# [2024-1 Digital Control]
# Chapter 8. Digital Controller Design

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# Introduction

## 8.2. Control System Specifications

We have learned about $\begin{cases}\text{discrete-time}\\ \text{sampled-data}\end{cases}$ system, modeled by

**Transfer function**

$$G^{\mathsf{d}}(z) = \frac{b_{n-1}z^{n-1} + \cdots + b_0}{z^n + a_{n-1}z^{n-1} + \cdots + a_0}$$

**State-variable model**

$$\mathbf{x}^{\mathsf{d}}(k+1) = \mathbf{A}^{\mathsf{d}}\mathbf{x}^{\mathsf{d}}(k) + \mathbf{B}^{\mathsf{d}}u^{\mathsf{d}}(k),$$
$$y^{\mathsf{d}}(k) = \mathbf{C}^{\mathsf{d}}\mathbf{x}(k)$$

and the related concepts, such as stability.

In Chapters 8–9, 11, we will study

- (Chapter 8) controller design with transfer function models, including phase-lag, phase-lead, lag-lead compensators, and PID controllers.
- (Chapter 9) controller design with state-variable model, including state-feedback control, state observer.
- (Chapter 11) optimal controller design for state-feedback control (i.e., best among candidates)

# Types of spec.: Steady-state accuracy



Remind that for the unit-step input whose $z$-transform is computed by

$$R^{\mathsf{d}}(z) = \frac{z}{z-1} = \frac{1}{1-z^{-1}}$$

one has the steady-state error of the closed-loop system as

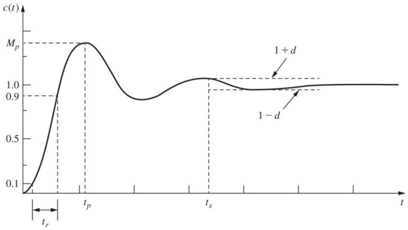
$$\begin{aligned} e_{\mathrm{ss}}^{\mathsf{d}} &= \lim_{k\to\infty} e^{\mathsf{d}}(k) \\ &= \lim_{z\to 1}(z-1)E^{\mathsf{d}}(z) = \lim_{z\to 1}\frac{(z-1)R^{\mathsf{d}}(z)}{1+G^{\mathsf{d}}(z)} = \frac{1}{1+\lim G^{\mathsf{d}}(z)} \\ &= \begin{cases} \text{constant} \neq 0, & \text{if } N = 0 \\ 0, & \text{if } N \geq 1 \end{cases} \end{aligned}$$

Control objective: Make the steady-state error $e_{\mathrm{ss}}^{\mathsf{d}}$ as small as possible.

# Types of spec.: Transient response

## 8.2. Control System Specifications

Control objective: Make the transient response fast enough and have little oscillation.




Parameters that specify the transient response:

- Rise time $t_r$: The time required for the step response to rise from $10$ percent to $90$ percent of the final value.
- Settling time $t_s$: The time required for the response to settle to within a certain percent of the final value.
- Time-to-peak overshoot $t_p$ and peak overshoot $M_p$

# Types of spec.: Transient response

## 8.2. Control System Specifications

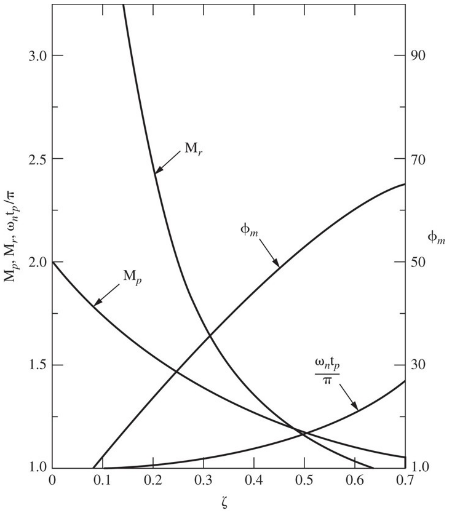
The figure represents the relation between

- rise time $t_r$
- settling time $t_s$
- time-to-peak overshoot $t_p$
- peak overshoot $M_p$

of the step response of

$$G_p(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

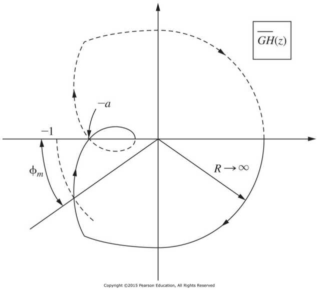
- $\omega_n$: natural frequency
- $\zeta$: damping ratio.



