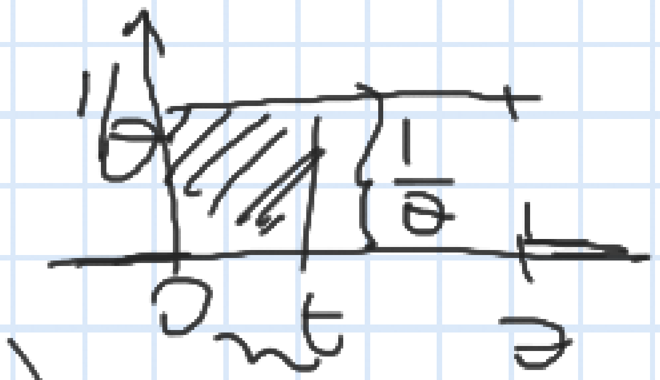
Sea (X1,X2, ..., Xn) una m.a. de una población con distribución uniforme en el intervalo (0, theta). Hallar un IC del 95% para theta.

$\theta > 0$

$X_1, \dots, X_n = \underline{X}$ $\qquad X_i \overset{iid}{\sim} \mathcal{U}(0,\theta)$ $\qquad f_\theta(x) = \frac{1}{\theta} \mathbb{I}\{0 < x < \theta\}$



$\hat{\theta} = \max(\underline{X})$

$F_{\hat{\theta}/\theta}(t) = P\left(\frac{\hat{\theta}}{\theta} \le t\right) = P\left(\frac{\max(\underline{X})}{\theta} \le t\right) = P\left(\frac{X_1}{\theta} \le t, \frac{X_2}{\theta} \le t, \dots, \frac{X_n}{\theta} \le t\right) =$

$\overset{iid}{=} P\left(\frac{X_1}{\theta} \le t\right) P\left(\frac{X_2}{\theta} \le t\right) \dots P\left(\frac{X_n}{\theta} \le t\right)$

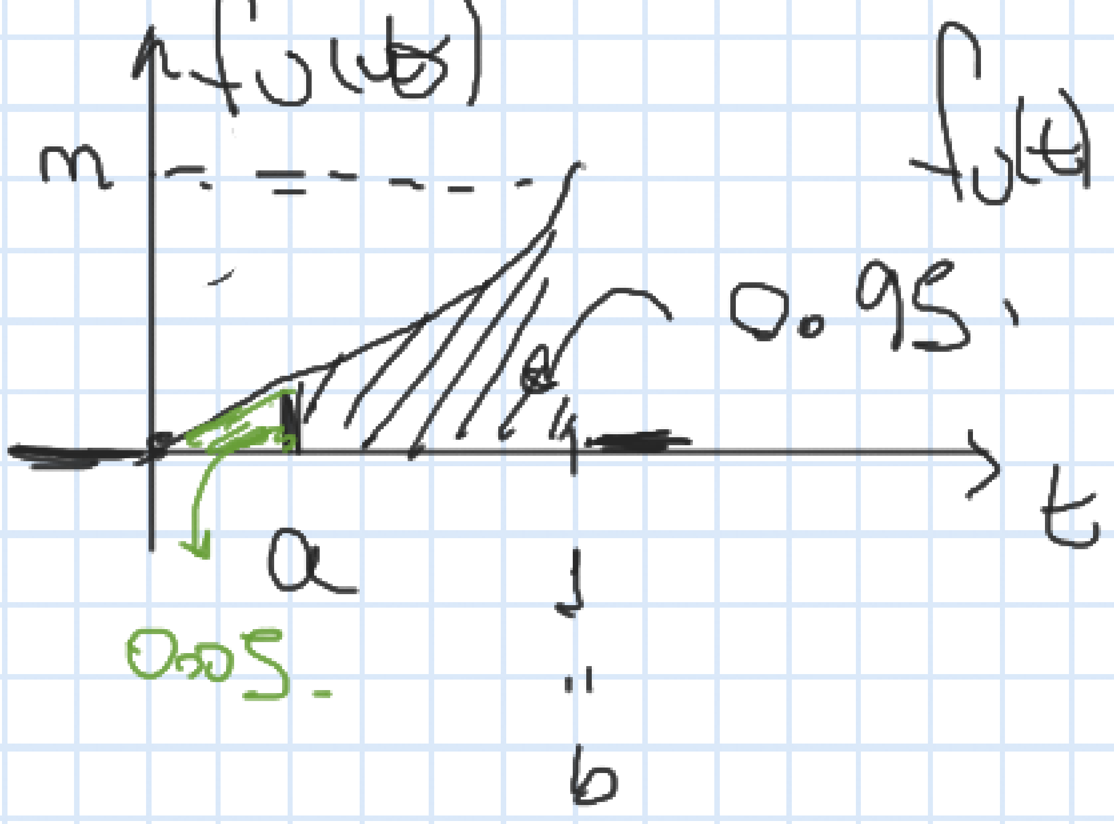
$P\left(\frac{X}{\theta} \le t\right)^n = P(X \le t\theta)^n = \begin{cases} 0 & t\theta < 0 \\ \left(\frac{t\theta}{\theta}\right)^n & 0 \le t\theta < \theta \\ 1 & \theta t \ge \theta \end{cases} = \begin{cases} 0 & t < 0 \\ t^n & 0 \le t < 1 \\ 1 & t \ge 1 \end{cases}$

