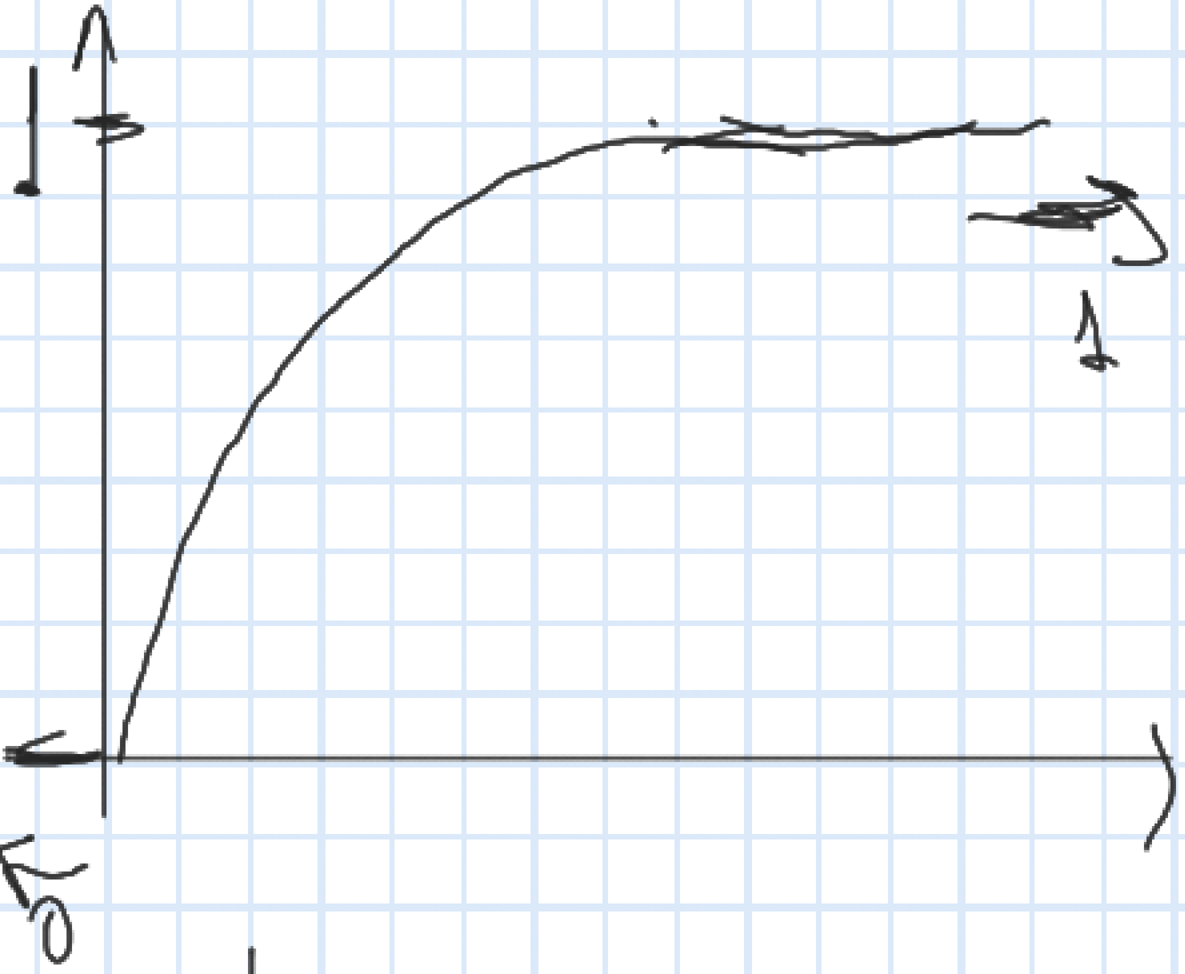
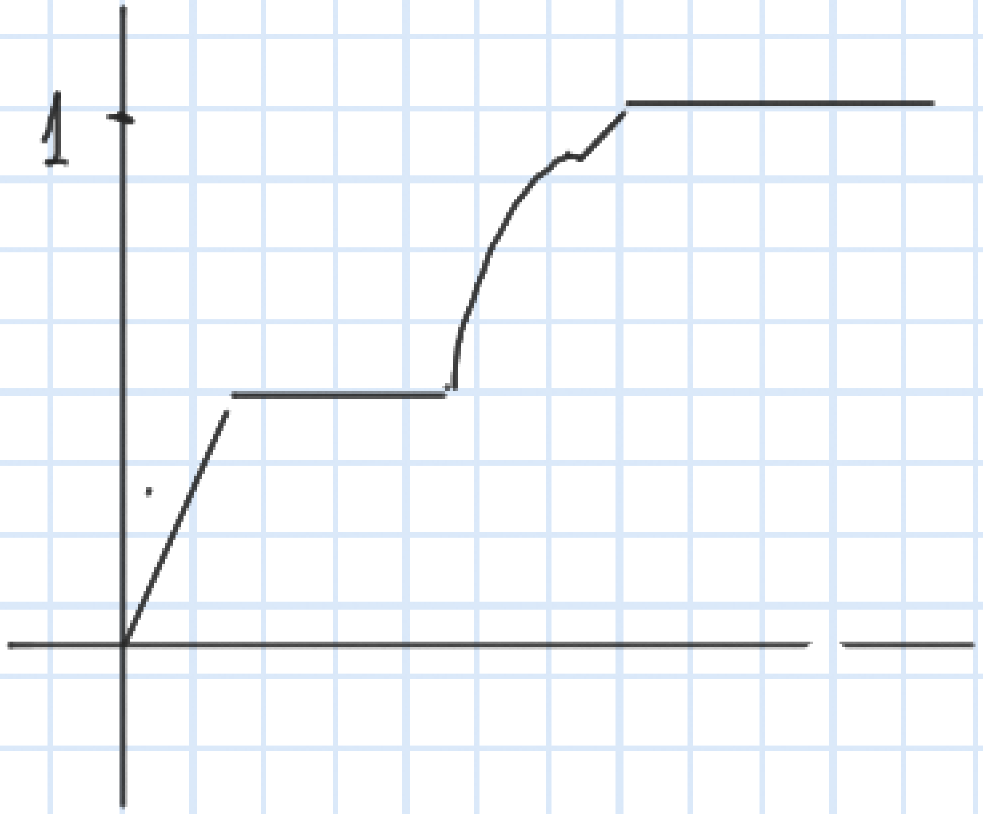
