$$(N_1, N_2, N_3) \sim \mathcal{M}_e(10, 0.3, 0.2, 0.5)$$

$$(N_2, N_3) \mid N_1 = 4 \sim \mathcal{M}_e\left(6, \frac{0.2}{0.7}, \frac{0.5}{0.7}\right)$$

$$N_2 \mid N_1 = 4 \sim \mathrm{Bin}\left(6, \frac{0.2}{0.7}\right)$$

$$\varphi(n_1) = \mathbb{E}\left[\underbrace{N_2 \mid N_1 = n_1}\right] = (10 - n_1)\frac{0.2}{0.7}$$

La func. de regresión

$$\sim \mathrm{Bin}\left(\underbrace{10 - n_1}_{n}, \underbrace{\frac{0.2}{0.7}}_{p}\right)$$

$$f_{X,Y}(x,y) = 2e^{-y/x}\, \mathbb{I}\{y>0,\ 0<x<1\}$$

$$Y|X=x \sim \mathcal{E}(1/x),\quad 0<x<1.$$

$$\varphi(x) = \mathbb{E}[Y|X=x] = x \quad , 0<x<1$$

$$= \int_{\mathbb{R}} y\, f_{Y|X=x}(y)\, dy.$$

$$f_{Y|X=x} = \frac{y}{x}\, \mathbb{I}\{0<y<\sqrt{2x}\}$$

$$\varphi(x) = \mathbb{E}[Y|X=x] = \int_0^{\sqrt{2x}} y \cdot \frac{y}{x}\, dy = \frac{y^3}{3x}\Big|_0^{\sqrt{2x}} = \frac{(2x)^{3/2}}{3x}.$$

$$\mathbb{E}[Y|X] = \varphi(X) = \frac{2^{3/2}\sqrt{X}}{3} \rightarrow \text{esp} \rightarrow \mathbb{E}[Y]$$

$\rightarrow$ probas

$$Y|X=x \sim \mathcal{E}(1/x) \Rightarrow \varphi(x) = x \rightarrow \mathbb{E}[Y|X] = X$$

$$\hookrightarrow \sigma(x) = \operatorname{var}(Y|X=x) = x^2$$

$$\hookrightarrow \mathbb{V}(Y|X) = X^2$$

$$\operatorname{var}(Y) = \operatorname{var}(\mathbb{E}[Y|X]) + \mathbb{E}[\mathbb{V}(Y|X)]$$

$$= \operatorname{var}(X) + \mathbb{E}[X^2]$$

$$= \mathbb{E}[X^2] - \mathbb{E}[X]^2 + \mathbb{E}[X^2] = 2\,\frac{1}{2} - \left(\frac{2}{3}\right)^2.$$

$$= 5/9$$

$$f_X(x) = 2x\, I\{0 < x < 1\}$$

$$\mathbb{E}[X^2] = \int_0^1 x^2\, 2x\, dx = \frac{2}{4} x^4 \Big|_0^1 = 1/2.$$

$$\mathbb{E}[X] = \int_0^1 x\, 2x\, dx = \frac{2}{3} x^3 \Big|_0^1 = 2/3.$$

Ejemplo de estimador $\underline{X} = \{X_1, \dots, X_n\}$

EA: "tiro 10 veces una moneda y miro si sale cara"

$$X_i = \begin{cases} 1 & \text{si sale cara} \\ 0 & \text{si no} \end{cases} \qquad X_i \sim Ber(p)$$

$\hookrightarrow P(X_i = 1)$

$$\delta(\underline{X}) = \frac{\sum_{i=1}^{10} X_i}{10}$$

$$\mathbb{E}[\delta(\underline{X}) - p] = \mathbb{E}\left[\frac{1}{10}\sum_{i=1}^{10} X_i\right] - p$$

$$= \frac{1}{10}\underbrace{\sum_{i=1}^{10}\underbrace{\mathbb{E}[X_i]}_{p}}_{10 \cdot p} - p = p - p = 0$$

$\Rightarrow$ Es un estimador insesgado

$$\hat{p} = \frac{1}{n}\sum_{i=1}^{n} x_i \qquad \sum_{i=1}^{n} x_i \text{ es un estadístico}$$

$F_{\underline{X}|T=t}(\underline{x})$ no dep. de los parámetros

Est. sufi

$$P_{\underline{X}}(\underline{x}) \underset{\text{ind.}}{=} \prod_{i=1}^{n} \underbrace{f_{X_i}(x_i)}_{p^{x_i}(1-p)^{1-x_i}} = \underbrace{p^{\sum x_i}(1-p)^{n-\sum x_i}}_{g(T(\underline{x}),p)} \quad \underbrace{\mathbb{1}\{x_i = \{0,1\}\}}_{h(\underline{x})}$$

$$T(\underline{x}) = \sum x_i \Rightarrow \text{est. sufi}$$

$$T = \sum X_i^2$$

MLE

$\underline{X} = (X_1, \dots, X_n)$ uma m.a. de uma v.a. X

ponto referente a uma família de

dist. $Ber(p)$, $p \in (0,1)$ desconhecido.