# Types of spec.: Gain and phase margins



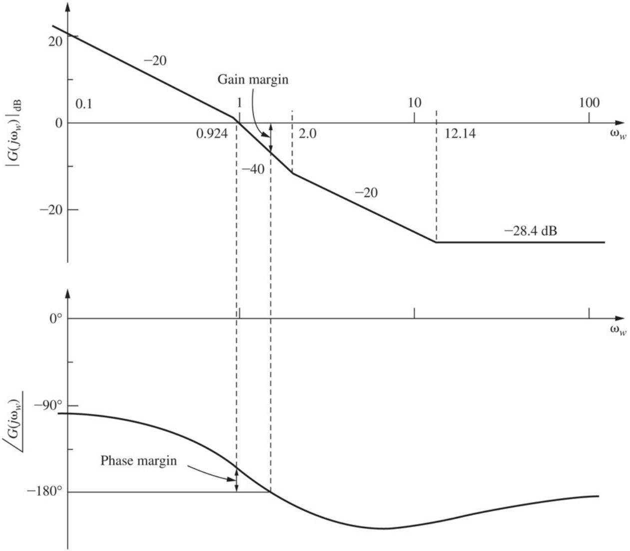
- Phase-crossover point = The point of $\angle G^{\mathsf{d}}(\mathrm{e}^{j\omega T}) = -\pi$.
- Gain-crossover point = The point of $|G^{\mathsf{d}}(\mathrm{e}^{j\omega T})| = 1$
- Gain margin $a = -20\log|G^{\mathsf{d}}(\mathrm{e}^{j\omega T})|$ dB at the phase-crossover point
- Phase margin $\phi_m = \angle G^{\mathsf{d}}(\mathrm{e}^{j\omega T}) - (-\pi)$ at the gain-crossover point




Control objective: Make the gain and phase margins of the system as large as possible.

# Types of spec.: Gain and phase margins

## 8.2. Control System Specifications

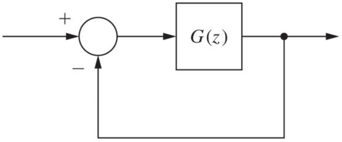


# Types of spec.: Sensitivity function

## 8.2. Control System Specifications

Consider the closed-loop transfer function given by

$$T^{\mathsf{d}}(z) = \frac{G^{\mathsf{d}}(z)}{1 + G^{\mathsf{d}}(z)}.$$



(a)

The sensitivity function of $T^{\mathsf{d}}$ with respect to $G^{\mathsf{d}}$ is given by

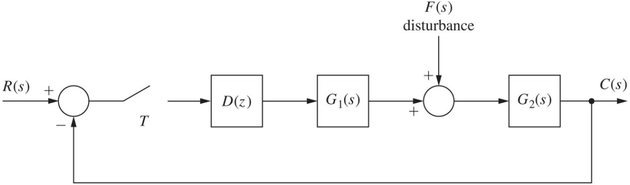
$$\begin{aligned} S^{\mathsf{d}}(z) &= \frac{\Delta T^{\mathsf{d}}/T^{\mathsf{d}}}{\Delta G^{\mathsf{d}}/G^{\mathsf{d}}} = \frac{\partial T^{\mathsf{d}}}{\partial G^{\mathsf{d}}} \frac{G^{\mathsf{d}}}{T^{\mathsf{d}}} \\ &= \frac{1 + G^{\mathsf{d}} - G^{\mathsf{d}}}{(1 + G^{\mathsf{d}})^2} \frac{G^{\mathsf{d}}}{G^{\mathsf{d}}/(1 + G^{\mathsf{d}})} = \frac{1}{1 + G^{\mathsf{d}}(z)}. \end{aligned}$$

Control objective: Make $|S^{\mathsf{d}}(e^{j\omega T})|$ as small as possible (at least in the frequency range of interest).

# Types of spec.: Disturbance rejection



Disturbance? A signal generated from an external source that needs to be compensated by a controller.




(b)



In the system above, if $r^{\mathsf{d}}(k) \equiv 0$, then the output $C^{\mathsf{d}}(z)$ is computed by

$$C^{\mathsf{d}}(z) = \frac{\overline{G_2 F}^{\mathsf{d}}(z)}{1 + D^{\mathsf{d}}(z)\overline{G_1 G_2}^{\mathsf{d}}(z)}$$

Control objective: Make $|C^{\mathsf{d}}(e^{j\omega T})|$ small enough in the frequency range where the disturbance $f(t)$ is dominant.

