Community Supported Quasi-Monte Carlo (QMC) Software

Fred Hickernell (hickernell@iit.edu), Sou-Cheng Choi (schoi32@iit.edu), Aleksei Sorokin (asorokin@hawk.iit.edu)

Illinois Instute of Technology

Software Objectives

To provide QMC software that is:

- Comprised of free open source tools
- Designed for development and support
- Easy to use for non-experts
- The recognized standard

The QMC Problem

Original Problem

$$\int_T g(t)\lambda(t)dt \tag{1}$$

$g : T \to \mathbb{R}$ = original integrand
λ = original weight

Convenient Form

$$\int_X f(x)\rho(x)dx = \int_X f(x)\hat{\gamma}(dx) \tag{2}$$

γ = well defined probability measure
$\phi : X \to T$ = change of variables
$f : X \to \mathbb{R}$ = integrand after change of variables

(quasi-)Monte Carlo Approximation

$$\sum_{i=1}^{n} f(x_i)w_i \tag{3}$$

$\hat{\gamma}_n \approx \gamma$ = discrete probability distribution
n guarantees

$$\left|\int_X f(x)\gamma(dx) - \sum_{i=1}^{n} f(x_i)w_i\right| \le \epsilon \tag{4}$$

Integrate

Specify and generate values $f(\hat{x})$ for $\hat{x} \in \hat{\gamma}$
Arguments

- funObj
- measureObj
- distribObj
- stopcritObj

Function

Specify and generate values $f(\hat{x})$ for $\hat{x} \in \hat{\gamma}$
Concrete Classes

- Keister
- Asian Call

Discrete Distribution

Specify and generate $a_n \sum_{i=1}^{n} w_i \delta_{\hat{x}_i}(\cdot)$
Concrete Classes

- IID
- Mesh

Stopping Criterion

Finds n such that Equation (4) holds
Concrete Classes

- Central Limit Theorem (IID)
- Mean Variance (Mesh)

Measure

Specify components of a general sampling method
Implemented Functions

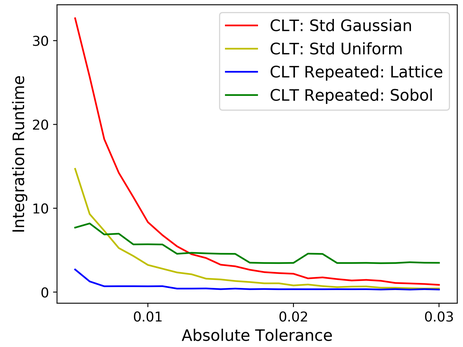
- Standard Uniform
- Standard Gaussian
- IID Zero Mean Gaussian
- Brownian Motion
- Lattice base 2
- Sobol base 2

Accumulate Data

Accumulated data required in the computation of the integral

Results

Integration Time Comparison



Future Work

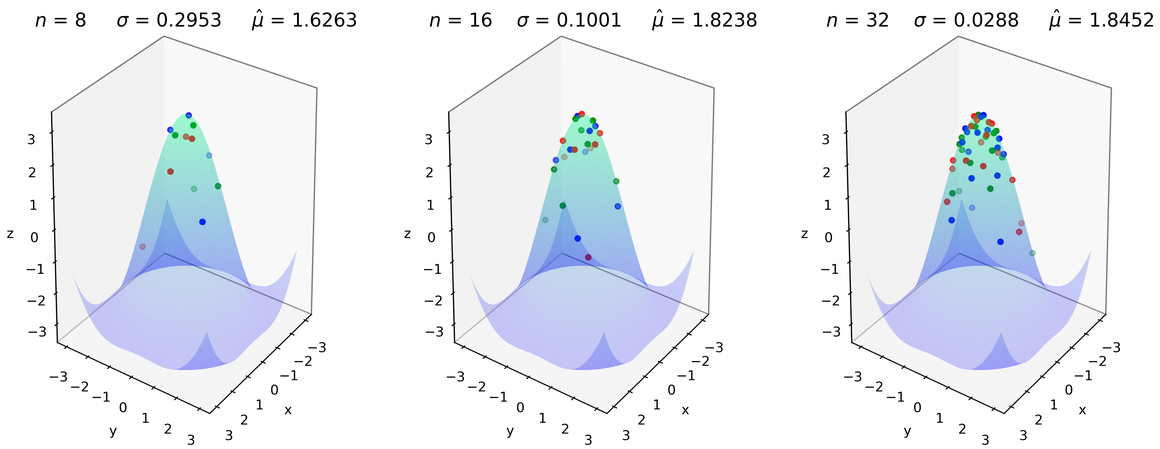
- Enhance testing and examples library
- Incorporate existing components
- Expand community of contributors

References

1. S.-C. T. Choi, Y. Ding, F. J. Hickernell, L. Jiang, Ll. A. JimÃlnez Rugama, D. Li, R. Jagadeeswaran, X. Tong, K. Zhang,Y. Zhang, and X. Zhou,"GAIL: Guaranteed Automatic Integration Library (versions 1.0-2.2)," **http://gailgithub.github.io/GAIL_Dev/**, MATLAB software, 2013-2019.

Python Example

```
dim = 3
stopObj = CLT_Rep(
    nInit=4,nMax=2**15,absTol=.05,J=3)
measureObj = measure().IIDZMeanGaussian(
    dimension=[dim],variance=[1/2])
distribObj = Mesh(trueD=measure().mesh(
    dimension=[dim],meshType='lattice'))
sol,out = integrate(KeisterFun(),
    measureObj,distribObj,stopObj)
```



Acknowledgements

github.com/QMCSoftware/QMCSoftware.git
Other References and Aknowledgements
Larger ∫ and Σ symbols
Colors and other styling
Better captions on figures
IIT and QMC logo
Spacing of Original Problem
Fix python example to match fixed figure
Integrating over unit cube?