2. Sean $X \sim \mathcal{P}(\lambda)$ y $Y \sim \mathcal{P}(\mu)$ dos variables independientes. Hallar la distribución de $X|(X+Y)=m$. *Sugerencia: usar el resultado del ejercicio 5. de Transformaciones de variables.*

$$M = X+Y \Rightarrow X|M=m \sim Bin\left(m, \frac{\lambda}{\lambda+\mu}\right)$$

$$\varphi(m) = \mathbb{E}[X|M=m] = m\frac{\lambda}{\lambda+\mu} \Rightarrow \mathbb{E}[X|M] = M\frac{\lambda}{\lambda+\mu}$$

α

$\parallel$

recta de reg.

$$cov(X,M) = cov(X,X+Y)$$

$$= \underbrace{cov(X,X)}_{} + cov(X,Y)$$

$$= var(X) + 0$$

$$cov(X,Y) = \mathbb{E}[X \cdot Y] - \mathbb{E}[X]\mathbb{E}[Y].$$

$$= \mathbb{E}[\underbrace{\mathbb{E}[X \cdot Y|Y]}_{Y\,\mathbb{E}[X|Y]}] - \mathbb{E}[X]\mathbb{E}[Y]$$

Cant. de accidentes	0	1	2	3	4	5
Frecuencia	10	29	25	17	13	6

Dada una muestra de tamaño n de una población X tal que $X|M = \mu \sim \mathcal{N}(\mu, \sigma_X^2)$, hallar el estimator MAP de M si se considera a priori que $M \sim \mathcal{N}(\mu_0, \sigma_\mu^2)$.

$\sigma_X^2 \to \text{var}(X \mid M = \mu)$

$\sigma_\mu^2 \to \text{var}(M)$

$\Theta = \{ \frac{2}{5}, \frac{4}{5} \}$

$P = \{ 2/5 ; 4/5 \}$

$p = 2/5$

$p = 4/5$

$$L(p) = \prod_{i=1}^{n} p^{x_i} (1-p)^{1-x_i}$$

Se realiza un estudio para estimar la proporción de residentes en una ciudad y en sus suburbios que están a favor de la construcción de una planta de energía nuclear. Para ello, se entrevistaron 100 personas, de las cuales 58 dijeron estar a favor. Hallar el estimador Bayesiano correspondiente al error cuadrático para la proporción de residentes a favor de construir la planta de energía nuclear. Suponer una distribucón a priori Beta(2,5) para el parámetro. ¿Qué ocurre si considerábamos una Beta(5,2)? Analizar que significa cada una de esas dstribuciones a priori.

