Problem Chosen	2021	Team Control Number
E	MCM/ICM	
	Summary Sheet	

Optimize food system & Better life

Summary

The unsoundness of a food system plays a vital role in the development of human society. The current food system still faces many challenges, the number of hungry people has been increasing slowly in the world, so it is crucial to re-optimize a strong food system. In order to help more people to get nutritious food, we have made the following efforts:

- First, we have identified 6 primary indicators and 17 secondary indicators to measure the food system. The specific 6 primary indicators are *Consumption, Ecosystem Stability, Waste & Loss, Value Chain, Social Structure, Resilience*.
- Secondly, we have established a **comprehensive evaluation model**, **optimization model** and time **prediction model**. In the comprehensive evaluation model of the food system, we use the **entropy method** and the **coefficient of variation method** to find the weights of the primary and secondary indicators, so that these indicators can be attributed to one data, named as the **comprehensive evaluation index of the food system *FSI***. In the optimization model, we optimize **the weights of secondary indicators** related to fairness and sustainability. The weights will fluctuate continuously by 5%, then we calculate the adjusted primary indicator values. We can calculate the optimized FSI, draw **the radar chart** of the first-level indicators before and after optimization, then calculate **the ratio** of the front and rear areas, the ratio is the difference degree, and find **the largest difference degree** as the final optimization target value. Provide a radar chart to visually see which indicators have changed and how much has changed. In the time prediction model, we use **the gray prediction model**. we can predict the index through the annual value, find the time interval, and determine the time of implementation.
- Next , in order to analyze the benefits and costs. We have established a **benefits and costs model**. First of all, we must change the priority. The first option is giving priority to fairness and the second option is giving priority to sustainability, then we change the related indicators and recalculate the weight and FSI. Next, we find four indicators directly related to costs and benefits, and establish a **regression equation** set between FSI and these four indicators. Then, using the **Gauss-Seidel iteration method**, we can get the costs and benefits. Finally, the time prediction model is also used to predict the occurrence time.
- Then, we selected **developed country Germany** and **developing country Niger** as the research objects, we substituted the data of Germany and Niger into our model respectively, then we calculated FSI, changes and time after and before changing priority. Our model can **truly reflect the actual situation.**
- Finally, we used the data of the United States and China to calculate the FSI and time, it proves that the model can be **adapted to different sizes of food systems and other regions**, it also proves the **scalability and adaptability** of the model. We conducted a sensitivity analysis and gave some **suggestions.**

In summary, our model completely solves all problems.

Keywords: Food Systems, Impact Indicators, Time Series, Radar Chart

Contents

1 Introduction

1.1 Background

For a long time, potential problems have become increasingly apparent in the global food system. The current food system is composed of massive national and international food producers and distributers. This system prioritizes efficiency and profitability. Despite the high efficiency of the current system, according to the latest statistics [1], nearly 690 million people are hungry, accounting for 8.9% of the world's population. At the same time, as the COVID-19 pandemic continues to evolve, it will have a further impact on the existing food system. As a result, more people fall into poverty and food crisis.

At the same time, the current food system leaves a large environmental footprint: greenhouse gas emissions, loss of biodiversity, deforestation, and freshwater use. Climate change is affecting every country on every continent and destroying the national economy. [2]

In addition, the food issue is the core of the Sustainable Development Goals (SDG) of the United Nations' 21st Century Development Agenda [3]. The United Nations' Sustainable Development Goals (SDG) emphasizes Goal 1: End poverty in all its forms everywhere and Goal 2: Zero Hunger. It is of major significance for eliminating hunger, achieving food security, improving nutritional status and promoting sustainable development. Today, there are only less than 10 years left to achieve this goal. Therefore, we will be committed to establishing a fair, reasonable, stable, efficient, and sustainable food system model to ensure that human beings have fair access to healthy, safe and sustainable food.

1.2 Restatement of Problem

Taking full account of the instability of the current global food system, the environmental crisis and the future survival and development of mankind, we need to solve the following problems:

- We should re-imagine and reprioritize the existing food system to provide a sufficiently robust food system model, it can be adjusted according to different levels of efficiency, profitability, sustainability and fairness to achieve optimization.
- We should explain what happens when a food system is optimized in terms of fairness and sustainability. What is the difference between the optimized food system and the existing system? We should estimate the time needed to realize such a food system.
- We should explain the benefits and costs of changing the priority of the food system and estimate when these benefits and costs will occur. What are the differences in benefits and costs between developed and developing countries?
- We should apply the established food model to one developed country and one developing country to support the established model.
- We should discuss the scalability and adaptability of the established model.
- We can give feasibility opinions or suggestions according to the current situation.

1.3 Overview of Our Work

In order to effectively optimize the current food systems, especially according to different levels of efficiency, profitability, sustainability, and fairness, we have established a relatively complete model that can be estimated the changes in the indicators of the optimized system and the time, it can be able to judge the benefits and costs of changing the priority of the food system, time, and the differences between developing and developed countries. Using concrete examples to support our model, and discussing scalability, adaptability, and sensitivity.

The model building process is shown in Figure 1.

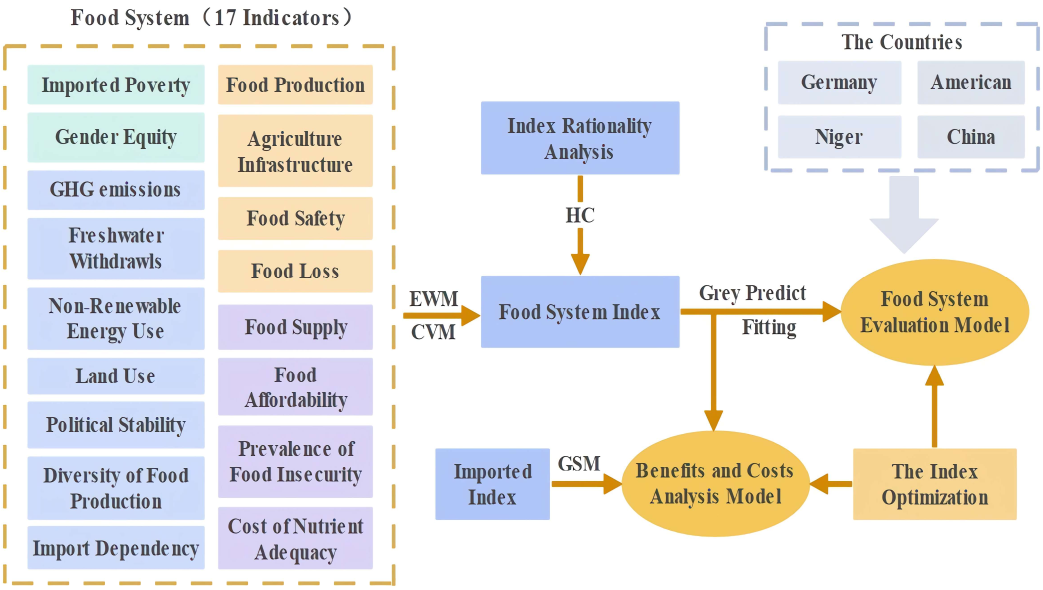

Figure 1: The structure of our paper

First, we have identified 6 primary indicators and 17 secondary indicators related to the evaluation of the food system. The specific 6 primary indicators are *Consumption, Ecosystem Stability, Waste & Loss, Value Chain, Social Structure, and Resilience.*

Then, We have established a comprehensive evaluation model, optimization model and time prediction model. In the food system comprehensive evaluation model, we use the entropy method and the coefficient of variation method to find the weights of the primary and secondary indicators, so that these measurement indicators can be attributed to one data, which is named the comprehensive evaluation index of the food system FSI. In the optimization model, optimize the weights of secondary indicators related to fairness and sustainability. The weights will fluctuate continuously by 5%, and then calculate the adjusted primary indicator values. Then we should calculate the optimized FSI, draw the radar chart of the first-level indicators before and after optimization, and calculate the ratio of the front and rear areas and find the largest difference degree as the final optimization target value. In the time prediction model, we use the gray prediction model. Because FSI can comprehensively represent the food system, we can predict the index through the annual value and find the time interval where the optimized FSI is located, and determine the time for realization.