$\Rightarrow U = \frac{\hat{\theta}}{\theta} = \frac{\max(\underline{X})}{\theta}$ es un pivote porque $F_{\hat{\theta}/\theta}(t)$ no depende de θ

$f_U(t) = n t^{n-1}$

$$IC_{0.95}(\underline{X}) = \{\theta : a < U < b\}$$

$$f_U(t) = n t^{n-1} \mathbb{1}\{0<t<1\}$$



$$P(a < U < b) = 0.95$$

$b = 1$

$$a : \underbrace{P(U \le a) = 0.05}$$

$$a^n = 0.05 \to a = \sqrt[n]{0.05}$$

$$IC_{0.95}(\underline{X}) = \left\{\theta : \sqrt[n]{0.05} < \frac{\max(\underline{X})}{\theta} < 1\right\} = \left\{\theta : \frac{1}{\sqrt[n]{0.05}} > \frac{\theta}{\max(X)} > 1\right\}$$

$$= \left\{\theta \in \left(\frac{\max(\underline{X})}{1}, \frac{\max(\underline{X})}{\sqrt[n]{0.05}}\right)\right\}$$

cuando tenemos las muestras $IC(\underline{x}) = \left(\frac{\max(\underline{x})}{1}, \frac{\max(\underline{x})}{\sqrt[n]{0.05}}\right)$

$\underline{X}$, $X \sim N(\mu, \sigma^2)$

$$\hat{\sigma}^2_{MV} = \frac{\sum (X_i - \bar{X})^2}{n} =$$

$$S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1}$$

es un estimador insesgado p/ la var de dist. $N(\mu, \sigma^2)$ c/ μ, σ desconocidos

Sea X1,..., Xn una m.a de una población con distribución nomal(mu, sigma^2), ambos desconocidos. Hallar un IC de nivel 0.99 para mu.

Un pivote p/ la media de la normal.

$$U = \frac{\overline{X} - \mu}{S}\sqrt{n} \sim t_{n-1} \qquad S = \sqrt{S^2}$$

$$\overline{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\mathbb{E}[\overline{X}] = \mathbb{E}\left[\frac{1}{n}\sum X_i\right] = \frac{1}{n}\sum \underbrace{\mathbb{E}[X_i]}_{\mu} = \frac{n}{n}\mu = \mu$$