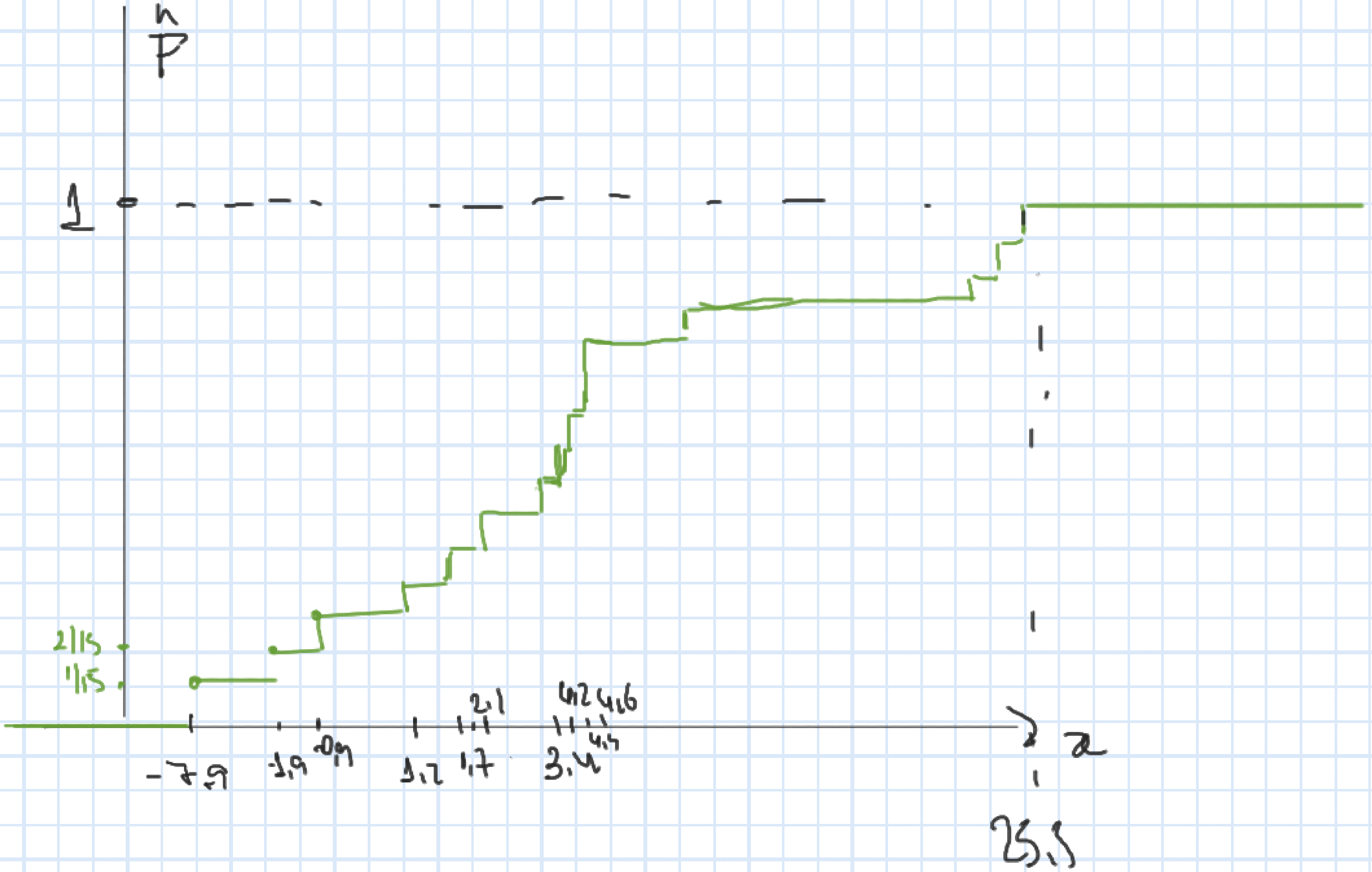
$$X = \begin{cases} 1 & \text{a favor} \\ 0 & \text{en contra} \end{cases}$$

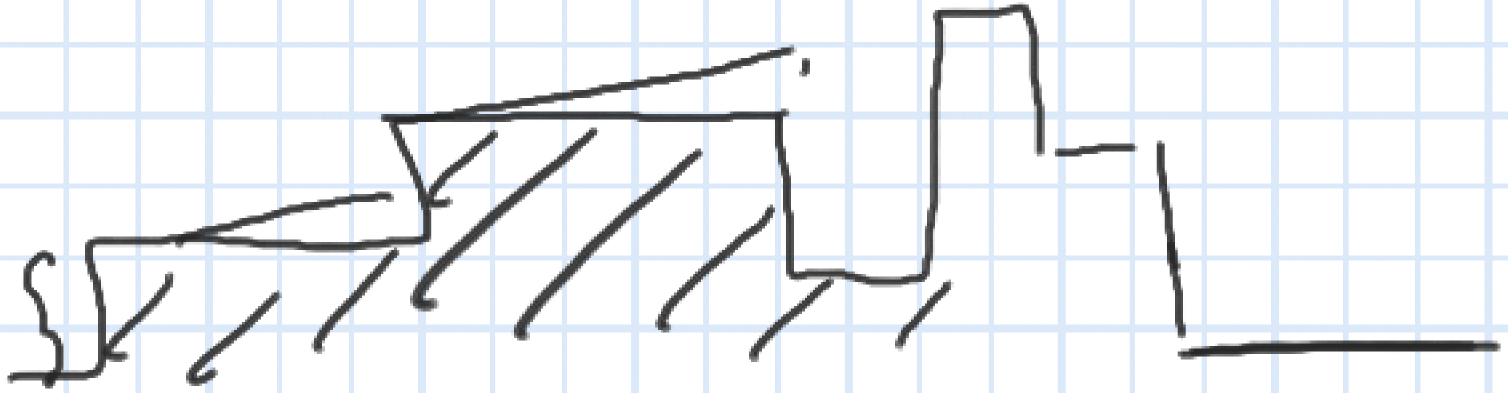
$\hookrightarrow X \sim Ber(p)$

$$\propto \theta^{\overset{\alpha-1}{k_1}}(1-\theta)^{\overset{\beta-1}{k_2}} \qquad \theta \in (0,1)$$

$$\Rightarrow \alpha = k_1 + 1 \qquad \beta = k_2 + 1$$

Es 1. 1.2, 4.6, 4.3, 4.2, -7.9, 7.8, 3.4, 19.8, 25.5, -1.9, 2.1, -0.9, 4.6, 21.1, 1.7