# Types of spec.: Control effort



The control effort is the input $u^{\mathsf{d}}(k)$ of the plant used for control of a system.

Control objective: Minimize the (squared) energy of the control effort

$$J = \int_0^{t_f} |u(t)|^2 \mathrm{d}t, \qquad \text{in continuous time,}$$

$$\text{or} \quad J = \sum_{i=0}^{N_f} |u^{\mathsf{d}}(i)|^2, \qquad \text{in discrete time.}$$

Note: A generalization of $J$ has the form

$$J = \sum_{i=0}^{N_f} \mathbf{x}^{\mathsf{d}}(i)^\top \mathbf{Q} \mathbf{x}^{\mathsf{d}}(i) + u^{\mathsf{d}}(i)^\top \mathbf{R} u^{\mathsf{d}}(i)$$

where $\mathbf{Q} \geq 0$ and $\mathbf{R} > 0$. (We will go further in Chapter 11.)

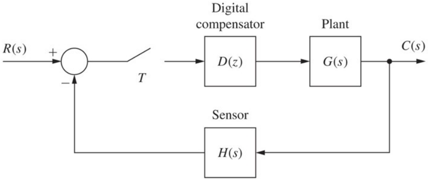
# Two types of compensation

## 8.3. Compensation

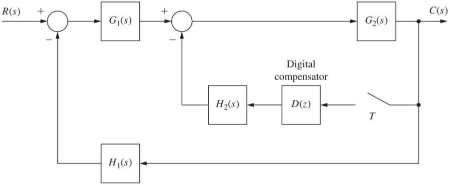
We will design a digital controller $D^{\mathsf{d}}(z)$ (or equivalently, compensator)

- for single-input singpue-output systems, and
- with the control objectives above taken into account.

1. Cascade (or series) compensation:




2. Feedback (or parallel) compensation:




(b)

We for now consider the first-order compensator

$$D^{\mathsf{d}}(z) = K_{\mathrm{C}} \frac{z - z_0}{z - z_{\mathrm{p}}}$$ where $K_{\mathrm{C}}$, $z_0$ and $z_{\mathrm{p}}$ are needed to be selected.

# Frequency response of the first-order compensator



Since $D^{\mathsf{d}}(z)$ is of first order, its frequency response will have the form

$$D^{\mathsf{w}}(\mathrm{w}) = a_0 \frac{1 + \mathrm{w}/\omega_{\mathrm{w0}}}{1 + \mathrm{w}/\omega_{\mathrm{wp}}}$$

where $a_0$ is DC gain, $\omega_{\mathrm{w0}}$ and $\omega_{\mathrm{wp}}$ are design parameters.

- IF $\omega_{\mathrm{w0}} < \omega_{\mathrm{wp}}$, THEN the compensation is called phase lead;
- IF $\omega_{\mathrm{w0}} > \omega_{\mathrm{wp}}$, THEN the compensation is called phase lag.

Note: Once $a_0$, $\omega_{\mathrm{w0}}$ and $\omega_{\mathrm{wp}}$ are determined, then

$$D^{\mathsf{d}}(z)\Big|_{z=\frac{1+(T/2)\mathrm{w}}{1-(T/2)\mathrm{w}}} = a_0 \frac{1 + \mathrm{w}/\omega_{\mathrm{w0}}}{1 + \mathrm{w}/\omega_{\mathrm{wp}}}$$

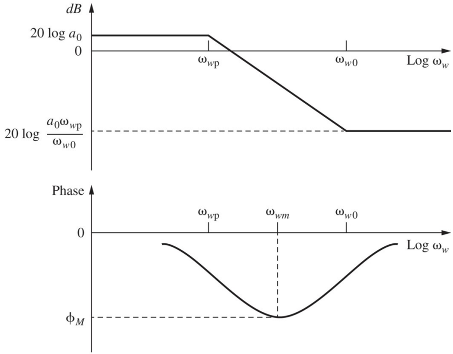
with

$$K_{\mathrm{C}} = a_0 \left( \frac{\omega_{\mathrm{wp}}(\omega_{\mathrm{w0}} + 2/T)}{\omega_{\mathrm{w0}}(\omega_{\mathrm{wp}} + 2/T)} \right), \quad z_0 = \frac{2/T - \omega_{\mathrm{w0}}}{2/T + \omega_{\mathrm{w0}}}, \quad z_{\mathrm{p}} = \frac{2/T - \omega_{\mathrm{wp}}}{2/T + \omega_{\mathrm{wp}}}.$$

