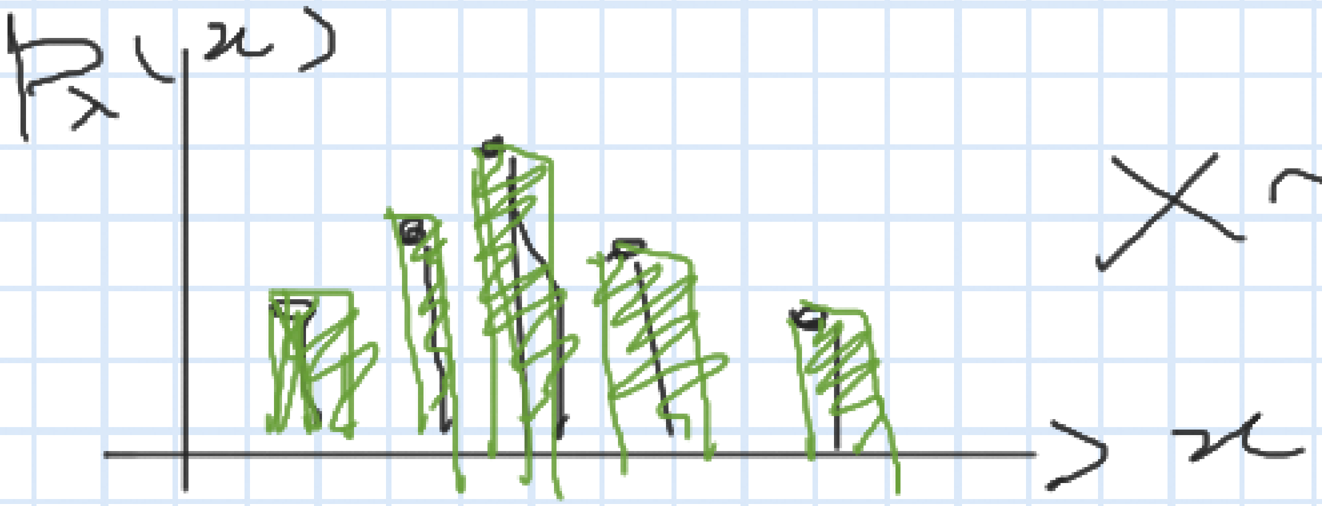
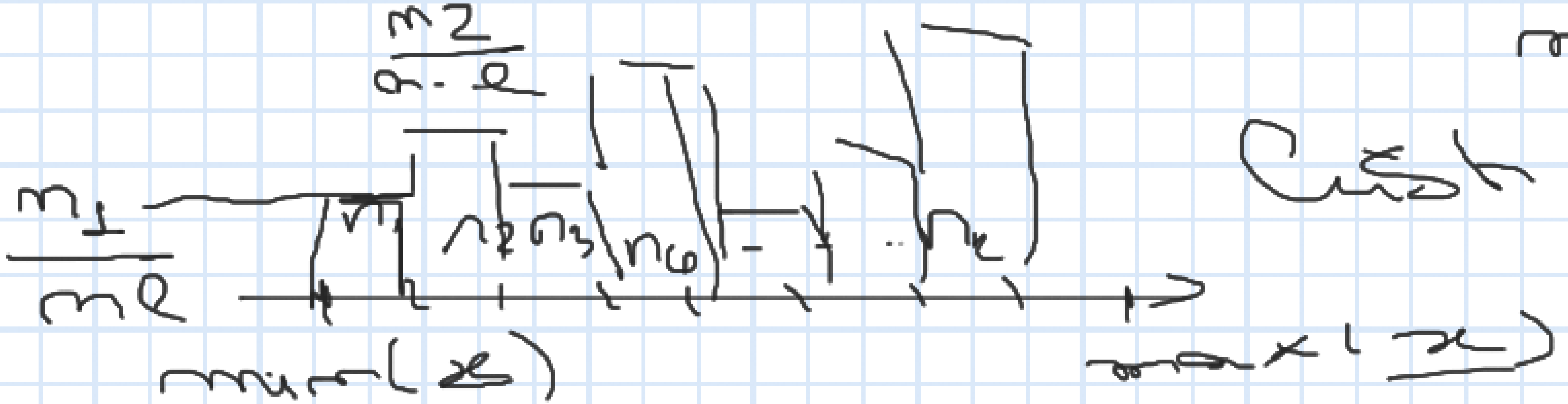
Pmf = función de masa (función de probabilidad)

