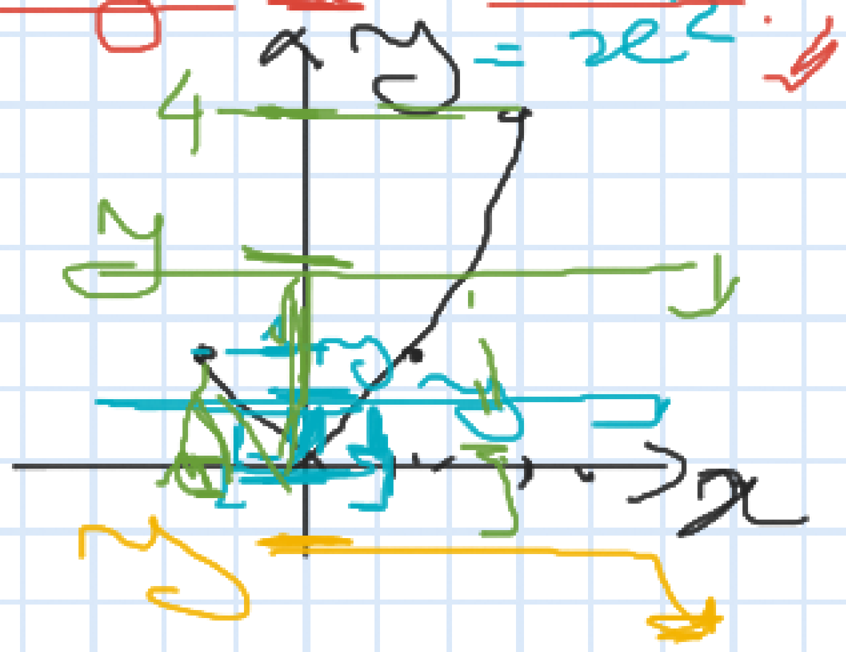
X con $f_X(x) = \frac{2(x+1)}{9} I_{\{-1<x<2\}} = \begin{cases} \frac{2x+1}{9} & -1<x<2 \\ 0 & \text{e.o.c.} \end{cases}$

$Y = g(x) = X^2$

$F_Y(y) = P(Y \leq y) = P(X^2 \leq y) =$

$I_{\{x \in A\}} = \begin{cases} 1 & \text{si } x \in A \\ 0 & \text{e.o.c.} \end{cases}$