Secondly, we have established a benefits and costs model. First of all, we must change the priority. The first option is giving priority to fairness and the second option is giving priority to sustainability. we should change the related indicators. Recalculate the weight and FSI. Next, we find four indicators directly related to costs and benefits, and use data from previous years to establish a regression equation which is established between FSI and these four indicators. Then, using the Gauss-Seidel iteration method, we can solve for costs and benefits. Through data changes, we can describe the benefits and costs. Finally, we use time prediction model to predict time.

Then, we selected developed country Germany and developing country Niger as the research objects, we substituted the data of Germany and Niger into our model respectively, then we calculated FSI, changes and time after and before changing priority. Our model can truly reflect the actual situation.

Finally, we used the data from the United States and China to calculate the FSI and the corresponding time in China to prove that the model can be adapted to different sizes of food systems and other regions. We also conducted a sensitivity analysis and gave some suggestions.

2 Assumptions and Notations

2.1 Assumption and Justifications

In order to simplify the model, this article makes the following basic assumptions, and each of them is reasonable.

- We assume that no major natural disasters occurred during the optimization forecast period.
- We assume that the country has no major policies issued to change the weight of our indicators.
- We assume that the relative importance of indicators at all levels has not changed over time.
- We assume that the data obtained is accurate and reliable. We obtain data from trusted websites and papers.

2.2 Definitions and Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description	Unit
FSI	Comprehensive Evaluation Index	——
FCC	Consumption	——
ESC	Ecosystem Stability	——
WLC	Waste & Loss	——
FVC	Value Chain	——
SSC	Social Structure	——
RCC	Resilience	——

Food System : the food system includes the cultivation, harvesting, processing, transportation, sale and consumption of food, covering the interaction between humans and the natural world, such as the exchange of material and energy, as well as all the services, institutions, infrastructure, and the effects of eating habits and cultural customs on results [3].

3 Evaluation of Food Systems & Optimization & Forecast

In this section, by analyzing different factors that affect the food system, we have established a comprehensive evaluation model of the food system, an optimization model and a time prediction model. Through the comprehensive evaluation model of the food system, we can determine a comprehensive evaluation index FSI; through the optimization model, we can determine what changes the food system will have after changes in fairness and sustainability; through the time prediction model, we can determine the time required for the optimization of the system .

3.1 Indicators to Measure the Food Systems

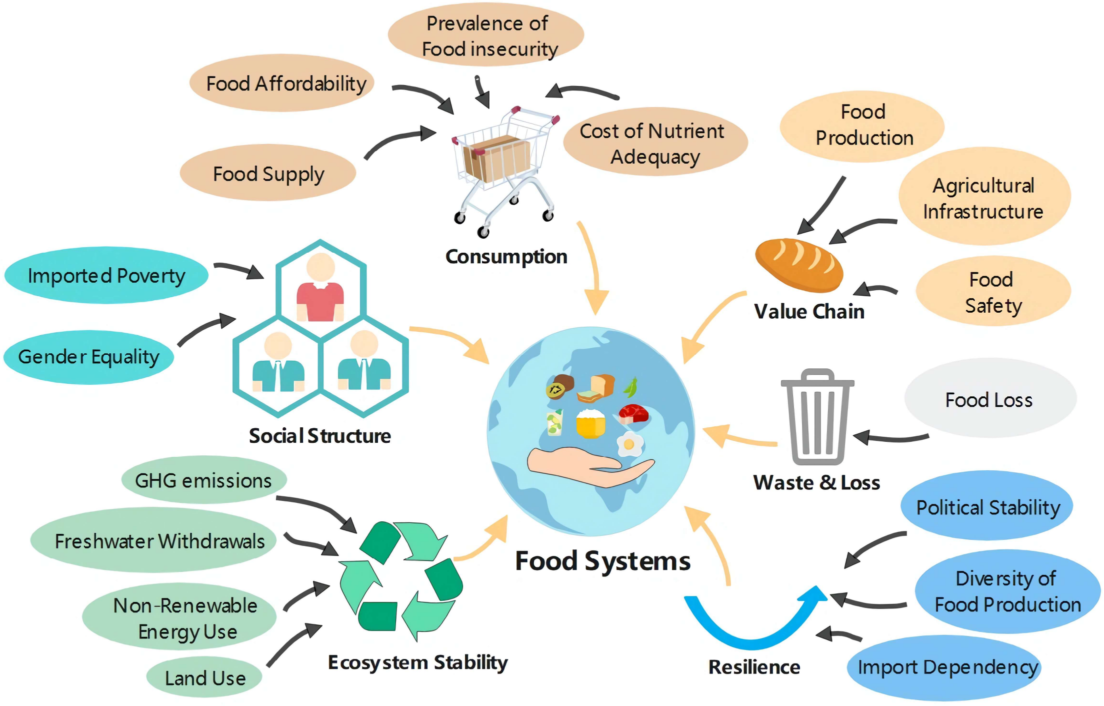

Figure 2 : Food system indicator map

3.1.1 Consumption

- Food Supply

This indicator is used to measure the diversity of food supply indicators. When the supply of all indicators is equal, the index is equal to $\ln(N)$, where , N represents the total number of foods. The general formula [4] is as follows:

$$Food\ Supply = -\sum_i s_i \ln(s_i)$$

Among them, s_i represents the share of the first i food in the food supply. The more unequal the food distribution, the smaller the index value. Food diversity helps to change the diet structure and promotes the development of sustainable diets.

- Food Affordability

Food affordability is expressed as the share of money that can be used for food. If a family or a country's food burden decreases year by year, it means that the food system at this time is more reasonable and more people do not need to worry about food shortages.

- Cost of Nutrient Adequacy

If people spend more on nutritious and reasonably priced food, it indicates that people are willing to improve their diet and change their lifestyle. If the cost of this item is high, it means that the food system is paying enough attention to reasonable diet.

- Prevalence of Food Insecurity

Food insecurity reflects that some people cannot obtain enough and affordable food necessary for maintaining production and health, indicating that the food system at this time cannot provide enough food, and some people still cannot get enough food.

3.1.2 Ecosystem Stability

- GHG(Greenhouse Gas) Emissions

It is generally believed that various social activities of humans emit a large amount of greenhouse gases, such as carbon dioxide, nitric oxide, methane and other gases containing carbon and nitrogen elements, which contribute to the increase in global climate temperature and the melting of glaciers.

