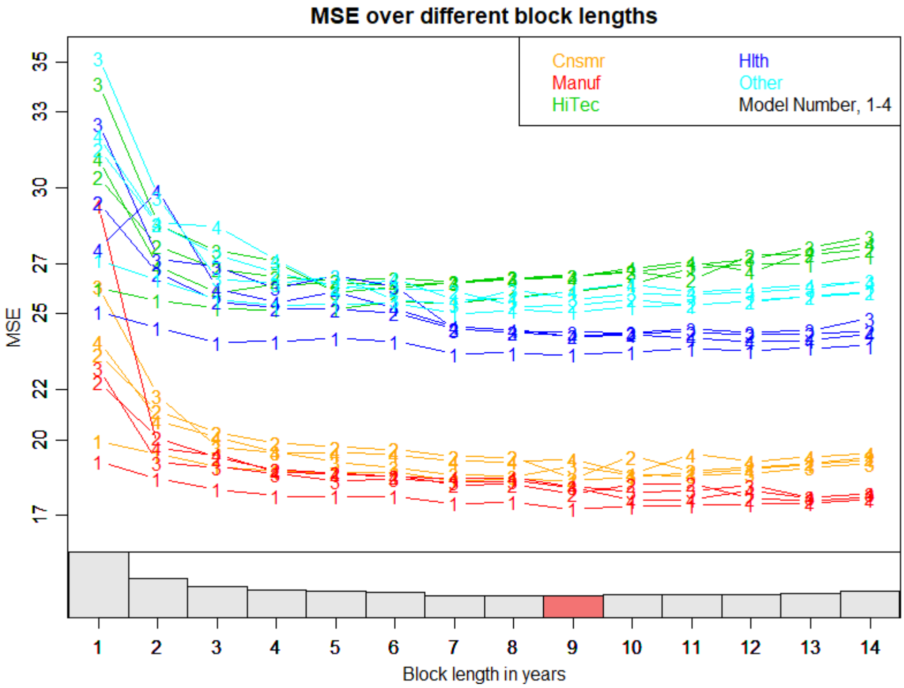


Figure 4.4: The MSE calculated for different block sizes in years for the 5 Industry Portfolio using four different models. The histogram shown below indicates the mean of the above errors. The red block indicates the lowest mean error.

Figure 4.4 shows that the lowest MSE was found when the block size is 9 years. The CRPS can be seen in Figure 4.5.

Figure 4.5 shows almost the same pattern as in Figure 4.4, resulting in the same years to be the optimal block size, hence 9 years. The block size should therefore span over 9 years of observations. This is used in the sliding model.

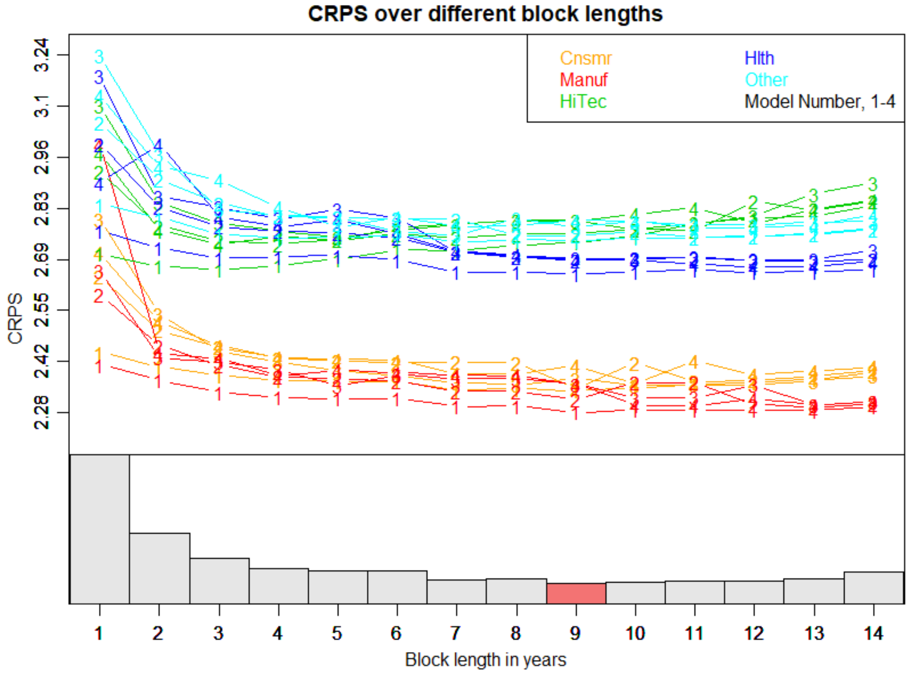


Figure 4.5: The CRPS calculated for different block sizes in years for the 5 Industry Portfolio using four different models. The histogram shown below indicates the mean of the above errors. The red block indicates the lowest mean error.

4.2.1.2 A Sliding Model

Not all predictors are significant in the VAR(1) model and some of them might be significant at different sub-periods of time. To give an overview of the behaviour of the VAR(1) model over time the method explained in section 3.2 is used. For the 5 Industry Portfolio the behaviour of the predictors over time can be observed in Figure 4.6.

The first thing to notice in Figure 4.6 is that the x-axis starts from 1948-12. This is because the blocks are spanning over 9 years (remember that the data starts in 1940-01). Thus the dates in the x-axis are only there for guidance. It can be seen that all response variables are clearly dominated by the intercept `Const`. However there are also prominent lagged predictors. To get a precise overview of the appearance of the predictors the number of times a predictor is significant in the sliding model can be seen in Table 4.2.

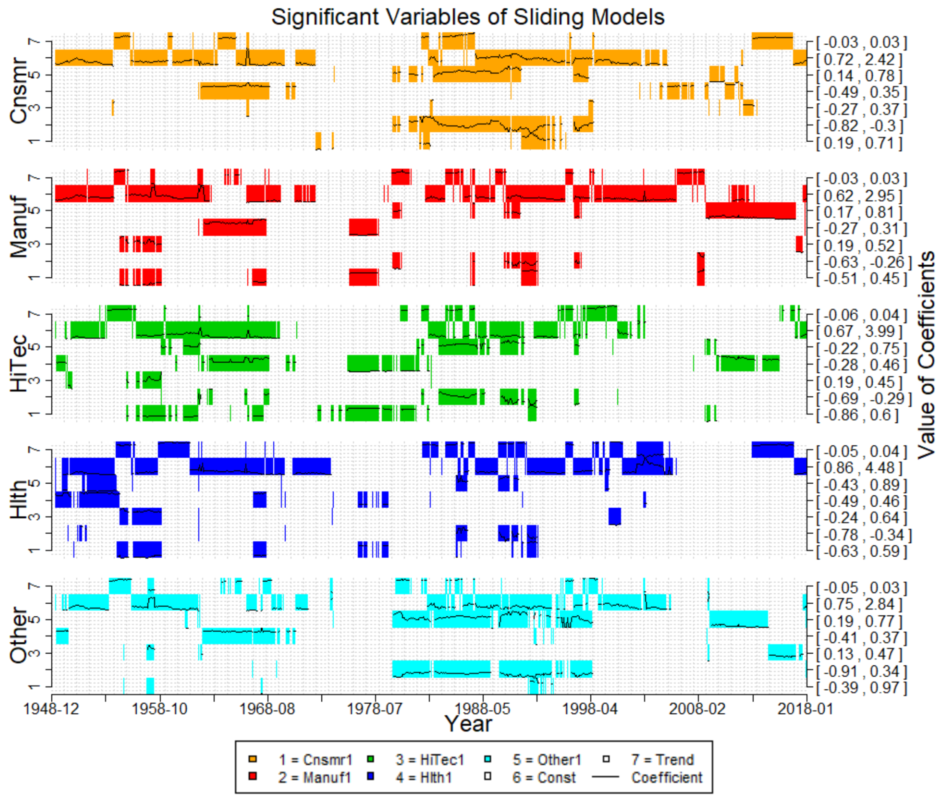


Figure 4.6: Significant variables found by translating the model one month using a block size of 9 years. Each date at the x-axis represents the last date in one block of 9 years. Thus the dates are only there for guidance. The left axis shows the response variable and the corresponding predictors are given by the numbers from 1 to 7. The right axis is the range of the values of the coefficients for the significant predictors. The black lines within the coloured blocks shows how the coefficient values fluctuate. Finally the coloured blocks indicate that the predictor is significant and is included in the model.

Table 4.2: The number of times each predictor variable is significant for each response variable.

Response\Predictor	Cnsmr1	Manuf1	HiTec1	Hlth1	Other1	Const	Trend
Cnsmr	55	184	29	136	152	456	142
Manuf	113	61	43	103	135	467	138
HiTec	167	107	35	225	122	345	144
Hlth	97	55	57	105	81	411	237
Other	28	196	48	110	260	386	124

It can be seen in Table 4.2 that there are stock market returns, which seems to be significant as lagged predictors for each of the response variables at different times. Further on some predictor variables seem to appear significant more often than others, e.g. `Manuf` is the most often occurring predictor for `Other` when comparing by the other industries. Based on this, it is assumed that `Manuf`, seems to be a superior predictor for `Other` in general. Also notice the empty regions in the sliding model. This indicates that no predictors was found to be significant for this time period.

4.2.2 Model Candidates

As described in section 3.3 a tabulator is made of the top used models in the sliding model, Figure 4.6. This is to give an overview of the sliding model and to assist with the process of choosing a model.

The most common combinations of predictors can be observed in Table 4.3:

Table 4.3: The top five overall used combinations of predictors acquired by the sliding model. Note that the appearance number is the amount of time the specific predictor combination can be found i.e. `Const` are found *alone* 1242 times but can be found more times in combinations with other predictors.

Predictor	**Appearance Nb.**
Const	1242
Trend	425
Manuf1 Other1 Const	274
Hlth1	228
Other1	173

In Table 4.3 it can be seen that clearly (as also can be seen in Figure 4.6) the intercept appears to be significant remarkably more times than the other predictors, but some of the lagged industries are also present.

To get a more specific view the most common significant predictor for each industry is identified. This can be seen in Table 4.4. A threshold of 5% is used, which means that only the combinations of significant predictors appearing more than 5% of the times in the sliding model, will be taken into consideration. This leaves the following models in Table 4.4.

Table 4.4: The top 5% used combinations of predictors to each response variable, acquired by the sliding model.

Model	**Cnsmr**	**Manuf**	**HiTec**	**Hlth**	**Other**
1	Const	Const	Const	Const	Const
2	Manuf1 Other1 Const	Trend	Trend	Trend	Manuf1 Other1 Const
3	Trend	Other1	Hlth1	Hlth1 Other1 Const	Other1
4	Hlth1	Cnsmr1 Hlth1	Cnsmr1 Hlth1	Const Trend	Hlth1
5	Hlth1 Const	Hlth1	Manuf1 Other1 Const	Cnsmr1 HiTec1 Const	Manuf1 Other1 Trend
6	Manuf1 Other1 Trend	Cnsmr1 HiTec1 Const	Hlth1 Const	Cnsmr1 Hlth1	HiTec1
7	Cnsmr1 Manuf1 Const	Manuf1 Other1 Const	Manuf1 Other1 Trend	Manuf1 Other1 Trend	Trend

Table 4.4 shows that the most common combination of predictors for each response variable, are to only use an intercept, `Const`, as seen in Model 1. After that the models contain different response variables for each model. These are the models that will be analysed further.

4.2.3 Predictions & Variance of the Predictions

The models presented in Table 4.4 are the ones which appears most frequently in the sliding model after the VAR(1) model have been restricted for each given block of years. The models are therefore only chosen by appearance and it is now time to look at how well these models actually perform starting off by looking at their predictions, described in section 3.1.3. The only focus here will be on one step predictions that will be calculated using the data in blocks of 9 years and predicting one month into the future. The predictions and their 95% confidence interval for Model 1, from Table 4.4, can be seen in Figure 4.7 below:

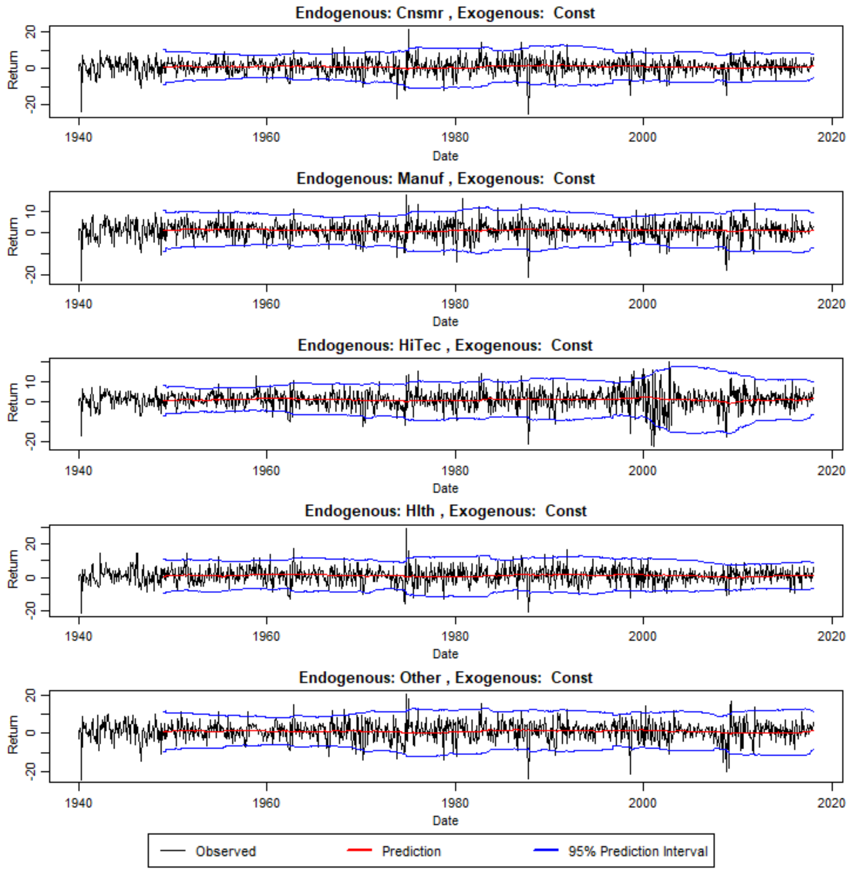


Figure 4.7: Predictions and its 95% prediction interval for *Model 1*.

Figure 4.7 shows that using the intercept gives predictions that do not diverge a lot. It is more or less a constant value, with very little fluctuations in variance. This means the model only containing an intercept will not be very good for predicting extreme cases. Introducing the lagged industries as variables will introduce a more volatile prediction. To show an example Figure 4.7 can be compared with the predictions of Model 6, Figure 4.8.

Figure 4.8 shows much more fluctuation in the predictions compared to Figure 4.7 due to including the lagged industries. The rest of the models predictions can be found in Appendix B.2.

Further on the variance of the predictions are investigated in Figure 4.9 for Model 1. The figures of the variance for the other models are almost identical to Figure 4.9 and are therefore omitted.

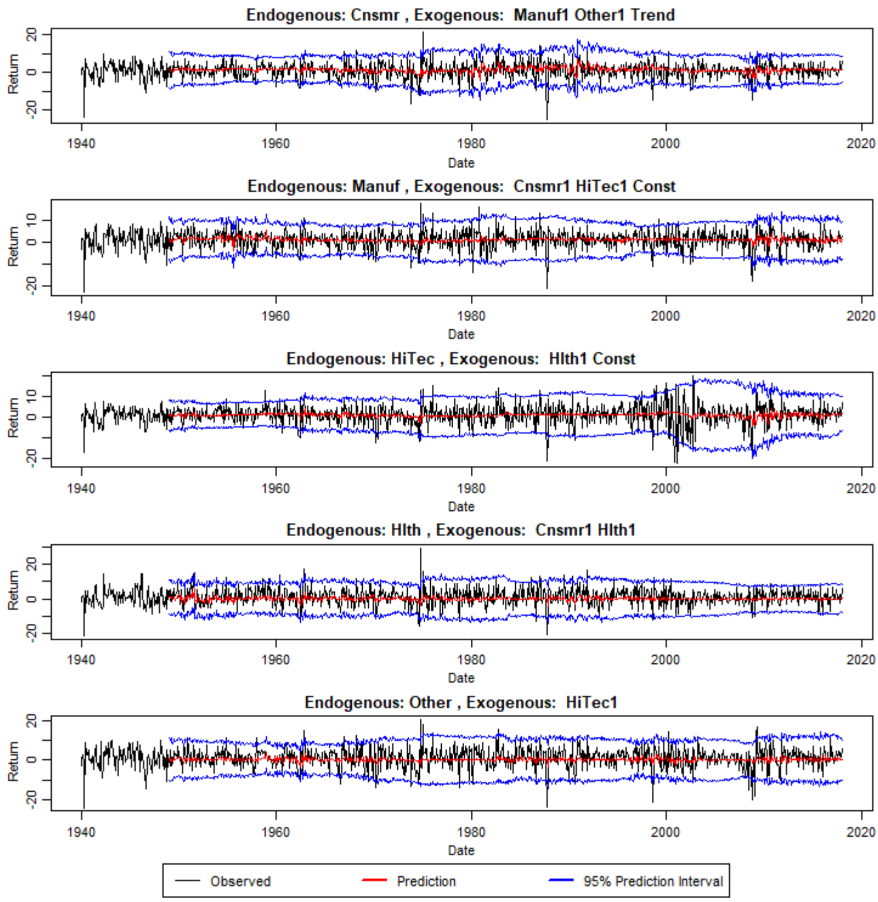


Figure 4.8: Predictions and its 95% prediction interval for *Model 6*.

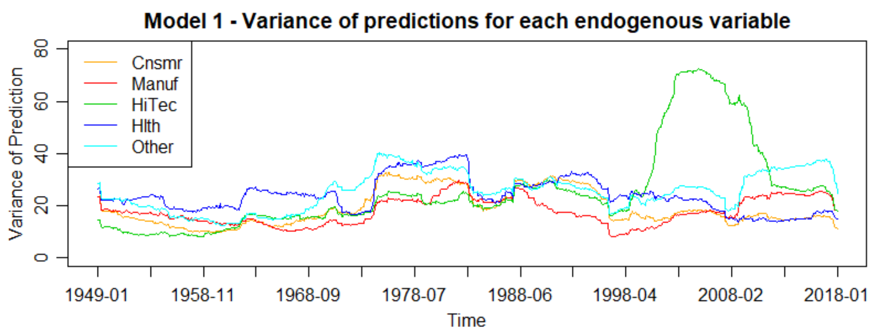


Figure 4.9: The variance of the predictions for Model 1.

Figure 4.9 shows that the five industries variance of predictions are not constant over time especially for `HiTec`. Notice that the high variance of `HiTec` starts to spike around 2001 and onward. This is a consequence of the 9/11 event in 2001, World Trade Center collapsing, also mentioned in chapter 2. Remember that the model is trained on 9 years of data. Therefore around 2001 and approximately 9 years forward, the models will contain the data of this event.

Looking at the variance it is clear that when an unexpected event happens, compare with the time series Figure 2.1, the prediction gets a lot higher risk (variance) and the predictions will be worse for the volatile sub-period. This will be discussed in further detail in Chapter 5.

4.2.4 Verification of Predictions

As presented in section 3.4 the models in Table 4.4 are compared by their accuracy and skill.

4.2.4.1 Accuracy

The accuracy of the model will be investigated using the SSE, see section 3.4.1. The figure below, Figure 4.10, shows an overview of the SSE for Model 1 from Table 4.4:

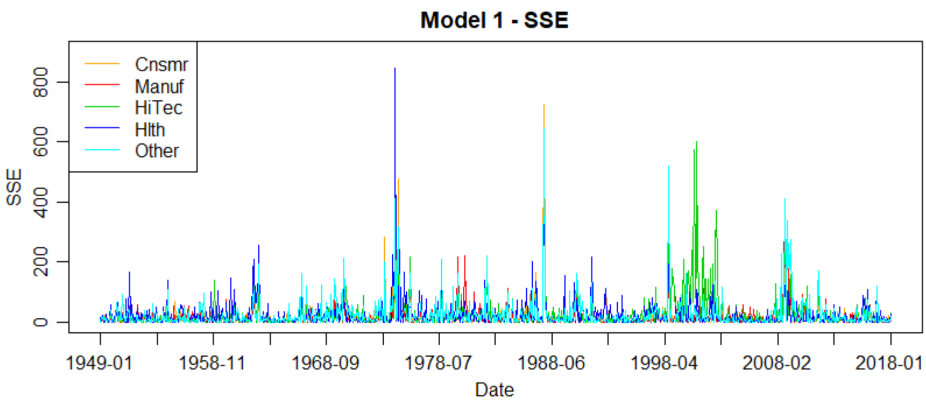


Figure 4.10: The SSE of Model 1 for all response variables.

Figure 4.10 shows that there are a few times where the SSE peaks high. Comparing this figure with the time series, Figure 2.1, it can be seen that these peaks are matching areas with higher variance. Hence it is clear that outliers in the data set is punished a lot by the SSE. The SSE figures for the other models are almost identical to Figure 4.10 and are therefore omitted.

The MSE is now calculated for each model and presented below. Notice that *Ref* is the reference error presented in section 3.4.1, equation (3.13).

Table 4.5: The MSE for each model and response variable where *Ref* is the reference error, see equation (3.13).

Model	**Cnsmr**	**Manuf**	**HiTec**	**Hlth**	**Other**
1	18.39	17.26	25.86	23.38	25.07
2	18.46	17.87	26.49	24.13	25.37
3	18.97	18.14	26.52	24.27	25.58
4	19.23	18.15	26.45	24.11	25.92
5	18.54	18.23	26.47	24.09	26.06
6	19.03	17.63	26.06	25.03	25.66
7	18.45	17.66	27.16	24.81	25.72
Ref	32.29	33.42	48.28	45.59	43.50

The first thing to notice in Table 4.5 is that all the models have a lower MSE than the reference error just as desired. Remember that the model numbers also describes how many times the model has been used in the sliding model. Therefore it is interesting to see that appearing many times in the sliding model does not necessarily imply a better accuracy. E.g. for `Cnsmr` the second lowest accuracy is for Model 7.

To get a more specific look on how the accuracy changes for each model the SSE is ranked against the SSE of the other models for each date. For the response `Cnsmr` the rank plots of the seven models can be seen in Figure 4.11.

Figure 4.11 shows that the model that has been ranked the best most of the time is Model 4. However it has also been ranked the worst resulting in a higher variance looking at the LOESS regression. This is a common trade off which also can be observed for the other response variables (found in Appendix B.3); if a model predicts best a lot of the times, it often also predicts the worst a lot of the times.

Looking at the rank mean (the dashed black line), in Figure 4.11, it looks like it is only Model 1 and 2 that has a mean under 4, both of them with low variance of the LOESS regression. The goal is to choose a model that performs well (low rank mean), but is also consistent (low variance of LOESS regression). This is however hard to pick out from the figures above and will be compared more exact later on in Table 4.7. In the same table the results for the other response industries also gets compared and no further depth is taken into their figures. Thus as mentioned, they are found in Appendix B.3, Figure B.8.

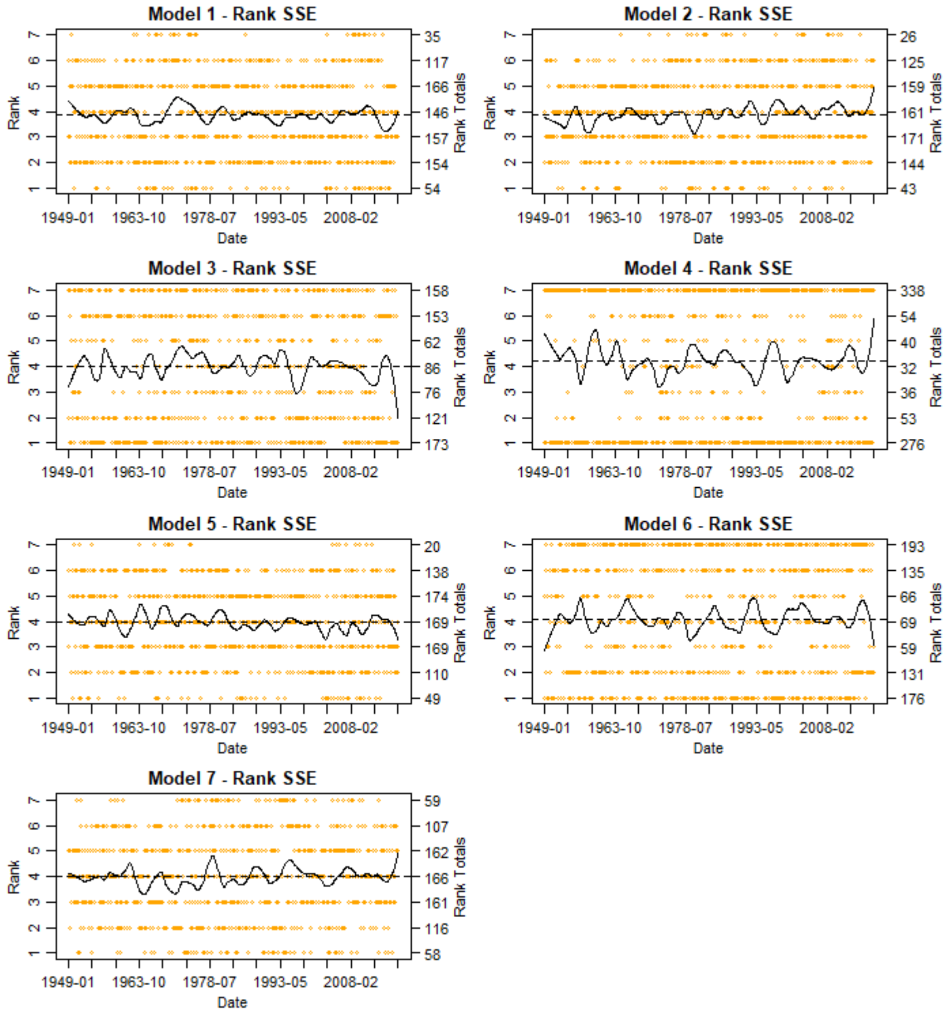


Figure 4.11: The figures show how each model have been ranked compared to each other, where a Rank=1 is considered the best rank. The right axis shows the number of times a given model have had that specific rank. The dashed black line represent the mean of the rank and the black line represents a locally weighted scatterplot smoothing, LOESS regression, of the ranks. The coloured circles represents the rank of a model at a given date.

4.2.4.2 Skill Score

The last part of the verification section will be focused on the skill of each model measured by the CRPS presented in section 3.4.2. The figure below, Figure 4.12, shows an overview of the CRPS for Model 1, from Table 4.4.

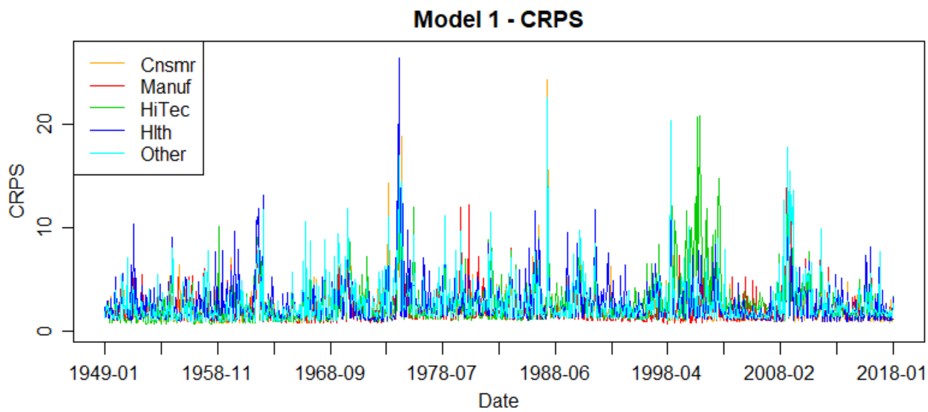


Figure 4.12: The CRPS for Model 1.

As for the SSE (Figure 4.10) the CRPS in Figure 4.12 also shows that these peaks are matching areas with higher variance in Figure 2.1. Hence it is clear that variance in the data set affects the CRPS, but the CRPS does not punish outliers as much as the SSE did. The CRPS figures for the other models are almost identical to Figure 4.12 and are therefore omitted.

Further on the $\overline{CRPS}$ of each model are presented below:

Table 4.6: The $\overline{CRPS}$ for each model.

Model	**Cnsmr**	**Manuf**	**HiTec**	**Hlth**	**Other**
1	2.340	2.281	2.737	2.652	2.740
2	2.347	2.316	2.760	2.690	2.754
3	2.372	2.357	2.796	2.695	2.779
4	2.407	2.357	2.794	2.689	2.796
5	2.351	2.361	2.762	2.690	2.785
6	2.379	2.306	2.748	2.753	2.786
7	2.349	2.306	2.789	2.725	2.769

Table 4.6 shows a few examples where models further down the table have a lower skill score than the models in the top of the table, just as for the MSE in Table 4.5. E.g. Model 7 has the second lowest skill score of `Cnsmr`. It is therefore again clear that appearing many times in the sliding model does not necessarily imply a better skill score.

To get a more specific idea on how the skill score changes for each model a plot with ranks are made. For the response `Cnsmr` the rank plots of the 7 models can be seen in Figure 4.13.

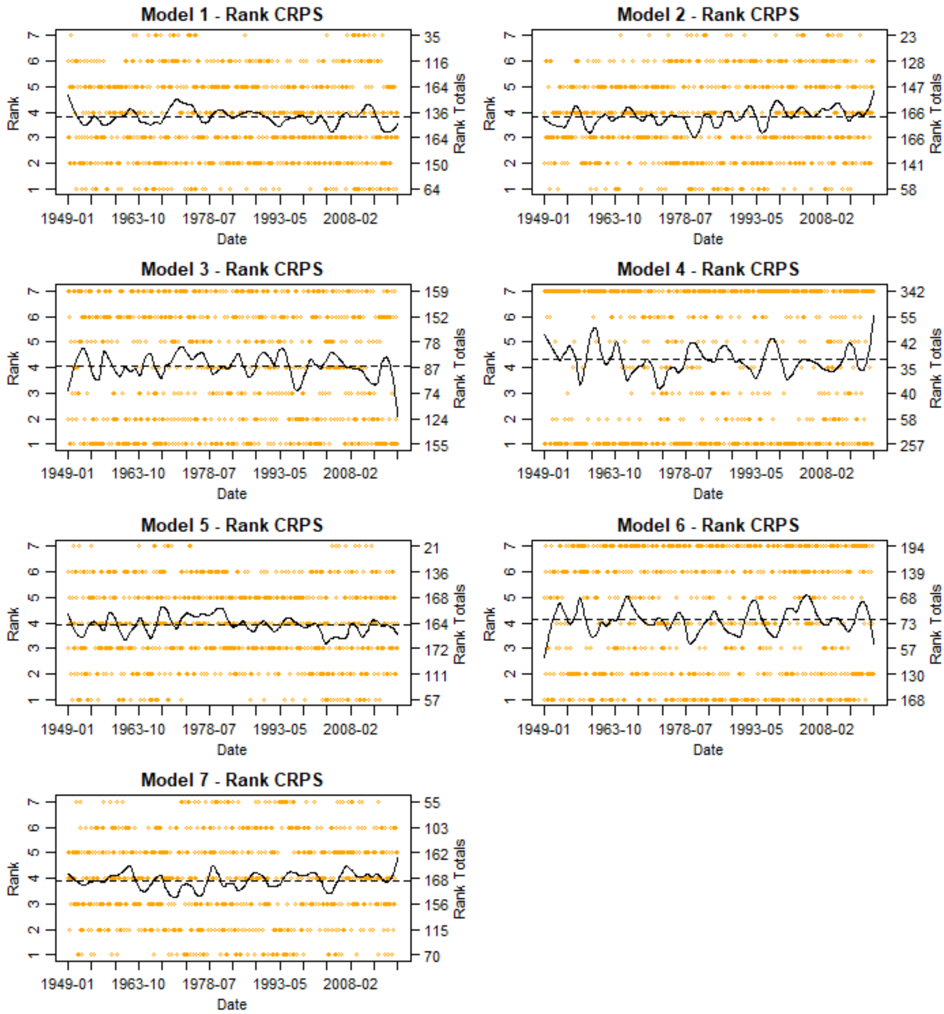


Figure 4.13: These figures show how each model have been ranked compared to each other where Rank=1 is the best rank. The right axis shows the number of times a given model have had that specific rank.The dashed black line represent the mean of the rank and the black line represents a locally weighted scatterplot smoothing, LOESS regression, of the ranks. The coloured circles represents the rank of a model at a given date.

A lot of the same behaviour as was observed for the MSE rankings, in Figure 4.11, is seen for the CRPS. I.e. models with a lot of best ranks often also have a lot of worst ranks. As for the MSE ranking, the figures of the other response variables for the CRPS can be found in Appendix B.4 and these figures will be further investigated in Table 4.7.

4.2.5 The Top Picks and Consistency of Predictions

By using the MSE and CRPS presented in section 3.4 the models will be compared, such that three models can be picked out to be used in the adaptive recursive least squares (ARLS) model. How to choose these models are explained in section 3.6. To get an overview over the choosing process the top three models of the different categories are presented in the table below. Remember that Model 1-7 refers to the models presented in Table 4.4.

Table 4.7: The top three models for each response variable of each category; MSE, Mean CRPS, Mean Rank SSE & CRPS, and lastly the standard deviation of the LOESS regression of the rank of the SSE & CRPS.

