Week 1

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Chapter 4

E1

In the model below, which line is the likelihood:

$$y_i \sim Normal(\mu, \sigma)$$
$$\mu \sim Normal(0, 10)$$
$$\sigma \sim Exponential(1)$$

Top line (the other lines are priors).

E2

In the model definition above, how many parameters are in the posterior distribution?

Two. Mu and sigma

E3

Using the model definition above, write down the appropriate form of Bayes' theorem that includes the proper likelihood and priors.

$$Pr(\mu, \sigma|y_i) = \frac{Normal(y_1|\mu, \sigma).Normal(p|0, 10)}{\int Normal(y_1|\mu, \sigma).Normal(p|0, 10)d\mu}$$

I have no clue, to be honest

E4

In the model definition below, which line is the linear model?

$$y_i \sim Normal(\mu, \sigma)$$
$$\mu_i = \alpha + \beta x_i$$
$$\alpha \sim Normal(0, 10)$$
$$\beta \sim Normal(0, 1)$$
$$\sigma \sim Exponential(2)$$

Line two is the linear model.

E5

In the model above, how many parameters are in the posterior distribution? Three- α, β, and σ.

M1

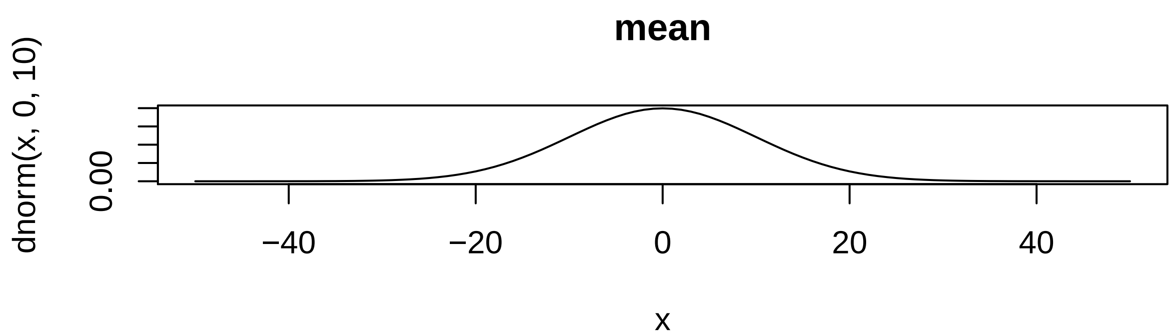
For the following model, simulate observed values from the prior:

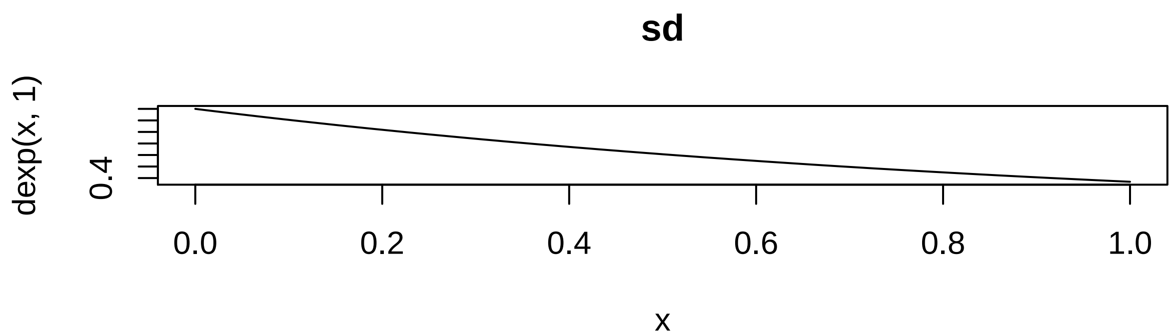
$$y_i \sim Normal(\mu, \sigma)$$
$$\mu \sim Normal(0, 10)$$
$$\sigma \sim Exponential(1)$$

Plot it first:

```r
par(mfrow = c(2, 1))

curve(dnorm(x, 0, 10), from = -50, to = 50, main = "mean")
curve(dexp(x, 1), from = 0, to = 1, main = "sd")
```