- Freshwater Withdrawals

The growth of crops requires the irrigation of freshwater resources, and the world is currently short of freshwater resources. If we can adjust the structure of food production, it will help promote the effective use of freshwater resources and reduce waste.

- Non-Renewable Energy Use

Food processing and use require the use of a certain amount of non-renewable energy. High-income countries use fossil energy in transportation and industrial production. Low-income countries are more inclined to use resources in food cooking.

- Land Use

The cultivation of crops requires arable land, and the production of milk and other products requires pasture. How to rationally use land will have a vital impact on the human food system. Land use should be allocated rationally to prevent food crises.

3.1.3 Waste & Loss

- Food Loss

Nearly one-third of the world's food is wasted every year. If the amount of food wasted each year can be effectively reduced, the utilization rate of food will be improved and the wasted food will be rationally distributed to areas where it is needed.

3.1.4 Value Chain

- Food Production

To establish a stable food system, it is necessary to ensure sufficient food sources. The production capacity of a country or region determines the amount of supply that can be deployed, determines the amount of imports and exports.

- Agriculture Infrastructure

The level of the agriculture in a country or region determines the human, physical and financial resources that need to be spent on agricultural production.

- Food Safety

This indicator is used to measure the difficulty of obtaining food. When the food system of a country or region can effectively distribute food and respond to food crises, it can indicate that the system has strong resistance to shocks and resistance to risks.

3.1.5 Social Structure

- Gender Equality

In low-income areas [4], women and children are more likely to face the problems of hunger and poverty. When a social structure has a relatively equal gender ratio, it plays a vital role in promoting social progress and sustainability.

- Proportion of Imported Production

This indicator indicates the proportion of the poor, and a large part of the poor cannot maintain a basic daily food source, let alone nutritious food.

3.1.6 Resilience

- Political Stability

This indicator means that there is no overall political turmoil and social unrest, the regime does not undergo sudden changes, and it has the ability to resist some emergencies. When violence or terrorist incidents occur, the current food system will inevitably be affected.

- Diversity of Food Production

The more diverse the food, the richer the types of food, the stronger the ability to lower the risk, which is conducive to improving people's dietary structure and achieving a balanced diet, thereby indirectly promoting the improvement of environmental problems.

- Import Dependency

This indicator can indicate that the food import dependence is too high. At this time, the food system cannot provide enough food. Once outside supply is not available, it may have a certain degree of impact on the current food system.

3.2 Comprehensive Evaluation Model of Food Systems

This section will use the entropy method and the coefficient of variation method to calculate the corresponding weights of the secondary indicators and the primary indicators, and finally calculate the comprehensive evaluation indicator FSI of the food system, which will be used as a standard to measure a country or region.

3.2.1 Determine the Weight

We first process the data matrix and determine the information entropy of each secondary indicator through the entropy method (to avoid subjective factors affecting the results), and then calculate the weight; then we will use the coefficient of variation method (to highlight the difference) to calculate the weights corresponding to the first-level indicators. Finally, the comprehensive indicator FSI of the food system evaluation is obtained.

Step1： Entropy method to determine the weight of the secondary index

- Dimensionless processing of data [5]

The 17 secondary indicators are subjected to dimensionless data processing, $X_1,X_2,\cdots,X_m$ represents the quantity of the food system being evaluated is m , where $X_i=\{x_{i1},x_{i2},\cdots,x_{in}\}$, represents the quantity of the indicators is n .Suppose the original data matrix is $X=(x_{ij})_{m\times n}$, and the processed matrix is

$$z_{ij}=\frac{x_{ij}-\min(x_i)}{\max(x_i)\text{-}\min(x_i)}$$

Where, $j=1,2,\cdots,n$

- Find the information entropy of the secondary index

$$y_j=\frac{z_{ij}}{\sum_{j=1}^{n} z_{ij}}$$

$$e_j=-\frac{1}{\ln m}\sum_{i=1}^{m} y_{ij}\ln y_{ij}$$

Where, y_{ij} Is the proportion of the first i food system indicator to be evaluated,, $y_{ij}\in[0,1]$; e_j is represent Information entropy.

- Find the weight coefficient of the secondary index

$$w_j=\frac{1-e_j}{\sum_{j=1}^{n}(1-e_j)}$$

where, w_j is the index weight .

So, we will *get Consumption*、*Ecosystem Stability*、*Waste & Loss*、*Value Chain*、*Social Structure*、*Resilience* secondary indicator weight:

$$\begin{cases} FCC_j=w_1z_{1j}+w_2z_{2j}+w_3z_{3j}+w_4z_{4j} \\ ESC_j=w_5z_{5j}+w_6z_{6j}+w_7z_{7j}+w_8z_{8j} \\ WLC_j=w_9z_{9j} \\ FVC_j=w_{10}z_{10j}+w_{11}z_{11j}+w_{12}z_{12j} \\ SSC_j=w_{13}z_{13j}+w_{14}z_{14j} \\ RCC_j=w_{15}z_{15j}+w_{16}z_{16j}+w_{17}z_{17j} \end{cases}$$

where, $w_i,i=1,2,\cdots,17$ Represents the weight of each corresponding secondary indicator.

Table 1：Secondary indicator weight

Indicators(I)	Indicators(II)	Weights
Consumption	Food Supply	0.05808
	Food Affordability	0.06504
	Prevalence of Food Insecurity	0.01876
	Cost of Nutrient Adequacy	0.04512
Value Chain	Food Production	0.08779
	Agriculture Infrastructure	0.07922
	Food Safety	0.05099
Waste & Loss	Food Loss	0.20500
Resilience	Political Stability	0.04912
	Diversity of Food Production	0.01759
	Import Dependency	0.01829
Ecosystem Stability	GHG Emissions	0.03252
	Freshwater Withdrawals	0.02962
	Non-Renewable Energy Use	0.02206
	Land Use	0.04480
Social Structure	Imported Poverty	0.10451
	Gender Equality	0.07149

Step2： Coefficient of variation method to determine the weight of first-level indicators

Next, the coefficient of variation method [6] will be used to determine the weights of the first-level indicators .we can obtain the comprehensive index *FSI* for food system evaluation. Analyze the different performance of the grain system through this indicator for subsequent optimization processing.

Due to the different dimensions of the evaluation indicators, the coefficient of variation formula is selected to express the degree of difference between the indicators. The definition formula is as follows:

$$v_i = \frac{\sigma_i}{x_i} (i = 1, 2, \cdots, n)$$

where， v_i represents the coefficient of variation, σ_i is the standard deviation of the index, and v_i is the average value of the index.

