Se arrojan dos dados de 6 caras equilibrados. Hallar la esperanza de la suma de los resultados.

D_1: "resultado del dado 1"
D_2: " " " " 2"

Esp equip

$d_2 \backslash d_1$	1	2	3	4	5	6	P_{D_2}
1	1/36	1/36	1/36	1/36	1/36	1/36	1/6
2	1/36	1/36	1/36	1/36	1/36	1/36	1/6
3	1/36	1/36	1/36	1/36	1/36	1/36	1/6
4	1/36	1/36	1/36	1/36	1/36	1/36	1/6
5	1/36	1/36	1/36	1/36	1/36	1/36	1/6
6	1/36	1/36	1/36	1/36	1/36	1/36	1/6
P_{D_1}	1/6	1/6	1/6	1/6	1/6	1/6	

$$P(D_1 = d_1, D_2 = d_2) = P_{D_1,D_2}(d_1,d_2) = \frac{1}{36} \quad \forall \{d_1, d_2\} \in \{1,\dots,6\}$$

$$P_{D_1}(d_1) = \sum_{d_2} P_{D_1,D_2}(d_1,d_2)$$

$X = D_1 + D_2$ X: "suma de ambos resultados"

$$\mathbb{E}[X] = \sum_{d_1=1}^{6}\sum_{d_2=1}^{6} \underbrace{g(d_1,d_2)}_{d_1+d_2} P_{D_1,D_2}(d_1,d_2) = \mathbb{E}[D_1+D_2] = \mathbb{E}[D_1] + \mathbb{E}[D_2]$$

$= 3,5 + 3,5$

Obs: $\mathbb{E}[X]$ va a estar dentro del rango de la v.a.

Hallar la esperanza de la suma
sabiendo que D1>3

$$\mathbb{E}[X \mid D_1 > 3] = \frac{\mathbb{E}[X \mathbb{1}\{D_1 > 3\}]}{\mathbb{P}(D_1 > 3)}$$

$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(B)}.$$

$$\mathbb{E}[X \mid X \in A] = \frac{\mathbb{E}[X \mathbb{1}\{X \in A\}]}{\mathbb{P}(A)}$$

$$I\{X \in A\} = \mathbb{1}\{X \in A\} = \begin{cases} 1 & X \in A \\ 0 & \text{c.c.} \end{cases}$$

$$P_{D_1, D_2 \mid D_1 > 3}(d_1, d_2) = \frac{1}{18} \mathbb{1}\left\{ \begin{array}{l} d_1 \in \{4,5,6\} \\ d_2 \in \{1, \dots, 6\} \end{array} \right\}$$

$$P_{D_1, D_2 \mid D_1 > 3}(d_1, d_2) = \mathbb{P}(D_1 = d_1, D_2 = d_2 \mid D_1 > 3)$$

$$= \frac{\mathbb{P}(D_1 = d_1, D_2 = d_2, D_1 > 3)}{\mathbb{P}(D_1 > 3)}$$

Sea R el radio de un círculo, tal que R ~U(3,4).
Hallar la esperanza de área del círculo

A: "área del círculo"

$R \sim U(3,4) \rightarrow f_R(r) = \frac{1}{4-3} I_{\{3<r<4\}}$

$A = \pi R^2$

$\mathbb{E}[A] = \mathbb{E}[\pi R^2] = \pi\, \mathbb{E}[R^2] = \pi \int_{\mathbb{R}} g(r)\, f_R(r)\, dr = \pi \int_3^4 r^2 \cdot 1 \, dr$

$g(r) = r^2$

$R^2 = g(R)$

$= \pi \left(\frac{r^3}{3} \Big|_3^4 \right) = \pi \left(\frac{4^3 - 3^3}{3} \right)$

$= 12{,}3\,\pi$

?

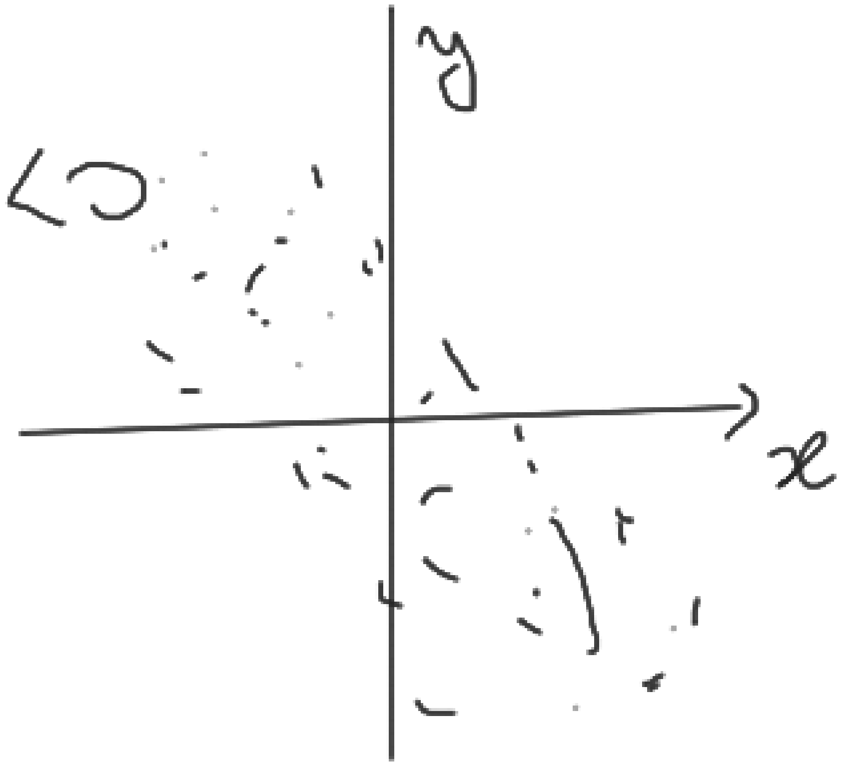
$\text{var}(R) = \mathbb{E}[R^2] - \mathbb{E}[R]^2$

$\hookrightarrow \mathbb{E}[R^2] = \text{var}(R) + \mathbb{E}[R]^2$

$\frac{(4-3)^2}{12} + \left(\frac{(4+3)}{2}\right)^2 = \frac{1}{12} + (3.5)^2 \Rightarrow \mathbb{E}[R^2] = 12.3$