$P_X(x)$ → para v. a.



$X \sim Bin(4, 1/2)$

$\underline{x} = [x_1, \ldots, x_n]$ desde una v.a. X con dist. $f_X(x)$ (X es cont.)
no lo conozco



Hist

Ej 1 X = " # caras en 10 tiradas"

$$X \sim Bin(10, 1/2)$$

a) $\mathbb{P}(X=3) = \binom{10}{3} 1/2^{3} (1/2)^{10-3}$

$= 0,1172$

b) $\mathbb{P}(X \geq 3) = 1 - \mathbb{P}(X < 3)$

$X \sim Bin(10, 0.4) = 1 - \mathbb{P}(X \leq 2)$

$= 1 - cdf(2) = 0.833$

$\hookrightarrow \mathbb{P}(X \leq 2)$

c) Y = "Resultado de una tirada"

$$\mathbb{P}(X \geq 3 \mid Y=0) = \frac{\mathbb{P}(X \geq 3, Y=0)}{\mathbb{P}(Y=0)} =$$

Eg. 3

$Sop(X,Y)$

→ indep e identicamente dist.

$X, Y \overset{ind}{\sim} U(0,1)$

$f_{X,Y}(x,y) \overset{ind}{=} f_X(x) f_Y(y)$

$= \mathbb{1}\{(x,y) \in [0,1] \times (0,1)\}$

$\mathbb{P}(X > 0.7, Y < 0.4) = \mathbb{P}(X > 0.7)\,\mathbb{P}(Y < 0.4)$

$= 0.12$

$= (1 - 0.7)(0.4)$

$(1-0.7)$

0 0.7 1

$$\underline{X} = (X_1, X_2) \qquad \Sigma = \begin{pmatrix} \sigma_{x_1}^2 & \operatorname{cov}(x_1, x_2) \\ \operatorname{cov}(x_1 x_2) & \sigma_{x_2}^2 \end{pmatrix} \qquad \underline{\mu} = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$$

$$f_{\underline{x}}(\underline{x}) = \frac{1}{2\pi |\Sigma|^{1/2}} e^{-\frac{1}{2}(x_1, x_2)\Sigma^{-1}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}}$$

$$\sigma_x^2 \sigma_y^2 - \operatorname{cov}^2(x_1, x_2)$$

$$= \sigma_x^2 \sigma_y^2 \left(1 - \rho^2\right)$$

$$\hookrightarrow \rho = \frac{\operatorname{cov}(x_1, x_2)}{\sigma_x \sigma_y}$$

(coef. de correlación)

$$f_{\underline{x}}(x) = \frac{1}{2\pi\sqrt{\sigma_x^2 \sigma_y^2 (1 - \rho^2)}} e^{-\left(\frac{1}{2(1-\rho^2)}\left[\frac{(x_1 - \mu_1)^2}{\sigma_1^2} + \frac{(x_2 - \mu_2)^2}{\sigma_2^2} - 2\rho\frac{(x_1 - \mu_1)(x_2 - \mu_2)}{\sigma_1 \sigma_2}\right]\right)}$$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{bmatrix} \Rightarrow \Sigma^{-1} = \begin{bmatrix} 1/\sigma_1^2 & 0 \\ 0 & 1/\sigma_2^2 \end{bmatrix}$$

