Community Supported Quasi-Monte Carlo (QMC) Software

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Software Objectives

To provide QMC software that is:

- Comprised of free open source tools
- Designed for development and support
- Easy to use for non-experts
- The recognized standard

The QMC Problem

Original Form

$$\mu = \int_T g(t)\,\lambda(\mathrm{d}t)$$

$g : T \to \mathbb{R}$ = original integrand
λ = original measure

Convenient Form

$$\mu = \int_X f(x)\rho(x)dx = \int_X f(x)\,\nu(\mathrm{d}x)$$

ν = well defined probability measure
$\phi : X \to T$ = change of variables
$f : X \to \mathbb{R}$ = integrand after change of variables

(quasi-)Monte Carlo Approximation

$$\hat{\mu}_n = a_n \sum_{i=1}^{n} f(x_i)w_i = \int_X f(x)\,\hat{\nu}(\mathrm{d}x)$$

$$\nu \approx \hat{\nu}_n = a_n \sum_{i=1}^{n} w_i \delta_{\hat{x}_i}(\cdot)$$

= discrete probability measure

Integrate

Find $\hat{\mu}_n$ such that $|\mu - \hat{\mu}| \le \epsilon$

Arguments

- Funciton Instance
- Measure Instance
- Discrete Distribution Instance
- Stopping Criterion Instance

Function

Specify and generate values $f(\hat{x})$ for $\hat{x} \in \hat{\nu}$

Concrete Classes

- Keister
- Asian Call

Discrete Distribution

Specify and generate $a_n \sum_{i=1}^{n} w_i \delta_{\hat{x}_i}(\cdot)$

Concrete Classes

- IID
- QMC

Stopping Criterion

Determine n

Concrete Classes

- Central Limit Theorem (IID)
- Mean Variance (Mesh)

Measure

Specify components of a general sampling method

Implemented Functions

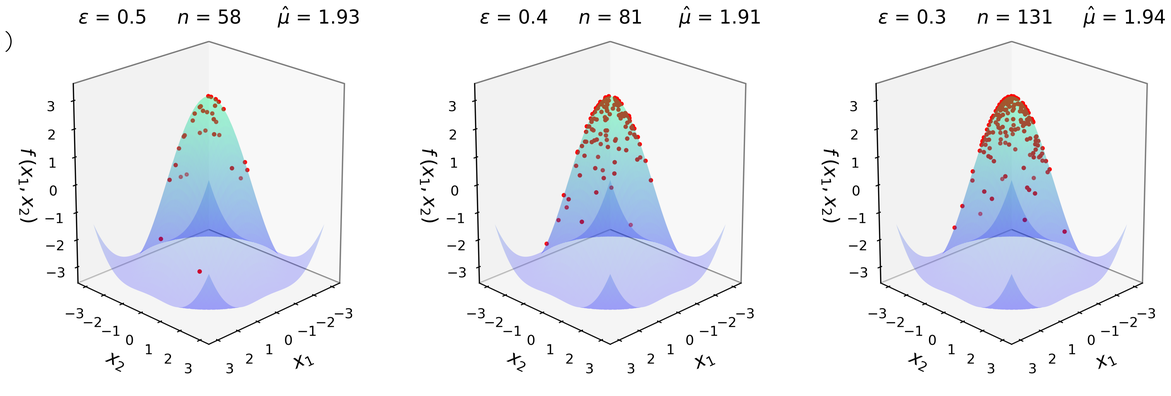
- Standard Uniform
- Standard Gaussian
- IID Zero Mean Gaussian
- Brownian Motion
- Lattice base 2
- Sobol base 2

Accumulate Data

Accumulated data required in the computation of the integral

Python Example

```
dim = 2
stopObj = CLTStopping(nInit=16,absTol=.5)
measureObj = measure().IIDZMeanGaussia(
    dimension=[dim],variance=[.5])
distribObj = IIDDistribution(
    trueD=measure().stdGaussian(
        dimension=[dim]))
sol,out = integrate(KeisterFun(),
    measureObj,distribObj,stopObj)
```



Results

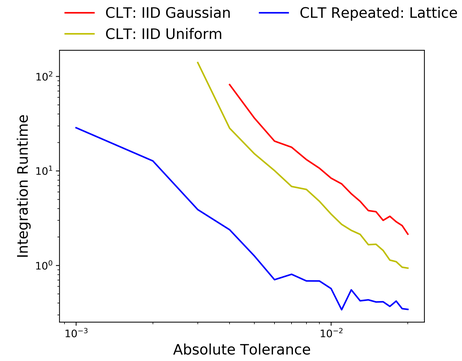


Figure 1:Multi-dimensional Asian Call Option integrated with respect to Brownian Motion

Future Work

- Enhance testing and examples library
- Incorporate existing components
- Expand community of contributors

References

1. **github.com/QMCSoftware/QMCSoftware.git**
2. S.-C. T. Choi, Y. Ding, F. J. Hickernell, L. Jiang, Ll. A. Jiménez Rugama, D. Li, R. Jagadeeswaran, X. Tong, K. Zhang,Y. Zhang, and X. Zhou,"GAIL: Guaranteed Automatic Integration Library (versions 1.0-2.2)," **http://gailgithub.github.io/GAIL_Dev/**, MATLAB software, 2013-2019.
3. F.Y. Kuo & D. Nuyens. Application of quasi-Monte Carlo methods to elliptic PDEs with random diffusion coefficients - a survey of analysis and implementation, Foundations of Computational Mathematics, 16(6):1631-1696, 2016.

Acknowledgements

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