Sean X, Y dos v.a. con función de densidad conjunta

$$f_{X,Y}(x,y) = \frac{e^{-x/2y}}{4y}\mathbf{1}\{0 < x, 1 < y < 3\}$$

Hallar la función de densidad de X|Y=y

$$\lambda e^{-\lambda x} \mathbb{1}\{x \geq 0\}.$$

$$f_{X,Y}(x,0) = f_{X|Y=y}(x)\ f_Y(y)$$

$$f_{X,Y}(x,y) = \frac{e^{-\frac{x}{2y}}}{4y}\ \underbrace{\mathbb{1}\{0<x,\ 1<y<3\}}_{\mathbb{1}\{x\geq 0\}\ \mathbb{1}\{1<y<3\}}$$

$$\mathbb{1}\{0<x<y<3\}$$

$$\mathbb{1}\{0<x<3\}\ \underbrace{\mathbb{1}\{x<y<3\}}$$

$$= \underbrace{\frac{1}{2y} e^{-\frac{x}{2y}}\ \mathbb{1}\{x\geq 0\}}_{X|Y=y \sim \mathcal{E}\left(\frac{1}{2y}\right)}\ \underbrace{\frac{1}{2}\ \mathbb{1}\{1<y<3\}}_{Y \sim \mathcal{U}(1,3)}.$$

Ej 5 (Clase 2) T = "tiempo de viaje"

$$M = \begin{cases} 1 & \text{tren} \\ 0 & \text{subte} \end{cases} \qquad \begin{array}{l} \mathbb{P}(M=1) = 0{,}6 \\ \mathbb{P}(M=0) = 0{,}4 \end{array}$$

$$T|_{M=1} \sim \mathcal{U}(0{,}8\, ;\, 1.25) \qquad T|_{M=0} \sim \mathcal{U}(0.75\, ;\, 1)$$

$$f_T(t) = \mathbb{P}(M=1)\frac{1}{\underbrace{1.25-0.8}_{0{,}45}}\mathbb{1}\{0{,}8 < x < 1{,}25\} + \mathbb{P}(M=0)\frac{1}{1-0.75}\mathbb{1}\{0{,}75 < x < 1\}$$

$$\mathbb{P}(\overbrace{M=1}^{\text{tren}} \mid T > 0.9) \overset{\text{Bayes}}{=} \frac{\mathbb{P}(T > 0{,}9 \mid M=1)\,\mathbb{P}(M=1)}{\mathbb{P}(T > 0{,}9 \mid M=1)\,\mathbb{P}(M=1) + \mathbb{P}(T > 0{,}9 \mid M=0)\,\mathbb{P}(M=0)} = \frac{\frac{1.25-0.9}{0{,}45} \cdot 0{,}6}{\frac{1{,}25-0{,}9}{0{,}45}0{,}6 + \frac{1-0{,}9}{0{,}25}0{,}4}$$

$$= 0{,}74$$

$$\mathbb{P}(M=1 \mid T=0{,}9) = \frac{f_{T|M=1}(0{,}9)\,\mathbb{P}(M=1)}{f_{T|M=1}(0{,}9)\,\mathbb{P}(M=1)+f_{T|M=0}(0{,}9)\,\mathbb{P}(M=0)} = \frac{1/0{,}45 \cdot 0{,}6}{1/0{,}45 \cdot 0{,}6 + 1/0{,}25 \cdot 0{,}1}$$

$$= 5/11 = 0{,}\widehat{45}$$

Ej. 1 (clase 3) $X \sim Bin(10, 1/3)$ $Y \sim Ber(1/3)$

\# de aciertos en últimos 9 tiros

1. $X \mid Y = y$:

\# aciertos en el 1º tiro $\quad W \sim Bin(9, 1/3)$

\# aciertos en 10 tiros

$$p_{X|Y=0}(x) = p_W(x) \quad x = 0, \dots, 9 \qquad X \mid Y=0 \sim Bin(9, 1/3)$$

