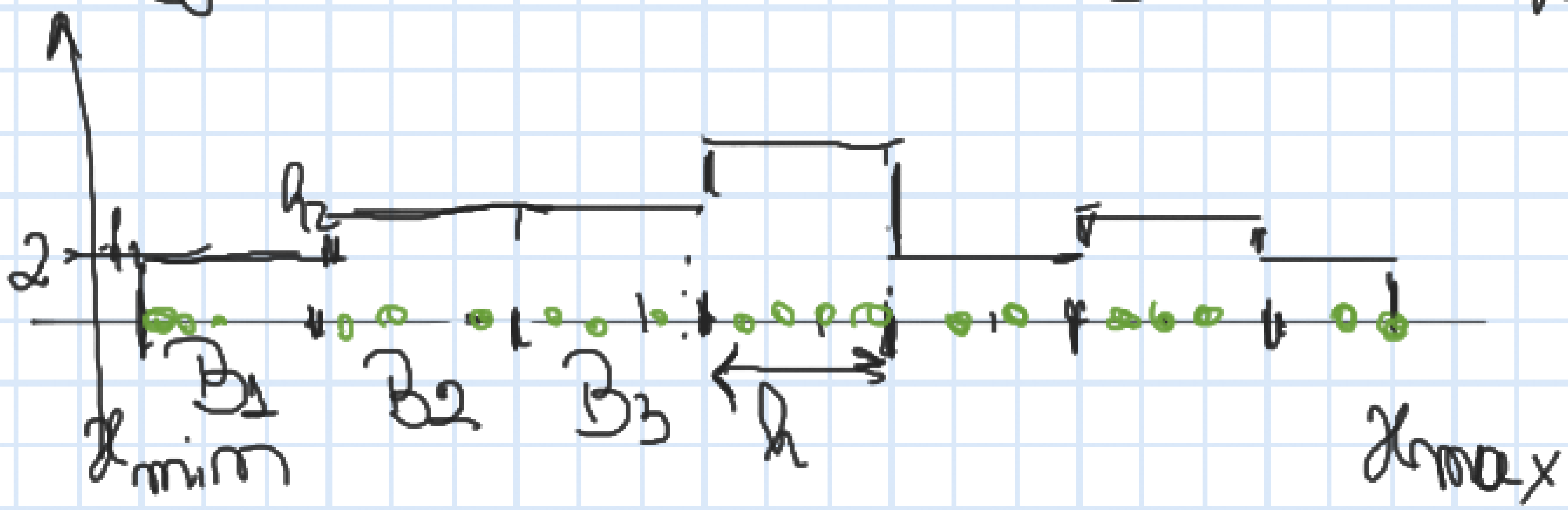
$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \qquad LGN: \quad \bar{X}_n \xrightarrow[n\to\infty]{} \mu$$

Histograma $\quad x_1 \dots x_n \qquad x_{min} = \min_{i=1\dots n} x_i \qquad x_{max} = \max_{i=1\dots n} x_i$



$$h_i = \frac{\# B_i}{n\,h}$$

$n = \#$ de muestras

Mínimos Cuadrados lineal.

a y b / $\underset{a,b}{\arg\min}\ \mathbb{E}\left[\left(X-(aY-b)\right)^2\right] =$

$ECM = \mathbb{E}\left[\left(X-(aY+b)\right)^2\right] = \mathbb{E}\left[X^2 + a^2Y^2 + b^2 - 2aXY - 2bX + 2abY\right]$

linealidad $= \mathbb{E}[X^2] + a^2\mathbb{E}[Y^2] + b^2 - 2a\mathbb{E}[XY] - 2b\mathbb{E}[X] + 2ab\mathbb{E}[Y]$

$$\frac{\partial ECM}{\partial a} = 2a\mathbb{E}[Y^2] - 2\mathbb{E}[XY] + 2b\mathbb{E}[Y] \overset{1}{=} 0$$

$$\frac{\partial ECM}{\partial b} = 2b - 2\mathbb{E}[X] + 2a\mathbb{E}[Y] = 0 \rightarrow b = \mathbb{E}[X] - a\mathbb{E}[Y] \quad (2)$$

(2) en (1) $\quad 2a\mathbb{E}[Y^2] - 2\mathbb{E}[XY] + 2\left(\mathbb{E}[X] - a\mathbb{E}[Y]\right)\mathbb{E}[Y] = 2a\,\mathrm{Var}(Y) - 2\,\mathrm{Cov}(X,Y) = 0$