	MSE	$\overline{\textbf{CRPS}}$	$\overline{\textbf{Rank SSE}}$	$\overline{\textbf{Rank CRPS}}$	**SE LOESS SSE**	**SE LOESS CRPS**
	Model 1	Model 1	Model 1	Model 1	Model 5	Model 5
`Cnsmr`	Model 7	Model 2	Model 2	Model 2	Model 2	Model 2
	Model 2	Model 7	Model 5	Model 7	Model 1	Model 1
	Model 1	Model 1	Model 1	Model 1	Model 1	Model 1
`Manuf`	Model 6	Model 7	Model 2	Model 6	Model 6	Model 6
	Model 7	Model 6	Model 6	Model 7	Model 7	Model 7
	Model 1	Model 1	Model 1	Model 1	Model 6	Model 6
`HiTec`	Model 6	Model 6	Model 5	Model 5	Model 1	Model 1
	Model 4	Model 2	Model 2	Model 6	Model 5	Model 5
	Model 1	Model 1	Model 1	Model 1	Model 1	Model 1
`Hlth`	Model 5	Model 4	Model 7	Model 3	Model 5	Model 5
	Model 4	Model 5	Model 4	Model 4	Model 3	Model 3
	Model 1	Model 1	Model 2	Model 2	Model 1	Model 1
`Other`	Model 2	Model 2	Model 1	Model 1	Model 2	Model 2
	Model 3	Model 7	Model 7	Model 7	Model 6	Model 6

In Table 4.7 it can be seen that e.g. for `Cnsmr` all categories contains both Model 1 and Model 2. However, there is a tie between Model 5 and 7. Since model 5 has the lowest model number, Model 5 will be chosen as the last model to represent `Cnsmr`. Doing this for all the response variables, the models which will be used for the ARLS are presented in Table 4.8.

Table 4.8: The top three overall best models for each response variable of the 5 Industry Portfolio. Note that these are the models that will be used in the ARLS.

Model	**Cnsmr**	**Manuf**	**HiTec**	**Hlth**	**Other**
1	Const	Const	Const	Const	Const
2	Manuf1 Other1 Const	Cnsmr1 HiTec1 Const	Hlth1 Const	Const Trend	Manuf1 Other1 Const
3	Hlth1 Const	Manuf1 Other1 Const	Manuf1 Other1 Const	Cnsmr1 HiTec1 Const	Trend

4.2.6 Adaptive Recursive Least Squares

In section 3.7 it was mentioned, why it would be a good idea to use an adaptive method. The chosen models in Table 4.8 will now be estimated using the ARLS method. First the forgetting factor λ has to be optimised. This is done by minimising the cost function given by the SSE for each given λ, as explained in section 3.7.1, and are presented in Table 4.9:

Table 4.9: Optimal forgetting factor λ for the response variables for each of the top three models presented in Table 4.8.

Model	**Cnsmr**	**Manuf**	**HiTec**	**Hlth**	**Other**
1	1	1	1	1	1
2	1	1	1	0.99778	1
3	1	1	1	1	0.99121

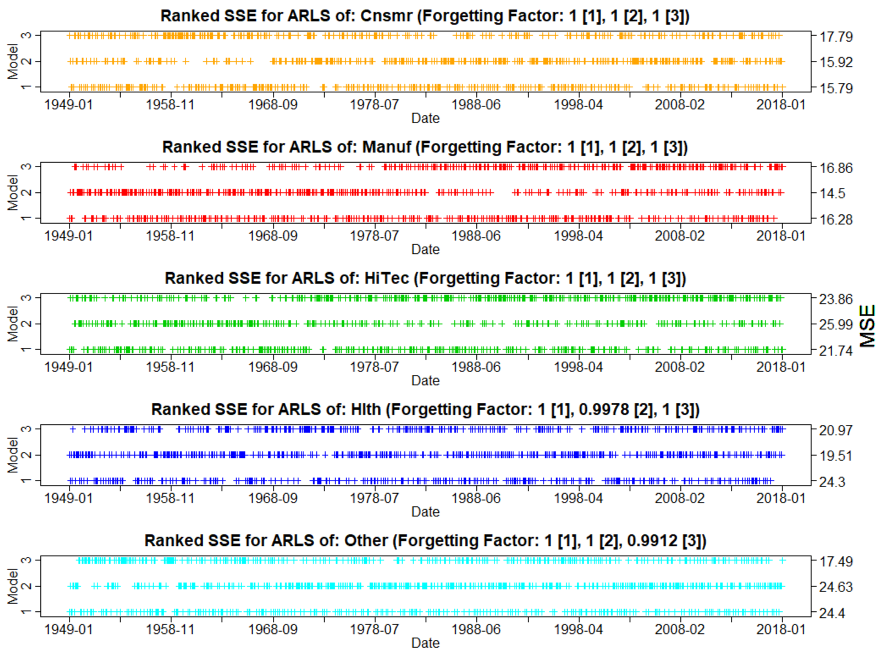


Figure 4.14: The best performing model over time when continuously introducing a new observation. Each + sign indicates the best model at a given date according to the SSE. Note that the x-axis starts in 1949-01 due to the burn-in period of 9 years. The left axis shows the model number according to Table 4.8 and the right axis shows the MSE of the corresponding plotted signs. The forgetting factor can be seen at the top of each plot, where the number inside the brackets is the model number.

In Table 4.9 very high forgetting factors are rounded of to 1 which can be seen to be the case the majority of times. The two times where the forgetting factor is slightly smaller then 1 are however used to further optimise the model.

A beneficial property of the ARLS is that the coefficients will be updated with newly introduced observations. This makes it possible to see how the models perform along with introducing new observations to the models. To compare the models, the method explained in section 3.7.1, will be used. Shortly the models will be compared by their SSE and CRPS and the model with the best value at a given time will be ranked the best and plotted. The top three models from Table 4.8 and the forgetting factors in Table 4.9 are used for this procedure. For the SSE the result can be seen in Figure 4.14.

Looking at Figure 4.14 it can be seen that all three models seem to be dominating at different time periods for all the response variables. As an example, looking at `Cnsmr`, it seems like Model 1 dominates at the start, then around year 1958 it changes to Model 3 and then changes back to Model 1 around 1965. Further on the same type of figures, but calculated with the CRPS can be seen in Figure 4.15.

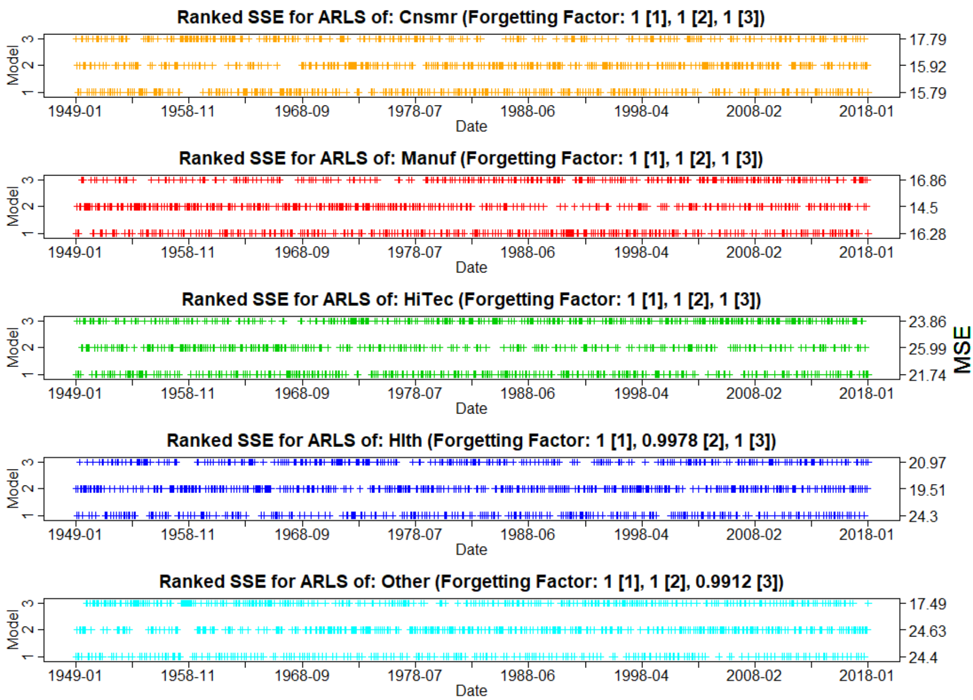


Figure 4.15: The best performing model over time. See Figure 4.14 for figure details.

Looking at Figure 4.15 the same pattern seems to appear as in Figure 4.14, only with slight changes. However these figures still make it hard to interpret if there is one model that is more consistent than the others. To make it clearer the number of changes between each model will be looked at, first for the SSE figures:

Table 4.10: Number of times the preferred model changes over the entire time series for Figure 4.14. As an example, for `Cnsmr` Model 1 changes to Model 1, 104 times, and Model 1 changes to Model 2, 91 times.

Cnsmr				Manuf				HiTec				Hlth				Other			
From\To	1	2	3	F\T	1	2	3	F\T	1	2	3	F\T	1	2	3	F\T	1	2	3
1	104	91	92	1	87	94	99	1	95	82	100	1	55	103	80	1	20	102	90
2	109	133	60	2	111	123	68	2	79	73	86	2	121	124	89	2	68	149	107
3	75	78	86	3	82	85	79	3	103	83	127	3	62	106	88	3	123	73	96

And for the CRPS:

Table 4.11: Number of times the preferred model changes over the entire time series for Figure 4.15.

Cnsmr				Manuf				HiTec				Hlth				Other			
From\To	1	2	3	F\T	1	2	3	F\T	1	2	3	F\T	1	2	3	F\T	1	2	3
1	96	85	95	1	84	95	91	1	90	81	95	1	61	95	85	1	36	110	83
2	104	144	58	2	106	122	71	2	76	77	88	2	116	119	85	2	83	161	97
3	77	77	92	3	79	82	98	3	100	83	138	3	64	105	98	3	109	70	79

Table 4.10 and Table 4.11 show no huge differences in the consistent of each model. It can be seen that Model 2 seems to be most consistent for `Cnsmr`, `Hlth`, `Other` and `Manuf` by looking at their superior values of the diagonal in both tables. However Model 3 seems to be the most consistent for `HiTec`. To get an overview, these models are therefore picked out from Table 4.8 and displayed below:

Table 4.12: The final chosen model for the 5 Industry Portfolio.

Model	**Cnsmr**	**Manuf**	**HiTec**	**Hlth**	**Other**
F5	Manuf1 Other1 Const	Cnsmr1 HiTec1 Const	Manuf1 Other1 Const	Const Trend	Manuf1 Other1 Const

Notice in Table 4.12 that `Hlth` seems to be unpredictable by any of the other lagged stock market returns. There also seems to be very few significant coefficients for `Hlth` when looking back at the sliding model in Figure 4.6.

4.2.7 Residual Analysis

As explained in section 3.8 the VAR model assumes that the residuals resemble white noise, are serially uncorrelated and that the covariance matrix is time invariant. These assumptions are checked for the final chosen model, Model F5 in Table 4.12.

Only one model will be tested, since it is expected that the tests will not differ a lot between the different models. Also, it was seen in chapter 2 that the time series have more or less the same characteristics. Therefore the residual analysis will **not** be made for the 17 and 48 Industry Portfolios.

To check randomness and if the variance are homoscedastic the residuals are plotted over time in Figure 4.16.

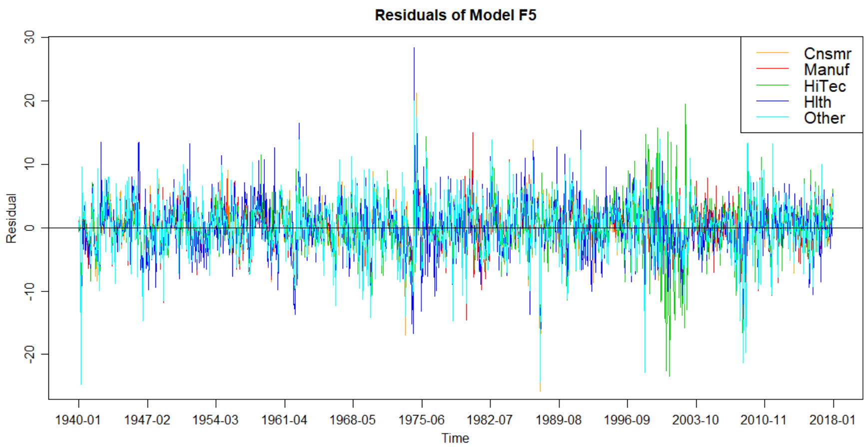


Figure 4.16: Residual plot of the final model. Volatility can be observed in the residuals.

Looking at Figure 4.16 it can be seen that the high volatility in the time series seem to be affecting the residuals, although they do look random. Especially for `HiTec` a clear volatile pattern can be seen around 2001 matching the high volatility of the time series at that time, Figure 2.1. However, to get a clearer picture of this, the residuals are plotted against the fitted values and shown in Figure 4.17.

Figure 4.17 indicates that the residuals have a *heteroscedastic* variance. Thus the assumption of time invariant covariance of the VAR Model F5 does **not** seem to be fulfilled.

To further investigate if the residuals are random, a sign test is made and presented in Table 4.13.

Table 4.13: A sign test of the final model in Table 4.12.

Sign Test	**Cnsmr**	**Manuf**	**HiTec**	**Hlth**	**Other**
95% CI	[0.46, 0.52]	[0.47, 0.53]	[0.46, 0.53]	[0.46, 0.53]	[0.47, 0.53]

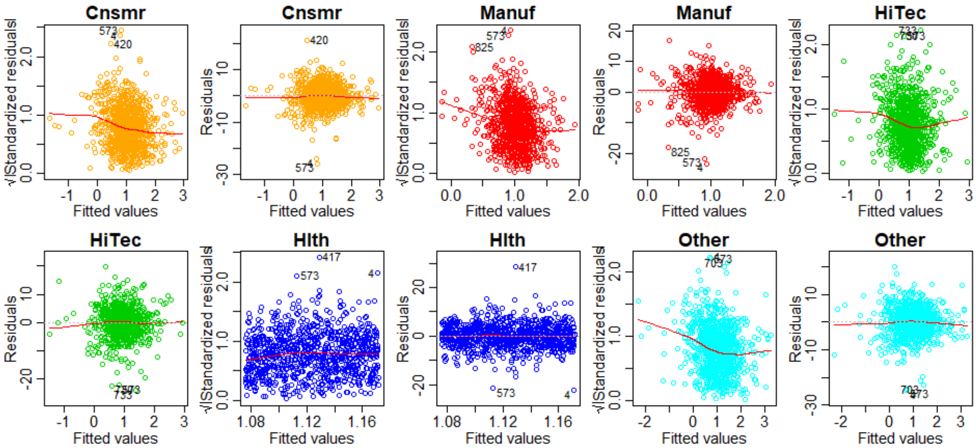


Figure 4.17: Studentised residuals and residuals against Fitted values. It can clearly be seen by the red line, that the variance is not homogeneous. Only for `Hlth` it seems like the variance could be homoscedastic, however a few outliers is still present.

The sign test shows that the confidence intervals, for each response variable, includes 0.5. Hence the null-hypothesis can not be rejected and therefore the residuals are assumed to be *random* (with a 95% confidence level) as desired.

To check if the residuals are serially uncorrelated the ACF is shown in Figure 4.18:

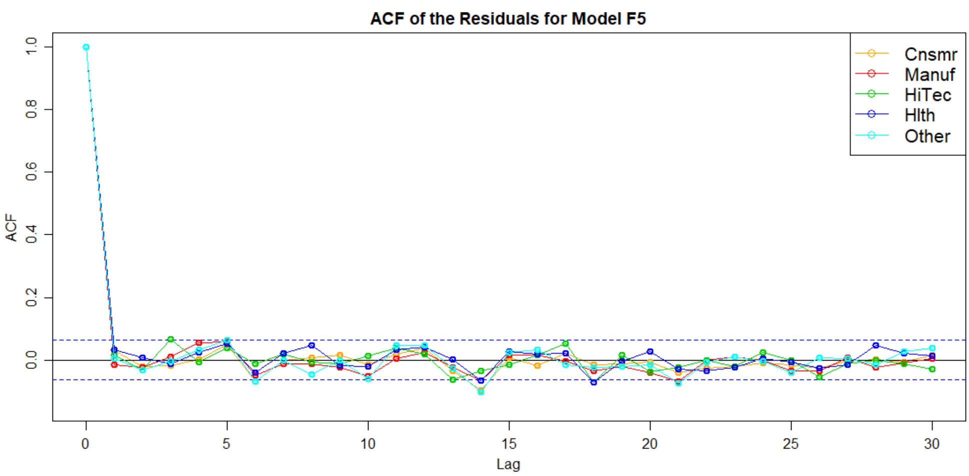


Figure 4.18: The ACF of the final model. A few unexplained lags in the ACF.

The ACF, in Figure 4.18, shows very little significance and therefore the residuals seem to be *serially uncorrelated* as desired.

The normality is checked by Q-Q-plots, Figure 4.19:

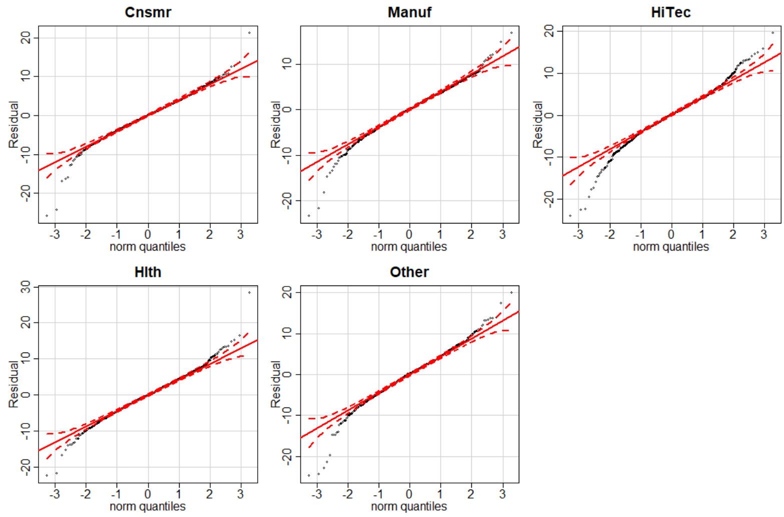


Figure 4.19: The Q-Q-plots for the final model. The residuals got heavy tails, hence they are non-normal.

Figure 4.19 shows that although most of the data is within the confidence interval, the residuals clearly show heavy tails in the figures. Hence the residuals are **not** normally distributed.

Due to clear signs of heteroscedastic variance it becomes interesting to know how much does the volatile periods influence the time series, Figure 2.1. If there is a high influence, it gives stronger reasons to believe that it indeed are the outliers that are affecting the residuals. This is tested by using Cook's distance, Figure 4.20.

From Figure 4.20 it is clear that a few observations seem to have an excessively large influence in the VAR model.

To conclude improvements can still be made. The residuals can not be deemed as white noise, due to kurtosis being present as seen from the Q-Q-plots in Figure 4.19 and heteroscedasticity seems to be present.

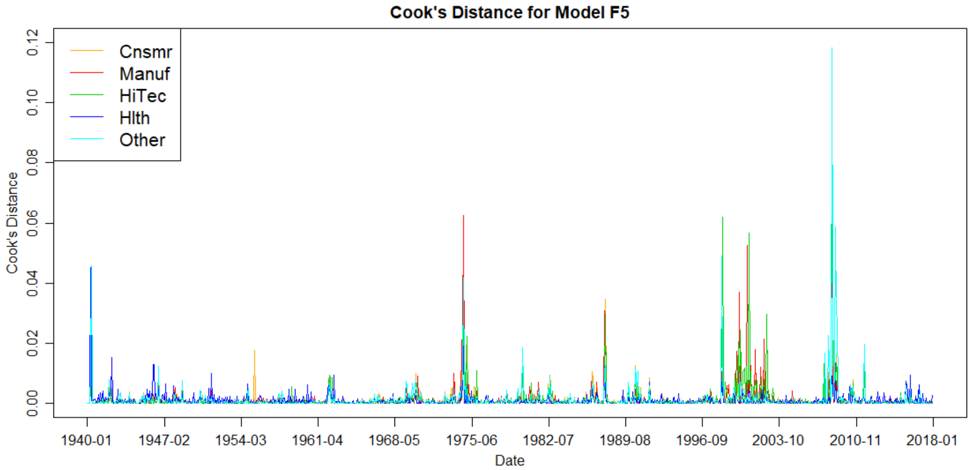


Figure 4.20: Cook's distance of Model F5. This shows the influence of the observations in the VAR model for each of the response variables.

4.3 Results for the 17 and 48 Industry Portfolios

In this section the results when using 17 and 48 Industry Portfolios will be shown. The approaches to analysing the industries will be, to a certain degree, identical to the analysing process of the 5 Industry Portfolio, section 4.2. Therefore parts of the results will only be introduced shortly and less relevant results will be omitted and/or placed in Appendix C.

4.3.1 The Sliding Models

As was determined in section 4.1 a VAR model with order 1 is used. However a new investigation for each Industry Portfolio must be made to find the optimal block size for the sliding model, see section 3.5. The optimal block length for the 17 Industry Portfolio becomes 9 years and for 48 it becomes 19 years, as can be seen in in Appendix C.1, Figure C.1-C.2.

Using these block sizes the sliding model figures for each portfolio can be made. Since these figures becomes rather cumbersome to look at they are placed in Appendix C.2, Figure C.4-C.10. Based on these figures, it is clear that there are significant variables for each of the response variables during different periods.

As described in section 3.3 the top 5% most used models for each response variable, in each portfolio, are elected for further analysis. These models are compared by their accuracy and skill as described in section 3.4. Due to the very large output it is chosen to not look into each of the steps towards the goal of finding the top three best models based on this analysis. Thus the analysis described in section 3.4 and section 3.6 are automatically done using the software `R`, [24], leading up to next section.

4.3.2 The Models

By comparing the results of the analysis described in section 3.6 the top three best models for the 17 Industry Portfolio becomes as seen in Table 4.14.
It can be seen in Table 4.14 that `Const` is usually included as a predictor both alone or together with other predictors, just as for the 5 Industry Portfolio, Table 4.8. It is noticed that the predictors start to get more specific. This also means certain variables might be the only predictor for the response variable, such as `Trans` predicting subsequent returns for `FabPr` in Model 3. Finally some sectors seem to dominate as predictors such as finances, `Finan`, appearing as predictor for 7 different industries.

Table 4.14: The top three best models for each response variable of the 17 Industry Portfolio. Note that these are the models that will be used in the ARLS.

Model	**Food**	**Mines**	**Oil**	**Clths**	**Durbl**	**Chems**
1	Const	Const	Const	Clths1 Const	Finan1	Const
2	Oil1 Const	Trend	Utils1 Finan1 Const	Oil1 Finan1 Const	Const	Utils1 Other1 Const
3	Cnsum1 Cnstr1 Const	FabPr1	Cnsum1 Trans1 Const	Clths1	Cnsum1 Rtail1 Const	Trend

Model	**Cnsum**	**Cnstr**	**Steel**	**FabPr**	**Machn**	**Cars**
1	Const	Const	Finan1	Finan1	Const	Const
2	Clths1 Steel1 Const	Oil1 Clths1 Const	Finan1 Other1	Const	Cnsum1 Rtail1 Const	Utils1 Finan1 Const
3	Steel1 Cars1 Other1 Const	Cars1 Other1 Const	Durbl1 Finan1	Trans1	Durbl1 Finan1	Rtail1

Model	**Trans**	**Utils**	**Rtail**	**Finan**	**Other**
1	Const	Steel1 Other1 Const	Const	Steel1 Other1 Const	Const
2	Oil1 Finan1 Const	Const	Cnsum1 Const	Oil1 Clths1 Const	Oil1 Finan1 Const
3	Finan1	Oil1 Const	Clths1 Const	Const	Oil1 Clths1 Const

The 48 industries top three best models have been placed in Appendix C.3 due to the large complexity of each model. What is interesting to notice is that the top models are no longer dominated by just a single `Const` compared to the other portfolios. As also can be seen in the sliding model figure, Figure C.10, `Const` is however still a part of the majority of all the chosen models.

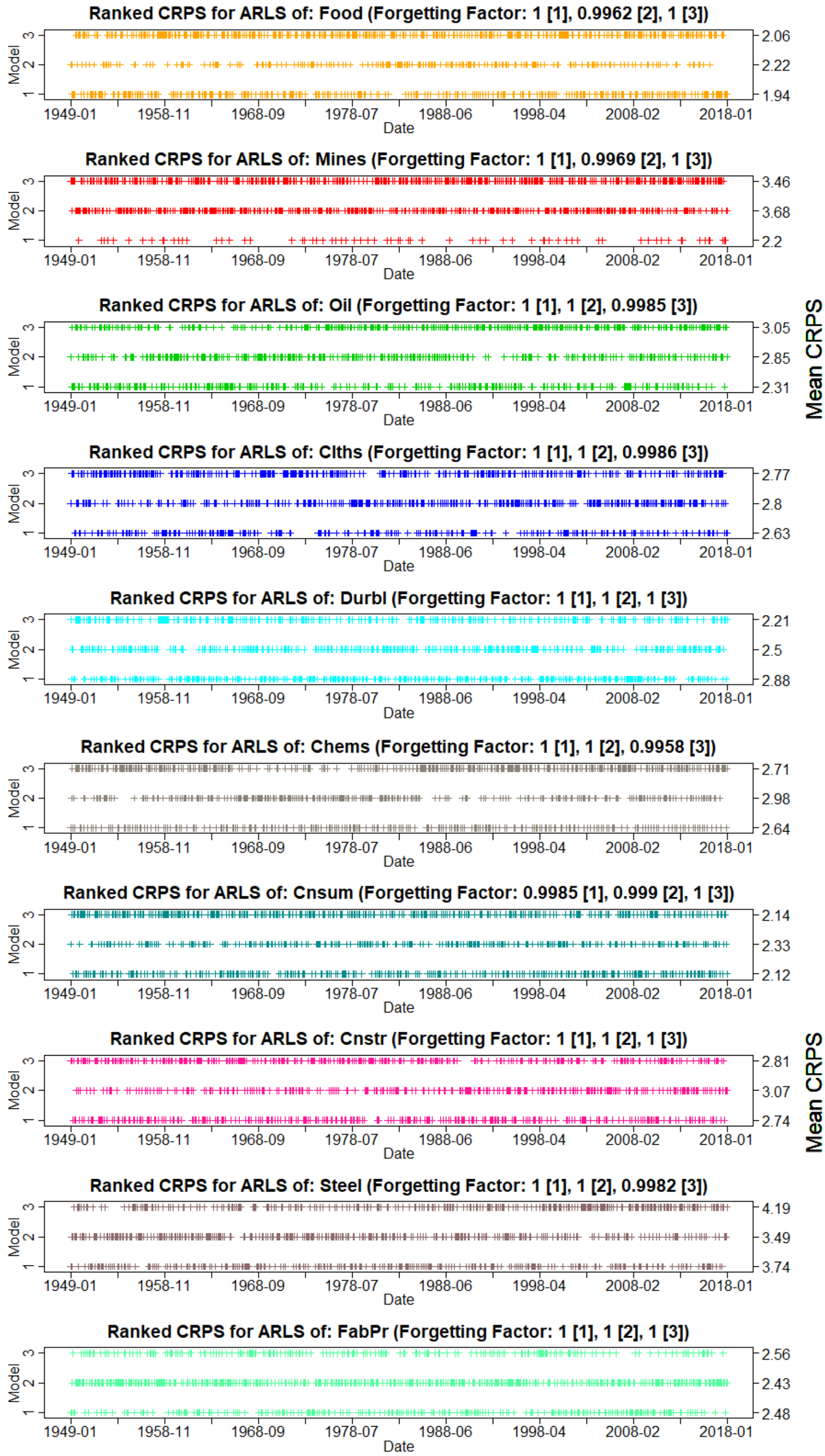
4.3.3 Adaptive Recursive Least Squares

The optimal forgetting factors have been calculated for the 17 and 48 Industry Portfolio. Tables with the forgetting factors can be found in Appendix C.4, Table C.4-C.5. It can be seen that a lot of the forgetting factors are so high, that they simply gets rounded up to 1. However for the 48 Industry Portfolio about half of the forgetting factors becomes slightly lower than 1. Those times these smaller forgetting factors are found, they are used for the next calculations.

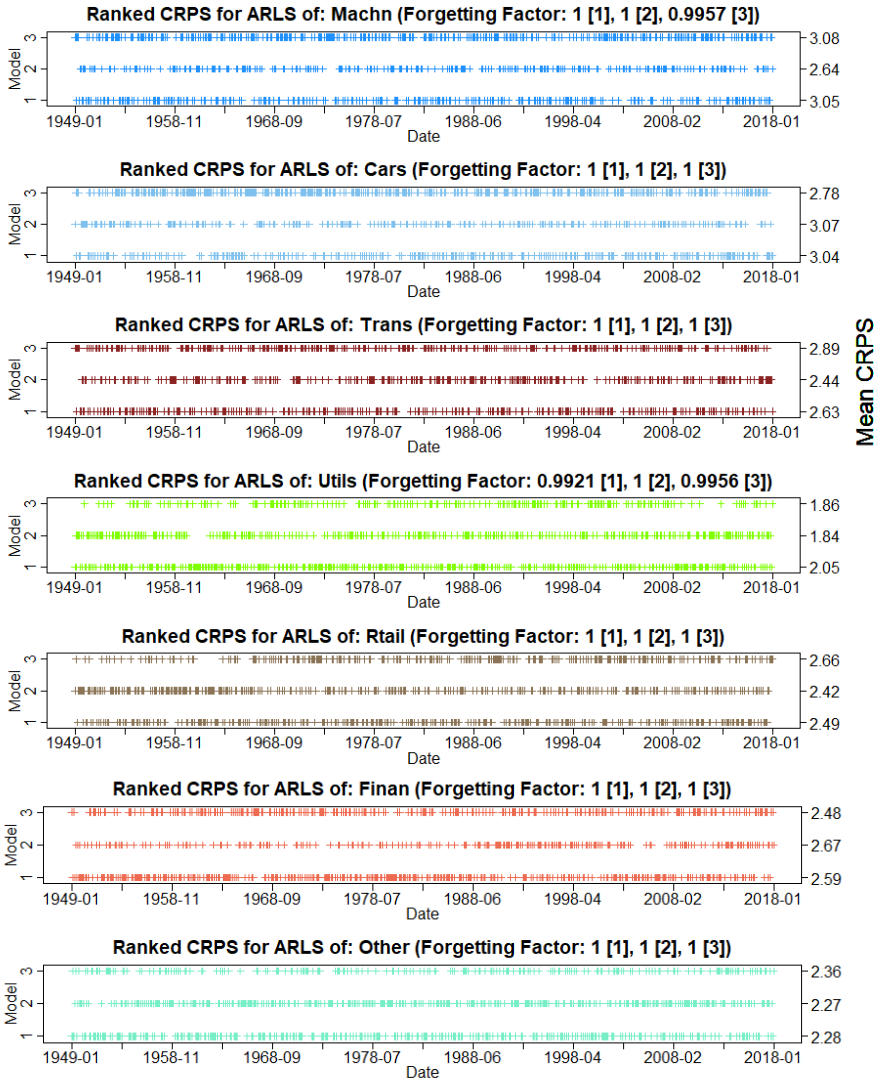
As for the 5 Industry Portfolio, the top three chosen models for each portfolio (see Table 4.14 and Table C.1-C.3) gets compared by using and ranking the ARLS as further explained in section 3.7.1. The result by the CRPS for the 17 Industry Portfolio are presented in Figure 4.21 (a-b).
It can be seen in Figure 4.21 (a-b) that it is not possible to single one model out to be the model that on average predicts the best. This is for neither of the response variables. The three different models seem to be equally good at different periods without giving a pattern on when a specific model performs the best.

Since the results based on the SSE and the CRPS are so much alike, the results based on the SSE can be found in Appendix C.4, Figure C.11. Further on the results for the 48 Industry Portfolio are so complex it can also be found in Appendix C.4, Figure C.12-C.13. The same interpretation that has been made for the 17 Industry Portfolio can be made for the 48.

The results from these figures are made more specific by looking at the number of changes between each model. This is to see how consistent each model is. The tables with this can be found in Appendix C.4. It can be seen that for some of the response variables one model can clearly be picked out as being more consistent than the others, for both portfolios. For the 48 Industry Portfolio this gets more prominent for some variables.


Ranked CRPS for ARLS of: Food (Forgetting Factor: 1 [1], 0.9962 [2], 1 [3])
2.06
2.22
1.94
Ranked CRPS for ARLS of: Mines (Forgetting Factor: 1 [1], 0.9969 [2], 1 [3])
3.46
3.68
2.2
Ranked CRPS for ARLS of: Oil (Forgetting Factor: 1 [1], 1 [2], 0.9985 [3])
3.05
2.85
2.31
Ranked CRPS for ARLS of: Clths (Forgetting Factor: 1 [1], 1 [2], 0.9986 [3])
2.77
2.8
2.63
Ranked CRPS for ARLS of: Durbl (Forgetting Factor: 1 [1], 1 [2], 1 [3])
2.21
2.5
2.88
Ranked CRPS for ARLS of: Chems (Forgetting Factor: 1 [1], 1 [2], 0.9958 [3])
2.71
2.98
2.64
Ranked CRPS for ARLS of: Cnsum (Forgetting Factor: 0.9985 [1], 0.999 [2], 1 [3])
2.14
2.33
2.12
Ranked CRPS for ARLS of: Cnstr (Forgetting Factor: 1 [1], 1 [2], 1 [3])
2.81
3.07
2.74
Ranked CRPS for ARLS of: Steel (Forgetting Factor: 1 [1], 1 [2], 0.9982 [3])
4.19
3.49
3.74
Ranked CRPS for ARLS of: FabPr (Forgetting Factor: 1 [1], 1 [2], 1 [3])
2.56
2.43
2.48
Model
Date
1949-01 1958-11 1968-09 1978-07 1988-06 1998-04 2008-02 2018-01
Mean CRPS

(a)



(b)

Figure 4.21: The best performing model over time, for the **17 Industry Portfolio**, when continuously introducing a new observation. Each + sign indicates the best model at time t according to the **CRPS**. The left axis shows the model number according to Table 4.14 and the right axis shows the $\overline{CRPS}$ of the corresponding plotted signs. The used forgetting factor can be seen at the top of each plot, where the number inside the brackets is the model number.

4.4 Including the Predictor Data

In this section it is investigated whether the Predictor data set, described in section 3.9, should be included. Note that when referring to the criteria it means the criteria for being included or not, as stated in section 3.9.