$Cov(X,Y) > 0$

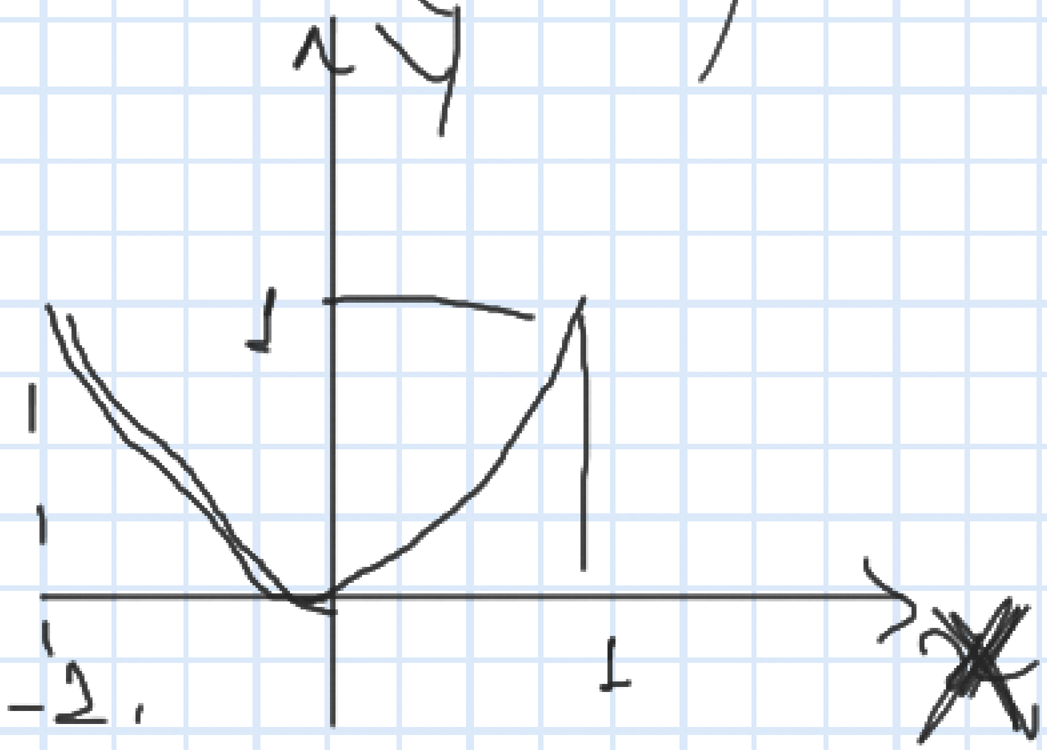
$Cov(X,Y) < 0$



$$\rho = \frac{Cov(X,Y)}{\sqrt{var(X)}\sqrt{var(Y)}}$$

$-1 < \rho < 1$

$X \sim \mathcal{U}(-1,1)$ $\qquad$ $Y = X^2$



$$\text{Cov}(X,Y) = \text{Cov}(X, X^2)$$

$$= \mathbb{E}[\underbrace{X \cdot X^2}_{X^3}] - \cancel{\underbrace{\mathbb{E}[X]}_{\substack{\| \\ 0}}\,\mathbb{E}[X^2]}$$

$$= \int_{-1}^{1} x^3 \cdot \frac{1}{2}\, dx$$

$$= \frac{1}{2}\frac{x^4}{4}\Big|_{-1}^{1} = 0$$

$\Rightarrow \text{Cov}(X,Y) = 0$ $\quad$ pero X, Y <u>NO</u> son indep

$X \sim Poi(\mu)$

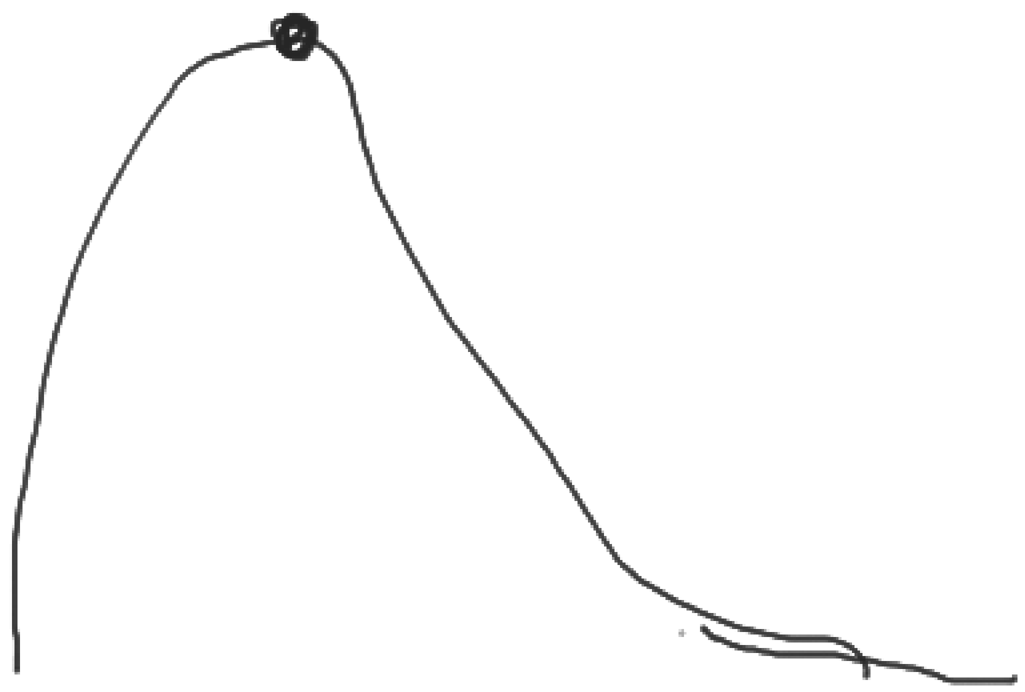
$$M_X(t) = \mathbb{E}\big[\underbrace{e^{tX}}_{g(X)}\big] = \sum_{n=0}^{\infty} \underbrace{e^{tn}}_{(e^t)^n} \frac{\mu^n e^{-\mu}}{n!} = e^{-\mu} \sum_{n=0}^{\infty} \frac{(\mu e^t)^n}{n!}$$

$$= e^{-\mu} e^{\mu e^t} = e^{\mu(e^t-1)}$$

$$e^x = \sum_{n=0}^{\infty} e^0 \frac{x^n}{n!} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