$b =$

$$\cancel{2}a\,\mathrm{var}(Y) - \cancel{2}\,\mathrm{Cov}(X,Y) = 0 \rightsquigarrow a = \frac{\mathrm{Cov}(X,Y)}{\mathrm{var}(Y)}$$

$$\Rightarrow b = \mathbb{E}[X] - \frac{\mathrm{Cov}(X,Y)}{\mathrm{var}(Y)}\,\mathbb{E}[Y].$$

$$\hat{X} = aY + b = \frac{\mathrm{Cov}(X,Y)}{\mathrm{var}(Y)}\,Y + \mathbb{E}[X] - \frac{\mathrm{Cov}(X,Y)}{\mathrm{var}(Y)}\,\mathbb{E}[Y]$$

$$= \frac{\mathrm{Cov}(X,Y)}{\mathrm{var}(Y)}\left(Y - \mathbb{E}[Y]\right) + \mathbb{E}[X].$$

$$B = \mathbb{E}\left[(\hat{X} - X)\right] \rightsquigarrow \mathbb{E}[\hat{X}] \overset{?}{=} \mathbb{E}[X]$$

$$\mathbb{E}[\hat{X}] = \frac{\mathrm{Cov}(X,Y)}{\mathrm{var}(Y)}\,\underbrace{\mathbb{E}\left[Y - \mathbb{E}[Y]\right]}_{0} + \mathbb{E}[X].$$

$\Rightarrow$ insesgado

$$ECM = var[\hat{X}] + \underbrace{B(\hat{X})^2}_{=0}$$

$$var(\hat{X}) = var\left(\frac{cov(X,Y)}{var(Y)}\left(Y - \mathbb{E}[Y]\right) + \mathbb{E}[X]\right)$$

$$= \left(\frac{cov(X,Y)}{var(Y)}\right)^2 var(Y) = \frac{cov(X,Y)^2}{var(Y)}$$

$$\hat{X} = aY + b \qquad (y_1 \dots y_m)$$

$$\begin{aligned} x_1 &= a y_1 + b \\ x_2 &= a y_2 + b \\ &\vdots \\ x_n &= a y_n + b \end{aligned} \quad = \begin{bmatrix} y_1 & 1 \\ y_2 & 1 \\ \vdots & \\ y_n & 1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Est. suficiente.

$\underline{X} = (X_1, \dots, X_n)$ m.a. $\quad X \sim \mathcal{E}(\lambda) \quad F_\mu(x) = [1 - e^{-\lambda x}] \mathbb{1}\{x>0\}$.

$\hookrightarrow X_i$ son iid $\quad \theta = \lambda \quad f_\mu(x) = \lambda e^{-\lambda x} \mathbb{1}\{x>0\}$.

$$f_{\underline{X}}(\underline{x}) \overset{\text{indep}}{=} \prod_{i=1}^{n} f_{X_i,\lambda}(x_i) = \prod_{i=1}^{n} \lambda e^{-\lambda x_i} \mathbb{1}\{x_i>0\} = \underbrace{\lambda^n e^{-\lambda \sum_{i=1}^{n} x_i}}_{h(T(\underline{x}),\lambda)} \underbrace{\mathbb{1}\{\min(x_i) > 0\}}_{g(x)}$$

$g(x)$? $\quad h(T(x), \lambda)$?

$\Rightarrow T(\underline{X}) = T = \sum_{i=1}^{n} X_i$

es un estad. sufi

$\underline{X} = (X_1 \ldots, X_n)$ $\quad X \sim Ber(p)$ $\quad P_X(x) = \begin{cases} p & si\ x=1 \\ 1-p & si\ x=0 \end{cases} = p^x (1-p)^{1-x}$

$$P_{\underline{X}}(\underline{x}) = \prod_{i=1}^{n} P_{X_i;p}(x_i) = \prod_{i=1}^{n} p^{x_i}(1-p)^{1-x_i} = \underbrace{p^{\sum x_i}(1-p)^{n-\sum x_i}}_{h(T(\underline{x}),\, p)} \cdot \underbrace{1\{x = \{0,1\}\}}_{g(x)}$$

$\hookrightarrow \sum_{i=1}^{n} X_i = T$ es est. sufi

Ejemplo de máximo verosimilitude

$$\underline{X} = (\underline{X}_1, \dots, X_n) \qquad X \sim Ber(p)$$

$$L(p) = \prod_{i=1}^{n} P_{X_i}(x_i;\theta) = p^{\sum x_i} \cdot (1-p)^{n-\sum x_i}$$

$$\Rightarrow \underbrace{\log L(p)}_{\text{log-likelihood}} = l(p) = \ln\left(p^{\sum x_i}\right) + \ln\left((1-p)^{n-\sum x_i}\right)$$