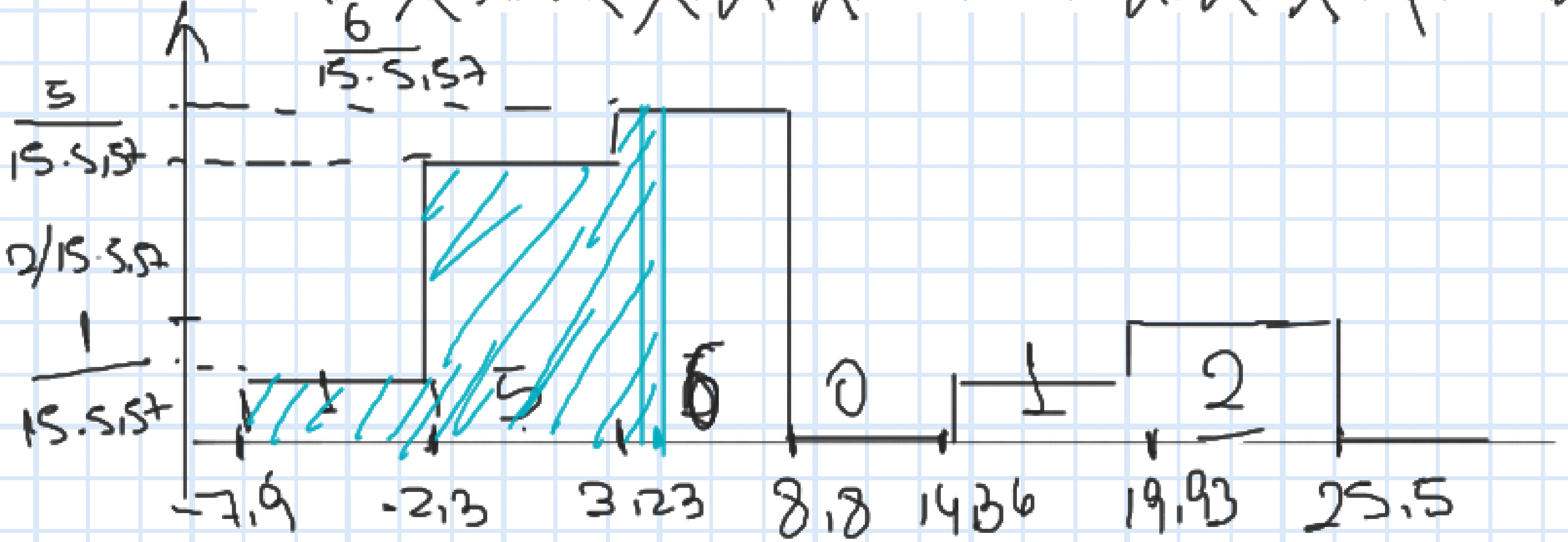




$$\frac{\#B_i}{mh}$$

$$\sum \cancel{h} \cdot \frac{\#B_i}{m \cdot \cancel{h}} = \frac{\sum \#B_i}{m} \quad m \quad = 1.$$

~~1.2~~, ~~4.6~~, ~~4.3~~, ~~4.2~~, ~~−7.9~~, ~~7.8~~, ~~3.4~~, 19.8, 25.5, ~~−1.9~~, ~~2.1~~, ~~−0.9~~, ~~4.6~~, 21.1, ~~1.7~~



$R = 5,57$

$P(X \leq 3,5)$

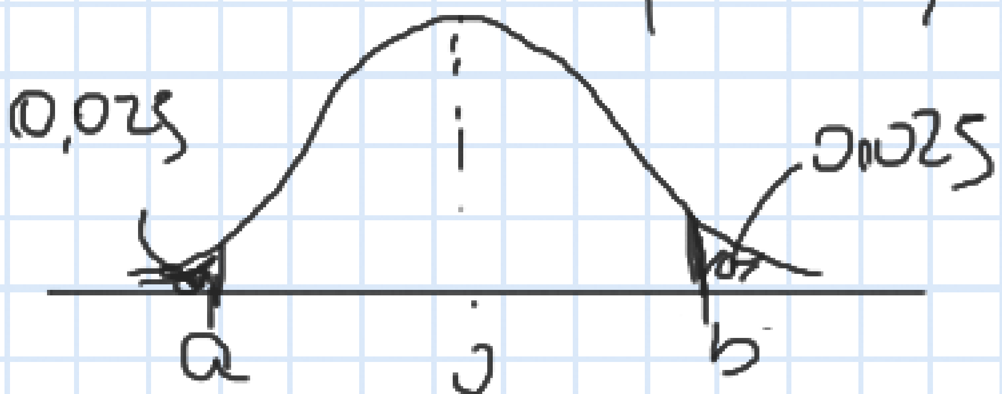
$\underline{X}$, $X \sim N(\mu, 1)$ $\qquad \hat{\mu} = \bar{X} = \frac{1}{n} \sum X_i$ estad. suficiente

