1. (1) 原式等价于

$$\begin{aligned}
\max\quad & 3x_1 + x_2 + 5x_3 + 4x_4 \\
\text{s.t.}\quad & 3x_1 - 3x_2 + 2x_3 + 8x_4 + x_5 = 50 \\
& 4x_1 + 6x_2 - 4x_3 - 4x_4 + x_6 = 40 \\
& 4x_1 - 2x_2 + x_3 + 3x_4 + x_7 = 20 \\
& x_1, \cdots, x_7 \geq 0
\end{aligned}$$

$$\begin{pmatrix} 3 & -3 & 2 & 8 & 1 & 0 & 0 & 50 \\ 4 & 6 & -4 & -4 & 0 & 1 & 0 & 40 \\ 4 & -2 & \textcircled{1} & 3 & 0 & 0 & 1 & 20 \\ -3 & -1 & -5 & -4 & 0 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} -5 & \textcircled{1} & 0 & 2 & 1 & 0 & -2 & 10 \\ 20 & -2 & 0 & 8 & 0 & 1 & 4 & 120 \\ 4 & -2 & 1 & 3 & 0 & 0 & 1 & 20 \\ 17 & -11 & 0 & 11 & 0 & 0 & 5 & 100 \end{pmatrix} \to$$

$$\begin{pmatrix} -5 & 1 & 0 & 2 & 1 & 0 & -2 & 10 \\ \textcircled{10} & 0 & 0 & 12 & 2 & 1 & 0 & 140 \\ -6 & 0 & 1 & 7 & 2 & 0 & -3 & 40 \\ -38 & 0 & 0 & 33 & 11 & 0 & -17 & 210 \end{pmatrix} \to \left(\begin{array}{ccccccc|c} 0 & 1 & 0 & 8 & 2 & \frac{1}{2} & -2 & 80 \\ 1 & 0 & 0 & \frac{6}{5} & \frac{1}{5} & \frac{1}{10} & 0 & 14 \\ 0 & 0 & 1 & \frac{71}{5} & \frac{16}{5} & \frac{3}{5} & -3 & 124 \\ 0 & 0 & 0 & \frac{198}{5} & \frac{93}{5} & \frac{19}{5} & -17 & 742 \end{array}\right)$$

从而最优解无界！

(2) 原式等价于

$$\begin{aligned}
\min\quad & 2x_1 - x_2 + x_3 + 5x_4 \\
\text{s.t}\quad & x_1 + x_2 + x_3 + x_4 = 4 \\
& 2x_1 + 3x_2 - 4x_3 + 2x_4 + x_5 = 5 \\
& -x_1 - 2x_2 + 5x_3 - x_4 + x_6 = -2 \\
& x_1, \cdots, x_6 \geq 0
\end{aligned}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 4 \\ 2 & 3 & -4 & 2 & 1 & 0 & 5 \\ -1 & \textcircled{-2} & 5 & -1 & 0 & 1 & -2 \\ 2 & -1 & 1 & 5 & 0 & 0 & 0 \end{pmatrix} \to \begin{pmatrix} \frac{1}{2} & 0 & \frac{7}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 3 \\ \frac{1}{2} & 0 & \textcircled{\frac{7}{2}} & \frac{1}{2} & 1 & \frac{3}{2} & 2 \\ \frac{1}{2} & 1 & -\frac{5}{2} & \frac{1}{2} & 0 & -\frac{1}{2} & 1 \\ \frac{5}{2} & 0 & -\frac{3}{2} & \frac{11}{2} & 0 & -\frac{1}{2} & 1 \end{pmatrix} \to$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ \frac{1}{7} & 0 & 1 & \frac{1}{7} & \frac{2}{7} & \frac{3}{7} & \frac{5}{7} \\ \frac{6}{7} & 1 & 0 & \frac{6}{7} & \frac{5}{7} & \frac{4}{7} & \frac{17}{7} \\ \frac{19}{7} & 0 & 0 & \frac{40}{7} & \frac{3}{7} & \frac{1}{7} & \frac{33}{7} \end{pmatrix}$$

从而最优解为 $(0, \frac{17}{7}, \frac{5}{7}, 0)$

最优值为 $\frac{-13}{7}$，不符合题意！

该问题无解

(3) 原式化为

$$\begin{aligned}
\min\quad & x_1 - 2x_2 + x_3 + 3x_4 \\
\text{s.t.}\quad & 2x_1 - x_2 + x_3 - x_4 + x_5 = 10 \\
& -5x_1 + 2x_2 - 2x_3 + x_4 + x_6 = 20 \\
& 3x_1 - 4x_2 + 4x_3 - 2x_4 + x_7 = 30 \\
& x_1, \cdots, x_7 \geq 0
\end{aligned}$$

$$\begin{pmatrix} 2 & -1 & 1 & -1 & 1 & 0 & 0 & 10 \\ -5 & \textcircled{2} & -2 & 1 & 0 & 1 & 0 & 20 \\ 3 & -4 & 4 & -2 & 0 & 0 & 1 & 30 \\ 1 & -2 & 1 & 3 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \boxed{-\frac{1}{2}} & 0 & 0 & -\frac{1}{2} & 1 & \frac{1}{2} & 0 & 20 \\ \boxed{-\frac{5}{2}} & 1 & -1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 10 \\ \boxed{-7} & 0 & 0 & 0 & 0 & 2 & 1 & 70 \\ \boxed{-4} & 0 & -1 & 4 & 0 & 1 & 0 & 20 \end{pmatrix}$$

$\Rightarrow$ 从而最优解无界!

