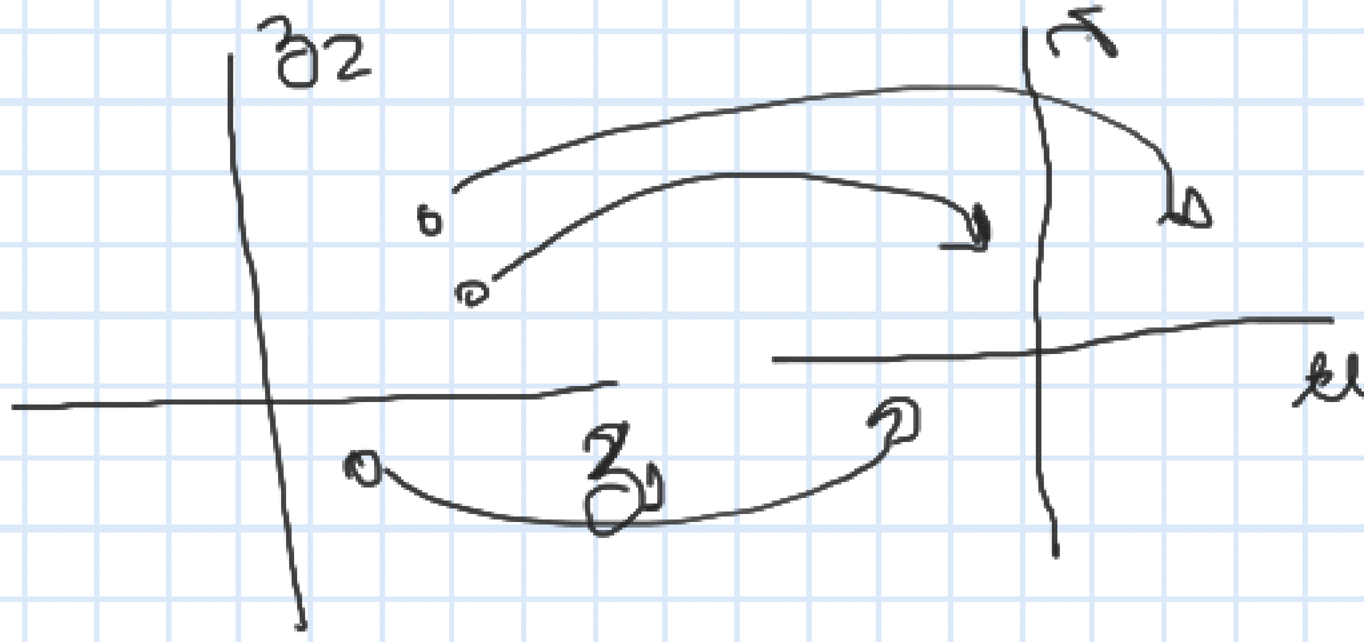
$Z_1, Z_2 \overset{i.i.d}{\sim} N(0,1)$

$U = Z_1^2 + Z_2^2$

$\rightarrow V = Z_2 / Z_1$



Es 1 a 1 la transf.

$u = 5 \rightarrow z_1 = 1, z_2 = 2 \rightarrow v = 1/2$

$z_1 = -1 \quad z_2 = 2 \rightarrow v = -1/2$

$z_1 = 2, z_2 = 1 \rightarrow v = 2$

$u = g_1(z_1, z_2) = z_1^2 + z_2^2$

$g_2(z_1, z_2) = z_1 / z_2$

$z_1 v = z_2$

$u = z_1^2 v^2 + z_2^2 = z_1^2(1+v^2)$

$z_1 = \sqrt{\frac{u}{1+v^2}}, \quad z_2 = v\sqrt{\frac{u}{1+v^2}}$

$$J = \begin{bmatrix} \frac{\partial u}{\partial z_1} & \frac{\partial u}{\partial z_2} \\ \frac{\partial v}{\partial z_1} & \frac{\partial v}{\partial z_2} \end{bmatrix} = \begin{bmatrix} 2z_1 & 2z_2 \\ -\frac{z_2}{z_1^2} & 1/z_1 \end{bmatrix}$$