Table 2: Coefficient of variation and weight

Indicators(I)	Consumption	Value Chain	Waste & Loss	Resilience	Ecosystem Stability	Social Structure
Coefficient of Variation	1.583	2.170	1.761	1.129	1.465	1.500
Weights	0.165	0.226	0.183	0.117	0.153	0.156

- The weight of each indicator is

$$W_i = \frac{v_i}{\sum_{i=1}^{n} v_i}.$$

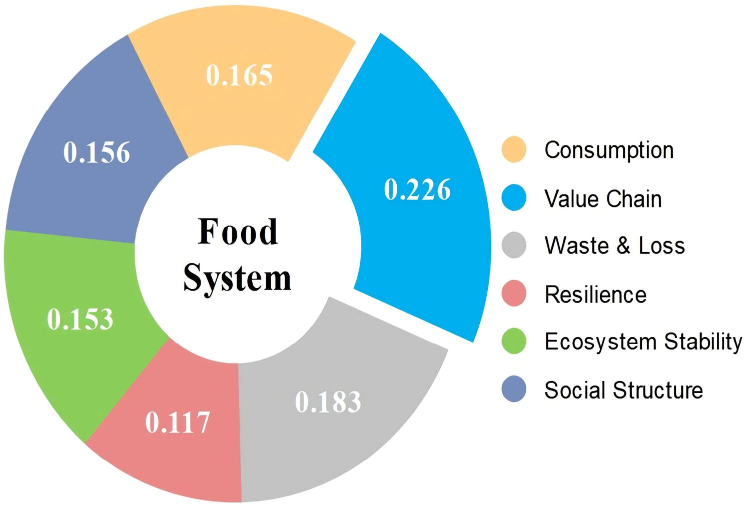

Figure 3: First-level indicator weight

3.2.2 Comprehensive evaluation index of food system(FSI)

Using the weights of 17 secondary indicators and 6 primary indicators that have been calculated so far, we define the formula for the comprehensive evaluation indicator FSI of the food system as follows:

$$FSI = (W_1 \times FCC + W_2 \times ESC + W_3 \times WLC + W_4 \times FVC + W_5 \times SSC + W_6 \times RCC) \times 100$$

This indicator comprehensively considers multiple perspectives of the food system and can evaluate the current food system.

3.3 Optimization Model

This section will optimize the indicators of the food system for fairness and sustainability. By updating the weight values, drawing a comparison chart before and after optimization, and determining the target value by analyzing the difference in area changes.

3.3.1 Optimization Weight

First, select the 7 indicators that reflect sustainability and fairness from the above 17 secondary indicators. The indicators that represent sustainability are: *GHG (Greenhouse Gas) Emissions, Freshwater Withdrawals, Non-Renewable Energy Use, Land Use and Food Loss*. The indicators that can represent fairness are: *Gender Equality, Imported Poverty.*

Secondly, in order to reflect that the food system has been optimized, we will optimize the weights w_i, $i = 5,6,7,8,9,13,14$, considering that the larger the value of some indicators, the better the promotion of fairness and sustainability. The smaller the index, the better it is to promote fairness and sustainability, so the index is adjusted in two ways: increasing or decreasing the weight. At the same time, a robust food system will not suddenly change by a large extent. We consider continuously changing the value of the weight within the range of 5% of the current weight. By analyzing the image of the weight change and finding the target value, we can consider the target The value is the optimized weight value. The specific formula is as follows:

$$\widetilde{w}_i = w_i(1 \pm 5\%)$$

Where, $\widetilde{w}_i$ is the optimized weight. After the weights of the secondary indicators are changed, use the coefficient of variation method to recalculate the weights of the primary indicators. Still have to guarantee:

$$\sum_{i=1}^{6} W_i = 1$$

3.3.2 Analyze the Degree of Difference

Step1：Draw the radar chart which is not optimized food system corresponding to the 6 weights, calculate the area contained in the graphic, and record it as $S_{current}$.

Step2：Draw the radar chart which is optimized food system corresponding to the 6 weights, calculate the area contained in the graphic, and record it as $S_{optimize}$.

The area formula is calculated as follows

$$S = \sum_{i=1}^{p} \frac{1}{2} r_i r_{i+1} \sin(\frac{2\pi}{6})$$

where, r_i represents the value of the first i indicator on the radar chart.

Step3：By calculating the difference between the areas of the two graphics, the weight that maximizes the difference is selected as the target weight. The calculation formula of the difference degree is as follows:

$$BZ = \frac{S_{optimize}}{S_{current}}$$

Step4： Bring the target weight value into formula (8) and formula (11), and recalculate the optimized comprehensive evaluation index of the food system. The specific calculation formula is as follows:

$$FSI' = (\widetilde{W}_1 \times FCC + \widetilde{W}_2 \times ESC + \widetilde{W}_3 \times WLC + \widetilde{W}_4 \times FVC + \widetilde{W}_5 \times SSC + \widetilde{W}_6 \times RCC) \times 100$$

3.3.3 Analysis of Results

We will analyze the food indicators, policy changes, distribution methods, and diet structure in specific countries or regions.

3.4 Time Prediction Model

We will use the gray time prediction model to predict the time it takes for the optimized food system to appear. By judging the time interval of the optimized comprehensive index, we can get the time to achieve the goal.

3.4.1 Grey Prediction Model

The essence of the model [6] [7] is to accumulate the original data once to make the generated sequence show a certain law, and then establish a first-order linear differential equation to solve the fitted curve for prediction.

- Data inspection and processing

In order to ensure the feasibility of the forecasting model, the annual FSI indicator series is first checked. Suppose the reference data is $x^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n))$ we can calculate the grade ratio of the series

$$\lambda(k)=\frac{x^{(0)}(k-1)}{x^{(0)}(k)},k=2,3,\cdots,n$$

If all the ratios fall within the tolerable coverage $(e^{-\frac{2}{n+1}},e^{\frac{2}{n+2}})$, then the series can be used for predictive analysis; otherwise, the series needs to be transformed to fall into the tolerable coverage by the following transformation

$$y^{(0)}(k)=x^{(0)}(k)+c$$

Where c is a constant, then the ratio of the series $y^{(0)}=(y^{(0)}(1),y^{(0)}(2),\cdots,y^{(0)}(n))$ of the serie

$$\lambda_y(k)=\frac{y^{(0)}(k-1)}{y^{(0)}(k)}\in X,k=2,3,\cdots,n$$

- Build the model

Let the original sequence be $x^{(0)}=(x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n))$, and make an accumulation to generate:

$$x^{(1)}=(x^{(1)}(1),x^{(1)}(2),\cdots,x^{(1)}(n))$$

where， $x^{(1)}=\sum_{j=1}^{i}x^{(0)}(j)(i=1,2,\cdots,n)$

Establish the first-order linear whitening differential equation

$$\frac{dx^{(1)}}{d_t}+ax^{(1)}=b$$

Using the least squares method to solve for the parameters

$$a^{\wedge}=\begin{bmatrix}a\\u\end{bmatrix}=(B^tB)^{-1}B^Ty_n$$

where，

$$B=\begin{bmatrix}-\frac{1}{2}(X^{(1)}(1)+X^{(1)}(2)) & 1\\ -\frac{1}{2}(X^{(1)}(2)+X^{(1)}(3)) & 1\\ \vdots & \vdots\\ -\frac{1}{2}(X^{(1)}(n-1)+X^{(1)}(n+1)) & 1\end{bmatrix}$$

The gray prediction model of $x^{(1)}$ is:

$$\hat{x}^{(1)}(k+1)=[x^{(0)}(1)-\frac{b}{a}]e^{-ak}+\frac{b}{a}(k=0,1,\cdots)$$

The actual predicted value can be derived from the following equation:

$$x^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k)$$

- Test the predicted value

$$\varepsilon(k)=\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)},k=1,2,\cdots,n$$

If $\varepsilon(k)<0.2$ then it can be considered to meet the general requirements. If $\varepsilon(k)<0.1$ then it can be considered to meet the higher requirements.