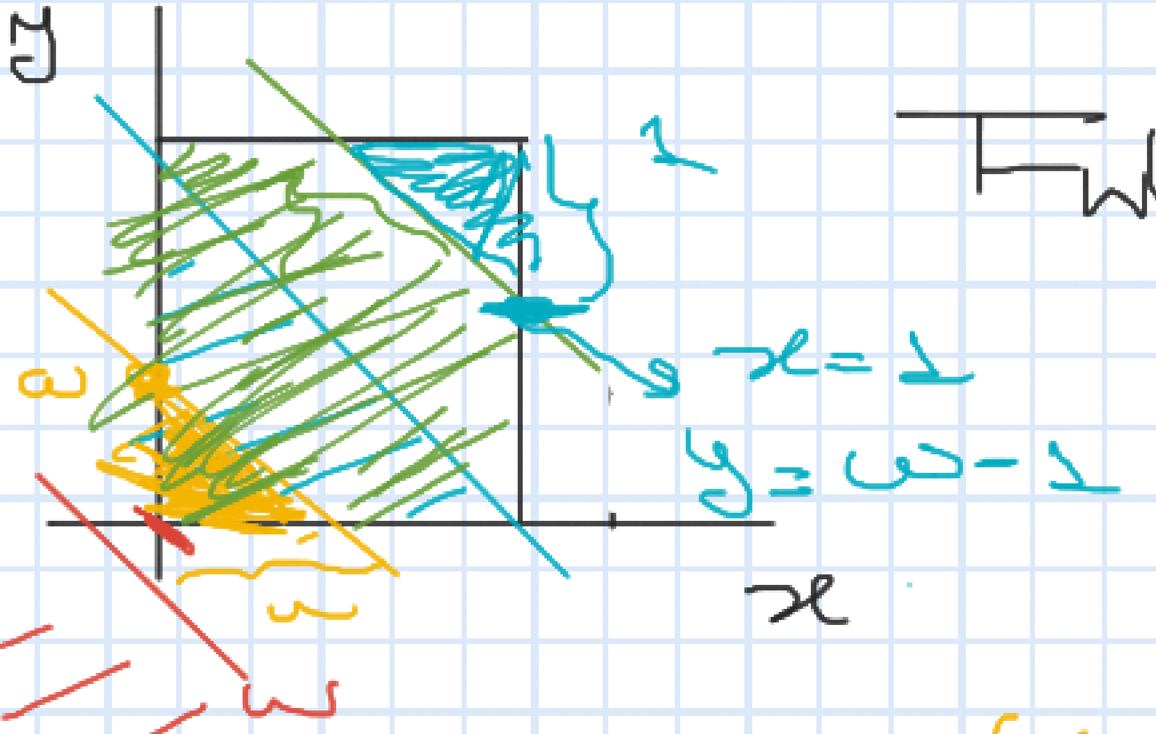


$$= \begin{cases} 0 & y<0 \\ P(-\sqrt{y} < X < \sqrt{y}) & 0 \leq y < 1 \\ P(-1 < X < \sqrt{y}) & 1 \leq y < 4 \\ 1 & y \geq 4 \end{cases} = \begin{cases} 0 & y<0 \\ F_X(\sqrt{y}) - F_X(-\sqrt{y}) & 0 \leq y < 1 \\ F_X(\sqrt{y}) & 1 \leq y < 4 \\ 1 & y \geq 4 \end{cases}$$

$$f_Y(y) = \frac{\partial F_Y(y)}{\partial y} = \begin{cases} f_X(\sqrt{y}) \frac{1}{2\sqrt{y}} + f_X(-\sqrt{y}) \frac{1}{2\sqrt{y}} & 0 \leq y < 1 \\ f_X(\sqrt{y}) \frac{1}{2\sqrt{y}} & 1 \leq y < 4 \\ 0 & \text{e.o.c.} \end{cases}$$

$X, Y \overset{iid}{\sim} U(0,1)$ (i.i.d - indipendente e identicamente distribuite)

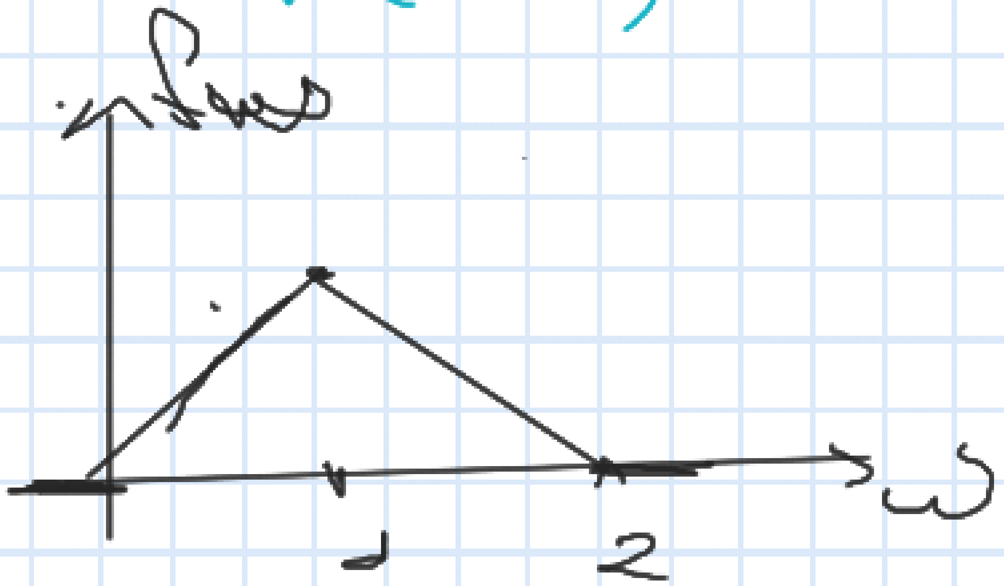
$W = X+Y$



$$F_W(w) = P(W \le w) = P(X+Y \le w) = P(Y \le w - x) = \begin{cases} 0 & w<0 \\ 1 \cdot w^2/2 & 0 \le w < 1 \\ 1 - \frac{(2-w)^2}{2} & 1 \le w < 2 \\ 1 & w \ge 2 \end{cases}$$

$\iint f_{X,Y}(x,y)\,dx\,dy$

$$f_W(w) = \begin{cases} w & 0 \le w < 1 \\ (2-w) & 1 \le w < 2 \\ 0 & \text{else} \end{cases}$$