$$p_{X|Y=1}(x) = p_W(x-1) \quad x = 1, \dots, 10 \qquad X \mid Y=1 = W+1$$

$$\mathbb{P}(W+1 = x) = \mathbb{P}(W = x-1) = p_W(x-1)$$

$$\mathbb{E}[X \mid Y=0] = 9/3 = 3$$

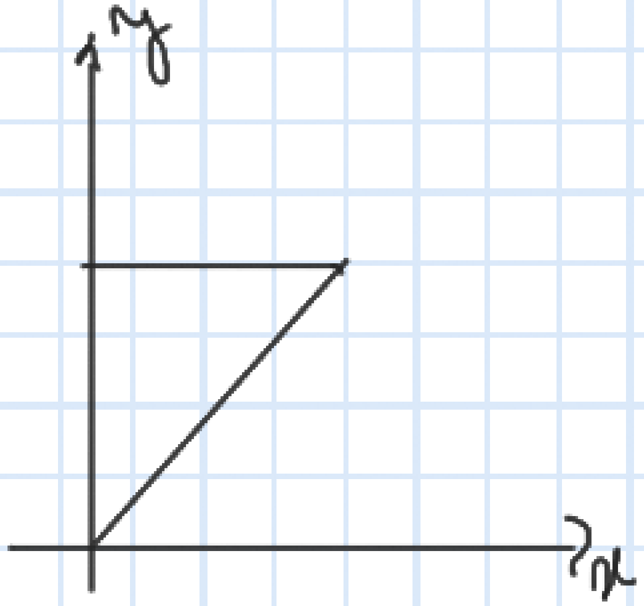
$$\mathbb{E}[X \mid Y=1] = \mathbb{E}[W+1] = \mathbb{E}[W] + 1 = 9/3 + 1 = 4$$

$$\varphi(y) = \begin{cases} 3 & \text{si } y = 0 \\ 4 & \text{si } y = 1. \end{cases}$$

$$\Rightarrow \mathbb{E}[X \mid Y] = \varphi(Y) = \begin{cases} 3 & \text{si } Y = 0 \\ 4 & \text{si } Y = 1 \end{cases}$$

$Y|X=x \sim \mathrm{Ber}(x/10) \longrightarrow \varphi(x) = \mathbb{E}[Y|X=x] = x/10$

$\mathbb{E}[Y|X] = \varphi(X) = X/10$

2. 

$Y|X=x \sim \mathcal{U}(x, 2)$

$\varphi(x) = \mathbb{E}[Y|X=x] = \dfrac{2+x}{2}$

$\mathbb{E}[Y|X] = \varphi(X) = 1 + X/2$

3. $X|Y=y \sim \mathcal{E}(1/2y)$

$\varphi(x) = \mathbb{E}[Y|X=x] = 2y$

$\mathbb{E}[Y|X] = 2Y$

$\mathbb{E}[\underbrace{X^2}_{r(x)} \cdot \underbrace{3Y}_{s(Y)} | X] = \varphi(X) = \underbrace{X^2}_{r(X)} \mathbb{E}[\underbrace{3Y}_{s(Y)} | X].$

$\varphi(x) = \mathbb{E}[X^2 \cdot 3Y | X=x] = \mathbb{E}[x^2\, 3Y | X=x]$

$= x^3\, \mathbb{E}[3Y | X=x]$

Ej 3 (Clase 3)

$\underline{X} = (X_1, \dots, X_n)$ $\quad X_i \sim N(\mu, 9)$ $\quad \hat{\mu} = \delta(\underline{X}) = \frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}$

$$ECM = \mathbb{E}\left[(\bar{X}-\mu)^2\right] = \mathbb{E}\left[\bar{X}^2 + \mu^2 - 2\mu\bar{X}\right] = \mathbb{E}[\bar{X}^2] + \mu^2 - 2\mu\underbrace{\mathbb{E}[\bar{X}]}_{\mu}$$

$\bar{X} \sim N\left(\mu, \frac{9}{n}\right)$

$$= \frac{9}{n} + \mu^2 + \mu^2 - 2\mu^2 = \frac{9}{n}$$

$$\mathbb{E}[\bar{X}] = \mathbb{E}\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right] = \frac{1}{n}\sum_{i=1}^{n}\overbrace{\mathbb{E}[X_i]}^{\mu} = \mu$$

$$var(\bar{X}) = var\left(\frac{1}{n}\sum X_i\right) = \frac{1}{n^2} var\left(\sum X_i\right) \underset{\text{indep}}{=} \frac{1}{n^2}\sum \overbrace{var(X_i)}^{9} = \frac{9}{n}$$