# Frequency response of phase-lag filter

## 8.4. Phase-lag Compensation

- DC gain $= a_0$;
- High-frequency gain $= a_0 \dfrac{\omega_{\text{wp}}}{\omega_{\text{w0}}}$;
- Maximum phase shift $= \phi_M$ (satisfying $-90^\circ < \phi_M < 0^\circ$)

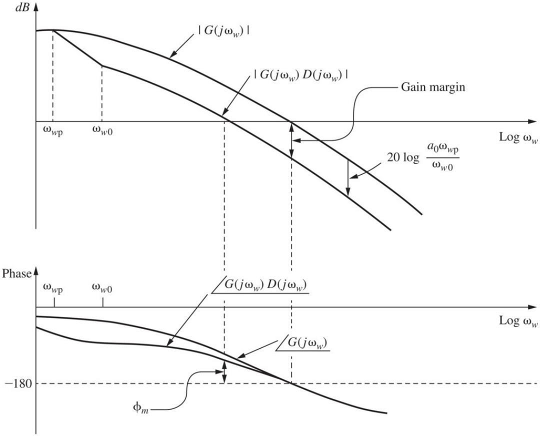


# Design objective for phase-lag compensation

## 8.4. Phase-lag Compensation

Design objective: Set the phase margin as a given value $\phi_m$.

Idea: Decrease the magnitude $|G^{\mathsf{w}}(j\omega_{\mathrm{w}})D^{\mathsf{w}}(j\omega_{\mathrm{w}})|$ of the resulting open-loop transfer function.



# Design steps for phase-lag compensator



Design objective: Set the phase margin as a given value $\phi_m$.

- Step 1: Determine $\omega_{\mathrm{w1}}$ at which $\angle G(j\omega_{\mathrm{w1}}) \approx -180^\circ + \phi_m + 5^\circ$.
  Note: This $\omega_{\mathrm{w1}}$ will be the gain-crossover frequency of the controlled system.
- Step 2: Choose $\omega_{\mathrm{w0}} = 0.1\omega_{\mathrm{w1}}$.
- Step 3: Set $\omega_{\mathrm{wp}}$ as

$$\omega_{\mathrm{wp}} = \frac{\omega_{\mathrm{w0}}}{a_0|G(j\omega_{\mathrm{w1}})} = \frac{0.1\omega_{\mathrm{w1}}}{a_0|G(j\omega_{\mathrm{w1}})}$$

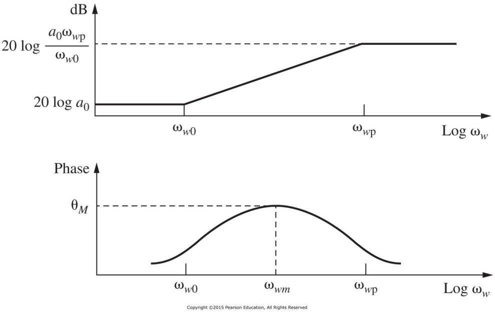
  (so that $|G^{\mathsf{w}}(j\omega_{\mathrm{w1}})D^{\mathsf{w}}(j\omega_{\mathrm{w1}})| = 1$).

Example: See Example 8.1.

# Frequency response of phase-lead filter



- DC gain $= a_0$;
- High-frequency gain $= a_0 \dfrac{\omega_{\text{wp}}}{\omega_{\text{w0}}}$;
- Maximum phase shift $= \theta_M$



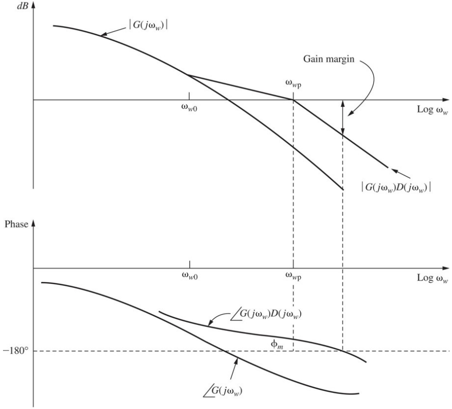
# Design objective for phase-lead compensation

## 8.5. Phase-lead Compensation

Design objective: Set the phase margin as a given value $\phi_m$.

Idea: Increase the angle $\angle G^{\mathsf{w}}(j\omega_{\mathrm{w}})D^{\mathsf{w}}(j\omega_{\mathrm{w}})$ of the resulting open-loop transfer function.

(We skip the detailed guideline for controller design...)