$\mathbb{E}[\bar{X}] = \mu \qquad var(\bar{X}) = \frac{var(X)}{n} \qquad \bar{X} \sim N(\mu, 1/n)$

$U = \dfrac{\bar{X} - \mu}{1/\sqrt{n}} \sim N(0, 1)$ es pivote para μ

Hallar un IC de nivel 0.95 para μ

$\mathbb{P}(a < U < b) = 0.95$



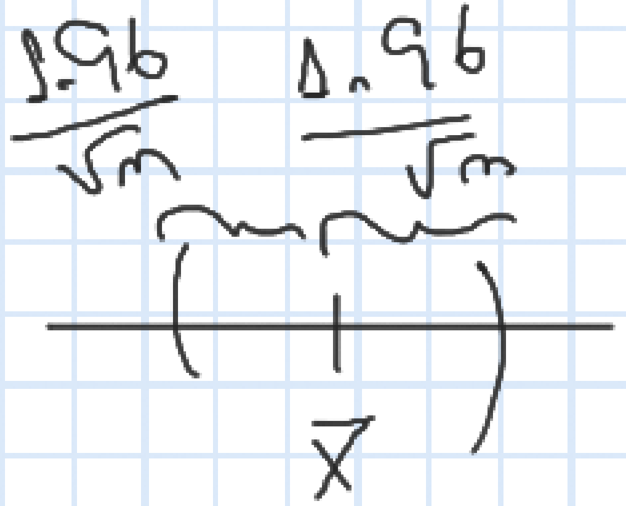
$\Rightarrow a = z_{0.025} = -b$

$b = z_{0.975}$

$$b = 1{,}96 \rightarrow a = -1{,}96$$

$$IC(\underline{X}) = \left\{ \mu : -1.96 < (\overline{X} - \mu)\sqrt{n} < 1.96 \right\}$$

$$-\frac{1.96}{\sqrt{n}} - \overline{x} < -\mu < \frac{1.96}{\sqrt{n}} - \overline{x}$$



$$\frac{1.96}{\sqrt{n}} + \overline{X} > \mu > -\frac{1.96}{\sqrt{n}} + \overline{X}$$

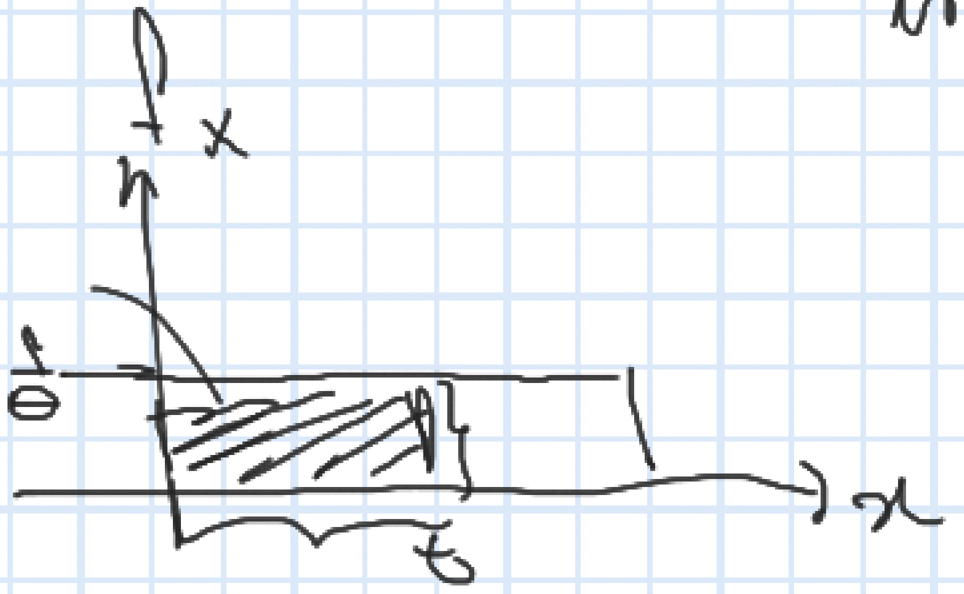
$$IC(\underline{X}) = \left(-\frac{1.96}{\sqrt{n}} + \overline{x},\ \frac{1.96}{\sqrt{n}} + \overline{x} \right) = \overline{X} \pm \frac{1.96}{\sqrt{n}}$$