Si n es grande

$$\Rightarrow \frac{\overline{X} - \mu}{S}\sqrt{n} \overset{a}{\sim} N(0,1)$$

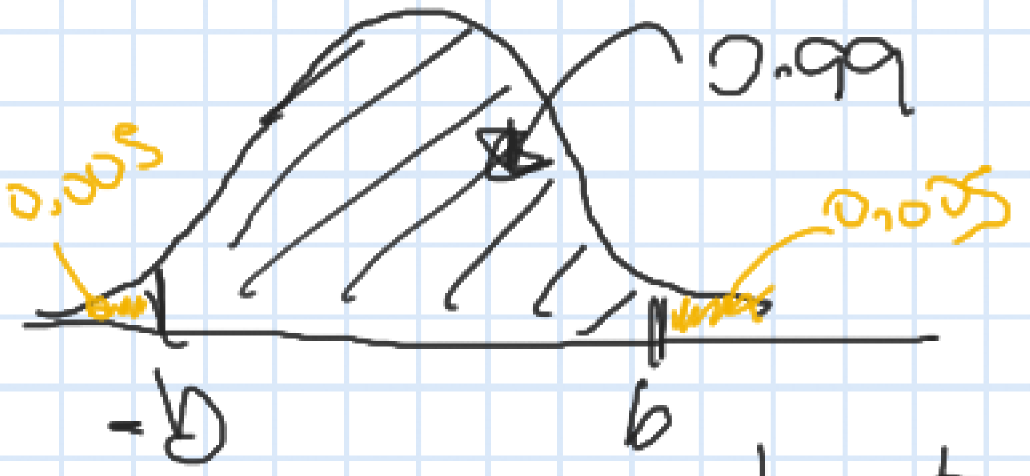
$$\operatorname{var}(\overline{X}) = \operatorname{var}\left(\frac{1}{n}\sum X_i\right) = \frac{1}{n^2}\operatorname{var}\left(\sum X_i\right)$$

$$\overset{ind}{=} \frac{1}{n^2}\sum \underbrace{\operatorname{var}(X_i)}_{\sigma^2} = \frac{n\sigma^2}{n^2} = \frac{\sigma^2}{n}$$

$a, b \,/\, \mathbb{P}(a < U < b) = 0{,}99$

(busco un intervalo simétrico)

$\mathbb{P}(-b < U < b) = 0{,}99$



$\hookrightarrow b = t_{n-1;\,0{,}995}$

$= 2{,}7564$

$\downarrow$

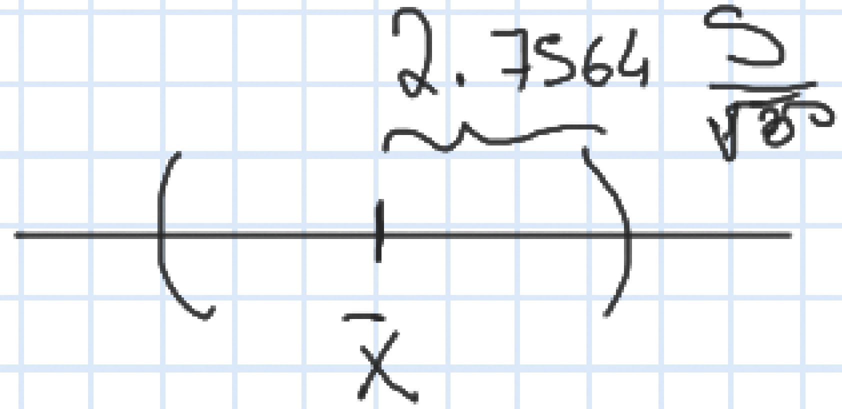
$t_{n-1;\,0{,}005}$

$$-2{,}7564 < \frac{\bar{X} - \mu}{S}\sqrt{30} < 2{,}7564$$

$$-2{,}7564\frac{S}{\sqrt{30}} - \bar{X} < -\mu < 2{,}7564\frac{S}{\sqrt{30}} - \bar{X}$$

$$2{,}7564\frac{S}{\sqrt{30}} + \bar{X} > \mu > -2{,}7564\frac{S}{\sqrt{30}} + \bar{X}$$

$$IC_{0{,}99}(\underline{x}) : \bar{X} \pm 2{,}7564\frac{S}{\sqrt{30}}$$



Sea X1,..., Xn una m.a de una población con distribución nomal(mu, sigma^2), ambos desconocidos. Hallar un IC de nivel 0.99 para var.

Sabemos que $W = \dfrac{\sum (X_i - \bar{X})^2}{\sigma} \sim \chi^2_{n-1}$ es un pivote p/ σ^2

$$\sum (X_i - \bar{X})^2 = \sum (X_i^2 + \bar{X}^2 - 2X_i\bar{X})$$
$$= \sum X_i^2 + n\bar{X}^2 - 2\bar{X}\underbrace{\sum X_i}_{n\bar{X}}$$
$$= \sum X_i^2 - n\bar{X}^2$$

$$P(a < W < b) = 0.99$$

$a = \chi_{n-1;\,0.005}$

$b = \chi_{n-1;\,0.995}$

$$\chi_{n-1;\,0.005} < \frac{\sum (X_i - \bar{X})^2}{\sigma} < \chi_{n-1;\,0.995}$$

$$IC(\underline{X}) = \left(\frac{\sum (X_i - \bar{X})^2}{\chi_{n-1;\,0.995}} , \frac{\sum (X_i - \bar{X})^2}{\chi_{n-1;\,0.005}} \right) \to IC(\underline{x}) = (\quad , \quad)$$