2. (1) 由等价可以得出 $3x_3 + 4x_4 = 0 \Rightarrow x_3 = x_4 = 0$

$\Rightarrow$ 解为 $(1, 2, 0, 0)$，最优值为 30.

(2)

$$\begin{pmatrix} 1 & 0 & 1 & \textcircled{1} & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 3 & 2 & 5 & 5 & 7 \\ -10 & -10 & -20 & -10 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ -1 & \textcircled{1} & 0 & 0 & 1 \\ -2 & 2 & 0 & 0 & 2 \\ 20 & -10 & 10 & 0 & 30 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 10 & 0 & 40 \end{pmatrix}$$

最优解为 $(0, 1, 0, 1)$，最优值为 40

3. (1) $b_1 \geq 0$, $b_2 \geq 0$

(2) $c_1 \geq 0$, $b_1, b_2 \geq 0$

(3) $c_1 > 0$, $b_1, b_2 \geq 0$

(4) $c_1 \leq 0$, $a_1 \leq 0$, $a_2 \leq 0$ $b_1, b_2 \geq 0$

(5) $c_1 < 0$; $\frac{b_1}{a_1} > 0$, $\frac{b_2}{a_2} < 0$ 或 $0 < \frac{b_1}{a_1} \leq \frac{b_2}{a_2}$

$b_1, b_2 \geq 0$

4. 由矩阵变换可推出初始矩阵为 $\begin{pmatrix} -2-2\lambda & -2 & -1-2a & 1 & 0 & 0 & -4 \\ 2\lambda & 2 & 2a & 0 & 1 & 0 & 12 \\ 0 & 0 & 3 & 0 & 0 & 1 & 20 \\ -2 & -5 & -8 & 0 & 0 & 0 & 0 \end{pmatrix}$

基矩阵 $\begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 1 & 20 \\ 2\lambda & 2 & 2a & 0 & 1 & 0 & 12 \\ -2-2\lambda & -2 & -1-2a & 1 & 0 & 0 & -4 \\ -2 & -5 & -8 & 0 & 0 & 0 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 1 & 20 \\ 2\lambda & 2 & \textcircled{2a} & 0 & 1 & 0 & 12 \\ -2-2\lambda & -2 & -1-2a & 1 & 0 & 0 & -4 \\ -2 & -5 & -8 & 0 & 0 & 0 & 0 \end{pmatrix} \longrightarrow \begin{pmatrix} -\frac{3\lambda}{a} & -\frac{3}{a} & 0 & 0 & -\frac{3}{2a} & 1 & 20-\frac{18}{a} \\ \frac{\lambda}{a} & \textcircled{\frac{1}{a}} & 1 & 0 & \frac{1}{2a} & 0 & \frac{6}{a} \\ -2+\frac{\lambda}{a} & \frac{1}{a} & 0 & 1 & \frac{2a+1}{2a} & 0 & 8+\frac{6}{a} \\ -2+\frac{5\lambda}{a} & -5+\frac{8}{a} & 0 & 0 & \frac{4}{a} & 0 & \frac{48}{a} \end{pmatrix} \longrightarrow$

$\begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 1 & 20 \\ \lambda & 1 & a & 0 & \frac{1}{2} & 0 & 6 \\ -2 & 0 & -1 & 1 & 1 & 0 & 8 \\ 5\lambda-2 & 0 & 5a-8 & 0 & \frac{5}{2} & 0 & 30 \end{pmatrix}$

(1) 基变量：x_2, x_4, x_6

非基变量：x_1, x_3, x_5

基本可行解：$(0, 6, 0, 8, 0, 20)$

(2) $5a-8=2$, $a=2$. $b=5\lambda-2$ $c=0$ $d=0$ $e=\frac{5}{2}$

$f=0$, $g=30$

(3) 原矩阵为 $\begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 1 & 20 \\ \lambda & 1 & 2 & 0 & \frac{1}{2} & 0 & 6 \\ -2 & 0 & -1 & 1 & 1 & 0 & 8 \\ 5\lambda-2 & 0 & 2 & 0 & \frac{5}{2} & 0 & 30 \end{pmatrix}$

$\Rightarrow \lambda \leq 0$ 时，最优解无界！

$\lambda \geqslant \frac{2}{5}$ 时，当前单纯形表为最优解.