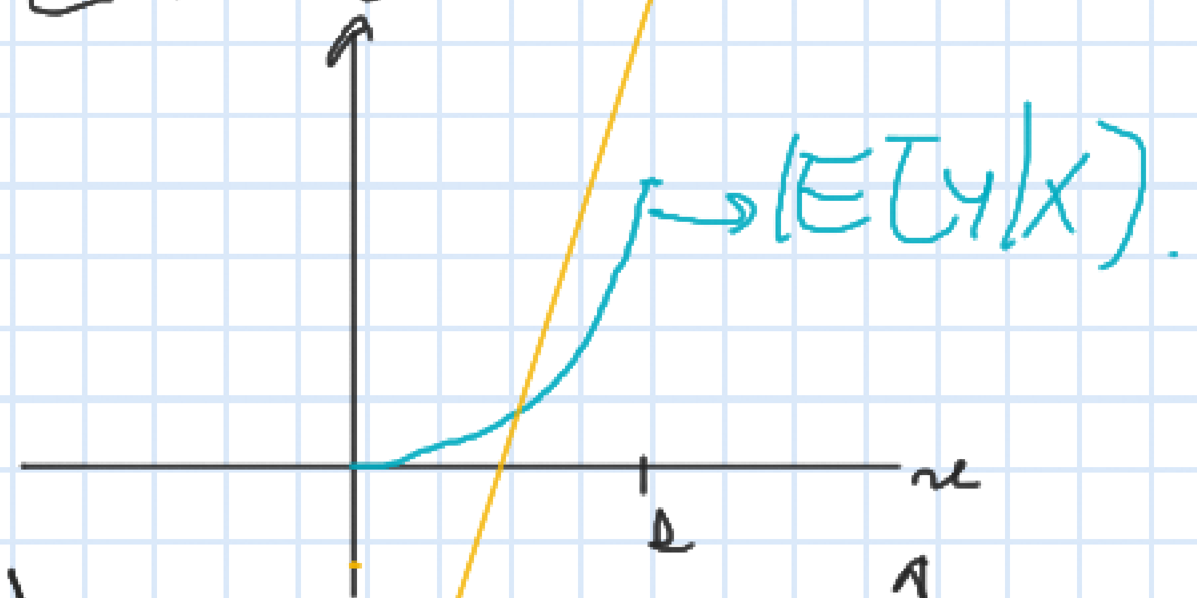
$$var(\bar{X}) = \mathbb{E}[\bar{X}^2] - \mathbb{E}[\bar{X}]^2 \Rightarrow \mathbb{E}[\bar{X}^2] = var(\bar{X}) + \mathbb{E}[\bar{X}]^2$$

$$\hat{X} = \frac{\mathrm{Cov}(X,Y)}{\mathrm{Var}(X)} \left(Y - \mathbb{E}(Y) \right) + \mathbb{E}(X)$$

Ej 4 (Clase 3) $f_X(x) = 1 \, \mathbb{1}\{0<x<1\}$

$Y = X^2$ $X \sim \mathcal{U}(0,1)$

$\mathbb{E}[Y|X] = X^2$



$$\hat{Y} = \frac{cov(X,Y)}{var(X)}(X - \mathbb{E}[X]) + \mathbb{E}[Y]$$

$$cov(X,Y) = cov(X,X^2) = \mathbb{E}[X^3] = \int_0^1 x^3 \cdot 1\,dx = \frac{x^4}{4}\Big|_0^1 = 1/4$$

$var(X) = 1/12$ $\mathbb{E}[X] = 1/2$ $\mathbb{E}[Y] = \mathbb{E}[X^2] = \frac{1}{12} + \left(\frac{1}{2}\right)^2 = 1/3$

$$\hat{Y} = 3(X - 1/2) + 1/3 = 3X - 7/6$$

(x_1, y_1)

(x_2, y_2)

$\vdots$

$$y_1 = ax_1 + b$$
$$y_2 = ax_2 + b$$
$$\vdots$$
$$y_m = ax_m + b$$

$$\rightsquigarrow \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} = \underbrace{\begin{bmatrix} x_1 & 1 \\ x_2 & 1 \\ x_3 & 1 \\ \vdots & \\ x_n & 1 \end{bmatrix}}_{A} \begin{bmatrix} a \\ b \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \end{bmatrix} = (A^T A)^{-1} A^T \begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix}$$