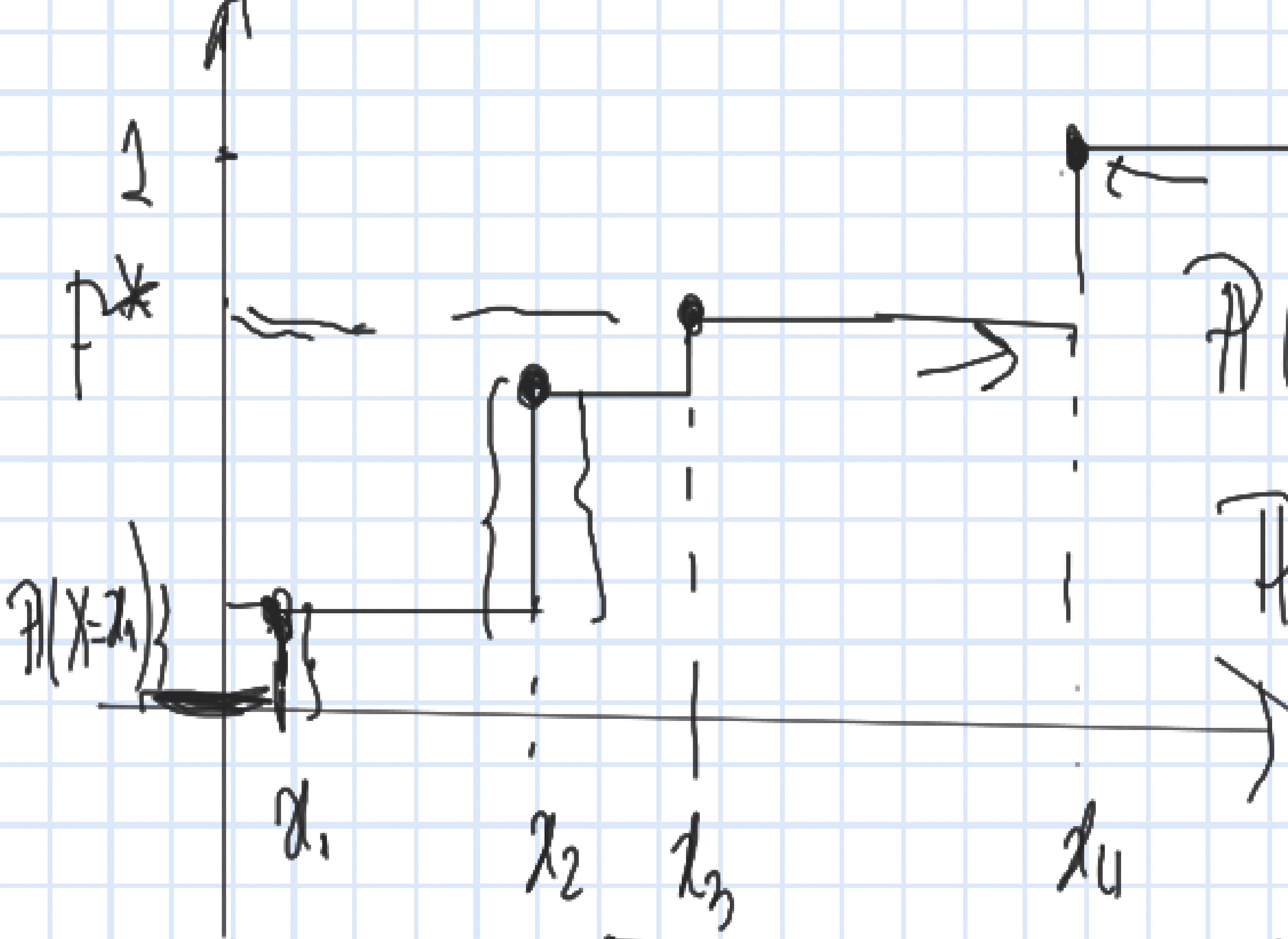
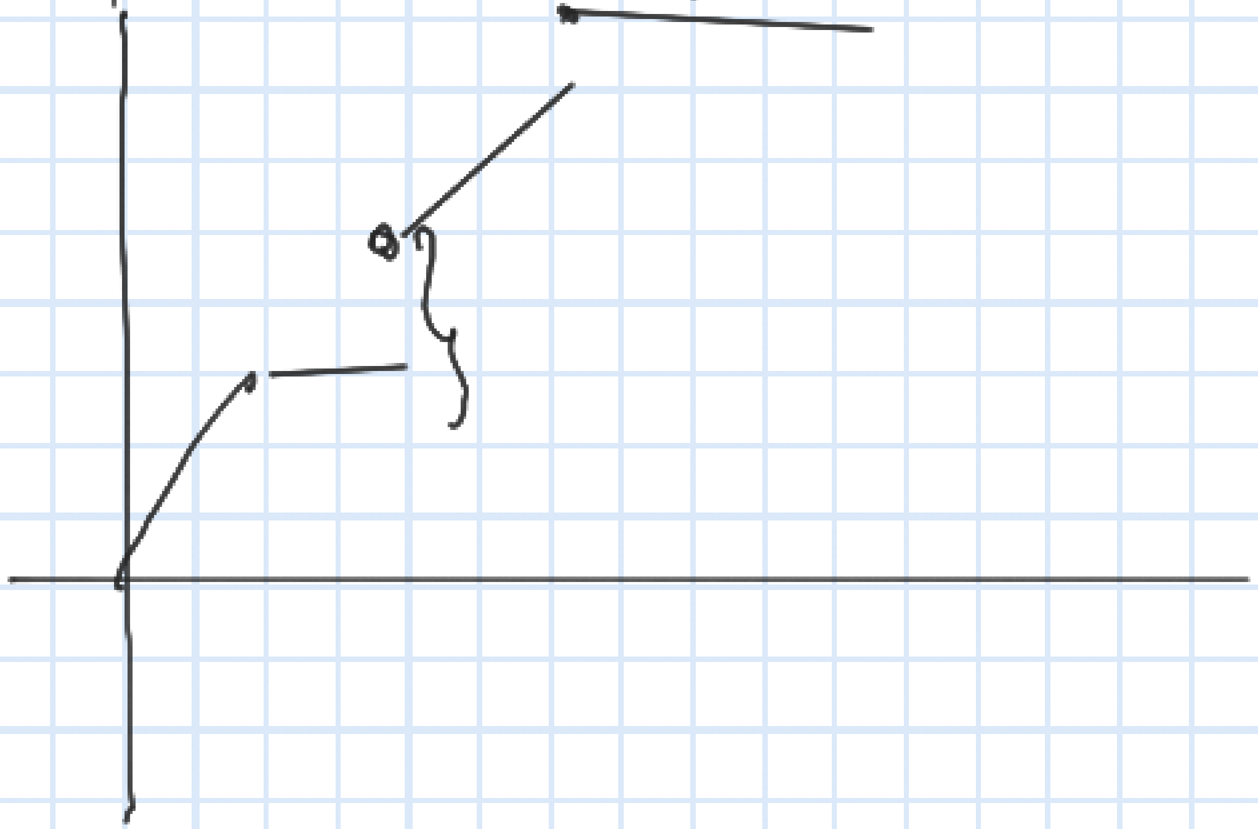






