概 率 论
Probability Theory

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Probability theory originated from calculations of probabilities in games of chance.

Where does the chance come from? If every thing is deterministic, we have no chance.

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There are normally two classes of phenomena in nature and human society.

- Deterministic phenomenon
- Random phenomena

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sure event (Ω) and impossible events ($\emptyset$)

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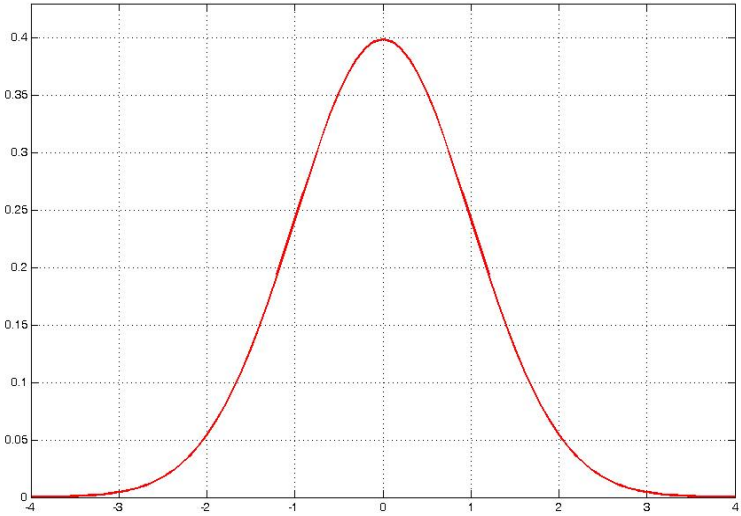
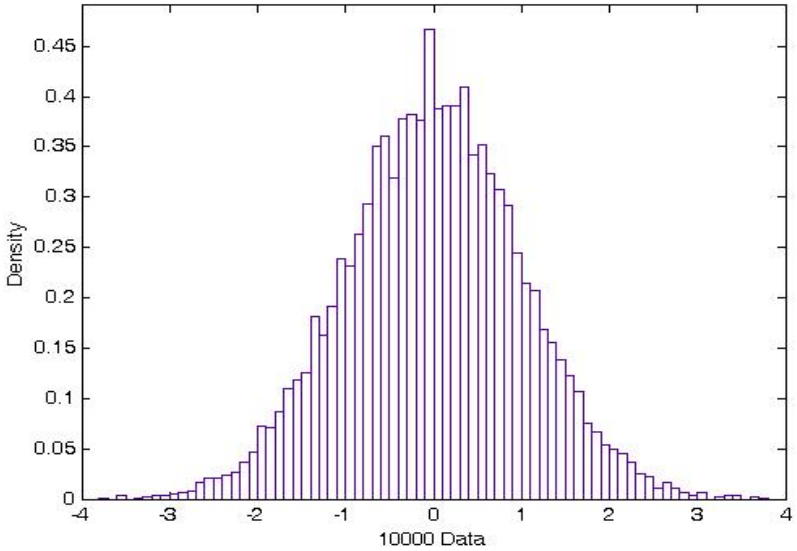
概率统计学家倾向于用带有随机误差项的数学公式描述自然界和人类社会中的规律：

$$y = f(x) + \epsilon$$



$$\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$-\infty < x < \infty$$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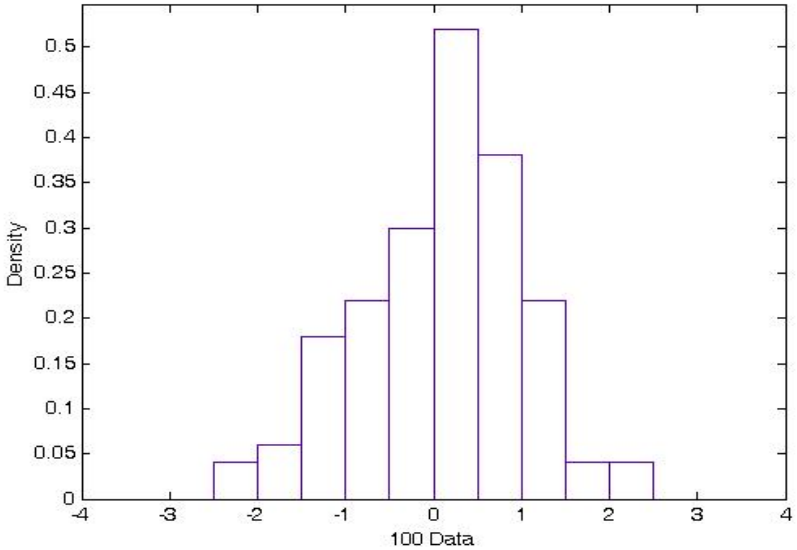
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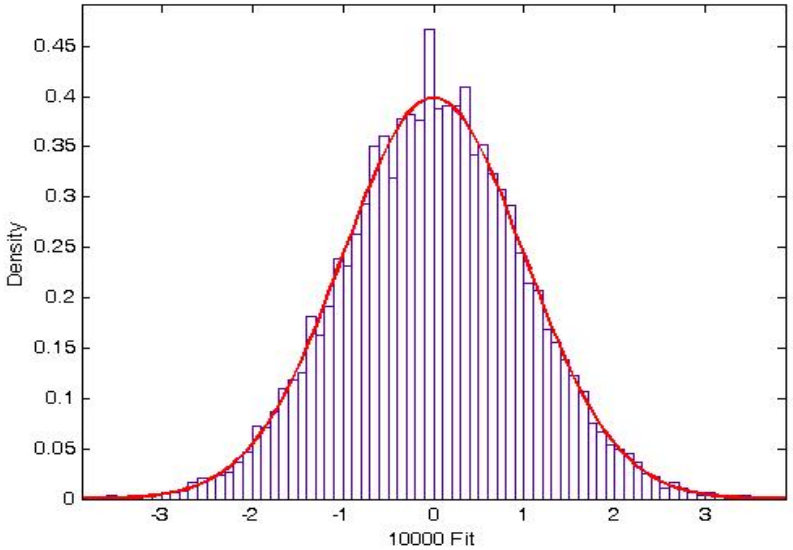