$|J| = 2 + 2 z_2^2 / z_1^2$

$$f_{U,V}(u,v) = \frac{f_{Z_1,Z_2}(z_1,z_2)}{|J|}\Bigg|_{\substack{z_1 = v\sqrt{\frac{u}{1+v}} \\ z_2 = \sqrt{\frac{u}{1+v}}}} = \frac{\frac{1}{\cancel{\sqrt{2\pi}}} e^{-z_1^2/2} \cdot \frac{1}{\cancel{\sqrt{2\pi}}} e^{-z_2^2/2}}{\cancel{2}\left(z_1^2/z_2^2 + 1\right)}\Bigg|_{\substack{z_1 = v\sqrt{\frac{u}{1+v^2}} \\ z_2 = \sqrt{\frac{u}{1+v^2}}}}$$

$$f_{Z_1,Z_2}(z_1,z_2) = f_{Z_1}(z_1)\, f_{Z_2}(z)$$

$$= \frac{\frac{1}{\pi} e^{-v^2 \frac{u}{1+v^2}\frac{1}{2}} e^{-\frac{1}{2}\frac{u}{1+v^2}} \mathbb{1}_{\{u>0\}}}{v^2+1}$$

$$|J| = \left| -2z_1 \, z_1/z_2^2 - 2 \right| = 2\left(z_1^2/z_2^2 + 1\right)$$

$$= \frac{1}{\pi} \frac{e^{-\frac{1}{2}\frac{u}{\cancel{v^2+1}}\cancel{(v^2+1)}} \mathbb{1}_{\{u>0\}}}{v^2+1}$$

Reviso show u²:

$$\underbrace{\left[2\frac{1}{\pi}\frac{1}{v^2+1}\right]}_{f_V(v)} \underbrace{\left[\frac{1}{2} e^{-\frac{1}{2}u} \mathbb{1}_{\{u>0\}}\right]}_{U \sim \mathcal{E}(1/2)} \sim = \frac{1}{\pi} \frac{e^{-\frac{1}{2}u}}{v^2+1} \mathbb{1}_{\{u>0\}}$$

$\Rightarrow U, V$ son indep

$$\underline{X} = (X_1, \dots, X_n)^T \qquad \underline{\mu} = [\mathbb{E}(X_1) \dots \mathbb{E}[X_n]]^T$$

$$\Sigma = \begin{bmatrix} \sigma_1^2 & cov(X_1, X_2) & \dots & cov(X_1 . X_n) \\ & \sigma_2^2 & & \\ & & \ddots & \\ & & & \sigma_n^2 \end{bmatrix} \qquad cov(\underline{X}, \underline{X}) = \mathbb{E}[(x-\mu)(x-\mu)^T]$$