Método del Jacobiano → 1) Es p/ vars cont.
→ 2) $\mathbb{R}^n \to \mathbb{R}^n$
→ 1 a 1 de $(y_1, y_2) \to (u_1, u_2)$
→ g_1, g_2 derivables

$$Z_1, Z_2 \overset{iid}{\sim} N(0,1)$$

$$U_1 = Z_1 + Z_2 = g_1(z_1, z_2)$$
$$U_2 = Z_1 - Z_2 = g_2(z_1, z_2)$$

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \Rightarrow \begin{array}{l} g_1(z_1, z_2) = z_1 + z_2 \\ g_2(z_1, z_2) = z_1 - z_2 \end{array}$$

$$\begin{bmatrix} g_1^{-1}(u_1, u_2) \\ g_2^{-1}(u_1, u_2) \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Z_1 ~~va~~
z_1 = un nº (escalar)

$$z_1 = u_1/2 + u_2/2$$
$$z_2 = u_1/2 - u_2/2.$$

$$J = \begin{bmatrix} \frac{\partial u_1}{\partial z_1} & \frac{\partial u_1}{\partial z_2} \\ \frac{\partial u_2}{\partial z_1} & \frac{\partial u_2}{\partial z_2} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} \Rightarrow |J| = |\det(J)| = 2$$

$$-2$$

$$f_{z_1,z_2}(z_1,z_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z_1^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{z_2^2}{2}}$$

$$\left(\frac{u_1}{2}\right)^2 + \left(\frac{u_2}{2}\right)^2 + 2\frac{u_1 u_2}{4}$$

$$f_{u_1,u_2}(z_1, z_2) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\left(\frac{u_1}{2}+\frac{u_2}{2}\right)^2}{2}} \frac{1}{\sqrt{2\pi}} e^{-\frac{\left(\frac{u_1}{2}-\frac{u_2}{2}\right)^2}{2}}$$

$$N(0,2)$$

$$= \frac{\frac{1}{\sqrt{2\pi}} \frac{1}{\sqrt{2\pi}} e^{-\left(\frac{u_1^2}{2} + \frac{u_2^2}{4} 2\right)}}{2} = \frac{1}{\sqrt{2\pi}\sqrt{2}} e^{-\frac{u_1^2}{2\cdot 2}} \frac{1}{\sqrt{2}\sqrt{2\pi}} e^{-\frac{u_2^2}{2\cdot 2}}$$

$$N(0,2)$$

$$\Rightarrow u_1, u_2 \overset{\text{iid}}{\sim} N(0,2)$$

$var(U_1) = var(Z_1 + Z_2) \underset{\text{ind.}}{=} var(Z_1) + var(Z_2) = 2 \cdot 1$

$f_{X,Y}(x,y) = f_X(x) f_Y(y) \Rightarrow$ indep

$\rightarrow f_{U_1,U_2}(u_1, u_2) = f_{U_1}(u_1) f_{U_2}(u_2) \Longrightarrow$ indep

$cov(X,Y) = 0 \quad \Longleftarrow$ indep

$Y_1; Y_2$ queremos $U_1 = g_1(Y_1, Y_2)$

me invento $U_2 = g_2(Y_1, Y_2)$

$f_{U_1,U_2}(u_1, u_2)$

$\Rightarrow f_{U_1}(u_1) = \int_{\mathbb{R}} f_{U_1,U_2}(u_1, u_2)\,du_2$

$Y \rightarrow U = g(Y)$

$$f_U(u) = \frac{f_Y(y)}{\left|\frac{\partial g(y)}{\partial y}\right|} \Bigg|_{y = g^{-1}(u)}$$

V.A. condicionadas discretas

$Bin(n, p) \to$ "la # de exitos en n ensayos"
$p = \mathbb{P}$("exito")

n piezas producidas por 3 máquinas

$\hookrightarrow N_1$ = # de piezas prod por la maq 1 (de las n)

N_2 = " " " " " 2

N_3 = " " " " " 3

$(N_1, N_2, N_3) \sim \mathcal{M}(n, p_1, p_2, p_3)$

$p_1 \to \mathbb{P}(M_1)$, $p_2 \to \mathbb{P}(M_2)$, $p_3 \to \mathbb{P}(M_3)$

$$\mathbb{P}_{N_1, N_2, N_3}(n_1, n_2, n_3) = \frac{n!}{n_1! \cdot n_2! \cdot n_3!} \cdot p_1^{n_1} p_2^{n_2} p_3^{n_3}$$

$$\binom{n}{n_1}\binom{n-n_1}{n_2}\underbrace{\binom{n-n_1-n_2}{n_3}}_{\binom{n_3}{n_3}=1}$$