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random event(随机事件)

Properties of random phenomenon (random events)

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(1) uncertainty(不确定性)

In a random phenomenon, the outcome is not predicted with certainty in each individual experiment.

If we repeatedly observe a random phenomenon under a certain condition, the outcomes may be different from places to places.

Remark 1：But all possible outcomes are known.

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Remark 2. A random phenomenon can be repeated under the same condition.

(2) statistical regularity(统计规律性)

The outcomes of a great deal of repeated experiments under a certain condition turn out to have some regularity (statistical regularity)

—The probability of each outcome to occur is deterministic.

Probability theory is a branch of mathematics that focuses on the study of quantity regularity of various random phenomena.

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Universal existence of random phenomena determines the importance of this subject.

Various applications: Probability theory can be used in almost all disciplines and professions, such as

- statistics;
- operations research;
- biology;
- economics;
- psychology
- .....

2021职业排行(CareerCast网站)

	Jobs	Income	Projected Growth
1	Data Scientist	$98,230	33.00%
2	Genetic Counselor	$85,700	21.00%
3	Statistician	$92,270	35.00%
4	Medical Services Manager	$104,28	32.00%
5	Mathematician	$110,860	33.00%
6	University Professor	$80,790	9.00%
7	Operations Research Analyst	$86,200	25.00%
8	Information Security Analyst	$99,730	31.00%
9	Actuary	$111,030	18.00%
10	Software Engineer	$110,140	22.00%

2019职业排行(CareerCast网站)

	Jobs	Overall Rating	Median Salary	Projected Growth
1	Data Scientist	97	$114,520	19%
2	Statistician	110	$84,760	33%
3	University Professor	112	$76,000	15%
7	Information Security Analyst	126	$95,510	28%
8	Mathematician	127	$84,760	33%
9	Operations Research Analyst	128	$81,390	27%
10	Actuary	141	$101,560	22%

2018职业排行(CareerCast网站)

	Jobs	Rated Score	Income	Growth Outlook
1	Genetic Counselor	100	$74,120	29%
2	Mathematician	102	$81,950	33%
3	University Professor	105	$75,430	15%
5	Statistician	111	$84,060	33%
7	Data Scientist	121	$111,840	19%
8	Information Security Analyst	126	$92,600	28%
9	Operations Research Analyst	127	$79,200	27%
10	Actuary	135	$100,610	22%

2017职业排行(CareerCast网站)

	Jobs	Rated Score	Income	Growth Outlook
1	Statistician	93.00	$80,110	34%
3	Operations Research Analyst	102.00	$79,200	30%
4	Information Security Analyst	104.00	$90,120	18%
5	Data Scientist	105.00	$111,267	15.75%
6	University Professor	110.00	$72,416	15.24%
7	Mathematician	128.00	$111,298	22.31%
8	Software Engineer	129.00	$100,690	17%

2016职业排行(CareerCast网站)

	Jobs	Rated Score	Income	Growth Outlook
1	Data Scientist	91.00	$128,240	16%
2	Statistician	96.00	$79,990	34%
3	Information Security Analyst	94.00	$88,890	18%
6	Mathematician	126.00	$103,720	21%
7	Software Engineer	131.00	$97,990	17%
8	Computer Systems Analyst	133.00	$82,710	21%
10	Actuary	138.00	$96,700	18%

The definition of probability

The statistical definition of probability

Frequency:

If event A occurs n_A times in N repeated experiments under a certain conditions, then frequency of A occurring in N experiments is defined as:

$$F_N(A) = \frac{n_A}{N}.$$

When N is large enough, the frequency turns out to have a kind of stability,

i.e., the values of $F_N(A)$ show fluctuations which become progressively weaker as N increases, until ultimately $F_N(A)$ stabilizes to a constant.

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Buffon	4040	2048	0.5069
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Frequency stabilizes to–$P(A) = \frac{1}{2}$

The statistical definition of probability:

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Example:The use frequency of each English letter: E—0.105; ....; J, Q, Z—0.001

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1. $F_N(A) \geq 0$;
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3. Suppose that A and B will never come up simultaneously and that $A + B$ stands for the event that A, or B, or both come up, then $F_N(A + B) = F_N(A) + F_N(B)$.

Properties of probability:

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