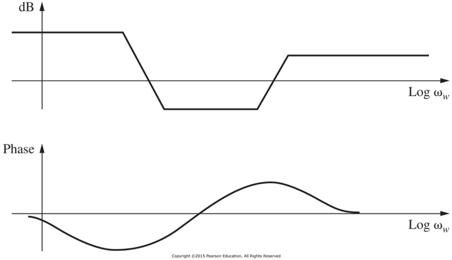


# Lag-lead compensator

## 8.7. Lag-lead Compensation

A lag-lead compensator

$= $ A phase-lag compensator $\times$ A phase-lead compensator



# Overview of PID control

## 8.8. Integration and Differentiation Filters

A PID controller in $s$-domain has the form

$$D_{\mathrm{PID}}(s) = K_{\mathrm{P}} + \frac{K_{\mathrm{I}}}{s} + K_{\mathrm{D}} s \qquad \text{(= Ideal version)}$$

$$\text{or} \quad D_{\mathrm{PID}}(s) = K_{\mathrm{P}} + \frac{K_{\mathrm{I}}}{s} + K_{\mathrm{D}} \frac{s}{T_{\mathrm{F}} s + 1} \qquad \text{(= Modified version with LPF).}$$

In the time-domain, the resulting control input is given by (e.g., ideal version):

$$m_{\mathrm{PID}}(t) = K_{\mathrm{P}} \cdot e(t) + K_{\mathrm{I}} \cdot \int_0^t e(\tau) \mathrm{d}\tau + K_{\mathrm{D}} \dot{e}(t).$$

Note: Each terms play the role of:

- P term $K_{\mathrm{P}}$: The present control effort for reducing the tracking error
- I term $\frac{K_{\mathrm{I}}}{s}$: The accumulated control effort with the history of the tracking error taken into account
- D term $K_{\mathrm{D}} s$: The predictive control effort to improve the tendency of the future trajectory.

# Numerical integration of a continuous-time signal



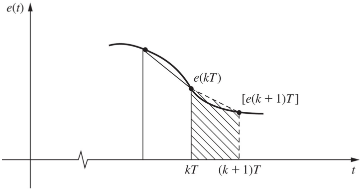
Let $p_{\mathrm{I}}^{\mathsf{d}}(k)$ be a numerical integral of a continuous-time signal $e(t)$.

- Forward Euler method:

$$p_{\mathrm{I}}^{\mathsf{d}}(k+1) = p_{\mathrm{I}}^{\mathsf{d}}(k) + T \cdot e^{\mathsf{d}}(k) \qquad \text{where } e^{\mathsf{d}}(k) := e(kT)$$

- Backward method
- Trapezoidal rule (= Tustin's method, bilinear transformation):

$$p_{\mathrm{I}}^{\mathsf{d}}(k+1) = p_{\mathrm{I}}^{\mathsf{d}}(k) + \frac{T}{2}\left(e^{\mathsf{d}}(k+1) + e^{\mathsf{d}}(k)\right)$$



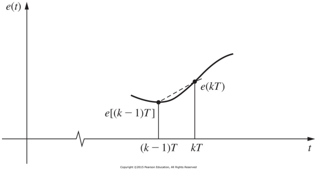
# Numerical differentiation of a continuous-time signal



Let $p_{\mathrm{D}}^{\mathsf{d}}(k)$ be a numerical derivative of of a continuous-time signal $e(t)$.

- Forward Euler method
- Backward Euler method:

$$p_{\mathrm{D}}^{\mathsf{d}}(k) = \frac{e^{\mathsf{d}}(k) - e^{\mathsf{d}}(k-1)}{T}$$

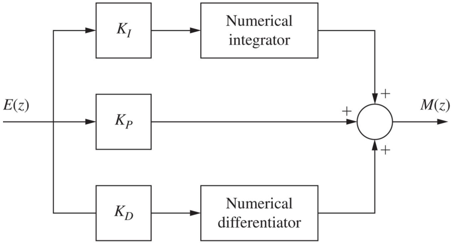



- Tustin's method:

$$\frac{p_{\mathrm{D}}^{\mathsf{d}}(k) + p_{\mathrm{D}}^{\mathsf{d}}(k-1)}{2} = \frac{e^{\mathsf{d}}(k) - e^{\mathsf{d}}(k-1)}{T}$$

# Discrete-time PID control law

## 8.9. PID Controllers





A discretized version of the PID controller above is

$$m^{\mathsf{d}}_{\mathrm{PID}}(k) = \begin{cases} +\ \text{P term} \\ +\ \text{I term} \\ +\ \text{D term} \end{cases} = \begin{cases} +\ K_{\mathrm{P}} \cdot e^{\mathsf{d}}(k) \\ +\ K_{\mathrm{I}} \cdot (\text{numerical integration of } e(t)) \\ +\ K_{\mathrm{D}} \cdot (\text{numerical differentiation of } e(t)) \end{cases}$$

where $K_{\mathrm{P}} > 0$, $K_{\mathrm{I}} > 0$, and $K_{\mathrm{D}} > 0$ are design parameters.