$$P(X \leq x_4) = 1$$

$$P(X < x_4) = p^*$$



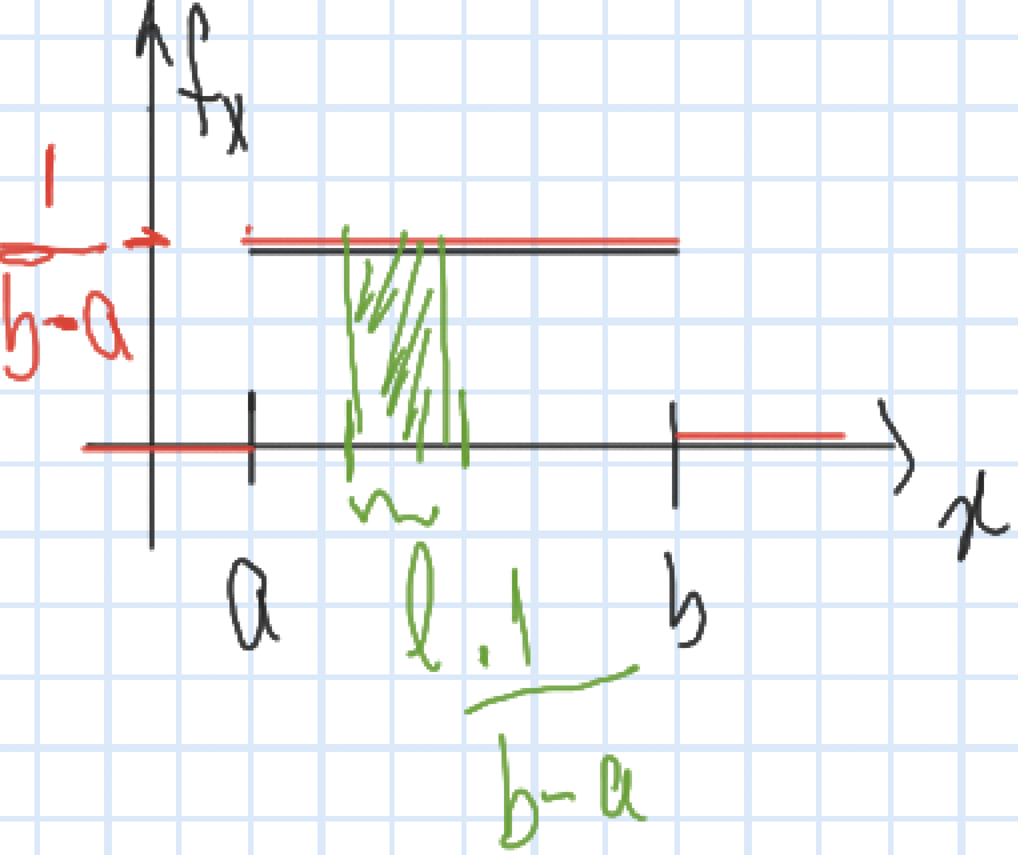
$\mathrm{cov} > 0 \mid \rho > 0$

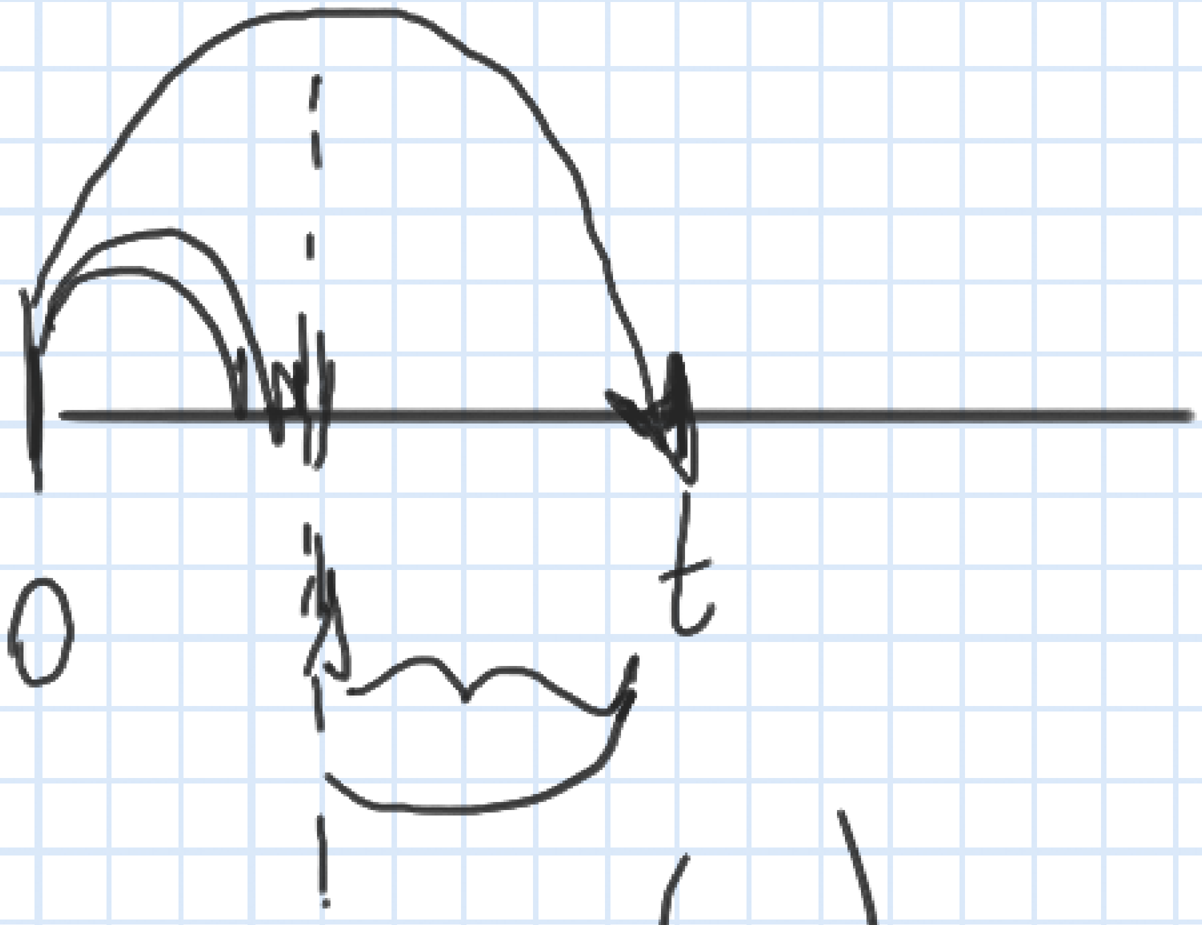
$$|\rho| = 1 \Rightarrow Y = aX + b$$