若 $\lambda < 0$，则 $5\lambda-2 < 0$，$\frac{6}{\lambda} < 0$

若 $\lambda = 0$，$5\lambda-2 < 0$，$\frac{8}{-2} < 0$

当 $\lambda = \frac{2}{5}$ 时，存无穷多最优解：

4. $P\{w_1X_1+X_2\geq 7,\ w_2X_1+X_2\geq 4\}\geq\alpha$

5. (1)

$$\begin{pmatrix} 1&0&0&0&0&1&1&2&1&3&3\\ 0&1&0&0&0&-1&1&2&3&4&2\\ 0&0&1&0&0&2&5&-4&0&2&4\\ 0&0&0&1&0&1&3&1&1&2&1\\ 0&0&0&0&1&\textcircled{5}&4&3&-2&1&3\\ 0&0&0&0&0&-3&-2&2&-1&-2&0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1&0&0&0&-\frac{1}{5}&-\frac{1}{5}&-\frac{2}{5}&-\frac{1}{5}&\frac{1}{5}&\frac{9}{5}\\ 0&1&0&0&-\frac{1}{5}&\frac{2}{5}&-\frac{1}{5}&\frac{2}{5}&\frac{7}{5}&\frac{13}{5}\\ 0&0&1&0&-\frac{2}{5}&-\frac{1}{5}&\frac{2}{5}&-\frac{1}{5}&\frac{1}{5}&\frac{9}{5}\\ 0&0&0&1&-\frac{1}{5}&\frac{2}{5}&\frac{1}{5}&\frac{2}{5}&\frac{7}{5}&\frac{13}{5}\\ 0&0&0&0&1&\textcircled{\frac{7}{5}}&-\frac{2}{5}&\frac{3}{5}&-\frac{2}{5}&\frac{4}{5}\\ 0&0&0&0&0&-\frac{2}{5}&\frac{1}{5}&-\frac{2}{5}&-\frac{1}{5}&\frac{2}{5} \end{pmatrix} \longrightarrow$$

$$\begin{pmatrix} 1&0&0&-1&0&0&-2&1&0&1&2\\ 0&1&0&-\frac{13}{7}&\frac{4}{7}&0&-\frac{16}{7}&\frac{13}{7}&0&\frac{6}{7}&\frac{13}{7}\\ 0&0&1&-\frac{4}{7}&-\frac{2}{7}&0&\frac{15}{7}&-\frac{38}{7}&0&\frac{4}{7}&\frac{18}{7}\\ 0&0&0&\frac{5}{7}&-\frac{1}{7}&0&\frac{11}{7}&\frac{2}{7}&1&\frac{9}{7}&\frac{2}{7}\\ 0&0&0&\frac{2}{7}&\frac{1}{7}&1&\frac{10}{7}&\frac{5}{7}&0&\frac{5}{7}&\frac{5}{7}\\ 0&0&0&\frac{11}{7}&\frac{2}{7}&0&\frac{27}{7}&\frac{31}{7}&0&\frac{10}{7}&\frac{17}{7} \end{pmatrix}$$

故最优解为 $(2,\frac{13}{7},\frac{18}{7},0,0,\frac{5}{7},0,0,\frac{2}{7},0)$

(2) 新的单纯形表中基向量没有发生改变. (都为5(1)相同结果)

$$\begin{pmatrix} 1&0&0&0&0&1&1&2&1&3&3&90\\ 0&1&0&0&0&-1&1&2&3&4&2&155\\ 0&0&1&0&0&2&5&-4&0&2&4&62\\ 0&0&0&1&0&1&3&1&1&2&1&70\\ 0&0&0&0&1&\textcircled{5}&4&3&-2&1&3&70\\ 0&0&0&0&0&-3&-2&2&-1&-2&0&0 \end{pmatrix} \longrightarrow \begin{pmatrix} 1&&&&-\frac{1}{5}&-\frac{1}{5}&-\frac{2}{5}&-\frac{1}{5}&\frac{1}{5}&\frac{9}{5}&76\\ &1&&&-\frac{1}{5}&\frac{2}{5}&-\frac{1}{5}&\frac{2}{5}&\frac{7}{5}&\frac{13}{5}&169\\ &&1&&-\frac{2}{5}&-\frac{1}{5}&\frac{2}{5}&-\frac{1}{5}&\frac{1}{5}&\frac{9}{5}&34\\ &&&1&-\frac{1}{5}&\frac{2}{5}&\frac{1}{5}&\frac{2}{5}&\frac{7}{5}&\frac{13}{5}&56\\ &&&&1&\textcircled{\frac{7}{5}}&-\frac{2}{5}&\frac{3}{5}&-\frac{2}{5}&\frac{4}{5}&14\\ &&&&&-\frac{2}{5}&\frac{1}{5}&-\frac{2}{5}&-\frac{1}{5}&\frac{2}{5}&42 \end{pmatrix}$$

最后右下角的数字为 $42+56\times\frac{11}{7}=130$

$\Rightarrow$ 最优值为 -130