To investigate the first criterion, the significance of the variables, a sliding model is made for the 5 Industry Portfolio including the Predictor data set. A block length of 9 years will be used to make it possible to compare these results by the ones made without the Predictor data:

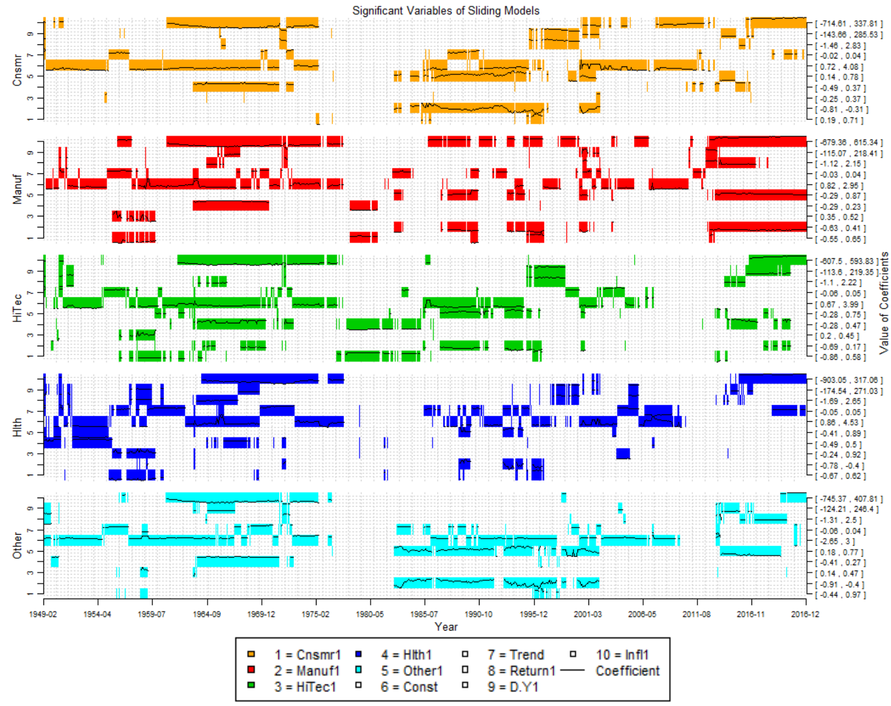


Figure 4.22: The sliding model including the predictor data set. Each date at the x-axis represents the last date in one block of 9 years. Thus the dates are only there for guidance. The left axis shows the response variable and the corresponding predictors are given by the numbers from 1 to 7. The right axis is the range of the values of the coefficients for the significant predictors. The black lines within the coloured blocks shows how the coefficient values fluctuate. Finally the coloured blocks indicate that the predictor is significant and is included in the model.

Looking at the sliding model in Figure 4.22, it is clear that the variables in the Predictor data is significant at certain periods. However it can be seen that the data from the 5 Industry Portfolio are also highly significant. Hence including the Predictor data does not exclude the significance of the variables from the industry portfolio.

Criterion 2, the normality assumptions, are tested by using a Jarque-Bera (JB) test explained in section 3.8.3. The following figure shows the results of the JB test with and without including the Predictor data to the 5 Industry Portfolio:

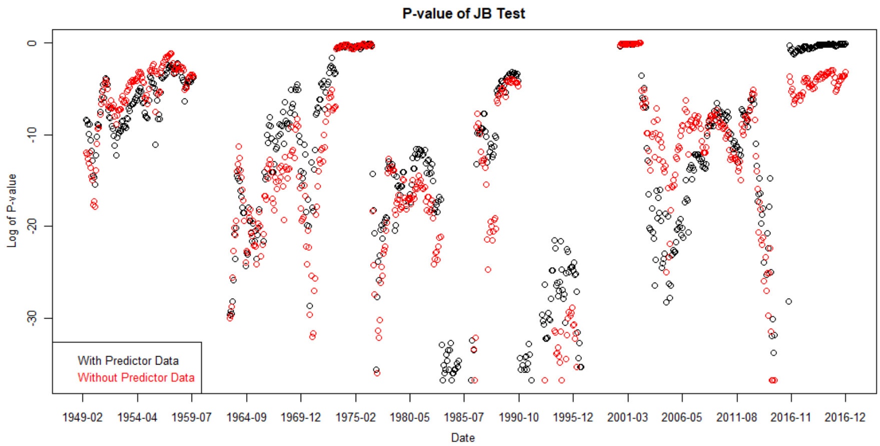


Figure 4.23: The p-values of each of the models in the sliding model with and without the Predictor data. Note that it is the logarithm of the p-values that are plotted to make it easier to interpret this figure.

It is clear by looking at the p-values of the JB test, in Figure 4.23, that the p-value for both with and without the Predictor data are more or less the same. The p-values are very significant in both cases and only for a few values the null-hypothesis, of the data being normally distributed, can not be rejected. For both data sets only slight and subtle differences can be noticed.

Finally the third criterion is investigated by comparing the accuracy and skill for different block sizes of the sliding model with and without the Predictor data as can be seen in Figure 4.24.

Figure 4.24 shows that overall the error is better when excluding the Predictor data, but only a very slight difference can be observed. However the lowest error was still found when excluding the Predictor data (the red bar).

Note that no more results including the Predictor data will be presented due to the discussion in section 5.1.7.

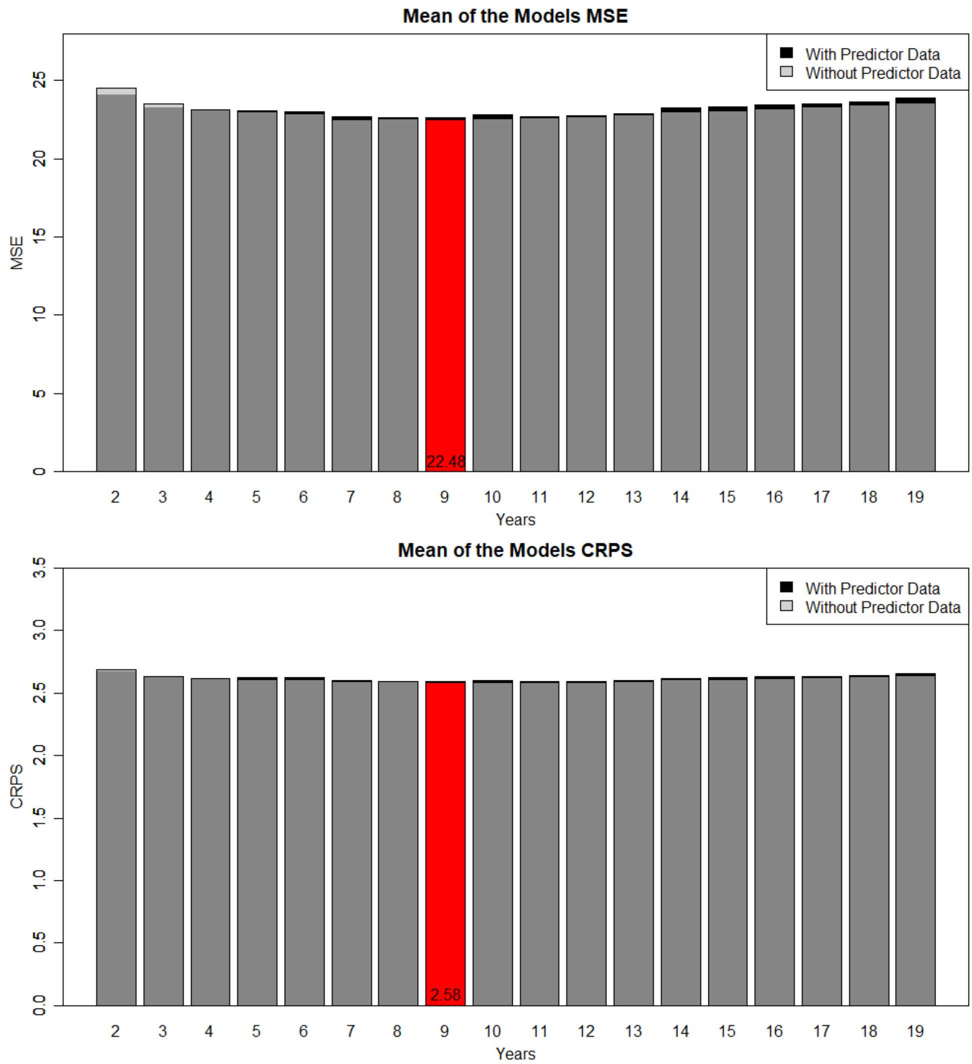


Figure 4.24: For each block size, the mean of the errors for the top four used models of all response variables, are calculated and presented as a bar in this barplot. The red bar indicates the lowest error, with the corresponding value of the error given in the bar. The lowest error is found when excluding Predictor data. Furthermore notice the grey colour is transparent, to show the bars behind it, therefore it might appear darker at places.

CHAPTER 5

Discussion

Throughout the report a number of simplifications are made, due to time limits and out of scope material. In this chapter a discussion of improvements and ideas will be made, to address problems and suggestions for future work. Furthermore the results found will be discussed. It is emphasised that these results only apply to the American stock markets introduced in chapter 2.

5.1 The Results

This part of the discussion will be focused on the results that have been found throughout chapter 4.

5.1.1 The Sliding Model

Looking at the sliding model for the 5 Industry Portfolio, in Figure 4.6, it is seen that adding an intercept (`Const`, 6) seems to be significant a lot of times. When the intercept is not significant it is often replaced by a very small trend ranging from $[-0.05, 0.04]$, which means that the time series are expected to only increase very little within the different sub periods including the trend. The trend can be explained by the local volatile periods within the industries where a significant increase or decrease of returns are observed, see Figure 2.1.

From the sliding model figure it is further on noticed how coefficients seem to follow in pairs. When `Other` is present, then `Manuf` is also present, as is also seen in the table where the five most significant combinations are counted, Table 4.3. The same goes for `Cnsmr` and `HiTec`, which also seems to follow each other. It is therefore expected to see pairs of these in the chosen models to occur. Another interesting part is to see how the response variables depends on different predictors. As an example `Cnsmr` seems mainly to be predicted by the lagged variables of `Manuf`, `Hlth`, `Other` and the `Const` and `Trend`. Further on it can be seen that the black coefficient lines in the sliding model, Figure 4.6, seem to be fairly stable, which indicates that the coefficients are stable for the significant periods.

Empty regions within the sliding model, Figure 4.6, can be observed. This is periods where no significant variables, within a 5% significance level, were found to be present. This corresponds to fitting a model to the error term only. A link between extreme volatile periods in the time series of the observations, Figure 2.1, and the empty regions are found. The empty regions are often found when these very volatile periods occur.

When doing the same interpretation of the sliding models for the 17 and 48 Industry Portfolios the same results are found, Figure C.4-C.10. Predictors are rarely isolated, which means that often several significant predictors are present at the same time. Further on the intercept `Const` does not get as prominent as for the 5 Industry Portfolio. This can especially be seen at times for the 48 Industry Portfolio. Once again empty regions can be observed, mostly for the highly volatile period around 2001.

Overall indications of predictability are found to be present for each of the 5, 17 and 48 Industry Portfolios.

5.1.2 Predictability

Looking at the sliding models, Figure 4.6 and Figure C.4-C.10, it can be seen that certain markets are found to be significant as predictors. Summarising the 5 Industry Portfolio, it looks like all of the variables are found to be significant at some intervals and is able to act as a predictor for one another, although some are more prominent then others. To illustrate the results of the sliding model the significant variables have been mapped for a single response variable, based on the number of times a predictor is shown to be significant in the sliding model, Table 4.2. Because the interest is in supply chain predictability between industries, Figure 5.1 will focus on the industry predictors and will not show the `Const` and `Trend`.

By looking at the thickness of the arrows in Figure 5.1 it can be seen that `HiTec` is in general *rarely* used to predict any of the other industries. The lagged return of `Other` seems to be good at predicting itself and `Cnsmr`. Also in general `Other` seems to be a good predictor appearing significant a lot of the times for all the response variables. `Other` includes a lot of different markets and this might explain why it seems like an all around good predictor.

`Manuf` seems to be mainly good at predicting `Other` and `Cnsmr`. `Cnsmr` seems to be good at predicting `Manuf`, `HiTec` and `Hlth`, but is rarely the most significant predictor. Overall `Hlth` itself does not seem to be very predictable, with the lowest total amount of significant predictor variables compared to the other markets and the lagged return of itself as the best predictor. The lagged return of `Hlth` however seems to be very good at predicting `HiTec`. This is further emphasised from the top 3 models in Table 4.8, where it can be seen that the second model actually contains `Hlth` as the only predictor for HiTec.

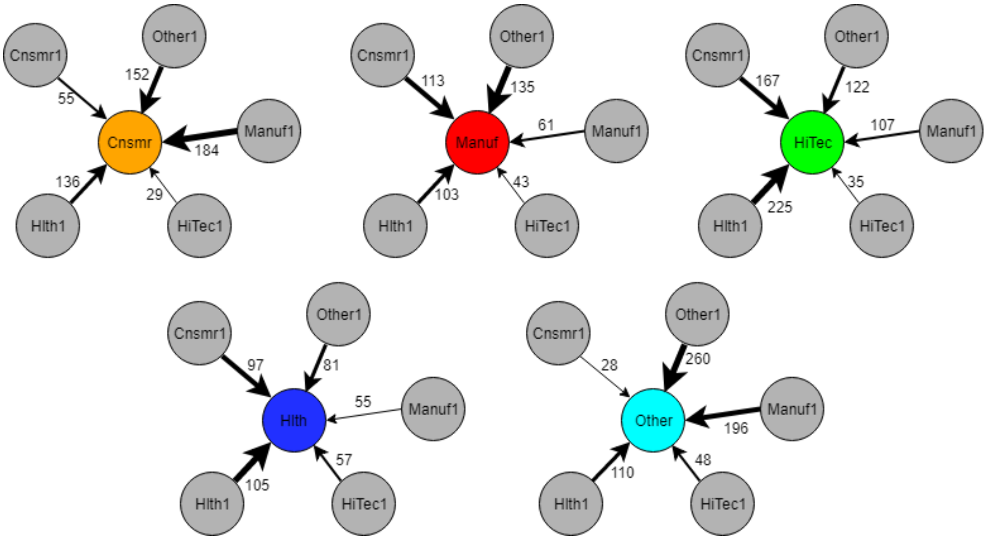


Figure 5.1: Illustration of the most significant predictors for each industry based on Table 4.2. A thicker arrow means the lagged return is significant more times, with the thickest arrow being the predictor significant most times. The arrows ranks the number of times a predictor is significant, but do not illustrate the relative significance. Therefore the number of times the variable appears significant for the response variable is indicated by a number next to the arrows, based on Table 4.2.

The sectors are as mentioned very general for the 5 Industry Portfolio and is composed by a large range of companies. This means it is not very clear if there is a specific industry of e.g. Manuf that seems to be good at predicting the different sectors of Cnsmr. To get an idea of this, larger and more specific data sets (e.g. 17 and 48 portfolios) should be used.

To give a more general view of the supply chain between the sectors an illustration of the supply chain for the last chosen model for the 5 Industry Portfolio, Model F5 in Table 4.12, can be seen in Figure 5.2.

In Figure 5.2 the supply chain of Model F5 (Table 4.12) can be observed. It is seen that according to Model F5 Hlth is unpredictable by the other industries. As an example look at Other, there seems to be autocorrelated returns. This means that the lagged returns of Other predict subsequent returns of itself. Furthermore lagged returns of Other seems to have significant statistical power in predicting HiTec and Cnsmr. Even though the 5 Industry Portfolio is very general, it can be confirmed that some industries returns seems to be predictable by others.

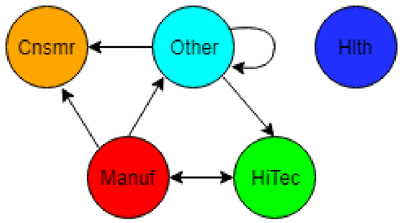


Figure 5.2: An illustration of the supply chain for the 5 Industry Portfolio based on Model F5, Table 4.12. Arrows indicate if the returns of one sector predicts the returns of another the following month in a statistically significant manner.

Figure 5.2 is based on a a single model, however by looking at the ARLS rankings over time, Figure 4.14-4.15, it is seen that it is not possible to single one model out. The different models are good during different periods. This is further emphasised by looking at the tables showing the changes between each model, Table 4.10-4.11. No single model seems to be remarkably better compared to the others. It is however possible to notice that a few of the models does not change as often and remains as the best predictor for a longer period of time. These models are the ones that where picked out and shown in Table 4.12. As seen in Model F5 all of the industries use other industries as predictors except for `Hlth`, as mentioned earlier, having only intercepts and a trend as predictors. However since these models, in the majority of the cases, contains other industries this gives further reasons to believe that there is statistically significant predictability between stock market returns.

Lastly, when looking at the table with the top 3 overall best models, Table 4.8, it is evident that when using the industries as predictors for the 5 Industry Portfolio they are often accompanied by another predictor. This could be an intercept, trend or other industries. Thus there are no industries that seems to be exceptionally good at predicting another industry by itself.

Summarising the 17 and 48 Industry Portfolios the same conclusions can more or less be made. Without making the same illustration as for the 5 industries, it can still be seen, by looking at the sliding models, that each industry can be explained by lagged returns of other industries. By looking at the top 3 overall best models for each industry it can also be seen that most models contain more than a single industry as a predictor, although there are times when a single industry are used, Table 4.14 and Table C.1-C.3. Furthermore it is also noticed that most of the predictors are dominated by certain sectors, such as `Finan`.

When comparing these models with the ARLS ranking method, Figure C.11 and Figure 4.21-C.13, it is evident that all models are good during different periods. Neither with these data sets is it possible to single one model out as being remarkably better than the others. The interesting part comes when looking at the tables with the changes over time, Figure C.14-C.16. Here it becomes clearer that a few industries indeed have models that seems better than the other models, by comparing consistency of the models. It is also noticed that the superior models rarely contains only intercepts. Thus using a data set composed of more specific sectors seems to emphasise the predictability between certain industries, which in turn might improve predictions of subsequent monthly returns.

5.1.3 Performance of the Models

The only part where the accuracy of the predictability are measured is when calculating and displaying the MSE for the 5 Industry Portfolio in section 4.2.4.1, Table 4.5. Here the accuracy of the predictions get compared with a reference error as explained in section 3.4.1, equation (3.13). From the results in the MSE table it is clear to see that the reference error is much higher than for the other fitted models. Thus this shows that the found models predict better than simply guessing that the next prediction will be the same as the last observation.

5.1.4 The `Const` Model

Looking at the top 3 models for the 5 and 17 Industry Portfolio, Table 4.8 and Table 4.14, they almost always contain only an intercept, `Const`. The reason to this could be that the time series are fairly stationary with only periods of volatility. An intercept will therefore be highly significant at most times.

From the SSE and CRPS ranking figures for the sliding model, Figure 4.11 and Figure 4.13, it is noticed that the model with only intercepts usually predicts average. It is rarely the worst nor the best. This makes sense since the time series fluctuates around a fairly constant mean value, making the predictions with the intercept never too far off from the real observations. However the model containing only intercepts is a rather uninteresting model based on the goals of finding predictability between markets. Therefore not much weight have been put on this model.

5.1.5 Problem with the Assumptions

The display of little autocorrelation and non-normal kurtosis is such a common phenomena of financial data that it is often termed "*stylised facts*" within the financial world [35]. The different portfolios are no exception to this phenomena. It is seen in the Q-Q plots of the observations for the portfolios, Figure 2.3, that the data is non-normal due to the heavy tails, which indicates excessive kurtosis. From Figure 4.1

almost no autocorrelation can further on be observed.

The cause of the excessive kurtosis are from outliers in the time series found in the volatile periods. This will make the OLS residuals non-normal in the VAR model. The OLS estimate is still a reasonable estimator in the face of non-normal errors, based on the Gauss-Markov theorem, which does not assume normality [22]. An issue is that the tests used (e.g. the t-test) assumes normality within the residuals. The p-values will therefore be approximations and the inference might not be robust. In this thesis it is assumed that, based on the Central Limit Theorem as mentioned in chapter 3, the p-values will allow for an reliable inference. The same goes for the confidence intervals of the predictions, which also becomes approximations.

A bigger problem is the heteroscedastic variance as is observed in the time series plot, Figure 4.16, and in the figure showing the residuals against the fitted values, Figure 4.17. If residuals exhibit heteroscedasticity, then the variance of the estimators will be biased and therefore the assumptions of the Gauss-Markov theorem, [22], will no longer be satisfied. This means that the OLS will no longer have the smallest variance among best linear unbiased estimators, [36]. The usual t-statistics and confidence intervals will therefore be invalid, even in large samples, [36].

The heteroscedasticity comes from outliers in the time series. This also makes the OLS put unreasonable weight on certain observations as is seen in the Cook's distance, Figure 4.20. This might be especially apparent for smaller samples, where the estimate can be driven by a few highly volatile observations.

Hence the assumption of homoscedastic variance and normality were not fulfilled and the results are therefore not to be completely trusted.

5.1.6 The Forgetting Factor

One of the reasons of using a forgetting factors was to see if a lower emphasis on volatile observations could be made and thereby make the estimators more efficient. This however does not seem to be very beneficial in all cases. As can be seen in the table of optimal forgetting factors, Table 4.9 and Table C.4-C.5, many of the forgetting factors have values equal to 1. In [22] it is mentioned that if there is a sudden and total change in the dynamics, then using a method with a forgetting factor is inappropriate. This is due to that $\boldsymbol{R}_t$ is based on prior observations, see equation (3.19). Therefore $\boldsymbol{R}_t$ will be containing information on the direction of the parameter changes. If the observations make radical changes, then $\boldsymbol{R}_t$ will make a wrong estimate, since it is also based on observations prior to the change [22]. Thus this gives reason to believe that the quite volatile time series will make it difficult for the SSE to decrease, which can be a reason for the high values of the forgetting factors.

5.1.7 Inclusion of the Predictor Data

From the investigation made in section 4.4 it can be seen that criterion 1 (the variables should be significant) is fulfilled to an extent. It can be seen in the sliding model containing the Predictor data, Figure 4.22, that the Predictor data indeed are significant, but it does not exclude the significance of the predictor variables from the 5 Portfolio Industries. Therefore it is concluded that the Predictor data set is not an *alternative* explanation of why some industries might predict other industries returns.

Looking at Figure 4.23 it can be seen that the inclusion of the Predictor data does not make any considerable improvements of the p-values. At some dates it gets better and at others it does not. These changes are not enough to believe that there has been a large enough improvement of the normality residual assumptions. Hence criterion 2 is not fulfilled.

Lastly Figure 4.24 shows that including the Predictor data the errors are in general higher, albeit only by a tiny amount. Due to the added complexity of adding the predictor and the very slightly worse error, it is concluded that neither the third criterion is fulfilled.

Since all the criteria are not fulfilled it is chosen to **not** include the Predictor data when using the 5 Industry Portfolio. Further on since it is already excluded using only the 5 Industry Portfolio, it is concluded to also be excluded in the 10, 17 and 48 Industry Portfolio.

5.2 Future Work

There are a few simplifications and results that lay ground to future work. This section will go through possible ideas on how this thesis can be further developed.

5.2.1 Forgetting Factor Over Time

The forgetting factors displayed in Table 4.9 and Table C.4-C.5 is fairly high a lot of the times. A small investigation were made by looking at the forgetting factor over time for the 5 Industry Portfolio giving following figure:

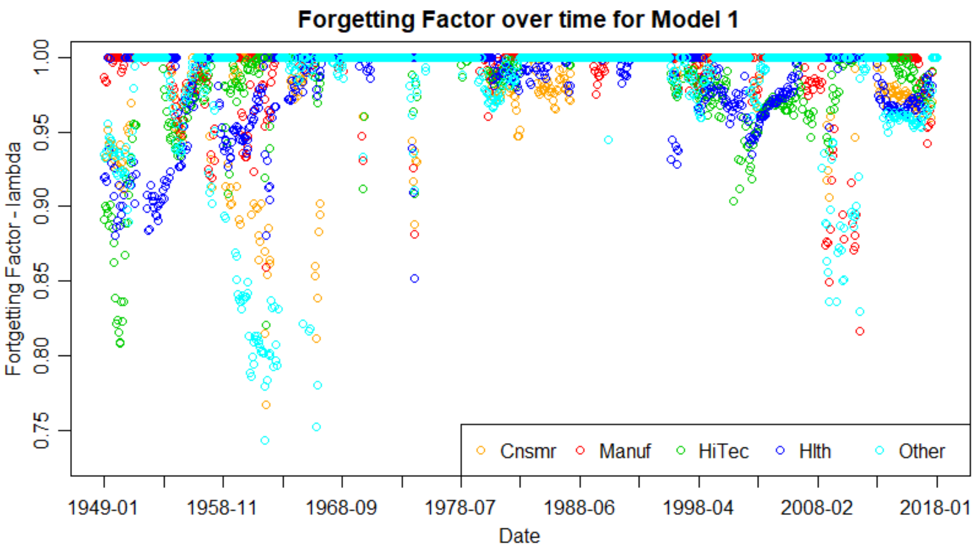


Figure 5.3: The forgetting factor, λ, calculated over time with blocks of 9 years, for Model 1 (Table 4.8) using the 5 Industry Portfolio.

Looking at Figure 5.3, it is clear that there are periods where including a forgetting factor would make sense since it is getting a lot smaller at periods. It would be interesting to include a lower forgetting factor ($\lambda < 1$) for specific time intervals and see how the models perform based on this.

5.2.2 Hidden Markov Model

As seen in the sliding models, Figure 4.6 and Figure C.4-C.10, variables are found to be significant during different sub-periods. Further in section 5.1.2 it is discussed that different models seem to be optimal at different times. A hidden Markov model will allow optimal transitions between the models based on the sub-periods [37]. This might help improving predictions, since the optimal model is chosen by the state of the time series and hereby avoids using non-significant predictors for the sub-period [38].

5.2.3 ARCH Models

Throughout the thesis it is assumed that the covariance matrix of the distribution is time invariant. However as mentioned before, the variance of the time series are not constant but changes over time. To account for the time dependent variance an autoregressive conditional heteroscedasticity (ARCH) model could be used [26].

This model incorporates the conditional variance and will therefore take into account volatile periods as the ones present in this data, Figure 2.1.

5.2.4 Investigating Higher Order

In section 4.1 and Appendix B.1 all the results of the criteria showed that a VAR model of order 1 should be used for each of the data sets. However when looking at the figures containing the ACF, PACF and CCF for the different portfolios it is clear that there are a few significant lags for higher orders as well, $k > 1$. Therefore including a higher order would be interesting to investigate. There might be industries where a higher lag order would benefit the predictability. To show an example the sliding model of the 5 industries with a higher lag order is presented in Appendix E, Figure E.1. It is clear that some of the higher order industries are significant making this an interesting subject.

5.2.5 Block Sizes

When looking at the block size in the sliding model, it is chosen to look at full years to keep it computational simplistic. If the sliding model were to be given its full potential, a consideration of allowing more than strictly full years should be made. Hence optimise on a monthly basis.

5.2.6 Significance of Variables

When evaluating the significance of the predictors in the VAR model, a fixed threshold is used for the t-test. The threshold should be updated when removing a predictor from the model, due to having 1 additional degree of freedom for each predictor parameter removed. This is a minor issue for models containing few variables, however when having a lot of variables, e.g. for the 48 dataset the degrees of freedom will vary more. An updated script, based on the `Restrict` function in [23], has been implemented using a dynamic threshold for the t-test. A slight difference in some of the results was noticed, but due to time restrictions, this has not been used in the results.

5.2.7 Additional Models

Throughout the results, it can be seen that it is not always the case that the most occurring model, based on the sliding models, is the most optimal. When choosing the top 3 models, Table 4.7, it is seen that Model 7 is included, which is the model with least appearance in the sliding model of the chosen top 5%. Since the last model of the chosen top 5% seems relatively good, looking at more of the unique combinations of models could be interesting.

CHAPTER 6

Conclusion

This thesis has investigated if it is possible to find predictability between monthly American stock market returns. This is done by using a VAR(1) model that are fitted over sub-periods of the entire time series and restricted such that it is possible to observe the lagged predictors significance over time, i.e. the sliding model. The most occurred predictors are then compared by their accuracy and skill, such that three models can be singled out as the best performing ones. Lastly these three models are compared by ranking their MSE and CRPS calculated by using ARLS. This is done to find the predictors that consistently predicts a response variable the best.

The results show that monthly lagged returns have predictive power of forecasting future returns of others. By the sliding model it was shown that there are statistically significant predictors for all stock market returns, in the 5, 17 and 48 Industry Portfolios, with some industries having a stronger predictive power than others. This indicates that a supply chain between the different industries are present. By the use of the ARLS it is found that different models seem to be consistent and best at different times throughout the time series.

APPENDIX A

Appendix A

A.1 Summary Statistics

Table A.1: Summary statistics of 10 industry portfolios for data between 1940-01 to 2018-01

	Date	NoDur	Durbl	Manuf	Enrgy	HiTec	Telcm
Min.	1940-01-01	-21.030	-32.630	-27.330	-24.470	-25.960	-16.3600
1st Qu.	1959-07-01	-1.230	-2.480	-1.790	-2.150	-2.460	-1.2300
Median	1979-01-01	1.080	0.970	1.310	1.050	1.190	0.9000
Mean	1978-12-31	1.033	1.002	1.014	1.106	1.094	0.8543
3rd Qu.	1998-07-01	3.600	4.480	4.120	4.420	4.780	3.1700
Max.	2018-01-01	18.880	42.630	17.510	24.560	20.760	21.3600

	Shops	Hlth	Utils	Other
Min.	-28.230	-21.21	-19.9400	-24.010
1st Qu.	1.800	-1.84	-1.3000	-1.820
Median	1.150	1.10	1.0600	1.380
Mean	1.066	1.12	0.8986	0.999
3rd Qu.	3.880	3.99	3.3200	3.910
Max.	25.860	29.52	18.8400	20.220

Table A.2: Summary statistics of 17 industry portfolios for data between 1940-01 to 2018-01

	Date	Food	Mines	Oil	Clths	Durbl	Chems
Min.	1940-01-01	-22.170	-32.740	-24.390	-31.490	-29.3200	-27.9500
1st Qu.	1959-07-01	-1.060	-2.880	-2.080	-2.200	-2.1900	-2.2600
Median	1979-01-01	1.090	0.840	1.160	1.060	1.1000	1.0200
Mean	1978-12-31	1.044	1.007	1.111	1.055	0.9658	0.9699
3rd Qu.	1998-07-01	3.360	5.210	4.390	4.350	4.4600	4.1800
Max.	2018-01-01	20.420	22.450	24.060	26.730	29.2400	22.3000

	Cnsum	Cnstr	Steel	FabPr	Machn	Cars	Trans
Min.	-19.240	-29.180	-32.9100	-28.5800	-28.320	-28.430	-28.950
1st Qu.	-1.470	-2.360	-3.3200	-2.0300	-2.330	-2.590	-2.110
Median	1.270	1.120	1.0200	1.1300	1.290	0.910	1.300
Mean	1.061	1.062	0.8833	0.9256	1.078	1.031	1.052
3rd Qu.	3.740	4.380	4.9000	4.0300	4.570	4.750	4.370
Max.	29.000	25.870	30.6700	18.8200	19.320	31.740	18.060

	Utils	Rtail	Finan	Other
Min.	-19.9400	-28.16	-26.130	-22.8100
1st Qu.	-1.3000	-1.84	-1.550	-1.4200
Median	1.0600	1.08	1.280	1.2000
Mean	0.8986	1.07	1.068	0.9449
3rd Qu.	3.3200	4.01	4.140	3.7200
Max.	18.8400	26.71	21.120	15.3200

Table A.3: Summary statistics of 48 industry portfolios for data between 1969-07 to 2018-01.