Se arroja 50 veces una una moneda con probabilidad p de salir cara. Hallar una cota superior asintotica de nivel 0.95 para p basado en la observacion x = 35

X = "# de caras en 50 tiros" $X \sim Bin(50, p)$

$X = \sum_{i=1}^{50} Y_i$ $Y = \begin{cases} 1 & \text{si sale cara} \\ 0 & \text{si sale ceca} \end{cases}$

Podemos aprox por TCL $Y \sim Ber(p)$

Suma de casos iid y $E[Y] = p$ $var(Y) = p(1-p)$

$$P(X \leq x) = P\Bigg(\underbrace{\frac{X - 50p}{\sqrt{50\hat{p}(1-\hat{p})}}}_{\tilde{Z}} \leq \frac{x - 50p}{\sqrt{50\hat{p}(1-\hat{p})}}\Bigg) \approx \Phi\left(\frac{x - 50p}{\sqrt{50\hat{p}(1-\hat{p})}}\right)$$

func. de dist de una normal estándar.

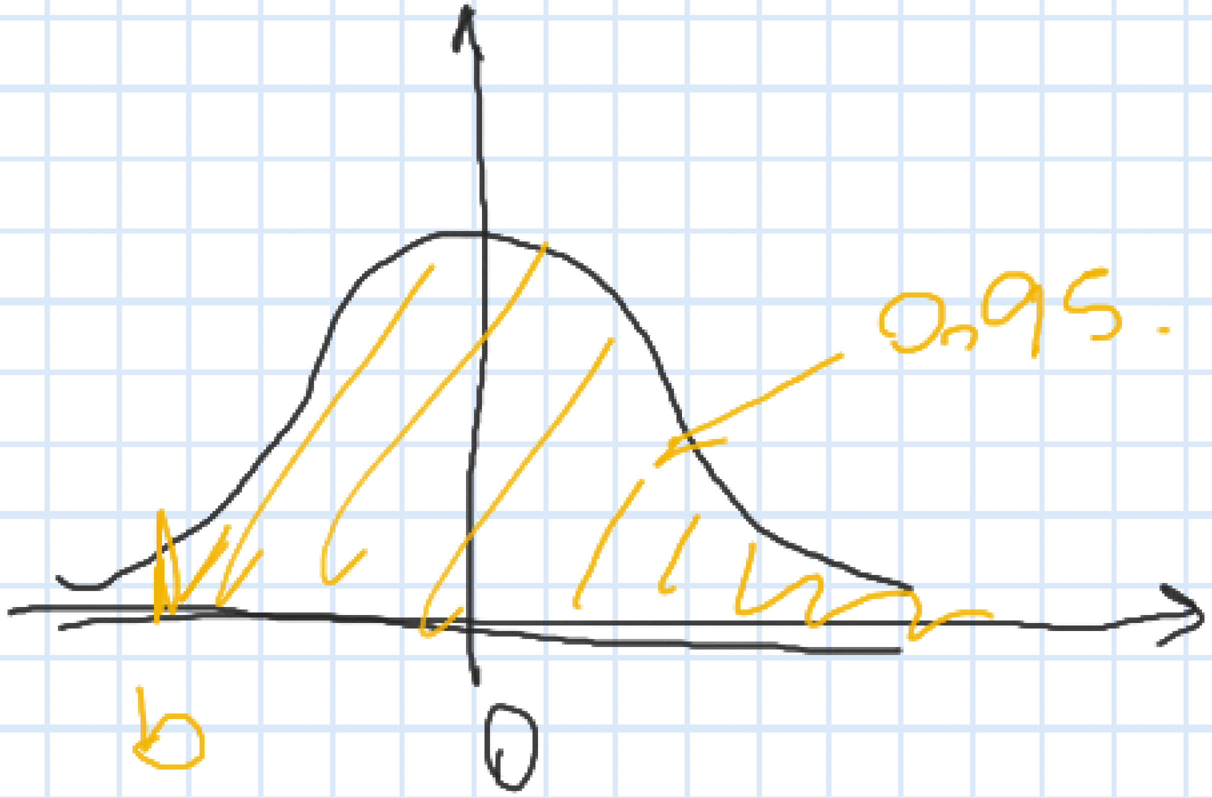
$\hat{p} = \frac{X}{50} = \bar{Y}$ = Promedio de los Yes