3.4.2 Time of Realization

Based on the data in recent years, we can calculate the comprehensive evaluation index of the food system, and use the model to predict the data and its trend in the next few years, and judge the time range of the optimized comprehensive evaluation index of the food system, and get the realization time.

We use the data from recent years to calculate a comprehensive evaluation indicator *FSI* for the food system. We also use the model GM(1,1) to predict the data and its direction for the next years. We determine the time range of the optimized comprehensive evaluation indicators *FSI of* the food system and obtain the time to achieve.

4 Analysis and Forecast of Benefits and Costs

4.1 Prioritization

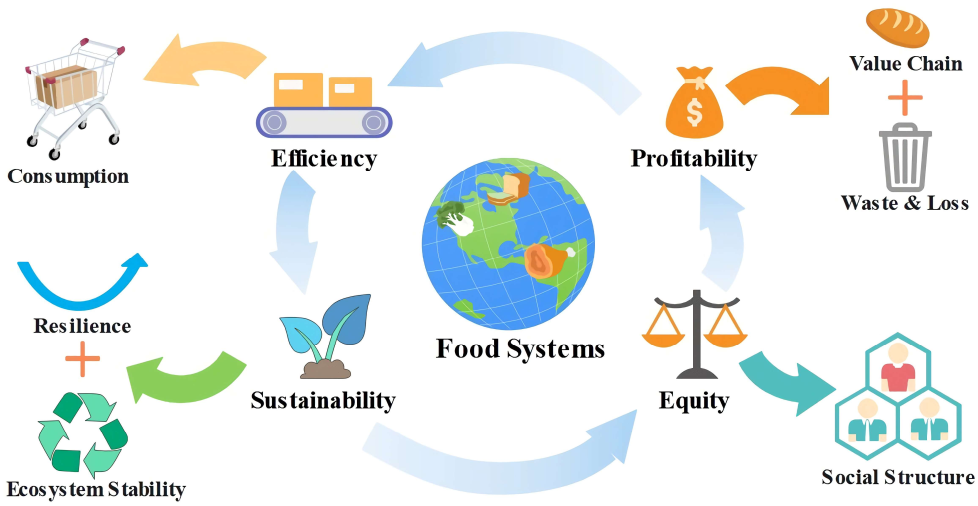

Figure 4: Four different priorities

The current food system prioritizes efficiency and profitability, but it is clear that the current system does not meet current human needs. We are changing the food system so that it can be more equitable and sustainable in the future. In the above food system model, the indicators representing sustainability are: *GHG Emissions, Freshwater Withdrawals, Non-Renewable Energy Use, Land Use, Food Loss*; the indicators representing equity are: *Gender Equality*、 *Imported*

Poverty, we will optimize them in different orders and calculate the values of the corresponding *FSI* indicators in different priorities.

- Prioritize fairness

Only the weights corresponding to the fairness indicators *Gender Equality, Imported Poverty* are optimized. Similar to the above optimization process, the following equation is applied to each weight

$$\widetilde{w}_i = w_i(1 \pm 5\%)$$

Recalculate the corresponding FSI_j -index under this weight.

- Prioritize sustainability

Only the weights corresponding to the sustainability indicators *GHG Emissions*, *Freshwater Withdrawals*, *Non-Renewable Energy Use*, *Land Use*, and *Food Safety* are optimized. Similar to the above optimization process, the following equation is applied to each weight

$$\widetilde{w}_i = w_i(1 \pm 5\%)$$

Recalculate the corresponding FSI_j index under this weight.

4.2 The Model of the Benefits and Costs

4.2.1 Determining the relationship between cost benefit indicators and composite indicators

By reviewing relevant information [9], we selected four indicators as measures of cost benefits, namely: *Agricultural Value Added* (total agricultural output - intermediate inputs) Y_1, *Grain Production* Y_2 , *Fertilizer Use Rate* Y_3 and *Labor* Y_4 .

- Establish a regression relationship

$$Y_1 = f(FSI, Y_2, Y_3, Y_4)$$
$$Y_2 = f(FSI, Y_1, Y_3, Y_4)$$
$$Y_3 = f(FSI, Y_1, Y_2, Y_4)$$
$$Y_4 = f(FSI, Y_2, Y_3, Y_1)$$

The above establishes the regression relationship between *FSI* and Y_i for each year. Finally, a significance analysis is performed to determine that our regression is valid.

The regression equation obtained is as follows:

$$\begin{cases} Y_1 = -0.414FSI + 0.006Y_2 - 0.001Y_3 + 6.354 \times 10^{-6} Y_4 - 28.450 \\ Y_2 = 64.186FSI + 0.102Y_3 - 1554.628 \\ Y_3 = 165.941FSI - 104.734Y_1 + 6.539Y_2 + 6.282 \times 10^{-5} Y_4 - 8319.751 \\ Y_4 = 142730.606FSI + 70551.835Y_1 + 2099.524Y_2 + 8.791Y_3 + 791173.133 \end{cases}$$

4.2.2 The Benefits and Costs of Changing the Order

Now, we only need to calculate $\overline{\overline{Y_1}}, \overline{\overline{Y_2}}, \overline{\overline{Y_3}}, \overline{\overline{Y_4}}, \overline{\overline{FSI}}$ after one iteration. We construct a linear system of equations from the relationships established by the above regression equations, and solve the results using the Gaussian Seidel iteration method.

$$AX=\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1,n+1} \\ a_{21} & a_{12} & \cdots & a_{1,n+1} \\ \vdots & & \ddots & \\ a_{n1} & a_{n2} & \cdots & a_{n,n+1} \\ a_{n+11} & a_{n+12} & \cdots & a_{n+1,n+1} \end{bmatrix}\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \\ FSI \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}=B$$

The Gaussian Seidel iteration is calculated in the format

$$x_i^{(k+1)}=\frac{1}{a_{ii}}(b_i-\sum_{j=1}^{i-1}a_{ij}x_j^{(k+1)}-\sum_{j=i+1}^{n}a_{ij}x_j^{(k)}),i=1,2,\cdots,n\,,\quad n=4$$

We take the known recent data as the iterative initial value and calculate it by one Gauss-Seidel iteration method to obtain $\overline{\overline{Y_1}},\overline{\overline{Y_2}},\overline{\overline{Y_3}},\overline{\overline{Y_4}},\overline{\overline{FSI}}$.

Make $\Delta_i=\overline{\overline{Y_i}}-Y_i, i=1,2,3,4$, Analyze the benefits and costs by looking at the changes in Δ_i .

For example, $\Delta Y_1>0$ or $\Delta Y_2>0$ indicates that value added in agriculture and food production both increased, representing benefits.

4.3 Time Forecast Analysis

In this section, we also use a gray prediction model to predict the time of occurrence.

- Prioritize fairness

We calculate the *FSI* by changing the weight of the fairness index, and use the prediction model of 3.5 to obtain the time of occurrence T_1 .

- Prioritize sustainability

We calculate the *FSI* by varying the sustainability indicator weights and use the predictive model of 3.5 to obtain the time of occurrence T_2 .