$\Downarrow$

$cov(X_1, X_2) = 0 \Rightarrow \rho = 0$

$\hookrightarrow$ descorrelacionadas

$\Downarrow$ (sólo xq son normales)

indep

si Σ es diag $\Rightarrow X_1, \dots, X_m$ son indep.

$$\frac{1}{2\pi\sqrt{\sigma_x^2\sigma_y^2(1-0)}}\, e^{-\frac{1}{2\cdot 1}\left(\frac{(x_1-\mu_1)^2}{\sigma_1^2} + \frac{(x_2-\mu_2)^2}{\sigma_2^2} + 0\right)}$$

$$1 \quad \underbrace{\frac{1}{\sqrt{2\pi}\,\sigma_x}\, e^{-\frac{1}{2}\frac{(x_1-\mu_1)^2}{\sigma_1^2}}}_{\sim N(\mu_1,\sigma_1^2)} \quad \underbrace{\frac{1}{\sqrt{2\pi}\,\sigma_y}\, e^{-\frac{1}{2}\frac{(x_2-\mu_2)^2}{\sigma_2^2}}}_{N(\mu_2,\sigma_2^2)}$$

$$f_{x,y}(x,y) = f_x(x)\, f_y(y)$$

$\Rightarrow$ indep

$$f_X(x) = \int_{-\infty}^{\infty} f_{X,Y}(x,y)\,dy$$

↳ gassiamo en y

$$\mathbb{E}[w^T X] = w^T \mathbb{E}[X] = w^T \mu$$

$$\mathrm{cov}(w^T X) = \mathrm{cov}\Big((w^T X)\underbrace{(w^T X)^T}_{X^T w}\Big) =$$

$$= w^T \underbrace{\mathrm{cov}(X X^T)}_{\Sigma}\, w \quad = w^T \Sigma w$$

$X \sim F_X(x)$ estrictamente creciente

a) $T = G(\cancel{X}) \Rightarrow G$ tiene inversa

$$F_T(t) = \mathbb{P}(T \leq t) = \mathbb{P}(G(X) \leq t) =$$
$$= \mathbb{P}(X \leq G^{-1}(t)) =$$
$$F_X(G^{-1}(t))$$

b) $U = F_X(\cancel{X})$

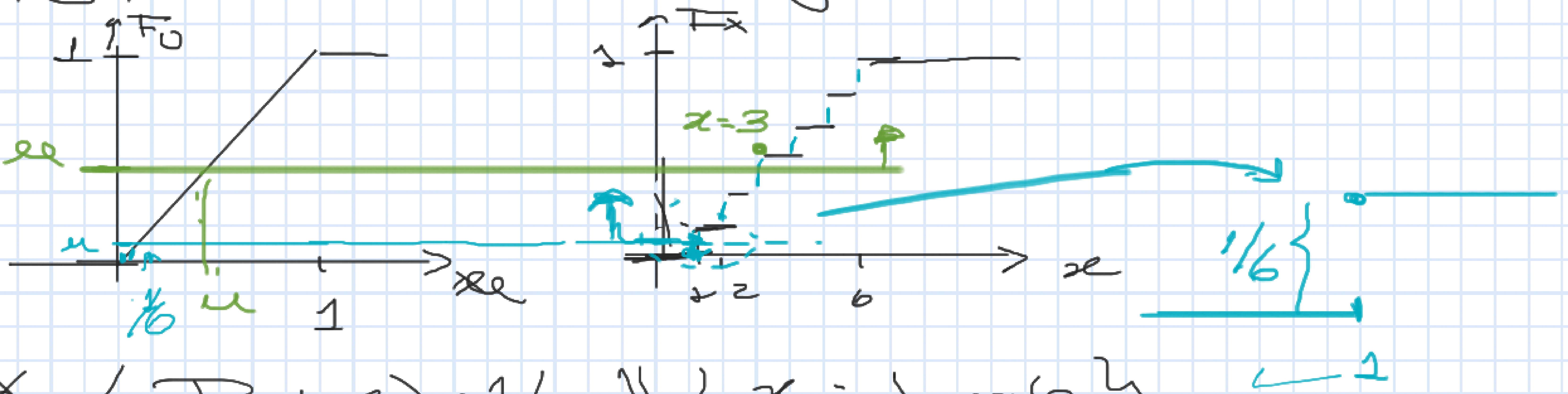
$$F_U(u) = \mathbb{P}_X(F_X(X) \leq u) = u \mathbb{1}\{0 < u < 1\}$$
$$= F_X(X \leq F^{-1}(u)) + \mathbb{1}\{u \geq 0\}$$