Produzione 10 pezzi $P(M_1) = 0,3$

$P(M_2) = 0,2$

$P(M_3) = 0,5$

$(N_2 N_3) | N_1 = 4 \quad P_{N_2, N_3 | N_1 = 4}(m_2, m_3) = P(N_2 = m_2, N_3 = m_3 | N_1 = 4)$

$$= \frac{P(N_2 = m_2, N_3 = m_3, N_1 = 4)}{P(N_1 = 4)} \qquad N_i \sim Bin(n, P_i)$$

$$= \frac{\cancel{\binom{n}{4}} \binom{n-4}{m_2} \underbrace{\binom{n-4-m_2}{m_3}}_{1} \cancel{P_1^4} P_2^{m_2} P_3^{m_3}}{\cancel{\binom{n}{4}} \cancel{P_1^4} (1-P_1)^{n-4}}$$

$$P_{N_2,N_3|N_1=4}(n_2,n_3) = \frac{\binom{n-4}{n_2} P_2^{n_2} P_3^{(n-4)-n_2}}{(1-P_1)^{n-4}}$$

$(1-P_1) = (P_2+P_3)$

$(P_2+P_3)^{n_2} (P_2+P_3)^{(n-4)-n_2}$

$$= \binom{n-4}{n_2} \left(\frac{P_2}{P_2+P_3}\right)^{n_2} \left(\frac{P_3}{P_2+P_3}\right)^{(n-4)-n_2}$$

$\left\{ N_2 \sim Bin\left(n-4, \frac{P_2}{P_2+P_3}\right) \right.$

$N_3 = n-4-N_2$

Suman 1

$\frac{P_2}{P_2+P_3} + \frac{P_3}{P_2+P_3}$

$$(N_2, N_3) \sim M_e\left(n-4, \frac{P_2}{P_2+P_3}, \frac{P_3}{P_2+P_3}\right)$$

$\hookrightarrow P_2, P_3$ son los datos originales del problema

X, Y ...

$$f_{X,Y}(x,y) = 2e^{-y/x}\, I\{0<y,\ 0<x<1\}$$

$$f_{Y|X=x}(y) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{\cancel{2}e^{-y/x}}{\cancel{2}x} I\{0<y\},\ 0<x<1$$

$$f_X(x) = \int_{\mathbb{R}} f_{X,Y}(x,y)\,dy = \int_0^\infty 2e^{-y/x}\,dy$$

$$= 2x\int_0^\infty \underbrace{\frac{1}{x}e^{-y/x}}_{\mathcal{E}(1/x)}\,dy = 2x\, I\{0<x<1\}$$

$$\Rightarrow f_{Y|X=x}(y) = \frac{1}{x}e^{-y/x}\, I\{y>0\} \Rightarrow Y|X=x \sim \mathcal{E}(1/x)$$

$$f_{X,Y}(x,y) = f_X(x)\, f_{Y|X=x}(y)$$

$$= \cancel{x}2 \frac{1}{x} e^{-y/x} I\{y>0\} I\{0<x<1\}$$

es una func de densidad?

propongo $Y|X=x \sim \mathcal{E}(1/x)$

$$\int_0^1 x2\, dx = x^2\Big|_0^1 = 1 \checkmark$$

$X \sim \mathcal{E}(\lambda)$

$\Rightarrow f_X(x) = \lambda e^{-\lambda x}$

$\hookrightarrow f_X(x) = 2x I\{0<x<1\}$

Mezcla. Tengo una v.a.d M que toma valores $1, 2, \ldots, n$

Busco $X|_{M=1}$ → M_1 $(M=1)$

Tengo un ensayo clínico → M_2 $(M=2)$
p/ obtención medicamento → Placebo $(M=3)$
p/ hipertensión

X = "presión del paciente"

$X|_{M=1} \sim N(12, 1)$

$X|_{M=2} \sim N(11, 1)$

$X|_{M=3} \sim N(15, 1)$

$$f_X(x) = f_{X|M=1}(x) P(M=1) + f_{X|M=2}(x) P(M=2) + f_{X|M=3}(x) P(M=3)$$

$$\mathbb{P}(M=m \mid X=x) = \frac{f_{X|M=m}(x)\,\mathbb{P}(M=m)}{f_X(x)}$$

Bayes Probabilidades

Por ejemplo $X = 11{,}8$

$\mathbb{P}(M=1) = 0.3$

$\mathbb{P}(M=2) = 0.25$

$\mathbb{P}(M=3) = 0.45$

$$\mathbb{P}(M=2 \mid X=11{,}8) = \frac{\frac{1}{\sqrt{2\pi}} e^{-\frac{(11{,}8-11)^2}{2}} \cdot 0.25}{\frac{1}{\sqrt{2\pi}} e^{-\frac{(11{,}8-11)^2}{2}}\, 0{,}25 + \frac{1}{\sqrt{2\pi}} e^{-\frac{(11{,}8-12)^2}{2}}\, 0.3 + \frac{1}{\sqrt{2\pi}} e^{-\frac{(11{,}8-14)^2}{2}}\, 0.45}$$