Ejercicio 4

$\underline{X} = (X_1 \dots X_n)$ $\qquad X_i \overset{iid}{\sim} \mathcal{U}(0,\theta)$ $\qquad$ Cota inferior del 95%

$$\mathbb{P}\left(\frac{\max(\underline{X})}{\theta} \le t\right) = \mathbb{P}\left(\frac{X_1}{\theta} \le t, \frac{X_2}{\theta} \le t, \dots, \frac{X_n}{\theta} \le t\right) =$$

$$\overset{indep}{=} \mathbb{P}(X_1 \le t\theta) \cdots \mathbb{P}(X_n \le t\theta) \underset{i.i.d}{=} \mathbb{P}(X_1 \le t\theta)^n$$



$$= \begin{cases} 0 & t\theta < 0 \\ \left(\frac{t\theta}{\theta}\right)^n & 0 \le t\theta < \theta \\ 1 & t\theta \ge \theta \end{cases}$$

$$\Rightarrow U = \frac{\max(\underline{X})}{\theta} \text{ es un pivote} \qquad F_U(u) = \begin{cases} 0 & u \le 0 \\ u^n & 0 \le u < 1 \\ 1 & u \ge 1. \end{cases}$$

$$\mathbb{P}(U \leq \underbrace{a}_{u_{0.95}}) = 0.95 \qquad \theta \geq a(\cancel{\underline{X}})$$

$$\frac{\max(\underline{X})}{\theta} < a$$

$$0.95 = a^n \rightarrow a = \sqrt[n]{0.95}$$

$$\frac{\max(\underline{X})}{\theta} < \sqrt[n]{0.95}$$

$$\frac{\max(\underline{X})}{\sqrt[n]{0.95}} < \theta \rightarrow \text{cota inferior}$$

$$\text{si } n = 20 \quad \boxed{\theta > \frac{\max(\underline{X})}{0.997}}$$

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n-1}$$

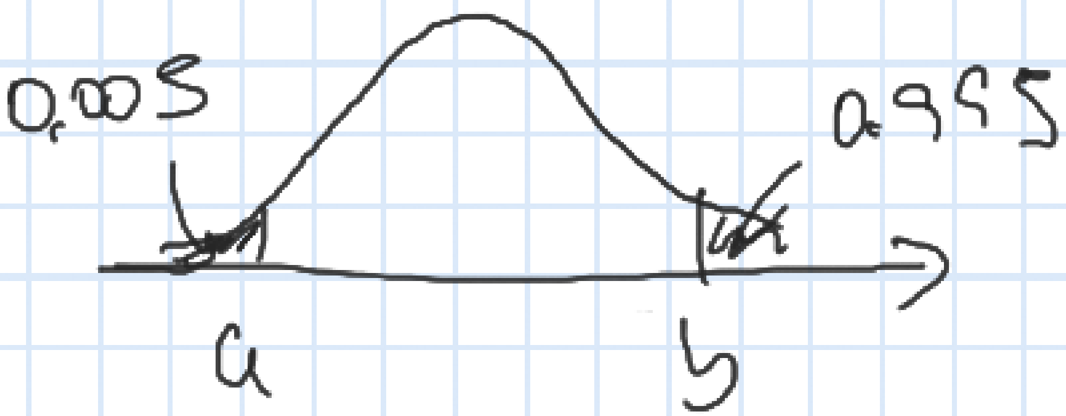
Ejercicio 5

IC para μ de nivel $0{,}99$

$\underline{X} = (X_1, \dots, X_n)$ $\quad X_i \overset{iid}{\sim} N(\mu, \sigma^2)$

$$U = \frac{\bar{X} - \mu}{S}\sqrt{n} \sim t_{n-1} \qquad S^2 = \frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}$$

$$P(a < U < b) = 0{,}99$$



μ: $-t_{n-1,0.995} < \frac{\bar{X} - \mu}{S}\sqrt{n} < t_{n-1,0.995}$

$a = -b$

$b = t_{n-1,0.995}$

$$IC(\underline{X}) = \bar{X} \pm t_{n-1,0.995} \frac{S}{\sqrt{n}}$$