$$f_U(u) = \mathbb{1}\{0 < u < 1\}$$

$$U \sim \mathcal{U}(0,1)$$

$\Rightarrow F_X^{-1}(U) = X$ se dist como F_X

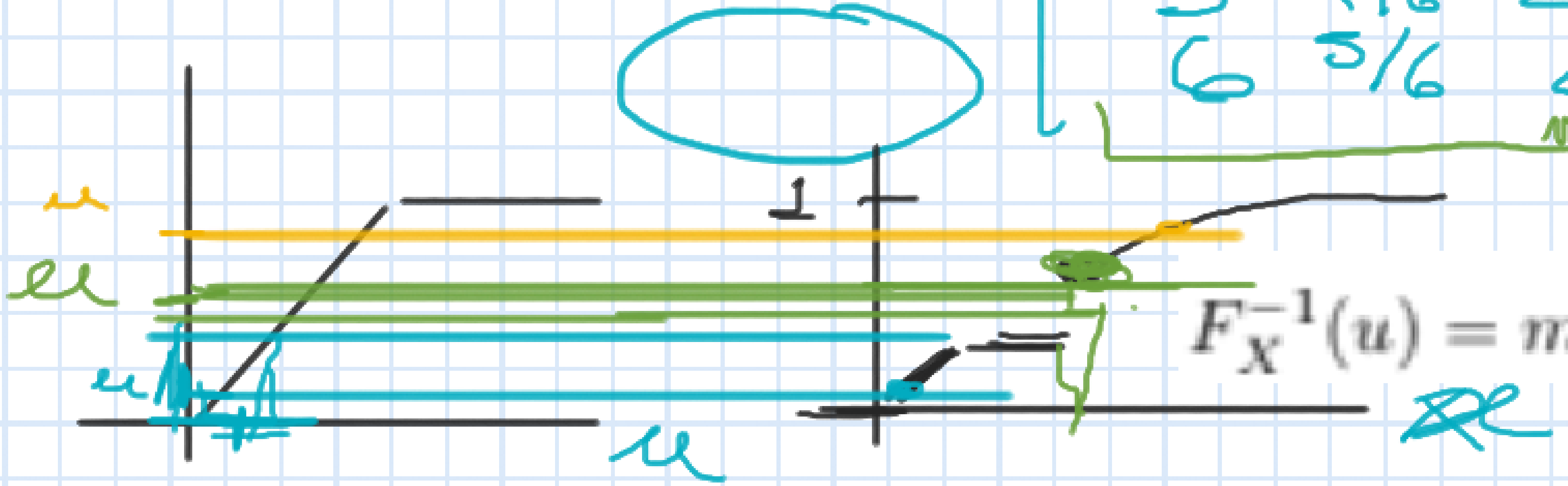
Método de la transformada inversa



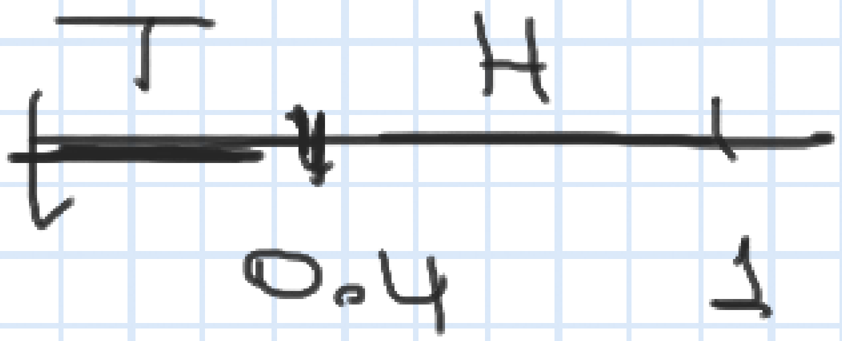