Θ

$X_i \overset{iid}{\sim} Ber(p)$

1) Sup X não depende de p

2) Θ é aberto

3) $L(p)$ é derivável

$$L(p) = p^{\sum x_i} (1-p)^{n-\sum x_i}$$

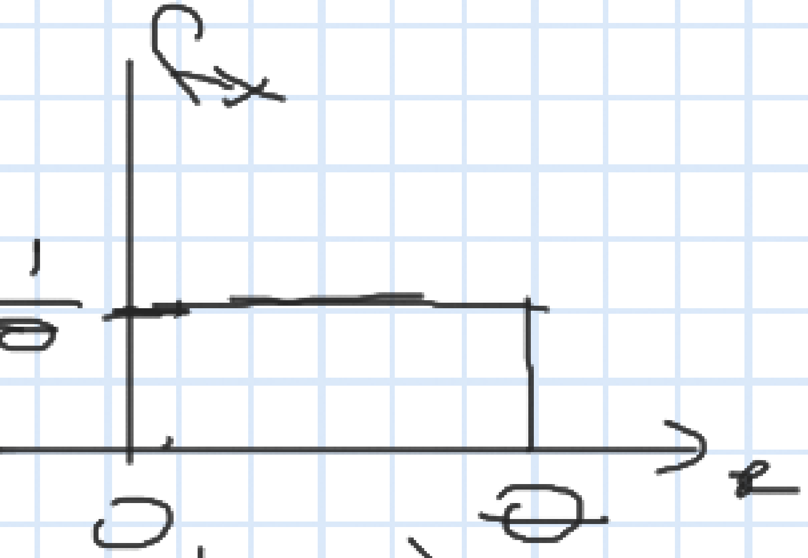
$$\hookrightarrow \ln(L(p)) = \left(\sum x_i\right) \ln(p) + \left(n - \sum x_i\right) \ln(1-p)$$

$$\frac{\partial \ln(L(p))}{\partial p} = \frac{\sum x_i}{p} = \frac{n - \sum x_i}{1-p} \Rightarrow \sum x_i - p\cancel{\sum x_i} = np - \cancel{p\sum x_i}$$

$$p = \frac{\sum x_i}{n} \rightarrow \hat{p} = \frac{\sum X_i}{n}$$

$\underline{X} = (X_1, \dots, X_n)$, $X_i \overset{iid}{\sim} \mathcal{U}(0, \theta)$

$$L(\theta) = \prod_{i=1}^{n} f_{X_i}(x_i) = \prod_{i=1}^{n} \frac{1}{\theta} \underbrace{\mathbb{I}\{0 < x_i < \theta\}}$$



$$= \frac{1}{\theta^n} \mathbb{I}\{\theta > \max(x_i)\}$$

$$\hat{\theta} = \max(\underline{X})$$

$$ECM(\hat{\theta}) = Var(\hat{\theta}) + B^2(\theta)$$

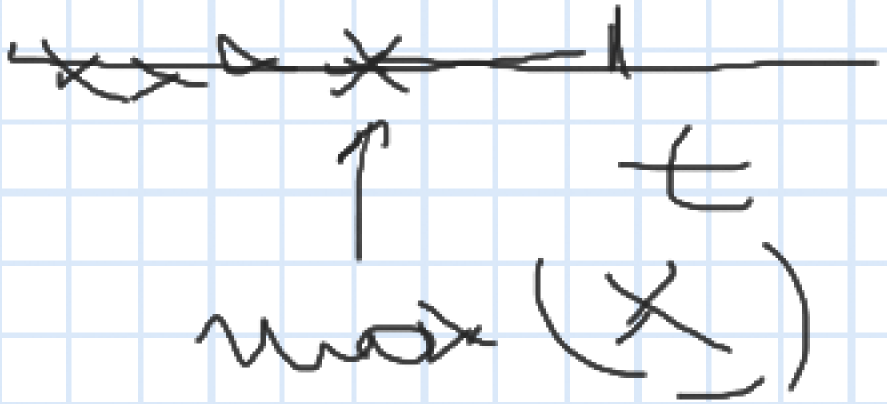
$$B(\theta) = \mathbb{E}[\hat{\theta}] - \theta$$

$$\mathbb{E}[\hat{\theta}] = \mathbb{E}[\max(\underline{X})] = \int_0^{\theta} t\frac{n}{\theta^n} t^{n-1}\,dt = \frac{n}{\theta^n}\left.\frac{t^{n+1}}{n+1}\right|_0^{\theta} = \frac{n}{n+1}\theta$$