Coeficiente de correlación.

$$\rho = \frac{\mathrm{cov}(X,Y)}{\sqrt{\mathrm{var}(X)\,\mathrm{var}(Y)}}$$

$$-1 \leq \rho \leq 1$$





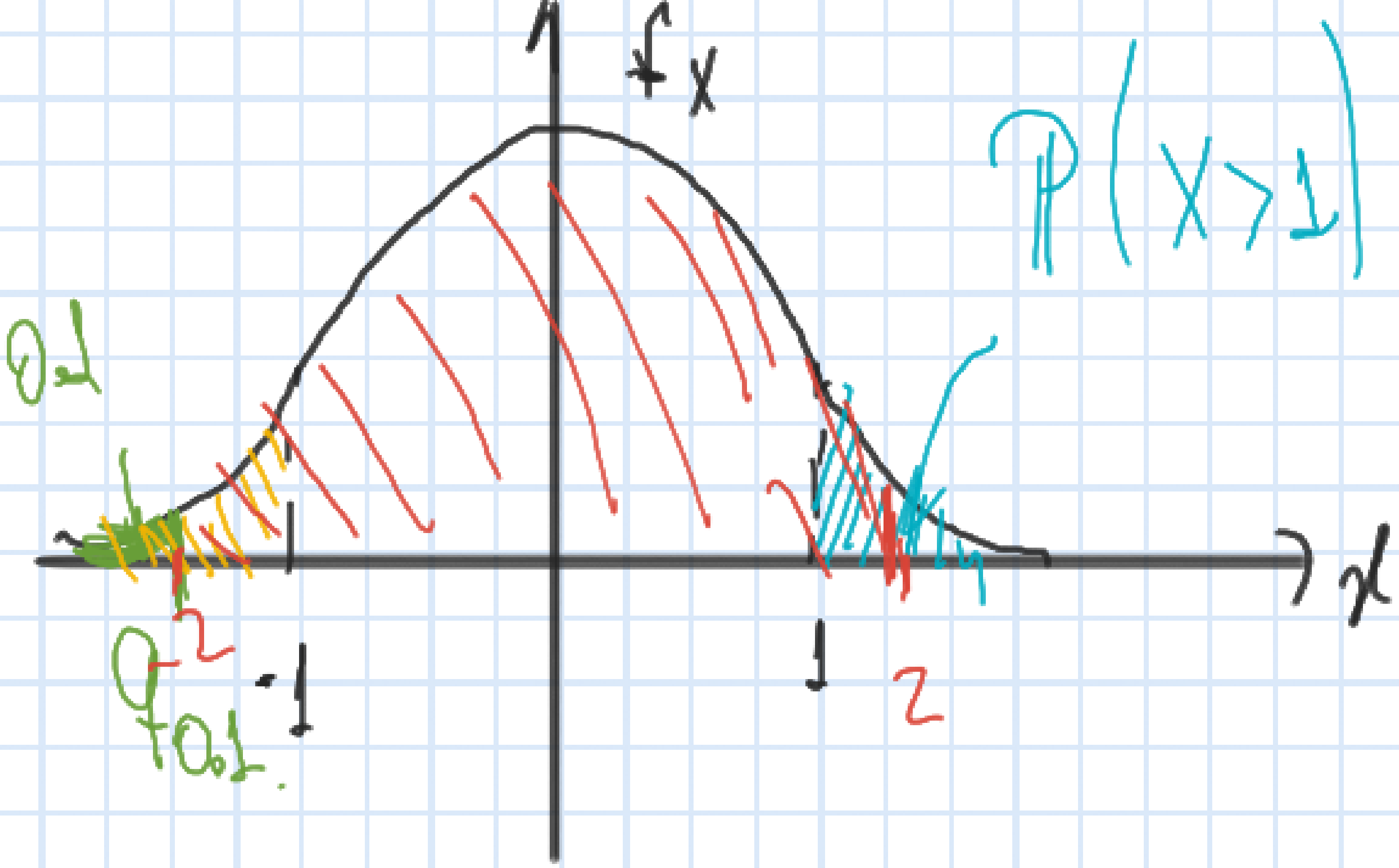
Si X es una v.a. c/ dist. $\mathcal{E}(\lambda)$