$X \ / \ P_X(x) = 1/6 \ \forall \{x = 1, \dots 6\}$

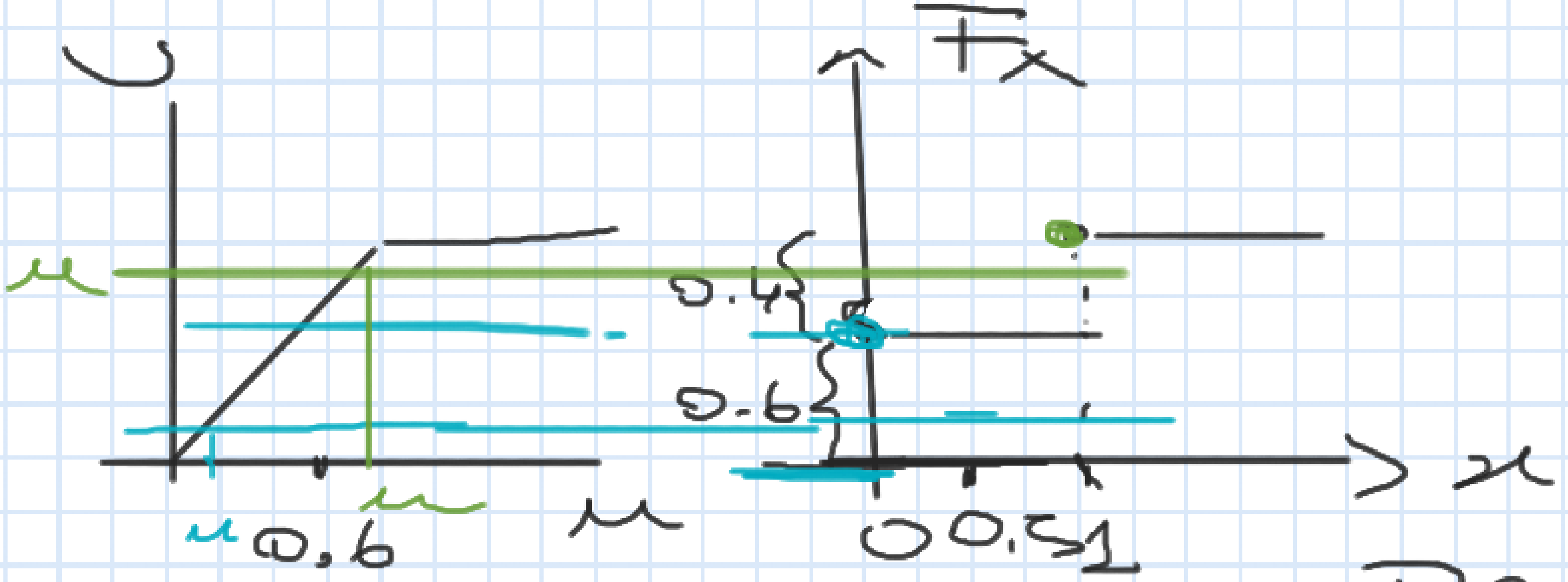
$$F_X^{-1}(u) = \begin{cases} 1 & 0 < u < 1/6 \\ 2 & 1/6 < u < 2/6 \\ 3 & 2/6 < u < 3/6 \\ 4 & 3/6 < u < 4/6 \\ 5 & 4/6 < u < 5/6 \\ 6 & 5/6 < u < 6/6 \end{cases}$$