	Date	Agric	Food	Soda	Beer	Smoke	Toys
Min.	1969-07-01	-29.060	-17.880	-26.260	-19.760	-24.930	-34.410
1st Qu.	1981-08-16	-2.675	-1.190	-2.195	-1.665	-2.425	-3.670
Median	1993-10-01	0.860	1.030	1.430	1.120	1.840	1.030
Mean	1993-09-30	1.000	1.113	1.140	1.153	1.471	0.750
3rd Qu.	2005-11-16	4.685	3.540	4.645	4.260	4.995	5.145
Max.	2018-01-01	28.880	19.590	38.270	26.090	32.470	26.880

	Fun	Books	Hshld	Clths	Hlth	MedEq	Drugs
Min.	-31.860	-25.1900	-21.6400	-30.900	-39.110	-20.560	-19.110
1st Qu.	-2.650	-2.4850	-1.8150	-2.740	-3.715	-1.945	-1.870
Median	1.440	0.5100	1.0600	1.080	1.080	1.310	1.130
Mean	1.362	0.8981	0.8538	1.066	1.023	1.062	1.102
3rd Qu.	5.885	4.4150	3.6300	4.825	5.540	4.435	4.175
Max.	38.770	30.7400	18.5400	32.370	36.470	21.030	31.800

	Chems	Rubbr	Txtls	BldMt	Cnstr	Steel	FabPr
Min.	-28.000	-30.570	-32.510	-30.670	-31.1000	-32.9100	-26.6700
1st Qu.	-2.345	-2.410	-2.855	-2.330	-3.4700	-3.8000	-3.6700
Median	1.110	1.300	1.120	1.210	0.8500	0.6700	0.7100
Mean	1.053	1.030	1.034	1.035	0.9126	0.7725	0.7486
3rd Qu.	4.330	4.395	4.895	4.435	5.3900	5.1950	5.2650
Max.	22.050	31.950	59.040	35.510	24.0100	30.6700	30.3800

	Mach	ElcEq	Autos	Aero	Ships	Guns	Gold
Min.	-31.190	-32.200	-36.4100	-30.230	-32.270	-30.080	-33.5700
1st Qu.	-2.790	-2.480	-3.0200	-2.370	-3.035	-2.535	-5.6600
Median	1.330	0.990	0.8200	1.560	1.130	1.450	0.5600
Mean	1.012	1.152	0.8614	1.227	1.102	1.321	0.9046
3rd Qu.	4.810	4.925	4.7500	5.085	5.235	5.145	6.7900
Max.	23.050	23.220	49.5700	25.330	29.150	32.640	78.6800

	Mines	Coal	Oil	Util	Telcm	PerSv	BusSv
Min.	-34.750	-37.940	-18.210	-12.6500	-16.3600	-28.2500	-27.540
1st Qu.	-3.530	-5.230	-2.440	-1.5600	-1.7300	-3.2300	-2.630
Median	0.650	0.710	0.890	1.0100	1.1600	0.9400	1.310
Mean	1.047	1.087	1.011	0.8931	0.9438	0.6356	1.119
3rd Qu.	5.405	6.925	4.640	3.4050	3.9000	4.7700	4.970
Max.	26.910	46.060	24.660	18.8400	21.3600	24.4700	25.430

	Comps	Chips	LabEq	Paper	Boxes	Trans	Whlsl
Min.	-32.3600	-32.230	-30.150	-26.350	-28.240	-27.9000	-28.6500
1st Qu.	-3.1050	-3.240	-3.310	-2.355	-2.155	-2.5000	-2.2550
Median	0.8200	1.570	1.270	1.130	1.120	1.2400	1.2200
Mean	0.8982	1.071	1.055	1.015	1.014	0.9807	0.9549
3rd Qu.	4.8900	5.800	5.110	4.185	4.390	4.5000	4.0150
Max.	24.2400	27.270	21.560	24.270	20.870	19.0200	18.1200

	Rtail	Meals	Banks	Insur	RlEst	Fin	Other
Min.	-29.140	-31.330	-27.250	-26.460	-39.4200	-25.910	-27.2000
1st Qu.	-2.130	-2.430	-2.190	-1.965	-3.0250	-2.460	-3.1000
Median	0.820	1.400	1.150	1.520	0.6400	1.480	0.7800
Mean	1.081	1.060	1.016	1.081	0.5549	1.130	0.4813
3rd Qu.	4.650	4.675	4.830	4.595	4.6050	5.055	4.5350
Max.	27.070	27.960	25.060	26.840	69.2300	19.560	21.2200

APPENDIX B

Appendix B

B.1 Verification of the VAR(1) model

Table B.1: Different test statistics of VAR(p) model for 17 industries data set, the lowest values is consistent for the model with order 1. The red colour indicates the minimum value.

p	1	2	3	4	5
AIC(p)	3.771e+01	3.790e+01	3.806e+01	3.825e+01	3.838e+01
HQ(p)	3.846e+01	3.923e+01	3.997e+01	4.073e+01	4.145e+01
SC(p)	3.968e+01	4.138e+01	4.306e+01	4.476e+01	4.641e+01
FPE(p)	2.393e+16	2.895e+16	3.398e+16	4.097e+16	4.720e+16

Table B.2: Different test statistics of VAR(p) model for 48 industries data set, the lowest values is consistent for the model with order 1. The red colour indicates the minimum value.

p	1	2	3	4	5
AIC(p)	1.146e+02	1.152e+02	1.157e+02	1.165e+02	1.169e+02
HQ(p)	1.182e+02	1.220e+02	1.257e+02	1.297e+02	1.334e+02
SC(p)	1.240e+02	1.330e+02	1.420e+02	1.511e+02	1.600e+02
FPE(p)	5.768e+49	1.064e+50	1.913e+50	4.507e+50	8.216e+50

Figure B.1: ACF, PACF and CCF for 17 industry portfolios

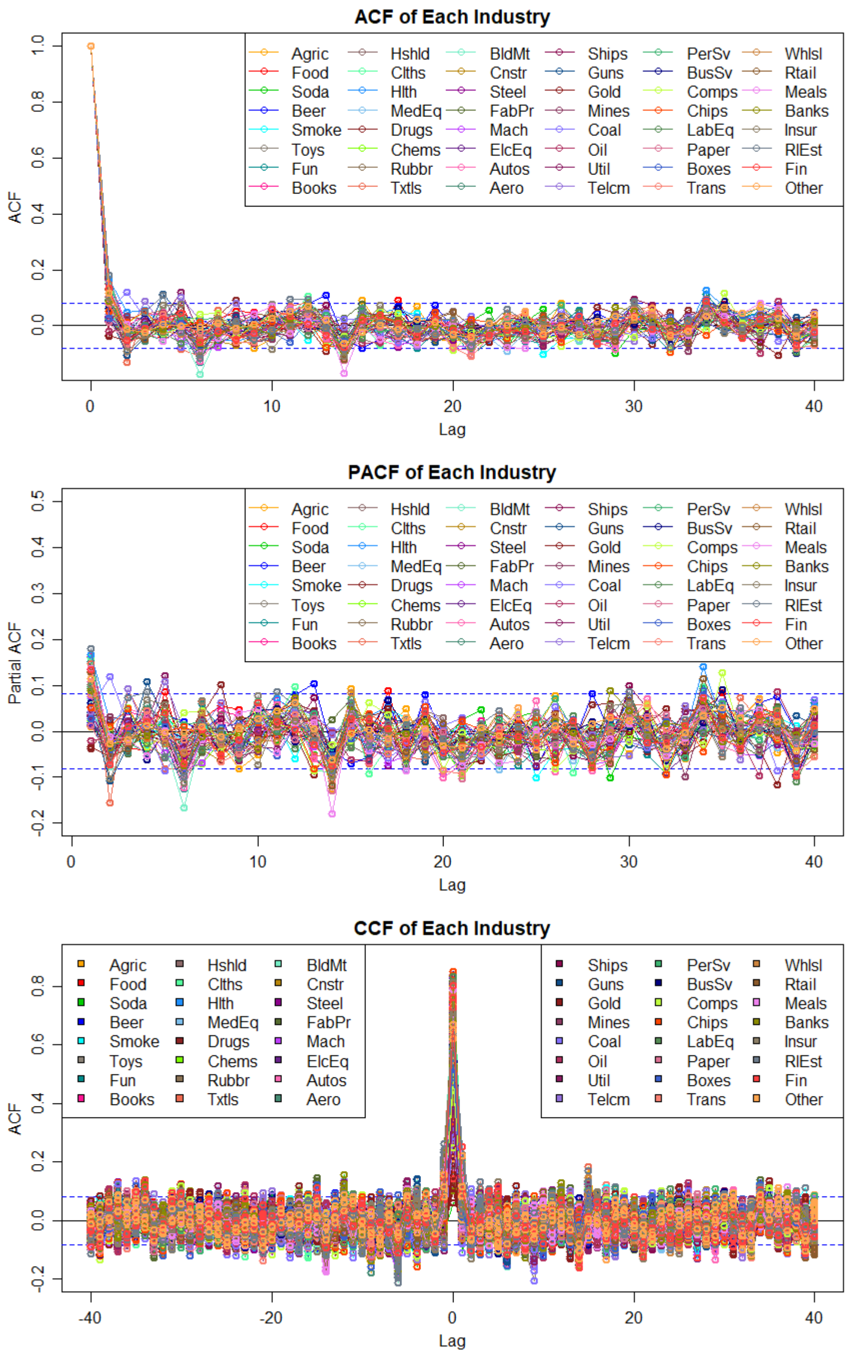


Figure B.2: ACF, PACF and CCF for 48 industry portfolios

B.2 Predictions

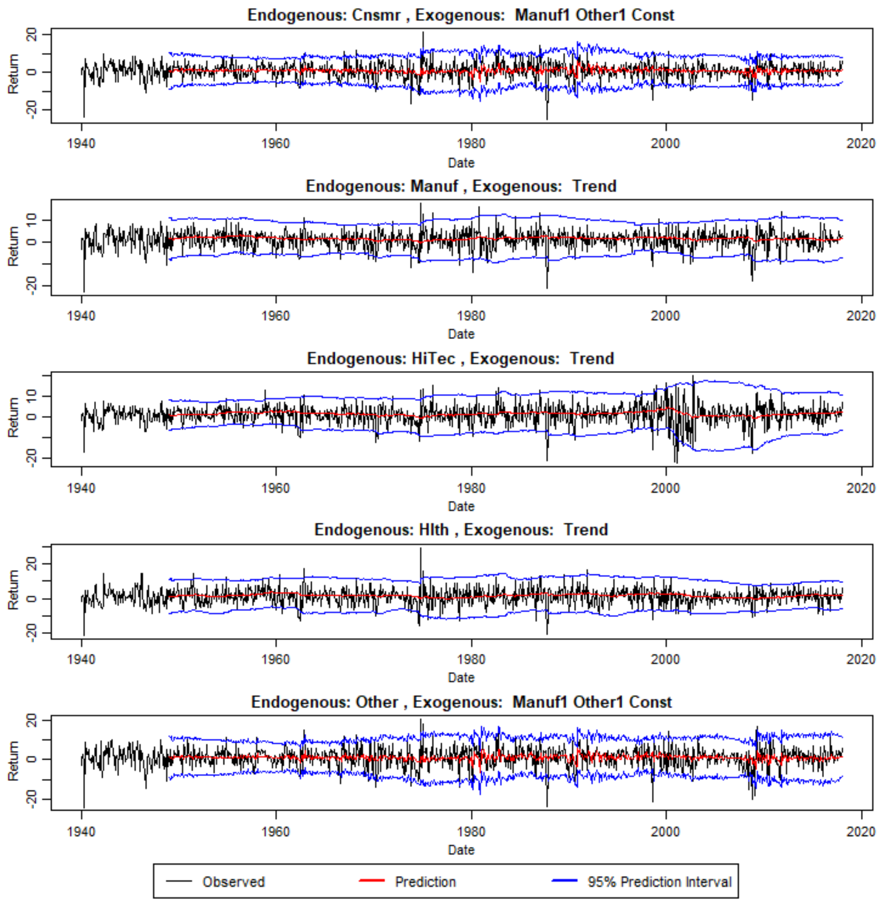


Figure B.3: Predictions and its 95% prediction interval for *Model 2* of the 5 Industry Portfolio.

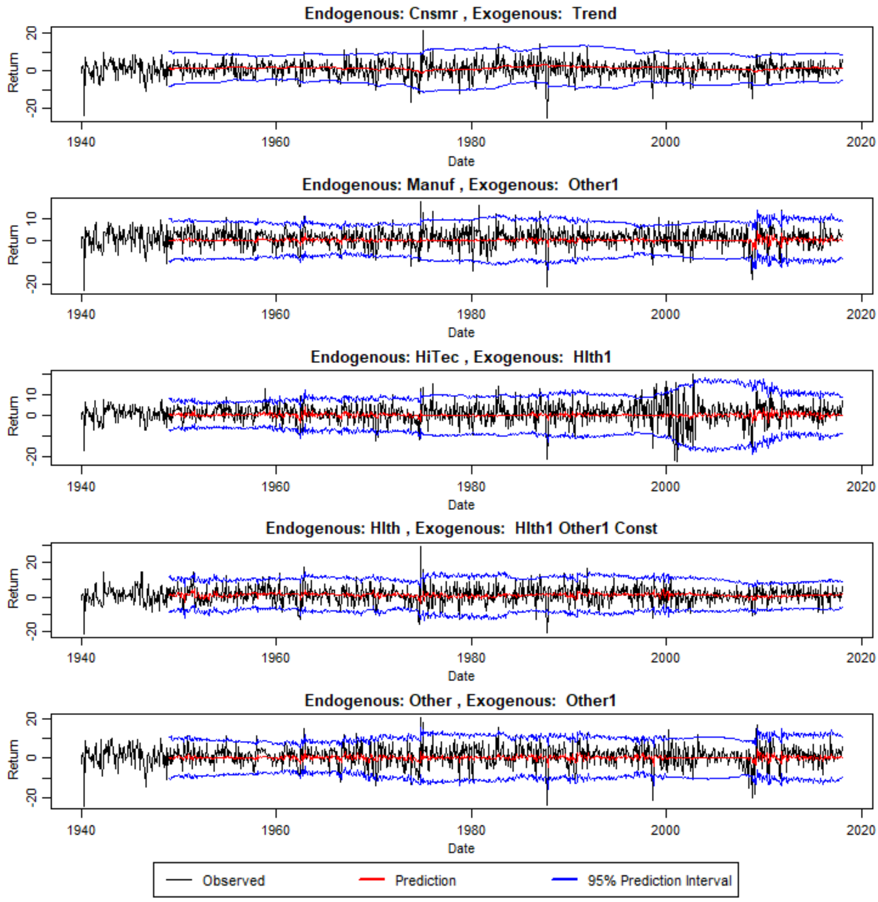


Figure B.4: Predictions and its 95% prediction interval for *Model 3* of the 5 Industry Portfolio.

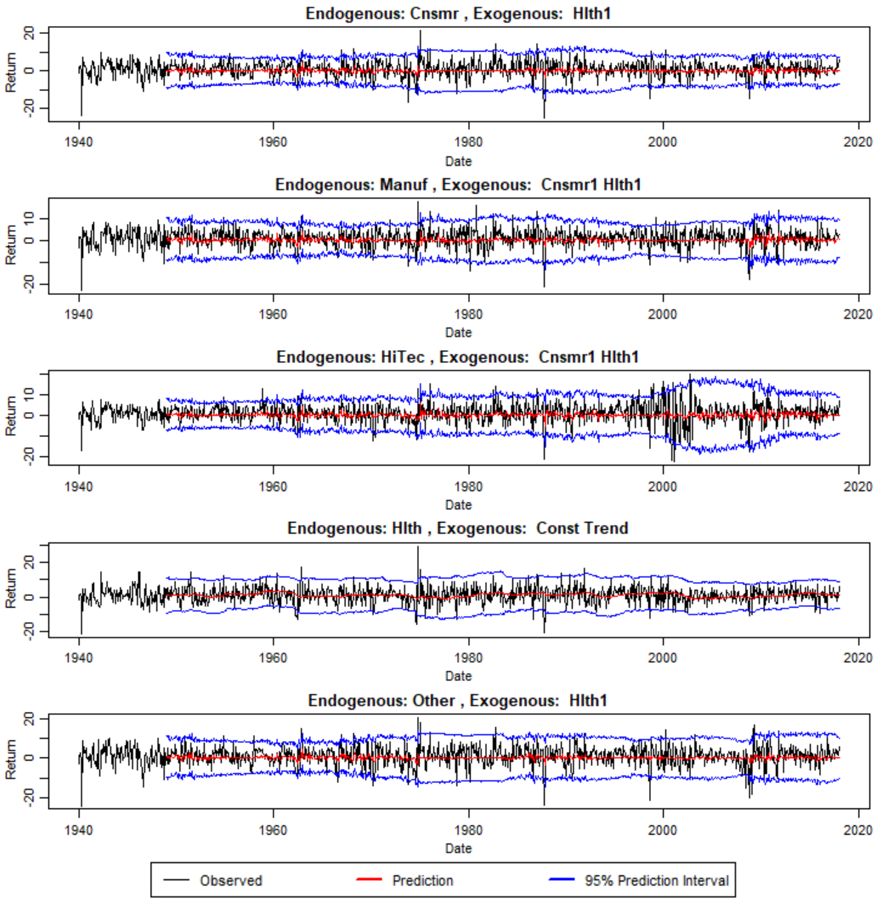


Figure B.5: Predictions and its 95% prediction interval for *Model 4* of the 5 Industry Portfolio.

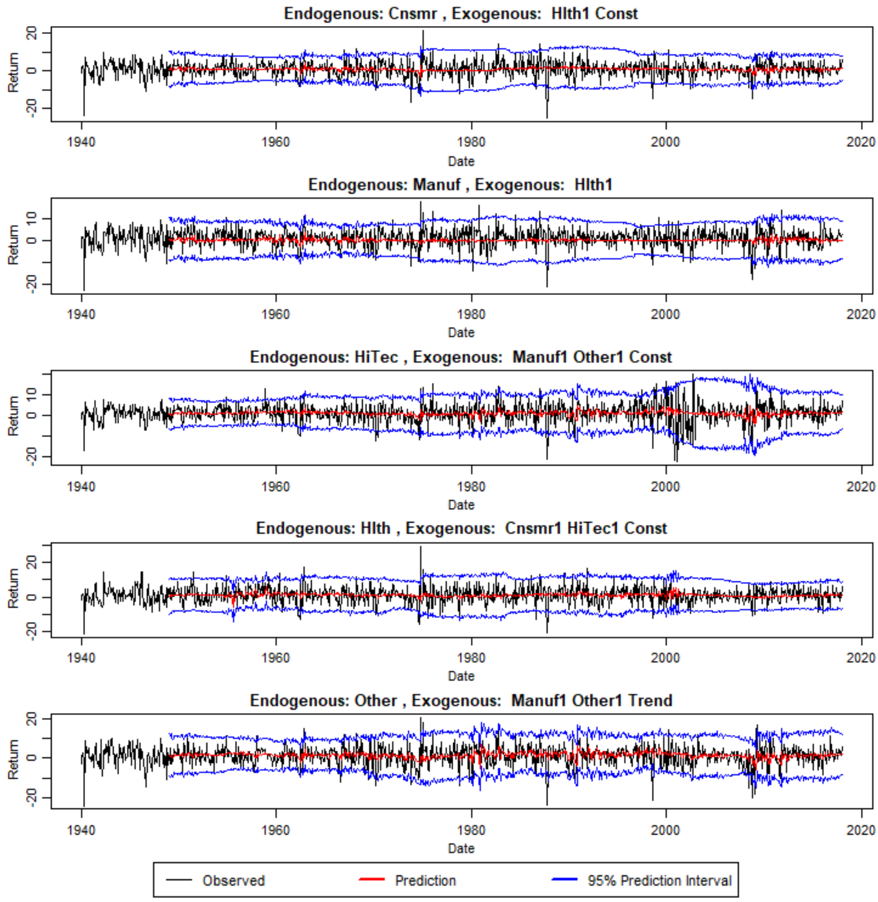


Figure B.6: Predictions and its 95% prediction interval for *Model 5* of the 5 Industry Portfolio.

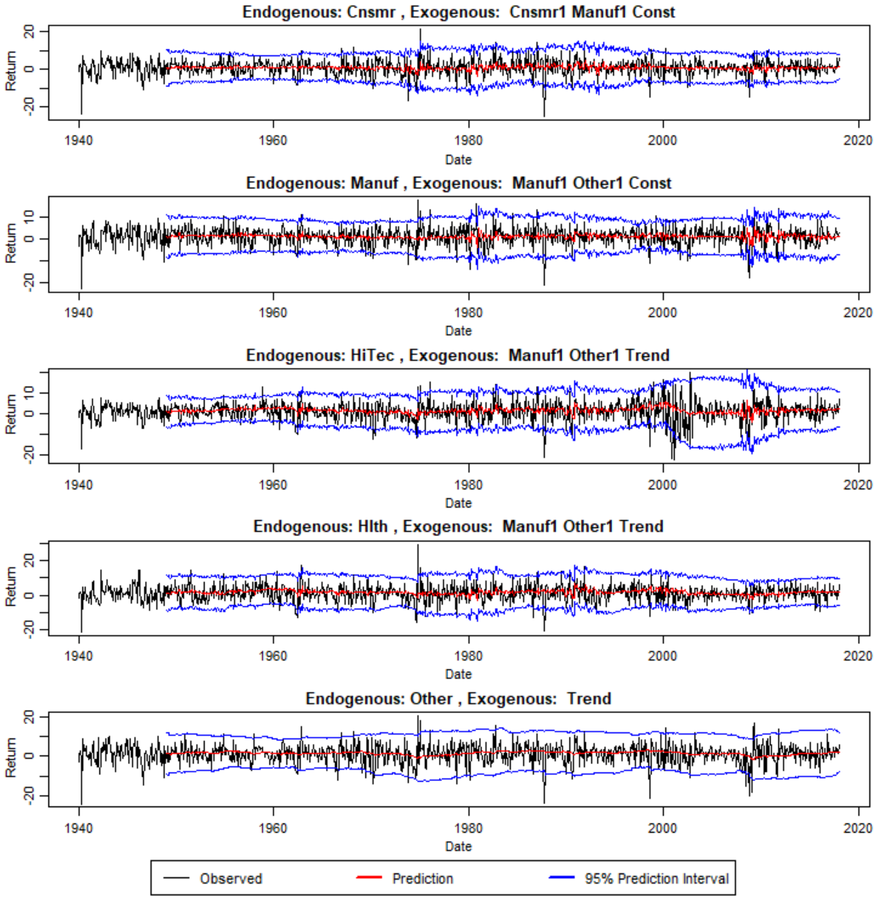
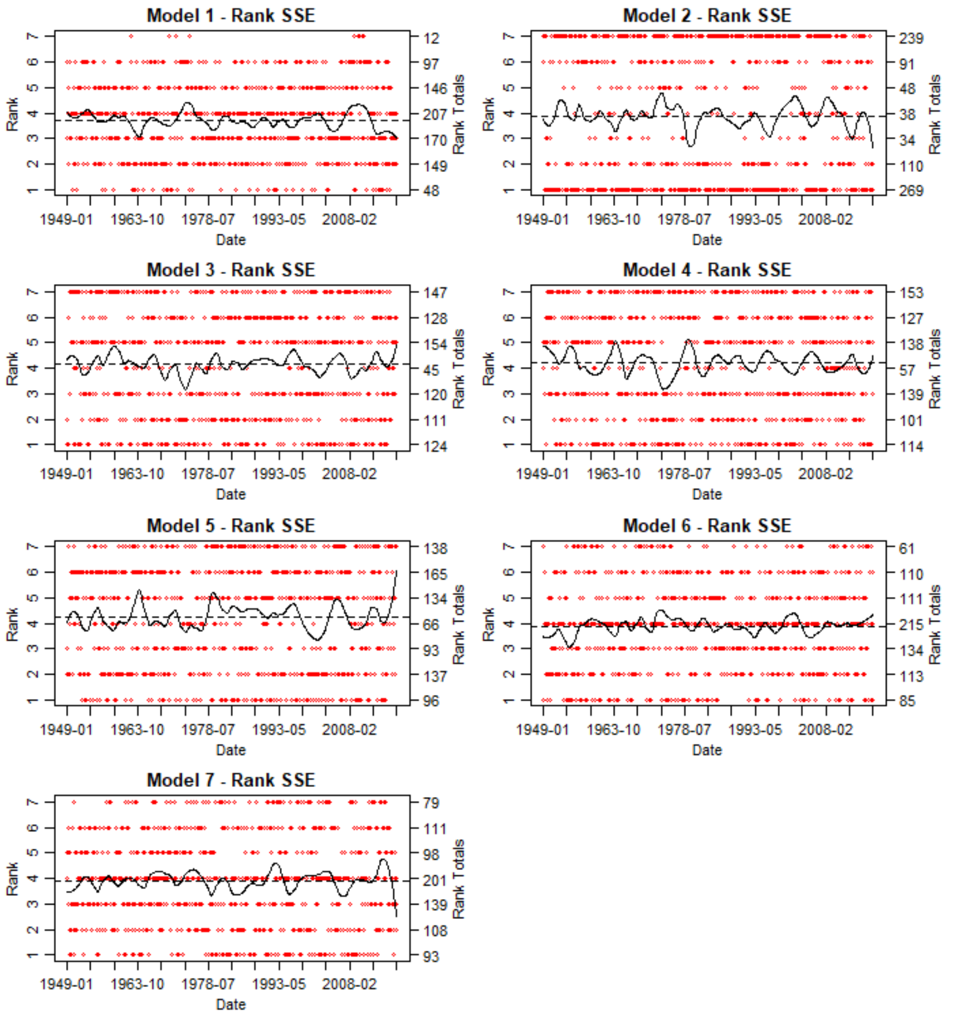
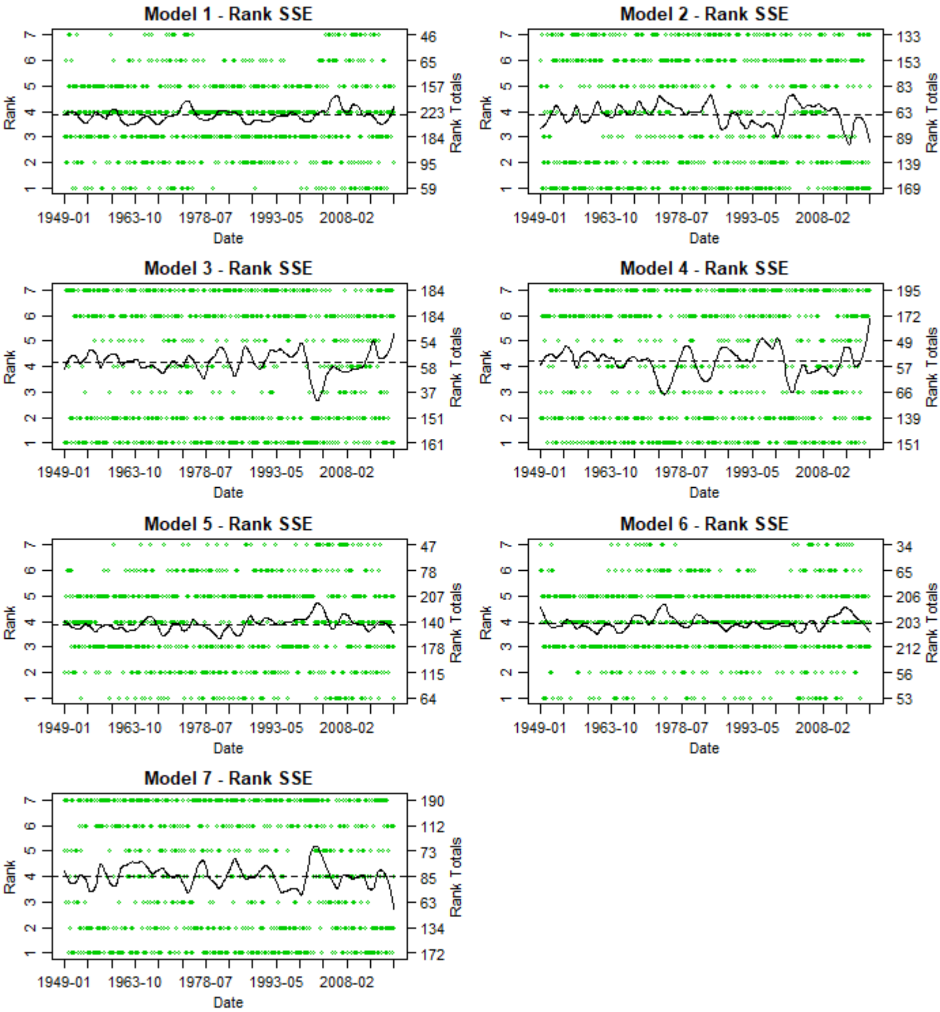


Figure B.7: Predictions and its 95% prediction interval for *Model 7* of the 5 Industry Portfolio.

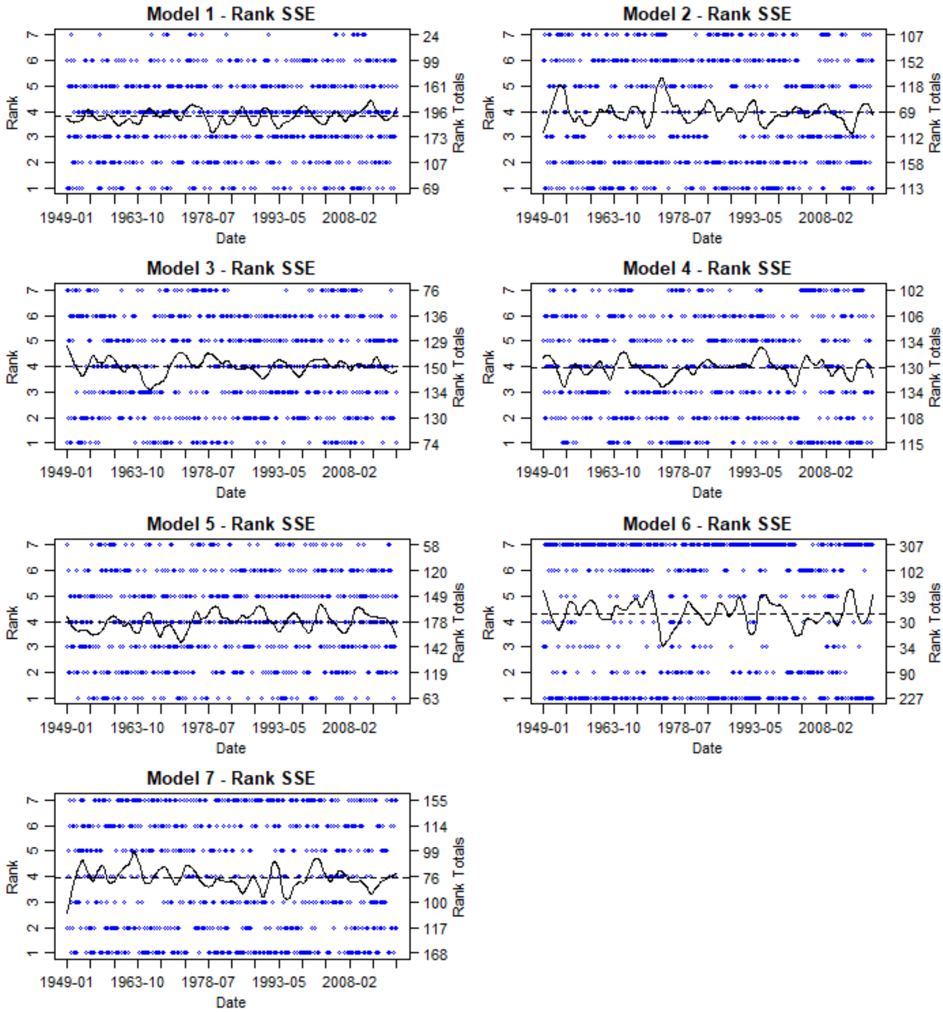
B.3 Accuracy



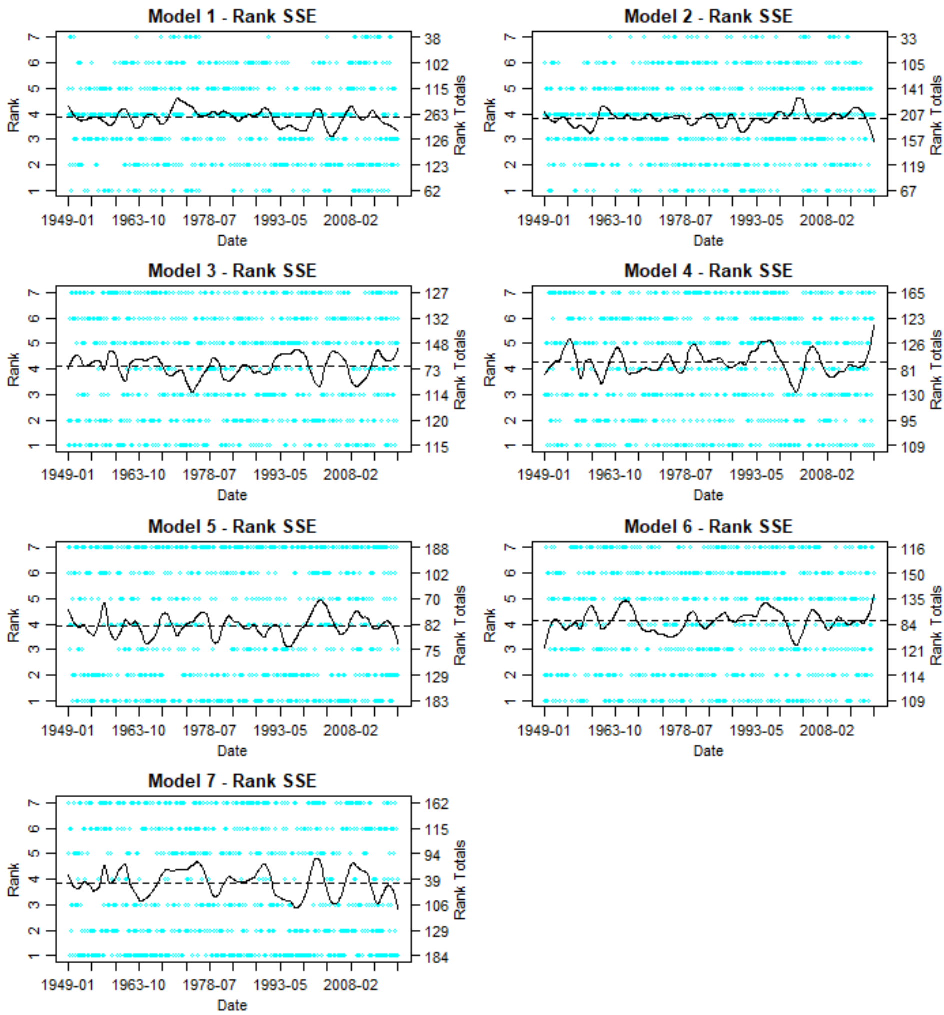
(a)


Model 1 - Rank SSE
Model 2 - Rank SSE
Model 3 - Rank SSE
Model 4 - Rank SSE
Model 5 - Rank SSE
Model 6 - Rank SSE
Model 7 - Rank SSE
Rank
Rank Totals
Date
1949-01
1963-10
1978-07
1993-05
2008-02

(b)


Model 1 - Rank SSE
Model 2 - Rank SSE
Model 3 - Rank SSE
Model 4 - Rank SSE
Model 5 - Rank SSE
Model 6 - Rank SSE
Model 7 - Rank SSE
Rank
Rank Totals
Date
1949-01
1963-10
1978-07
1993-05
2008-02

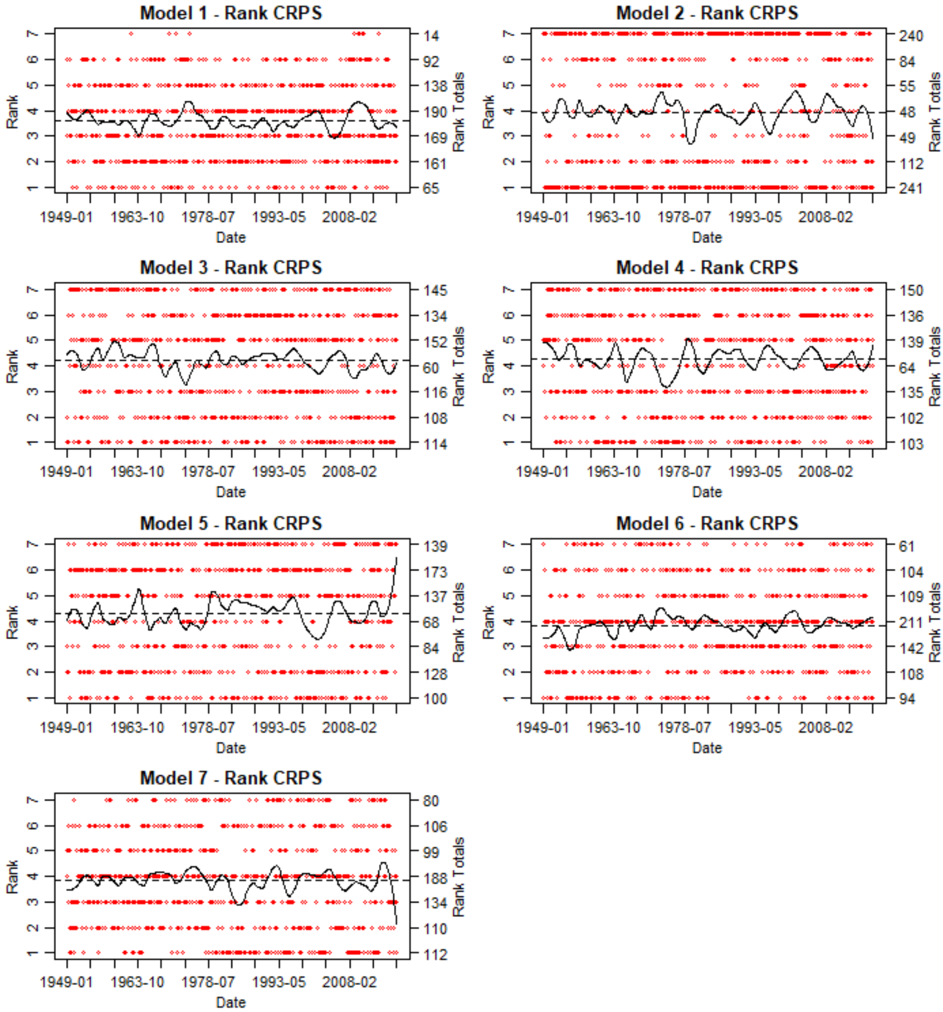
(c)



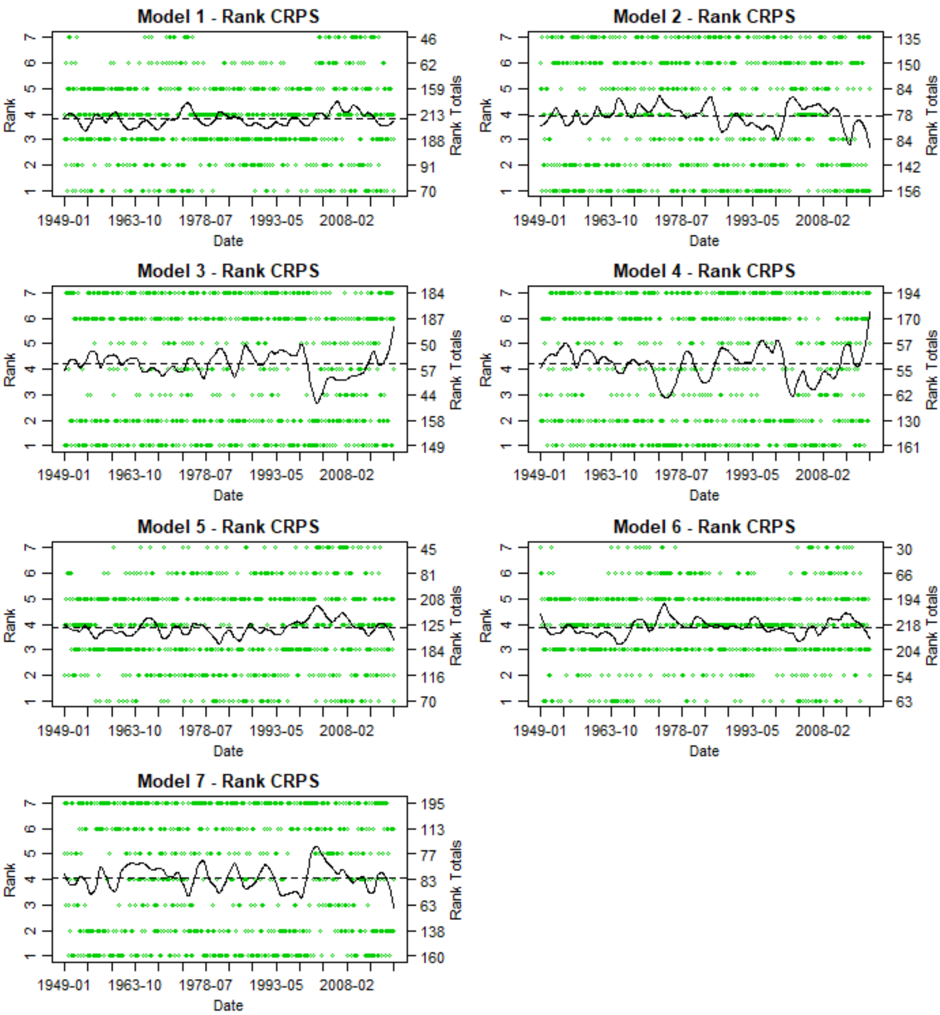
(d)

Figure B.8: The Figures (a)-(d) shows how each model have been ranked compared to each other where Rank=1 is the best rank. The extra axis to the right shows the number of times a given model have had that specific rank. The dashed black line represent the mean of the rank and the black line represents the density of the ranks. The coloured circles represents the rank of a model at a given date.