Taylor alrededor del cero

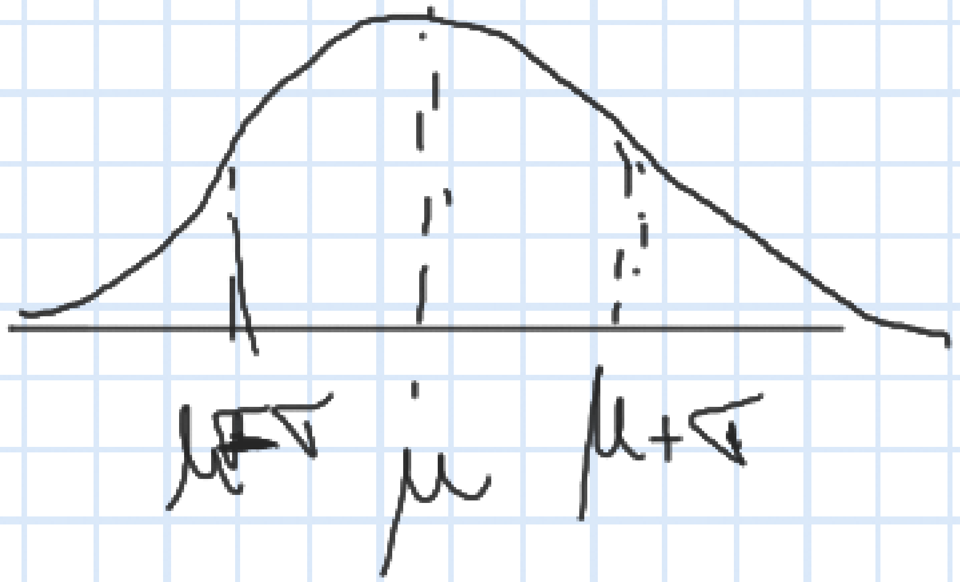
$$\mathbb{E}[X] = \frac{d}{dt} M_x(t)\Big|_0 = e^{\mu(e^t-1)} \cdot \mu e^t \Big|_0 = \mu$$



$$x_{0.1} : x_{0.1} \,/\, \mathbb{P}(X \leq x_{0.1}) = 0.1$$

$$x_{0.5} : x_{0.5} \,/\, \mathbb{P}(X \leq x_{0.5}) = 0.5$$

$X \sim N(\mu, \sigma^2)$ Si $f_x(x) = \frac{1}{\sqrt{2\pi}\sqrt{\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$