# Frequency response of numerical integration via B.T.



From the previous slides, we have

$$\text{(Numerical integration of } e(t)) \quad p_{\mathrm{I}}^{\mathsf{d}}(k+1) = p_{\mathrm{I}}^{\mathsf{d}}(k) + \frac{T}{2}\big(e^{\mathsf{d}}(k+1) + e^{\mathsf{d}}(k)\big)$$

$$\xrightarrow{\mathcal{Z}\text{-transform}} \quad zP_{\mathrm{I}}^{\mathsf{d}}(z) = P_{\mathrm{I}}^{\mathsf{d}}(z) + \frac{T}{2}\big(zE^{\mathsf{d}}(z) + E^{\mathsf{d}}(z)\big).$$

The transfer function from $e^{\mathsf{d}}$ to $p_{\mathrm{I}}^{\mathsf{d}}$ is computed by

$$\frac{P_{\mathrm{I}}^{\mathsf{d}}(z)}{E^{\mathsf{d}}(z)} = \frac{T}{2}\frac{z+1}{z-1}.$$

The frequency response in $\mathrm{w}$-domain:

$$\left.\frac{P_{\mathrm{I}}^{\mathsf{d}}(z)}{E^{\mathsf{d}}(z)}\right|_{z=\frac{1+(T/2)\mathrm{w}}{1-(T/2)\mathrm{w}}} = \frac{1}{\mathrm{w}}.$$

# Frequency response of numerical differentiation via B.T.



In a similar way, one can have

$$\text{(Numerical differentiation of } e(t)\text{)}\quad p^{\mathsf{d}}_{\mathrm{D}}(k) = -p^{\mathsf{d}}_{\mathrm{D}}(k-1) + \frac{2}{T}\big(e^{\mathsf{d}}(k) - e^{\mathsf{d}}(k-1)\big)$$

$$\xrightarrow{\mathcal{Z}\text{-transform}} P^{\mathsf{d}}_{\mathrm{D}}(z) = -z^{-1}P^{\mathsf{d}}_{\mathrm{D}}(z) + \frac{2}{T}\big(E^{\mathsf{d}}(z) - z^{-1}E^{\mathsf{d}}(z)\big).$$

The transfer function from $e^{\mathsf{d}}(k)$ to $m^{\mathsf{d}}_{\mathrm{D}}(k)$ is computed by

$$\frac{P^{\mathsf{d}}_{\mathrm{D}}(z)}{E^{\mathsf{d}}(z)} = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}} = \frac{2}{T}\frac{z-1}{z+1}.$$

The frequency response in $\mathrm{w}$-domain:

$$\left.\frac{P^{\mathsf{d}}_{\mathrm{D}}(z)}{E^{\mathsf{d}}(z)}\right|_{z=\frac{1+(T/2)\mathrm{w}}{1-(T/2)\mathrm{w}}} = \frac{1}{\mathrm{w}}.$$

# Frequency response of discrete-time PID control



Thus, the discrete-time PID controller

$$D^{\mathsf{d}}_{\mathrm{PID}}(z) = K_{\mathrm{P}} + K_{\mathrm{I}} \cdot \frac{T}{2}\frac{z+1}{z-1} + K_{\mathrm{D}} \cdot \frac{2}{T}\frac{z-1}{z+1}$$

has the frequency response in $\mathrm{w}$-domain as follows:

$$D^{\mathsf{d}}_{\mathrm{PID}}(\mathrm{w}) = K_P + K_I \frac{1}{\mathrm{w}} + K_D \mathrm{w}.$$

Note: If we use different methods for numerical differentiation and/or integration, then the frequency response should be slightly modified.