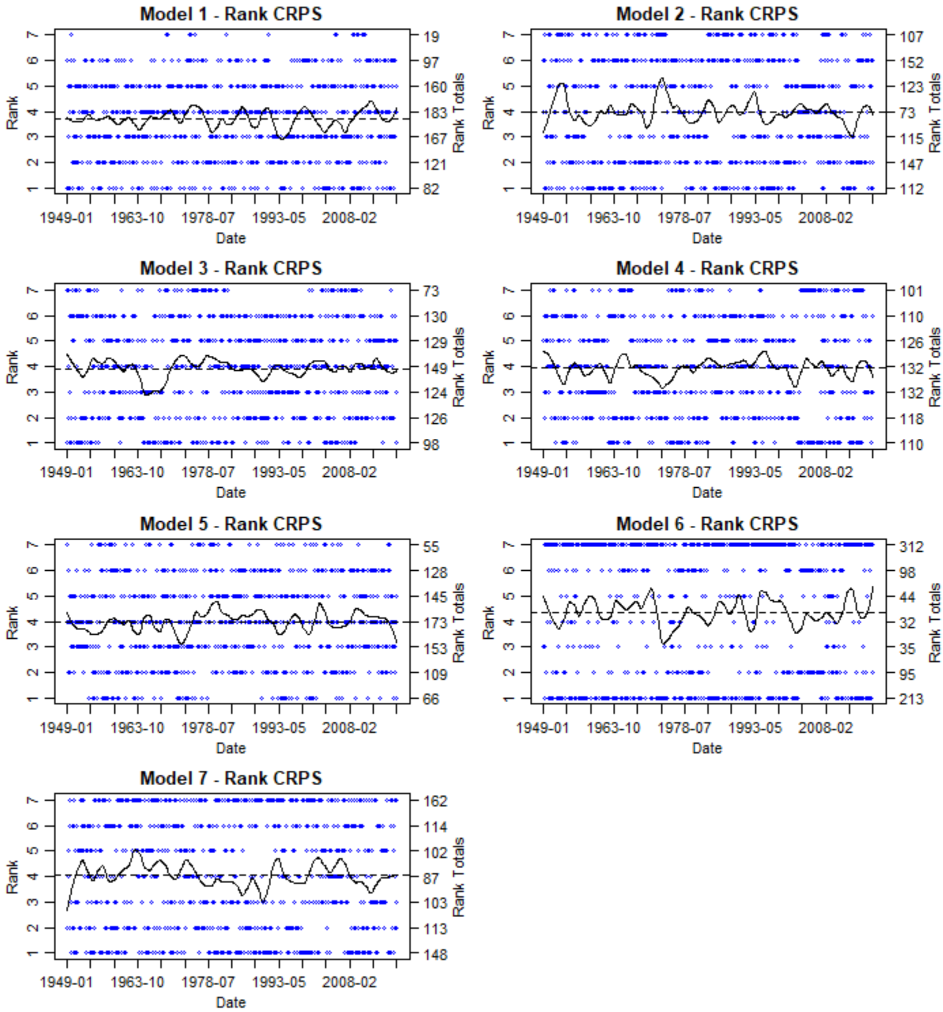
B.4 Skill Score



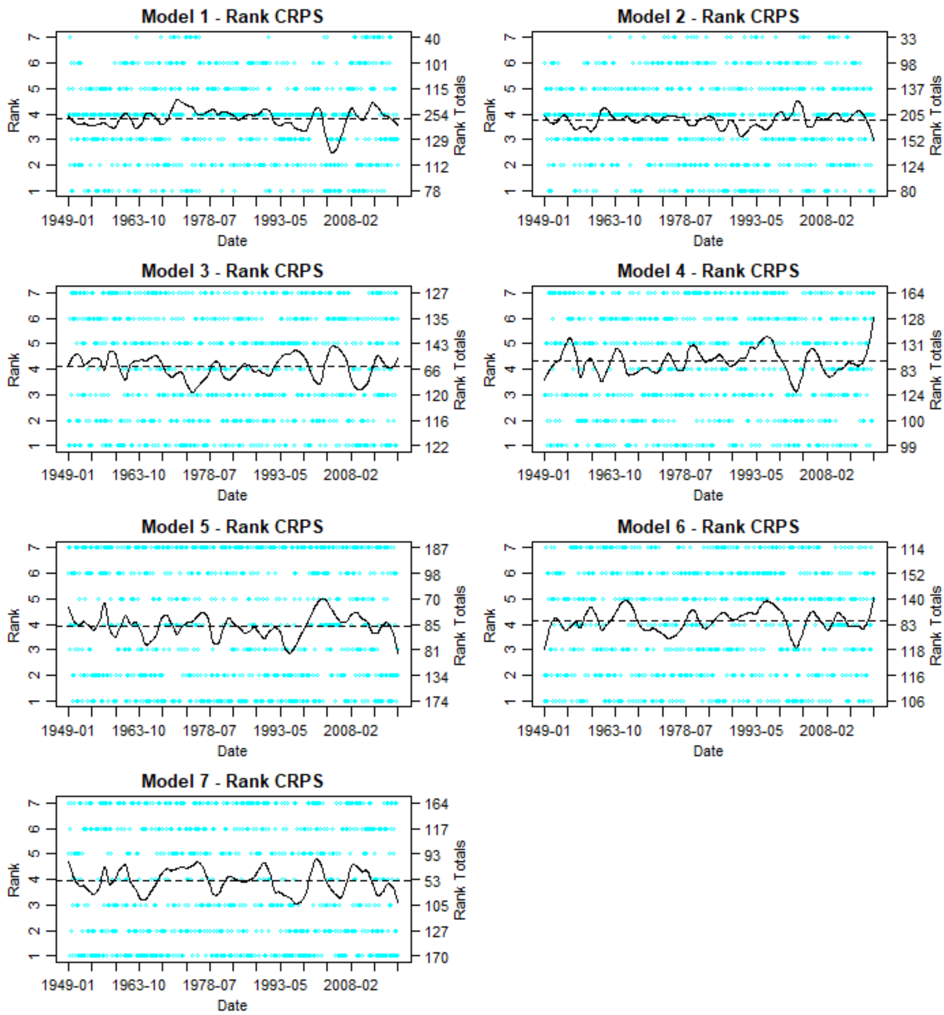
(a)


Model 1 - Rank CRPS
Rank
Rank Totals
46
62
159
213
188
91
70
Model 2 - Rank CRPS
135
150
84
78
84
142
156
Model 3 - Rank CRPS
184
187
50
57
44
158
149
Model 4 - Rank CRPS
194
170
57
55
62
130
161
Model 5 - Rank CRPS
45
81
208
125
184
116
70
Model 6 - Rank CRPS
30
66
194
218
204
54
63
Model 7 - Rank CRPS
195
113
77
83
63
138
160
1949-01
1963-10
1978-07
1993-05
2008-02
Date

(b)


Model 1 - Rank CRPS
Rank
Rank Totals
19
97
160
183
167
121
82
Model 2 - Rank CRPS
107
152
123
73
115
147
112
Model 3 - Rank CRPS
73
130
129
149
124
126
98
Model 4 - Rank CRPS
101
110
126
132
132
118
110
Model 5 - Rank CRPS
55
128
145
173
153
109
66
Model 6 - Rank CRPS
312
98
44
32
35
95
213
Model 7 - Rank CRPS
162
114
102
87
103
113
148
1 2 3 4 5 6 7
1949-01 1963-10 1978-07 1993-05 2008-02
Date

(c)



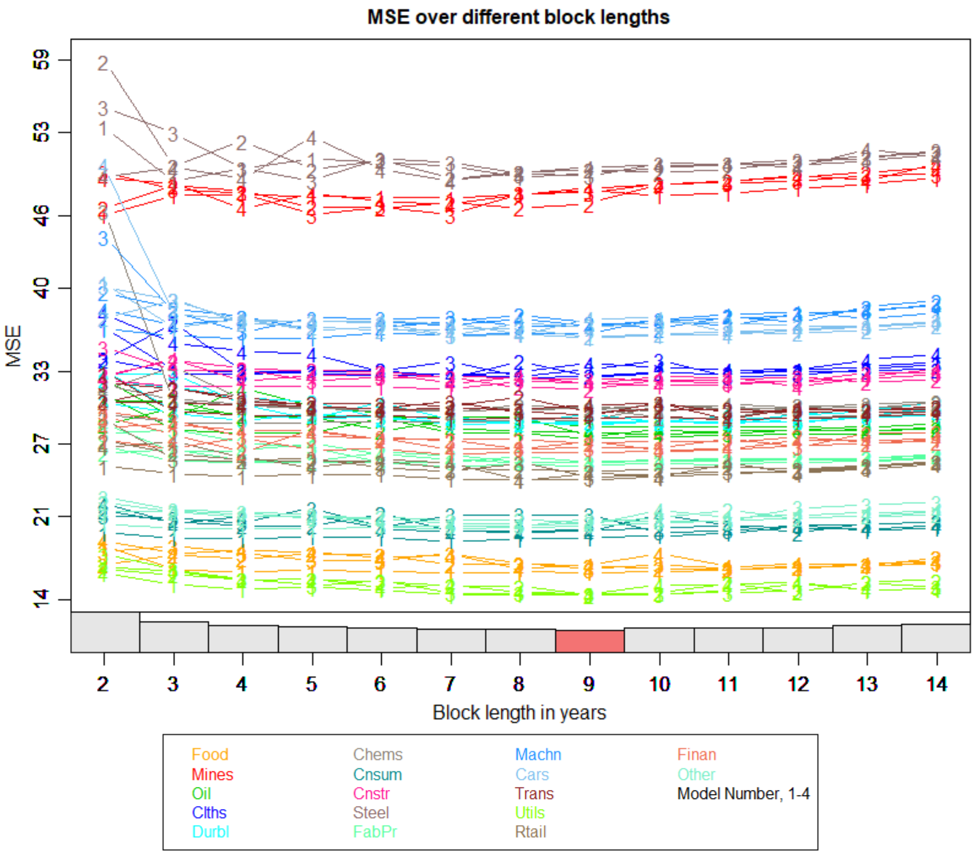
(d)

Figure B.9: The Figures (a)-(d) shows how each model have been ranked compared to each other where Rank=1 is the best rank. The extra axis to the right shows the number of times a given model have had that specific rank. The dashed black line represent the mean of the rank and the black line represents the density of the ranks. The coloured circles represents the rank of a model at a given date.

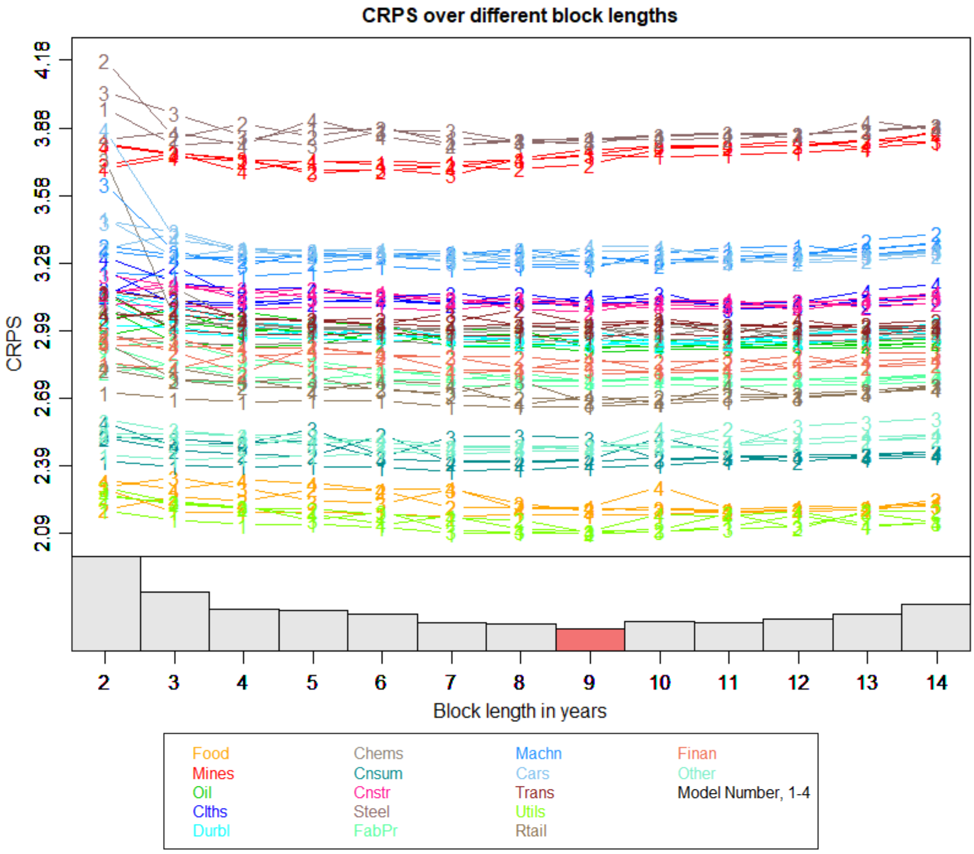
APPENDIX C

Appendix C

C.1 Block Sizes



(a)



(b)

Figure C.1: (a)-(b) The MSE and CRPS calculated for different sizes of block lengths in years for the 17 Industry Portfolio. The histogram shown below indicates the total error for a given block length where the red part indicates the lowest total error, hence 9 years. Note that due to overfitting the figure starts with a block size of 2 years.

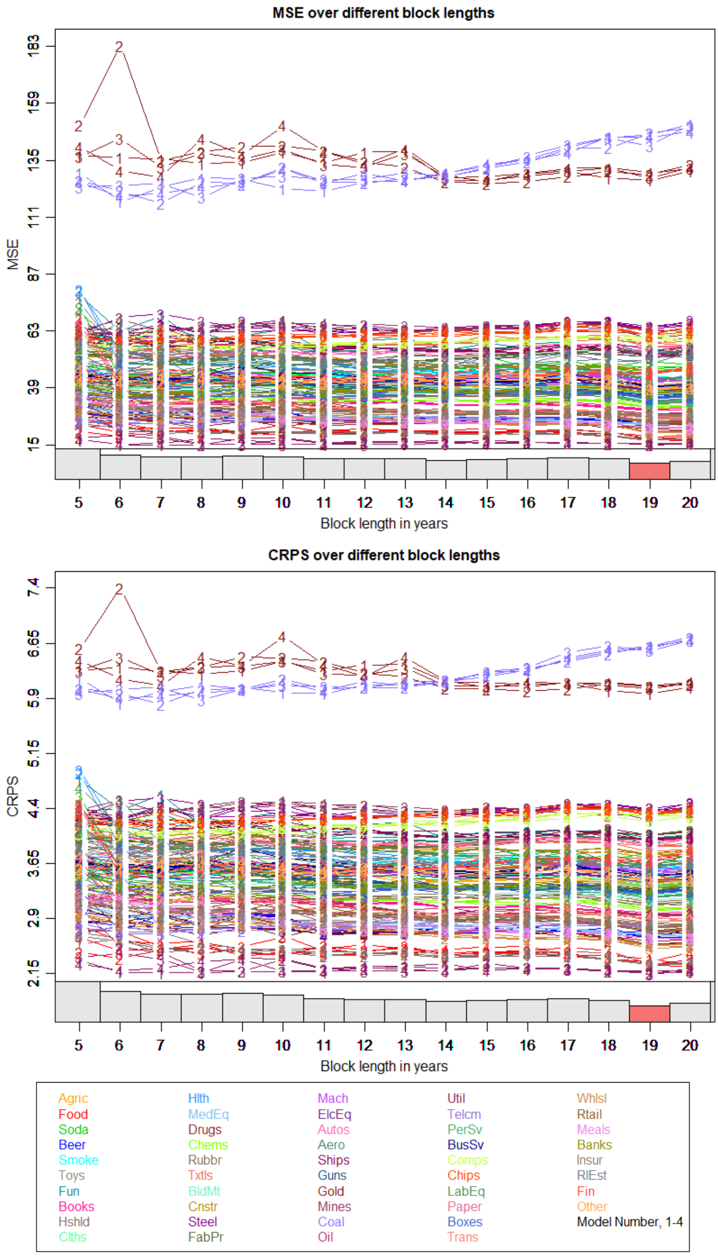
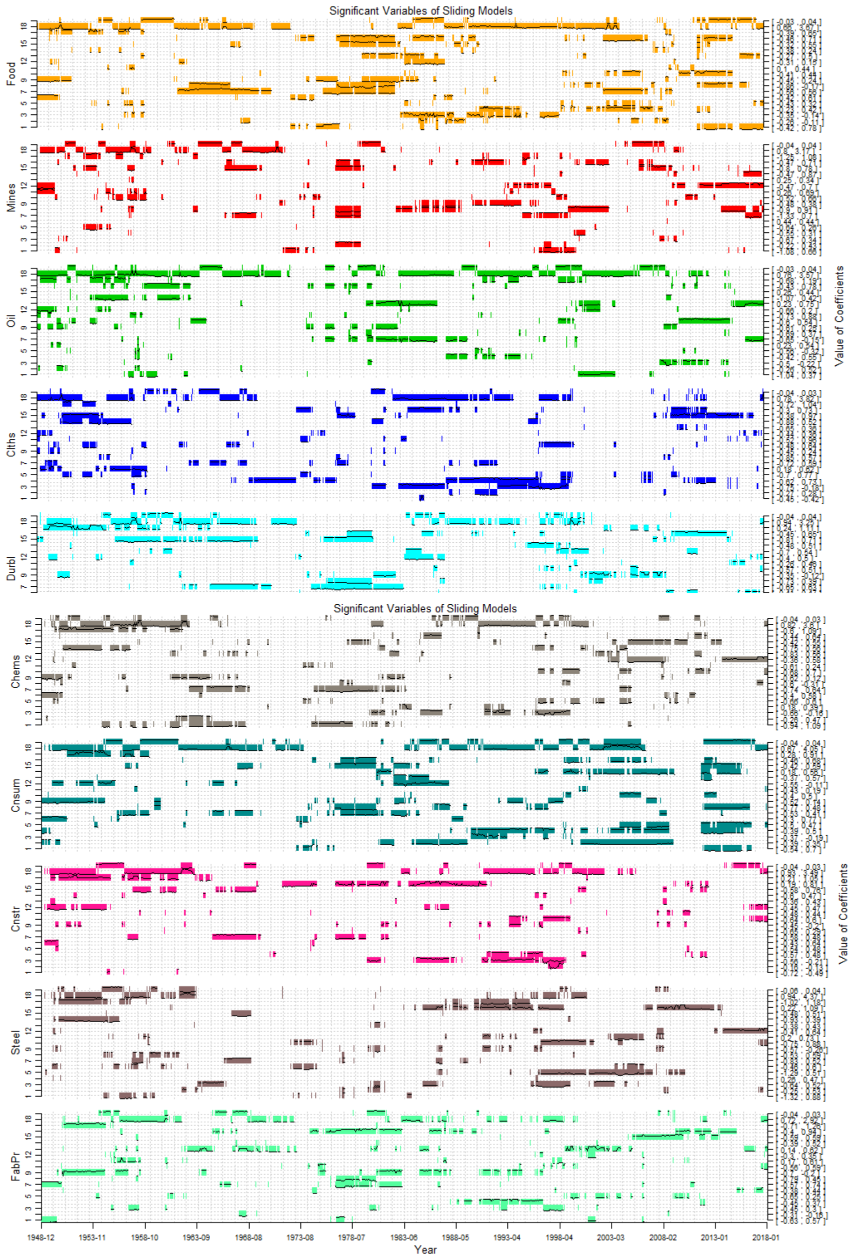
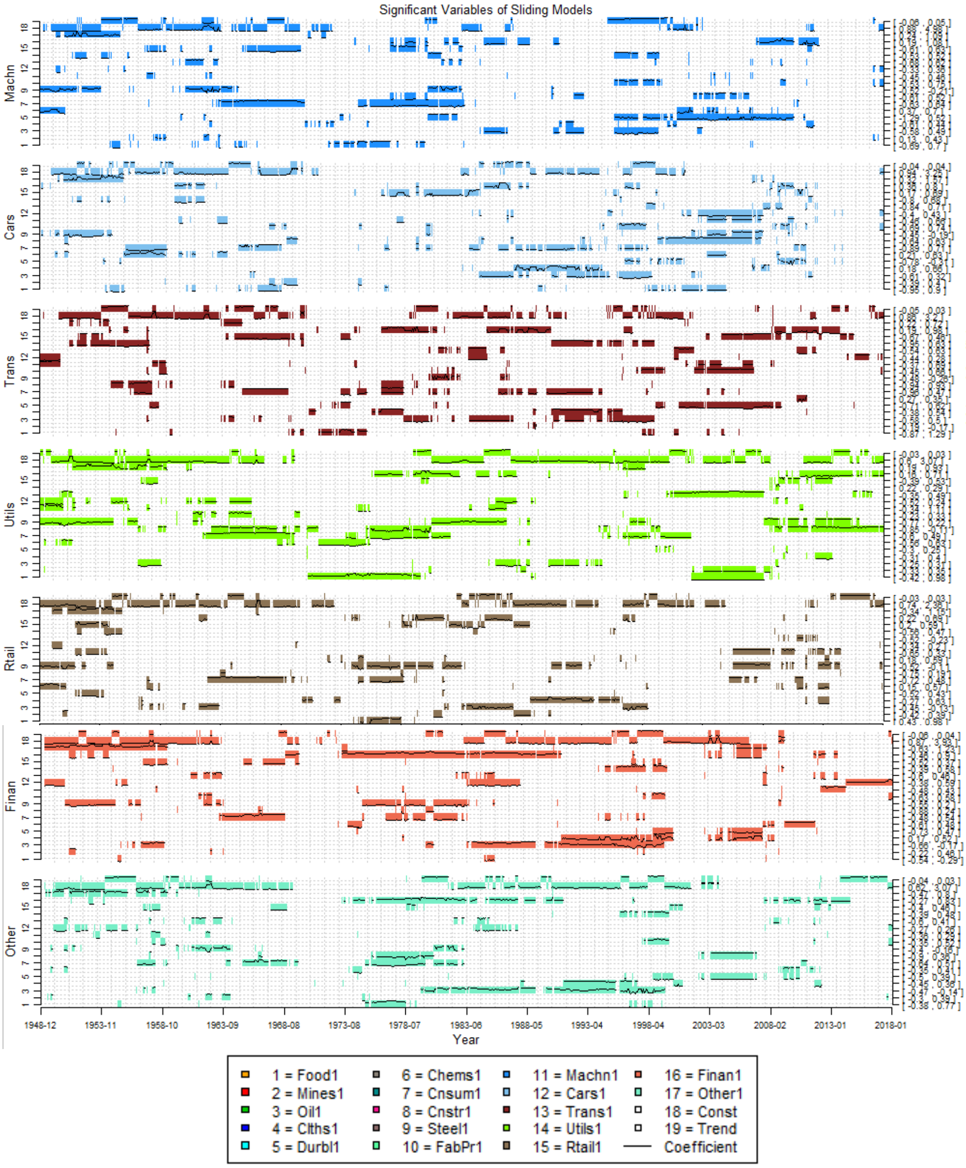


Figure C.2: The MSE and CRPS calculated for different sizes of block lengths in years for the 48 Industry Portfolio. The histogram shown below indicates the total error for a given block length where the red part indicates the lowest total error. Hence the optimal block length for the 48 Industry Portfolio is 9 years. Note that due to overfitting the figure starts with a block size of 5 years.

C.2 The Sliding Models

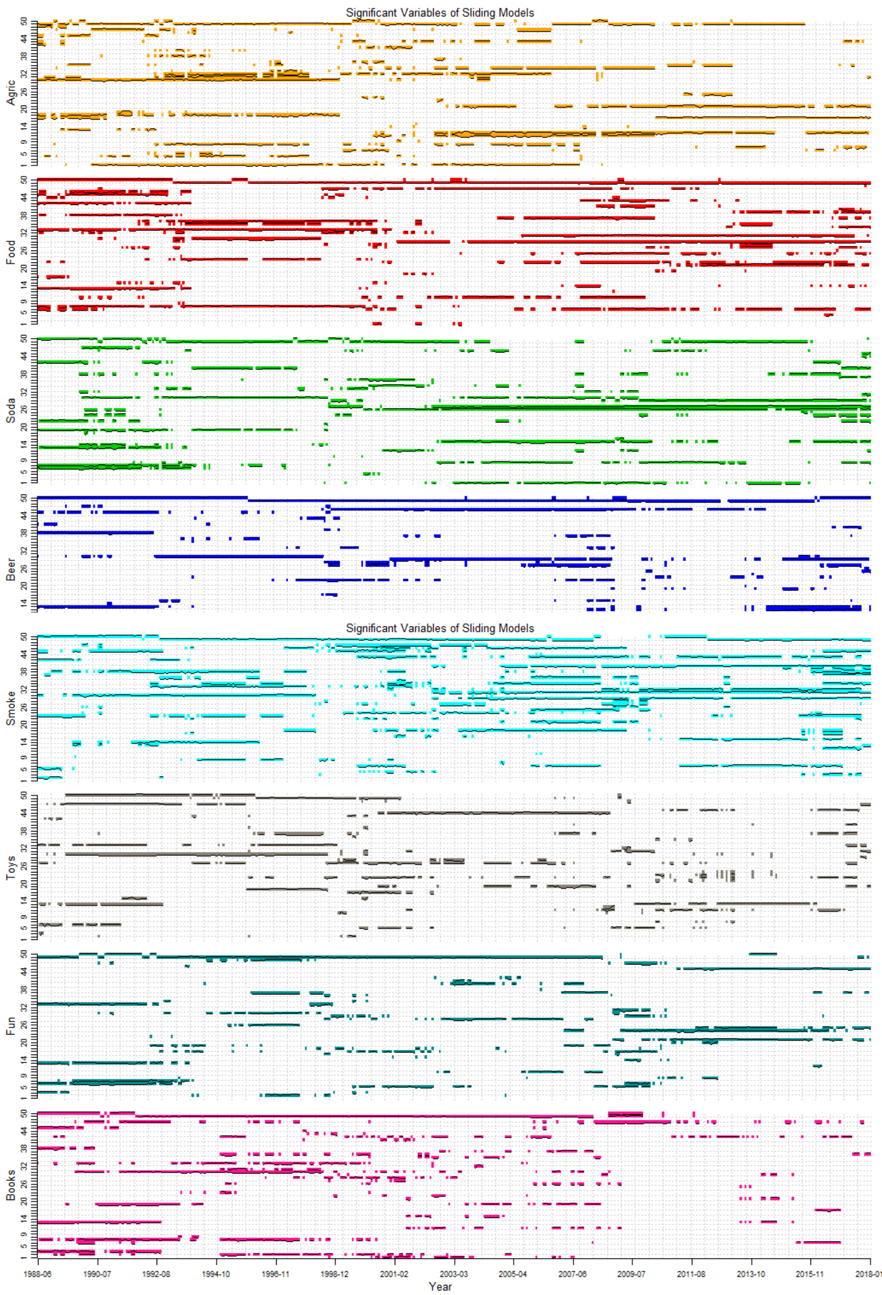


(a)

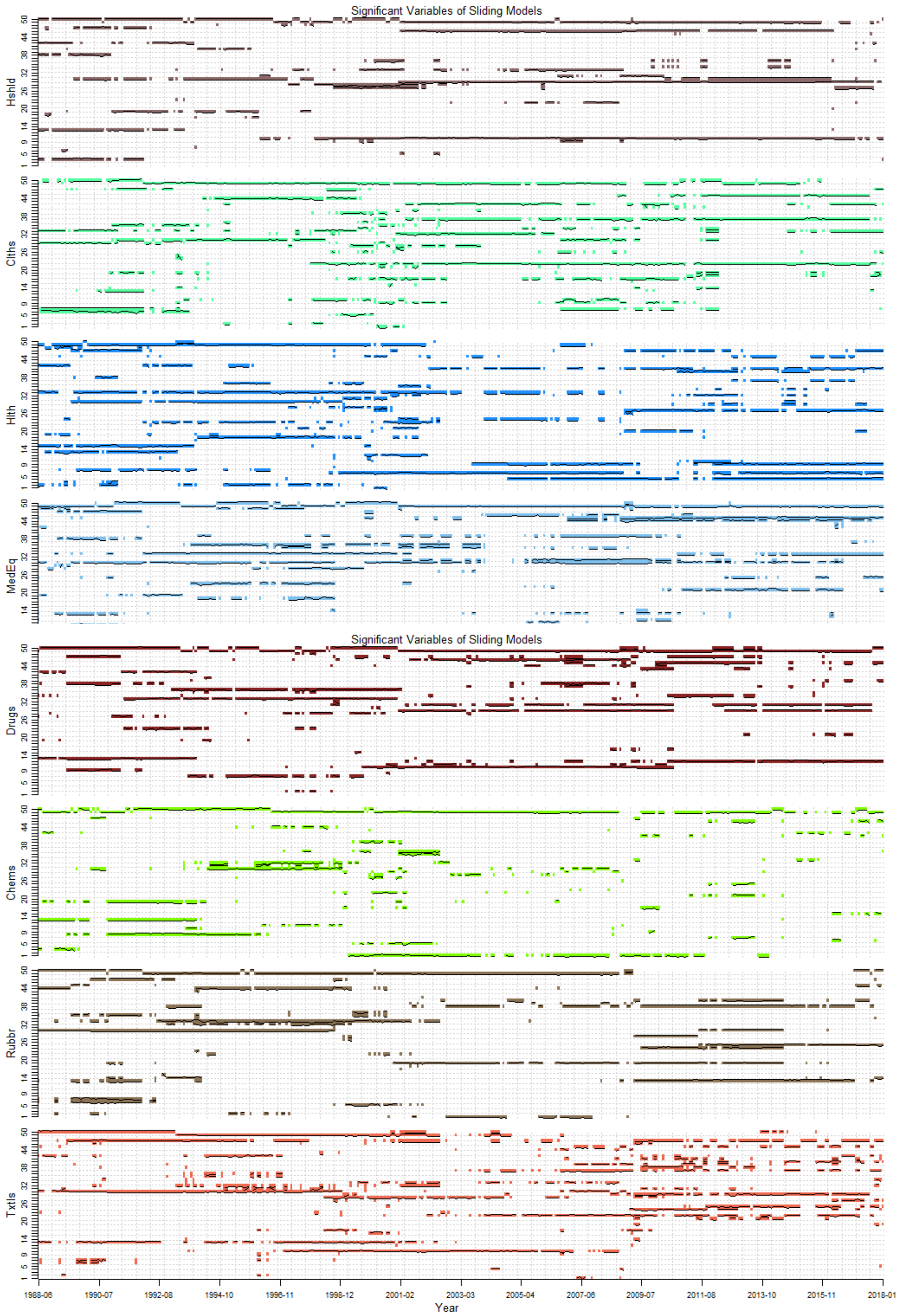


(b)

Figure C.4: (a-b) The sliding model for the 17 industries. Each date at the x-axis represents the last date in one block of 9 years. Thus the dates are only there for guidance. The left axis shows the response variable and the corresponding predictors are given by the numbers. The right axis is the range of the values of the coefficients for the significant predictors. The black lines within the coloured blocks shows how the coefficient values fluctuate.


