

1. (1) 原式等价于

$$\begin{aligned} \max \quad & 3x_1 + x_2 + 5x_3 + 4x_4 \\ \text{s.t.} \quad & 3x_1 - 3x_2 + 2x_3 + 8x_4 + x_5 = 50 \\ & 4x_1 + 6x_2 - 4x_3 - 4x_4 + x_6 = 40 \\ & 4x_1 - 2x_2 + x_3 + 3x_4 + x_7 = 20 \\ & x_1, \dots, x_7 \geq 0 \end{aligned}$$

$$\begin{pmatrix} 3 & -3 & 2 & 8 & 1 & 0 & 0 & 50 \\ 4 & 6 & -4 & -4 & 0 & 1 & 0 & 40 \\ 4 & -2 & 1 & 3 & 0 & 0 & 1 & 20 \\ -3 & -1 & -5 & -4 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} -5 & 1 & 0 & 6 & 1 & 0 & -2 & 10 \\ 20 & -2 & 0 & 8 & 0 & 1 & 4 & 120 \\ 4 & -2 & 1 & 3 & 0 & 0 & 1 & 20 \\ 17 & -11 & 0 & 11 & 0 & 0 & 5 & 100 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} -5 & 1 & 0 & 6 & 1 & 0 & -2 & 10 \\ 10 & 0 & 0 & 20 & 2 & 1 & 0 & 140 \\ -6 & 0 & 1 & 15 & 2 & 0 & -3 & 40 \\ -38 & 0 & 0 & 77 & 11 & 0 & -17 & 210 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & 1 & 0 & 16 & 2 & \frac{1}{2} & -2 & 80 \\ 1 & 0 & 0 & 2 & \frac{1}{5} & \frac{1}{10} & 0 & 14 \\ 0 & 0 & 1 & 27 & \frac{16}{5} & \frac{3}{5} & -3 & 124 \\ 0 & 0 & 0 & 153 & \frac{93}{5} & \frac{19}{5} & -17 & 742 \end{pmatrix}$$

从而最优解为 $(14, 80, 124, 0)$ ，最优值为 742

(2) 原式等价于

$$\begin{aligned} \min \quad & 2x_1 - x_2 + x_3 + 5x_4 \\ \text{s.t.} \quad & x_1 + x_2 + x_3 + x_4 = 4 \\ & 2x_1 + 3x_2 - 4x_3 + 2x_4 + x_5 = 5 \\ & -x_1 - 2x_2 + 5x_3 - x_4 + x_6 = -2 \\ & x_1, \dots, x_6 \geq 0 \end{aligned}$$

$$\begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 4 \\ 2 & 3 & -4 & 2 & 1 & 0 & 5 \\ -1 & -2 & 5 & -1 & 0 & 1 & -2 \\ 2 & -1 & 1 & 5 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \frac{1}{2} & 0 & \frac{7}{2} & \frac{1}{2} & 0 & \frac{1}{2} & 3 \\ \frac{1}{2} & 0 & \frac{7}{2} & \frac{1}{2} & 1 & \frac{3}{2} & 2 \\ \frac{1}{2} & 1 & -\frac{5}{2} & \frac{1}{2} & 0 & -\frac{1}{2} & 1 \\ \frac{5}{2} & 0 & -\frac{3}{2} & \frac{11}{2} & 0 & -\frac{1}{2} & 1 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & -1 & -1 & 1 \\ \frac{1}{7} & 0 & 1 & \frac{1}{7} & \frac{2}{7} & \frac{3}{7} & \frac{4}{7} \\ \frac{6}{7} & 1 & 0 & \frac{6}{7} & \frac{5}{7} & \frac{4}{7} & \frac{17}{7} \\ \frac{19}{7} & 0 & 0 & \frac{40}{7} & \frac{3}{7} & \frac{1}{7} & \frac{13}{7} \end{pmatrix}$$

从而最优解为 $(0, \frac{17}{7}, \frac{4}{7}, 0)$

最优值为 $-\frac{13}{7}$ ，不符合题意！

该问题无解

(3) 原式化为

$$\min \quad x_1 - 2x_2 + x_3 + 3x_4$$

$$\text{s.t.} \quad 2x_1 - x_2 + x_3 - x_4 + x_5 = 10$$

$$-5x_1 + 2x_2 - 2x_3 + x_4 + x_6 = 20$$

$$3x_1 - 4x_2 + 4x_3 - 2x_4 + x_7 = 30$$

$$x_1, \dots, x_7 \geq 0$$

$$\begin{pmatrix} 2 & -1 & 1 & -1 & 1 & 0 & 0 & 10 \\ -5 & 2 & -2 & 1 & 0 & 1 & 0 & 20 \\ 3 & -4 & 4 & -2 & 0 & 0 & 1 & 30 \\ 1 & -2 & 1 & 3 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & 1 & \frac{1}{2} & 0 & 20 \\ -\frac{5}{2} & 1 & -1 & \frac{1}{2} & 0 & \frac{1}{2} & 0 & 10 \\ -7 & 0 & 0 & 0 & 0 & 2 & 1 & 70 \\ -4 & 0 & -1 & 4 & 0 & 1 & 0 & 20 \end{pmatrix}$$

\Rightarrow 从而最优解为 $(0, 10, 0, 0)$, 最优值为 -20 .