X con $F_X(x)$ / $\exists F_X^{-1}$

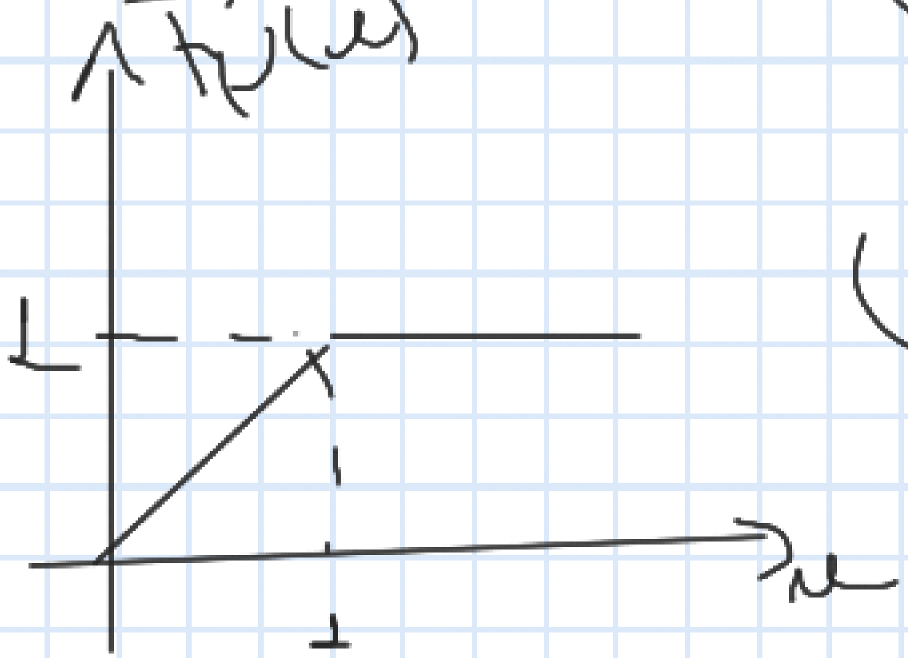
$u \in (0,1)$

$$U = F_X(X)$$

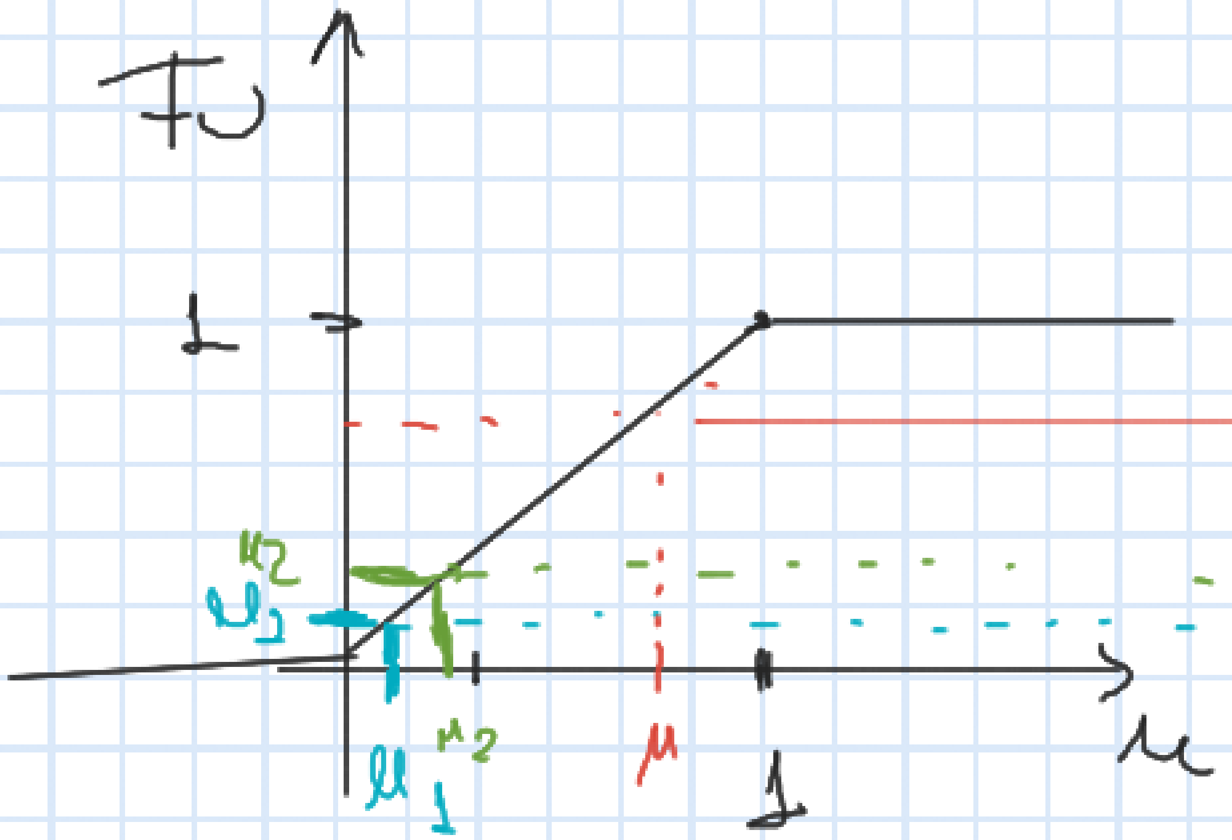
$$F_U(u) = \mathbb{P}(U \le u) = \mathbb{P}(F_X(X) \le u) = \mathbb{P}(\underbrace{F_X^{-1}(F_X(X))}_{X} \le F_X^{-1}(u))$$

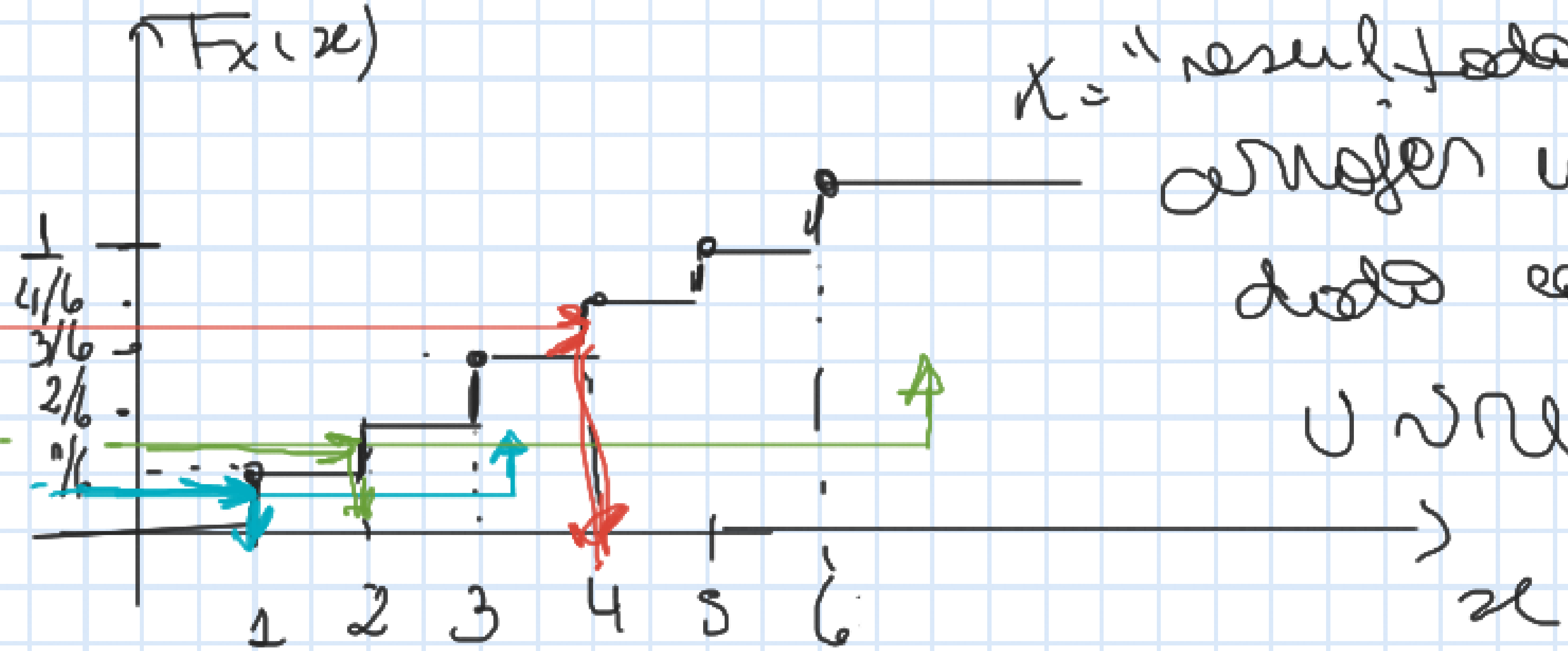
$$= \mathbb{P}(X \le F_X^{-1}(u)) = F_X(F_X^{-1}(u)) = u$$

$$F_U(u) = \begin{cases} 0 & u < 0 \\ u & 0 \le u < 1 \\ 1 & u \ge 1 \end{cases}$$