5 Application of the Model

We will select a developed country with a high level of agrarianization and a more developed food system: Germany and a less developed region with a harsh agricultural environment and a high rate of hunger: Niger to support the model.

5.1 Developed country—Federal Republic of Germany

5.1.1 Basic Introduction

Germany is a highly developed country with developed agriculture and a high degree of mechanization. Germany has a highly developed agriculture, and more than 80% of its agricultural products can be self-sufficient. The state provides certain financial subsidies to agriculture to ensure agricultural production. At the same time, Germany provides a lot of policy support for agricultural development.

5.1.2 Changes after Optimizing Food Systems

By reviewing a large amount of data, we find the food system indicator data and calculate the first level indicator weights and *FSI* before optimization.Represent them in a radar plot. Then we use the optimization model to optimize for equity and sustainability, and again calculate the first level indicator weights, and *FSI*, again represent by radar plots. The area variability ratio is 1.0787.

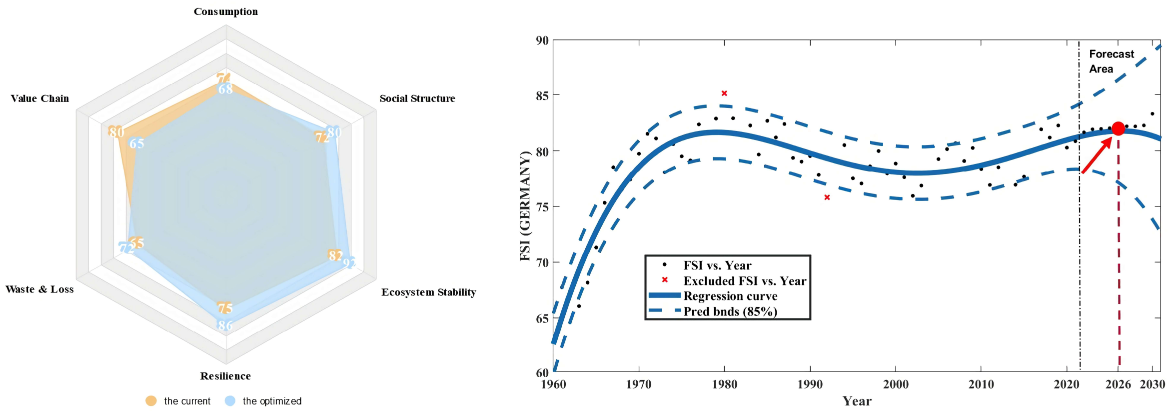

Figure 5：Radar plot and time projection of evaluation indicators in Germany

As you can see from the chart above

- Germany's overall performance is balanced across all dimensions and scores relatively high on all dimensions. This indicates that the country's food system is relatively stable in terms of efficiency, profitability, equity, and sustainability.
- After changing the food system, we can see that the food system is obviously improved in ecological aspects. There are significant improvements in *Ecosystem Sta-bility*, S*ocial Struc-ture*, *Resilience*, and *Waste & Loss* indicating that the improvements are effective in reduc-ing food waste. This is the result of reducing the environmental damage caused by growing food, slowing down environmental degradation, and promoting the rational distribution of food. Improvements in these indicators will help countries or regions that have the capacity to provide food or have a surplus of food to allocate food to countries or regions that have extreme food scarcity or are facing food insecurity, and will help change the diet structure and reduce obesity and disease rates.
- Faced with the decrease in the score of *Value Chain* and *Consumption* indicators, we believe that some of the economic benefits will be lost after more consideration of sustainability and fairness. It is possible that part of the food needs to be sold at a lower price to support agri-cultural development in poor areas as well as to give some poverty subsidies.
- Finally, using a time prediction model, we expect the optimized system to achieve in 2026.

5.1.3 Analysis of Cost and Benefits

We optimize for fairness and sustainability in sequential order. The optimization equations ob-tained and the calculated costs and benefits are as follows:

Table 3：German cost benefit table

Germany	Agricultural Value Added	Grain Production	Fertilizer Use Rate	Labor
Original values	211	7783	38500	43577709
After first optimizing sustainability	222.440223	7674.77746	35014.5	44696906
After first optimizing for equitability	222.721743	7631.13098	34586.77	44524314
Final Value	222.146283	7720.34952	35461.1	44877113
After first optimizing sustainability- Original values	11.44022299	-108.22254	-3485.5	1119197
After first optimizing for equitability- Original values	11.72174299	-151.86902	-3913.23	946605.3
Final Value- Original values	11.14628299	-62.65048	-3038.9	1299404

Benefits: increased agricultural value added; reduced fertilizer consumption, which is good for the environment; increased labor force.

Costs: reduced food production and some food crisis.

The expected time of realization is

Table 4：Germany Projected Time to Achievement

	Projected time	
Germany	Prioritize sustainability	2024 year
	Prioritize fairness	2023 year

5.2 Developing Country——Niger

5.2.1 Brief introduction

The Republic of Niger is located in west-central Africa. It is a landlocked country on the southern edge of the Sahara desert. And it is one of the least developed countries in the world. Niger infrastructure is relatively poor. It has a weak industrial base and erratic food production. With a food shortage that broke out in 2016 and nearly 2 million people facing food shortages, the country is facing a heavy crisis from a highly unstable food system. The area variability ratio is 1.2594.

5.2.2 Food system indicators

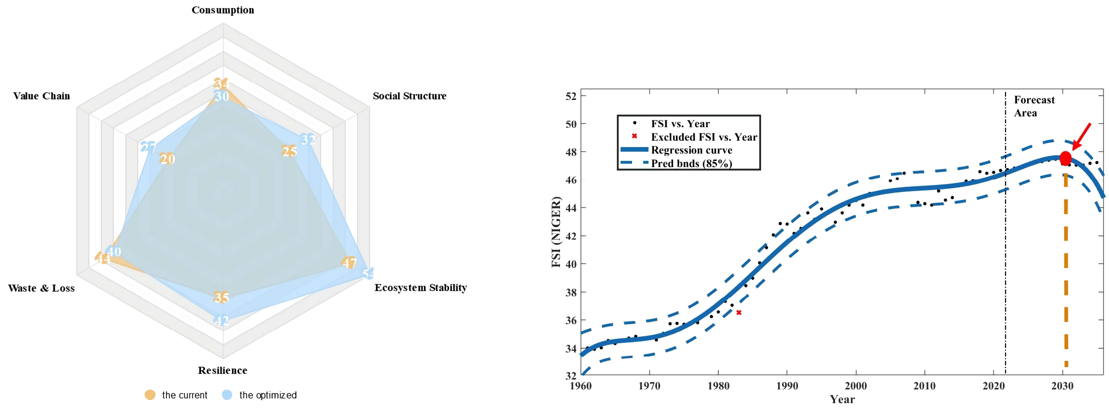

Figure 6： Radar and time projection charts for evaluation indicators in Niger

As you can see from the chart above

- The overall performance of Niger is very unsatisfactory in all aspects, especially in the Value Chain. This is in line with the country's situation and the current state of the system. The country does not have a well-developed agricultural system and policies to protect it. The local environment is a major constraint to agriculture, and the government needs to combine with other outside forces to build and improve its own national food system.
- After optimizing the food system, we find an improvement in most indicators and only a slight decrease in others. This shows that the optimization in terms of equity and sustainability not only benefits the ecological environment, but equally strongly enhances the stability of the society. This effectively reduces the number of hungry people, enables more people to create value through labor, renews the demographic structure, and guarantees the basic livelihood issues of the country.
- We substituted the country's data set into the time projection model to obtain the time to achieve the target food system: 2030.