2. (1) 由条件可以得出 $3x_3 + 4x_4 = 0 \Rightarrow x_3 = x_4 = 0$

\Rightarrow 解为 $(1, 2, 0, 0)$, 最优值为 30 .

$$(2) \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 2 \\ 3 & 2 & 5 & 5 & 7 \\ 10 & -10 & -20 & -10 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ -2 & 2 & 0 & 0 & 2 \\ 20 & -10 & 10 & 0 & 30 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 10 & 0 & 10 & 0 & 40 \end{pmatrix}$$

最优解为 $(0, 1, 0, 1)$, 最优值为 40

3. (1) $b_1 \geq 0, b_2 \geq 0$

(2) $C_1 \geq 0, b_1, b_2 \geq 0$

(3) $C_1 > 0, b_1, b_2 \geq 0$

(4) $C_1 \leq 0, a_1 \leq 0, a_2 \leq 0, b_1, b_2 \geq 0$

(5) $C_1 < 0; \frac{b_1}{a_1} > 0, \frac{b_2}{a_2} < 0$ 或 $0 < \frac{b_1}{a_1} \leq \frac{b_2}{a_2}$
 $\underbrace{\quad}_{b_1, b_2 \geq 0}$

4. 由矩阵变换可推出初始矩阵为

$$\begin{pmatrix} -2-2\lambda & -2 & -1-2a & 1 & 0 & 0 & -4 \\ 2\lambda & 2 & 2a & 0 & 1 & 0 & 12 \\ 0 & 0 & 3 & 0 & 0 & 1 & 20 \\ -2 & -5 & -8 & 0 & 0 & 0 & 0 \end{pmatrix}$$

其系数子

$$\begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 1 & 20 \\ 2\lambda & 2 & 2a & 0 & 1 & 0 & 12 \\ -2-2\lambda & -2 & -1-2a & 1 & 0 & 0 & -4 \\ -2 & -5 & -8 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 1 & 20 \\ 2\lambda & 2 & 2a & 0 & 1 & 0 & 12 \\ -2-2\lambda & -2 & -1-2a & 1 & 0 & 0 & -4 \\ -2 & -5 & -8 & 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} -\frac{2\lambda}{a} & -\frac{2}{a} & 0 & 0 & -\frac{3}{2a} & 1 & 20-\frac{18}{a} \\ \frac{\lambda}{a} & \frac{1}{a} & 1 & 0 & \frac{1}{2a} & 0 & \frac{6}{a} \\ -2+\frac{\lambda}{a} & -\frac{1}{a} & 0 & 1 & \frac{2a+1}{2a} & 0 & 8+\frac{6}{a} \\ -2+\frac{8\lambda}{a} & -5+\frac{8}{a} & 0 & 0 & \frac{4}{a} & 0 & \frac{48}{a} \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 1 & 20 \\ \lambda & 1 & a & 0 & \frac{1}{2} & 0 & 6 \\ -2 & 0 & -1 & 1 & 1 & 0 & 8 \\ 5\lambda-2 & 0 & 5a-8 & 0 & \frac{5}{2} & 0 & 30 \end{pmatrix}$$

故: (1) 基变量: x_2, x_4, x_6

非基变量: x_1, x_3, x_5

基本可行解: $(0, 6, 0, 8, 0, 20)$

(2) $5a-8=2, a=2, b=5\lambda-2, c=0, d=0, e=\frac{5}{2}$

$f=0, g=30$

(3) 原矩阵为

$$\begin{pmatrix} 0 & 0 & 3 & 0 & 0 & 1 & 20 \\ \lambda & 1 & 2 & 0 & \frac{1}{2} & 0 & 6 \\ -2 & 0 & -1 & 1 & 1 & 0 & 8 \\ 5\lambda-2 & 0 & 2 & 0 & \frac{5}{2} & 0 & 30 \end{pmatrix}$$

$\lambda \geq \frac{2}{5}$ 时, 当前单纯形表为最优解.

若 $\lambda < 0$, 则 $5\lambda-2 < 0, \frac{6}{\lambda} < 0$

若 $\lambda = 0$, $5\lambda-2 < 0, \frac{8}{-2} < 0$

$\Rightarrow \lambda \leq 0$ 时, 最优解无界!