$$= \left(\sum x_i\right)\ln(p) + \left(n - \sum x_i\right)\ln(1-p)$$

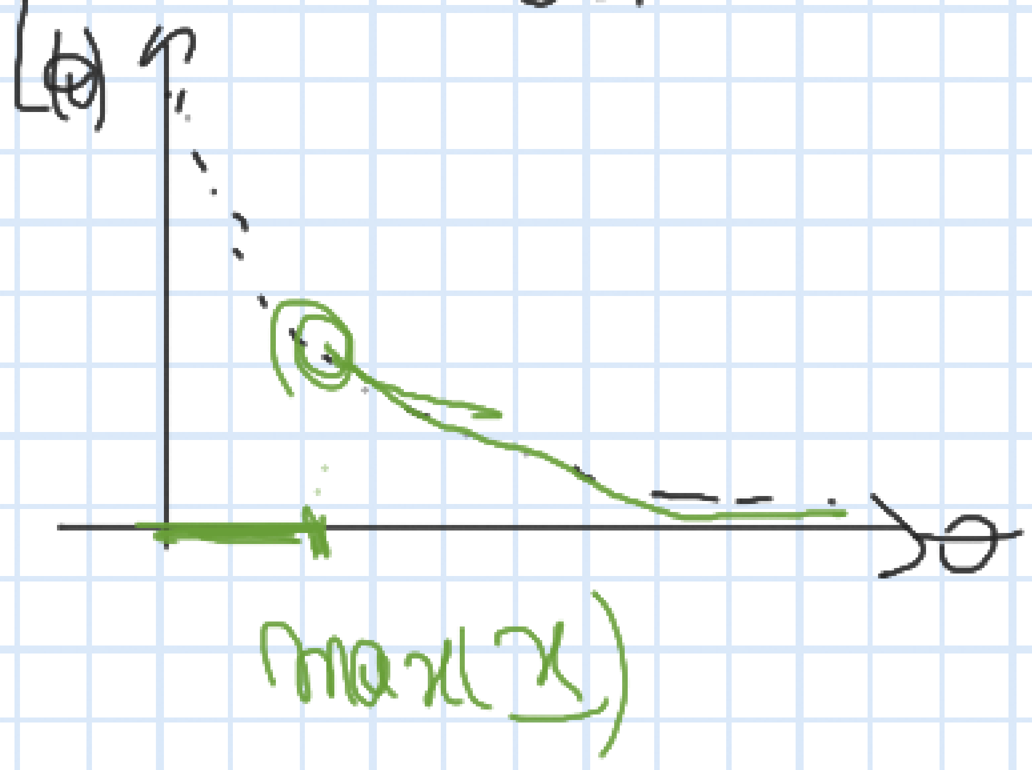
$$\frac{\partial l(p)}{\partial p} = \frac{\sum x_i}{p} - \frac{(n-\sum x_i)}{1-p} = 0 \Rightarrow \sum x_i - \cancel{\sum x_i (p)} = np - \cancel{\sum x_i p}$$

$$\hat{p} = \frac{\sum X_i}{n} = \bar{X}$$

$\underline{X} = (X_1, \dots, X_m)$ $\quad X \sim \mathcal{U}(0, \theta)$ $\quad f_X(x;\theta) = \frac{1}{\theta} \mathbb{1}\{0 < x < \theta\}$

$L(\theta) = \prod_{i=1}^{m} \frac{1}{\theta} \mathbb{1}\{0 < x_i < \theta\} = \frac{1}{\theta^n} \underbrace{\prod_{i=1}^{m} \mathbb{1}\{0 < x_i < \theta\}}_{\mathbb{1}\{\max(\underline{x}) < \theta\}} = \underbrace{\frac{1}{\theta^n} \mathbb{1}\{\max(\underline{x}) < \theta\}}_{h(T(\underline{x}), \theta)}$

$g(x) = 1$



$\hookrightarrow \hat{\theta} = \max(\underline{X})$

$\Rightarrow \max(\underline{X}) = T$

es est sufi

p/ θ.

$\hat{\theta} = 2{,}86$ (con mis muestras)

$P(X > 2) = \frac{\theta - 2}{\theta} \mathbb{1}\{2 < \theta\} = \lambda(\theta)$

$\widehat{P(X > 2)} = \lambda(\hat{\theta}) = \frac{\hat{\theta} - 2}{\hat{\theta}} \mathbb{1}\{2 < \theta\} = \frac{2{,}86 - 2}{2{,}86}$

Func de dist empírica

De un experimento en los efectos de un medicamento para la ansiedad, entre otras cosas se midió la diferencia (en segundos) entre el puntaje de un test de memoria antes y despues de tomar el medicamento, obteniendo los siguientes resultados:

~~1,2~~; ~~4,6~~; 4,3; ~~−4,2~~; ~~−7,9~~; ~~7,8~~; ~~3,4~~; ~~19,8~~; 25,5; ~~−1,9~~; ~~2,1~~; ~~−0,9~~; 4,6; 21,1; ~~1,7~~

−7.9, −4.2, −1.9, −0.9, 1.2, 1.7, 2.1,

3.4, 4.3, $\underbrace{4.6}_{\times 2}$, 7.8, 19.8, 21.1, 25.5