Significant Variables of Sliding Models
Agric
Food
Soda
Beer
Significant Variables of Sliding Models
Smoke
Toys
Fun
Books
1988-06
1990-07
1992-08
1994-10
1996-11
1998-12
2001-02
2003-03
2005-04
2007-06
2009-07
2011-08
2013-10
2015-11
2018-01
Year

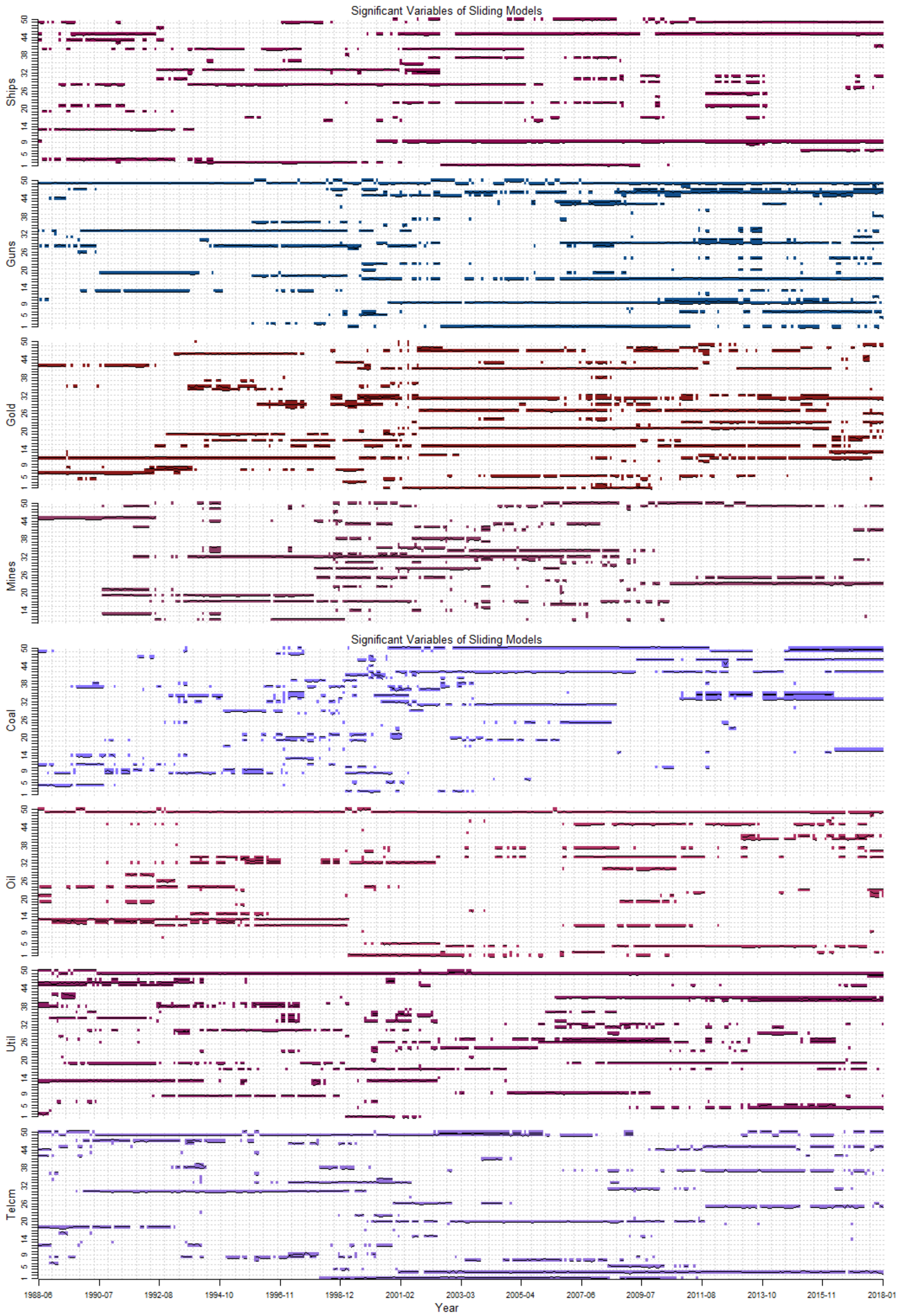
(a)


Significant Variables of Sliding Models
Hshld
Clths
Hlth
MedEq
Significant Variables of Sliding Models
Drugs
Chems
Rubbr
Txtls
1988-06
1990-07
1992-08
1994-10
1996-11
1998-12
2001-02
2003-03
2005-04
2007-06
2009-07
2011-08
2013-10
2015-11
2018-01
Year

(b)


Significant Variables of Sliding Models
BldMt
Cnstr
Steel
FabPr
Significant Variables of Sliding Models
Mach
ElcEq
Autos
Aero
1988-06
1990-07
1992-08
1994-10
1996-11
1998-12
2001-02
2003-03
2005-04
2007-06
2009-07
2011-08
2013-10
2015-11
2018-01
Year

(c)


Significant Variables of Sliding Models
Ships
Guns
Gold
Mines
Significant Variables of Sliding Models
Coal
Oil
Util
Telcm
1988-06
1990-07
1992-08
1994-10
1996-11
1998-12
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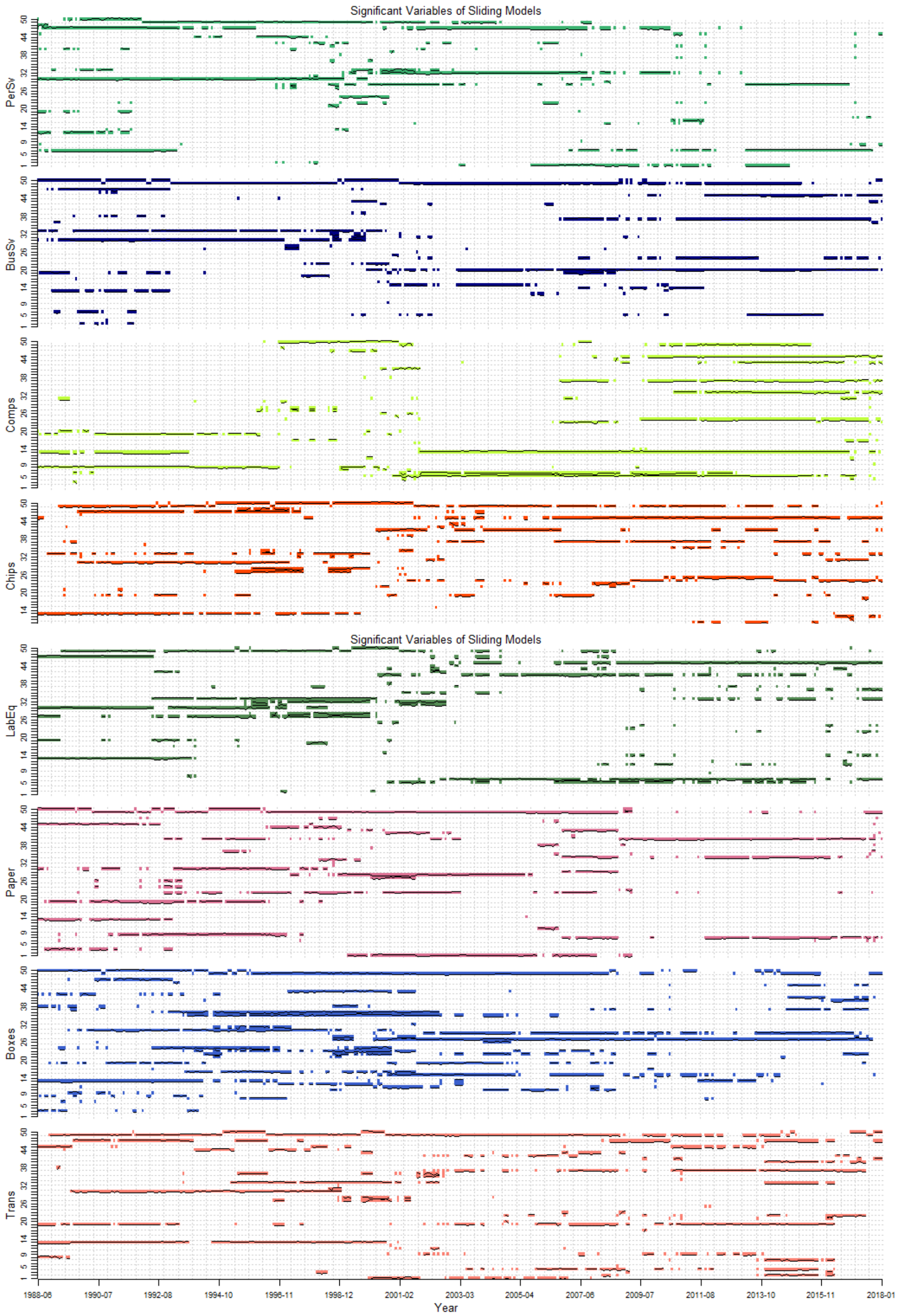
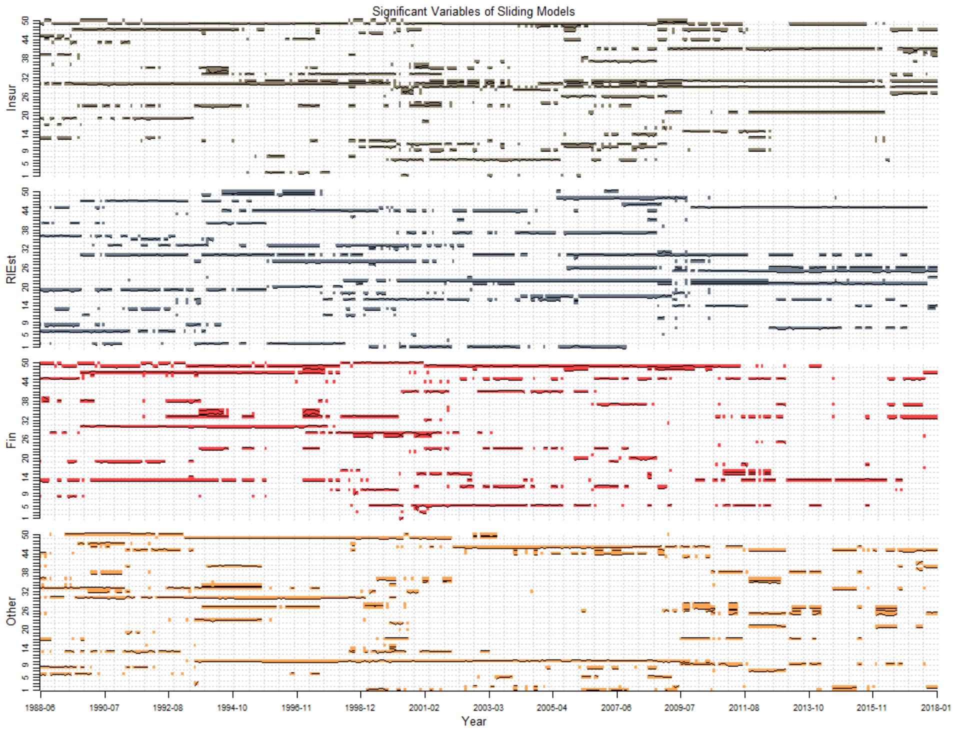
2003-03
2005-04
2007-06
2009-07
2011-08
2013-10
2015-11
2018-01
Year

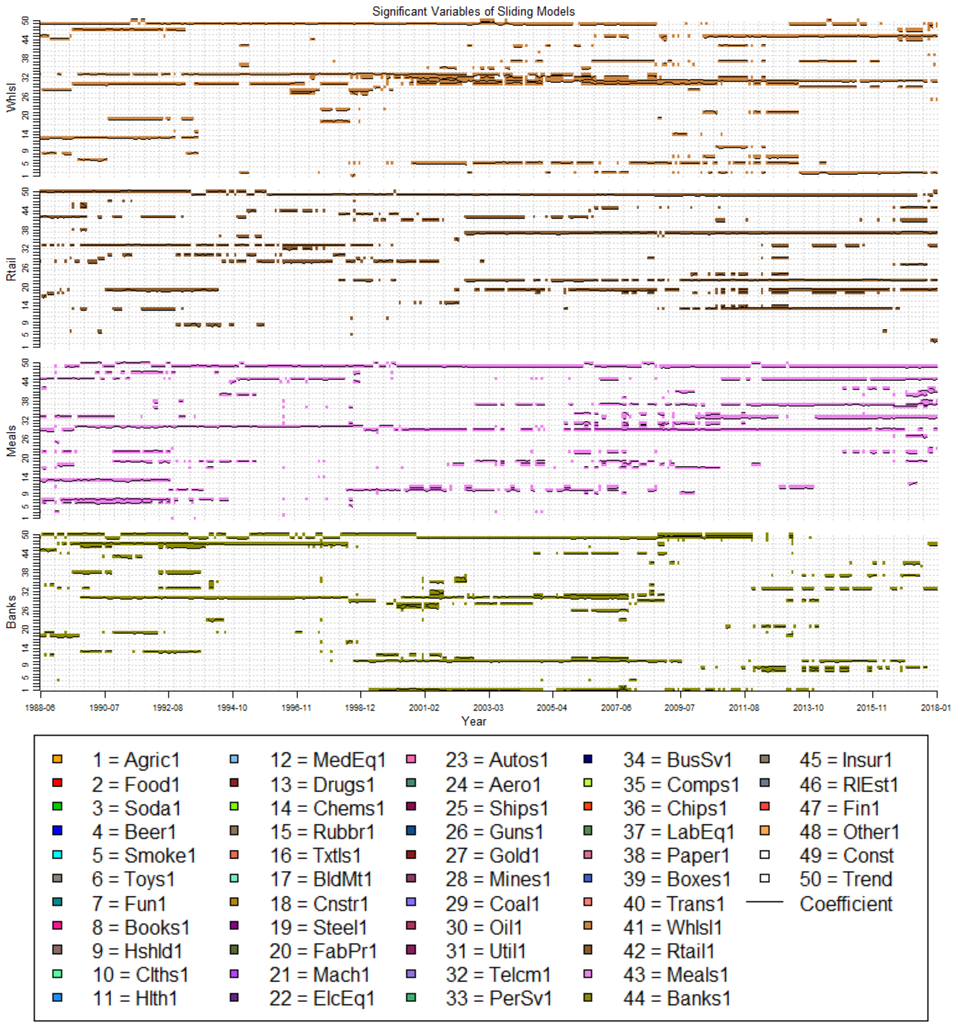
(d)


Significant Variables of Sliding Models
PerSv
BusSv
Comps
Chips
Significant Variables of Sliding Models
LabEq
Paper
Boxes
Trans
1988-06
1990-07
1992-08
1994-10
1996-11
1998-12
2001-02
2003-03
2005-04
2007-06
2009-07
2011-08
2013-10
2015-11
2018-01
Year

(e)


Significant Variables of Sliding Models
Insur
RlEst
Fin
Other
1988-06
1990-07
1992-08
1994-10
1996-11
1998-12
2001-02
2003-03
2005-04
2007-06
2009-07
2011-08
2013-10
2015-11
2018-01
Year

(f)



(g)

Figure C.10: (a)-(g) The sliding model for the 48 industries. Each date at the x-axis represents the last date in one block of 19 years. Thus the dates are only there for guidance. The left axis shows the response variable and the corresponding predictors are given by the numbers. The black lines within the coloured blocks shows how the coefficient values fluctuate. The coefficient values on the right axis have been omitted, due to not being visually satisfying.

C.3 Top 3 Choosen Models

Table C.1: The top three overall best models for each response variable of the 48 Industry Portfolio. Note that these are the models that will be used in the adaptive RLS.

Model	**Agric**	**Food**	**Soda**	**Beer**	**Smoke**	**Toys**	**Fun**	**Books**
1	Hlth1 MedEq1 Mach1 BusSv1 Const	Toys1 Mach1 ElcEq1 Coal1 Util1 Boxes1 Const	Steel1 Oil1 Trans1 Const	Clths1 RlEst1 Const	Toys1 Rubbr1 Coal1 Util1 Telcm1 Trans1 Meals1 Const	Steel1 Gold1 Banks1	Clths1 Mines1 Const	Fin1
2	Fun1 MedEq1 BldMt1 Mach1 Const	Toys1 Mach1 Gold1 Mines1 Coal1 Util1 Comps1 Boxes1 Const	Toys1 Fun1 Drugs1 Steel1 ElcEq1 Rtail1 Trend	Food1 Autos1 RlEst1 Const	Books1 Autos1 Oil1 PerSv1 BusSv1 Const	Gold1 Mines1 Banks1	Mines1 Trans1 Const	Rtail1
3	MedEq1 BldMt1 Mach1 Ships1 Comps1 Const	Clths1 Coal1 Fin1 Const	Agric1 Guns1 Gold1 Coal1 Const	Clths1 Coal1 RlEst1 Const	Toys1 Rubbr1 Coal1 Util1 Telcm1 Trans1 Meals1 Trend	Cnstr1 Oil1 PerSv1 LabEq1 Const	Food1 Guns1 Oil1 LabEq1 Other1 Const	Soda1 Fun1 Drugs1 Steel1 Oil1 Fin1 Const

Model	**Hshld**	**Clths**	**Hlth**	**MedEq**	**Drugs**	**Chems**	**Rubbr**	**Txtls**
1	Clths1 Coal1 Oil1 RlEst1 Const	ElcEq1 Telcm1 LabEq1 Rtail1 Const	Beer1 Toys1 Hshld1 Gold1 LabEq1 Whlsl1 Insur1	Food1 Mach1 Oil1 PerSv1 Banks1 Insur1 Const	MedEq1 Coal1 Util1 Const	Const	Agric1 Steel1 Paper1 Const	Oil1 Fin1 Trend
2	Clths1 Coal1 RlEst1 Const	ElcEq1 LabEq1 Insur1 Const	Beer1 Toys1 Hshld1	Books1 RlEst1 Const	Clths1 Coal1 Util1 Paper1 RlEst1 Const	Agric1 Oil1 Const	Steel1 PerSv1 Const	Drugs1 Oil1 Fin1 Trend
3	Drugs1 Steel1 Rtail1 Trend	ElcEq1 Mines1 Telcm1 LabEq1 Rtail1 Const	Beer1 Toys1 Hshld1 Aero1 PerSv1 Whlsl1	Steel1 Oil1 PerSv1 Trend	MedEq1 BusSv1 Insur1 Fin1 Const	Books1 Drugs1 Steel1 Trend	Oil1 Telcm1 PerSv1 Banks1 Const	Drugs1 Oil1 Comps1 Chips1 Rtail1 Fin1 Const

Table C.2: The top three overall best models for each response variable of the 48 Industry Portfolio. Note that these are the models that will be used in the adaptive RLS.

Model	**BldMt**	**Cnstr**	**Steel**	**FabPr**	**Mach**	**ElcEq**	**Autos**	**Aero**
1	Clths1 Coal1 Const	Insur1	Ships1 Insur1	PerSv1	Const	Oil1 Fin1 Const	Agric1 Books1 Hshld1 Guns1 Oil1 BusSv1	Beer1 Mines1 Comps1 Trans1 Const
2	Coal1 Rtail1 Const	Fun1 Autos1 Ships1 Insur1 Const	Insur1	Fun1 Chems1 Ships1 Guns1 Fin1	Drugs1 Insur1 Const	Const	Agric1 Books1 Hshld1	Books1 Drugs1 Steel1 Trans1 Const
3	ElcEq1 Coal1 Rtail1 Const	Books1 Drugs1 Steel1 Gold1 Insur1	Drugs1 Insur1	Ships1 Guns1 Fin1	Hshld1 Ships1 LabEq1 Insur1 Const	ElcEq1 Rtail1 Const	Toys1 Books1 Drugs1 Oil1 Fin1 Trend	Beer1 Mines1 Comps1 Chips1 Trans1 Const

Model	**Ships**	**Guns**	**Gold**	**Mines**	**Coal**	**Oil**	**Util**	**Telcm**
1	Hshld1 Insur1 Const	Mines1 PerSv1 Const	Toys1 Hlth1 Rtail1	Autos1 Ships1 Const	Ships1 Util1 Rtail1 Trend	Const	Hshld1 Aero1 Gold1 Const	Oil1 Fin1 Const
2	Toys1 Hshld1 Insur1 Const	Drugs1 Steel1 PerSv1 Const	Hlth1 Rubbr1 Mach1 Autos1 Gold1 Util1 Telcm1 Whlsl1	Insur1	Txtls1 PerSv1 Rtail1 RlEst1 Const Trend	Beer1 Trans1 Const	BldMt1 Aero1 Const	Soda1 Ships1 LabEq1 Insur1 Const
3	Agric1 Hshld1 ElcEq1 Mines1 LabEq1 Trans1 Insur1	Drugs1 Cnstr1 Mines1 PerSv1 Chips1 Const	Hlth1 Rubbr1 Mach1 Autos1 Gold1 Util1 Whlsl1 Fin1	BldMt1 Steel1 Telcm1	Steel1 Util1 Rtail1 Trend	Beer1 Insur1 Const	Hshld1 Steel1 Guns1 Gold1 Telcm1 Whlsl1 Const	Cnstr1 Oil1 Fin1 Const

Table C.3: The top three overall best models for each response variable of the 48 Industry Portfolio. Note that these are the models that will be used in the adaptive RLS.

Model	**PerSv**	**BusSv**	**Comps**	**Chips**	**LabEq**	**Paper**	**Boxes**	**Trans**
1	Oil1 Banks1 Const	Steel1 FabPr1 LabEq1 Const	Smoke1 Drugs1 Aero1 PerSv1 LabEq1 Meals1 Insur1 Const	Smoke1 Toys1 Aero1 LabEq1 Insur1 Const	Toys1 Insur1	Agric1 Mines1 Const	Rubbr1 Steel1 Gold1 Coal1 Const	Agric1 Const
2	Oil1 Fin1 Const	Smoke1 FabPr1 LabEq1 Insur1 Const	Smoke1 Toys1 Drugs1	Smoke1 Toys1 Whlsl1 Const	Smoke1 Toys1 Whlsl1 Insur1	Agric1 ElcEq1 Gold1 Mines1 Rtail1 Const	Rubbr1 Autos1 Gold1 Coal1 Const	Hshld1 Steel1 LabEq1 Insur1 Const
3	Toys1 Mines1	FabPr1 Aero1 LabEq1 Insur1 Const	Books1 Drugs1 Steel1	Smoke1 Clths1 Ships1 Whlsl1 Insur1 Const	Toys1 Whlsl1	Trans1	Gold1 Coal1 LabEq1 Whlsl1 Insur1 Const	Beer1 Steel1 LabEq1 Rtail1 Const

Model	**Whlsl**	**Rtail**	**Meals**	**Banks**	**Insur**	**RlEst**	**Fin**	**Other**
1	Oil1 PerSv1 Const	Drugs1 Steel1 ElcEq1 LabEq1 Const	Coal1 PerSv1 LabEq1 Insur1 Const	Oil1 Fin1 Trend	Mach1 Coal1 Util1 Whlsl1 Const	Chems1 Mach1 ElcEq1 Ships1 Oil1 Insur1	Drugs1 Steel1 Oil1 Fin1 Const	Clths1 Gold1 Oil1 Const
2	Food1 Coal1 Util1 LabEq1 Insur1 Const	Steel1 LabEq1 Const	Clths1 Coal1 Const	Oil1 Fin1 Const	Steel1 Oil1 Fin1 Const	Agric1 Mach1 ElcEq1 Oil1 Banks1	Oil1 Fin1 Const	Clths1 Oil1 Const
3	Smoke1 Oil1 Util1 Telcm1 PerSv1 Comps1 Const	Steel1 ElcEq1 LabEq1 Rtail1 Const	Hlth1 Coal1 LabEq1 Insur1 Const	Agric1 Clths1 Mines1 Oil1 Const	Autos1 Oil1 Fin1 Const	Mach1 ElcEq1 Ships1 Insur1	Other1 Const	Clths1 Autos1 Gold1 Oil1 PerSv1 BusSv1 Trans1 Const

C.4 ARLS Results for the 17 & 48 Industry Portfolios

Table C.4: Optimal forgetting factor λ for the predictor variables for each of the top three models presented in Table 4.14.

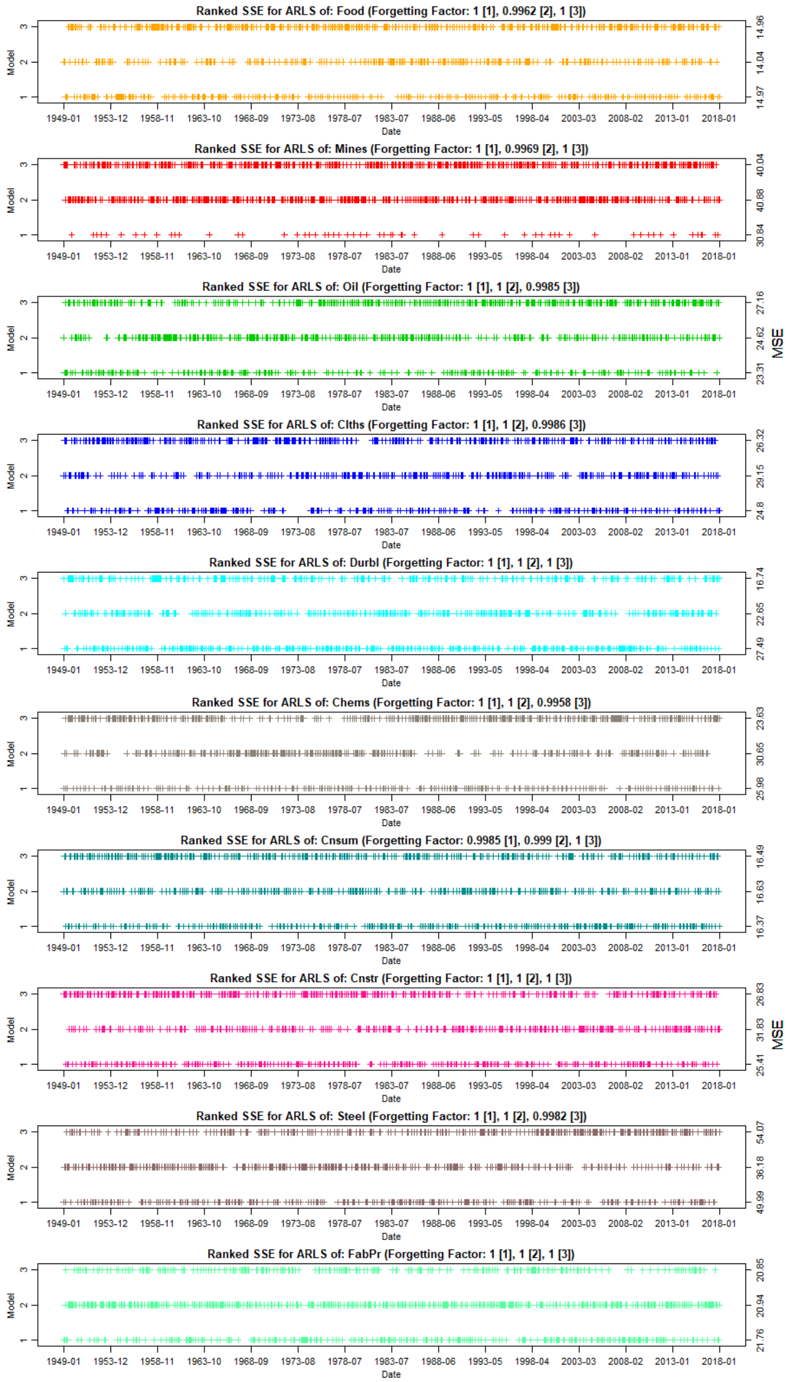
	1	2	3
Food	1	0.9962	1
Mines	1	0.9969	1
Oil	1	1	0.9985
Clths	1	1	0.9986
Durbl	1	1	1
Chems	1	1	0.9958
Cnsum	0.9985	0.9990	1
Cnstr	1	1	1
Steel	1	1	0.9982

	1	2	3
FabPr	1	1	1
Machn	1	1	0.9957
Cars	1	1	1
Trans	1	1	1
Utils	0.9921	1	0.9956
Rtail	1	1	1
Finan	1	1	1
Other	1	1	1

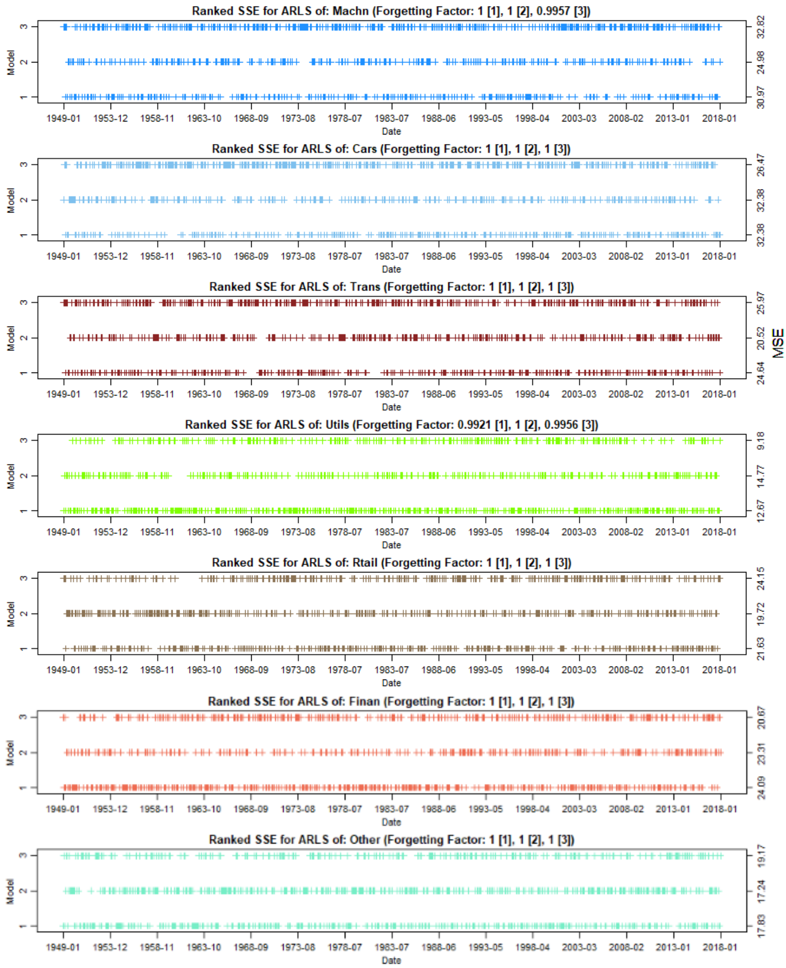
Table C.5: Optimal forgetting factor λ for the predictor variables for each of the top three models presented in Table C.1-C.3.