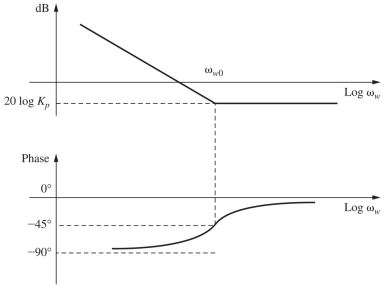
# Frequency-domain design of PID control

## 8.9. PID Controllers

A PI controller has the frequency response

$$D_{\mathrm{PI}}^{\mathsf{w}}(\mathrm{w}) = K_{\mathrm{P}} + K_{\mathrm{I}}\frac{1}{\mathrm{w}} = K_{\mathrm{P}}\frac{1 + (\mathrm{w}/\omega_{\mathrm{w0}})}{\mathrm{w}} \quad \text{where } \omega_{\mathrm{w0}} := K_{\mathrm{I}}/K_{\mathrm{P}} > 0.$$

Bode plot of $D_{\mathrm{PI}}^{\mathsf{w}}(\mathrm{w})$:





Lesson:

- Negative phase angle
- Increase gain margin
- ∴ PI controller works as a phase-lag controller.

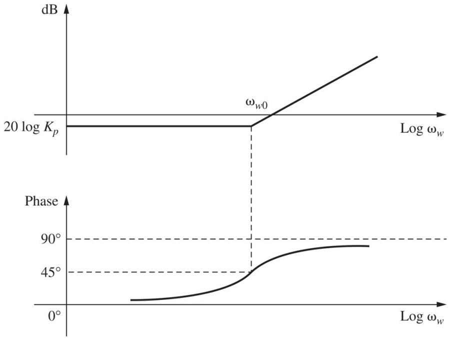
# Frequency-domain design of PID control

## 8.9. PID Controllers

A PD controller has the frequency response

$$D^{\mathsf{w}}_{\mathrm{PD}}(\mathrm{w}) = K_{\mathrm{P}} + K_{\mathrm{D}}\mathrm{w} = K_{\mathrm{P}}\left(1 + \frac{\mathrm{w}}{\omega_{\mathrm{w0}}}\right) \quad \text{where } \omega_{\mathrm{w0}} := \frac{K_{\mathrm{D}}}{K_{\mathrm{P}}} > 0.$$

Bode plot of $D^{\mathsf{w}}_{\mathrm{PD}}(\mathrm{w})$:



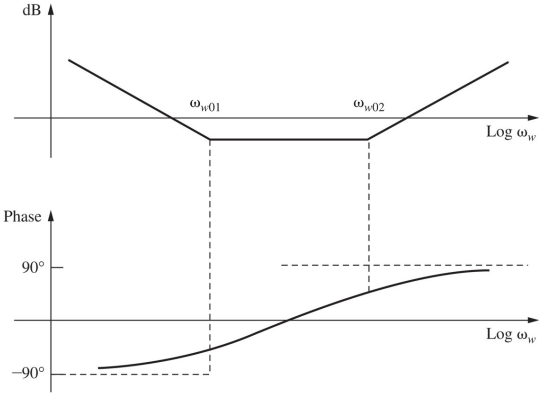



Lesson:

- Positive phase angle
- Increase phase margin
- $\therefore$ PD controller works as a phase-lead controller.

# Summarizing so far, PID ≈ Lead-lag compensator

## 8.9. PID Controllers





∴ The PID controller can be constructed in a similar way of lead-lag compensator (in Sections 8.4 – 8.6).

# Heuristic design of PID control: Ziegler-Nichols methods



- A widely-used heuristic approach to selecting $K_{\mathrm{P}}$, $K_{\mathrm{I}}$, $K_{\mathrm{D}}$.
- Useful particularly when we do not have system model.
- Basic idea: Find the control gains via trial-and-error approach.
- Two methods are available (while the second one is usually employed...)

In the following, we will discuss the ZN method for a continuous-time version

$$
\begin{aligned}
D_{\mathrm{PID}}(s) &= K_{\mathrm{P}} + K_{\mathrm{I}}\frac{1}{s} + K_{\mathrm{D}}\frac{s}{T_{\mathrm{F}}s+1} \\
&= K_P\left(1 + \frac{1}{T_I s} + \frac{T_{\mathrm{D}}s}{T_{\mathrm{F}}s+1}\right), \quad \text{where } T_{\mathrm{I}} = \frac{K_{\mathrm{P}}}{K_{\mathrm{I}}} \text{ and } T_{\mathrm{D}} = \frac{K_{\mathrm{D}}}{K_{\mathrm{P}}}
\end{aligned}
$$

(while its discrete-time counterpart can be derived in the same way.)

For more details, refer to other references, including

- K. Aström, T. Hägglund, *PID Controllers: Theory, Design, and Tuning (2nd ed.)*, ISA

# Heuristic design of PID control: First ZN method



Suppose that the plant to be controlled

- has overshoot-free step response, and
- has neither integrator nor complex-conjugate poles (in continuous time).