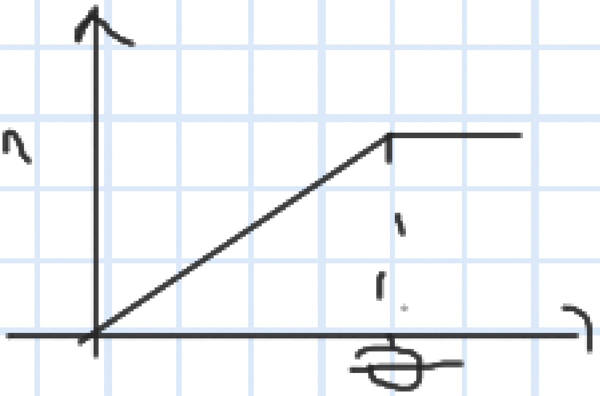
método de sucesos equivalentes

$$\mathbb{P}(\max(\underline{X}) \leq t) = \mathbb{P}(X_1 \leq t, X_2 \leq t, \dots, X_n \leq t)$$

$$\underset{\text{indep}}{=} \prod_{i=1}^{n} \mathbb{P}(X_i \leq t) \underset{\text{i.d.}}{=} \mathbb{P}(X_1 \leq t)^n = F_{\hat{\theta}}(t)$$



$$= \begin{cases} 0 & t<0 \\ (t/\theta)^n & 0<t<\theta \\ 1 & t \geq \theta \end{cases}$$



$$f_{\hat{\theta}}(t) = \frac{n}{\theta}\left(\frac{t}{\theta}\right)^{n-1} I_{\{0<t<\theta\}} = \frac{n}{\theta^n} t^{n-1} I_{\{\dots\}}$$

$$\mathbb{E}[\hat{\theta}] = \frac{n}{n+1}\,\theta \longrightarrow \text{asintóticamente insesgado.}$$

$$\mathbb{E}[\hat{\theta}] - \theta = B(\theta) = \left(\frac{n}{n+1} - 1\right)\theta = -\frac{1}{n+1}\,\theta$$

$$\operatorname{Var}(\hat{\theta}) = \mathbb{E}[\hat{\theta}^2] - \mathbb{E}[\hat{\theta}]^2$$

$$= \underbrace{\int t^2 \frac{n}{\theta^n} t^{n-1}\,dt}_{\frac{n}{\theta^n}\frac{\theta^{n+2}}{n+2}} - \left(\frac{n}{n+1}\theta\right)^2$$

$$= \frac{n}{n+2}\,\theta^2 - \left(\frac{n}{n+1}\right)^2\theta^2$$

$$ECM(\hat{\theta}) = Var(\hat{\theta}) - \underbrace{B(\theta)}_{E(\hat{\theta}) - \theta}{}^2$$

$$= \left(\frac{n}{n+2} - \frac{n^2}{(n+1)^2}\right)\theta^2 + \left(\frac{1}{n+1}\right)^2 \theta^2$$

$$= \left(\underbrace{\frac{n}{n+2}}_{\to 1} - \underbrace{\frac{n^2}{(n+1)^2}}_{\to 1} + \underbrace{\frac{1}{(n+1)^2}}_{\to 0}\right)\theta^2$$

$$ECM(\hat{\theta}) \xrightarrow[n \to \infty]{} 0 \Rightarrow$$ consistente en media cuadrática

Estimar $P(X \leq 2{,}5) = \frac{2{,}5}{\theta} I_{\{2{,}5 < \theta\}} + I_{\{2{,}5 \geq \theta\}}$

$$\widehat{\mathbb{P}(X < 2{,}5)} = \mathbb{P}_{\hat\theta}(X \le 2{,}5) = \frac{2{,}5}{\hat\theta} I\{2{,}5 < \hat\theta\} + 1\{2{,}5 > \hat\theta\}$$

$$\hat\theta_{100} = 2{,}91 \Rightarrow \widehat{\mathbb{P}(X<2{,}5)} = \frac{2{,}5}{2{,}91}$$

↳ muestra

$$\hat\theta_{10} = 2{,}29 \Rightarrow \widehat{\mathbb{P}}(X<2{,}5) = 1$$

ECDF

func. de dist

De un experimento en los efectos de un medicamento para la ansiedad, entre otras cosas se midió la diferencia (en segundos) entre el puntaje de un test de memoria antes y despues de tomar el medicamento, obteniendo los siguientes resultados:

~~1,2~~; 4,6; 4,3; ~~−4,2~~; ~~−7,9~~; 7,8; ~~3,4~~; 19,8; 25,5; ~~−1,9~~; ~~2,1~~; ~~−0,9~~; 4,6; 21,1; ~~1,7~~

$$\widehat{F}_n(x) = \frac{\sum_{i=1}^{n} I\{X_i \leq x\}}{n}$$

−7.9, −4.2, −1.9, −0.9, 1.2, 1.7, 2.1, 3.4, 4.3, 4.6, 7.8, 19.8, 21.1, 25.5 2.1