	1	2	3
Agric	0.9958	0.9962	0.9976
Food	0.9951	0.9937	0.9967
Soda	0.9985	0.9976	0.9959
Beer	0.9886	1	1
Smoke	0.9922	0.9942	0.9902
Toys	1	1	0.9953
Fun	1	1	1
Books	0.9766	0.9969	0.9929
Hshld	0.9975	1	0.9953
Clths	1	1	1
Hlth	0.9951	0.9899	0.9931
MedEq	0.9953	1	0.9913
Drugs	1	1	1
Chems	1	1	0.9968
Rubbr	1	1	1
Txtls	1	1	1
BldMt	1	1	1
Cnstr	1	1	0.9945
Steel	1	1	1
FabPr	1	0.9929	0.9978
Mach	1	1	1
ElcEq	0.9962	1	1
Autos	1	1	1
Aero	1	0.9956	0.9976

	1	2	3
Ships	1	0.9966	0.9974
Guns	0.9960	0.9962	0.9956
Gold	0.9919	0.9962	0.9963
Mines	1	1	1
Coal	0.9866	0.9913	0.9844
Oil	1	1	1
Util	0.9927	0.9944	0.9926
Telcm	1	0.9985	0.9981
PerSv	0.9931	0.9906	1
BusSv	1	1	1
Comps	0.9939	0.9895	0.9944
Chips	1	1	1
LabEq	0.9981	1	1
Paper	0.9980	1	1
Boxes	0.9980	0.9985	0.9984
Trans	1	1	1
Whlsl	0.9956	1	0.9990
Rtail	0.9976	1	0.9963
Meals	0.9951	0.9967	0.9953
Banks	0.9952	1	0.9986
Insur	1	0.9954	0.9957
RlEst	0.9975	1	0.9947
Fin	0.9981	1	1
Other	0.9927	0.9933	0.9941

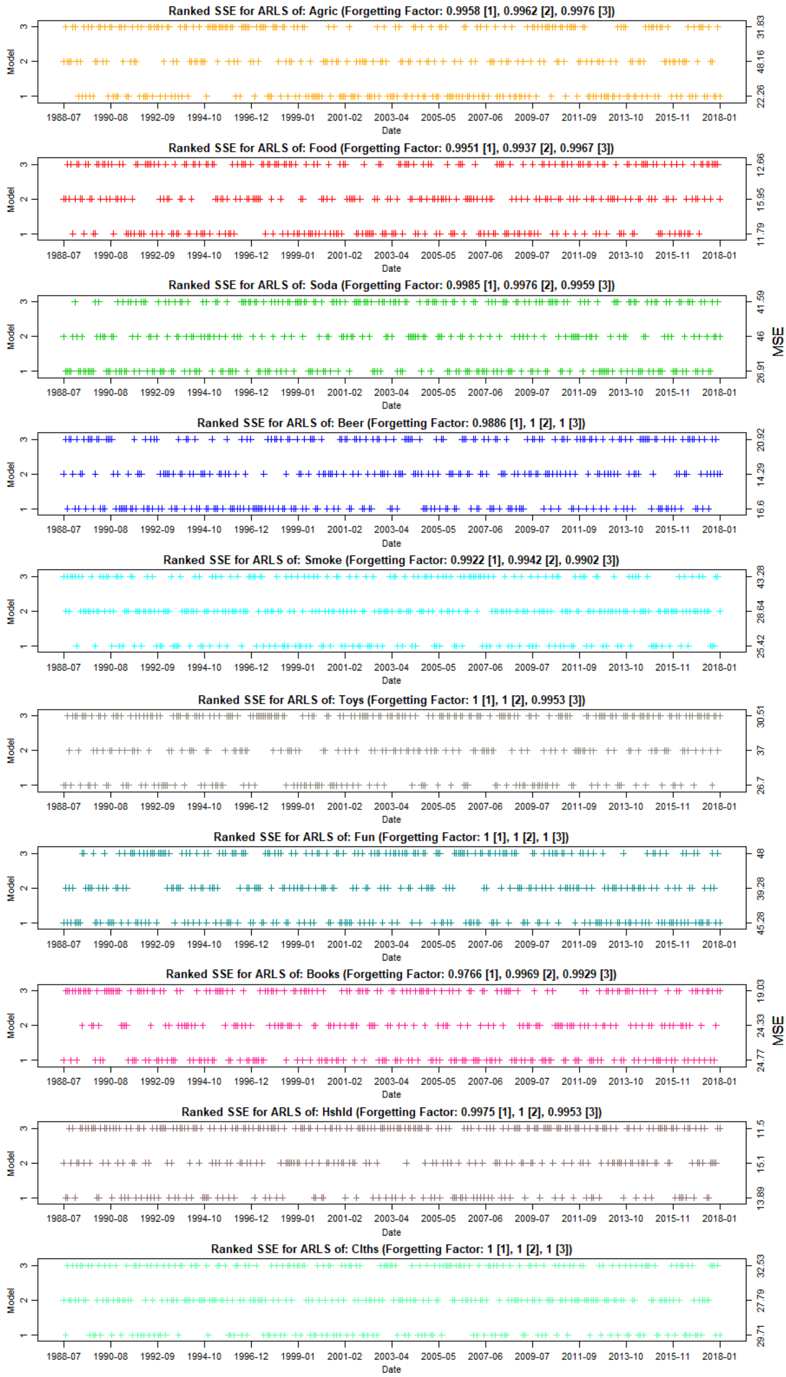

Ranked SSE for ARLS of: Food (Forgetting Factor: 1 [1], 0.9962 [2], 1 [3])
Ranked SSE for ARLS of: Mines (Forgetting Factor: 1 [1], 0.9969 [2], 1 [3])
Ranked SSE for ARLS of: Oil (Forgetting Factor: 1 [1], 1 [2], 0.9985 [3])
Ranked SSE for ARLS of: Clths (Forgetting Factor: 1 [1], 1 [2], 0.9986 [3])
Ranked SSE for ARLS of: Durbl (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: Chems (Forgetting Factor: 1 [1], 1 [2], 0.9958 [3])
Ranked SSE for ARLS of: Cnsum (Forgetting Factor: 0.9985 [1], 0.999 [2], 1 [3])
Ranked SSE for ARLS of: Cnstr (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: Steel (Forgetting Factor: 1 [1], 1 [2], 0.9982 [3])
Ranked SSE for ARLS of: FabPr (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Model
Date
MSE

(a)

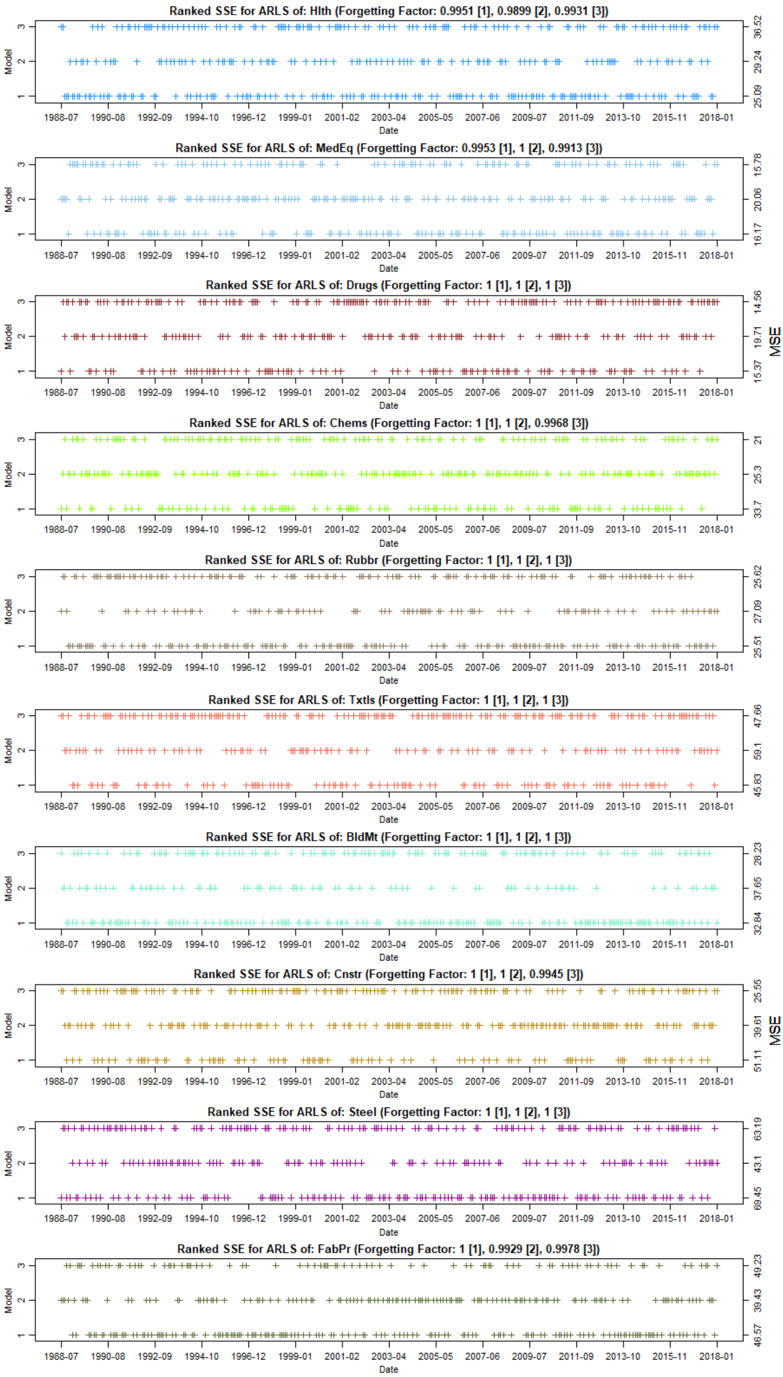


(b)

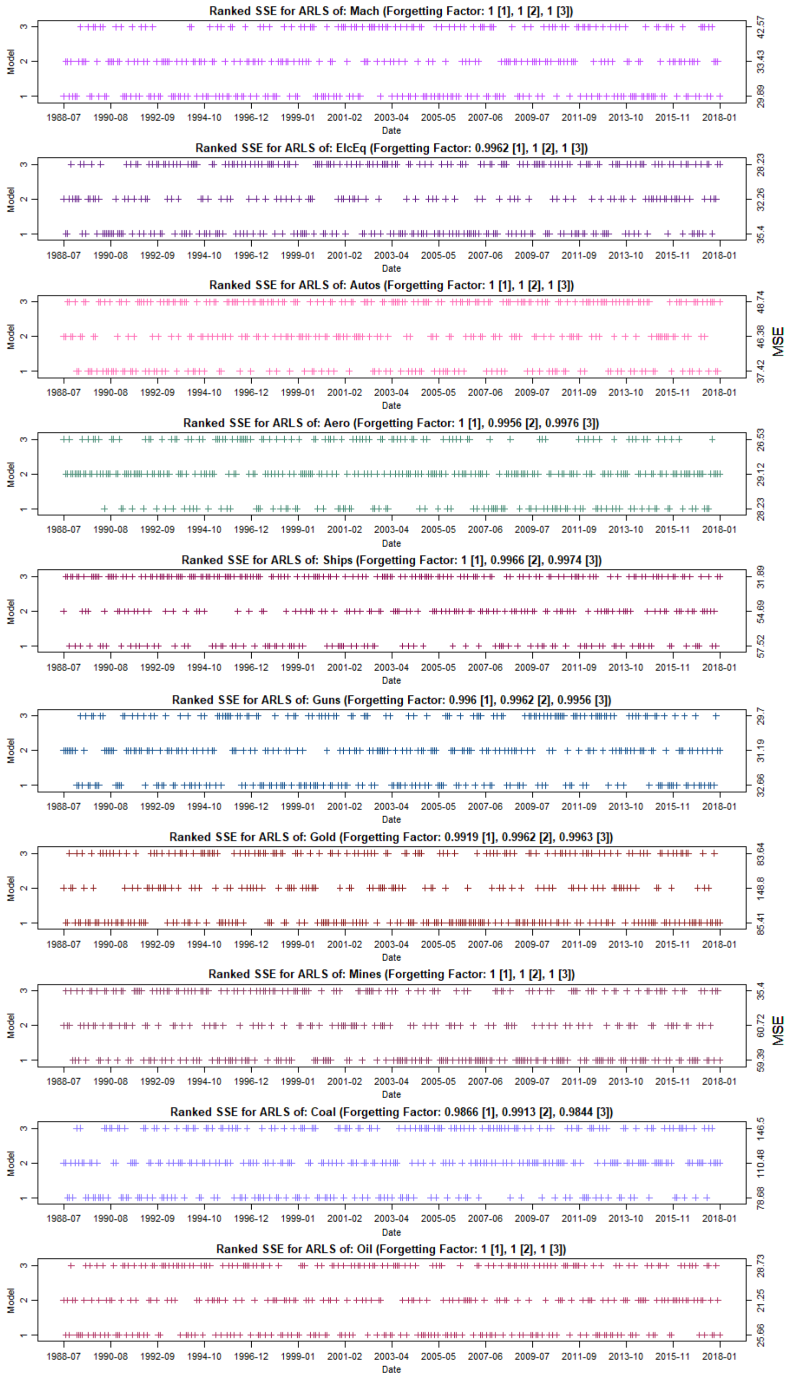
Figure C.11: The best performing model over time, for the **17 Industry Portfolio**, when continuously introducing a new observation. Each + sign indicates the best model at time t according to the **SSE**. The left axis shows the model number according to Table 4.8. Right axis shows the MSE of the corresponding plotted signs. The forgetting factor can be seen at the top of each plot, where the number inside the brackets is the model number.


Ranked SSE for ARLS of: Agric (Forgetting Factor: 0.9958 [1], 0.9962 [2], 0.9976 [3])
Ranked SSE for ARLS of: Food (Forgetting Factor: 0.9951 [1], 0.9937 [2], 0.9967 [3])
Ranked SSE for ARLS of: Soda (Forgetting Factor: 0.9985 [1], 0.9976 [2], 0.9959 [3])
Ranked SSE for ARLS of: Beer (Forgetting Factor: 0.9886 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: Smoke (Forgetting Factor: 0.9922 [1], 0.9942 [2], 0.9902 [3])
Ranked SSE for ARLS of: Toys (Forgetting Factor: 1 [1], 1 [2], 0.9953 [3])
Ranked SSE for ARLS of: Fun (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: Books (Forgetting Factor: 0.9766 [1], 0.9969 [2], 0.9929 [3])
Ranked SSE for ARLS of: Hshld (Forgetting Factor: 0.9975 [1], 1 [2], 0.9953 [3])
Ranked SSE for ARLS of: Clths (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Model
Date
MSE

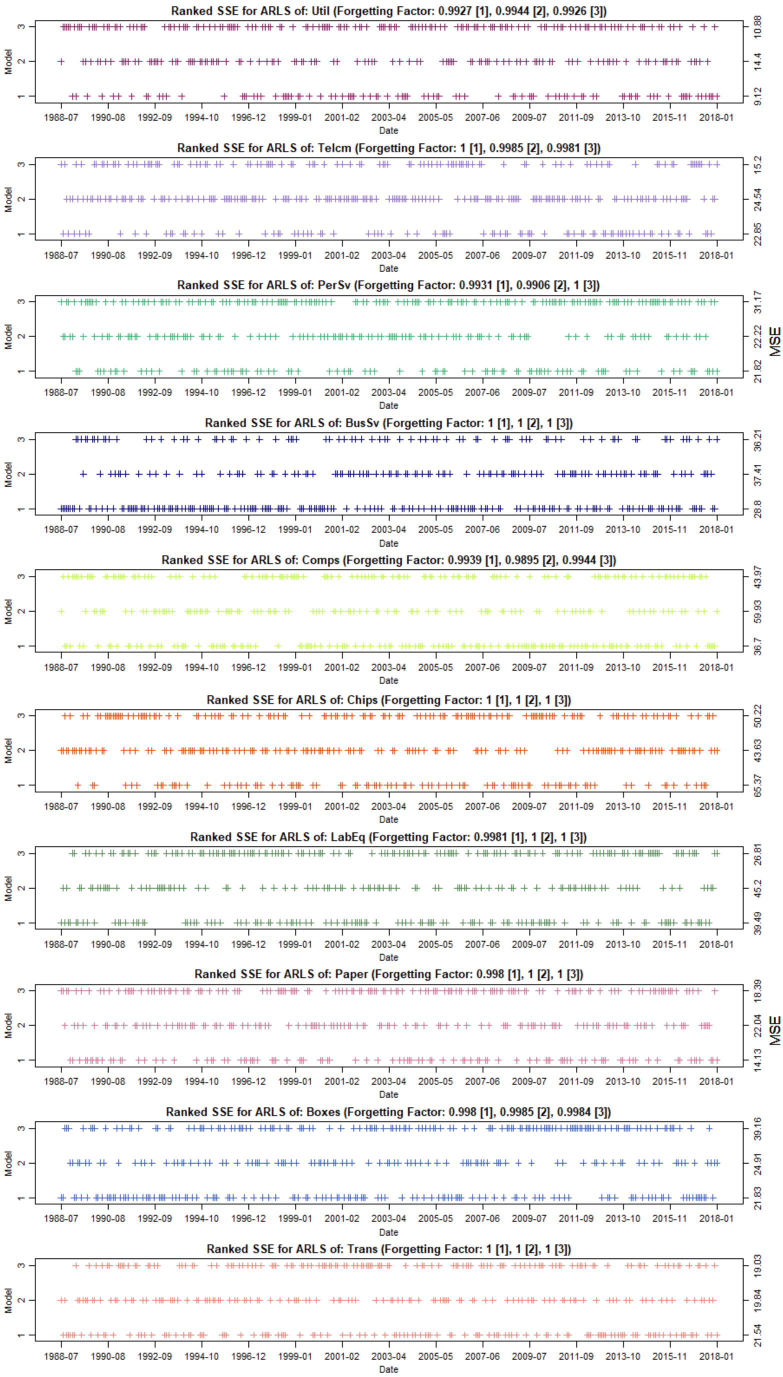
(a)


Ranked SSE for ARLS of: Hlth (Forgetting Factor: 0.9951 [1], 0.9899 [2], 0.9931 [3])
Ranked SSE for ARLS of: MedEq (Forgetting Factor: 0.9953 [1], 1 [2], 0.9913 [3])
Ranked SSE for ARLS of: Drugs (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: Chems (Forgetting Factor: 1 [1], 1 [2], 0.9968 [3])
Ranked SSE for ARLS of: Rubbr (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: Txtls (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: BldMt (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: Cnstr (Forgetting Factor: 1 [1], 1 [2], 0.9945 [3])
Ranked SSE for ARLS of: Steel (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: FabPr (Forgetting Factor: 1 [1], 0.9929 [2], 0.9978 [3])
Model
Date
MSE
1988-07
1990-08
1992-09
1994-10
1996-12
1999-01
2001-02
2003-04
2005-05
2007-06
2009-07
2011-09
2013-10
2015-11
2018-01

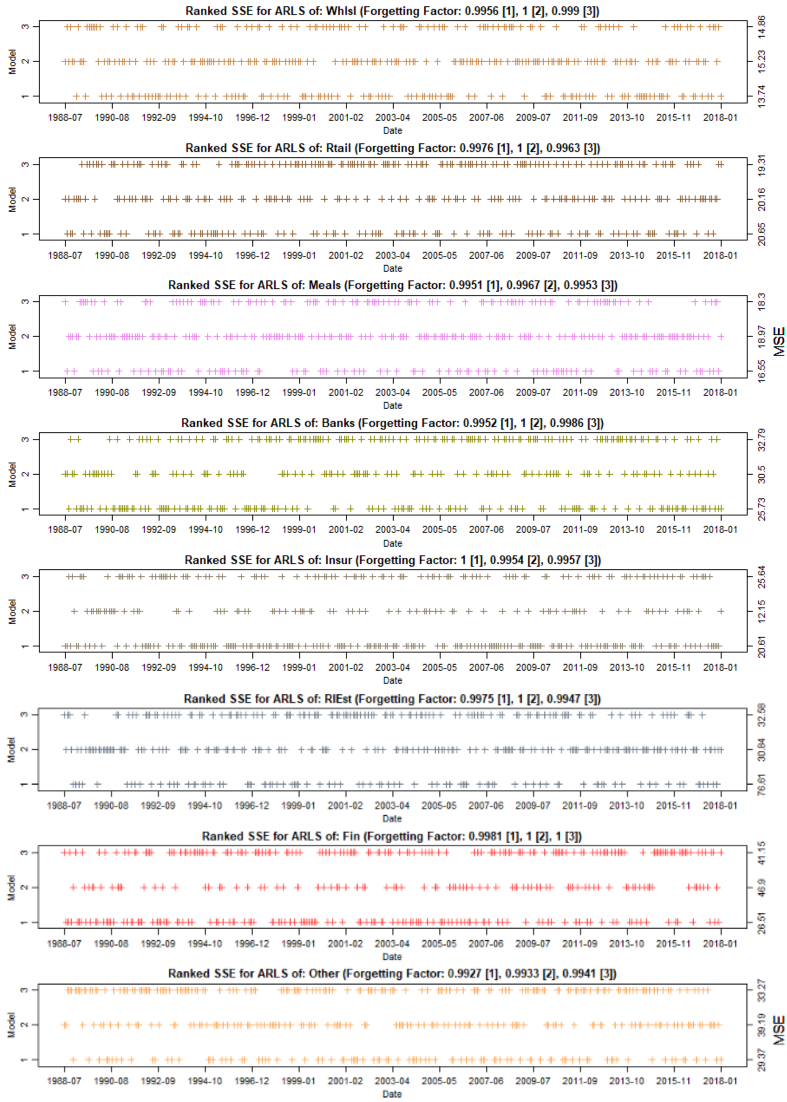
(b)


Ranked SSE for ARLS of: Mach (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Model
Date
42.57
33.43
29.89
Ranked SSE for ARLS of: ElcEq (Forgetting Factor: 0.9962 [1], 1 [2], 1 [3])
28.23
32.26
35.4
Ranked SSE for ARLS of: Autos (Forgetting Factor: 1 [1], 1 [2], 1 [3])
48.74
46.38
37.42
MSE
Ranked SSE for ARLS of: Aero (Forgetting Factor: 1 [1], 0.9956 [2], 0.9976 [3])
26.53
29.12
28.23
Ranked SSE for ARLS of: Ships (Forgetting Factor: 1 [1], 0.9966 [2], 0.9974 [3])
31.89
54.69
57.52
Ranked SSE for ARLS of: Guns (Forgetting Factor: 0.996 [1], 0.9962 [2], 0.9956 [3])
29.7
31.19
32.66
Ranked SSE for ARLS of: Gold (Forgetting Factor: 0.9919 [1], 0.9962 [2], 0.9963 [3])
83.64
148.8
85.41
Ranked SSE for ARLS of: Mines (Forgetting Factor: 1 [1], 1 [2], 1 [3])
35.4
60.72
59.39
MSE
Ranked SSE for ARLS of: Coal (Forgetting Factor: 0.9866 [1], 0.9913 [2], 0.9844 [3])
146.5
110.48
78.68
Ranked SSE for ARLS of: Oil (Forgetting Factor: 1 [1], 1 [2], 1 [3])
28.73
21.25
25.66
1988-07 1990-08 1992-09 1994-10 1996-12 1999-01 2001-02 2003-04 2005-05 2007-06 2009-07 2011-09 2013-10 2015-11 2018-01

(c)

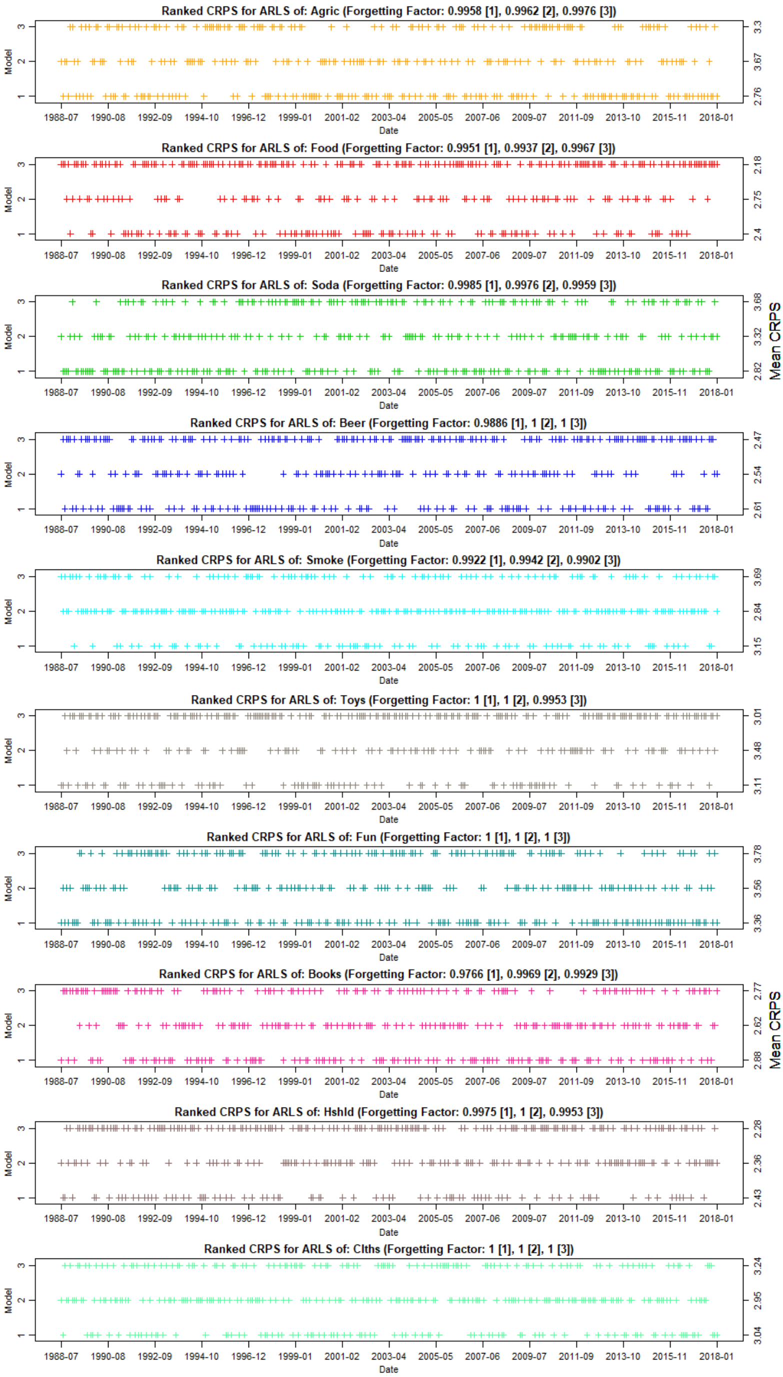

Ranked SSE for ARLS of: Util (Forgetting Factor: 0.9927 [1], 0.9944 [2], 0.9926 [3])
Ranked SSE for ARLS of: Telcm (Forgetting Factor: 1 [1], 0.9985 [2], 0.9981 [3])
Ranked SSE for ARLS of: PerSv (Forgetting Factor: 0.9931 [1], 0.9906 [2], 1 [3])
Ranked SSE for ARLS of: BusSv (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: Comps (Forgetting Factor: 0.9939 [1], 0.9895 [2], 0.9944 [3])
Ranked SSE for ARLS of: Chips (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: LabEq (Forgetting Factor: 0.9981 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: Paper (Forgetting Factor: 0.998 [1], 1 [2], 1 [3])
Ranked SSE for ARLS of: Boxes (Forgetting Factor: 0.998 [1], 0.9985 [2], 0.9984 [3])
Ranked SSE for ARLS of: Trans (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Model
Date
MSE

(d)

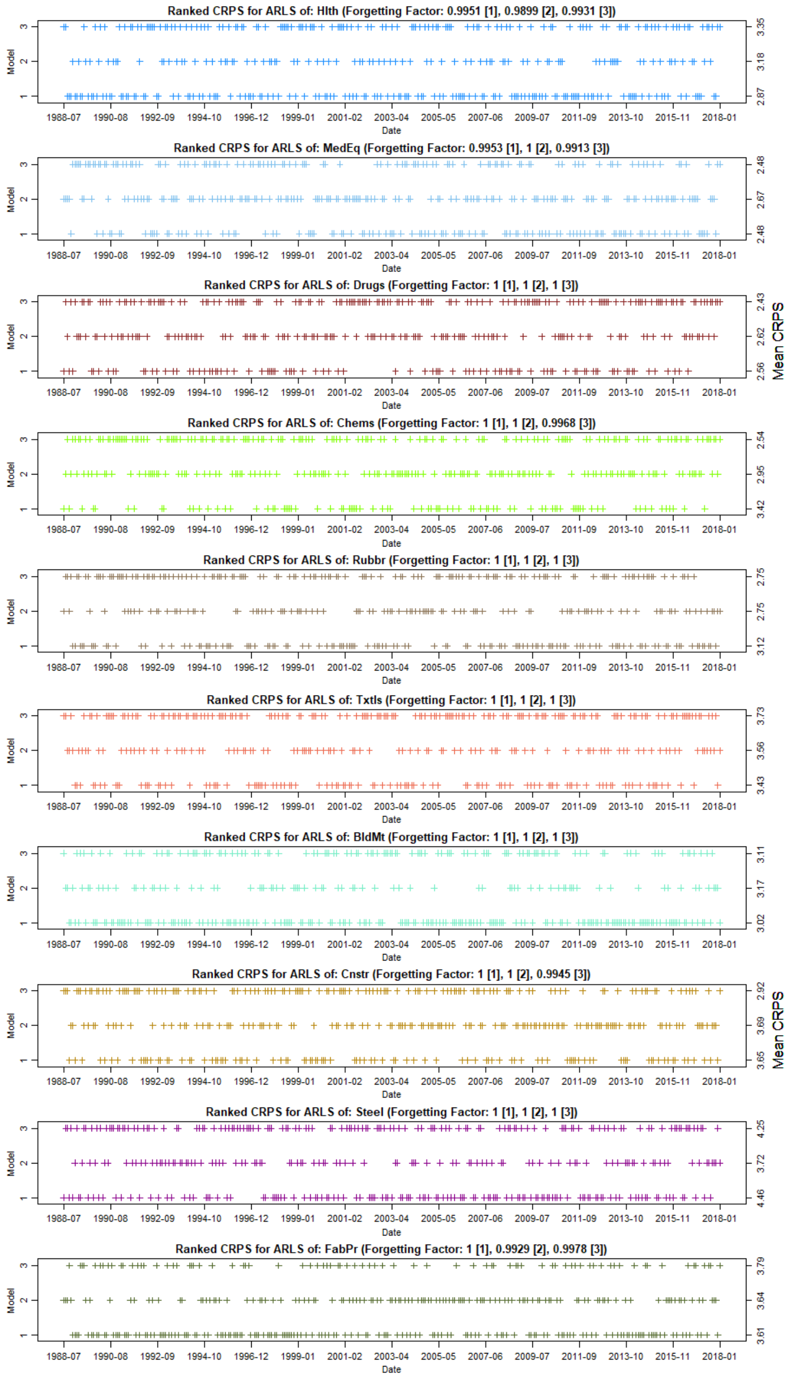


(e)

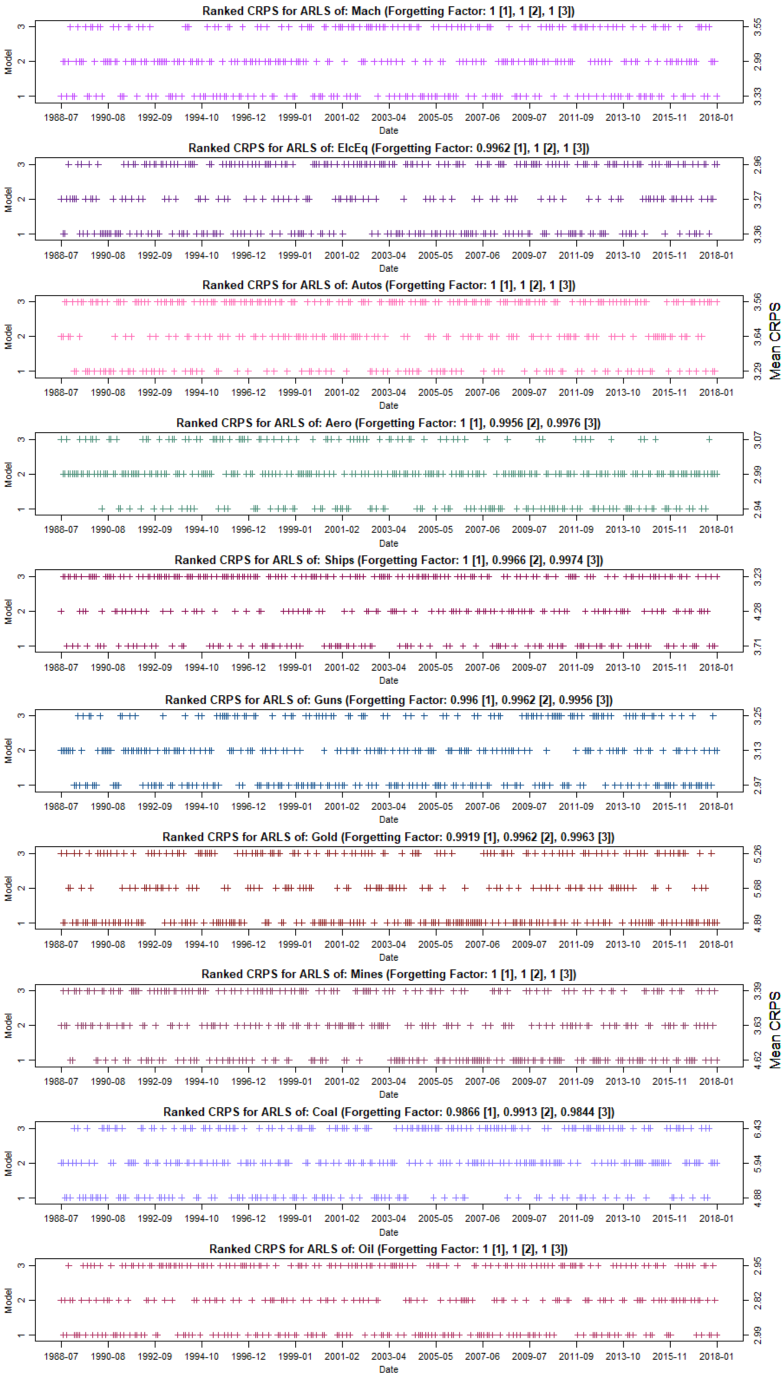
Figure C.12: (a-e) The best performing model over time, for the **48 Industry Portfolio**, when continuously introducing a new observation. Each + sign indicates the best model at time t according to the **SSE**. The left axis shows the model number according to Table 4.8. Right axis shows the MSE of the corresponding plotted signs. The forgetting factor can be seen at the top of each plot, where the number inside the brackets is the model number.