Then the first method suggests the PID gain as

$$K_{\mathrm{P}} = 1.2\frac{t_r}{t_i \cdot c_{\mathrm{ss}}}, \quad T_{\mathrm{I}} = \frac{K_{\mathrm{P}}}{K_{\mathrm{I}}} = 2t_i, \quad T_{\mathrm{D}} = \frac{K_{\mathrm{D}}}{K_{\mathrm{P}}} = 0.5t_i.$$

where

- $t_i$: The time required for the response to rise from the initial value to $10\ \%$ of the final response
- $t_r$: The rise time
- $c_{\mathrm{ss}}$: The steady-state step response

# Heuristic design of PID control: Second ZN method



The second method suggests to follow the steps below.

- Step 1: Increase $K_{\mathrm{P}}$ from $0$ to $\infty$ and apply the P control with no I and D terms.
- Step 2: Find a critical value $K_{\mathrm{P}} = K_{\mathrm{P}}^{\star}$, with which the output $c^{\mathsf{d}}(k)$ exhibits oscillation.
- Step 3: Let $T^{\star}$ be the period of oscillation that takes place with $K_{\mathrm{P}} = K_{\mathrm{P}}^{\star}$.
- Step 4: Set the PID gains as

$$K_{\mathrm{P}} = 0.6K_{\mathrm{P}}^{\star}, \quad T_{\mathrm{I}} = 0.5T^{\star}, \quad T_{\mathrm{D}} = 0.125T^{\star}.$$

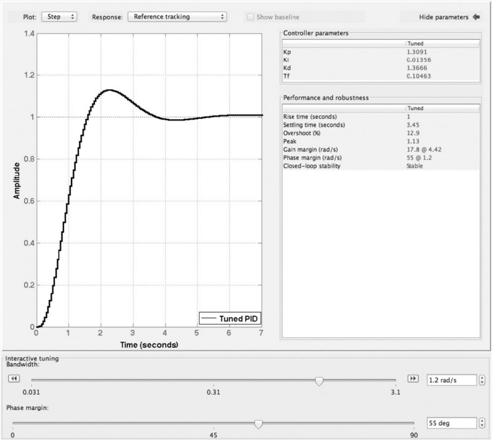
Note: The core of the second method is,

- determine $K_{\mathrm{P}}$ first only with P control, and
- add I and D terms if needed.

# Computer-aided design: MATLAB PID Toolbox



- MATLAB provides a toolbox for frequency and/or time-domain designs.
- To run the toolbox, enter `pidtool` in Command Window



# Root locus-based design of first-order compensator



The root locus technique provides yet another method for constructing a first-order compensator

$$D^{\mathsf{d}}(z) = K_{\mathrm{C}} \frac{z - z_0}{z - z_{\mathrm{p}}}.$$

For ease of explanation, we assume that the plant to be controlled has a discrete-time transfer function

$$G^{\mathsf{d}}(z) = K_{\mathrm{G}} \frac{z - z_1}{(z - z_2)(z - 1)}.$$

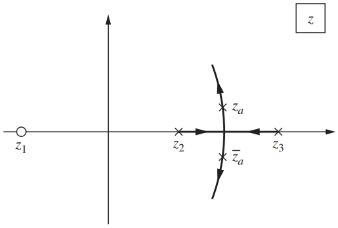
Design objective is to

- place the closed-loop pole to a desired location $z_a$, and
- set the control gain $K_{\mathrm{C}} K_{\mathrm{G}}$ as the desired value $K^{\star}$.

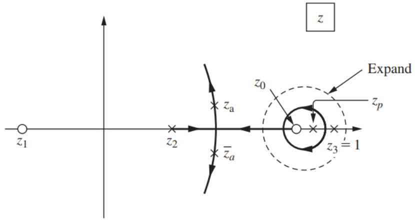
# Key idea of root locus-based design

8.11. Design by Root Locus

- First, find $K_\mathrm{u}$ such that the poles of $\dfrac{K_\mathrm{u}G^\mathsf{d}(z)}{1+K_\mathrm{u}G^\mathsf{d}(z)}$ are located at the desired values $z_a$. (Fig. (a))
- Second, add 1 zero $z = z_0$ and 1 pole $z = z_\mathrm{p} < 1$ to the open-loop transfer function. (Fig. (b))
  (Roughly speaking, this leads to a virtual shift of the open-loop pole $z = 1$ inside the unit circle.)
- Adjust the design parameters properly so that the design objectives are satisfied.



(a)



(b)