$$\mathbb{P}(X \leq t \mid X > s) = \mathbb{P}(X \leq t - s)$$

$$s < t$$

(propiedad de pérdida de memoria)

Eg 1



$P(X>1) = 1 - P(X \leq 1)$

$= 1 - cdf(1)$

≈ 0.1586

$F_X(x) = P(X \leq x)$

cdf $= F_X(x)$

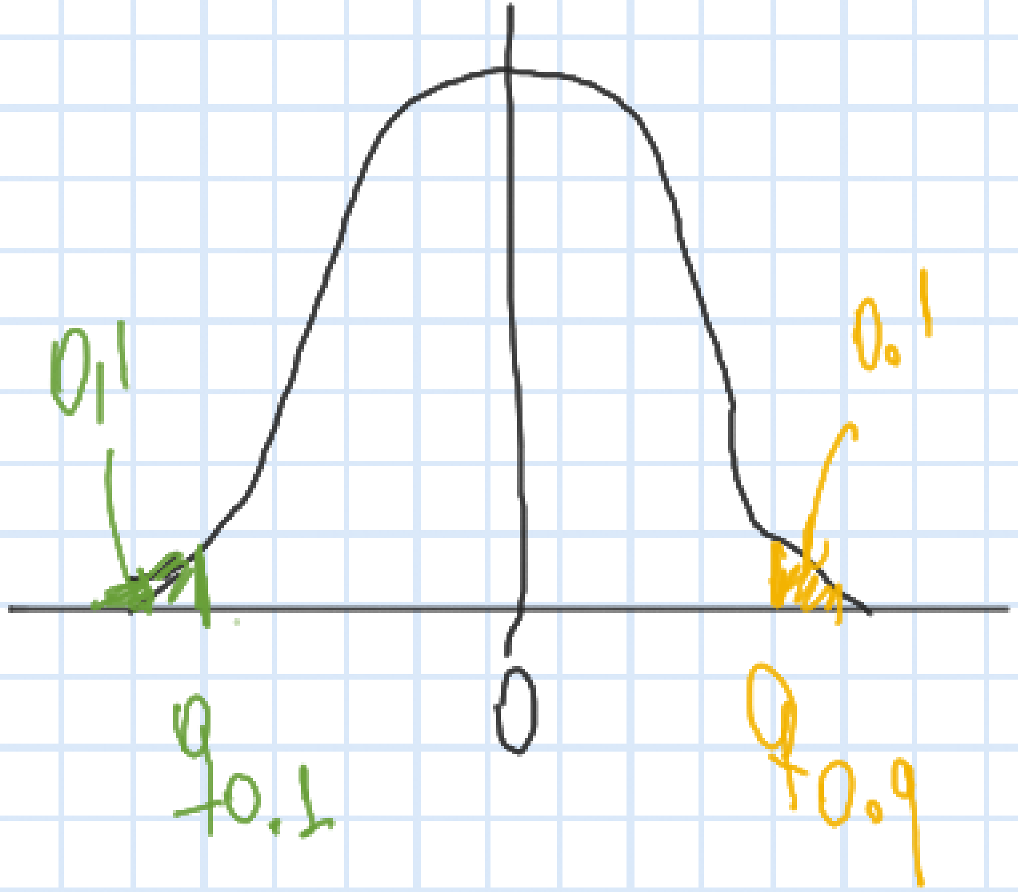
pdf $= f_X(x)$

ppf = Quantiles

ppf(0.1) → qual x acumula pelo 0.1. $q_{0.1}$

$P(X<-1) = cdf(-1) = P(X>1)$ por simetria

$P(|X|<2) = cdf(2) - cdf(-2)$



$q_{0.1}$: $\quad \mathbb{P}(X \leq q_{0.1}) = 0.1.$

$q_{0.9}$ / $\quad \mathbb{P}(X \leq q_{0.9}) = 0.9$

$$q_{0.1} = -q_{0.9} = -1.28$$

$$q_{0.9} = 1.28$$

$X \sim N(0,1)$, $\quad Y \sim N(2, 9)$

$W = 2X + Y \qquad \mathbb{P}(W < 5) = 0.7973 \qquad W \sim N\left(\underbrace{2 \cdot 0 + 2}_{2},\ \underbrace{2^2 \cdot 1 + 9}_{13}\right)$