Ranked CRPS for ARLS of: Agric (Forgetting Factor: 0.9958 [1], 0.9962 [2], 0.9976 [3])
Ranked CRPS for ARLS of: Food (Forgetting Factor: 0.9951 [1], 0.9937 [2], 0.9967 [3])
Ranked CRPS for ARLS of: Soda (Forgetting Factor: 0.9985 [1], 0.9976 [2], 0.9959 [3])
Ranked CRPS for ARLS of: Beer (Forgetting Factor: 0.9886 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: Smoke (Forgetting Factor: 0.9922 [1], 0.9942 [2], 0.9902 [3])
Ranked CRPS for ARLS of: Toys (Forgetting Factor: 1 [1], 1 [2], 0.9953 [3])
Ranked CRPS for ARLS of: Fun (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: Books (Forgetting Factor: 0.9766 [1], 0.9969 [2], 0.9929 [3])
Ranked CRPS for ARLS of: Hshld (Forgetting Factor: 0.9975 [1], 1 [2], 0.9953 [3])
Ranked CRPS for ARLS of: Clths (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Model
Date
Mean CRPS
1988-07 1990-08 1992-09 1994-10 1996-12 1999-01 2001-02 2003-04 2005-05 2007-06 2009-07 2011-09 2013-10 2015-11 2018-01

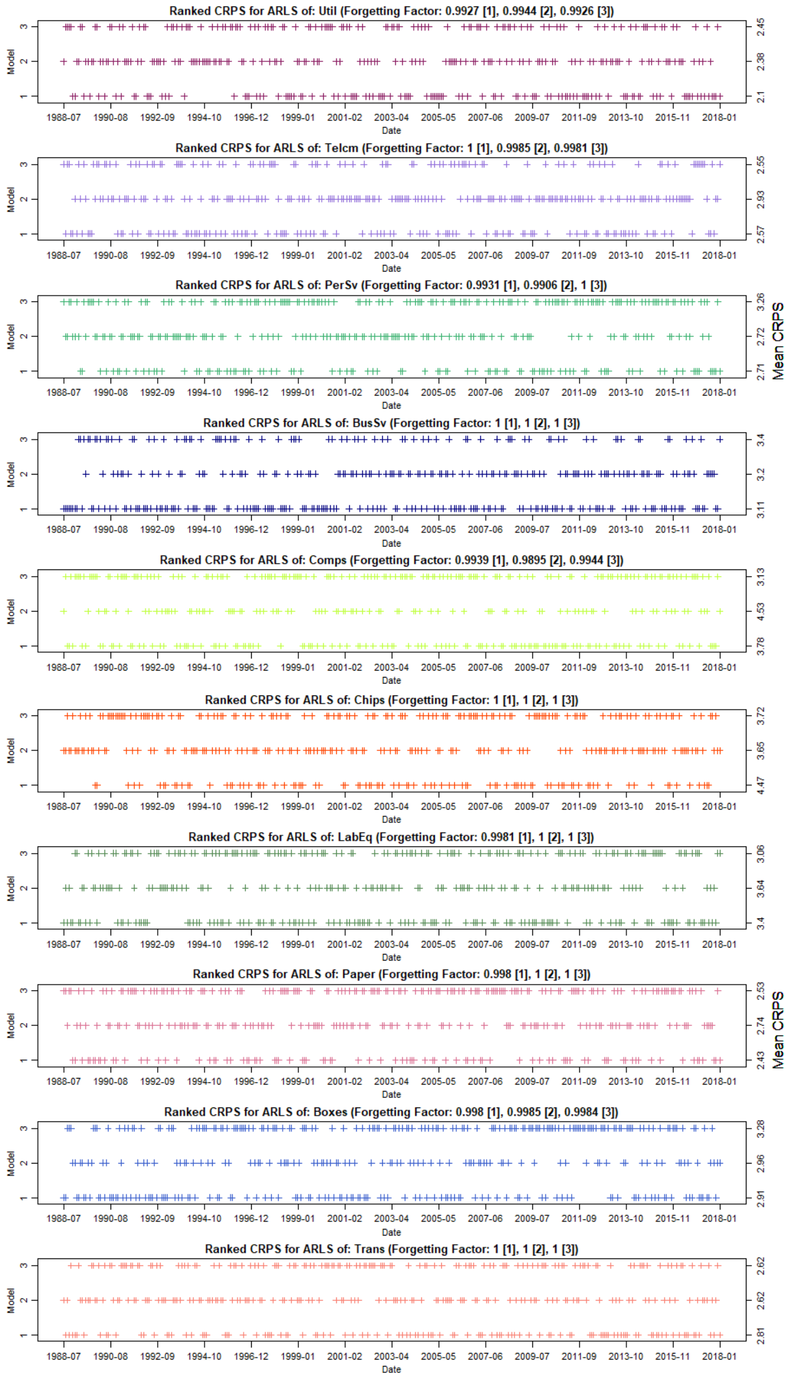
(a)


Ranked CRPS for ARLS of: Hlth (Forgetting Factor: 0.9951 [1], 0.9899 [2], 0.9931 [3])
Ranked CRPS for ARLS of: MedEq (Forgetting Factor: 0.9953 [1], 1 [2], 0.9913 [3])
Ranked CRPS for ARLS of: Drugs (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: Chems (Forgetting Factor: 1 [1], 1 [2], 0.9968 [3])
Ranked CRPS for ARLS of: Rubbr (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: Txtls (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: BldMt (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: Cnstr (Forgetting Factor: 1 [1], 1 [2], 0.9945 [3])
Ranked CRPS for ARLS of: Steel (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: FabPr (Forgetting Factor: 1 [1], 0.9929 [2], 0.9978 [3])
Model
Date
Mean CRPS

(b)

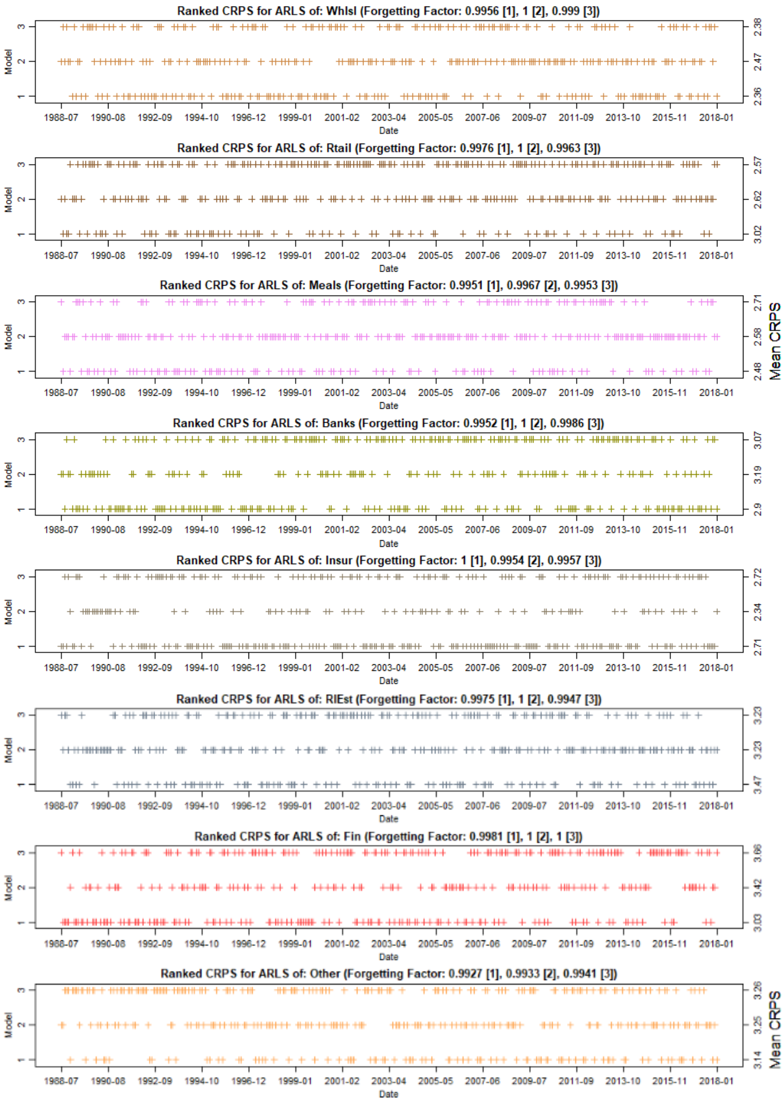

Ranked CRPS for ARLS of: Mach (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: ElcEq (Forgetting Factor: 0.9962 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: Autos (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: Aero (Forgetting Factor: 1 [1], 0.9956 [2], 0.9976 [3])
Ranked CRPS for ARLS of: Ships (Forgetting Factor: 1 [1], 0.9966 [2], 0.9974 [3])
Ranked CRPS for ARLS of: Guns (Forgetting Factor: 0.996 [1], 0.9962 [2], 0.9956 [3])
Ranked CRPS for ARLS of: Gold (Forgetting Factor: 0.9919 [1], 0.9962 [2], 0.9963 [3])
Ranked CRPS for ARLS of: Mines (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: Coal (Forgetting Factor: 0.9866 [1], 0.9913 [2], 0.9844 [3])
Ranked CRPS for ARLS of: Oil (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Model
Date
Mean CRPS
1988-07 1990-08 1992-09 1994-10 1996-12 1999-01 2001-02 2003-04 2005-05 2007-06 2009-07 2011-09 2013-10 2015-11 2018-01

(c)



Ranked CRPS for ARLS of: Util (Forgetting Factor: 0.9927 [1], 0.9944 [2], 0.9926 [3])
Ranked CRPS for ARLS of: Telcm (Forgetting Factor: 1 [1], 0.9985 [2], 0.9981 [3])
Ranked CRPS for ARLS of: PerSv (Forgetting Factor: 0.9931 [1], 0.9906 [2], 1 [3])
Ranked CRPS for ARLS of: BusSv (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: Comps (Forgetting Factor: 0.9939 [1], 0.9895 [2], 0.9944 [3])
Ranked CRPS for ARLS of: Chips (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: LabEq (Forgetting Factor: 0.9981 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: Paper (Forgetting Factor: 0.998 [1], 1 [2], 1 [3])
Ranked CRPS for ARLS of: Boxes (Forgetting Factor: 0.998 [1], 0.9985 [2], 0.9984 [3])
Ranked CRPS for ARLS of: Trans (Forgetting Factor: 1 [1], 1 [2], 1 [3])
Model
Date
Mean CRPS
1988-07
1990-08
1992-09
1994-10
1996-12
1999-01
2001-02
2003-04
2005-05
2007-06
2009-07
2011-09
2013-10
2015-11
2018-01

(d)



(e)

Figure C.13: (a-e) The best performing model over time, for the **48 Industry Portfolio**, when continuously introducing a new observation. Each + sign indicates the best model at time t according to the **CRPS**. The left axis shows the model number according to Table 4.8. Right axis shows the $\overline{CRPS}$ of the corresponding plotted signs. The forgetting factor can be seen at the top of each plot, where the number inside the brackets is the model number.

Food

	1	2	3
1	71	79	99
2	76	64	88
3	103	84	164

Mines

	1	2	3
1	2	27	31
2	32	177	175
3	26	181	177

Oil

	1	2	3
1	60	65	84
2	69	114	107
3	80	111	138

Clths

	1	2	3
1	91	67	82
2	74	97	99
3	76	105	137

Durbl

	1	2	3
1	134	129	62
2	101	86	82
3	90	54	90

Chems

	1	2	3
1	38	87	98
2	67	101	103
3	117	83	134

Cnsum

	1	2	3
1	81	87	99
2	84	77	98
3	103	94	105

Cnstr

	1	2	3
1	89	87	90
2	86	83	88
3	91	88	126

Steel

	1	2	3
1	82	94	77
2	88	110	100
3	82	94	101

FabPr

	1	2	3
1	78	104	62
2	122	162	98
3	43	117	42

Machn

	1	2	3
1	71	82	93
2	48	81	104
3	127	71	151

Cars

	1	2	3
1	61	87	92
2	56	75	89
3	124	57	187

Trans

	1	2	3
1	81	94	80
2	56	100	100
3	119	62	136

Utils

	1	2	3
1	163	112	105
2	118	54	61
3	98	67	50

Rtail

	1	2	3
1	109	105	78
2	102	103	86
3	81	83	81

Finan

	1	2	3
1	150	98	104
2	84	55	74
3	118	61	84

Other

	1	2	3
1	108	111	78
2	114	139	77
3	74	80	47

(a) SSE

Food

	1	2	3
1	100	67	109
2	67	48	80
3	110	79	168

Mines

	1	2	3
1	4	34	33
2	33	169	174
3	34	174	173

Oil

	1	2	3
1	85	73	84
2	78	104	95
3	79	100	130

Clths

	1	2	3
1	91	75	69
2	78	109	104
3	67	106	129

Durbl

	1	2	3
1	120	124	59
2	96	101	87
3	87	59	95

Chems

	1	2	3
1	39	88	102
2	74	97	98
3	115	84	131

Cnsum

	1	2	3
1	83	81	107
2	77	72	92
3	112	87	117

Cnstr

	1	2	3
1	92	82	94
2	87	78	86
3	89	92	128

Steel

	1	2	3
1	83	102	75
2	98	107	94
3	78	90	101

FabPr

	1	2	3
1	70	106	59
2	123	186	93
3	41	111	39

Machn

	1	2	3
1	72	82	92
2	54	89	100
3	120	73	146

Cars

	1	2	3
1	64	91	93
2	63	75	85
3	122	56	179

Trans

	1	2	3
1	80	97	81
2	65	108	94
3	114	62	127

Utils

	1	2	3
1	140	130	88
2	127	83	66
3	90	63	41

Rtail

	1	2	3
1	109	107	78
2	108	102	82
3	77	82	83

Finan

	1	2	3
1	155	98	105
2	85	53	73
3	118	61	80

Other

	1	2	3
1	103	114	73
2	117	149	76
3	69	79	48

(b) CRPS

Figure C.14: Tables with the number of times the preferred model changes over the entire time series for the **17 Industry Portfolio**.

Agric

	1	2	3
1	45	43	39
2	40	33	33
3	43	29	49

Food

	1	2	3
1	41	39	35
2	37	41	40
3	37	38	46

Soda

	1	2	3
1	56	33	38
2	29	29	42
3	42	38	47

Beer

	1	2	3
1	45	44	36
2	39	38	37
3	41	32	42

Smoke

	1	2	3
1	19	46	25
2	44	68	51
3	27	50	24

Toys

	1	2	3
1	30	29	36
2	23	31	47
3	41	41	76

Fun

	1	2	3
1	37	44	39
2	39	36	37
3	44	32	46

Books

	1	2	3
1	42	36	45
2	34	36	34
3	46	32	49

Hshld

	1	2	3
1	14	23	49
2	27	27	51
3	45	54	64

Clths

	1	2	3
1	16	34	40
2	40	57	49
3	35	54	29

Hlth

	1	2	3
1	47	44	43
2	35	25	41
3	52	32	35

MedEq

	1	2	3
1	40	39	34
2	36	64	39
3	37	35	30

Drugs

	1	2	3
1	29	30	41
2	31	38	44
3	39	45	57

Chems

	1	2	3
1	31	35	33
2	32	56	53
3	35	50	29

Rubbr

	1	2	3
1	57	32	51
2	39	23	26
3	44	33	49

Txtls

	1	2	3
1	27	30	41
2	31	22	46
3	40	48	69

BldMt

	1	2	3
1	76	30	53
2	33	18	26
3	51	29	38

Cnstr

	1	2	3
1	26	34	31
2	28	56	53
3	37	47	42

Steel

	1	2	3
1	40	46	41
2	37	23	43
3	49	35	40

FabPr

	1	2	3
1	51	57	37
2	60	37	30
3	34	32	16

Mach

	1	2	3
1	46	43	42
2	51	41	31
3	34	39	27

ElcEq

	1	2	3
1	47	36	48
2	30	18	42
3	54	35	44

Autos

	1	2	3
1	23	30	43
2	25	38	45
3	48	39	63

Aero

	1	2	3
1	25	40	25
2	44	81	46
3	21	51	21

Ships

	1	2	3
1	22	25	44
2	27	35	48
3	42	49	62

Guns

	1	2	3
1	40	43	33
2	45	50	36
3	31	38	38

Gold

	1	2	3
1	66	31	54
2	33	23	33
3	53	34	27

Mines

	1	2	3
1	69	32	42
2	35	24	36
3	40	38	38

Coal

	1	2	3
1	27	38	31
2	36	62	46
3	33	44	37

Oil

	1	2	3
1	34	42	46
2	36	36	41
3	53	34	32

Util

	1	2	3
1	30	24	39
2	24	40	50
3	40	49	58

Telcm

	1	2	3
1	21	37	26
2	41	82	43
3	22	47	35

PerSv

	1	2	3
1	25	25	37
2	17	35	60
3	46	52	57

BusSv

	1	2	3
1	81	49	38
2	51	29	26
3	35	28	17

Comps

	1	2	3
1	45	40	45
2	39	30	34
3	46	33	42

Chips

	1	2	3
1	24	32	34
2	38	53	44
3	28	50	51

LabEq

	1	2	3
1	36	33	45
2	33	27	40
3	44	40	56

Paper

	1	2	3
1	34	34	30
2	26	41	51
3	39	43	56

Boxes

	1	2	3
1	40	39	45
2	42	20	35
3	41	39	53

Trans

	1	2	3
1	47	35	35
2	36	34	45
3	35	45	42

Whlsl

	1	2	3
1	27	52	29
2	51	54	43
3	31	41	26

Rtail

	1	2	3
1	28	27	46
2	34	40	45
3	39	51	44

Meals

	1	2	3
1	19	44	28
2	36	75	44
3	36	37	35

Banks

	1	2	3
1	50	25	51
2	35	27	37
3	42	46	41

Insur

	1	2	3
1	79	34	52
2	39	21	22
3	46	28	33

RlEst

	1	2	3
1	20	37	31
2	34	68	45
3	34	43	42

Fin

	1	2	3
1	42	31	52
2	40	18	28
3	43	37	63

Other

	1	2	3
1	25	30	38
2	34	37	45
3	35	48	62

Figure C.15: Tables with the number of times the preferred model changes over the entire time series using SSE, for the **48 Industry Portfolio**.

Agric

	1	2	3
1	41	46	39
2	44	37	31
3	42	28	46

Food

	1	2	3
1	32	23	44
2	21	16	39
3	46	37	96

Soda

	1	2	3
1	66	43	36
2	35	29	38
3	44	30	33

Beer

	1	2	3
1	21	31	39
2	30	29	46
3	40	45	73

Smoke

	1	2	3
1	21	47	18
2	46	83	51
3	19	51	18

Toys

	1	2	3
1	28	25	35
2	20	31	50
3	39	45	81

Fun

	1	2	3
1	42	44	44
2	40	36	33
3	48	29	38

Books

	1	2	3
1	31	42	34
2	39	57	44
3	36	41	30

Hshld

	1	2	3
1	9	20	47
2	30	46	50
3	37	60	55

Clths

	1	2	3
1	20	37	34
2	40	64	49
3	32	51	27

Hlth

	1	2	3
1	54	39	50
2	32	19	36
3	57	29	38

MedEq

	1	2	3
1	36	37	35
2	40	70	39
3	32	41	24

Drugs

	1	2	3
1	24	30	37
2	29	42	48
3	37	47	60

Chems

	1	2	3
1	25	34	28
2	31	55	55
3	30	52	44

Rubbr

	1	2	3
1	45	29	45
2	35	32	36
3	39	42	51

Txtls

	1	2	3
1	31	32	41
2	34	19	44
3	39	47	67

BldMt

	1	2	3
1	87	33	51
2	34	19	23
3	51	24	32

Cnstr

	1	2	3
1	32	34	36
2	33	52	46
3	38	45	38

Steel

	1	2	3
1	41	43	43
2	33	22	43
3	52	34	43

FabPr

	1	2	3
1	63	59	37
2	57	30	28
3	38	26	16

Mach

	1	2	3
1	32	41	38
2	48	54	37
3	31	44	29

ElcEq

	1	2	3
1	41	33	49
2	29	17	42
3	53	37	53

Autos

	1	2	3
1	20	28	43
2	23	37	46
3	48	40	69

Aero

	1	2	3
1	38	52	18
2	52	88	36
3	18	37	15

Ships

	1	2	3
1	36	25	48
2	28	27	39
3	45	41	65

Guns

	1	2	3
1	52	48	35
2	50	50	28
3	33	30	28

Gold

	1	2	3
1	70	36	50
2	32	24	33
3	55	29	25

Mines

	1	2	3
1	56	33	37
2	34	34	45
3	37	45	33

Coal

	1	2	3
1	24	36	34
2	35	58	44
3	35	43	45

Oil

	1	2	3
1	35	35	53
2	31	32	40
3	58	35	35

Util

	1	2	3
1	42	45	32
2	48	59	37
3	30	39	22

Telcm

	1	2	3
1	35	51	28
2	47	75	33
3	32	29	24

PerSv

	1	2	3
1	25	28	39
2	20	39	50
3	48	42	63

BusSv

	1	2	3
1	73	50	33
2	52	39	27
3	30	29	21

Comps

	1	2	3
1	34	27	54
2	31	24	35
3	50	39	60

Chips

	1	2	3
1	25	31	35
2	39	48	45
3	27	53	51

LabEq

	1	2	3
1	41	35	45
2	36	30	38
3	43	39	47

Paper

	1	2	3
1	26	30	29
2	24	43	52
3	36	46	68

Boxes

	1	2	3
1	34	35	47
2	42	21	32
3	39	40	64

Trans

	1	2	3
1	41	33	34
2	34	42	47
3	34	47	42

Whlsl

	1	2	3
1	27	47	34
2	51	48	41
3	31	44	31

Rtail

	1	2	3
1	13	29	34
2	32	49	58
3	31	60	48

Meals

	1	2	3
1	20	46	23
2	40	86	43
3	29	38	29

Banks

	1	2	3
1	47	23	49
2	34	29	37
3	39	47	49

Insur

	1	2	3
1	76	30	55
2	36	20	22
3	48	29	38

RlEst

	1	2	3
1	23	44	32
2	38	72	43
3	38	38	26

Fin

	1	2	3
1	37	32	48
2	43	33	28
3	37	39	57

Other

	1	2	3
1	25	33	25
2	29	53	52
3	30	47	60

Figure C.16: Tables with the number of times the preferred model changes over the entire time series using CRPS, for the **48 Industry Portfolio**.

APPENDIX D

Appendix D

D.1 Top 3 Best Models

Table D.1: The top three overall best models for each response variable of the 10 Industry Portfolio. Note that these are the models that will be used in the adaptive RLS.

Model	**NoDur**	**Durbl**	**Manuf**	**Enrgy**	**HiTec**
1	Const	Shops1	Const	Const	Hlth1
2	Hlth1 Const	Other1	NoDur1 Other1 Const	NoDur1 HiTec1 Const	Const
3	Enrgy1 Shops1 Const	Durbl1 Shops1	Enrgy1 Shops1 Const	Durbl1 Shops1 Const	Trend

Model	**Telcm**	**Shops**	**Hlth**	**Utils**	**Other**
1	Const	Const	Const	Const	Const
2	Durbl1 HiTec1 Const	Shops1 Const	NoDur1 Telcm1 Const	Enrgy1 Const	Enrgy1 Other1 Const
3	NoDur1 Durbl1 Const	Enrgy1 Other1 Const	Trend	Other1 Const	Durbl1 Enrgy1 Other1 Const

D.2 The Sliding Model

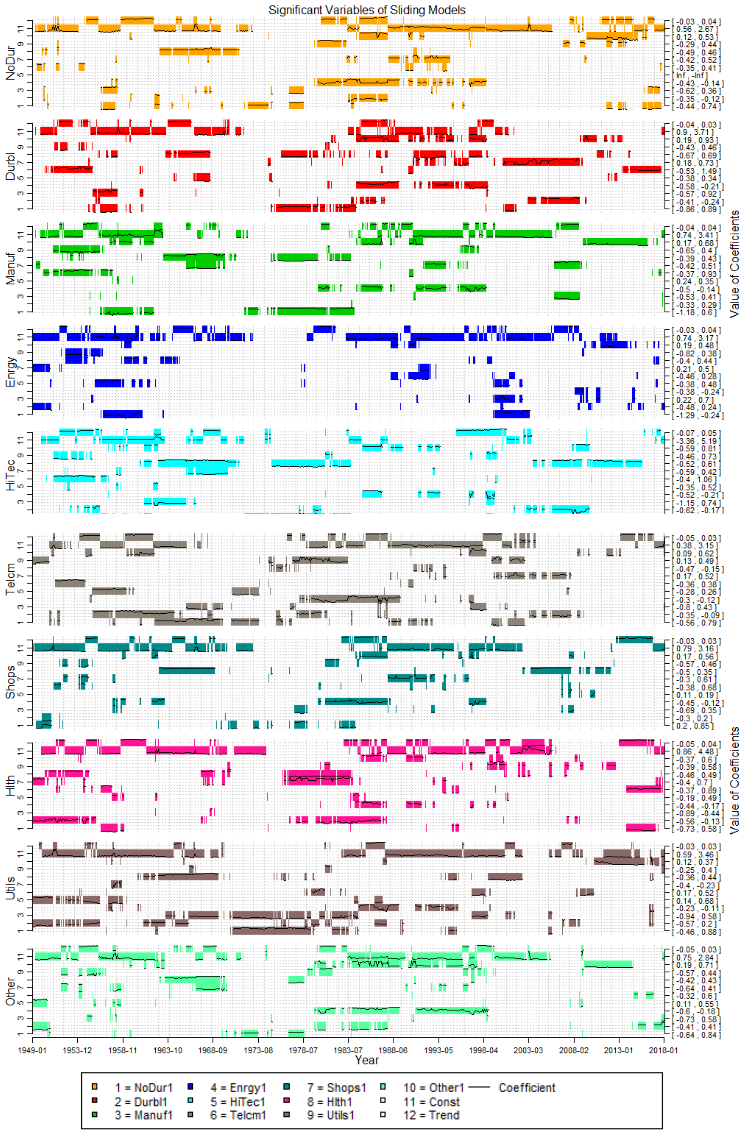


Figure D.1: The sliding model for the 10 industries. Each date at the x-axis represents the last date in one block of 9 years. Thus the dates are only there for guidance. The left axis shows the response variable and the corresponding predictors are given by the numbers. The right axis is the range of the values of the coefficients for the significant predictors. The black lines within the coloured blocks shows how the coefficient values fluctuate.

D.3 ARLS Results for the 10 Industry Portfolios

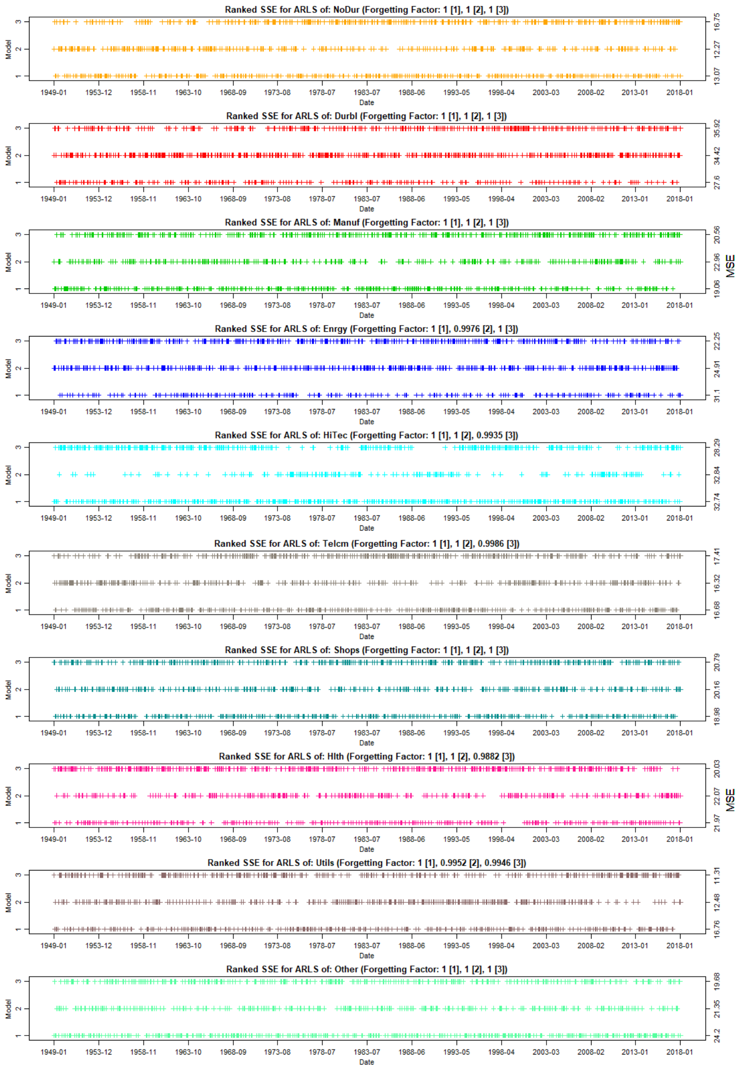


Figure D.2: The best performing model over time, for the **10 Industry Portfolio**, when continuously introducing a new observation. Each + sign indicates the best model at time t according to the **SSE**. The left axis shows the model number according to Table D.1. Right axis shows the MSE of the corresponding plotted signs. The forgetting factor can be seen at the top of each plot, where the number inside the brackets is the model number.

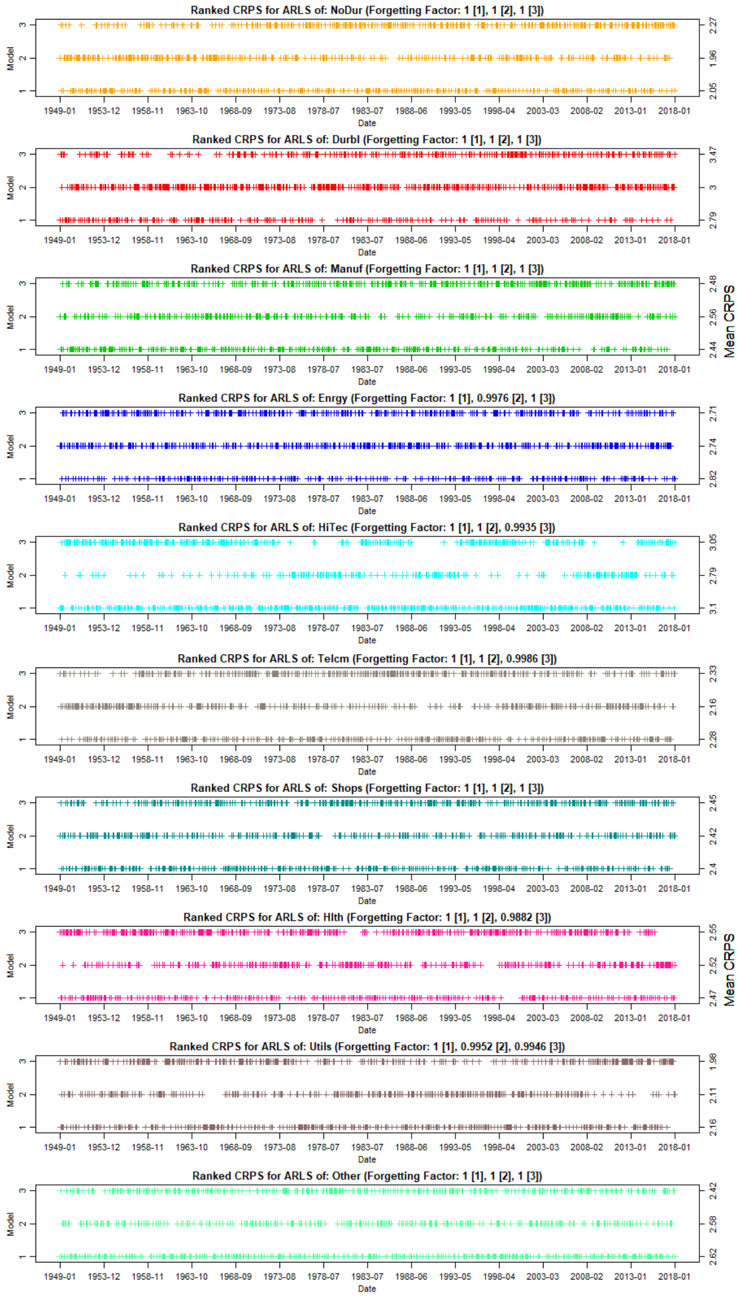


Figure D.3: The best performing model over time, for the **10 Industry Portfolio**, when continuously introducing a new observation. Each + sign indicates the best model at time t according to the **CRPS**. The left axis shows the model number according to Table 4.8. Right axis shows the $\overline{CRPS}$ of the corresponding plotted signs. The forgetting factor can be seen at the top of each plot, where the number inside the brackets is the model number.

APPENDIX E

Appendix E

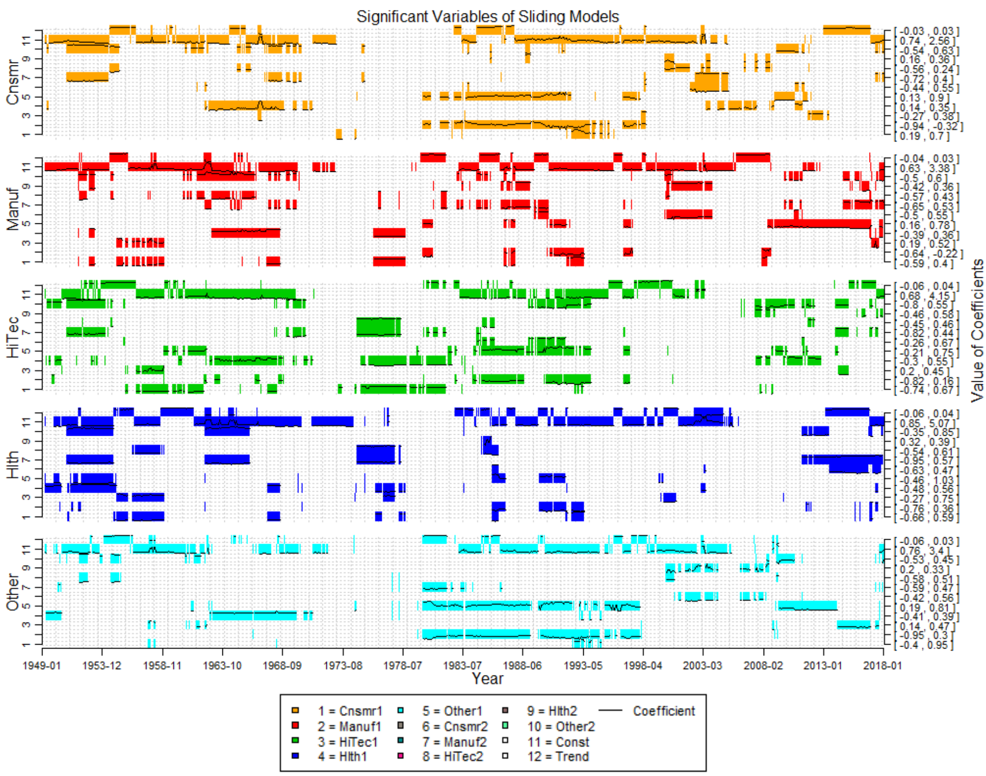


Figure E.1: This plot shows the results when including order of 2 in the VAR-model. Significant variables found by translating the model one month using a block length of 9 years. The left axis shows the response variable and the corresponding predictor variables given by the numbers from 1 to 12. The right axis is the range of the values of the coefficients for the significant predictor variables. The black lines within the coloured blocks shows how the coefficient values fluctuate. Finally the coloured blocks indicate that the variable is significant and is included in the model.

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