$$U \sim \mathcal{U}(0,1)$$

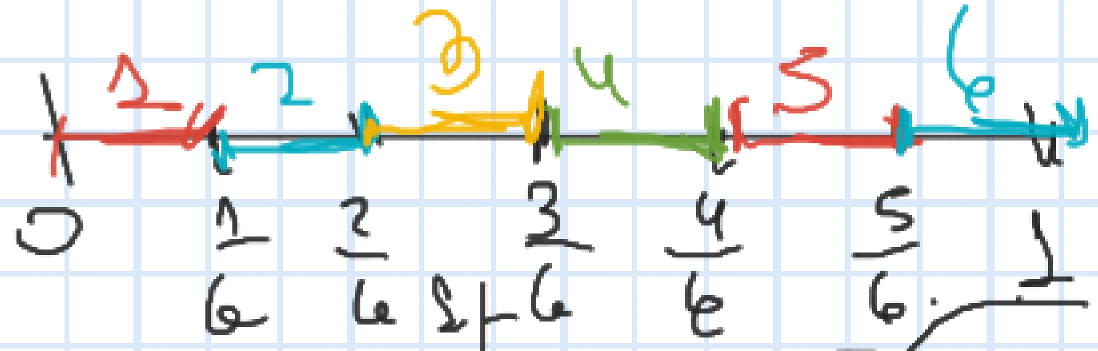


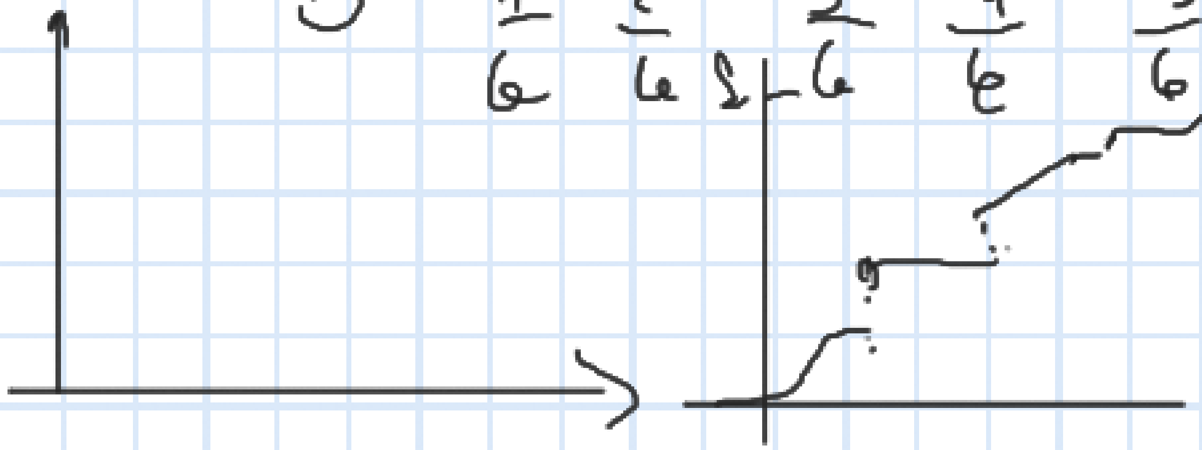


X = "resultado do arrojar um dado equil."

$U \sim \mathcal{U}(0,1)$

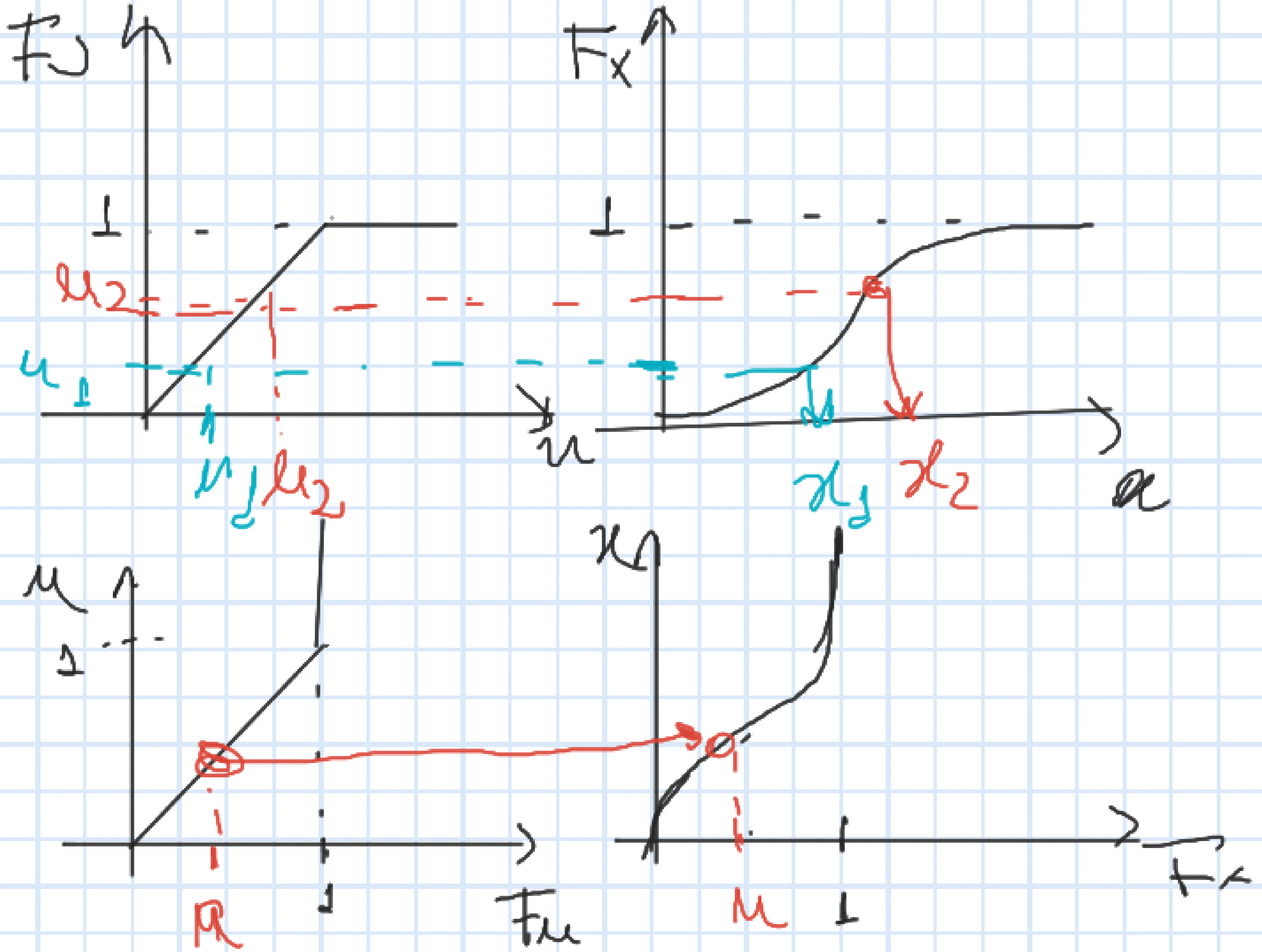
$$F_X^{-1}(u) = \min\{x \in \mathbb{R} : F_X(x) \geq u\} = \begin{cases} 1 & u \in (0, 1/6) \\ 2 & u \in (1/6, 2/6) \\ 3 & u \in (2/6, 3/6) \\ 4 & u \in (3/6, 4/6) \\ 5 & u \in (4/6, 5/6) \\ 6 & u \in (5/6, 6/6) \end{cases}$$