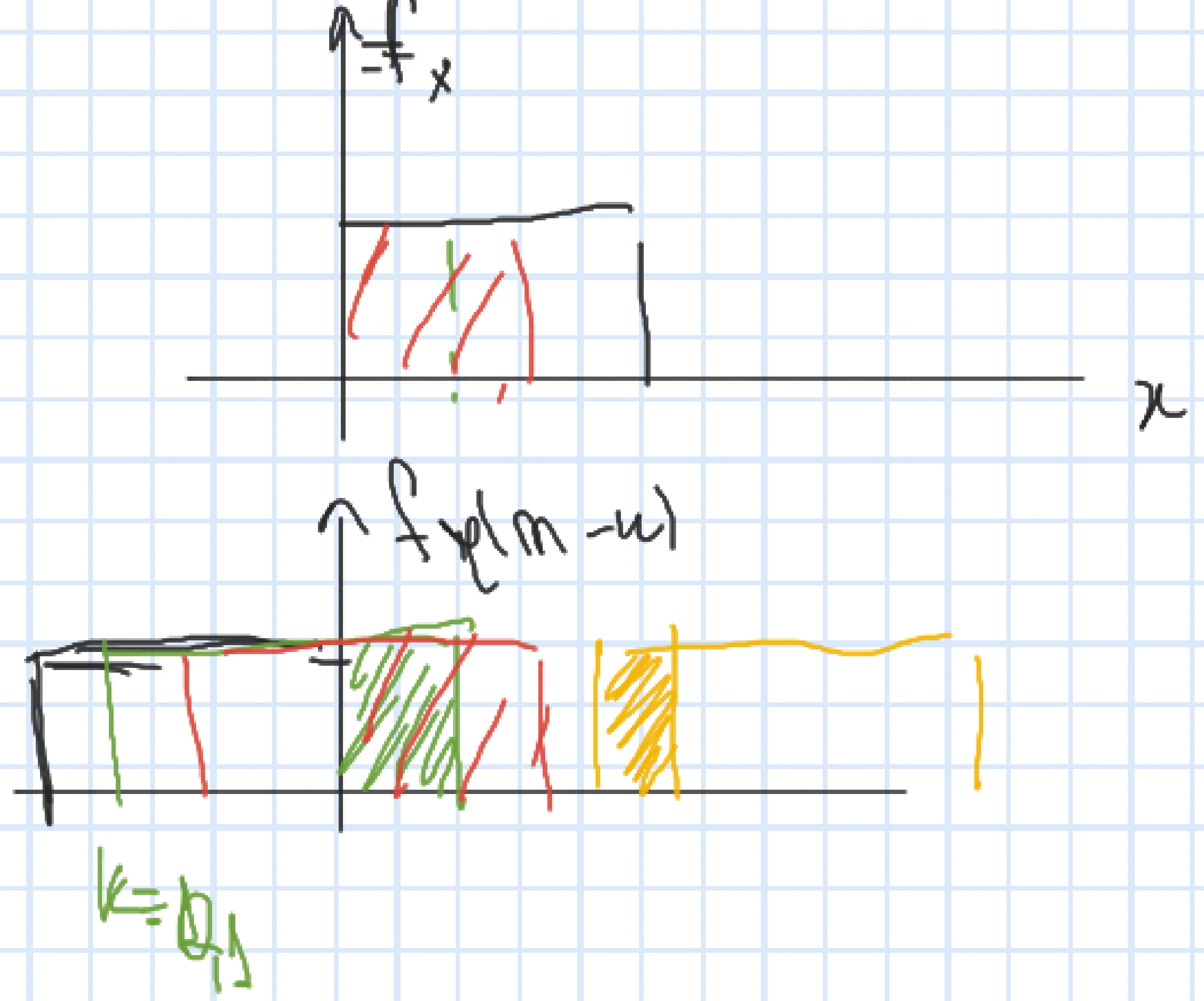
$$w = [w_1 \dots w_n]^T$$

$$\cancel{Z} = w^T \underline{X} = w_1 X_1 + w_2 X_2 + \dots + w_3 X_3 .$$

C.l de v.a normales es normales.

$$\mathbb{E}[w^T \underline{X}] \overset{\text{lineal}}{=} w^T \mathbb{E}[\underline{X}] = w^T \underline{\mu}$$

$$Var(w^T \underline{X}, w^T \underline{X}) = w^T \, \underline{Cov}(\underline{X}, \underline{X}) \, w = w^T \Sigma w \Rightarrow Z \sim N(w^T \mu . w^T \Sigma w)$$


f x
x
f y(m-u)
k=0,1

La probabilidad de acertar a un blanco es $\frac{1}{5}$. Se realizan 10 tiros independientes y se cuenta la cantidad de aciertos. Sean X la cantidad total de aciertos en los 10 tiros, e Y la cantidad de aciertos en el primer tiro. Hallar la distribución de $Y|X = x$