Ejercicio 2 | $X \sim \mathcal{E}(1/5)$ X = "Tiempo hasta la 1° llamada"

a) $\mathbb{P}(X>2) = \int_2^{+\infty} f_X(x)\,dx = 1 - \underbrace{\mathbb{P}(X \leq 2)}_{\text{cdf}} = 0{,}67$

b) $\mathbb{P}(X>5 \mid X>3) = \dfrac{\mathbb{P}(X>5,\ X>3)}{\mathbb{P}(X>3)} = \dfrac{\mathbb{P}(X>5)}{\mathbb{P}(X>3)} = 0{,}67$

pérdida de memoria

Normal multivariada

$$\Sigma = \begin{bmatrix} \sigma_1^2 & cov(X_1, X_2) & \cdots & cov(X_1, X_n) \\ cov(X_2, X_1) & \sigma_2^2 & \cdots & cov(X_2, X_n) \\ \vdots & & \ddots & \vdots \\ cov(X_n, X_1) & \cdots & & \sigma_n^2 \end{bmatrix}$$

Σ es simétrico y def. positivo

$\Downarrow$

Es invertible

$(\exists\ \Sigma^{-1})$

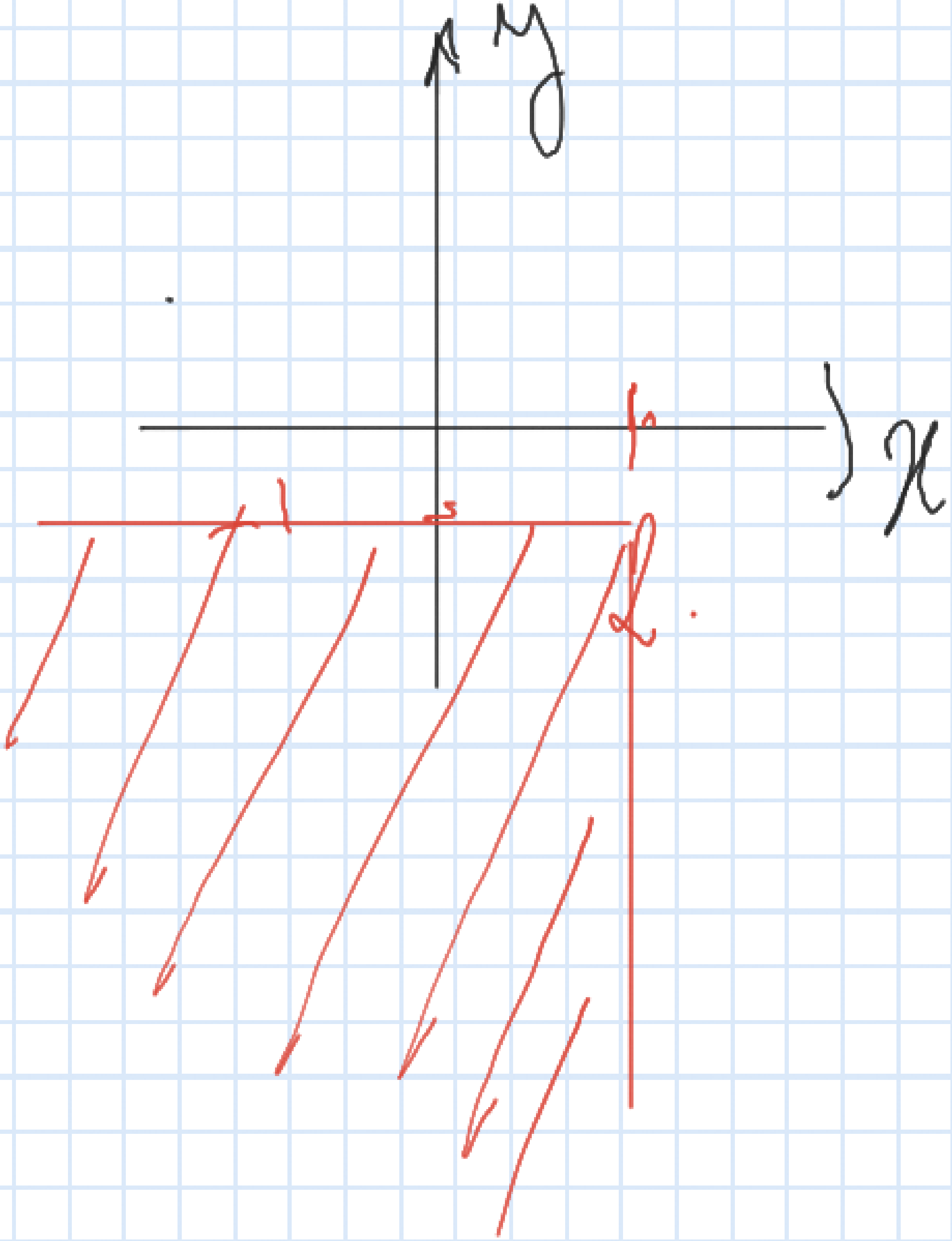
$$\frac{1}{(2\pi)^{n/2} |\Sigma|^{1/2}} e^{-\frac{1}{2}(\underline{x}-\underline{\mu})^T \Sigma^{-1} (\underline{x}-\underline{\mu})}$$