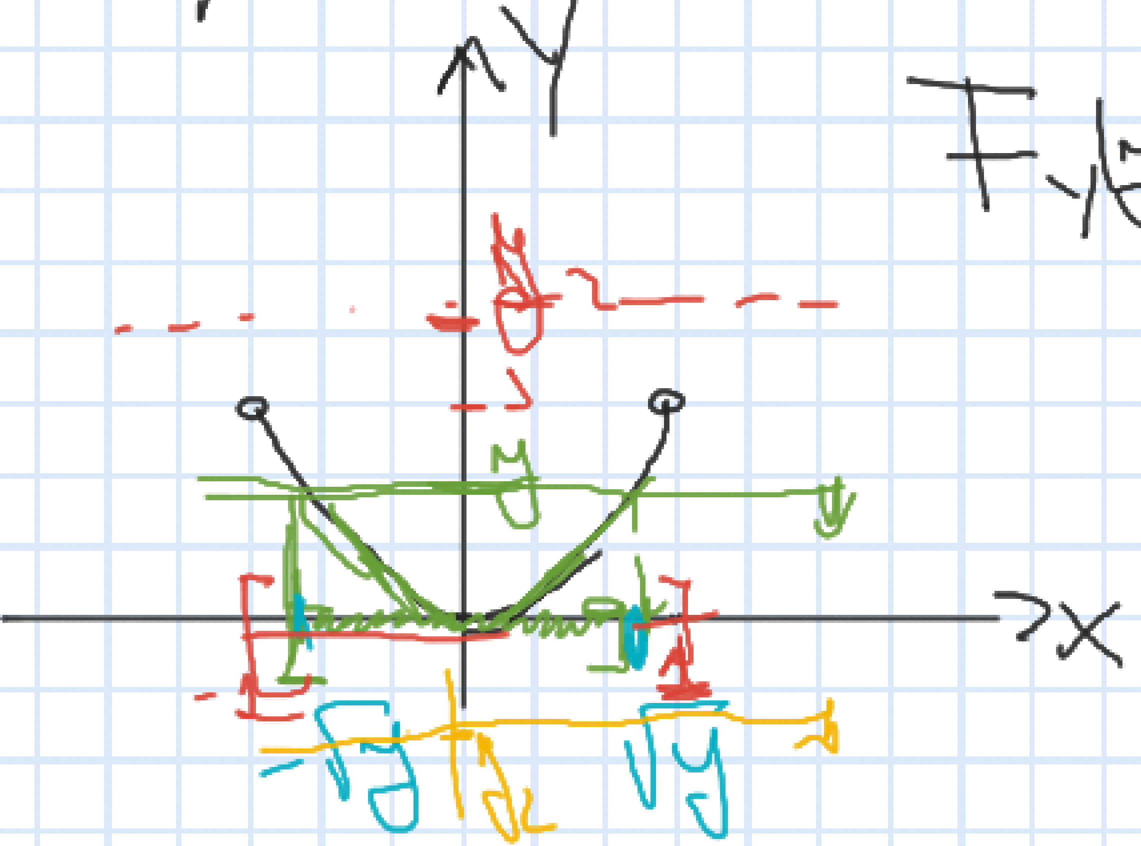


$F_U(u) = F_X(x)$ Si X es una v. a. c

$u = F_X(x)$ si $\exists F_X^{-1} \Rightarrow F_X^{-1}(u) = x$



$Y = X^2$, $X \sim \mathcal{U}(-1,1)$



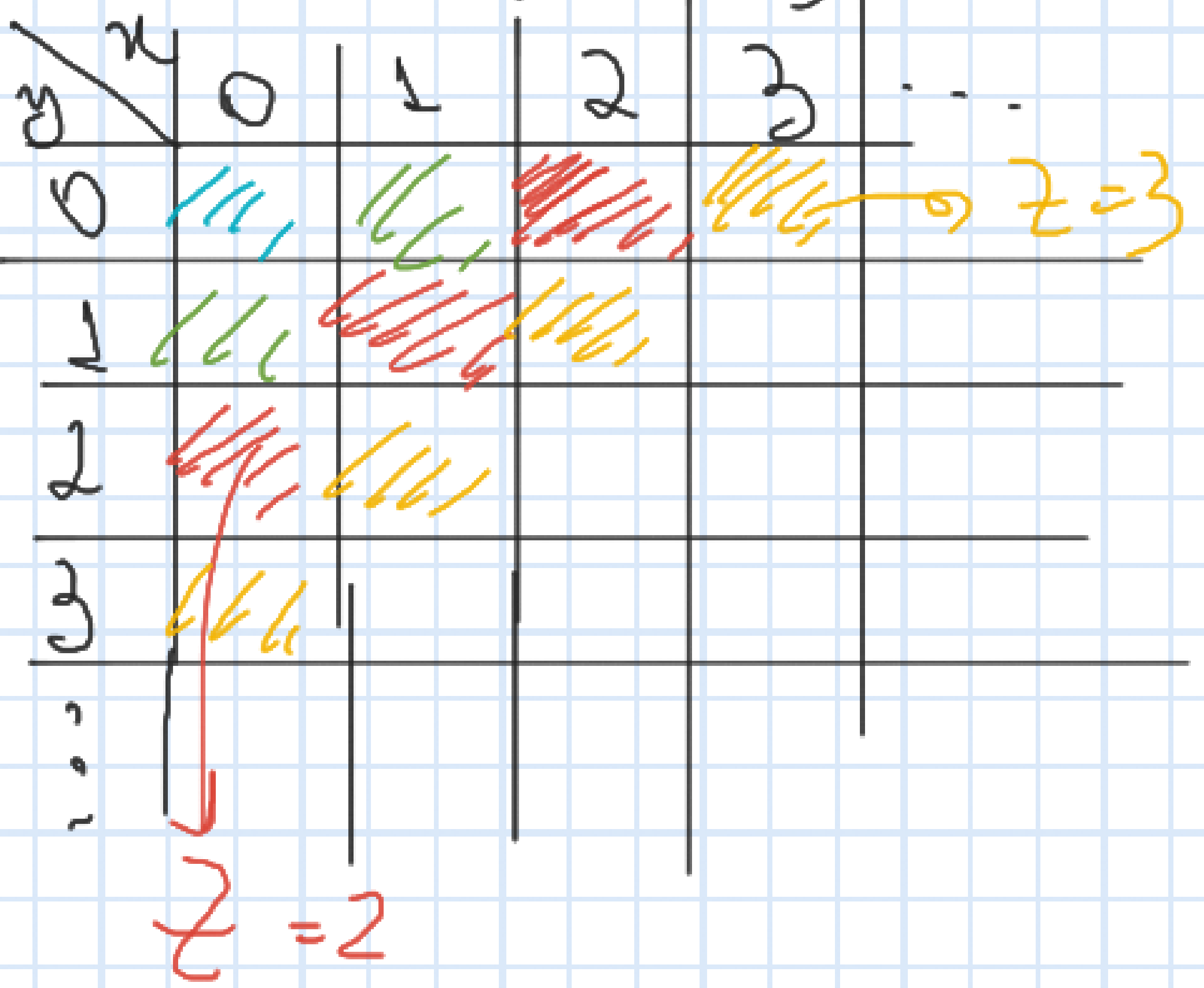
$F_Y(y) = P(Y \leq y) = P(X^2 \leq y) = P(|X| \leq \sqrt{y})$