X = "# de aciertos en 10 tiros"

Y = "# de aciertos en el 1º tiro"

p = "P("acierto") = 1/5

$\to X \sim Bin(10, 1/5)$

$Y \sim Ber(p)$

$$p_{Y|X=x}(y) = P(Y=y \mid X=x) = \frac{P(Y=y, X=x)}{P(X=x)}$$

$$= \frac{p_{X,Y}(x,y)}{p_X(x)} = \frac{P(Y=y, V=x-y)}{p_X(x)}$$

$$\underbrace{\text{F}}_{Y}\ \underbrace{\text{F F F F F F F F F}}$$

V = # de aciertos en los restos 9 tiros.

$\to V \sim Bin(9, 1/5)$

Y, V son indep

$y=1$

$x=3$

$$\begin{cases} \dfrac{\cancel{\frac{4}{5}} \cdot \binom{9}{x} \cancel{\frac{1}{5}}^{x} \cancel{\frac{4}{5}}^{9-x}}{\binom{10}{x} \cancel{\frac{1}{5}}^{x} \cancel{\left(\frac{4}{5}\right)}^{10-x}} & y=0 \\ \dfrac{\cancel{1/5} \cdot \binom{9}{x-1} \cancel{\frac{1}{5}}^{x-1} \cancel{\frac{4}{5}}^{9-(x-1)}}{\binom{10}{x} \cancel{1/5}^{x} \cancel{(4/5)}^{10-x}} & y=1 \end{cases}$$

ES 1

$$\frac{\binom{9}{x}}{\binom{10}{x}} = \frac{\frac{9!}{\cancel{x!}\,(9-x)!}}{\frac{10!}{\cancel{x!}\,(10-x)!}} = \frac{1}{10}(10-x)$$

$$\frac{\binom{9}{x-1}}{\binom{10}{x}} = \frac{\frac{9!}{(x-1)!\,\cancel{(10-x)!}}}{\frac{10!}{x!\,\cancel{(10-x)!}}} = \frac{1}{10}x$$

$$\Rightarrow P_{Y|X=x}(y) = \begin{cases} \frac{1}{10}(10-x) & y=0 \\ \frac{1}{10}x & y=1 \end{cases}$$

$$= \begin{cases} 1-\frac{x}{10} & y=0 \\ \frac{x}{10} & y=1 \end{cases}$$

$$\boxed{Y|_{X=x} \sim \mathrm{Ber}\left(\frac{x}{10}\right)}$$

$$(X_1, X_2, X_3) \sim \mathcal{M}_0(10, p_1, p_2, p_3)$$

Ej 2

$$(X_1, X_2) \mid X_3 = 3 \sim \mathcal{M}_0\left(7, \frac{p_1}{p_1+p_2}, \frac{p_2}{p_1+p_2}\right)$$