5.2.3 Benefit-cost analysis

We optimize for fairness and sustainability in sequential order. We can obtain the optimization equations and the calculated costs and benefits as follows:

Table 5：Niger cost benefit table

Niger	Agricultural Value Added	Grain Production	Fertilizer Use Rate	Labor
Original values	0.66	530	593	8423258
After first optimizing sustainability	8.101621332	1538.00464	9256.948	11415807
After first optimizing for equitability	8.229961332	1518.10698	9061.954	11337125
Final Value	7.836661332	1579.08368	9659.516	11578246
After first optimizing sustainability- Original values	7.441621332	1008.00464	8663.948	2992549
After first optimizing for equitability- Original values	7.569961332	988.10698	8468.954	2913867
Final Value- Original values	7.176661332	1049.08368	9066.516	3154988

Benefits: more food production; higher agricultural value added, which helps to reduce the number of hungry people.
Costs: fertilizer consumption becomes higher, causing environmental pollution; there will be some loss of labor and some people are expected to be engaged in non-agricultural production.
The expected time of realization is

Table 6：Niger Projected Time to Achievement

	Projected time	
Niger	Prioritize sustainability	2027 year
	Prioritize fairness	2024 year

5.3 Contrast Analysis

By combining and comparing the comprehensive evaluation indicators of the food systems of Germany and Niger, we get specific values for the costs and costs of different optimized sequential systems. We can get what are the different structures and what factors affect the development and stability of a country or region.

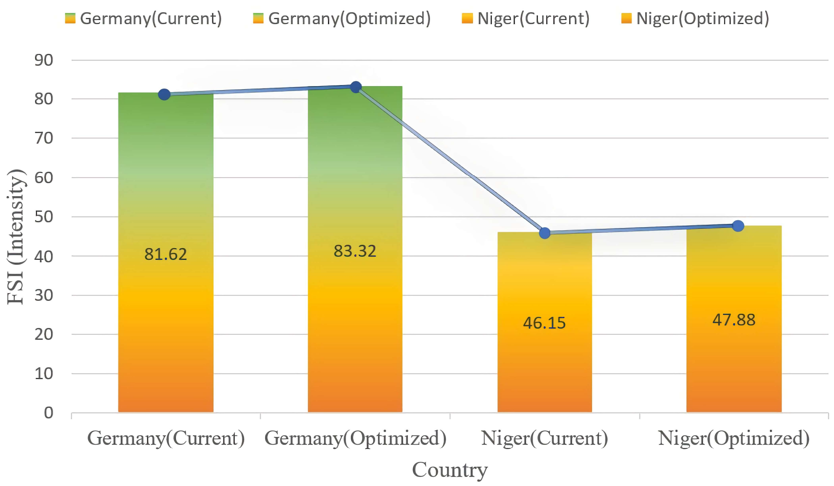

Figure 6：Contrasting evaluation indicators

As can be seen in the figure above

- We can see that developed countries have a higher degree of robustness relative to developing countries. This is because developed countries have explored their own national development paths. They have relatively more stable agricultural security measures and a certain amount of food reserves. And they can provide a certain amount of economic support in the fight against poverty.
- We can see that after optimization, developing countries have a higher increase relative to developed countries. This is because once the financial, human and material resources are invested will inject new dynamics to the less stable developing countries and help them to establish a documented food system.

6 Scalability& Adaptability &Sensitivity Analysis

- Model Scalability and Model Adaptability

In this section, we test the scalability and adaptability of the model by substituting data of the United States and China. The United States, a highly developed country, has a more robust food system. China, the world's most populous developing country, is also optimizing its own food system. First, we substitute the data into the model to obtain the direction of the *FSI* indicator from 1961-2018, as shown in the following figure:

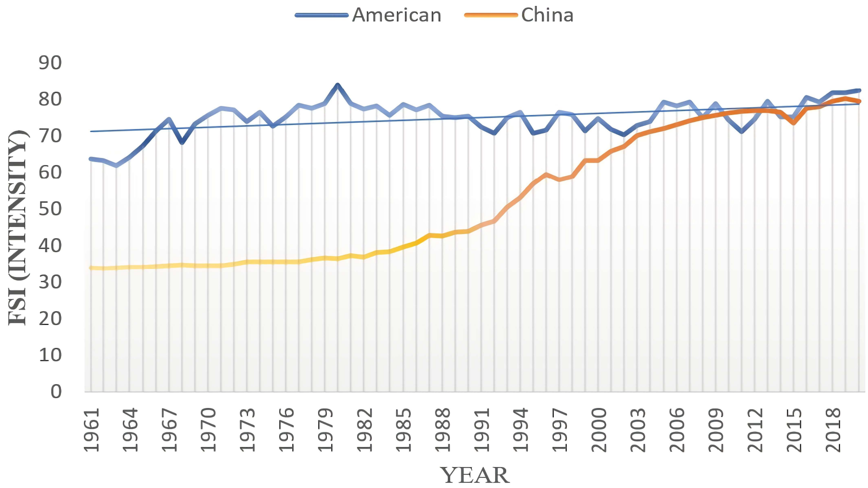

Figure 7：Evaluation indexes towards the map

From the figure above, we can see that it is more in line with the real-world situation. The indicator for the United States, a developed country, is more stable and remains at a high level, with only slight up and down fluctuations between adjacent years.China started out as a backward agricultural country, but has continued to improve by constantly updating and optimizing its food agriculture system. We choose to perform a time projection for China and apply the time projection model by optimizing in terms of equity and sustainability, and the expected time results are plotted as follows:

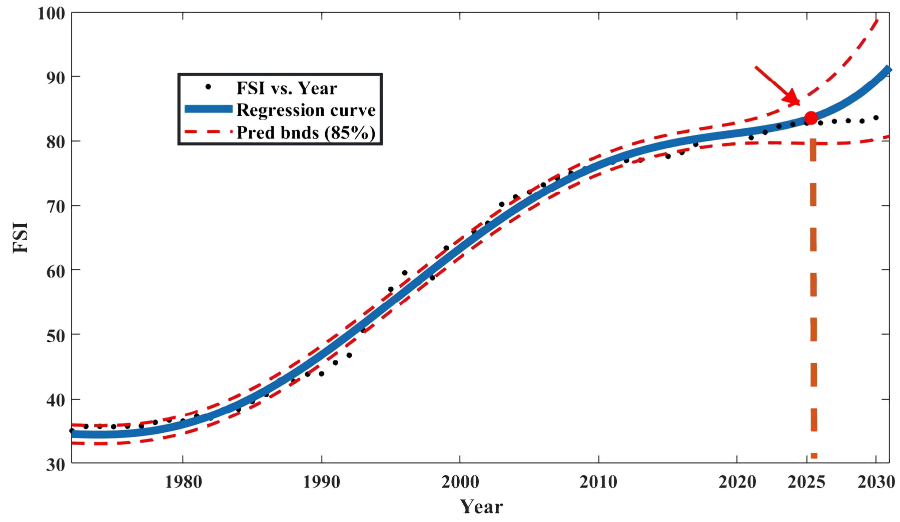

Figure 8：Time projection

As you can see from the chart above, we expect to see optimization results in 2025.