$F^{-1}(U) = X$



$F_X^{-1}(u) = min\{x \in \mathbb{R} : F_X(x) \geq u\}, u \in (0,1)$





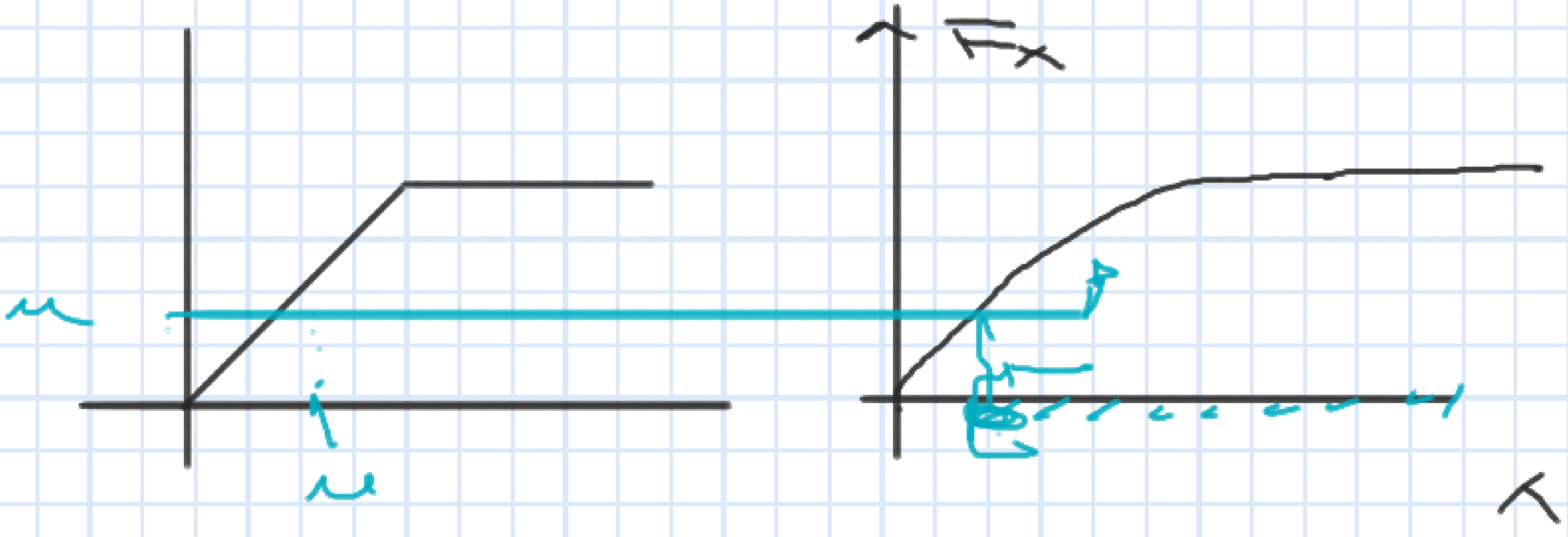
$X \sim Ber(0.4)$

$$F_X(x) = \mathbb{P}(X \le x) \Rightarrow \mathbb{P}(X \le -10) = 0$$

$$\mathbb{P}(X \le 0.5) = \mathbb{P}(X = 0)$$

$$\mathbb{P}(X \le 1.5) = 1$$

$$X = F^{-1}(U) = \begin{cases} 0 & 0 < U < 0.6 \\ 1 & 0.6 < U < 1 \end{cases}$$



$$F_X(x) = 1 - e^{-x}$$

$$\Rightarrow F_X(x) = \underbrace{F_U(u)}$$

$$F_X(x) = u$$