无论 λ 取何值, 该线性规划总有无穷多最优解, 因为 x_6 的检验数恒为 0.

$$5. (1) \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & 1 & 3 & 3 \\ 0 & 1 & 0 & 0 & 0 & -1 & 1 & 2 & 3 & 4 & 2 \\ 0 & 0 & 1 & 0 & 0 & 2 & 5 & -4 & 0 & 2 & 4 \\ 0 & 0 & 0 & 1 & 0 & 1 & 3 & 1 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 1 & \textcircled{5} & 4 & 3 & -2 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & -3 & -2 & 2 & -1 & -2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & 0 & -\frac{1}{5} & 0 & \frac{1}{5} & \frac{7}{5} & \frac{7}{5} & \frac{14}{5} & \frac{12}{5} \\ 0 & 1 & 0 & 0 & -\frac{1}{5} & 0 & \frac{9}{5} & \frac{13}{5} & \frac{13}{5} & \frac{21}{5} & \frac{13}{5} \\ 0 & 0 & 1 & 0 & -\frac{2}{5} & 0 & \frac{17}{5} & -\frac{26}{5} & \frac{4}{5} & \frac{8}{5} & \frac{14}{5} \\ 0 & 0 & 0 & 1 & -\frac{1}{5} & 0 & \frac{11}{5} & \frac{2}{5} & \textcircled{\frac{7}{5}} & \frac{2}{5} & \frac{3}{5} \\ 0 & 0 & 0 & 0 & -\frac{1}{5} & 1 & \frac{4}{5} & \frac{3}{5} & -\frac{2}{5} & \frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 0 & 0 & \frac{3}{5} & 0 & \frac{2}{5} & \frac{19}{5} & -\frac{11}{5} & -\frac{7}{5} & \frac{9}{5} \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & 0 & 0 & -1 & 0 & 0 & -2 & 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & -\frac{13}{7} & \frac{4}{7} & 0 & -\frac{16}{7} & \frac{13}{7} & 0 & \frac{6}{7} & \frac{13}{7} \\ 0 & 0 & 1 & -\frac{4}{7} & -\frac{2}{7} & 0 & \frac{15}{7} & -\frac{38}{7} & 0 & \frac{4}{7} & \frac{18}{7} \\ 0 & 0 & 0 & \frac{5}{7} & -\frac{1}{7} & 0 & \frac{11}{7} & \frac{2}{7} & 1 & \frac{9}{7} & \frac{2}{7} \\ 0 & 0 & 0 & \frac{2}{7} & \frac{1}{7} & 1 & \frac{10}{7} & \frac{5}{7} & 0 & \frac{5}{7} & \frac{5}{7} \\ 0 & 0 & 0 & \frac{11}{7} & \frac{2}{7} & 0 & \frac{27}{7} & \frac{31}{7} & 0 & \frac{10}{7} & \frac{17}{7} \end{pmatrix}$$

故最优解为 $(2, \frac{13}{7}, \frac{18}{7}, 0, 0, \frac{5}{7}, 0, 0, \frac{2}{7}, 0)$

(2) 新的单纯形表中基变量没有发生改变. (部分与(1)相同结果)

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & 1 & 3 & 3 & 90 \\ 0 & 1 & 0 & 0 & 0 & -1 & 1 & 2 & 3 & 4 & 2 & 155 \\ 0 & 0 & 1 & 0 & 0 & 2 & 5 & -4 & 0 & 2 & 4 & 62 \\ 0 & 0 & 0 & 1 & 0 & 1 & 3 & 1 & 1 & 7 & 1 & 70 \\ 0 & 0 & 0 & 0 & 1 & \textcircled{5} & 4 & 3 & -2 & 7 & 3 & 70 \\ 0 & 0 & 0 & 0 & 0 & -3 & -2 & 2 & -1 & -2 & -2 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & & & & -\frac{1}{5} & & \frac{1}{5} & \frac{7}{5} & \frac{7}{5} & \frac{12}{5} & 76 \\ & 1 & & & -\frac{1}{5} & & \frac{9}{5} & \frac{13}{5} & \frac{13}{5} & \frac{16}{5} & 169 \\ & & 1 & & -\frac{2}{5} & & \frac{17}{5} & -\frac{26}{5} & \frac{4}{5} & \frac{14}{5} & 34 \\ & & & 1 & -\frac{1}{5} & & \frac{11}{5} & \frac{2}{5} & \textcircled{\frac{7}{5}} & \frac{2}{5} & 56 \\ & & & & -\frac{1}{5} & 1 & \frac{4}{5} & \frac{3}{5} & -\frac{2}{5} & \frac{1}{5} & 14 \\ & & & & \frac{3}{5} & & \frac{2}{5} & \frac{19}{5} & -\frac{11}{5} & \frac{9}{5} & 42 \end{pmatrix}$$

最后右下角的数字为 $42 + 56 \times \frac{11}{7} = 130$

\Rightarrow 最优值为 -130