- Sensitivity Analysis

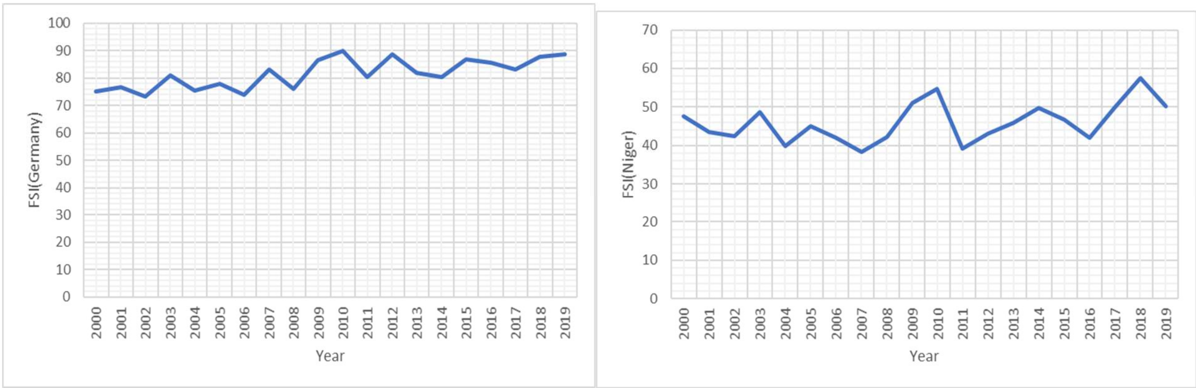

Figure 9：Sensitivity Analysis

We optimize the current food system by changing the weights of each indicator. To demonstrate the robustness of the model we developed, we analyze the annual *FSI* indices for the selected developed country: Germany and the developing country: Niger. We find that the *FSI* index for Germany moves towards a stable trend, but the *FSI* index for Niger fluctuates more sharply. Also, for different countries, the *FSI* index curve does not change dramatically relative to the current system in a short period of time. Such a trend situation of our fitted curve is also consistent with the current development of the country. This shows that our model can fit the current system very well. This can also prove the stability of our model and this can be practically used to evaluate the country's food system.

7 Model Evaluation and Further Discussion

7.1 Strengths

- Indicators are fully considered, data are relatively rigorous and applicable to a wide range of countries and regions.

- Using the entropy weighting method to determine the weight of each indicator is more objective and converts qualitative analysis into quantitative analysis, which is more convincing.
- By quantifying the results of the index, we can obtain the *FSI* index. It is more convincing to evaluate the grade of the index to judge the goodness of a food system.
- The model takes full account of the correlation between these indicators by establishing regression equations for the combined indicators *FSI* and cost effectiveness indicators, and iterative calculations.

7.2 Weaknesses

- Due to time constraints, we do not evaluate additional national food systems with different developmental profiles.
- Lack of consideration for the acceptability of the population in the face of food system optimization, e.g., diet structure.
- For the relatively few metrics that reflect costs and benefits, you can continue to add
- The model does not take into account the effects of national foreign relations, natural disasters, external funding, and other policies.

7.3 Further Discussion

- Applicability

We evaluated the model by setting 17 indicators. These indicators cover six scopes. They fully consider all aspects of a food system and are applicable for different countries and regions. The wide coverage of the food system and the high degree of utilization of the 17 indicators may only cover some of them in the face of regions with smaller food systems. The weight of the remaining indicators can be adjusted to zero when calculating the problem.
The results obtained by assessing the food systems of two countries, Germany and Niger, are consistent with the current development of both countries, indicating that the model is highly adaptable to countries with different levels of development in different regions.

- Innovation

The radar plot shows more visually the difference of each indicator of the food system before and after optimization. The quotient of the area of the radar plot after and before optimization is used to indicate the difference between the two systems, mathematizing the problem and facilitating understanding. A large quotient indicates that there is a large difference between the pre- and post-optimization systems. Indirectly, it reflects our optimization in the right direction and compensates for the misunderstanding of the graphs as well as the indicators by the lack of experience in the related fields.

- Recommendations

Developed countries have a more stable production situation and social structure, and their people have security of life Therefore, we propose to focus on solving the country's environmental problems and the population's eating habits to ensure a balanced and a healthy diet. For developing countries, the main objective is to feed and clothe the people, increase annual food production, focus on the construction of agricultural infrastructure and safeguarding the basic livelihood of the people.

- Future work

Under the premise of 17 indicators, we can also take into account the inclusion of consideration of the country being an importer or exporter. The whole food system will be closely related not only to the domestic production situation but also to foreign trade. The model results can give recommendations for the country's import and export policy.

8 Conclusion

We provide a method to optimize the food system for better equity and sustainability. By establishing a comprehensive evaluation model of the food system, an optimization model, and a time prediction model, we can determine a comprehensive evaluation index: *FSI*. By constantly updating the model, we can optimize the food system of a country or region. It is able to predict which aspects will change in the future and be able to predict when the system will be realized. Through the benefit-cost analysis, we can determine the priority order, update the weights of the corresponding indicators, and recalculate the values of the comprehensive evaluation indicators. Next, we establish a functional relationship between the indicators that actually represent the cost benefits and the comprehensive evaluation indicators. And determine the set of regression equations. By solving the Gaussian Seder iterative method, we can obtain the values of costs and benefits. The time of occurrence is again predicted using the time cost prediction model to obtain the final estimated time. Then, by reviewing the relevant data, we choose Germany and Niger to support the model. The results obtained from the calculation better support the model we established. Finally, we use data of the United States and China to discuss the scalability and adaptability of the model, and perform a sensitivity analysis with more satisfactory results. In conclusion, we have solved the current problem well and proposed corresponding countermeasures.

Reference

[1] Report on the State of World Food Security and Nutrition 2020 https://www.sohu.com/a/415413615_99900352

[2] Sustainable Development Goals , https://www.un.org/sustainabledevelopment/zh/climate-change-2/

[3] https://www.ifad.org/en/food-systems

[4] https://translate.googleusercontent.com/translate_f

[5] Zhang Peng. Research on Comprehensive Evaluation Based on Principal Component Analysi s [D]. Nanjing University of Science and Technology,2004.

[6] Zhao Hong, Ma Liyan, Jia Qing. Grey correlation analysis model based on coefficient of vari ation method and its application [J]. Heilongjiang Water Science and Technology,2007(02):26-27.

[7] Xuan Ruirui, Chen Yanping, Liu Chunju, Wang Lixia, Yuan Jianhua. Quality evaluation of fr esh glutinous corn based on entropy weight method and grey relational degree method [J/OL]. Sc ience and Technology of Food Industry :1-14[2021-02-]

[8] Lai Hongsong, Zhu Guorui, Dong Pinjie. Population forecasting based on grey forecasting an d neural network [J]. Economic Geography,2004(02):197-201.

[9] Long F. Research on China's food security issues in the new century [D]. Hunan Agric ultural University,2007.

Appendix

Food system hierarchy map