$$x = F_X^{-1}(u)$$

$$1 - e^{-x} = u \rightarrow e^{-x} = 1 - u$$

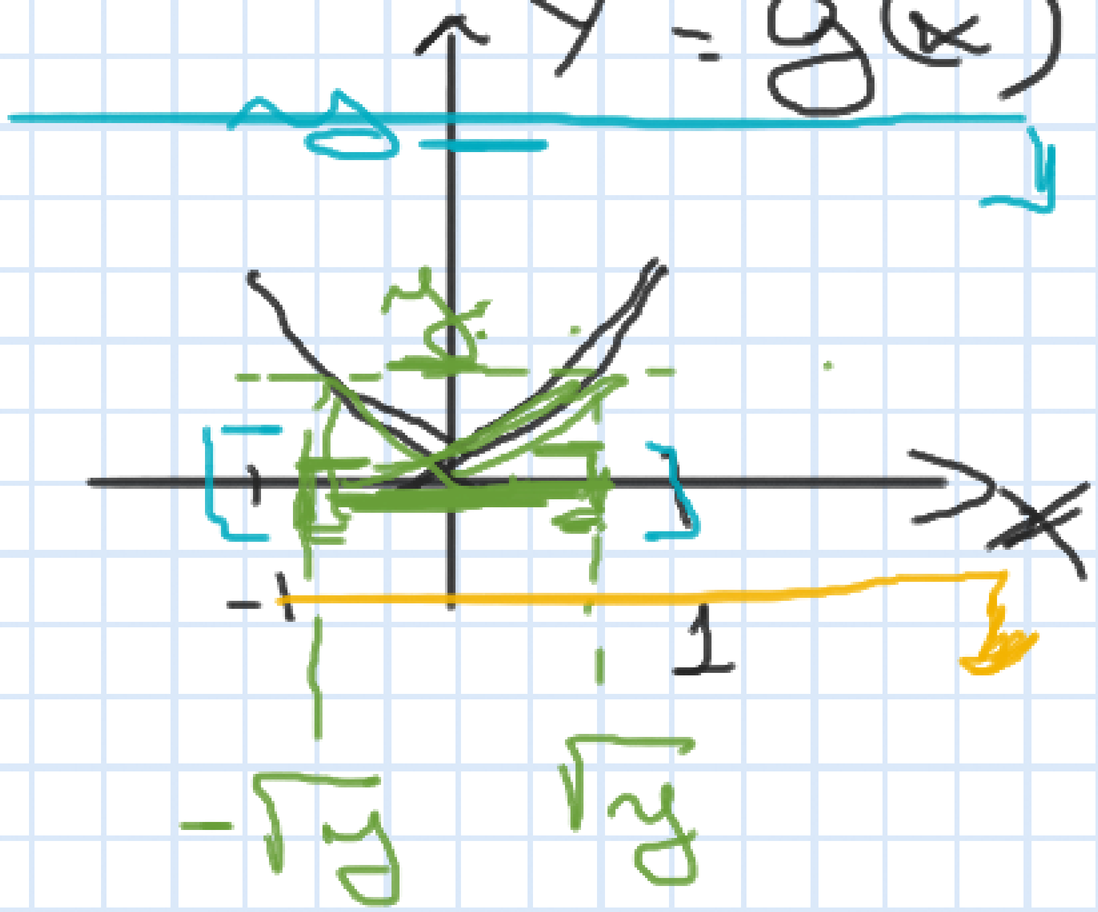
$$x = -\ln(1-u)$$

Tengo X con $F_X(x)$ conosciuto

Tengo $Y = g(X) \Rightarrow F_Y(y)$?

$$F_Y(y) = P(Y \le y) = P(g(X) \le y)$$

$$= P(-\sqrt{y} < X < \sqrt{y}) = \frac{\sqrt{y} - (-\sqrt{y})}{2} = \sqrt{y}$$



$$g(x) = x^2$$

$$X \sim U(-1, 1)$$

$$\Rightarrow F_Y(y) = \begin{cases} 0 & y < 0 \\ \sqrt{y} & 0 < y < 1 \\ 1 & y \geq 1 \end{cases}$$