$\tilde{Z}$ es un pivote (asintótico) para p

$\tilde{Z}$ → T. Slutzky $\frac{X - 5p}{\sqrt{50\hat{p}(1-\hat{p})}} \xrightarrow{P} N(0,1)$

$\mathbb{P}(Z > b) = 0.95$

$p < b(x)$



$b = z_{0.05}$

$\frac{X - 50p}{\sqrt{50\hat{p}(1-\hat{p})}} > z_{0.05}$

$\frac{\frac{1}{50}(X - 50p)}{\frac{1}{50}\left(\sqrt{50\bar{x}(1-\bar{x})}\right)} > z_{0.05}$

$\frac{\bar{X} - p}{\sqrt{\bar{x}(1-\bar{x})}}\sqrt{50} > z_{0.05}$

$p < \bar{X} - z_{0.05}\frac{\sqrt{\bar{x}(1-\bar{x})}}{\sqrt{50}}$

$\rightarrow CS(x) = \left\{ p < \underbrace{\frac{35}{50} - (-1.6446)\frac{\sqrt{\frac{35}{50}\cdot\frac{15}{50}}}{\sqrt{50}}} \right\}$

0.994

0.394

$z_{0.05} = -1.6446$

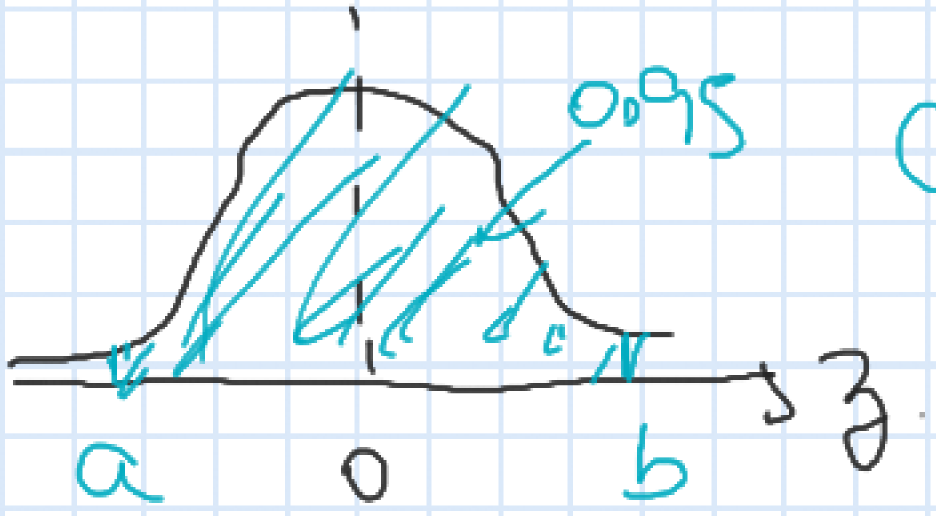
De un experimento en los efectos de un medicamento ara la ansiedad se midió el puntaje de un test de memoria antes y después de tomar el medicamento. A partir de los datos que se encuentran en el archivo Islander_data.csv, hallar un IC para la media del tiempo de respuesta despunes de consumir el medicamento.

X: "tiempo de rto del paciente después del medicamento"

↳ $\hat{\mu} = \bar{X}$

$$Z = \frac{\bar{X} - \mu}{S}\sqrt{n} \overset{(a)}{\sim} N(0,1) \qquad S^2 = \frac{\sum (X_i - \bar{X})^2}{n-1}$$

$$P(a < Z < b) = 0.95$$



$a = -b$

↳ $b = z_{0.975} = 1.96$

$a = z_{0.025} = -1.96$

$$-1.96 < \frac{\bar{X} - \mu}{S}\sqrt{n} < 1.96$$

$$\mu \in \bar{X} \pm 1.96 \frac{S}{\sqrt{n}}$$