$$X_3 \sim Bin(10, p_3)$$

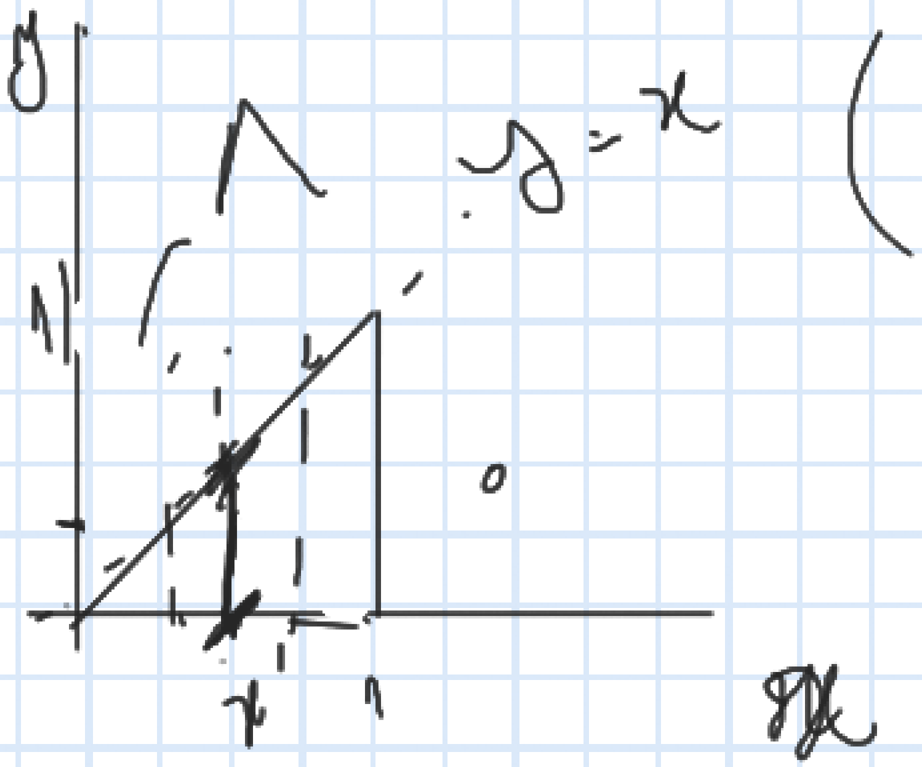
$$\underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_}_{\text{hay de tipo 2 y 1.}}\ \underline{3}\ \underline{3}\ \underline{3}$$

$$P(X_1 = x_1, X_2 = x_2 \mid X_3 = 3) = \frac{P(X_1 = x_1, X_2 = x_2, X_3 = 3)}{P(X_3 = 3)} = \frac{\frac{\cancel{10!}}{x_1!\, x_2!\, \cancel{3!}} p_1^{x_1} p_2^{x_2} \cancel{p_3^{3}}}{\frac{\cancel{10!}}{\cancel{3!}\,(10-3)!} \cancel{p_3^{3}} (1-p_3)^{7}}$$

$$= \frac{(10-3)!}{x_1!\, x_2!} \cdot \frac{p_1^{x_1}\, p_2^{x_2}}{\underbrace{\left(1-p_3\right)^{7}}_{\left(1-p_3\right)^{x_1}\left(1-p_3\right)^{x_2}}} = \underbrace{\frac{7!}{x_1!\, x_2!}} \left(\frac{p_1}{1-p_3}\right)^{x_1} \left(\frac{p_2}{1-p_3}\right)^{x_2}$$

Si $x_1 + x_2 = 7$

(sino 0)



$(X, Y) \sim \mathcal{U}(\Lambda)$ Hallar $f_{Y|X=x}(y)$

$$f_{Y|X=x}(y) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$$\Lambda = \{(x,y) : 0 < y < x < 1\}$$

$$= \frac{2\, \mathbb{1}\{0<y<x<1\}}{2x\, \mathbb{1}\{0<x<1\}} = \frac{1}{x} \mathbb{1}\{0<y<x\}$$

si $0<x<1$

$$f_X(x) = \int_0^x 2\, dy = 2x\, \mathbb{1}\{0<x<1\}$$

$$Y|X=x \sim \mathcal{U}(0, x)$$

Ej 3

$$f_{Y|X=x}(y) = \frac{f_{X,Y}(x,y)}{f_X(x)} \rightarrow f_{Y|X=x}(y)\, f_X(x) = f_{X,Y}(x,y)$$

$$\lambda e^{-\lambda x} \mathbb{1}\{x>0\}$$

Eg

$$f_{X,Y}(x,y) = \frac{e^{-x/2y}}{4y} \mathbb{1}\{0<x, 1<y\leq 3\}$$

$X|Y=y$?