$$var(X) = 1 = var(Y) \qquad \mathbb{E}[X] = 0 = \mathbb{E}[Y].$$

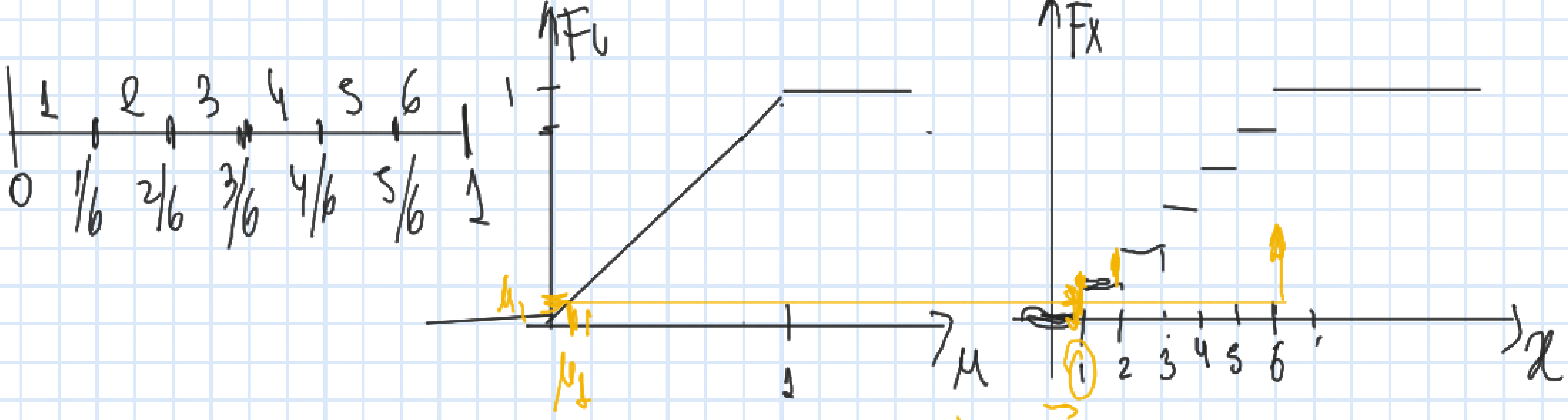
$$cov(X, Y) = -0{,}8$$

$X \sim N(0,1) \quad Y \sim N(0,1)$

$P(X < 2, Y < -1) = 0{,}14$



Ej 4



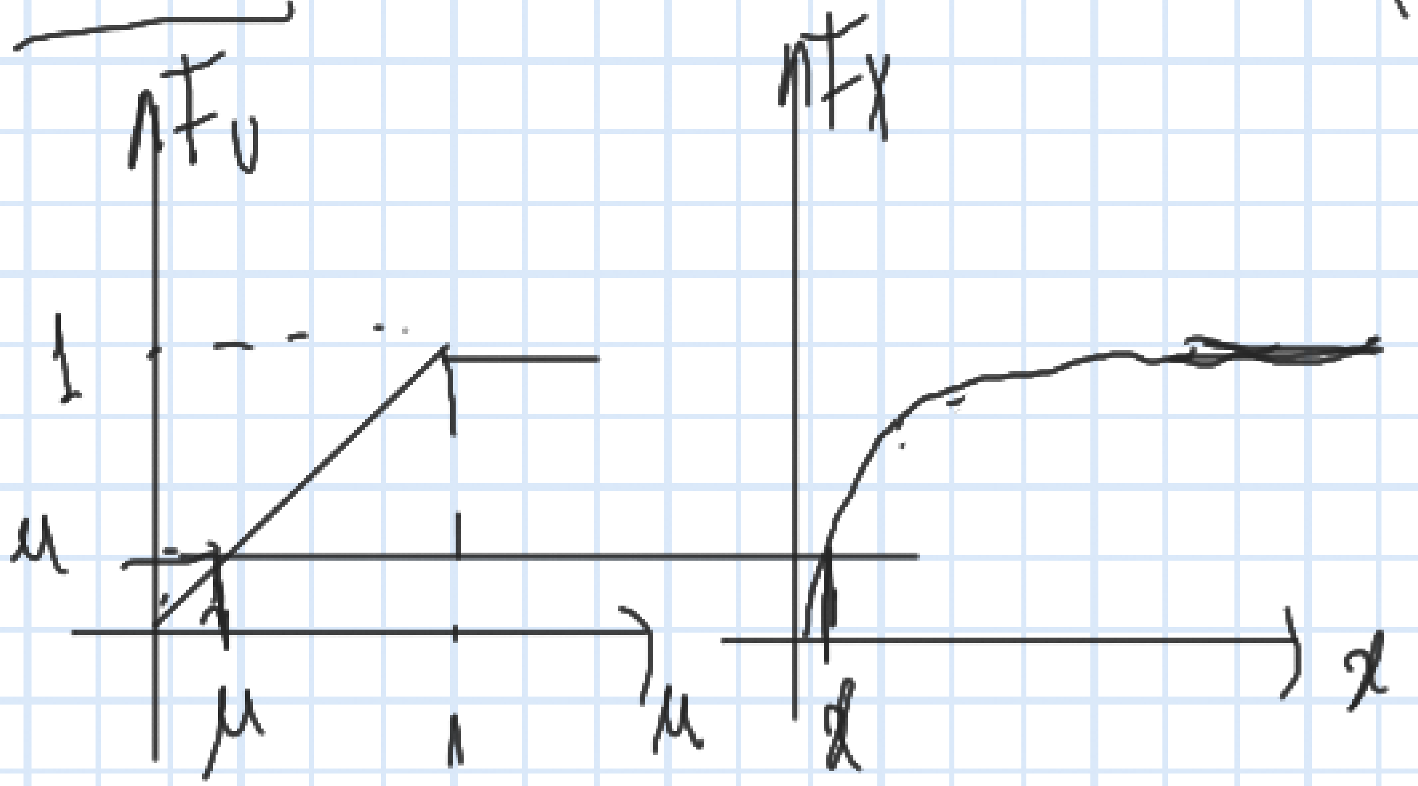
$u_1 < 1/6$ $\quad u \in (0, 1/6) \to 1$