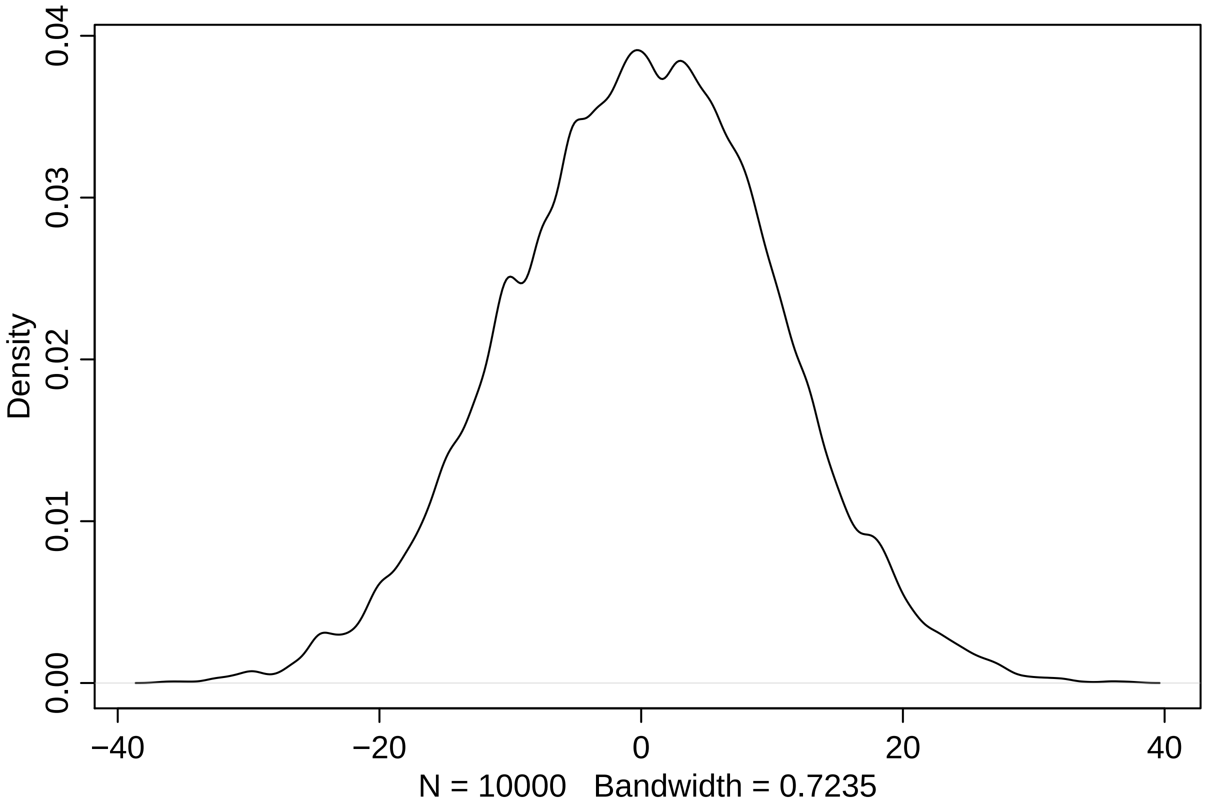



```r
par(mfrow = c(1, 1))
```

Now sample from it

```r
sample_mu <- rnorm(1e4, 0, 10)
sample_sigma <- rexp(1e4, 1)

prior <- rnorm(1e4, sample_mu, sample_sigma)
dens(prior)
```



M2

Translate the model above into a quap formula

```
flist <- alist(
  value ~ dnorm(mu, sigma),
  mu ~ dnorm(0, 10),
  sigma ~ dexp(1)
)
```

M3

Translate the following code into a mathematical formula

```
y ~ dnorm(mu, sigma)
mu <- a + b * x
a ~ dnorm(0, 10)
b ~ dunif(0, 1)
sigma ~ dexp(1)
```

$$y_i \sim Normal(\mu, \sigma)$$

$$\mu_i = a + \beta x_i$$

$$a \sim Normal(0, 10)$$

$$b \sim Uniform(0, 1)$$

$$\sigma \sim Exponential(1)$$

M4

A sample of students is measured for height each year for 3 years. After three years you want to fit a linear regression predicting height using year as a predictor. Write down the model and the priors.

$$y_i \sim Normal(\mu, \sigma)$$
$$\mu_i = a + \beta x_i$$
$$a \sim Normal(178, 30)$$
$$b \sim Normal(0, 2)$$
$$\sigma \sim Uniform(50)$$

M5

Suppose I reminded you that every student gets taller each year. How would that affect the prior?

$$b \sim Uniform(0, 5)$$

M6

Now suppose I tell you that the variance of height is never more than 64cm. How does this lead you to revise your priors?

$$\sigma \sim Uniform(8)$$

M7

Refit the model again without xbar for weight this time

Original code:

```
library(rethinking)
data(Howell1)
d <- Howell1
d2 <- d[ d$age >= 18 , ]

# define the average weight, x-bar
xbar <- mean(d2$weight)

# fit model
m4.3 <- quap(
    alist(
        height ~ dnorm( mu , sigma ) ,
        mu <- a + b*( weight - xbar ) ,
        a ~ dnorm( 178 , 20 ) ,
        b ~ dlnorm( 0 , 1 ) ,
        sigma ~ dunif( 0 , 50 )
    ) , data=d2 )

## R code 4.44
precis( m4.3 )
```

```
##              mean         sd        5.5%      94.5%
## a     154.6013707 0.27030767 154.1693668 155.033375
## b       0.9032809 0.04192363   0.8362789   0.970283
## sigma   5.0718810 0.19115480   4.7663788   5.377383
```

```
## R code 4.45
round( vcov( m4.3 ) , 3 )
```

```
##           a     b sigma
## a     0.073 0.000 0.000
## b     0.000 0.002 0.000
## sigma 0.000 0.000 0.037
```

New model

```
library(rethinking)
data(Howell1)
d <- Howell1
d2 <- d[ d$age >= 18 , ]

# fit model
m.new <- quap(
    alist(
        height ~ dnorm( mu , sigma ) ,
        mu <- a + b*( weight) ,
        a ~ dnorm( 178 , 20 ) ,
        b ~ dlnorm( 0 , 1 ) ,
        sigma ~ dunif( 0 , 50 )
    ) , data=d2 )

## R code 4.44
precis( m.new )
```

```
##              mean         sd        5.5%       94.5%
## a     114.5343082 1.89774682 111.5013422 117.5672742
## b       0.8907302 0.04175798   0.8239929   0.9574675
## sigma   5.0727177 0.19124883   4.7670652   5.3783703
```

```
## R code 4.45
round( vcov( m.new ) , 3 )
```

```
##            a      b sigma
## a      3.601 -0.078 0.009
## b     -0.078  0.002 0.000
## sigma  0.009  0.000 0.037
```