$$= \underbrace{\frac{e^{-x/2y}}{2y} \mathbb{1}\{0<x\}}_{X|Y=y \sim \mathcal{E}(1/2y)} \underbrace{\frac{1}{2} \mathbb{1}\{1<y\leq 3\}}_{Y\sim U(1,3)}$$

Eg 4

T = tiempo en horas de viaje

$$M = \begin{cases} 1 & \text{Si va en Subte} \\ 2 & \text{" " " colectivo} \\ 3 & \text{" " " tren} \end{cases}$$

$$T|_{M=1} \sim \mathcal{U}(3/4, 1)$$

$$T|_{M=2} \sim \mathcal{U}(1.5, 2)$$

$$T|_{M=3} \sim \mathcal{U}(3/4, 5/4)$$

$$\mathbb{P}(T \leq t) = \overbrace{\mathbb{P}(T \leq t \mid M=1)}^{F_{T|M=1}(t)}\mathbb{P}(M=1) + \mathbb{P}(T \leq t \mid M=2)\mathbb{P}(M=2) + \mathbb{P}(T \leq t \mid M=3)\mathbb{P}(M=3)$$

$$f_T(t) = F_T'(t) = \underbrace{f_{T|M=1}(t)}_{4\,\mathbb{1}\{3/4 \leq t \leq 1\}}\mathbb{P}(M=1) + f_{T|M=2}(t)\mathbb{P}(M=2) + f_{T|M=3}(t)\mathbb{P}(M=3)$$

$$\mathbb{P}(M=m \mid T=t) = \frac{f_{T|M=m}(t)\,\mathbb{P}(M=m)}{f_T(t)}$$

$$f_T(t) = \sum_{m=1}^{k} f_{T|M=m}(t)\,\mathbb{P}(M=m)$$

Func de regresión

Ej 1

$$Y|_{X=x} \sim Ber\left(\frac{x}{10}\right) \rightarrow \mathbb{E}[Y|X=x] = x/10 = \varphi(x)$$

Ej 2

$$(X_1, X_2)|_{X_3=x_3} \sim \mathcal{M}\left(10-x_3, \frac{p_1}{1-p_3}, \frac{p_2}{1-p_3}\right) \quad \mathbb{E}[(X_1, X_2)|X_3=x_3] =$$

$$\left((10-x_3)\frac{p_1}{1-p_3}, (10-x_3)\frac{p_2}{1-p_3}\right) = \varphi(x_3)$$

Ej 3

$$Y|X=x \sim \mathcal{N}(0, x) \rightarrow \mathbb{E}[Y|X=x] = x/2. = \varphi(x)$$

Ej 4

$$X|_{Y=y} \sim \mathcal{E}(1/2y) \rightarrow \mathbb{E}[Y|X=x] = 2y. = \varphi(y)$$

Esperanza condicional $\mathbb{E}[Y|X]$

↳ es una v.a.

Ej 1 $\mathbb{E}[Y|X] = \varphi(X) = X/10$

Ej 2 $\mathbb{E}[X_1|X_3] = \varphi(X_3) = (10 - X_3)\dfrac{p_1}{(1 - p_3)}$

Ej 3 $\mathbb{E}[Y|X] = \varphi(X) = X/2$

Ej 4 $\mathbb{E}[X|Y] = \varphi(Y) = 2Y$

Prop: $\mathbb{E}[\mathbb{E}[Y|X]] = \mathbb{E}[Y]$

Ej 2 (Se que $X_1 \sim Bin(10, p_1) \rightsquigarrow \mathbb{E}[X_1] = 10 \cdot p_1$)

$$\varphi(x_3) = \mathbb{E}[X_1 | X_3 = x_3] = (10 - x_3)\frac{p_1}{1-p_3} \to \mathbb{E}[X_1|X_3] = (10 - X_3)\frac{p_1}{1-p_3}$$