$= P(-\sqrt{y} < X < \sqrt{y})$

$$= \begin{cases} 0 & \text{si } y < 0 \\ P(-\sqrt{y} < X < \sqrt{y}) & \text{si } 0 \leq y < 1 \\ 1 & y \geq 1 \end{cases}$$

$$F_Y(y) = \begin{cases} 0 & \text{si } y < 0 \\ \frac{\cancel{2}\sqrt{y}}{\cancel{2}} & 0 \leq y < 1 \\ 1 & y \geq 1 \end{cases}$$

$X \sim Poi(\lambda)$, $Y \sim Poi(\mu)$ X, Y indep Hallar la func de prob de $Z = X+Y$



$$p_Z(z) = \mathbb{P}(Z=z) = \mathbb{P}(X+Y=z)$$

$$Sop_Z = 0, 1, \ldots, = \mathbb{N}_0$$

X,Y indep

$$p_Z(0) = \mathbb{P}(X=0, Y=0) = \mathbb{P}(X=0)\,\mathbb{P}(Y=0) = \frac{\lambda^0}{0!}e^{-\lambda} \cdot \frac{\mu^0}{0!}e^{-\mu}$$

$$p_Z(1) = \mathbb{P}(Z=1) = \mathbb{P}(X=0, Y=1) + \mathbb{P}(X=1, Y=0)$$

$$= \frac{\lambda^0}{0!}e^{-\lambda} \cdot \frac{\mu^1}{1!}e^{-\mu} + \frac{\lambda^1}{1!}e^{-\lambda}\frac{\mu^0}{0!}e^{-\mu}$$

$$p_Z(z) = \mathbb{P}(Z=z) = \sum_{x=0}^{z} \mathbb{P}(X=x, Y=z-x)$$

$$= \sum_{x=0}^{z} \frac{\lambda^x}{x!} e^{-\lambda} \cdot \frac{\mu^{z-x}}{(z-x)!} e^{-\mu} = \frac{e^{-\lambda} e^{-\mu}}{z!} \sum_{x=0}^{z} \underbrace{\frac{z!}{x!(z-x)!}}_{\binom{z}{x}} \lambda^x \mu^{(z-x)}$$

$$\binom{z}{x} = \frac{z!}{x!(z-x)!}$$

$$= \frac{(\lambda+\mu)^z}{z!} e^{-(\lambda+\mu)} \sum_{x=0}^{z} \frac{\binom{z}{x} \lambda^x \mu^{z-x}}{(\lambda+\mu)^z}$$