$u \in (1/6, 2/6) \to 2$

$\vdots$

$u \in (5/6, 1) \to 6$

Ej 5 | Quiero $X \sim \mathcal{E}(1/5)$

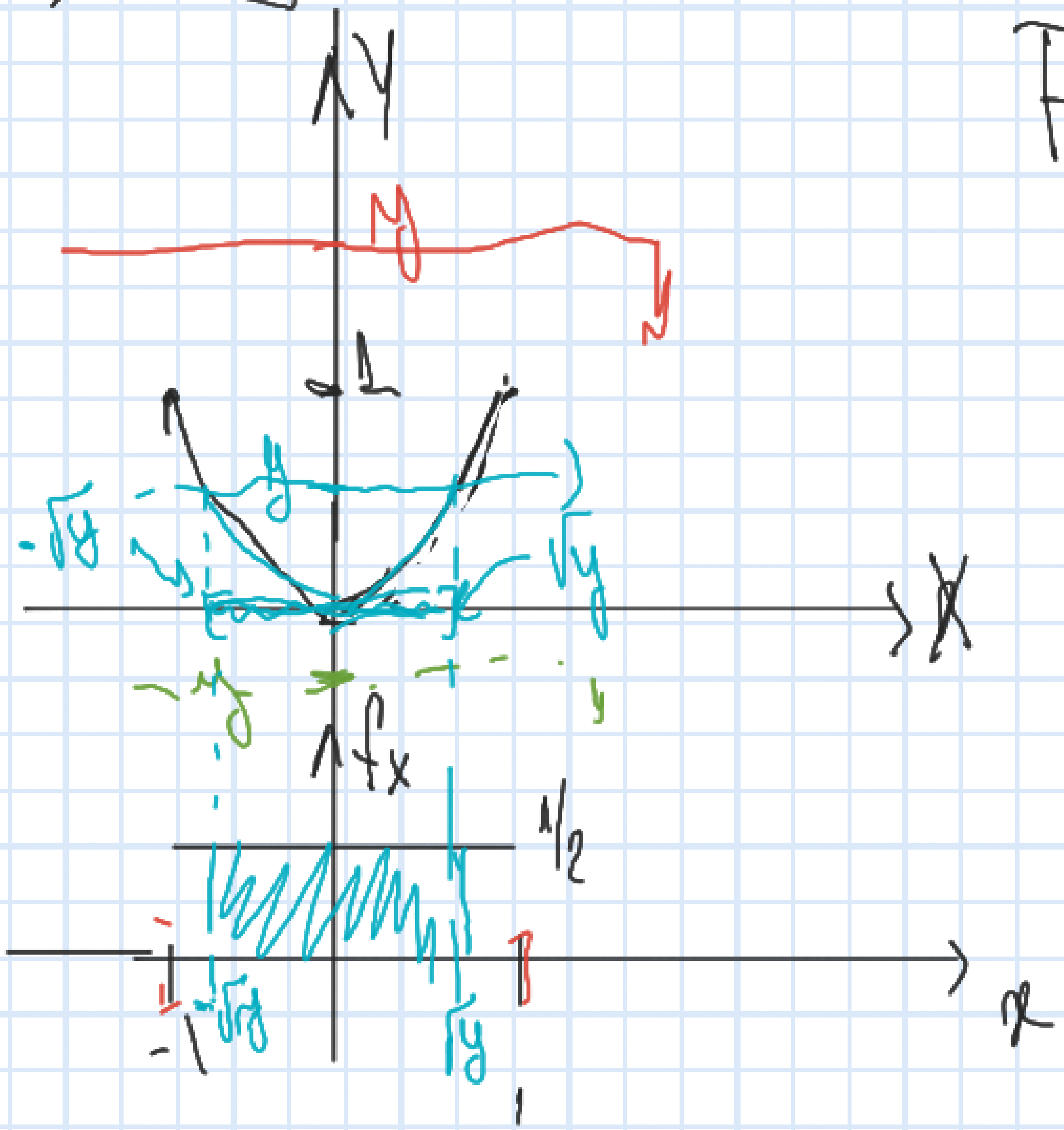


$$F_X(x) = \underbrace{\int_0^x 1/5\, e^{-1/5 t}\, dt}_{P(X \leq x)} = \left(1 - e^{-1/5 x}\right) \mathbb{1}_{\{x>0\}}$$

$$F_X(x) = u$$

$$1 - e^{-1/5 x} = u \rightsquigarrow x = -5 \ln(1-u)$$

Ej 6) $X \sim U[-1,1]$ $\quad Y = X^2$



$$F_Y(y) = \mathbb{P}(Y \leq y) = \mathbb{P}(X^2 \leq y) = \begin{cases} 0 & y < 0 \\ \mathbb{P}(-\sqrt{y} \leq X \leq \sqrt{y}) = \frac{\sqrt{y} - (-\sqrt{y})}{2} & 0 \leq y < 1 \\ 1 & y \geq 1 \end{cases}$$

$$= \begin{cases} 0 & y < 0 \\ \sqrt{y} & 0 \leq y < 1 \\ 1 & y \geq 1 \end{cases}$$