[2024-1 Digital Control]
Chapter 0. Introduction to Course

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About the course

Ch. 0. Introduction to Course

- Course title: 디지털제어 (Digital Control)
  - 수요일 10:00 – 12:50, 정보기술관 108/109호
  - 학부-대학원 공통 과목 (학부: 40067(01), 대학원: 49.671(01))
  - Prerequisite: 제어공학
- Lecturer: 박경훈 (Prof. Gyunghoon Park)
  - Office: 정보기술관 501호 (Rm. 501, IT bldg.)
  - Email: `gyunghoon.park@uos.ac.kr`
  - Phone number: 02-6490-2322
- Teaching assistant: 김준수 박사과정생
- Objectives of the course:
  - Digital device로 제어되는 제어 시스템의 안정성을 해석하고, 이를 바탕으로 이산 시간 제어기를 설계, 구현할 수 있습니다.
  - 연속 시간에서와 이산 시간에서의 제어 시스템 해석의 유사점과 차이점을 이해합니다.
- For details, see:

  https://cdsl-uos-wiki.notion.site/2024-1-40067-01-f2be00650e8f4a3db78b7f464b94be14?pvs=4

What is "control"?

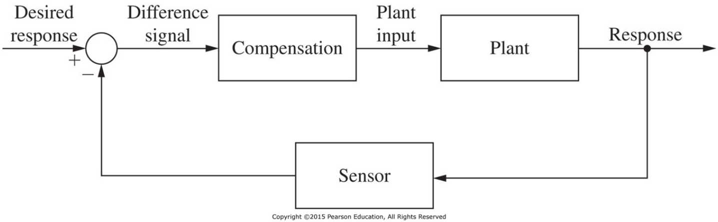

Figure: A configuration of a closed-loop control system

A control system may consist of

- Plant (to be controlled)
- Sensor and actuator
- Controller (to be constructed by our hand)

Main objective of the control engineering:

- Stabilize the closed-loop system
- Force the output $y(t)$ to follow a reference $r(t)$

“Control is everywhere!”: A beauty of control engineering

- Stable walking of legged robots

  `https://www.youtube.com/watch?v=tF4DML7FIWk`

Figure: Atlas AT Boston Dynamics (`https://www.bostondynamics.com/`)

- Landing of reusable rockets

  https://www.youtube.com/watch?v=Aq7rDQx9jns

- Pose stabilization of a missile

  https://www.youtube.com/watch?v=STWBZ904qUE

Figure: Falcon 9 and Heavy AT SpaceX

https://www.spacex.com/vehicles/falcon-9/

You may study in control engineering class . . .

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For given closed-loop system w/ plant $P(s)$ and controller $C(s)$,

- System description
  - Transfer function $P(s)$
  - (State-space model $\dot{x} = Ax + Bu$, $y = Cx + Du$)
- Notion of stability
  - Open-loop: "Poles" of $P(s) \in \mathbb{C}_{<0}$
  - Closed-loop: "Poles" of $PC/(1 + PC)(s) \in \mathbb{C}_{<0}$
- Stability criteria
  - Routh-Hurwitz criterion
  - Root locus technique
  - Nyquist plot
  - Gain/phase margins
- Performance & design
  - Overshoot & settling time
  - PID controller
  - Lead-lag compensator

A toy example that we want to study in this class

Consider a simple plant

$$P(s) = \frac{1}{s}$$

The classical control engineering class says that a PI controller

$$C(s) = K_p + K_i \frac{1}{s}, \quad K_p > 0,\ K_i > 0.$$

would stabilize the closed-loop system!

$$\because \quad \frac{PC}{1+PC} = \frac{\dfrac{sK_p + K_i}{s^2}}{1 + \dfrac{sK_p + K_i}{s^2}} = \frac{sK_p + K_i}{s^2 + sK_p + K_i}.$$

Unfortunately,
we have to do more for the implementation of $C(s)$ in real world...

Q1: How can we implement $C(s)$ in the code?

We CANNOT directly implement $C(s) = K_p + K_i/s$ or

$$u(t) = \mathcal{L}^{-1}(C(s)(r(s) - y(s))) = K_p(r(t) - y(t)) + \int_0^t K_i(r(\tau) - y(\tau))\mathrm{d}\tau$$

in the "digital" device. (WHY???)