$$\mathbb{E}[X_1] = \mathbb{E}[\mathbb{E}[X_1|X_3]] = \mathbb{E}\left[(10 - X_3)\frac{p_1}{1-p_3}\right] =$$

$$= (10 - \mathbb{E}[X_3])\frac{p_1}{1-p_3} = \underbrace{(10 - 10 \cdot p_3)}_{10(\cancel{1-p_3})}\frac{p_1}{\cancel{1-p_3}} = \underline{10\,p_1}$$

$$X_3 \sim Bin(10, p_3) \to \mathbb{E}[X_3] = 10 \cdot p_3$$

Var. condicional

Ej 1. $Y|X=x \sim Ber\left(\frac{x}{10}\right)$ $\quad g = var(Y|X=x) = \left(\frac{x}{10}\right)\left(1-\frac{x}{10}\right)$

$\Rightarrow V(Y|X) = g(X) = \left(\frac{X}{10}\right)\left(1-\frac{X}{10}\right)$

Ej 2 $X_1|X_3=x_3 \sim Bin\left(10-x_3, \frac{P_1}{1-P_3}\right)$ $\quad g = var(X_1|X_3=x_3) = (10-x_3)\frac{P_1}{1-P_3}\left(\frac{1-P_1}{1-P_3}\right)$

$\rightarrow V(Y|X) = (10-X_3)\left(\frac{P_1}{1-P_3}\right)\left(\frac{1-P_1}{1-P_3}\right)$

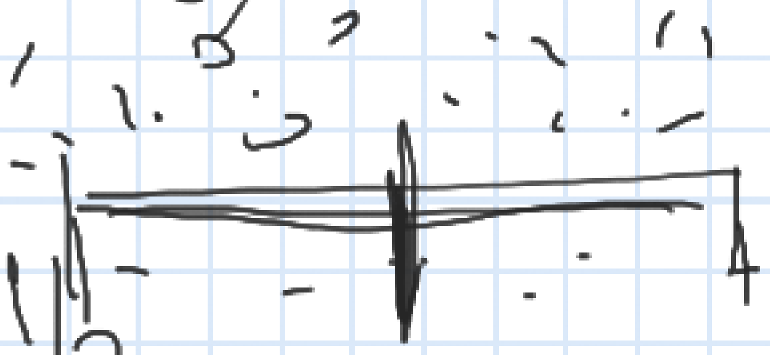
Ej 3 = $Y|X=x \sim U(0,x) \rightarrow g = var(Y|X=x) = \frac{x^2}{12}$

$\rightarrow V(Y|X) = X^2/12$

Ej 4 = $X|Y=y \sim \mathcal{E}(1/2y) \rightarrow g = var(X|Y=y) = (2y)^2$

$\rightarrow V(X|Y) = g(Y) = 2Y^2$

$$\operatorname{var}(X) = \mathbb{E}[V(X|Y)] + \operatorname{var}(\mathbb{E}[X|Y])$$



Es 4

$$Y \sim \mathcal{U}(1,3) \quad \begin{cases} \mathbb{E}[Y] = 2 \\ \operatorname{var}(Y) = \frac{(3-1)^2}{12} = 1/3 \end{cases}$$

$$\operatorname{var}(X) = \mathbb{E}[(2Y)^2] + \operatorname{var}(2Y)$$

$$= 4\mathbb{E}[Y^2] + 2^2 \operatorname{var}(Y)$$

$$= 4 \cdot \frac{13}{3} + 4 \cdot 1/3$$

$$= 4 \cdot \frac{14}{3} =$$

$$\operatorname{var}(Y) = \mathbb{E}[Y^2] - \mathbb{E}[Y]^2$$

$$\mathbb{E}[Y^2] = 1/3 + 2^2 = \frac{13}{3}$$

$\mathbb{E}[X|Y] = g(Y)$ / $\min \mathbb{E}[(X - g(Y))^2]$.