$$(\lambda+\mu)^z = (\lambda+\mu)^x (\lambda+\mu)^{z-x}$$

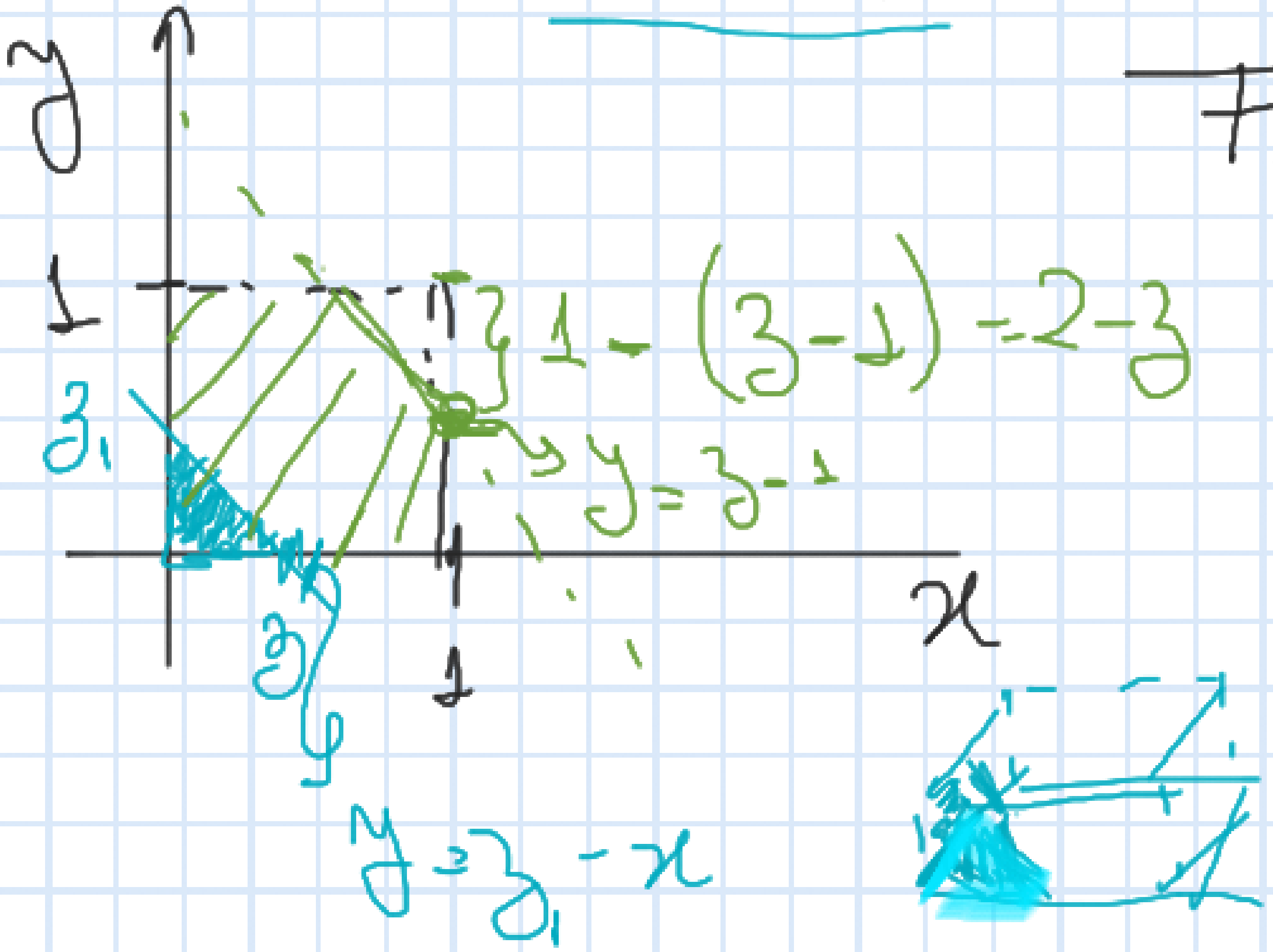
$$= \boxed{\frac{(\lambda+\mu)^z}{z!} e^{-(\lambda+\mu)}} \underbrace{\sum_{x=0}^{z} \binom{z}{x} \left(\frac{\lambda}{\lambda+\mu}\right)^x \underbrace{\left(\frac{\mu}{\lambda+\mu}\right)^{z-x}}}_{1}$$

$$\left(\frac{\mu}{\lambda+\mu}\right) = 1 - \frac{\lambda}{\lambda+\mu}$$

$$p_Z(z) = \frac{(\lambda+\mu)^z}{z!} e^{-(\lambda+\mu)} \quad , z \in \mathbb{N}_0$$

$$\Rightarrow Z = X + Y \sim \mathrm{Poi}(\lambda+\mu)$$

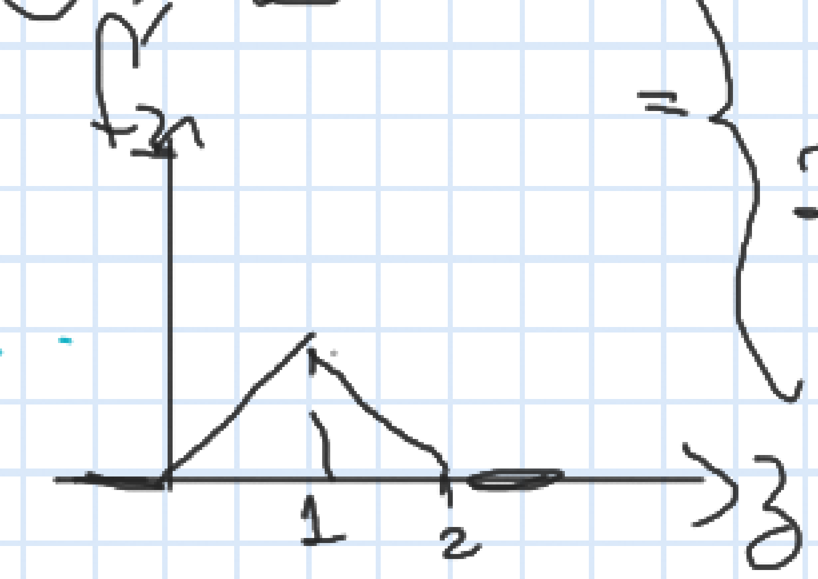
$X, Y \overset{iid}{\sim} \mathcal{U}(0,1)$ $\quad Z = X+Y$



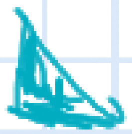
$$F_Z(z) = P(Z \le z) = P(X+Y \le z) = P(Y \le z - X)$$

$$= \begin{cases} 0 & z<0 \\ z^2/2 & 0<z<1 \\ 1-\underbrace{(2-z)^2/2}_{1} & 1<z<2 \\ 1 & z>2 \end{cases}$$

$$f_Z(z) = F_Z'(z) = \begin{cases} z & 0<z<1 \\ 2-z & 1<z<2 \end{cases}$$



$$\iint_{\triangle} f_X(x) f_Y(y)\,dx\,dy = \frac{\text{area}(\triangle)}{\text{area}(\square)} = \frac{z^2/2}{1}.$$



$$f_X(x) f_Y(y) = f_{X,Y}(x,y) = \frac{1}{1} I\{0<x<1\} \cdot \frac{1}{1} I\{0<y<1\}$$

$$= 1\, I\{0<x<1,\ 0<y<1\}$$

$X \sim \text{Poi}(\lambda) \quad Y \sim \text{Poi}(\mu) \qquad X, Y$ indep

$Z = X + Y \sim \text{Poi}(\mu + \lambda)$

$$M_Z(t) = \mathbb{E}[e^{tZ}] = \mathbb{E}[e^{t(X+Y)}] = \mathbb{E}[e^{tX} e^{tY}]$$

$$\overset{X,Y \text{ indep}}{=} \mathbb{E}[e^{tX}]\,\mathbb{E}[e^{tY}] = e^{\lambda(e^t - 1)} e^{\mu(e^t - 1)}$$

$$= e^{\lambda(e^t - 1) + \mu(e^t - 1)}$$

$$= e^{(\lambda+\mu)(e^t - 1)}$$

$$\Downarrow$$

$$Z \sim \text{Poi}(\lambda + \mu)$$