An alternative is to approximate $u(t)$ above in the discrete time:

$$u^{\mathsf{d}}(k) = K_p(r^{\mathsf{d}}(k) - y^{\mathsf{d}}(k)) + \sum_{j=0}^{k} K_i(r^{\mathsf{d}}(j) - y^{\mathsf{d}}(j))T$$

- $T > 0$: Small time period (called sampling period later)
- k: Discrete time
- $u^{\mathsf{d}}(k)$: Discrete-time control input
- $r^{\mathsf{d}}(k) = r(kT)$, $y^{\mathsf{d}}(k) = y(kT)$.

Q2: How can we put $u^{\mathsf{d}}(k)$ into the "continuous-time" plant?

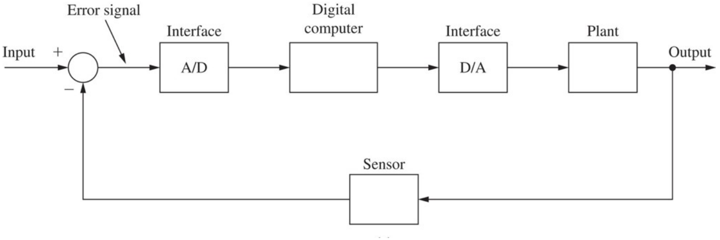

Figure: Sampled-data system

- The input/output of the controller are in discrete time;
- The input/output of the plant are in continuous time.

To deal with such a mismatch, we need two additional ingredients:

- Sampler (or D/A): $y^{\mathsf{d}}(k) = y(kT)$
- Hold (or A/D): $u(t) = u^{\mathsf{d}}(k)$, $\forall kT \leq t < (k+1)T$.

Simulation setting in MATLAB/Simulink

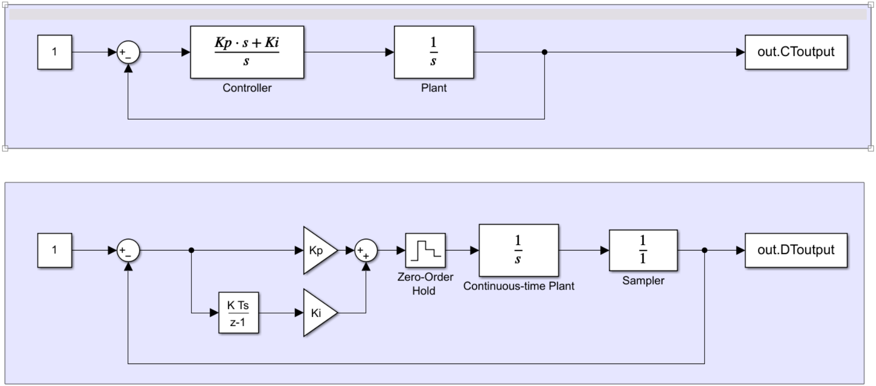

Figure: Implementation of PI-controlled systems in continuous time (above) in sampled-data setting (or in discrete time) (below)

Q3: Stability in continuous time $\not\Rightarrow$ Stability in discrete time

Step responses of

- continuous-time closed-loop system (blue)
- discrete-time closed-loop system (red)

in two cases:

- (Use of low gain, left) $(K_p, K_i) = (7, 12)$;
- (Use of high gain, right) $(K_p, K_i) = (30, 200)$.

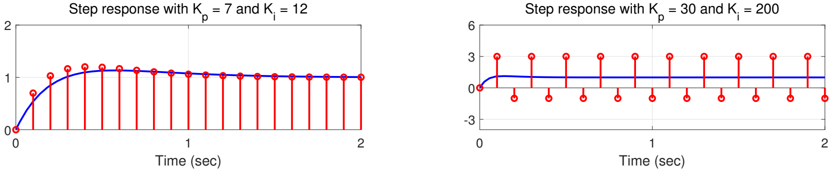

Figure: Step response under variation of K_i and K_p

"A discrete-time controller does not work like its continuous-time counterpart!"

Understanding these phenomena is the core of the class.

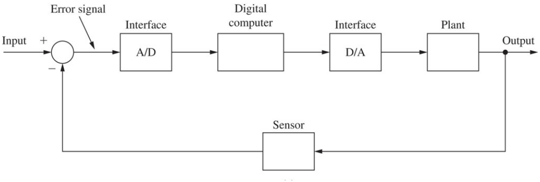

Figure: Overall configuration of sampled-data system

We will study

- System model in discrete time;
- Stability analysis of discrete-time system and sampled-data system;
  Intuition: A scalar system $x^{\mathsf{d}}(k+1) = \alpha x^{\mathsf{d}}(k) + \beta u^{\mathsf{d}}(k)$: stable when $|\alpha| < 1$!
- Relation between sample-and-hold process and stability
- Controller design in discrete time
  (e.g., pole placement, PID controller, LQR, Kalman filter)